Multi-channel Jost functions for describing resonances and collective modes in finite quantum many-body systems

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Duy Tan University





Founded: 1994 – Largest private university in Central Vietnam
Location: **Da Nang City**, Vietnam
Students: 20,000+ | Faculty: 1,500+ (30% PhDs)
Programs: Engineering, IT, Business, Tourism, Medicine, etc.
Global Partnerships: U.S. & U.K. universities (2+2, 3+1, Double Degrees)
Accreditations: ABET (Engineering/IT), UNWTO TedQual (Tourism)
Rankings: QS #482 (2026), THE 601–800 (2025), Vietnam Top 4 (Nature Index)
Graduate Employment: ~92% within 1 year; 100% in IT
Motto: 'We Do Everything for Student Rights and Future Careers'





Students taking the introductory physics course in the partnership program with Troy University

Contents

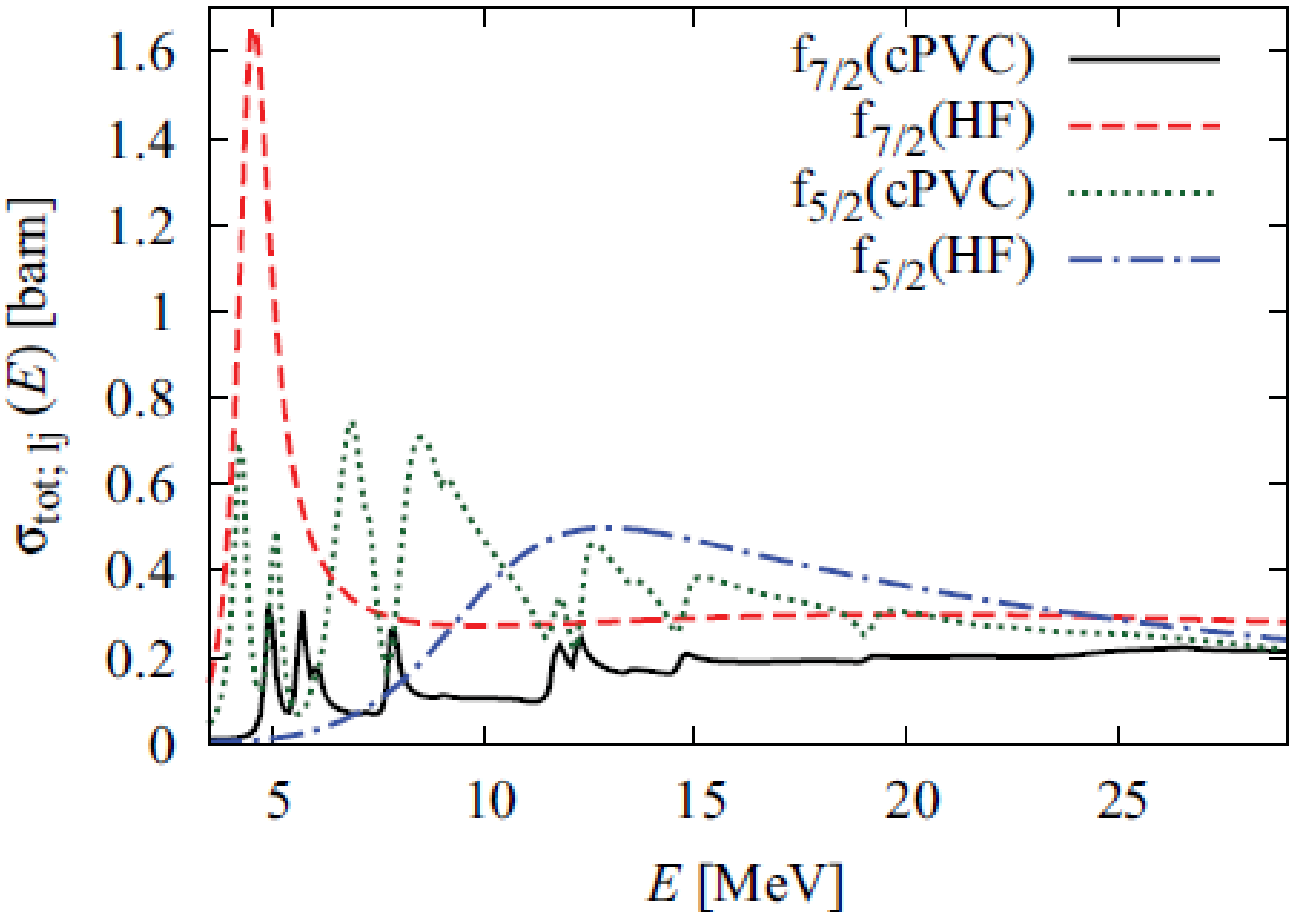
1. Motivation: What is resonance?
2. Jost function and Riemann sheet
3. Extension of the Jost Function to Multichannel Systems
   a. Hartree-Fock-Bogoliubov (HFB) Jost function
   b. Random-Phase-Approximation (RPA) Jost function
4. Analysis of resonance
   a. S-matrix and Green function
   b. Eigenphase-shift decomposition
   c. Mittag-Leffler theorem
5. Takagi factorization of the S-matrix residue
   a. Pole term of the transition amplitude
   b. Collectivity of resonance poles
   c. Application to ^{208}Pb
6. Summary

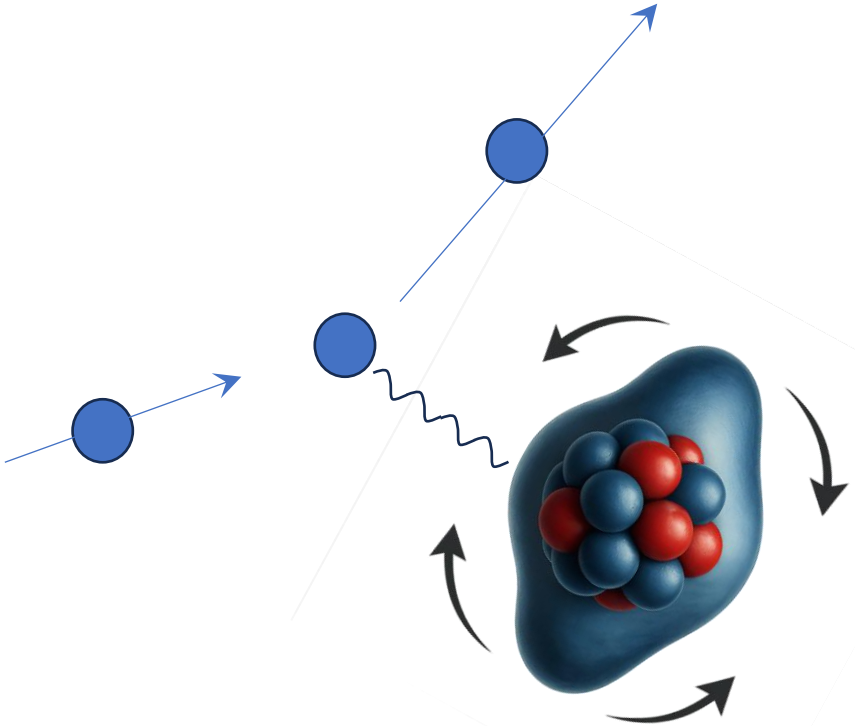
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1. What is resonance?

PRC86, 041603(R), 2012, KM and K. Ogata





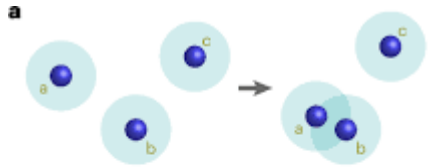
Confirmed that the coupling between incident neutron and collective excitation of the target nucleus produces a sharp, narrow resonances!!.

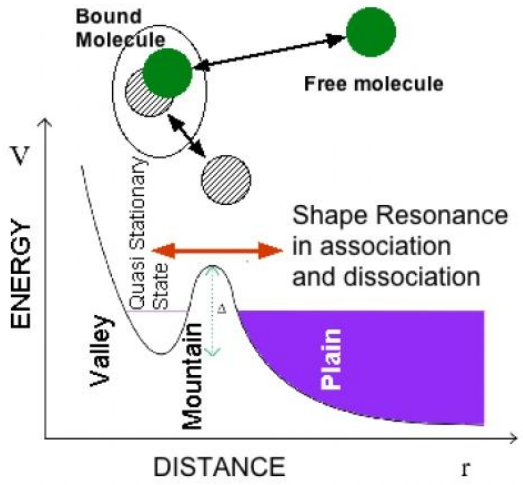
But Why?

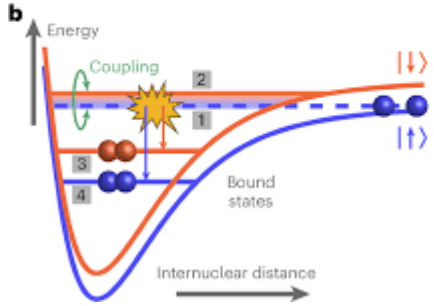
1. What is resonance?

- Many types of resonance
  Shape resonance, Feshbach resonance, Fano resonance, etc…
- Giant resonance in nuclear structure is also resonance?
- Why does the coupling between particles and collective modes of the target nucleus produce sharp resonance peaks?
- What is the framework of nuclear theory in which to solve these questions?
  - Extension of the Jost Function to Nuclear Many-Body Systems (HFB, RPA, PVC, etc…)

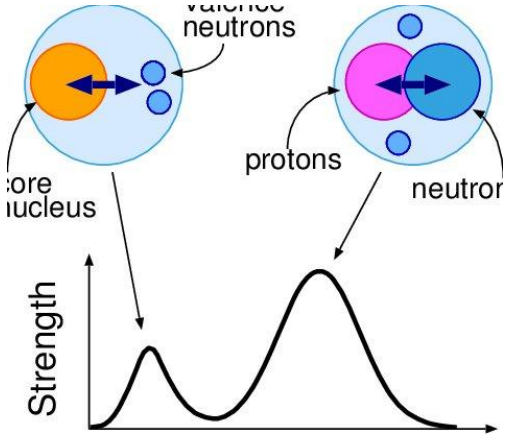
https://www.nature.com/articles/s41567-024-02726-3



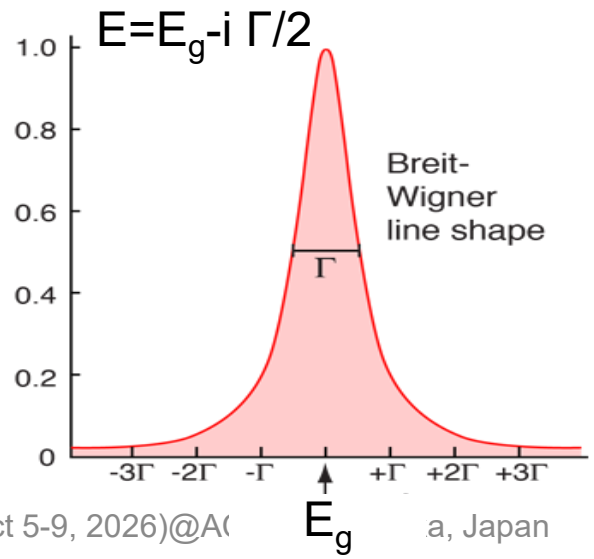


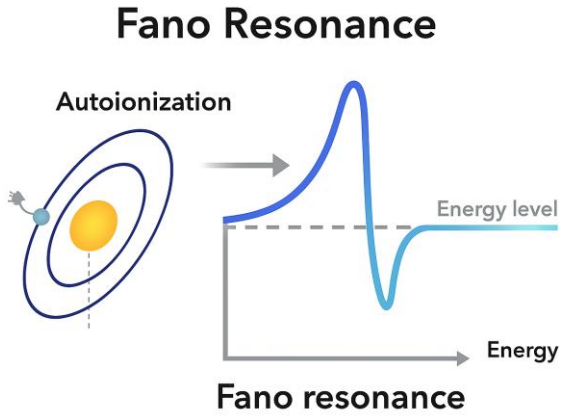


https://www.mdpi.com/1422-0067/12/10/6810



10.1007/978-3-642-13899-7_5





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2. Jost function and Riemann sheet

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}+U(r)\right]\phi(r;E)=\frac{\hbar^2 k^2(E)}{2m}\phi(r;E)$$

$$k(E)=\sqrt{\frac{2mE}{\hbar^2}}$$

$$\phi(r;E) = \begin{cases} \phi_{lj}^{(r)}(r) & \text{with} \quad \lim_{r\to 0}\phi_{lj}^{(r)} = r j_l(kr) \\ \phi_{lj}^{(\pm)}(r) & \text{with} \quad \lim_{r\to \infty}\phi_{lj}^{(\pm)}(r) = r h_l^{(\pm)}(kr) \end{cases}$$

←Regular solution

←Irregular solution

R. Jost and A. Pais, Phys. Rev. 82, 840 (1951).

$$\phi_{lj}^{(r)}(r)=\frac{1}{2}\left[J_{lj}^{(+)}(E)\phi_{lj}^{(-)}(r)+J_{lj}^{(-)}(E)\phi_{lj}^{(+)}(r)\right]$$

2. Jost function and Riemann sheet

$$J_{lj}^{(\pm)}(E) = \pm\frac{k}{i}\left[\phi_{lj}^{(r)}(r)\,\frac{d\phi_{lj}^{(\pm)}(r)}{dr} - \frac{d\phi_{lj}^{(r)}(r)}{dr}\,\phi_{lj}^{(\pm)}(r)\right]$$

← **Wronskian representation**

from $\phi_{lj}^{(r)}(r) = \frac{1}{2}\left[J_{lj}^{(+)}(E)\,\phi_{lj}^{(-)}(r) + J_{lj}^{(-)}(E)\,\phi_{lj}^{(+)}(r)\right]$ and $\phi_{lj}^{(+)}\frac{d\phi_{lj}^{(-)}}{dr} - \frac{d\phi_{lj}^{(+)}}{dr}\phi_{lj}^{(-)} = -\frac{2i}{k}$

$$J_{lj}^{(\pm)}(E) = 1 \pm i\,\frac{2mk}{\hbar^2}\int_0^\infty dr\; r\,h_l^{(\pm)}(kr)\;U_{lj}(r)\;\phi_{lj}^{(r)}(r)$$

← **Integral representation**

$U_{lj}(r)$: potential without the centrifugal term, $r\,h_l^{(\pm)}(kr)$: free solutions, $U_{lj} = 0 \;\Rightarrow\; J_{lj}^{(\pm)} = 1$

The HFB (2x2) and RPA (2Nx2N) Jost functions have the same Wronskian form.

2. Jost function and Riemann sheet

$$\psi_{lj}^{(+)}(r) \equiv \frac{\phi_{lj}^{(r)}(r)}{J_{lj}^{(+)}(E)} = \frac{1}{2}\Big[\underbrace{\phi_{lj}^{(-)}(r)}_{\text{incoming}} + S_{lj}(E)\ \underbrace{\phi_{lj}^{(+)}(r)}_{\text{outgoing}}\Big]$$

← Scattering wave function

$$S_{lj}(E) \equiv \frac{J_{lj}^{(-)}(E)}{J_{lj}^{(+)}(E)} \qquad \text{real } E: \quad |S_{lj}(E)| = 1, \quad S_{lj}(E) = e^{2i\delta_{lj}(E)}$$

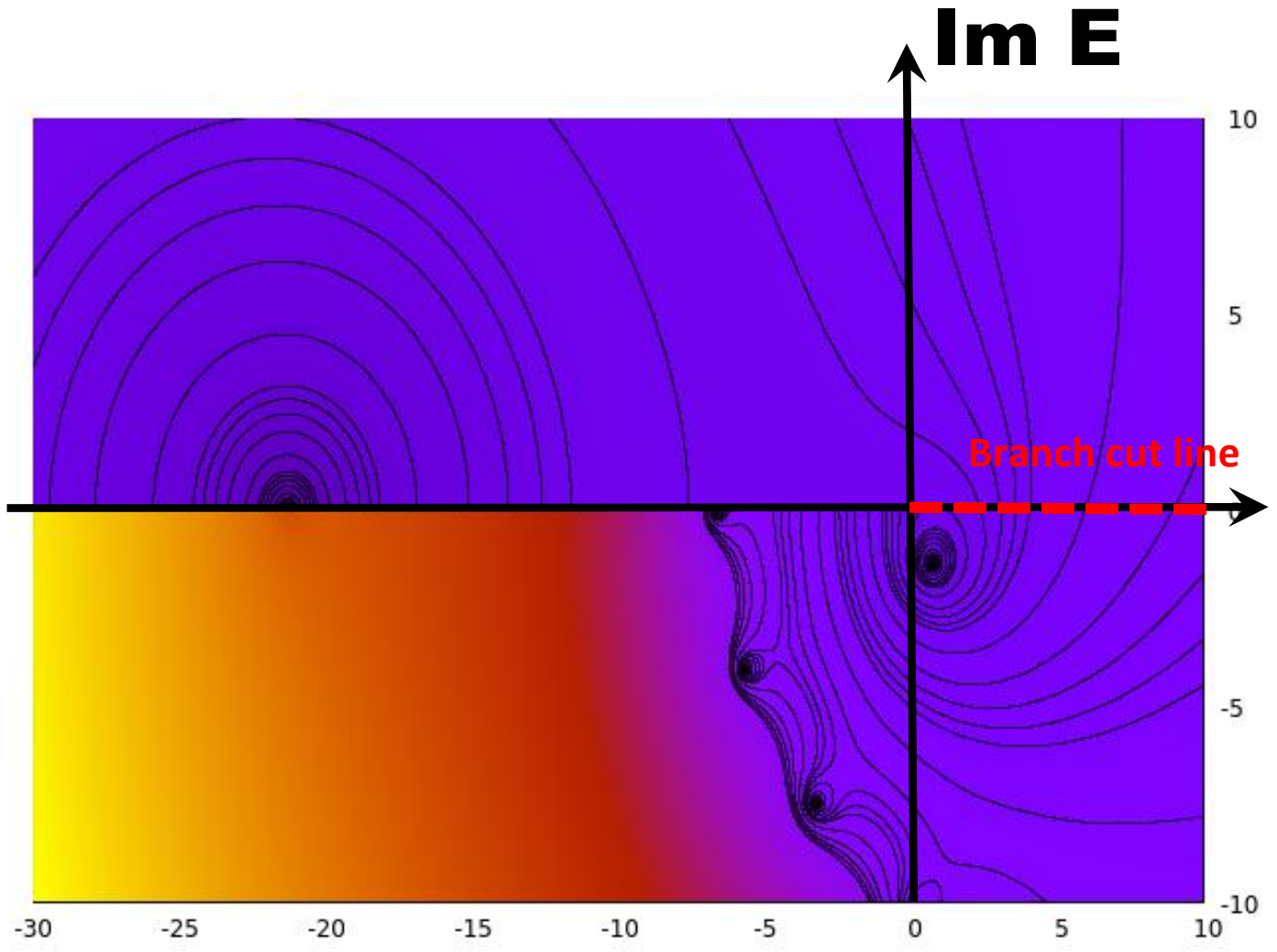
← S-matrix

$$\lim_{r\to\infty} \psi_{lj}^{(+)}(r) = \frac{i^{\,l+1}}{2k}\Big[e^{-ikr} + (-1)^{l+1}\, S_{lj}(E)\, e^{+ikr}\Big]$$

← asymptotic form

S-matrix poles: $J_{lj}^{(+)}(E) = 0$ → bound states and resonances

2. Jost function and Riemann sheet



1st Riemann Sheet of E(k) ;
Im k > 0
→ 0 < arg k < π

2nd Riemann Sheet of E(k) ;
Im k < 0
→ π < arg k < 2π

Neutron $p_{3/2}$ for ^{16}O with Woods-Saxon

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3-a. Hartree-Fock-Bogoliubov (HFB) Jost function

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2}-\frac{l(l+1)}{r^2}\right)+\sigma_z \boldsymbol{\mathcal{U}}_{lj}(r)\right]\vec{\phi}_{lj}(r;E)=\frac{\hbar^2}{2m}\begin{pmatrix}k_1^2(E) & 0\\ 0 & k_2^2(E)\end{pmatrix}\vec{\phi}_{lj}(r;E)$$

$$\boldsymbol{\mathcal{U}}_{lj}=\begin{pmatrix}U_{lj}(r) & \Delta(r)\\ \Delta(r) & -U_{lj}(r)\end{pmatrix}$$

$$\frac{\hbar^2 k_{1,2}^2(E)}{2m}=\lambda\pm E$$

2x2 matrix regular and irregular solutions

$$\Phi_{lj}^{(r)} \equiv \left(\vec{\phi}_{lj}^{(r1)} \quad \vec{\phi}_{lj}^{(r2)}\right)$$

$$\Phi_{lj}^{(\pm)} \equiv \left(\vec{\phi}_{lj}^{(\pm 1)} \quad \vec{\phi}_{lj}^{(\pm 2)}\right)$$

$$\Phi_{lj}^{(r)\mathsf{T}}=\frac{1}{2}\left[\mathcal{J}_{lj}^{(+)}(E)\Phi_{lj}^{(-)\mathsf{T}}+\mathcal{J}_{lj}^{(-)}(E)\Phi_{lj}^{(+)\mathsf{T}}\right]$$

HFB Jost function (2x2 matrix)

$$\mathcal{J}_{lj}^{(\pm)}(E)=\pm\frac{1}{i}\left[\Phi_{lj}^{(r)\mathsf{T}}(r)\,\sigma_z\frac{\partial}{\partial r}\Phi_{lj}^{(\pm)}(r)-\left(\frac{\partial}{\partial r}\Phi_{lj}^{(r)\mathsf{T}}(r)\right)\sigma_z\Phi_{lj}^{(\pm)}(r)\right]\mathcal{K}(E)$$

$$\det \mathcal{J}_{lj}(E_B)=0$$

$$\det \mathcal{J}_{lj}(E_R)=0$$

PRC **99**, 054607 (2019)

3-b. Random-Phase-Approximation (RPA) Jost function

$$\left[-\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}\mathbf{I}-\frac{\mathbf{L}^2}{r^2}\right)+\boldsymbol{\mathcal{V}}(r)\right]\vec{\phi}_{lj}(r;E)=\frac{\hbar^2}{2m}\boldsymbol{\mathcal{K}}^2(E)\,\vec{\phi}_{lj}(r;E)$$

$$\boldsymbol{\mathcal{V}}=\begin{pmatrix}\mathcal{U}+\mathcal{V}_{\rm res} & \mathcal{V}_{\rm res}\\ \mathcal{V}_{\rm res} & \mathcal{U}+\mathcal{V}_{\rm res}\end{pmatrix},\quad \boldsymbol{\mathcal{K}}=\mathrm{diag}\left(k_1^{(i)},k_2^{(i)}\right)$$

$$\frac{\hbar^2 k_{1,2}^{(i)2}(E)}{2m}=\varepsilon_{h_i}\pm E\quad\text{(forward / backward)}$$

2Nx2N matrix regular and irregular solutions

$$\Phi_{lj}^{(r)}\equiv\left(\vec{\phi}_{lj}^{(r1)}\quad\vec{\phi}_{lj}^{(r2)}\quad\cdots\quad\vec{\phi}_{lj}^{(r2N)}\right)$$

$$\Phi_{lj}^{(\pm)}\equiv\left(\vec{\phi}_{lj}^{(\pm1)}\quad\vec{\phi}_{lj}^{(\pm2)}\quad\cdots\quad\vec{\phi}_{lj}^{(\pm2N)}\right)$$

$$\Phi_{lj}^{(r)\mathsf{T}}=\frac{1}{2}\left[\mathcal{J}_{lj}^{(+)}(E)\Phi_{lj}^{(-)\mathsf{T}}+\mathcal{J}_{lj}^{(-)}(E)\Phi_{lj}^{(+)\mathsf{T}}\right]$$

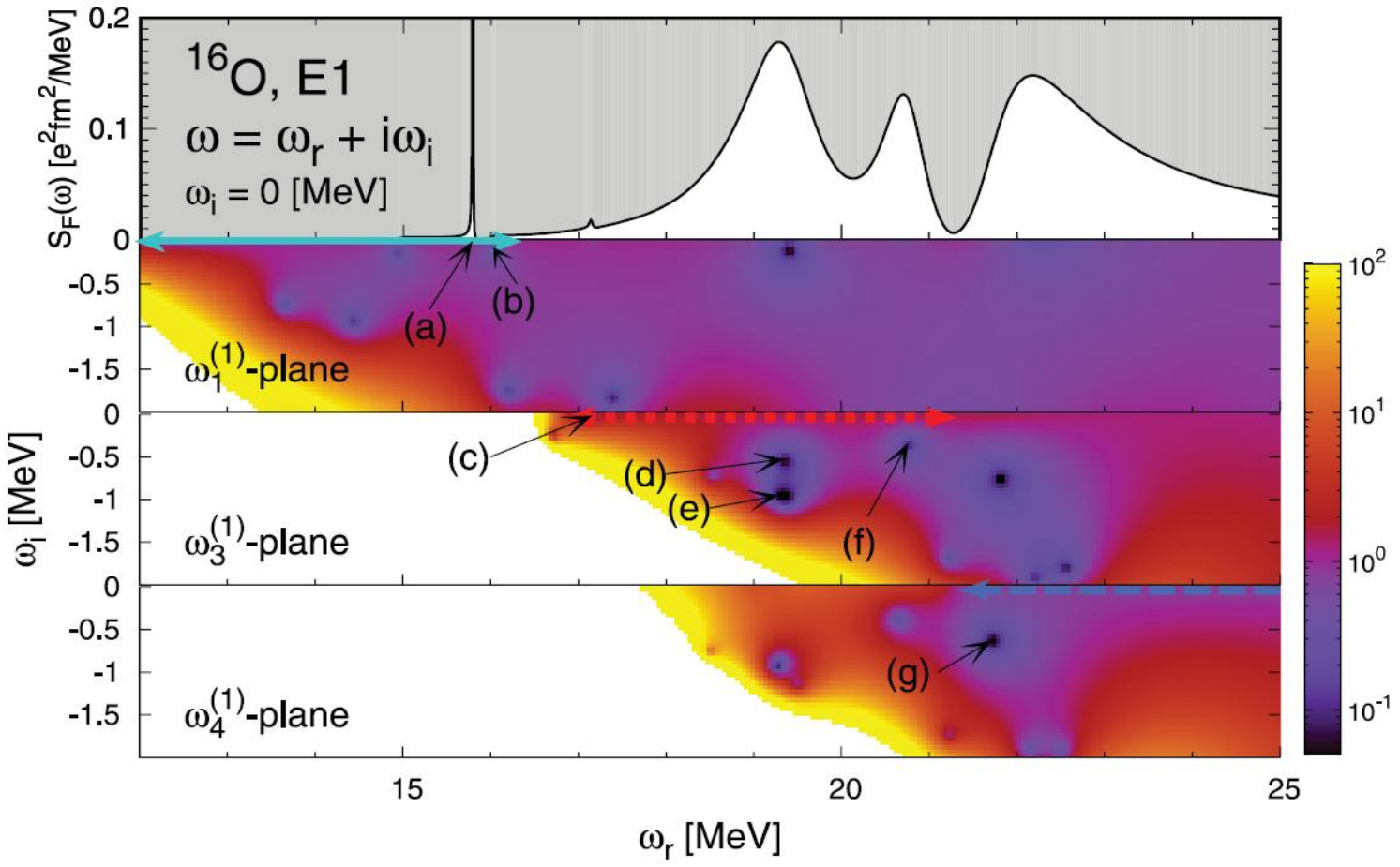
RPA Jost function (2Nx2N matrix)

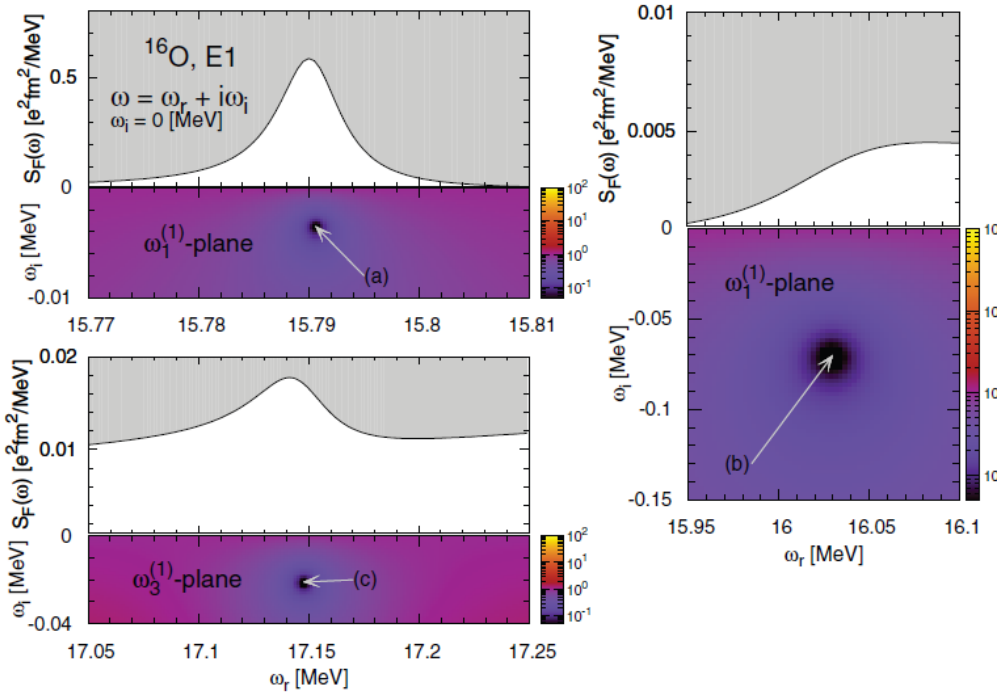
$$\mathcal{J}_{lj}^{(\pm)}(E)=\pm\frac{1}{i}\left[\Phi_{lj}^{(r)\mathsf{T}}(r)\frac{\partial}{\partial r}\Phi_{lj}^{(\pm)}(r)-\left(\frac{\partial}{\partial r}\Phi_{lj}^{(r)\mathsf{T}}(r)\right)\Phi_{lj}^{(\pm)}(r)\right]\mathcal{K}(E)$$

$$\det\mathcal{J}_{lj}(E_B)=0$$
$$\det\mathcal{J}_{lj}(E_R)=0$$

PRC **107**, 024303 (2023)

3-b. Random-Phase-Approximation (RPA) Jost function





PRC **107**, 024303 (2023)

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4-a. S-matrix and Green function

PRC **109**, 054304 (2024)

Jost-RPA Green function:

$$\left[\frac{\hbar^2}{2m}\boldsymbol{\mathcal{K}}^2-\left\{-\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2}\boldsymbol{I}-\frac{\mathbf{L}^2}{r^2}\right)+\boldsymbol{\mathcal{V}}(r)\right\}\right]\boldsymbol{\mathcal{G}}(r,r';E)=\boldsymbol{I}\,\delta(r-r')$$

S-matrix from the Jost functions (c: open channels, d: closed channels)

$$\boldsymbol{\mathcal{S}}(E)\equiv\boldsymbol{\mathcal{K}}^{\frac{1}{2}}\boldsymbol{\mathcal{J}}^{(+)-1}\boldsymbol{\mathcal{J}}^{(-)}\boldsymbol{\mathcal{K}}^{-\frac{1}{2}}=\begin{pmatrix}\boldsymbol{\mathcal{S}}_{cc} & \boldsymbol{\mathcal{S}}_{cd}\\ \boldsymbol{\mathcal{S}}_{dc} & \boldsymbol{\mathcal{S}}_{dd}\end{pmatrix}=\boldsymbol{\mathcal{S}}^{\mathsf{T}}(E)$$

Open-channel block S_{cc}: unitary

$$\boldsymbol{\mathcal{S}}_{cc}^{\dagger}(E)\,\boldsymbol{\mathcal{S}}_{cc}(E)=\boldsymbol{\mathcal{S}}_{cc}(E)\,\boldsymbol{\mathcal{S}}_{cc}^{\dagger}(E)=\mathbf{1}\quad(\text{real } E)$$

Scattering wave function:

$$\Psi_F^{(+)\mathsf{T}}(r;E)\equiv\boldsymbol{\mathcal{K}}^{\frac{1}{2}}\boldsymbol{\mathcal{J}}^{(+)-1}\Phi^{(r)\mathsf{T}}(r;E)=\frac{1}{2}\left[\Phi_F^{(-)\mathsf{T}}(r;E)+\boldsymbol{\mathcal{S}}(E)\,\Phi_F^{(+)\mathsf{T}}(r;E)\right],\qquad\Phi_F^{\mathsf{T}}\equiv\boldsymbol{\mathcal{K}}^{\frac{1}{2}}\Phi^{\mathsf{T}}$$

Green function with the scattering wave function and the S-matrix

$$\begin{aligned}\boldsymbol{\mathcal{G}}(r,r';E)&=-i\frac{2m}{\hbar^2}\left[\theta(r-r')\,\Phi_F^{(+)}(r)\,\Psi_F^{(+)\mathsf{T}}(r')+\theta(r'-r)\,\Psi_F^{(+)}(r)\,\Phi_F^{(+)\mathsf{T}}(r')\right]\\&=-i\frac{m}{\hbar^2}\left[\theta(r-r')\,\Phi_F^{(+)}(r)\,\Phi_F^{(-)\mathsf{T}}(r')+\theta(r'-r)\,\Phi_F^{(-)}(r)\,\Phi_F^{(+)\mathsf{T}}(r')+\Phi_F^{(+)}(r)\,\boldsymbol{\mathcal{S}}(E)\,\Phi_F^{(+)\mathsf{T}}(r')\right]\end{aligned}$$

4-a. “Imaginary part” of Green function

RPA response function and strength function

PRC **109**, 054304 (2024)

$$R_F(E) = \sum_{qq'} \iint dr\, dr'\, f_q^{(\tau)}(r)\, \xi_q^{\mathsf{T}}(r)\, \boldsymbol{\mathcal{G}}(r, r'; E)\, \xi_{q'}(r')\, f_{q'}^{(\tau)}(r'), \qquad S_F^{(\tau)}(E) = -\frac{1}{\pi} \operatorname{Im} R_F(E)$$

Symmetry of the solutions (c: open channels)

$$\boldsymbol{\mathcal{S}}_{cc}^{\dagger}(E) = \boldsymbol{\mathcal{S}}_{cc}^{-1}(E), \qquad \Psi_{F,c}^{(+)\mathsf{T}}(r; E) = \boldsymbol{\mathcal{S}}_{cc}(E)\, \Psi_{F,c}^{(+)\dagger}(r; E^*)$$

⇒ Only the open channels and S_{cc} remain

$$\begin{aligned} \operatorname{Im} \boldsymbol{\mathcal{G}}(r, r'; E) &= \frac{1}{2i}\Big[\boldsymbol{\mathcal{G}}(r, r'; E) - \boldsymbol{\mathcal{G}}^*(r, r'; E^*)\Big] \\ &= -\frac{2m}{\hbar^2}\, \Psi_{F,c}^{(-)}(r; E)\, \boldsymbol{\mathcal{S}}_{cc}(E)\, \Psi_{F,c}^{(-)\mathsf{T}}(r'; E), \qquad \Psi_{F,c}^{(-)}(r; E) \equiv \Psi_{F,c}^{(+)*}(r; E^*) \end{aligned}$$

The closed-channel blocks S_{cd}, S_{dc} and S_{dd} do not appear: the strength function $S_F = -(1/\pi)\,\mathrm{Im}\,R_F$ is carried by S_{cc} alone.

5-a. Strength Function through the Jost-RPA Green Function

Only the Open Channels Carry the Strength: $S_F(E)$ Is Built on $S_{cc}(E)$

1. Jost-RPA Green Function

$$\mathcal{G}(r,r') = -i\frac{2m}{\hbar^2}\Big[\theta(r-r')\mathbf{\Phi}^{(+)}(r)\mathcal{K}\mathcal{J}^{(+)-1}\mathbf{\Phi}^{(r)\mathsf{T}}(r') + \theta(r'-r)\mathbf{\Phi}^{(r)}(r)\big(\mathcal{J}^{(+)-1}\big)^{\mathsf{T}}\mathcal{K}\mathbf{\Phi}^{(+)\mathsf{T}}(r')\Big]$$

- $\mathbf{\Phi}^{(r)}(r)$: Regular solution ($\mathbf{\Phi}^{(r)} \to 0$ as $r \to 0$) $\mathbf{\Phi}^{(+)}(r)$: Outgoing Jost solution
- Jost matrix $\mathcal{J}^{(+)}(E) \implies$ **Poles of $\mathcal{G}$ occur at** $\det \mathcal{J}^{(+)}(E_g) = 0$

2. Strength Function Carried by the Open-Channel S-Matrix S_{cc}

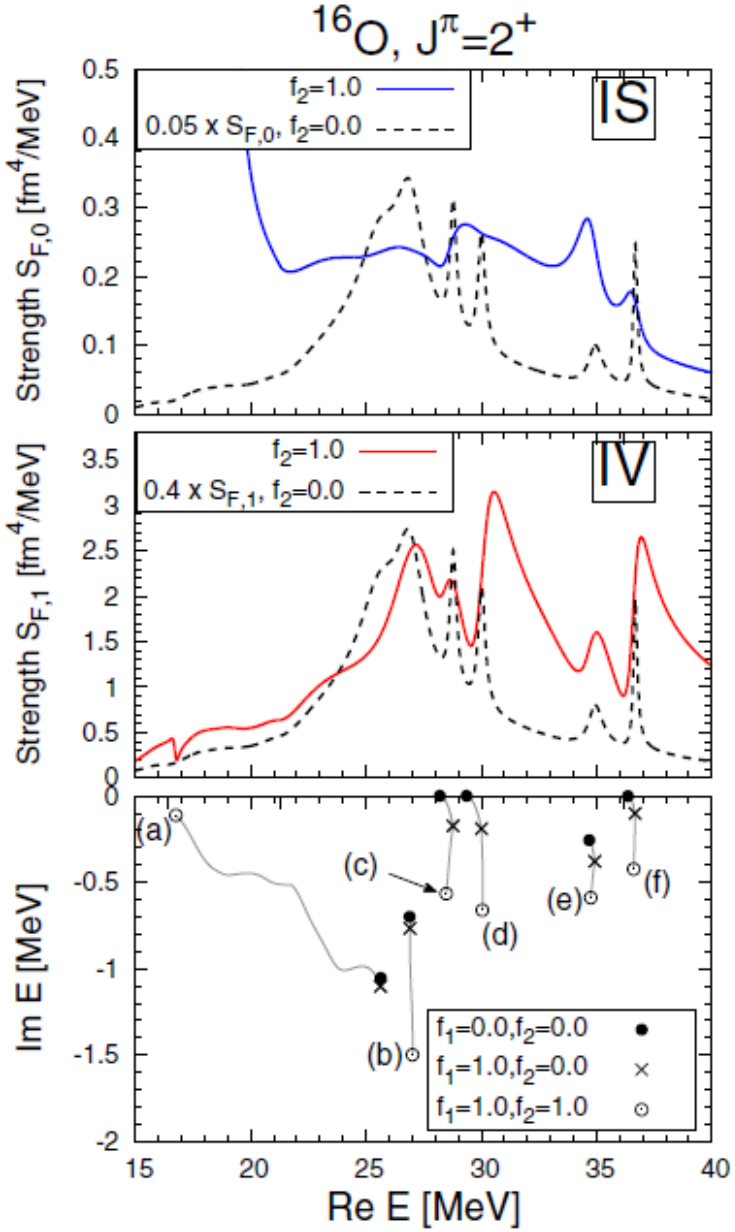
$$S_F^{(\tau)}(E) = \left(\sum_q \int_0^\infty dr\, f_q^{(\tau)}(r)\, \vec{\boldsymbol{\rho}}_q^{(-)\mathsf{T}}(r;E)\right) \boldsymbol{\mathcal{S}}_{cc}(E) \left(\sum_{q'} \int_0^\infty dr'\, \vec{\boldsymbol{\rho}}_{q'}^{(-)}(r';E)\, f_{q'}^{(\tau)}(r')\right)$$

$$\text{where} \quad \vec{\boldsymbol{\rho}}_q^{(-)}(r;E) \equiv \sqrt{\frac{2m}{\pi\hbar^2}}\, \mathbf{\Psi}_{F,c}^{(-)\mathsf{T}}(r;E)\, \boldsymbol{\xi}_q(r), \qquad \boldsymbol{\xi}_q(r) \equiv (\tilde{\varphi}_q(r),\, \tilde{\varphi}_q(r))^{\mathsf{T}}$$

Only the open channels c carry the strength: the poles of S_{cc} appear directly in S_F

4-b. Eigenphase-shift decomposition

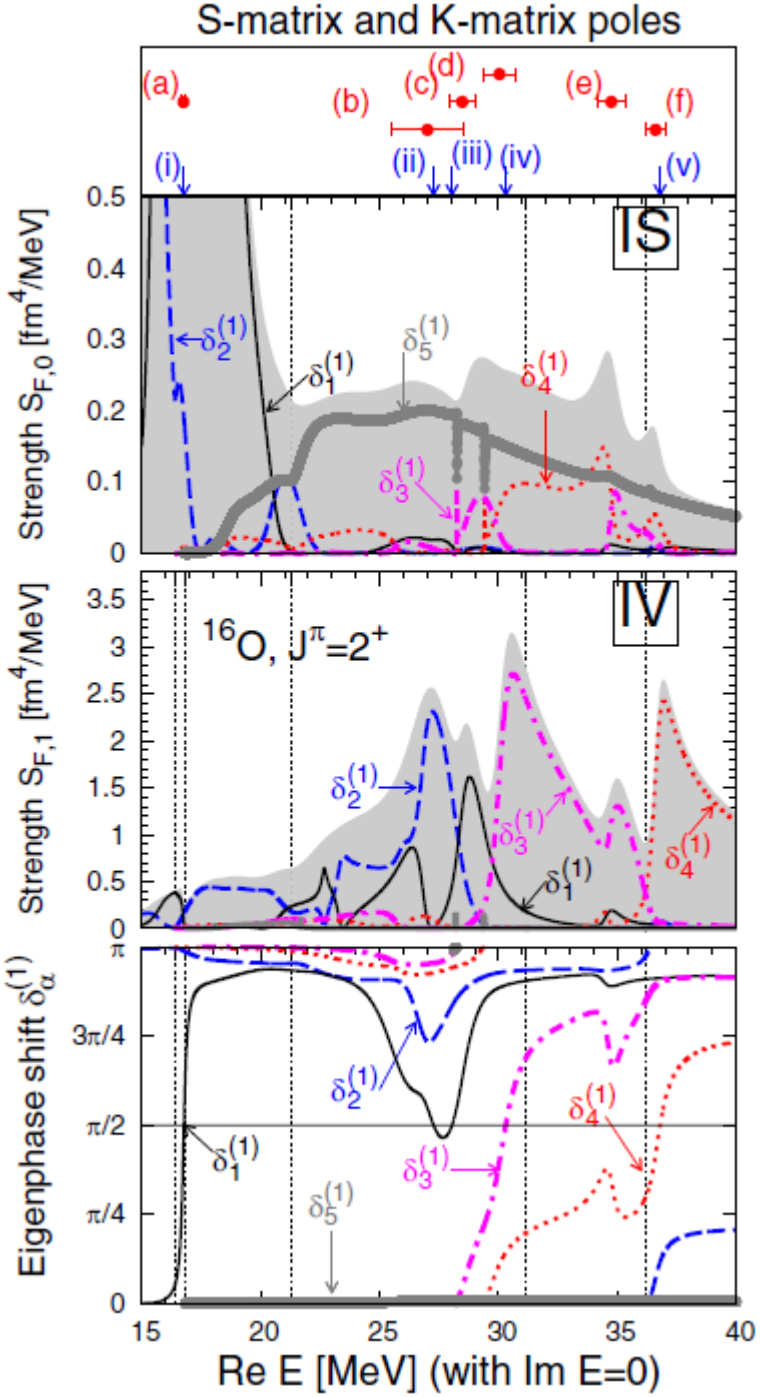
PRC **109**, 054304 (2024)



$$\mathcal{S}^{(1)} \equiv \mathcal{S}\mathcal{S}^{(0)\dagger}$$

$$\mathcal{S}_\delta^{(1)} \equiv \mathcal{U}^\dagger \mathcal{S}^{(1)} \mathcal{U} = \begin{pmatrix} e^{2i\delta_1^{(1)}} & & \\ & e^{2i\delta_2^{(1)}} & \\ & & \ddots \\ & & & \ddots \end{pmatrix}$$

1. Derived a unitary S-matrix within the Jost-RPA framework.
2. Proposed eigenphase shift decomposition of the RPA strength function.
3. Applied the method to quadrupole excitations in ^{16}O.
4. Showed that the Coulomb interaction strongly affects IS/IV mode mixing.



4-c. Mittag-Leffler theorem

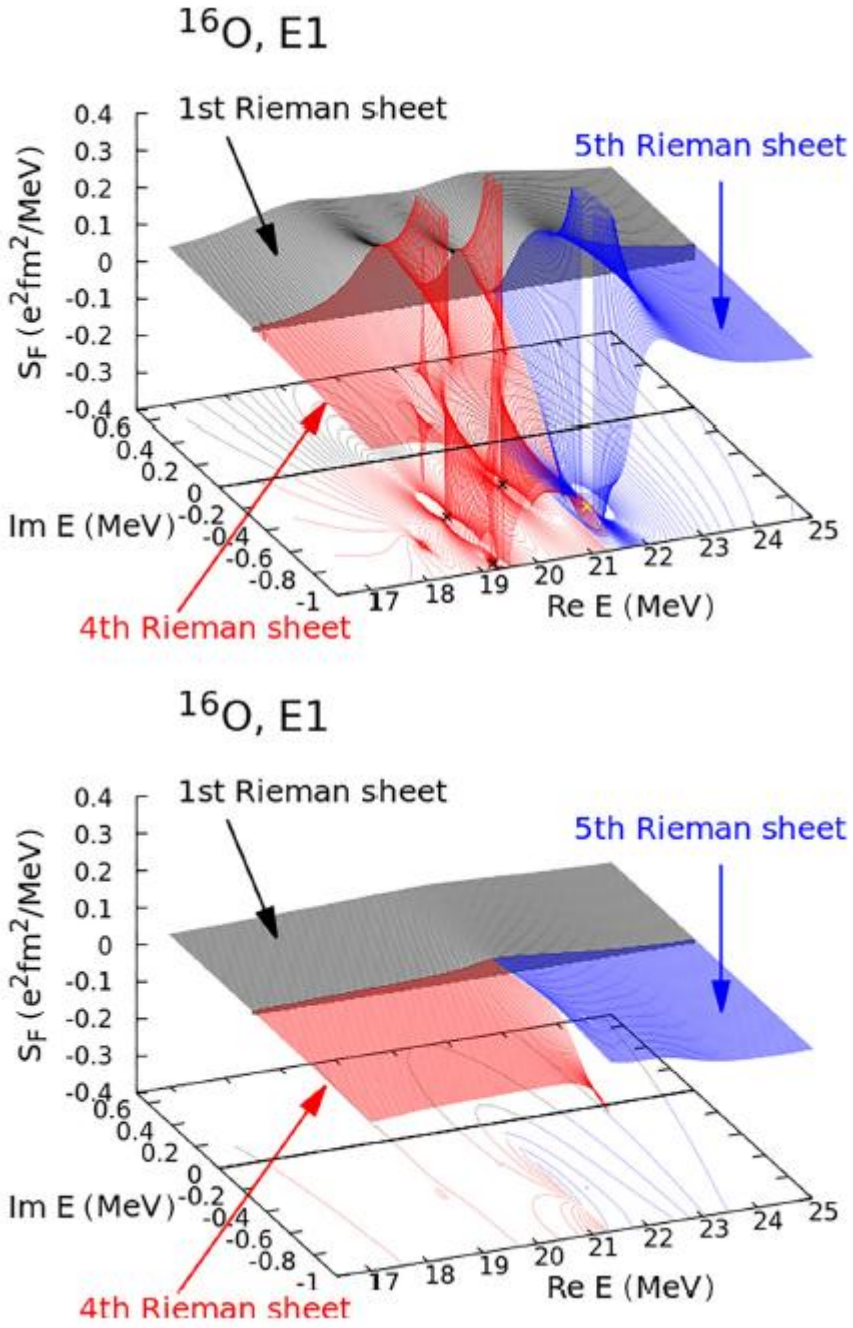
PRC **111**, 054323 (2025)

RPA response function

$$R_F(E) = R_F^{\mathrm{reg}}(E) + \sum_g \frac{\widetilde{B}_g}{E - E_g}$$

Residue of R_F for a pole $E=E_g$

$$\widetilde{B}_g \equiv \mathrm{Res}[R_F, E_g] = \oint_{C_g} \frac{dE'}{2\pi i} R_F(E')$$



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- $\boldsymbol{\Phi}^{(r)}(r)$: Regular solution ($\boldsymbol{\Phi}^{(r)} \to 0$ as $r \to 0$) $\quad \boldsymbol{\Phi}^{(+)}(r)$: Outgoing Jost solution
- Jost matrix $\boldsymbol{\mathcal{J}}^{(+)}(E) \implies$ **Poles of $\boldsymbol{\mathcal{G}}$ occur at** $\det \boldsymbol{\mathcal{J}}^{(+)}(E_g) = 0$

2. Strength Function Carried by the Open-Channel S-Matrix S_{cc}

$$S_F^{(\tau)}(E) = \left(\sum_q \int_0^\infty dr\, f_q^{(\tau)}(r)\, \vec{\boldsymbol{\rho}}_q^{(-)\mathsf{T}}(r;E)\right) \boldsymbol{\mathcal{S}}_{cc}(E) \left(\sum_{q'} \int_0^\infty dr'\, \vec{\boldsymbol{\rho}}_{q'}^{(-)}(r';E)\, f_{q'}^{(\tau)}(r')\right)$$

$$\text{where} \quad \vec{\boldsymbol{\rho}}_q^{(-)}(r;E) \equiv \sqrt{\frac{2m}{\pi\hbar^2}}\, \boldsymbol{\Psi}_{F,c}^{(-)\mathsf{T}}(r;E)\, \boldsymbol{\xi}_q(r), \qquad \boldsymbol{\xi}_q(r) \equiv (\tilde{\varphi}_q(r),\, \tilde{\varphi}_q(r))^{\mathsf{T}}$$

Only the open channels c carry the strength: the poles of S_{cc} appear directly in S_F

5-a. Takagi Factorization at Resonance Poles

At Each Pole the Residue of S_{cc} Has Rank 1: One Singular Value $\sigma_g^{(c)}$ and One Takagi Vector $\omega_g^{(c)}$

arXiv:2604.02237 (2026), accepted in PRC

Residue of the open-channel S-matrix at a pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

$$\mathrm{Res}\left[\boldsymbol{\mathcal{S}}_{cc}, E_g\right] = \sigma_g^{(c)}\, \vec{\boldsymbol{\omega}}_g^{(c)}\, \vec{\boldsymbol{\omega}}_g^{(c)\mathsf{T}}$$

rank 1: one non-zero singular value $\sigma_g^{(c)}$ (Takagi factorization)

$\vec{\boldsymbol{\omega}}_g^{(c)}$: normalized open-channel Takagi vector, $\|\vec{\boldsymbol{\omega}}_g^{(c)}\| = 1$

Pole term of the open-channel S-matrix

$$\boldsymbol{\mathcal{S}}_{cc}(E) = \boldsymbol{\mathcal{S}}_{\mathrm{B},g}(E) + \frac{\sigma_g^{(c)}\, \vec{\boldsymbol{\omega}}_g^{(c)}\, \vec{\boldsymbol{\omega}}_g^{(c)\mathsf{T}}}{E - E_g}$$

$\boldsymbol{\mathcal{S}}_{\mathrm{B},g}(E)$: everything except the pole term of g (including the other poles)

5-a. What is Takagi Factorization?

Teiji Takagi, Japanese Journal of Mathematics 1 (1924) 83–93

Takagi Factorization Theorem for Complex Symmetric Matrices

$$\boldsymbol{A} = \boldsymbol{W}\boldsymbol{\Sigma}\boldsymbol{W}^{\mathsf{T}}$$

$$\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$$

- $\boldsymbol{A} \in \mathbb{C}^{n\times n}$: **Complex symmetric** ($\boldsymbol{A} = \boldsymbol{A}^{\mathsf{T}}$)
- $\boldsymbol{W} \in \mathbb{C}^{n\times n}$: **Unitary matrix** ($\boldsymbol{W}^{\dagger}\boldsymbol{W} = \boldsymbol{I}$)
- $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$: **Singular values** of $\boldsymbol{A}$
- $\boldsymbol{w}_j$ (columns of $\boldsymbol{W}$): **Takagi vectors**



Teiji Takagi (1875–1960)

Why Takagi? — Comparison with Standard Matrix Decompositions

Decomposition	Form	Valid?	Physical Reason / Applicability to Continuum S-Matrix
Hermitian Spectral	$H = U\Lambda U^{\dagger}$	[×]	Requires $H = H^{\dagger}$. Fails: S-matrix is symmetric, **not** Hermitian ($S \neq S^{\dagger}$).
Standard SVD	$A = U\Sigma V^{\dagger}$	[△]	Two bases U, V. Symmetric form $V = U^{*}$ ($S_{ab} = S_{ba}$) not built in.
Takagi Factorization	$\boldsymbol{A = W\Sigma W^{\mathsf{T}}}$	[✓]	Single unitary W. **Exact match for complex symmetric $S = S^{\mathsf{T}}$!**

5-a. Pole Term of the Transition Amplitude

The Strength Function Is the Squared Norm of the Transition Amplitude Vector; the Pole Term of S_{cc} Gives Its Pole Term

Strength function and transition amplitude vector in the open channels c

$$S_F^{(\tau)}(E) = \left(\sum_q \int_0^\infty dr\, f_q^{(\tau)}(r)\, \vec{\boldsymbol{\rho}}_q^{(-)\mathsf{T}}(r;E) \right) \boldsymbol{\mathcal{S}}_{cc}(E) \left(\sum_{q'} \int_0^\infty dr'\, \vec{\boldsymbol{\rho}}_{q'}^{(-)}(r';E)\, f_{q'}^{(\tau)}(r') \right)$$

$$= \vec{\mathcal{F}}^{(\tau)\dagger}(E^*)\, \vec{\mathcal{F}}^{(\tau)}(E)$$

$$\vec{\mathcal{F}}^{(\tau)}(E) \equiv \sqrt{\frac{2m}{\pi\hbar^2}}\, \boldsymbol{\mathcal{S}}_{cc}(E) \sum_q \int_0^\infty dr\, \boldsymbol{\Psi}_{F,c}^{(+)*\mathsf{T}}(r;E^*)\, \boldsymbol{\xi}_q(r)\, f_q^{(\tau)}(r)$$

Pole term of the transition amplitude vector (inserting the pole term of $\boldsymbol{\mathcal{S}}_{cc}$)

$$\vec{\mathcal{F}}^{(\tau)}(E) = \vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E) + \frac{c_g^{(\tau)}\, \vec{\boldsymbol{\omega}}_g^{(c)}}{E - E_g}$$

$\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)$: everything except the pole term of g

5-a. Excitation and Decay Channel Vector of the Pole

Takagi Factorization Splits the Pole Term into the Excitation c_g (Depends on the External Field) and the Decay Channel Vector ω_g (Does Not)

Pole term of the transition amplitude vector $\dfrac{c_g^{(\tau)}\ \vec{\boldsymbol{\omega}}_g^{(c)}}{E - E_g}$

$c_g^{(\tau)}$: excitation by the external field $f^{(\tau)}$

$$c_g^{(\tau)} = \sqrt{\frac{i\sigma_g^{(c)}}{2\pi}}\,\mathcal{F}_g^{(\tau)}, \qquad \mathcal{F}_g^{(\tau)} = \sum_q \int_0^\infty dr\, \vec{\boldsymbol{\psi}}_g^{\mathsf{T}}(r)\, \boldsymbol{\xi}_q(r)\, f_q^{(\tau)}(r) = \sum_i F_{g,i}^{(\tau)}$$

$\Rightarrow$ depends on the external field (IS / IV)

$\vec{\omega}_g^{(c)}$: decay channel vector (Takagi vector)

$$\vec{\boldsymbol{\psi}}_g(r) \equiv \sqrt{\frac{4m\sigma_g^{(c)}}{i\hbar^2}}\ \boldsymbol{\Psi}_{F,c}^{(+)*}(r; E_g^*)\, \vec{\boldsymbol{\omega}}_g^{(c)} \ \text{(pole state of } g)$$

$\Rightarrow$ property of the pole: independent of the field

$\rightarrow$ collectivity from the channel amplitudes $F_{g,i}^{(\tau)}$

5-b. Collectivity of Resonance Poles

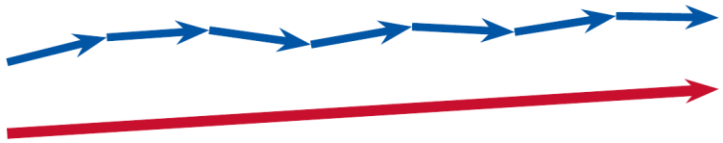
Collective excitation:
coherent superposition of many particle–hole excitations

G. Co', V. De Donno, M. Anguiano, A. M. Lallena, PRC 87, 034305 (2013)

$$\langle \nu | \hat{F} | 0 \rangle = \sum_i F_{\nu,i}, \qquad B(EL) = \left| \sum_i F_{\nu,i} \right|^2$$

Collective: many, in phase

$$\left| \sum_i F_{\nu,i} \right|^2 \gg \sum_i |F_{\nu,i}|^2$$



$F_{\nu,i}$ $\sum_i F_{\nu,i}$

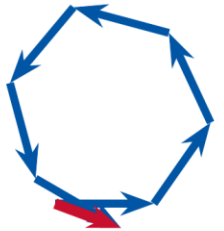
Single particle–hole

$$\left| \sum_i F_{\nu,i} \right|^2 = \sum_i |F_{\nu,i}|^2$$



Many, phases cancel

$$\left| \sum_i F_{\nu,i} \right|^2 \ll \sum_i |F_{\nu,i}|^2$$



5-b. Conventional Evaluation of Collectivity

Closest to This Work: Number of Participating p–h Pairs and Coherence of Discrete RPA States (Co' et al.)

(1) Weisskopf unit

strength only (fixed single-particle reference)

$$B(EL)/B_{\mathrm{W}}(EL) \gg 1$$

V. F. Weisskopf, Phys. Rev. 83, 1073 (1951)

(2) Participation ratio

number of configurations (no phases)

$$P = \left(\sum_i |c_i|^2\right)^2 \Big/ \sum_i |c_i|^4$$

F. Wegner, Z. Phys. B 36, 209 (1980); J. R. Armstrong *et al.*, PRE 86, 066204 (2012); F. Borgonovi *et al.*, Phys. Rep. 626, 1 (2016)

(3) Co' *et al.* (discrete RPA)

number and phase alignment (coherence)

$$D = \frac{N^*}{N_{ph}}, \qquad \mathcal{C} = \frac{B(EL)}{\left(\sum_{ph} |b_{ph}|\right)^2} = \frac{\left|\sum_{ph} b_{ph}\right|^2}{\left(\sum_{ph} |b_{ph}|\right)^2} \leq 1$$

N^*: number of p–h pairs with $X_{ph}^2 - Y_{ph}^2 \geq 1/N_{ph}$ (above the average weight)

b_{ph}: p–h contribution to the transition amplitude; $\mathcal{C} = 1$: all in phase

averaged over the pygmy and giant energy regions with weight $B(EL)$

G. Co' *et al.*, PRC 87, 034305 (2013); E. G. Lanza *et al.*, PRC 79, 054615 (2009)

All use the amplitudes of discrete states (eigenvectors)

5-b. Collectivity Indicators of Each Resonance Pole

Coherence C_g (Phase Alignment) and Participation η_g (How Evenly the Channels Share) of the Channel Amplitudes $F_{g,i}$

arXiv:2604.02237 (2026), accepted in PRC

Channel amplitudes of the pole g ($i = 1, \ldots, N_{\rm tot}$: channels)

$$F_{g,i}^{(\tau)} \equiv \int_0^\infty dr\, f_q^{(\tau)}(r)\, \vec{\psi}_{g,i}^{\,\mathsf{T}}(r)\, \boldsymbol{\xi}_{q,i}(r), \qquad \mathcal{F}_g^{(\tau)} \equiv \sum_{i=1}^{N_{\rm tot}} F_{g,i}^{(\tau)} = \left|\mathcal{F}_g^{(\tau)}\right| e^{i\Theta_g^{(\tau)}}$$

Collectivity indicators

$$C_g^{(\tau)} \equiv \frac{\left|\mathcal{F}_g^{(\tau)}\right|}{\sum_i \left|F_{g,i}^{(\tau)}\right|} \qquad \mathbf{0 \le C_g^{(\tau)} \le 1} \qquad \text{coherence index}$$

$$\eta_g^{(\tau)} \equiv \frac{\left(\sum_i \left|F_{g,i}^{(\tau)}\right|\right)^2}{N_{\rm tot} \sum_i \left|F_{g,i}^{(\tau)}\right|^2} \qquad \mathbf{\frac{1}{N_{tot}} \le \eta_g^{(\tau)} \le 1} \qquad \text{participation ratio}$$

$$N_{\rm eff} \equiv N_{\rm tot}\, \eta_g^{(\tau)} \qquad \mathbf{1 \le N_{eff} \le N_{tot}} \qquad \text{effective number of channels}$$

5-b. Complex Transition Probability of the Pole

$\tilde{B}_g$ = Res[R_F, E_g] = F_g^2 Factorizes into the Collectivity Enhancement $C_g^2 N_{tot} \eta_g$, the Single-Particle Scale and the Phase

arXiv:2604.02237 (2026), accepted in PRC

Complex transition probability of the pole (residue of the RPA response function R_F)

$$\widetilde{B}_g^{(\tau)} \equiv \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^{(\tau)2} = \left[C_g^{(\tau)2} N_{\mathrm{tot}} \eta_g^{(\tau)}\right] \left(\sum_i \left|F_{g,i}^{(\tau)}\right|^2\right) e^{2i\Theta_g^{(\tau)}}$$

$$\text{collectivity enhancement } C_g^{(\tau)2} N_{\mathrm{tot}} \eta_g^{(\tau)} = \frac{\left|\sum_i F_{g,i}^{(\tau)}\right|^2}{\sum_i \left|F_{g,i}^{(\tau)}\right|^2} \ (= 1\text{: one channel})$$

Strength function as a sum over the poles

$$S_F^{(\tau)}(E) = \sum_g \underbrace{C_g^{(\tau)2} N_{\mathrm{tot}} \eta_g^{(\tau)}}_{\textbf{collectivity enhancement}} \times \underbrace{\left(\sum_i \left|F_{g,i}^{(\tau)}\right|^2\right)}_{\textbf{single-particle scale}} \times \underbrace{\left(-\frac{1}{\pi} \mathrm{Im}\left[\frac{e^{2i\Theta_g^{(\tau)}}}{E - E_g}\right]\right)}_{\textbf{resonance line shape}} + S_{F,\mathrm{bg}}^{(\tau)}(E)$$

5-b. Collectivity Indicators: Comparison with Co' et al.

Same Idea — Coherence and Participation — Applied to Each Resonance Pole, with One Measure That Factorizes $|\tilde{B}_g|$

	Co' *et al.* (discrete RPA)	This work (Jost-RPA)
States	discrete states, averaged over an energy region with weight $B(EL)$	each resonance pole E_g itself
Amplitudes	real b_{ph}	complex $F_{g,i}$ from the Takagi vector $\vec{\omega}_g$
Coherence	$\mathcal{C} = \dfrac{\lvert\sum_{ph} b_{ph}\rvert^2}{(\sum_{ph} \lvert b_{ph}\rvert)^2}$	$C_g^2 = \dfrac{\lvert\sum_i F_{g,i}\rvert^2}{(\sum_i \lvert F_{g,i}\rvert)^2}$ (same form)
Participation	**L2**: counted with probabilities $\lvert X_{ph}\rvert^2$ $N^* = \#\{\text{p–h pairs with } \lvert X_{ph}\rvert^2 \geq 1/N_{ph}\}$ $\approx \dfrac{\left(\sum_{ph} \lvert X_{ph}\rvert^2\right)^2}{\sum_{ph} \lvert X_{ph}\rvert^4}$ (L2 participation) both give m for m equally shared pairs	**L1**: counted with amplitudes $\lvert F_{g,i}\rvert$ $N_{\rm tot}\eta_g = \dfrac{\left(\sum_i \lvert F_{g,i}\rvert\right)^2}{\sum_i \lvert F_{g,i}\rvert^2}$
Measure	two indices: $D = N^*/N_{ph}$ and $\mathcal{C}$	one: $C_g^2 N_{\rm tot}\eta_g = \dfrac{\lvert\sum_i F_{g,i}\rvert^2}{\sum_i \lvert F_{g,i}\rvert^2}$ ($= 1$: one channel)

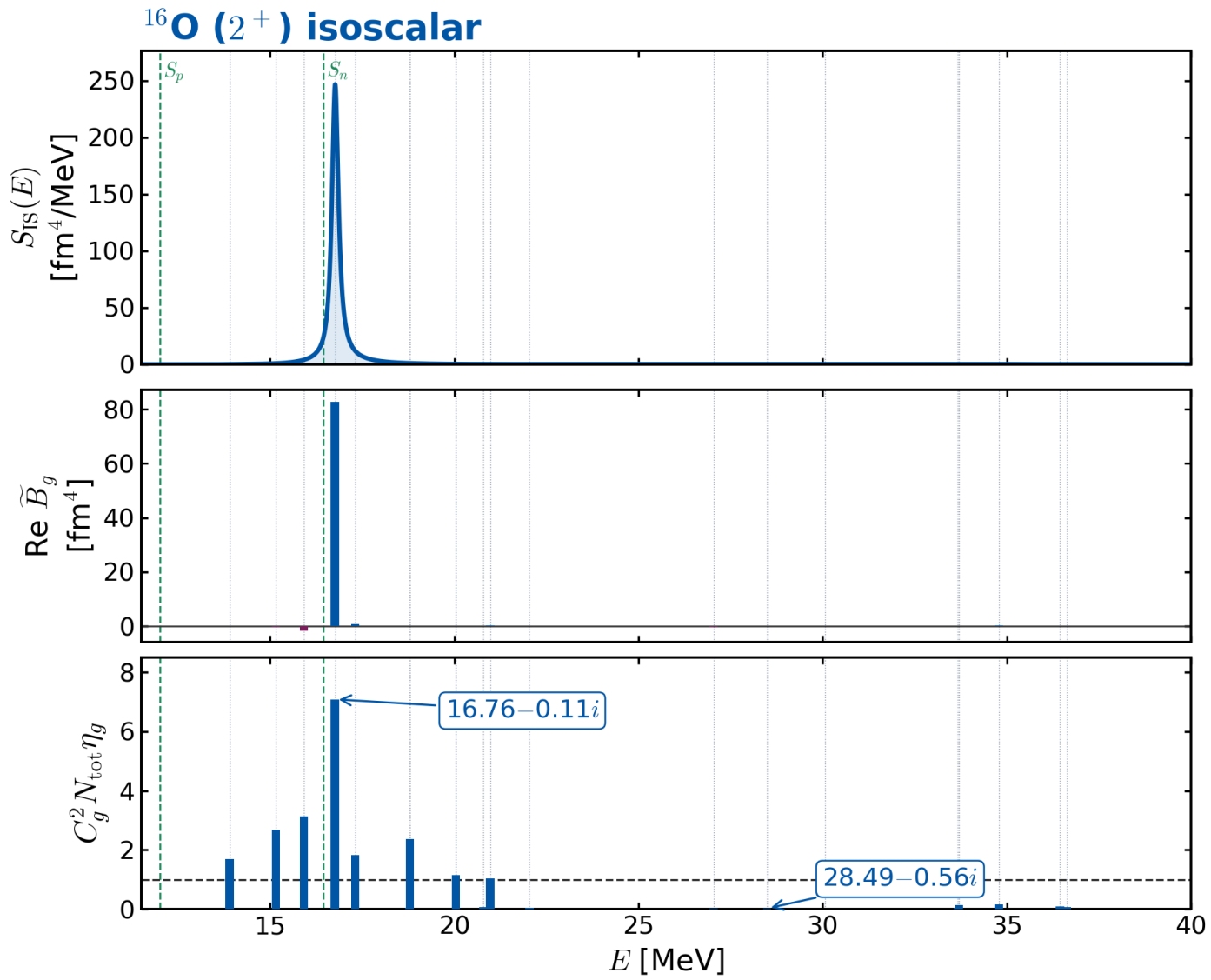
Why

- the collectivity of the resonance itself (each pole E_g), not of pseudostates
- participation from $\sum_i |F_{g,i}| \Rightarrow |\widetilde{B}_g| = \left|\sum_i F_{g,i}\right|^2 = \underbrace{C_g^2}_{\text{coherence}} \times \underbrace{N_{\rm tot}\eta_g}_{\text{participation}} \times \underbrace{\sum_i |F_{g,i}|^2}_{\text{own single-particle scale}}$

G. Co' *et al.*, PRC 87, 034305 (2013); Hoyer, JMLR 5, 1457 (2004); Hurley & Rickard, IEEE TIT 55, 4723 (2009)

5-b. Numerical Results: ^{16}O ($J^\pi = 2^+$) Collectivity Decomposition: Isoscalar

Strength of Each Pole (Re $\tilde{B}_g$) and Its Collectivity ($C_g^2 N_{tot} \eta_g$): ISGQR 16.76 – 0.11i MeV ($C_g^2 N_{tot} \eta_g$ = 7.1)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^2 = -2\pi i\, \mathrm{Res}\left[S_F, E_g\right]$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE\; 2\,\mathrm{Re}\left[\frac{\mathrm{Res}\left[S_F, E_g\right]}{E - E_g}\right]$$

area of the pole terms of S_F at E_g and E_g^* on the real axis; < 0: the pole lowers S_F (dip)

Bottom panel: collectivity measure

$$C_g^2 N_{\mathrm{tot}} \eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

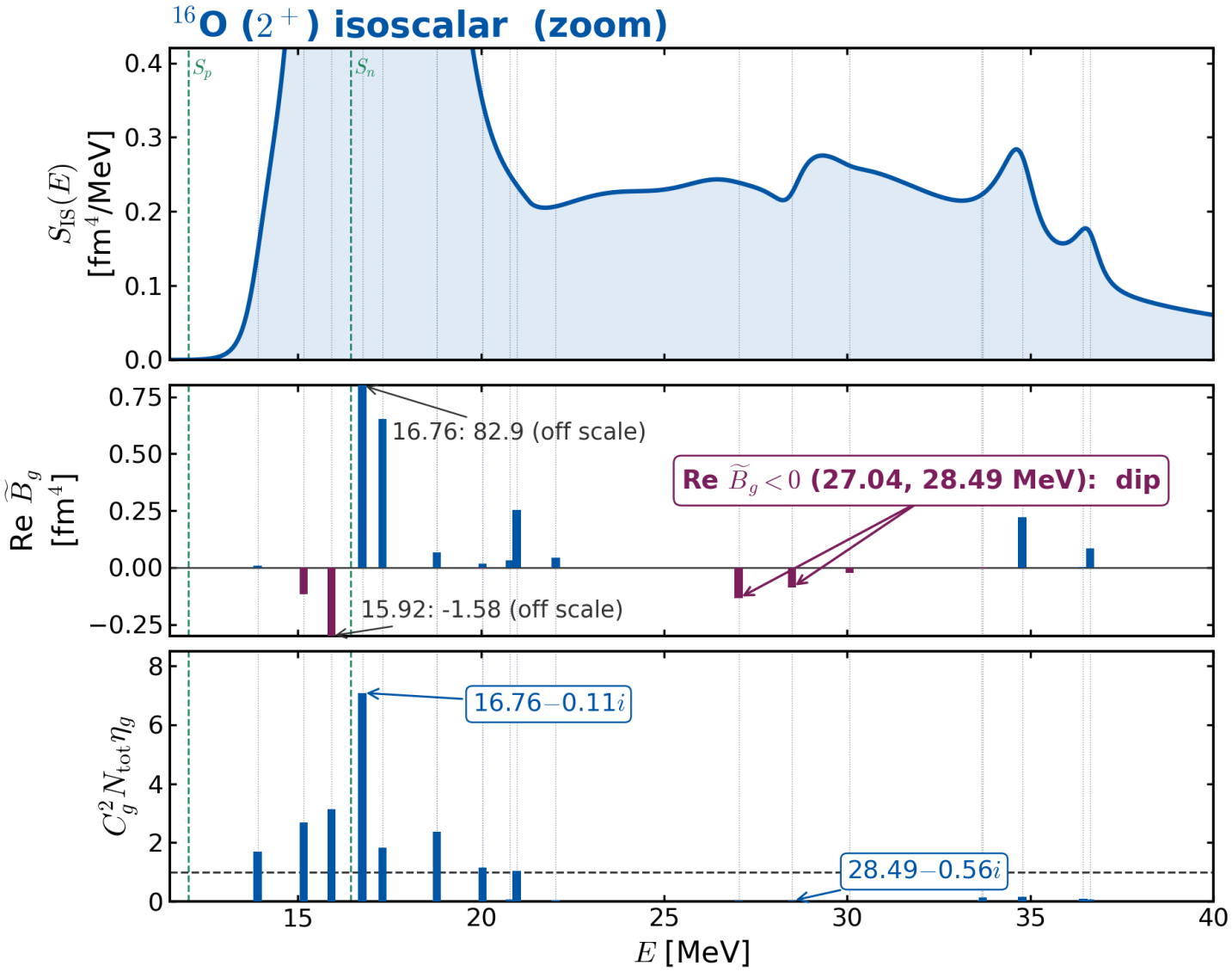
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\mathrm{tot}} \eta_g \sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\mathrm{tot}} \eta_g$

5-b. Numerical Results: ^{16}O ($J^\pi = 2^+$) Collectivity Decomposition: Isoscalar (Zoom)

Poles with Re $\tilde{B}_g < 0$ (27.04, 28.49 MeV) Make the Isoscalar Dips; Both Are Non-Collective ($C_g^2 N_{tot} \eta_g \approx 0.02$)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^2 = -2\pi i\, \mathrm{Res}\left[S_F, E_g\right]$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE\; 2\,\mathrm{Re}\left[\frac{\mathrm{Res}\left[S_F, E_g\right]}{E - E_g}\right]$$

area of the pole terms of S_F at E_g and E_g^* on the real axis; < 0: the pole lowers S_F (dip)

Bottom panel: collectivity measure

$$C_g^2 N_{\mathrm{tot}} \eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

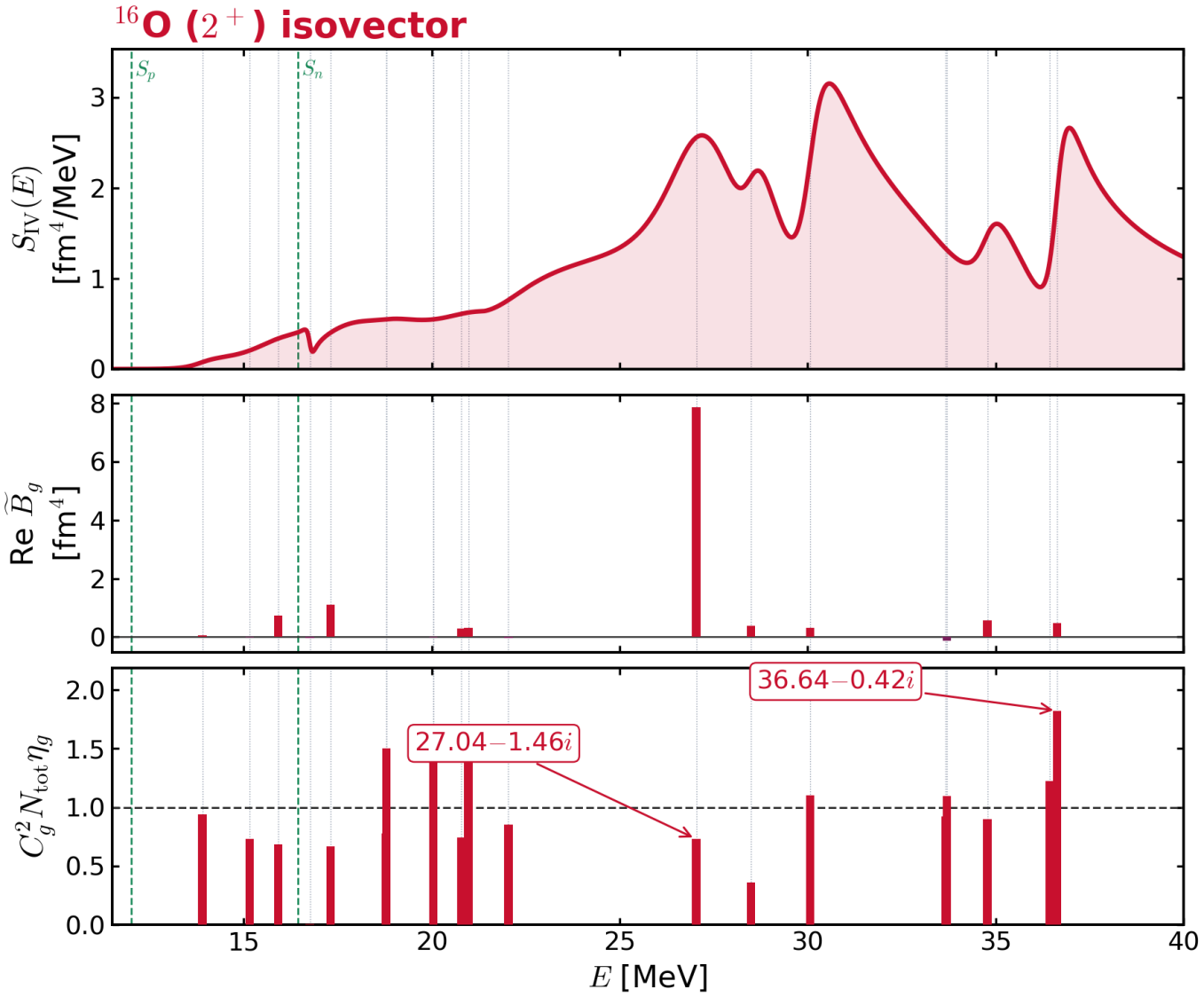
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\mathrm{tot}} \eta_g \sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\mathrm{tot}} \eta_g$

5-b. Numerical Results: ^{16}O ($J^\pi = 2^+$) Collectivity Decomposition: Isovector

Largest IV Strength 27.04 – 1.46i MeV Is Not Collective ($C_g^2 N_{tot} \eta_g = 0.73$); Most Collective IV Pole 36.64 – 0.42i MeV (1.8)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\big[R_F, E_g\big] = \mathcal{F}_g^2 = -2\pi i\,\mathrm{Res}\big[S_F, E_g\big]$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE\; 2\,\mathrm{Re}\left[\frac{\mathrm{Res}\big[S_F, E_g\big]}{E - E_g}\right]$$

area of the pole terms of S_F at E_g and E_g^* on the real axis; < 0: the pole lowers S_F (dip)

Bottom panel: collectivity measure

$$C_g^2 N_{\rm tot}\eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

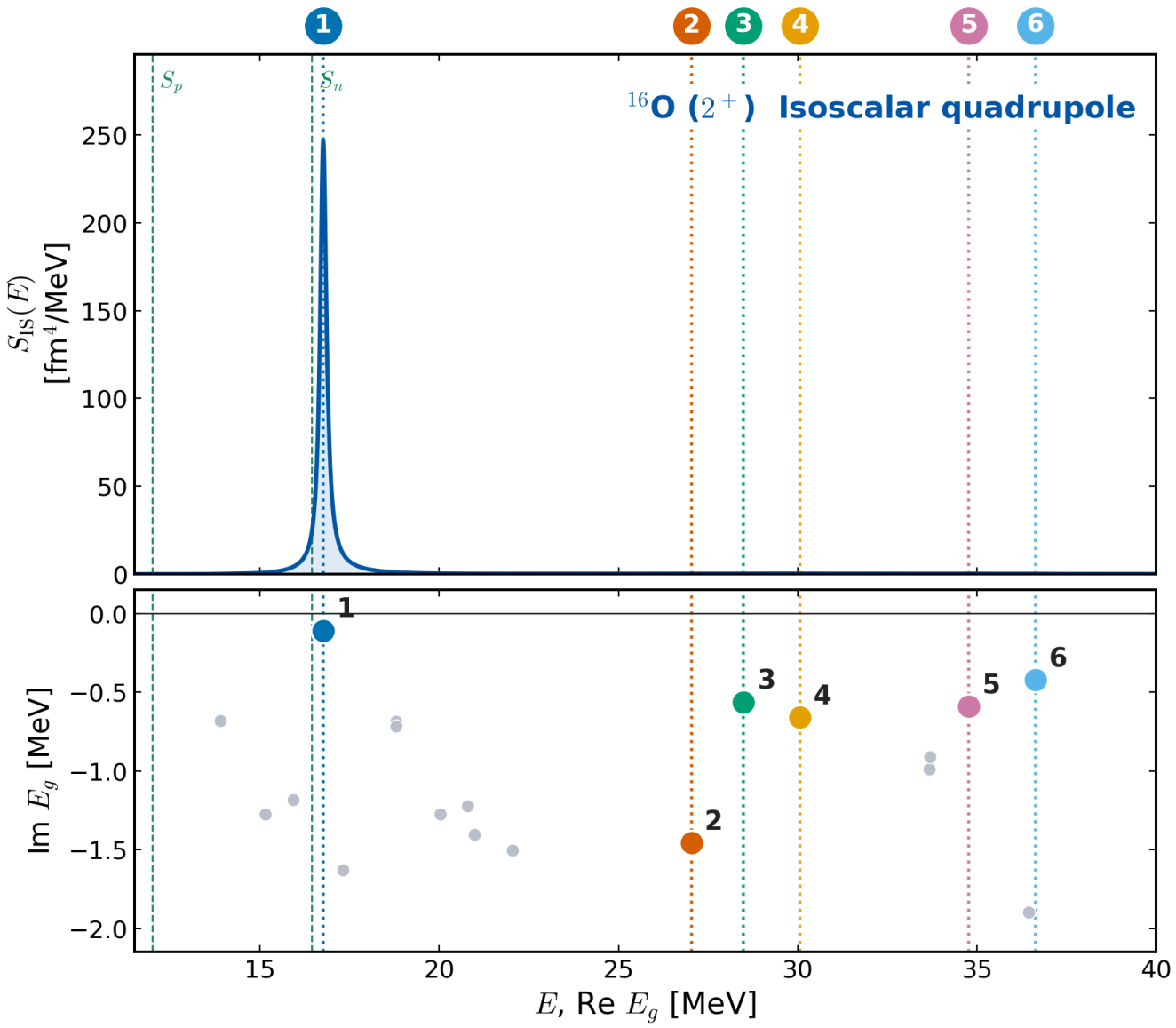
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

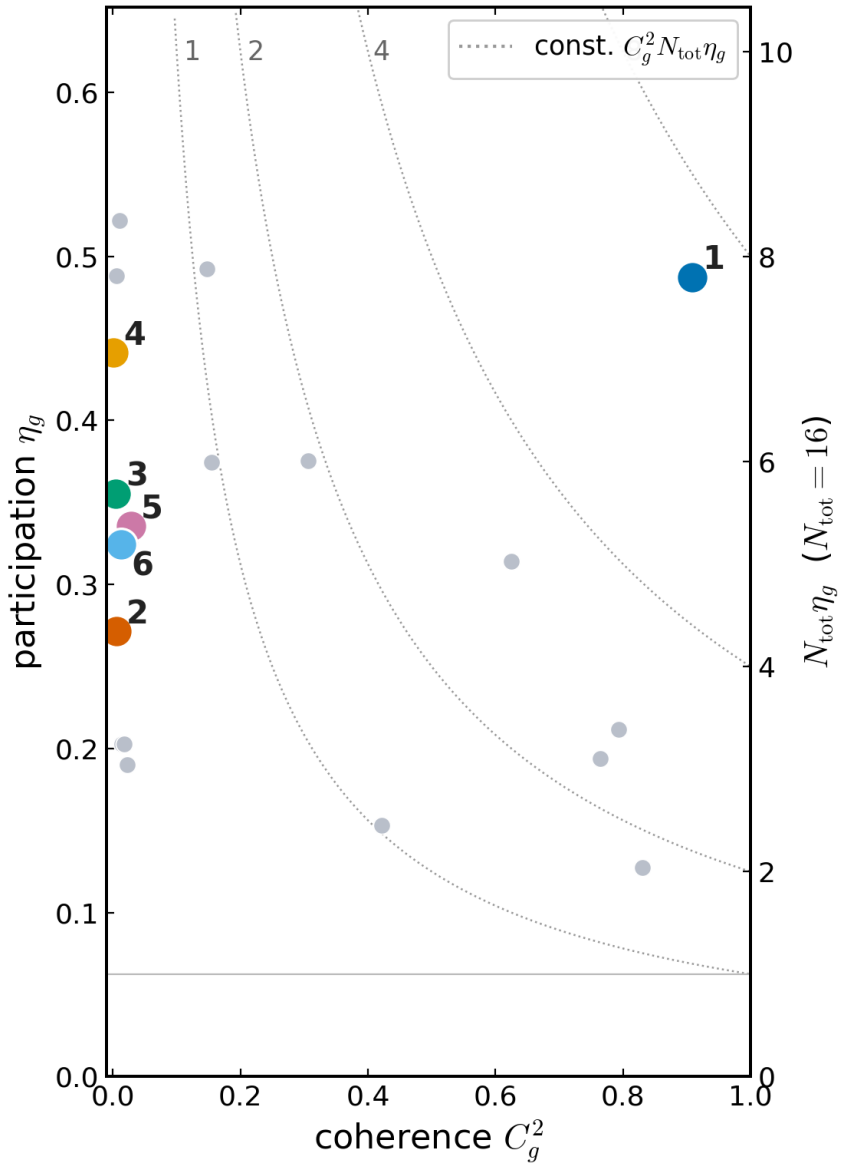
$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\rm tot}\eta_g \sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\rm tot}\eta_g$

5-b. Collectivity Indicator Correlation: ^{16}O ($J^\pi = 2^+$) Isoscalar (IS)

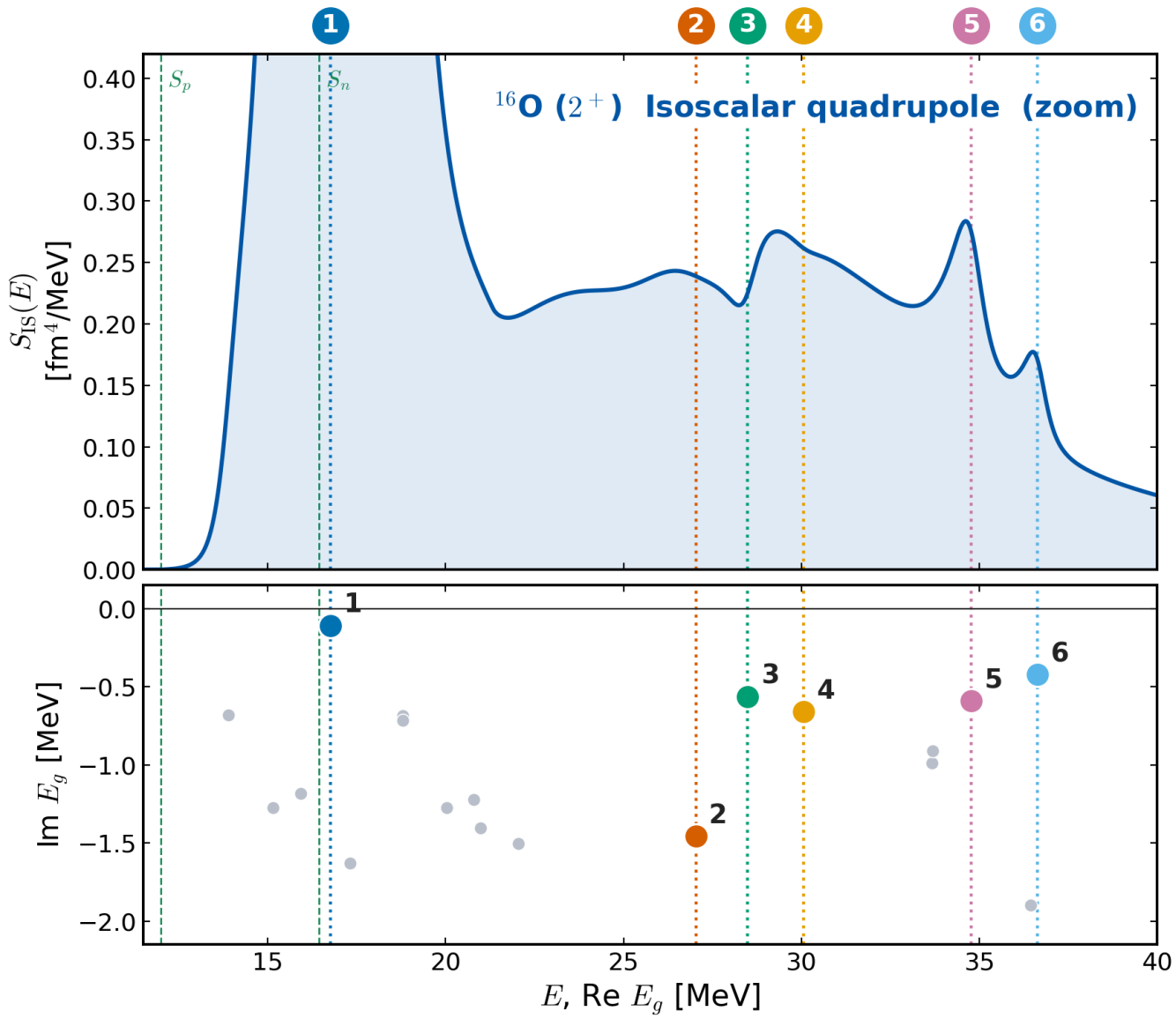
Participation η_{IS} vs. Coherence C_{IS}^2 ($N_{tot} = 16$); Dotted: Constant $C^2N_{tot}\,\eta$

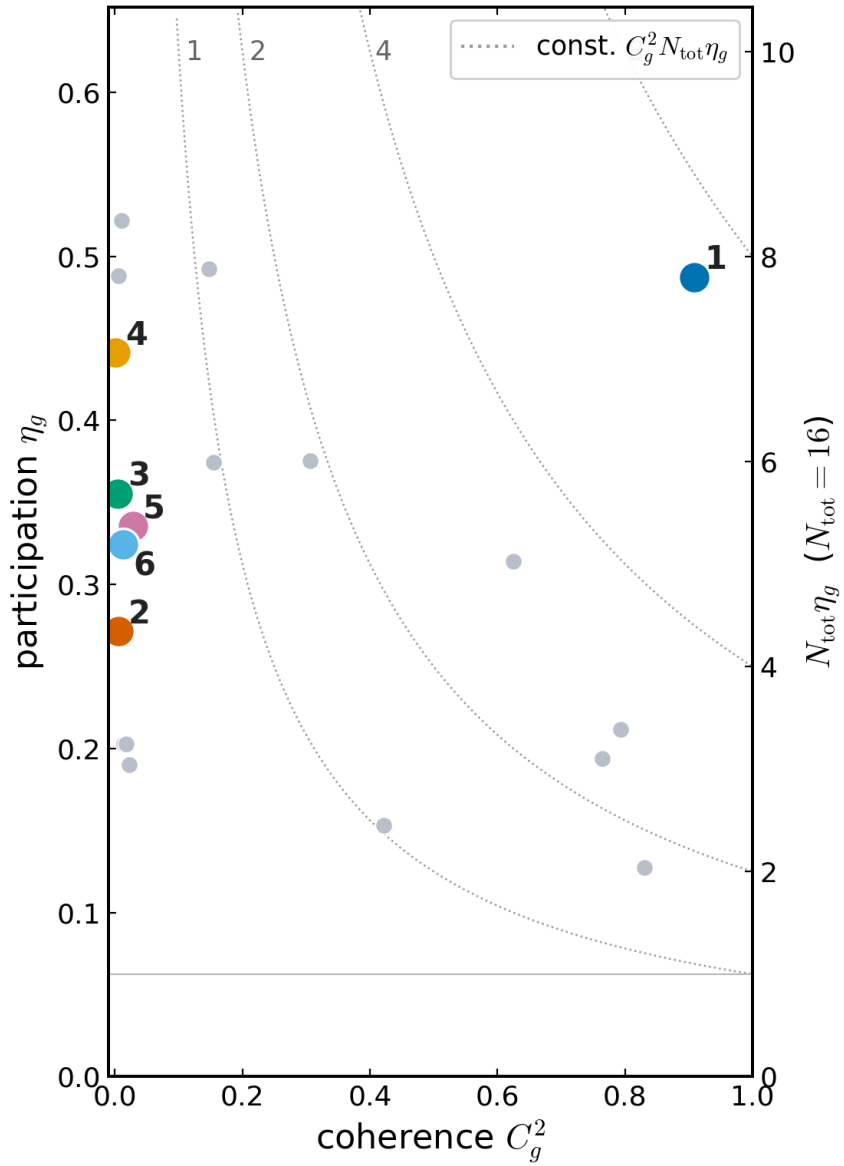




5-b. Collectivity Indicator Correlation: ^{16}O ($J^{\pi} = 2^{+}$) Isoscalar (IS, Zoom)

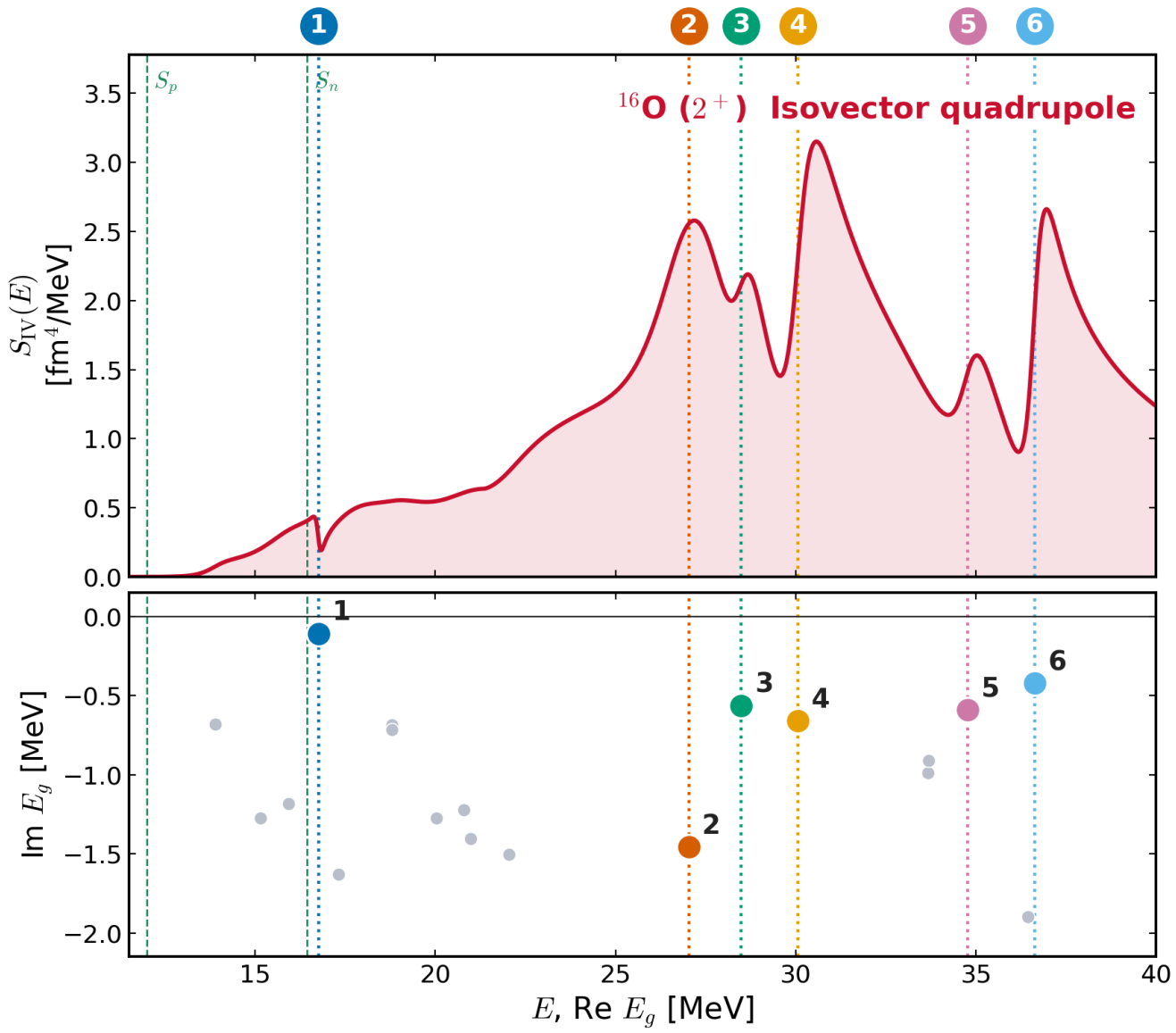
Poles 2–6 at High Energy: η_{IS} = 0.27–0.44 but $C_{IS}^2 \leq 0.03$, so Non-Collective ($C_g^2 N_{tot} \eta_g \leq 0.15$)

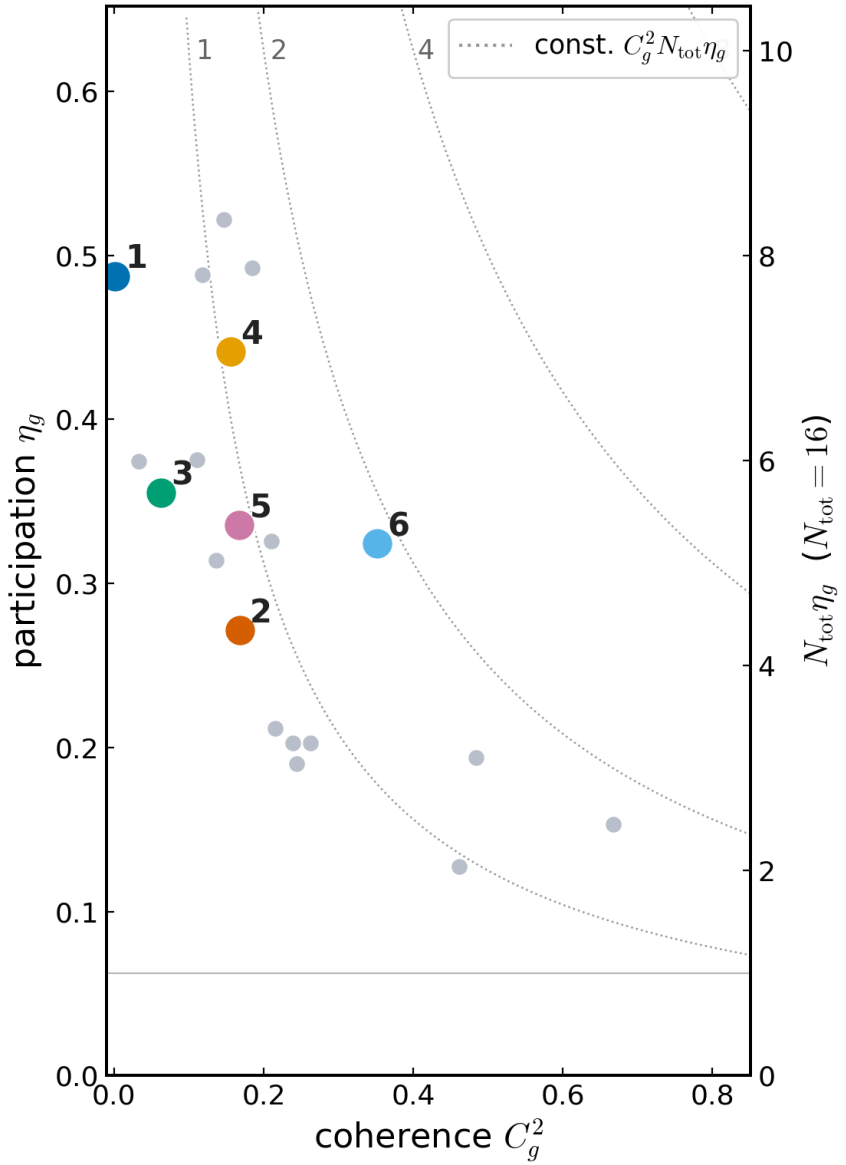




5-b. Collectivity Indicator Correlation: ^{16}O ($J^\pi = 2^+$) Isovector (IV)

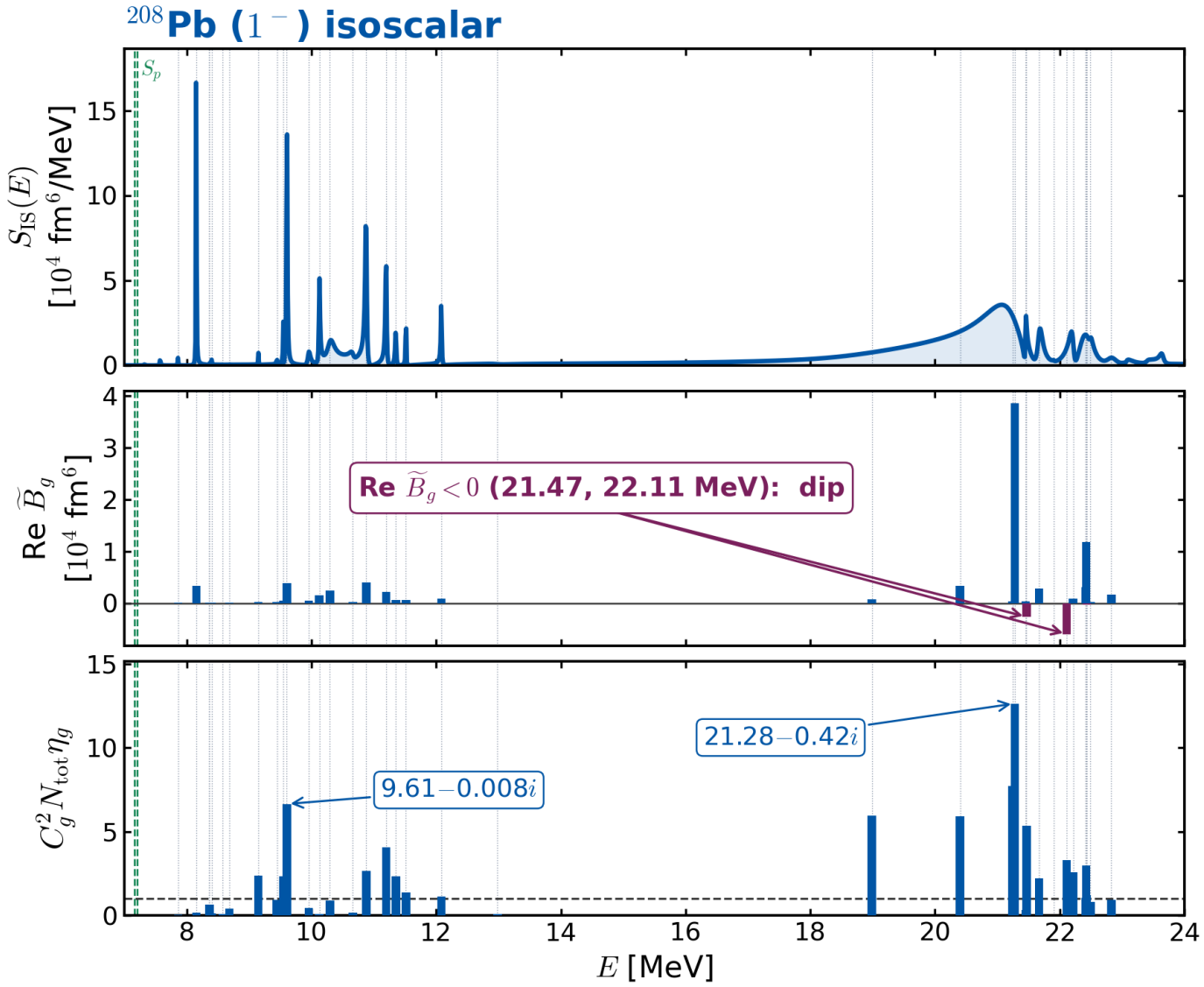
Participation η_{IV} vs. Coherence C_{IV}^2 (N_{tot} = 16); Dotted: Constant $C^2 N_{tot}\,\eta$





5-c. Numerical Results: ^{208}Pb ($J^\pi = 1^-$) Collectivity Decomposition: Isoscalar

Tallest IS Peak 8.15 MeV ($C_g^2 N_{tot} \eta_g = 0.18$) Is Not the Most Collective Pole; ISGDR 21.28 – 0.42i MeV ($C_g^2 N_{tot} \eta_g = 12.6$)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^2 = -2\pi i\, \mathrm{Res}\left[S_F, E_g\right]$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE\; 2\,\mathrm{Re}\left[\frac{\mathrm{Res}\left[S_F, E_g\right]}{E - E_g}\right]$$

area of the pole terms of S_F at E_g and E_g^* on the real axis; < 0: the pole lowers S_F (dip)

Bottom panel: collectivity measure

$$C_g^2 N_{\mathrm{tot}} \eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

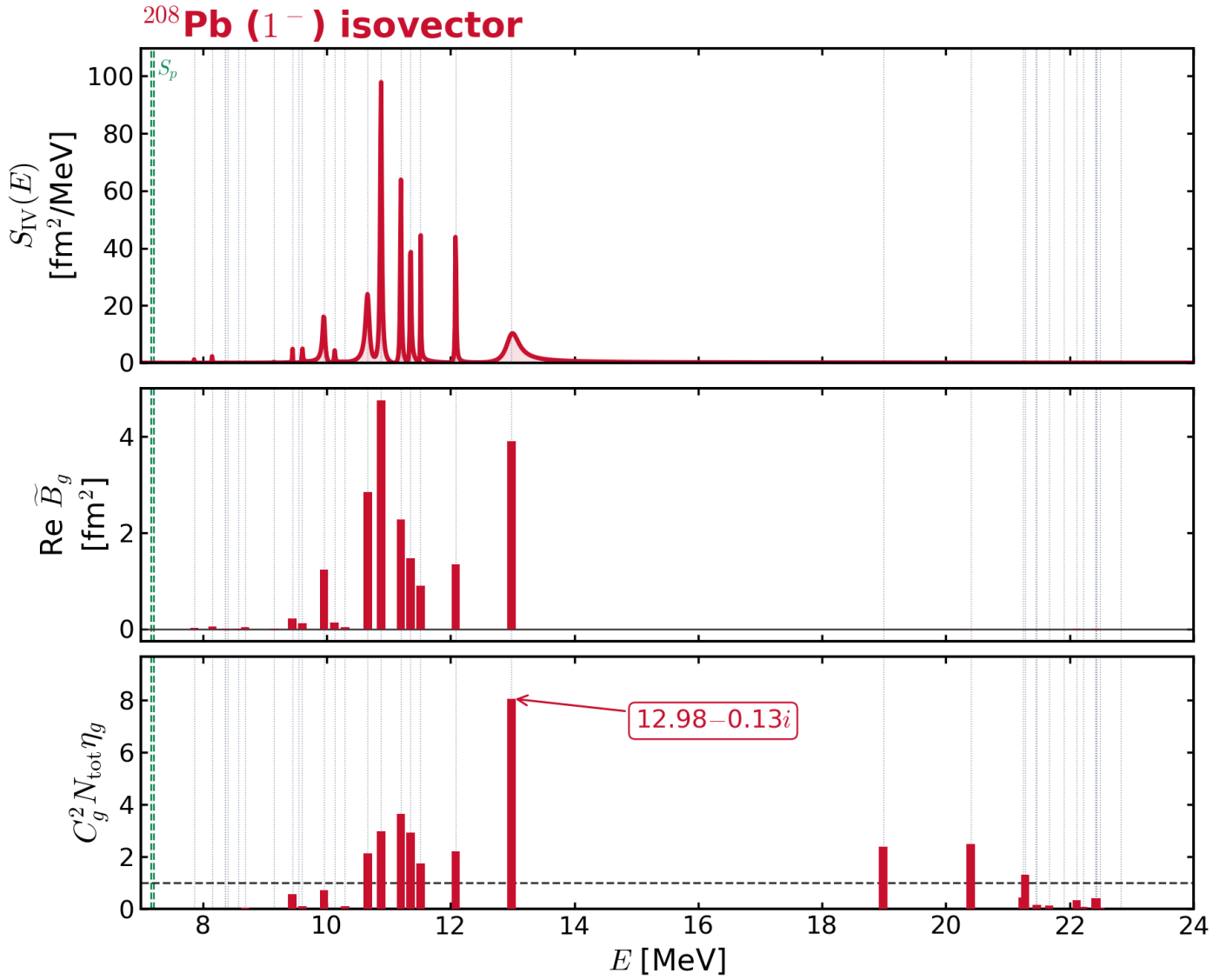
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\mathrm{tot}} \eta_g \textstyle\sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\mathrm{tot}} \eta_g$

5-c. Numerical Results: ^{208}Pb ($J^\pi = 1^-$) Collectivity Decomposition: Isovector

Tallest IV Peak 10.88 MeV ($C_g^2 N_{tot} \eta_g = 3.0$) Is Not the Most Collective Pole; IVGDR 12.98 – 0.13i MeV ($C_g^2 N_{tot} \eta_g = 8.1$)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^2 = -2\pi i\, \mathrm{Res}\left[S_F, E_g\right]$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE\; 2\,\mathrm{Re}\left[\frac{\mathrm{Res}\left[S_F, E_g\right]}{E - E_g}\right]$$

area of the pole terms of S_F at E_g and E_g^* on the real axis; < 0: the pole lowers S_F (dip)

Bottom panel: collectivity measure

$$C_g^2 N_{\mathrm{tot}} \eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

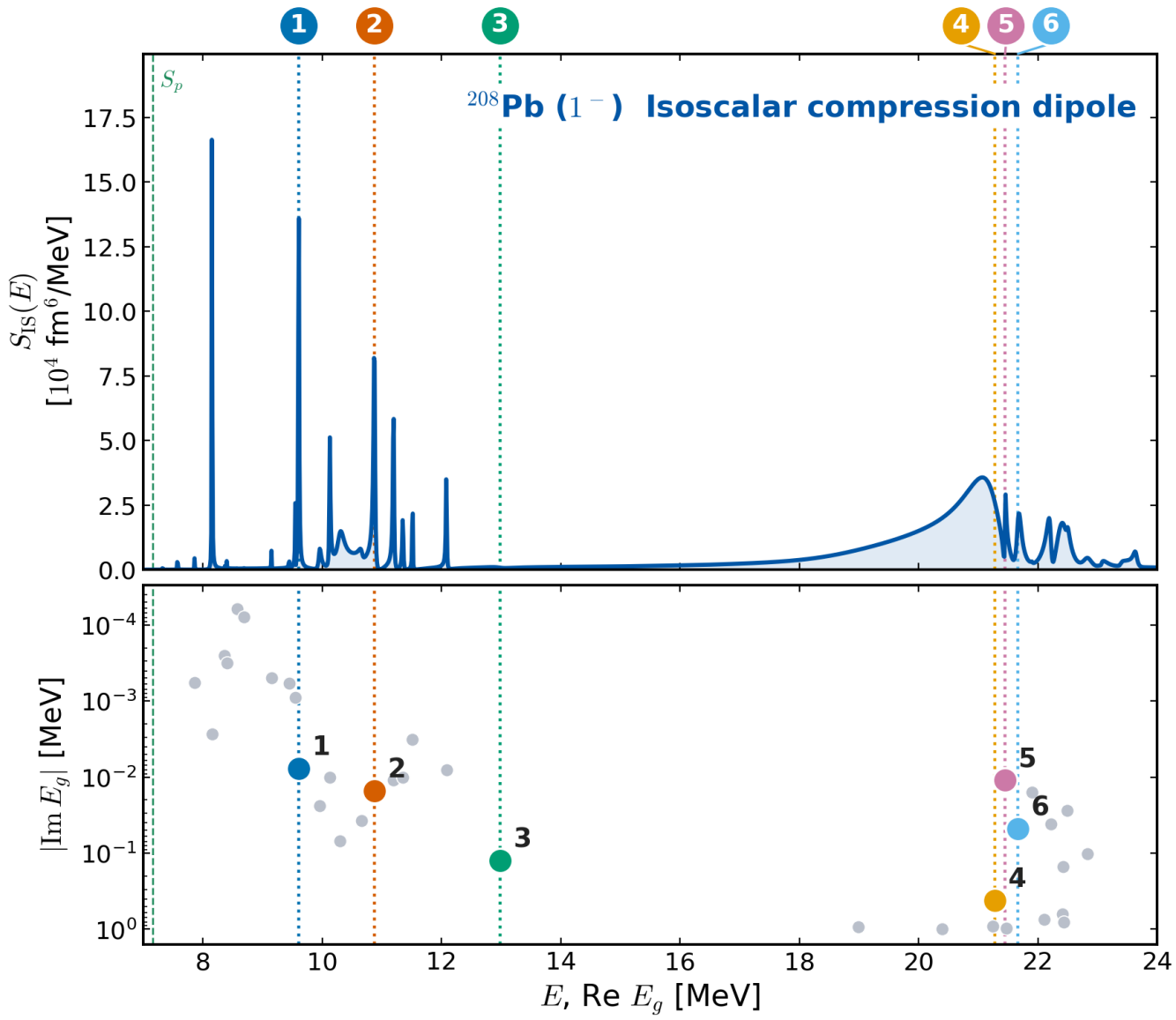
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

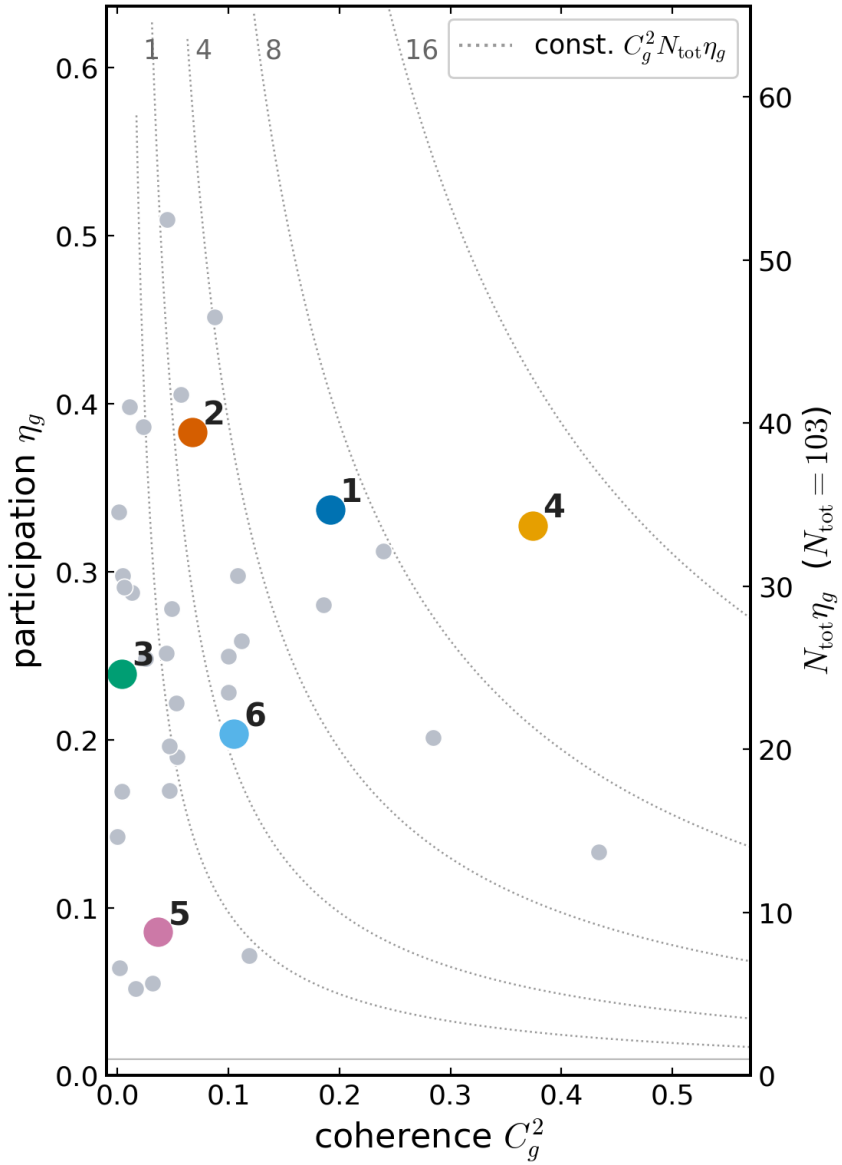
$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\mathrm{tot}} \eta_g \textstyle\sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\mathrm{tot}} \eta_g$

5-c. Collectivity Indicator Correlation: ^{208}Pb ($J^{\pi} = 1^{-}$) Isoscalar (IS)

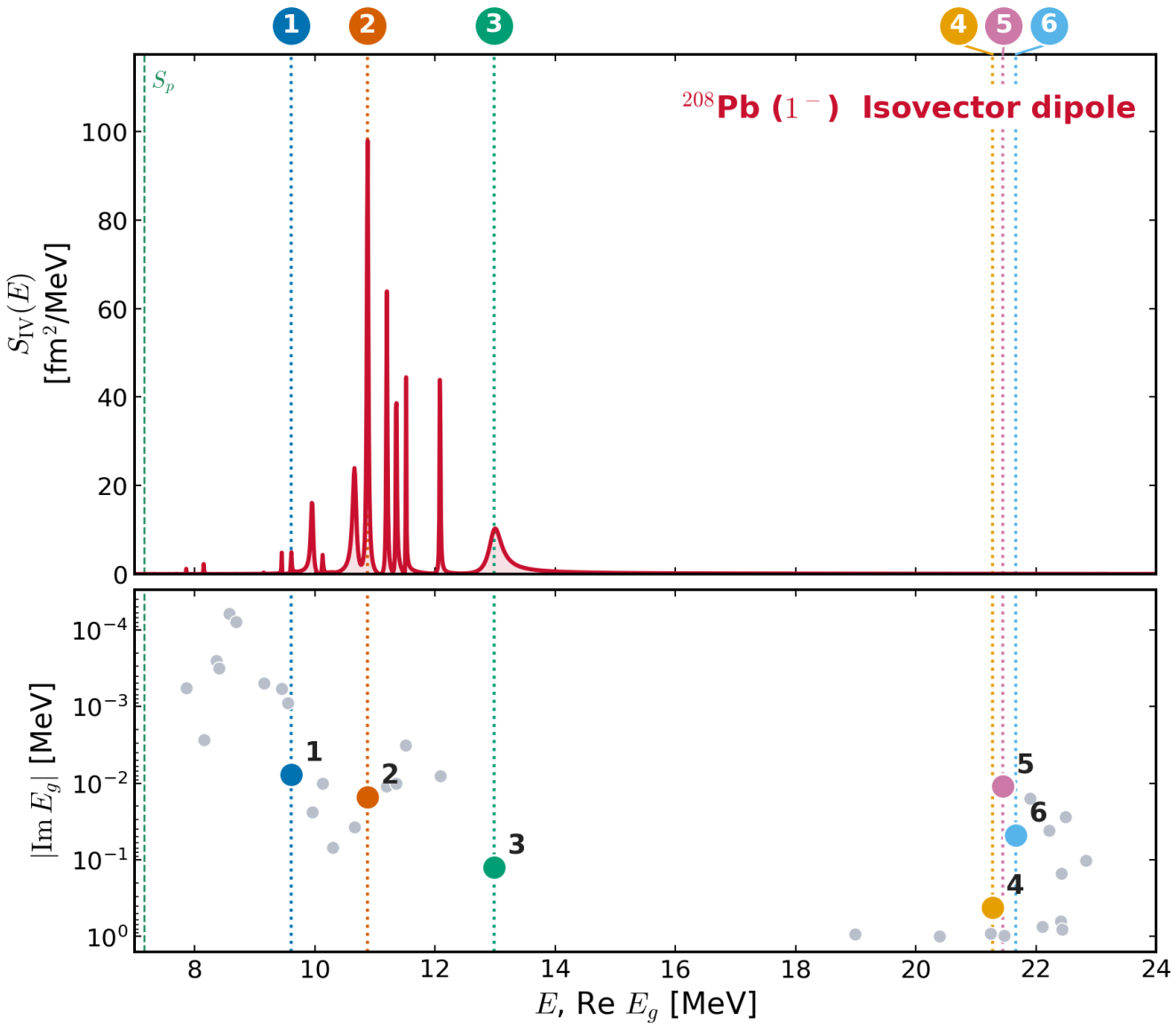
Participation η_{IS} vs. Coherence C_{IS}^2 (N_{tot} = 103); Dotted: Constant $C^2N_{tot}\,\eta$

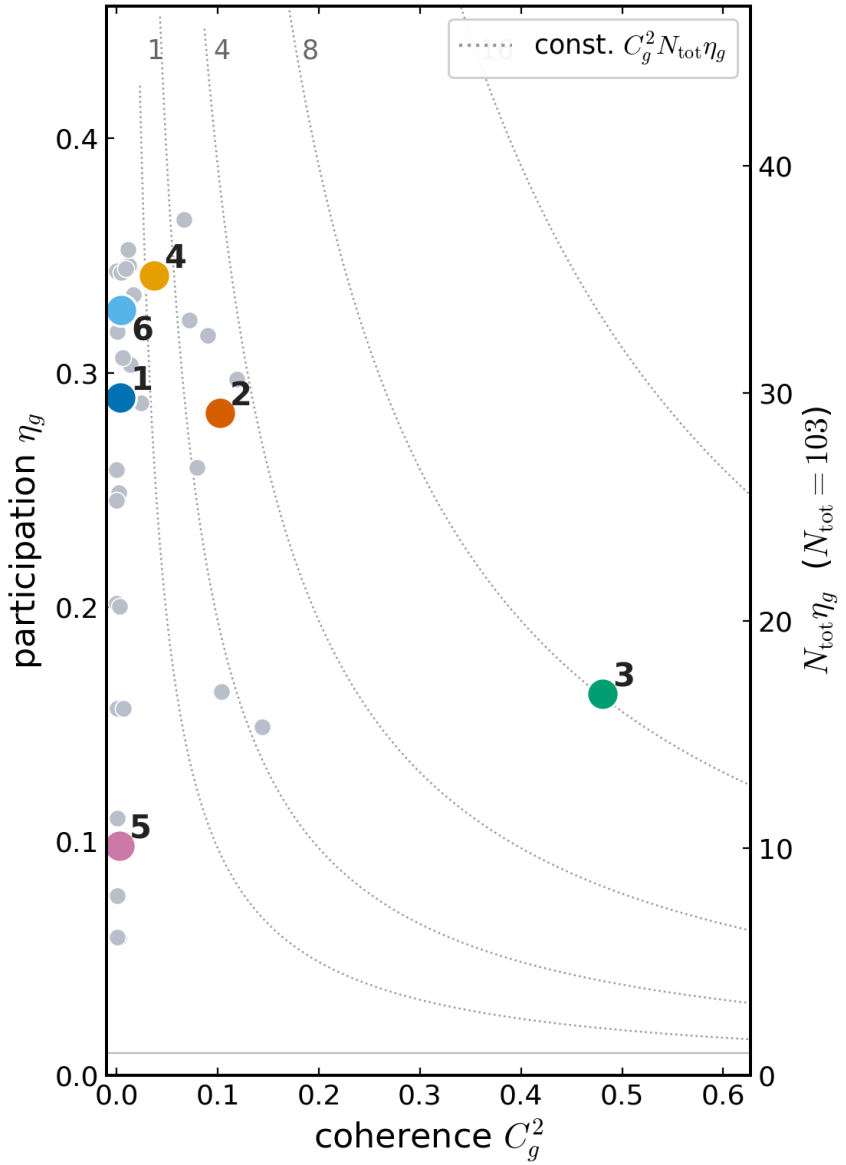




5-c. Collectivity Indicator Correlation: ^{208}Pb ($J^\pi = 1^-$) Isovector (IV)

Participation η_{IV} vs. Coherence C_{IV}^2 (N_{tot} = 103); Dotted: Constant $C^2 N_{tot}\,\eta$





6. Summary

✓Multichannel Jost functions

- Resonances are poles of the S-matrix built from the Jost functions; the HFB and RPA Jost functions locate them on the complex energy plane **PRC 99, 054607 (2019)**; **PRC 107, 024303 (2023)**

✓Analysis of resonances with the S-matrix

- **Only the open channels carry the strength**: the poles of the open-channel S-matrix appear directly in the strength function
- Eigenphase-shift decomposition **PRC 109, 054304 (2024)** and pole expansion by the Mittag-Leffler theorem **PRC 111, 054323 (2025)**

✓Takagi factorization of the S-matrix residue

- Collectivity of each pole **arXiv:2604.02237 (2026), accepted in PRC**: **the tallest peak is not always the most collective state**, and **non-collective poles can make dips**
- The same analysis works for the heavy nucleus ^{208}Pb

✓In progress

- Fano formula derived from the pole term of the transition amplitude: understanding the peaks and dips through the interference of each pole with the background


Thank you very much for your attention!!
* This is the view from our office in Da Nang
OPPO F11 Pro

Backup files

5-a. Takagi Factorization at Resonance Poles

Relation between Open-Channel Residue Res[S_{cc}] and Full Residue Res[S] at E_g = Re E_g – (i/2)Γ_g

1. Open-Channel S-Matrix Residue Res[S_{cc}, E_g] (Rank-1 Takagi Decomposition)

arXiv:2604.02237 (2026), accepted in PRC

$$\mathrm{Res}\left[\boldsymbol{\mathcal{S}}_{cc},\, E_g\right] = \sigma_g^{(c)}\, \vec{\boldsymbol{\omega}}_g^{(c)} \vec{\boldsymbol{\omega}}_g^{(c)\mathsf{T}}$$

$$\mathrm{rank}\left(\mathrm{Res}[\boldsymbol{\mathcal{S}}_{cc},\, E_g]\right) = 1 \implies \boldsymbol{\Sigma} = \mathrm{diag}\left(\sigma_g^{(c)},\, 0,\, \ldots,\, 0\right)$$

$\vec{\boldsymbol{\omega}}_g^{(c)} \in \mathbb{C}^{n_c}$: Normalized open Takagi vector $\quad (\|\vec{\boldsymbol{\omega}}_g^{(c)}\| = 1)$

$$\boldsymbol{\mathcal{S}}(E) = \begin{pmatrix} \boldsymbol{\mathcal{S}}_{cc} & \boldsymbol{\mathcal{S}}_{cd} \\ \boldsymbol{\mathcal{S}}_{dc} & \boldsymbol{\mathcal{S}}_{dd} \end{pmatrix}$$

- **Open channels** (c): outgoing waves: **decay into the continuum**
- **Closed channels** (d): exponentially decaying, no asymptotic flux
  (backward channels; forward channels below threshold)

$\boldsymbol{\mathcal{S}}_{cc} \in \mathbb{C}^{n_c \times n_c}$: **open-channel** S**-matrix** (unitary on the RS interval)

2. Reconstructing Full S-Matrix Residue Res[S, E_g] via Jost Algebraic Extension

1. Algebraic Extension $\vec{q}_g$	2. Normalization $(\vec{\omega}_g, \sigma_g)$	3. Full S-Matrix Residue $\mathrm{Res}[\boldsymbol{\mathcal{S}}, E_g]$

$$\vec{\boldsymbol{q}}_g \equiv \begin{pmatrix} \vec{\boldsymbol{\omega}}_g^{(c)} \\ -\boldsymbol{\mathcal{K}}_d^{\frac{1}{2}} \left(\boldsymbol{\mathcal{J}}_{dd}^{(+)}\right)^{-1} \boldsymbol{\mathcal{J}}_{dc}^{(+)} \boldsymbol{\mathcal{K}}_c^{-\frac{1}{2}}\, \vec{\boldsymbol{\omega}}_g^{(c)} \end{pmatrix}$$

Closed-channel part fixed by the open-channel $\vec{\boldsymbol{\omega}}_g^{(c)}$

$$\vec{\boldsymbol{\omega}}_g = \frac{\vec{\boldsymbol{q}}_g}{\|\vec{\boldsymbol{q}}_g\|}, \qquad \sigma_g = \sigma_g^{(c)} \|\vec{\boldsymbol{q}}_g\|^2$$

Normalized vector $(\|\vec{\boldsymbol{\omega}}_g\| = 1)$ and σ_g

$$\mathrm{Res}\left[\boldsymbol{\mathcal{S}},\, E_g\right] = \sigma_g\, \vec{\boldsymbol{\omega}}_g \vec{\boldsymbol{\omega}}_g^{\mathsf{T}} = \sigma_g^{(c)} \vec{\boldsymbol{q}}_g \vec{\boldsymbol{q}}_g^{\mathsf{T}}$$

Rank-1 residue in the full $2N_{\mathrm{tot}}$ channel space

5-a. Residue Width, Partial Widths & Branching Ratios

Pole Term of the Open-Channel S-Matrix and Decay Widths from the Takagi Vector

1. Pole Term of S_{cc} & Residue Width $\sigma_g^{(c)}$ vs. Pole Width Γ_g

arXiv:2604.02237 (2026), accepted in PRC

$$\boldsymbol{\mathcal{S}}_{cc}(E) = \boldsymbol{\mathcal{S}}_{\mathrm{B},g}(E) + \frac{\sigma_g^{(c)}\,\vec{\boldsymbol{\omega}}_g^{(c)}\vec{\boldsymbol{\omega}}_g^{(c)\mathsf{T}}}{E - E_g}$$

$$E_g = \mathrm{Re}\,E_g - \frac{i}{2}\Gamma_g, \qquad \|\vec{\boldsymbol{\omega}}_g^{(c)}\| = 1$$

$\boldsymbol{\mathcal{S}}_{\mathrm{B},g}(E)$: everything except the pole term of g (including the other poles), analytic at E_g and E_g^*

$$\frac{\sigma_g^{(c)}}{\Gamma_g} \simeq 1 - \frac{\Gamma_g}{2}\,\mathrm{Re}\left[\vec{\boldsymbol{\omega}}_g^{(c)\dagger}\frac{d\boldsymbol{\mathcal{S}}_{\mathrm{B},g}}{dE}(\mathrm{Re}\,E_g)\,\vec{\boldsymbol{\omega}}_g^{(c)*}\right]$$

- $\sigma_g^{(c)} = \Gamma_g$: $\boldsymbol{\mathcal{S}}_{\mathrm{B},g}$ flat across the pole (narrow, isolated pole)
- $\sigma_g^{(c)} \neq \Gamma_g$: energy dependence of $\boldsymbol{\mathcal{S}}_{\mathrm{B},g}$ (barriers, other poles)
- $\sigma_g^{(c)} < \Gamma_g$ is **not** a loss of flux to closed channels

2. Resonance Observables via Takagi Factorization

1. Residue Width

$$\sigma_g^{(c)} = \sum_{i=1}^{n_c} \Gamma_i$$

$\sigma_g^{(c)} \to \Gamma_g$ for a narrow isolated pole

2. Partial Widths

$$\Gamma_i = \sigma_g^{(c)}\,|\omega_{g,i}^{(c)}|^2$$

open channel i (particle $l_p j_p$ + hole h)

3. Branching Ratios

$$\mathrm{br}_i \equiv |\omega_{g,i}^{(c)}|^2 = \frac{\Gamma_i}{\sigma_g^{(c)}}, \qquad \sum_i \mathrm{br}_i = 1$$

absolute: $\mathrm{BR}_i \equiv \Gamma_i/\Gamma_g = (\sigma_g^{(c)}/\Gamma_g)\,\mathrm{br}_i$

5-a. Pole Term of the Transition Amplitude

Takagi Factorization Splits the Pole Term into the Excitation c_g (Depends on the External Field) and the Decay Channel Vector ω_g (Does Not)

Strength function and transition amplitude vector in the open channels c

$$S_F^{(\tau)}(E) = \left(\sum_q \int_0^\infty dr\, f_q^{(\tau)}(r)\, \vec{\boldsymbol{\rho}}_q^{(-)\mathsf{T}}(r;E) \right) \boldsymbol{\mathcal{S}}_{cc}(E) \left(\sum_{q'} \int_0^\infty dr'\, \vec{\boldsymbol{\rho}}_{q'}^{(-)}(r';E)\, f_{q'}^{(\tau)}(r') \right)$$

$$= \vec{\mathcal{F}}^{(\tau)\dagger}(E^*)\, \vec{\mathcal{F}}^{(\tau)}(E), \qquad \vec{\mathcal{F}}^{(\tau)}(E) \equiv \sqrt{\frac{2m}{\pi\hbar^2}}\, \boldsymbol{\mathcal{S}}_{cc}(E) \sum_q \int dr\, \boldsymbol{\Psi}_{F,c}^{(+)*\mathsf{T}}(r;E^*)\, \boldsymbol{\xi}_q(r)\, f_q^{(\tau)}(r)$$

Pole term of the transition amplitude vector ($\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)$: everything except the pole term of g)

$$\vec{\mathcal{F}}^{(\tau)}(E) = \vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E) + \frac{c_g^{(\tau)}\, \vec{\boldsymbol{\omega}}_g^{(c)}}{E - E_g}$$

$c_g^{(\tau)}$: excitation by the external field $f^{(\tau)}$

$$c_g^{(\tau)} = \sqrt{\frac{i\sigma_g^{(c)}}{2\pi}}\, \mathcal{F}_g^{(\tau)}$$

$$\mathcal{F}_g^{(\tau)} = \sum_q \int dr\, \vec{\boldsymbol{\psi}}_g^{\mathsf{T}}(r)\, \boldsymbol{\xi}_q(r)\, f_q^{(\tau)}(r) = \sum_i F_{g,i}^{(\tau)}$$

$\Rightarrow$ depends on the external field (IS / IV)

$\vec{\omega}_g^{(c)}$: decay channel vector (Takagi vector)

$$\vec{\boldsymbol{\psi}}_g(r) \equiv \sqrt{\frac{4m\sigma_g^{(c)}}{i\hbar^2}}\, \boldsymbol{\Psi}_{F,c}^{(+)*}(r;E_g^*)\, \vec{\boldsymbol{\omega}}_g^{(c)}$$

pole state of g

$\Rightarrow$ property of the pole: independent of the field

$\rightarrow$ branching ratios $\mathrm{br}_i = |\omega_{g,i}^{(c)}|^2$ $\rightarrow$ collectivity from $F_{g,i}^{(\tau)}$ $\rightarrow$ Fano analysis: interference with $\vec{\mathcal{F}}_{\rm bg}^{(\tau)}$

5-b. Transition Amplitudes at the Pole & Collectivity Indicators

Pole State ψ_g, Channel Amplitudes $F_{g,i}$ and the Complex Transition Probability $\tilde{B}_g$

1. Pole State, Channel Amplitudes & Collectivity Indicators

arXiv:2604.02237 (2026), accepted in PRC

$$\vec{\psi}_g(r) \equiv \sqrt{\frac{4m\sigma_g^{(c)}}{i\hbar^2}}\, \mathbf{\Psi}_{F,c}^{(+)*}(r; E_g^*)\, \vec{\boldsymbol{\omega}}_g^{(c)} \quad \left(\vec{\boldsymbol{\omega}}_g^{(c)} : \text{Takagi vector}\right)$$

$$\rho_{q,i}^{(g)}(r) \equiv \vec{\psi}_{g,i}^{\mathsf{T}}(r)\, \boldsymbol{\xi}_{q,i}(r) \quad \left(\boldsymbol{\xi}_q(r) : \text{hole-state vector}\right)$$

$$F_{g,i}^{(\tau)} \equiv \int_0^\infty dr\, f_q^{(\tau)}(r)\, \rho_{q,i}^{(g)}(r), \quad \mathcal{F}_g^{(\tau)} \equiv \sum_{i=1}^{N_{\rm tot}} F_{g,i}^{(\tau)} = |\mathcal{F}_g^{(\tau)}|\, e^{i\Theta_g^{(\tau)}}$$

$$C_g^{(\tau)} \equiv \frac{|\mathcal{F}_g^{(\tau)}|}{\sum_{i=1}^{N_{\rm tot}} |F_{g,i}^{(\tau)}|} \qquad 0 \le C_g^{(\tau)} \le 1 \qquad \textbf{(Coherence Index)}$$

$$\eta_g^{(\tau)} \equiv \frac{\left(\sum_{i=1}^{N_{\rm tot}} |F_{g,i}^{(\tau)}|\right)^2}{N_{\rm tot} \sum_{i=1}^{N_{\rm tot}} |F_{g,i}^{(\tau)}|^2} \qquad \frac{1}{N_{\rm tot}} \le \eta_g^{(\tau)} \le 1 \qquad \textbf{(Participation Ratio)}$$

$$N_{\rm eff} \equiv N_{\rm tot}\eta_g^{(\tau)} \qquad 1 \le N_{\rm eff} \le N_{\rm tot} \qquad \textbf{(Effective Channels)}$$

2. Complex Transition Probability $\tilde{B}_g$ & the Strength Function $S_F(E)$

$$S_F^{(\tau)}(E) = \sum_g \underbrace{C_g^{(\tau)2} N_{\rm tot}\eta_g^{(\tau)}}_{\textbf{Collectivity Enhancement}} \times \underbrace{\left(\sum_{i=1}^{N_{\rm tot}} |F_{g,i}^{(\tau)}|^2\right)}_{\textbf{Single-Particle Scale}} \times \underbrace{\left(-\frac{1}{\pi}\,\mathrm{Im}\left[\frac{e^{2i\Theta_g^{(\tau)}}}{E - E_g}\right]\right)}_{\textbf{Resonance Line Shape}} + S_{F,\rm bg}^{(\tau)}(E)$$

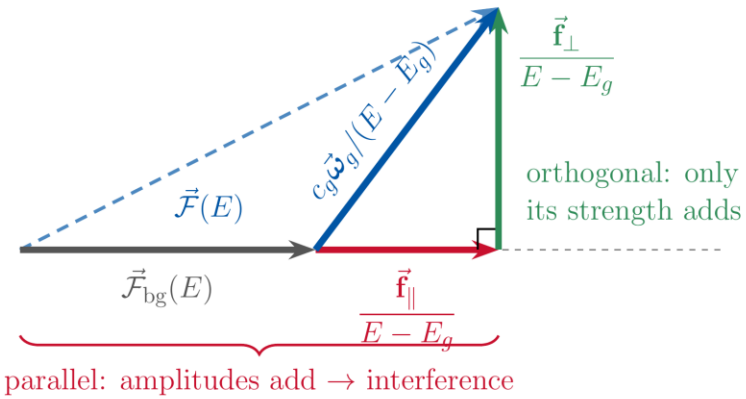
$$\widetilde{B}_g^{(\tau)} \equiv \mathrm{Res}\,[R_F, E_g] = \mathcal{F}_g^{(\tau)2} = \left[C_g^{(\tau)2} N_{\rm tot}\eta_g^{(\tau)}\right] \left(\sum_{i=1}^{N_{\rm tot}} |F_{g,i}^{(\tau)}|^2\right) e^{2i\Theta_g^{(\tau)}}$$

$$R_F(E) = \sum_{q,q'} \int_0^\infty dr \int_0^\infty dr'\, f_q^{(\tau)}(r)\, \boldsymbol{\xi}_q^{\mathsf{T}}(r)\, \boldsymbol{\mathcal{G}}(r, r'; E)\, \boldsymbol{\xi}_{q'}(r')\, f_{q'}^{(\tau)}(r')$$

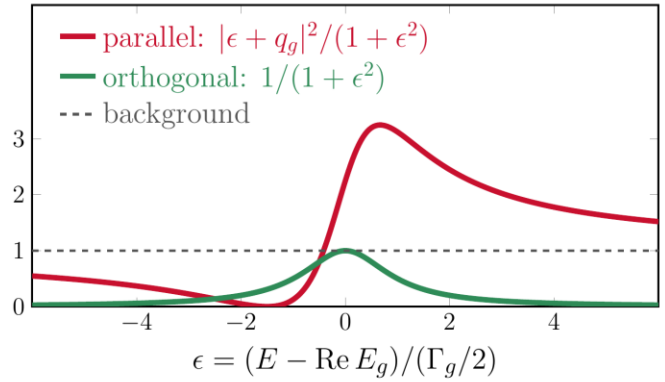
5-c. Fano Analysis: Essence of the Fano Formula

The Pole-Term Component Parallel to F_{bg} Interferes with F_{bg}; the Orthogonal Component Only Adds Its Strength (Breit–Wigner)

(a) split the pole term along $\vec{\mathcal{F}}_{\rm bg}$



(b) line shapes of the two parts ($q_g = 1.5$)



Split the pole term along $\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)$: $c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)} = \vec{\mathbf{f}}_\parallel(E) + \vec{\mathbf{f}}_\perp(E)$

$$\vec{\mathbf{f}}_\parallel(E) = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}}{\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}\,\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)$$

Pythagoras ($\vec{\mathbf{f}}_\perp \perp \vec{\mathcal{F}}_{\rm bg}^{(\tau)}$)

$$S_F^{(\tau)}(E) = \left\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)} + \frac{\vec{\mathbf{f}}_\parallel}{E-E_g}\right\|^2 + \left\|\frac{\vec{\mathbf{f}}_\perp}{E-E_g}\right\|^2$$

Parallel part (Fano's relation: defines q_g at each real E)

$$\vec{\mathcal{F}}_{\rm bg}^{(\tau)} + \frac{\vec{\mathbf{f}}_\parallel}{E-E_g} = \left(1 + \frac{q_g - i}{\epsilon + i}\right)\vec{\mathcal{F}}_{\rm bg}^{(\tau)} = \frac{\epsilon + q_g}{\epsilon + i}\,\vec{\mathcal{F}}_{\rm bg}^{(\tau)}$$

$$\Rightarrow \quad q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction of the pole term along $\vec{\mathcal{F}}_{\rm bg}^{(\tau)}$

$$p_g \equiv \frac{\|\vec{\mathbf{f}}_\parallel\|}{\|c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}\|}, \qquad 1 - p_g^2 = \frac{\|\vec{\mathbf{f}}_\perp\|^2}{\|c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}\|^2}$$

Fano formula

$$S_F^{(\tau)}(E) = \underbrace{\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}\|^2\,\frac{|\epsilon+q_g|^2}{1+\epsilon^2}}_{\text{background + interference + parallel pole part}} + \underbrace{(1-p_g^2)\,\frac{\|c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}\|^2}{|E-E_g|^2}}_{\text{non-interfering: Breit–Wigner}}$$

5-c. Fano Analysis: Interference Term and the Pole's Own Contribution

S_F = Background + Interference Term + Pole's Own Contribution; Their Ratio Im R_g + ε Re R_g Decides Which Builds the Structure

Strength function = background + interference term + the pole's own contribution

$$S_F^{(\tau)}(E) = \underbrace{\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}\|^2}_{\text{background}} + \underbrace{2\,\mathrm{Re}\,\frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}\, c_g^{(\tau)} \vec{\boldsymbol{\omega}}_g^{(c)}}{E - E_g}}_{\text{interference term}} + \underbrace{\frac{\|c_g^{(\tau)} \vec{\boldsymbol{\omega}}_g^{(c)}\|^2}{|E - E_g|^2}}_{\text{pole's own contribution}}$$

Their ratio at each E ($E - E_g = \frac{\Gamma_g}{2}(\epsilon + i)$)

$$\frac{\text{interference term}}{\text{pole's own contribution}} = \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g, \qquad R_g \equiv \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}\, c_g^{(\tau)} \vec{\boldsymbol{\omega}}_g^{(c)}}{\|c_g^{(\tau)} \vec{\boldsymbol{\omega}}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Reading

$|R_g| \ll 1$: the pole's own contribution (Breit–Wigner) dominates

$|R_g| > 1$: strong interference (within $|\epsilon| \le 1$)

$\mathrm{Im}\, R_g < -1$: S_F below the background at $\epsilon = 0$

$\mathrm{Re}\, R_g > 0$ (< 0): the side $\epsilon > 0$ ($\epsilon < 0$) is raised

$R_g = 0 \iff p_g = 0$: no interference

Example: ^{16}O 2^+ (IS), $E_g = 28.495 - 0.564i$ MeV

$R_g = 0.61 - 1.80i$, $|R_g| = 1.9 > 1$

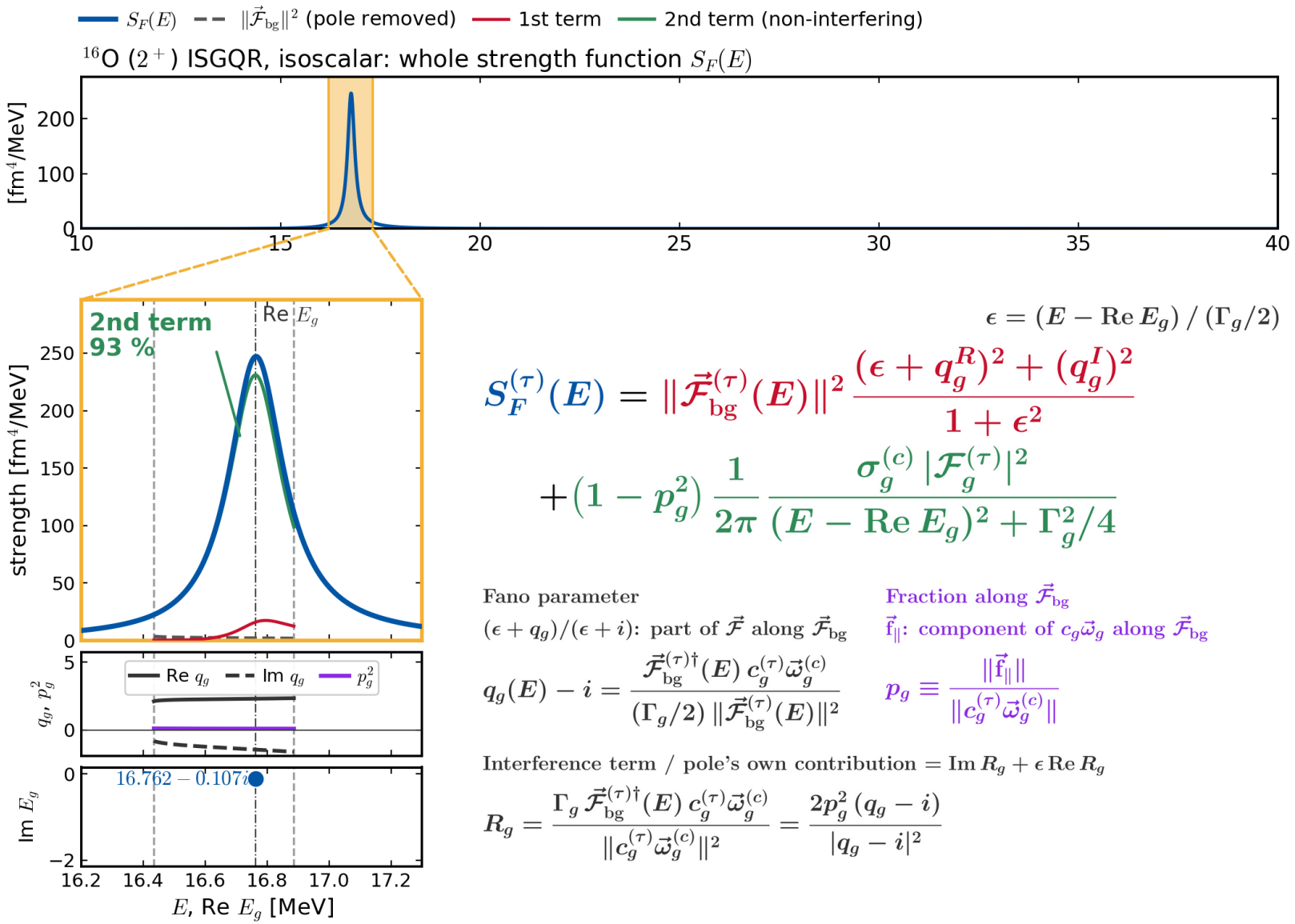
at $\epsilon = 0$: interference $= -1.80\times$ pole's own

$\rightarrow$ S_F below the background

$\mathrm{Re}\, R_g > 0$: the side $\epsilon < 0$ is lowered $\rightarrow$ dip at 28.24 MeV

5-c. Fano Analysis: ^{16}O ($J^\pi = 2^+$) Isoscalar Giant Quadrupole Resonance

Most Collective IS Pole, but $|R_g| \ll 1$: the Non-Interfering Breit–Wigner Term Dominates S_F at Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\, E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$16.762 - 0.107i$ MeV

Collectivity (IS): collective

C_g^2	0.9088
$N_{\rm tot}\eta_g$	7.80
$C_g^2 N_{\rm tot}\eta_g$	**7.09**

At E = Re E_g ($\epsilon = 0$)

q_g	$2.31 - 1.44i$
p_g^2	0.10
R_g	$0.039 - 0.041i$
$\lvert R_g\rvert$	$0.057 \ll 1$

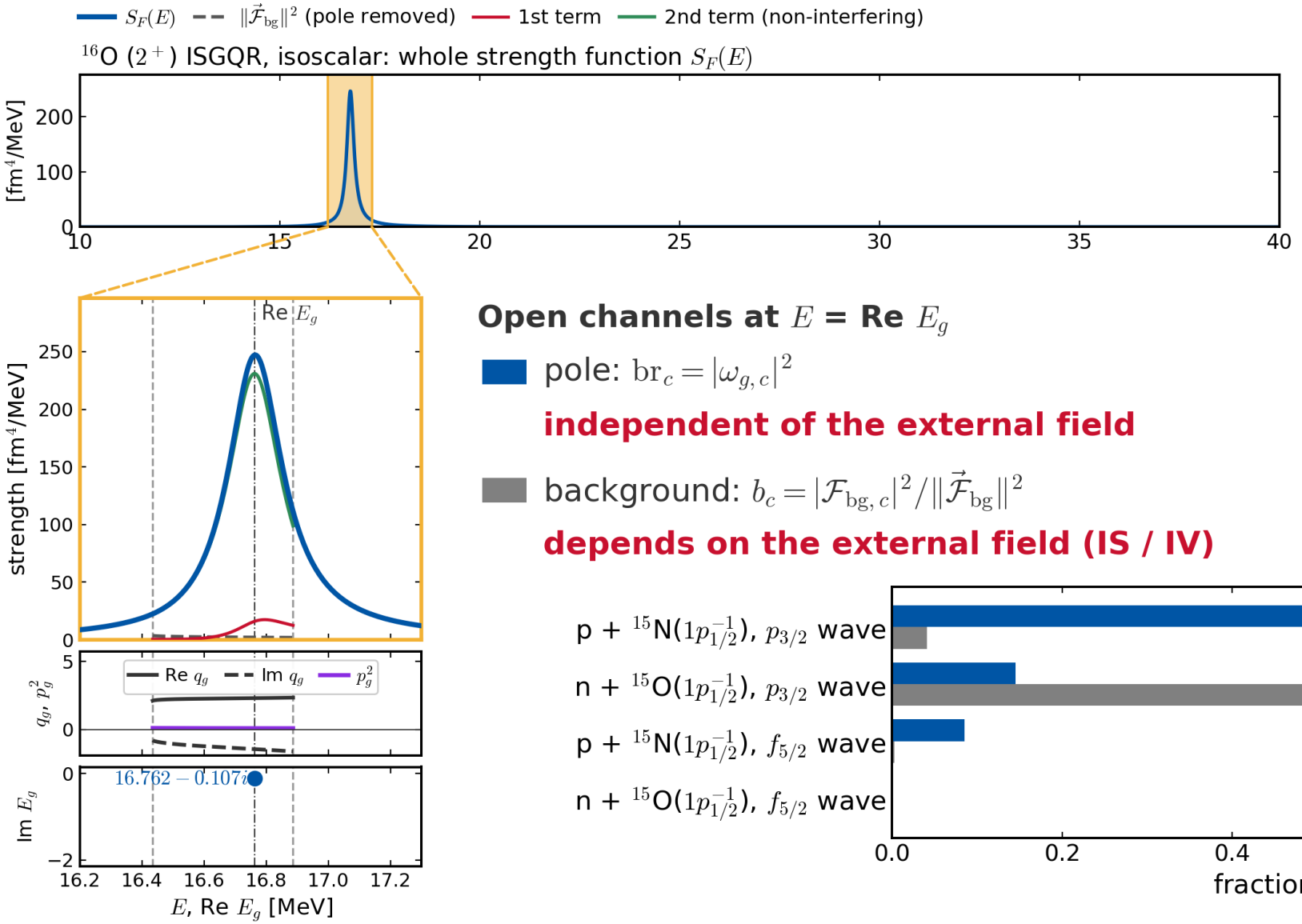
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak**

5-c. Branching Ratios: ^{16}O ($J^\pi = 2^+$) Isoscalar Giant Quadrupole Resonance

The Pole Decays Mainly into p + ^{15}N($1p_{1/2}^{-1}$), $p_{3/2}$ Wave; the IS Background Lies Mainly in n + ^{15}O($1p_{1/2}^{-1}$), $p_{3/2}$ Wave



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$16.762 - 0.107i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.04

Pole decays into (br_c)

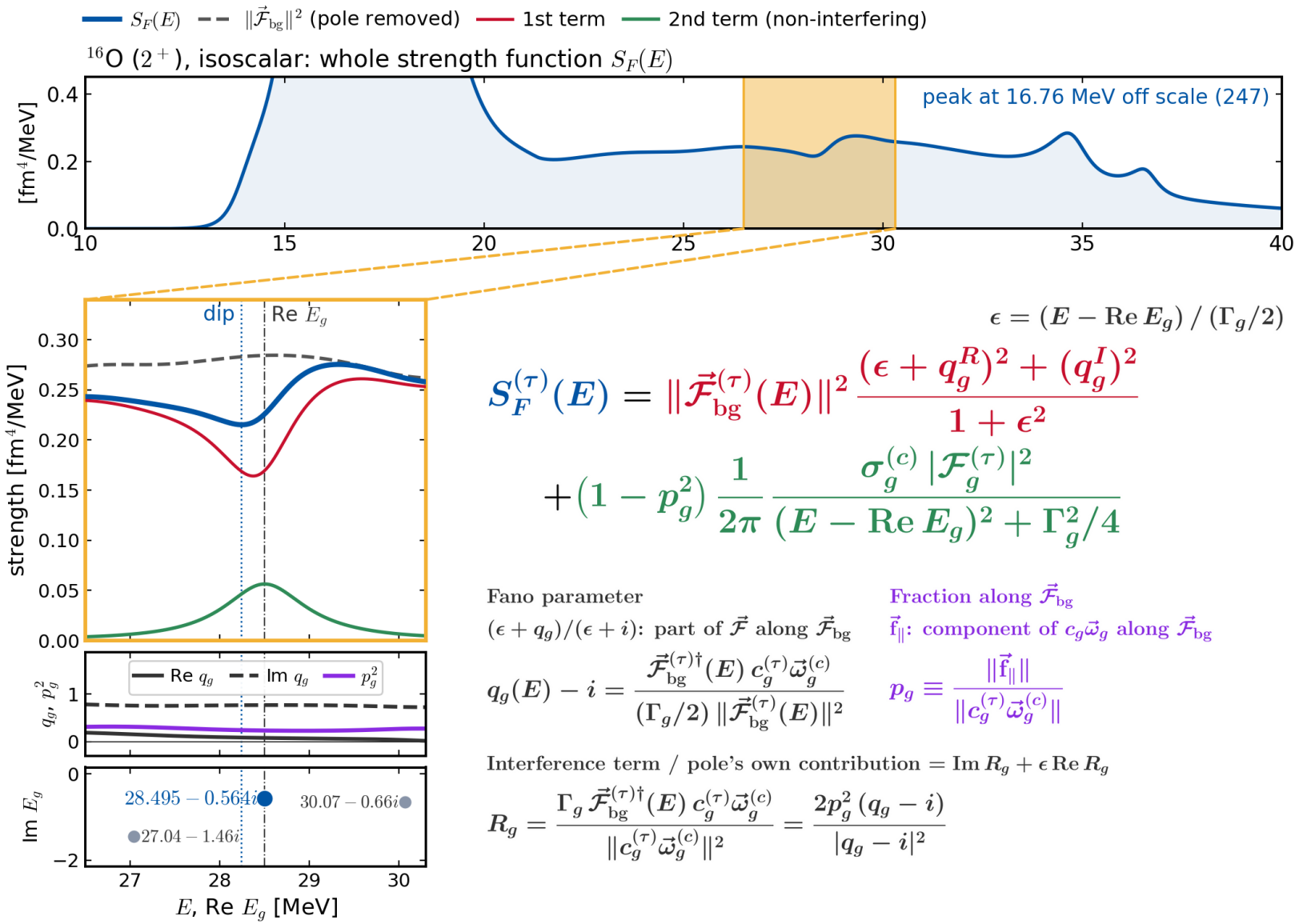
p + ^{15}N($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.77
n + ^{15}O($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.15

Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.95
p + ^{15}N($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.04

5-c. Fano Analysis: ^{16}O ($J^\pi = 2^+$) Isoscalar Dip at 28.24 MeV

Non-Collective Pole: $|R_g| > 1$ and Im $R_g < -1$ → S_F below the Background



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$28.495 - 0.564i$ MeV

Collectivity (IS): non-collective

C_g^2	0.0041
$N_{\rm tot}\eta_g$	5.68
$C_g^2 N_{\rm tot}\eta_g$	0.023

At E = Re E_g ($\epsilon = 0$)

q_g	$0.08 + 0.77i$
p_g^2	0.23
R_g	$0.61 - 1.80i$
$\|R_g\|$	1.90 > 1

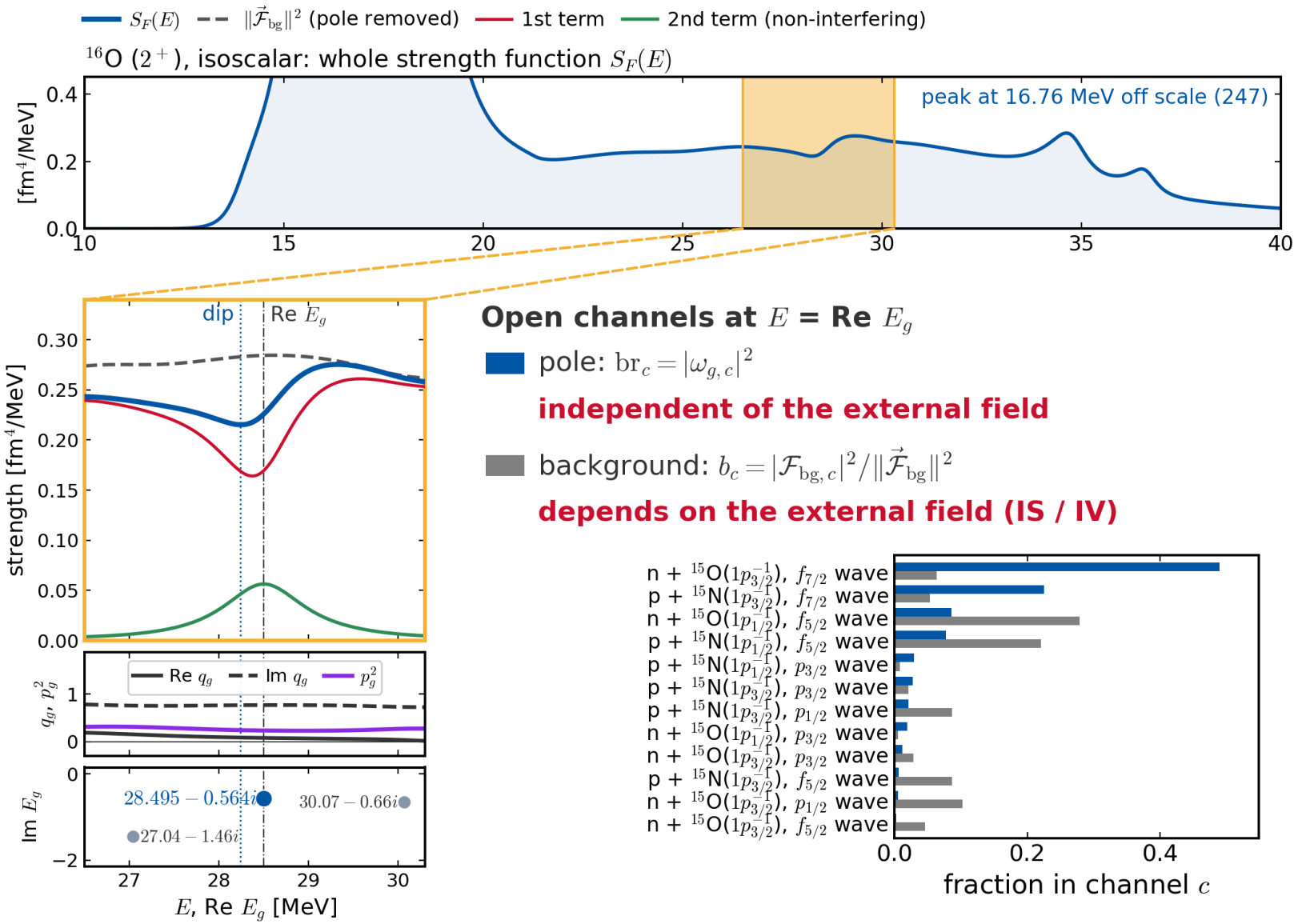
strong interference

Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ **dip (window-like)**

5-c. Branching Ratios: ^{16}O ($J^\pi = 2^+$) Isoscalar Dip at 28.24 MeV

The Pole Decays into Several Channels (Largest: n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ Wave); the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

- pole: $\mathrm{br}_c = |\omega_{g,c}|^2$
  independent of the external field
- background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$
  depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$28.495 - 0.564i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.10

Pole decays into (br_c)

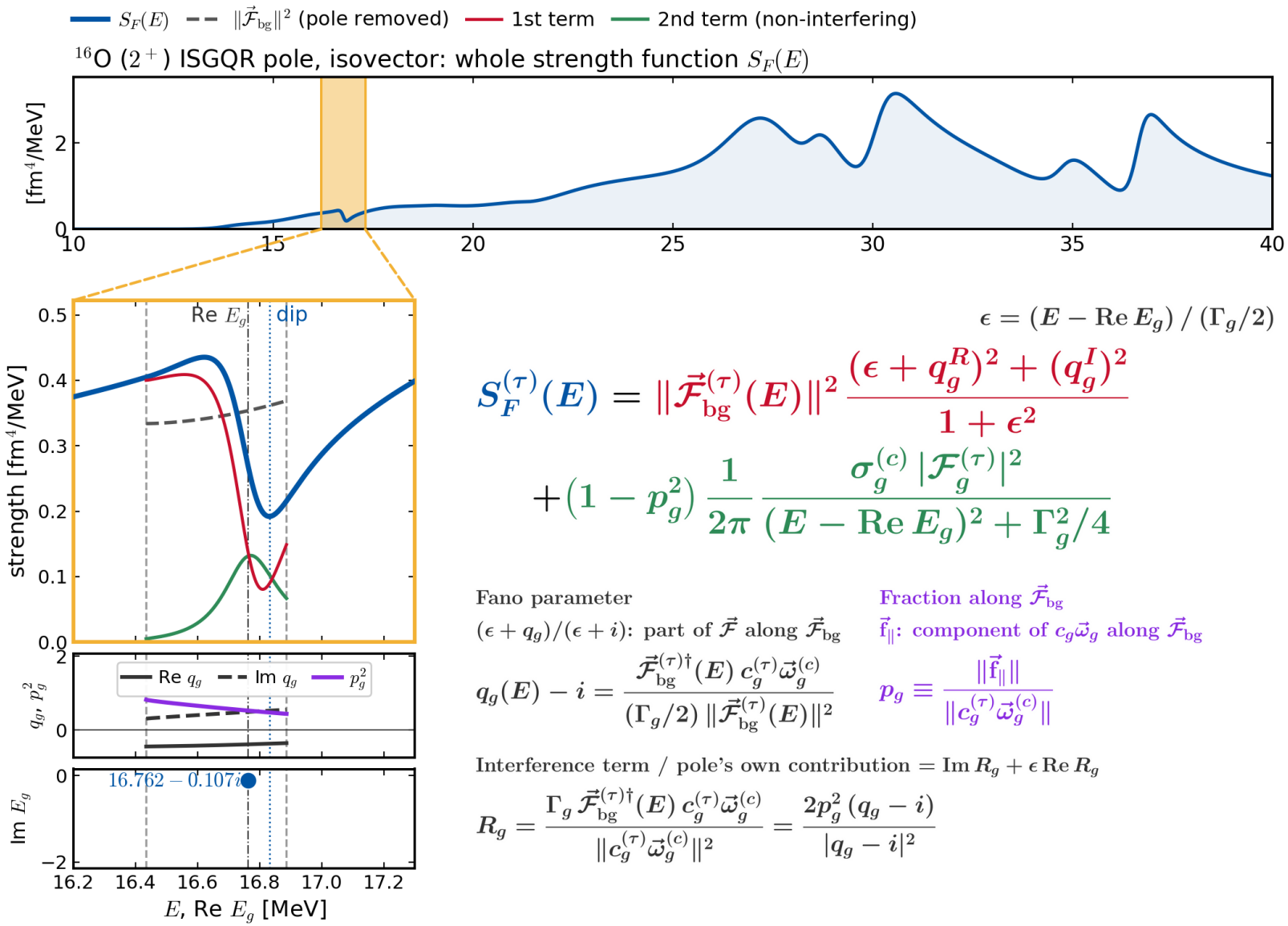
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.49
p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.23

Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.28
p + ^{15}N($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.22

5-c. Fano Analysis: ^{16}O ($J^\pi = 2^+$) ISGQR Pole in the Isovector Strength

Same Pole, Isovector Field (Non-Collective): $|R_g| > 1$ and Im $R_g < -1$ → S_F below the Background, Dip



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\frac{1}{2\pi}\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$16.762 - 0.107i$ MeV

Collectivity (IV): non-collective

C_g^2	0.0010
$N_{\rm tot}\eta_g$	7.80
$C_g^2 N_{\rm tot}\eta_g$	0.008

At E = Re E_g ($\epsilon = 0$)

q_g	-0.39 + 0.49i
p_g^2	0.53
R_g	-1.00 − 1.30i
$\|R_g\|$	1.64 > 1

strong interference

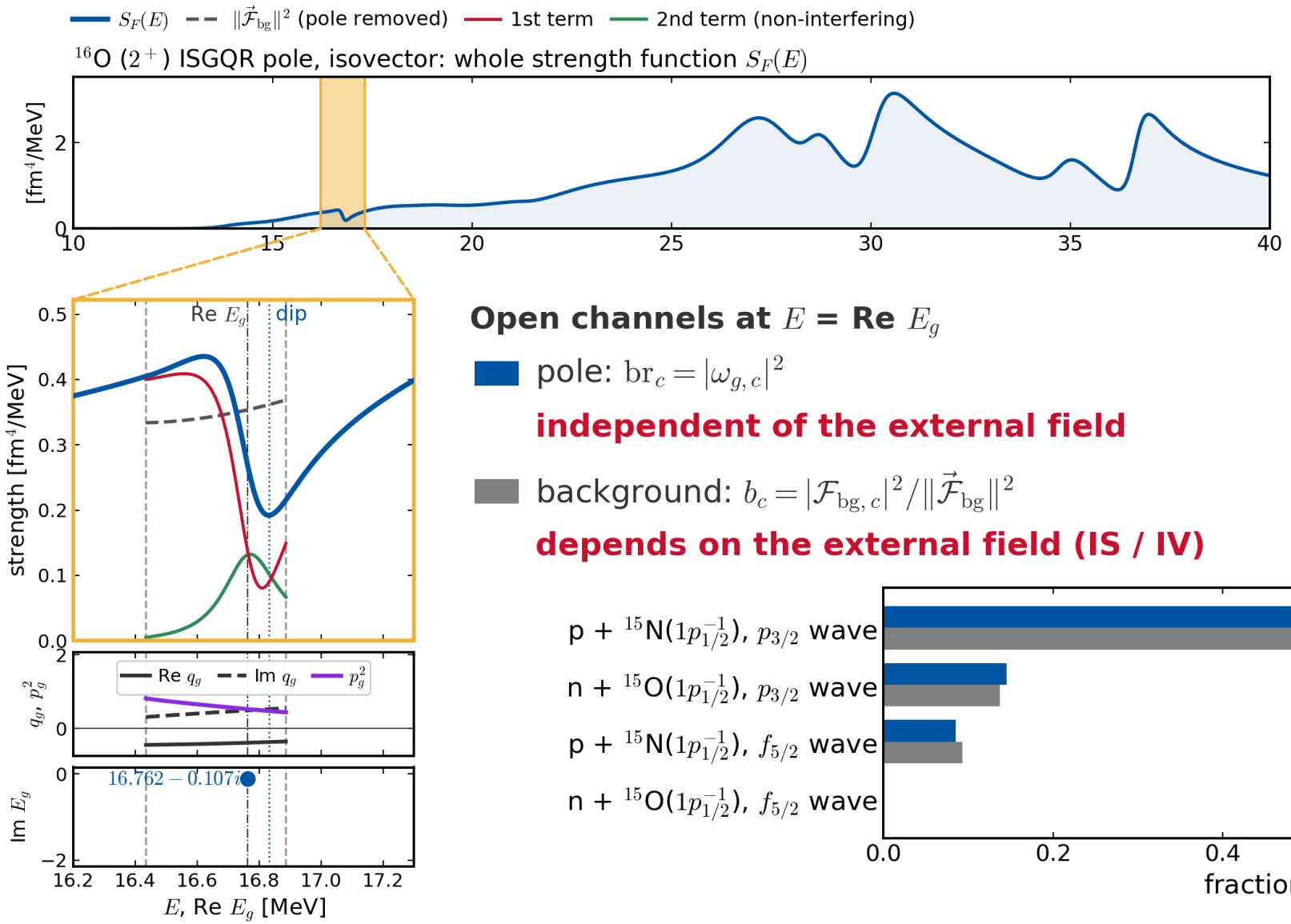
Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ **asymmetric Fano profile with a dip**

dip at 16.83 MeV

5-c. Branching Ratios: ^{16}O ($J^\pi = 2^+$) ISGQR Pole in the Isovector Strength

Same br_c as in the IS Field; the IV Background Lies Mainly in p + ^{15}N($1p_{1/2}^{-1}$), $p_{3/2}$ Wave, the Main Decay Channel of the Pole



Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$16.762 - 0.107i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.04

Pole decays into (br_c)

p + ^{15}N($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.77
n + ^{15}O($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.15

Background at Re E_g (b_c)

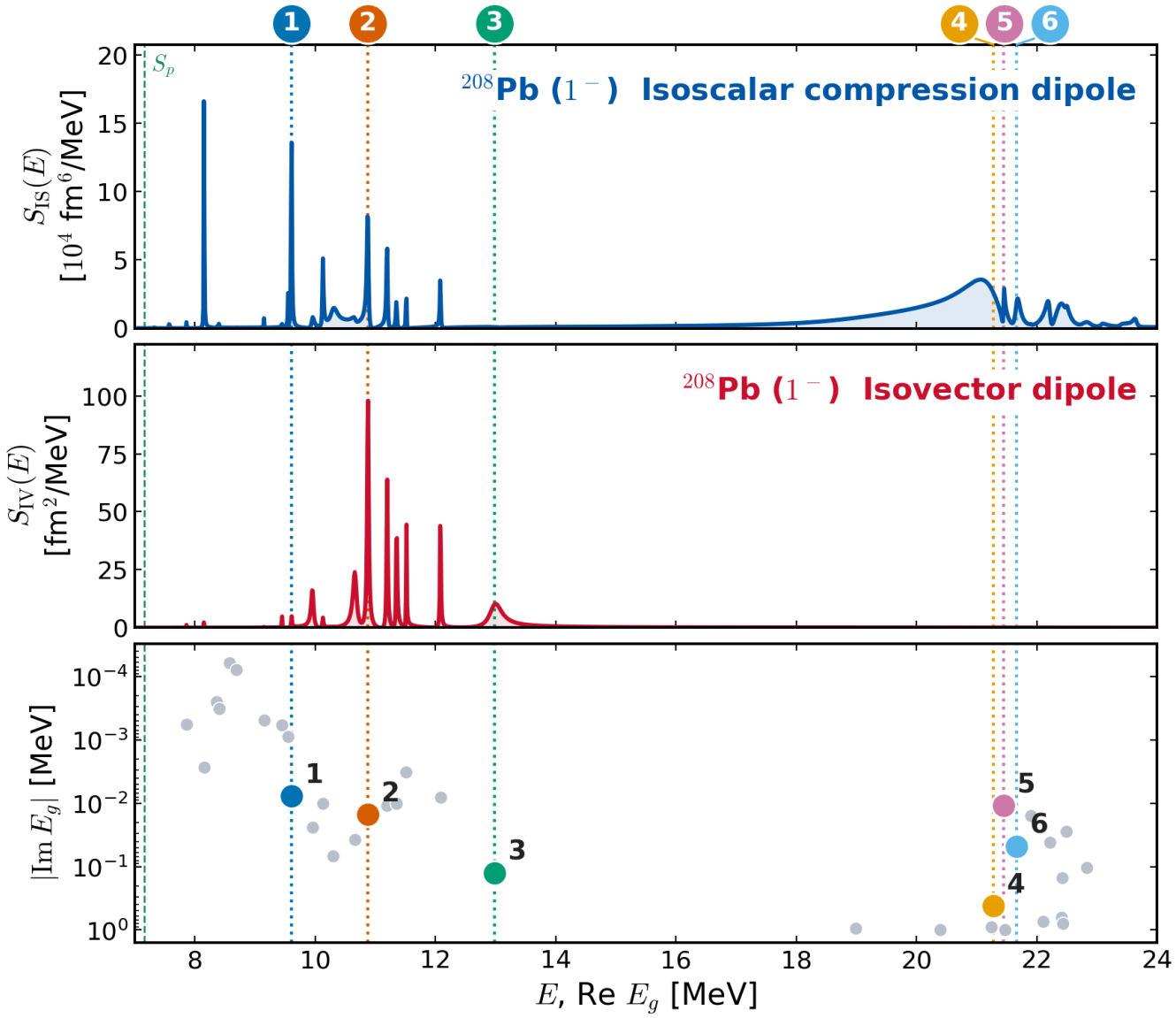
p + ^{15}N($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.77
n + ^{15}O($1p_{1/2}^{-1}$), $p_{3/2}$ wave	0.14

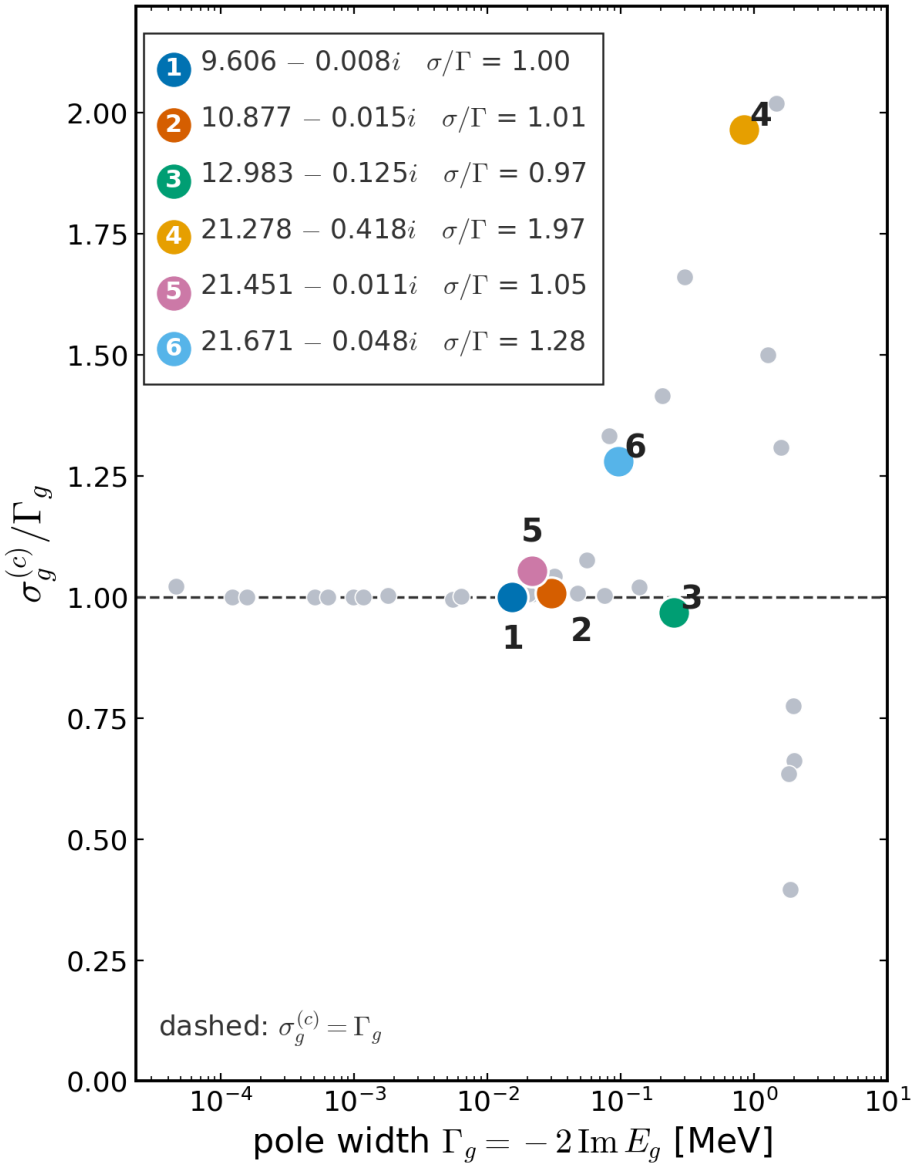
Open channels at E = Re E_g

- pole: $\mathrm{br}_c = |\omega_{g,c}|^2$
  independent of the external field
- background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$
  depends on the external field (IS / IV)

5-d. Numerical Results: ^{208}Pb ($J^\pi = 1^-$) Strength Functions & Resonance Poles

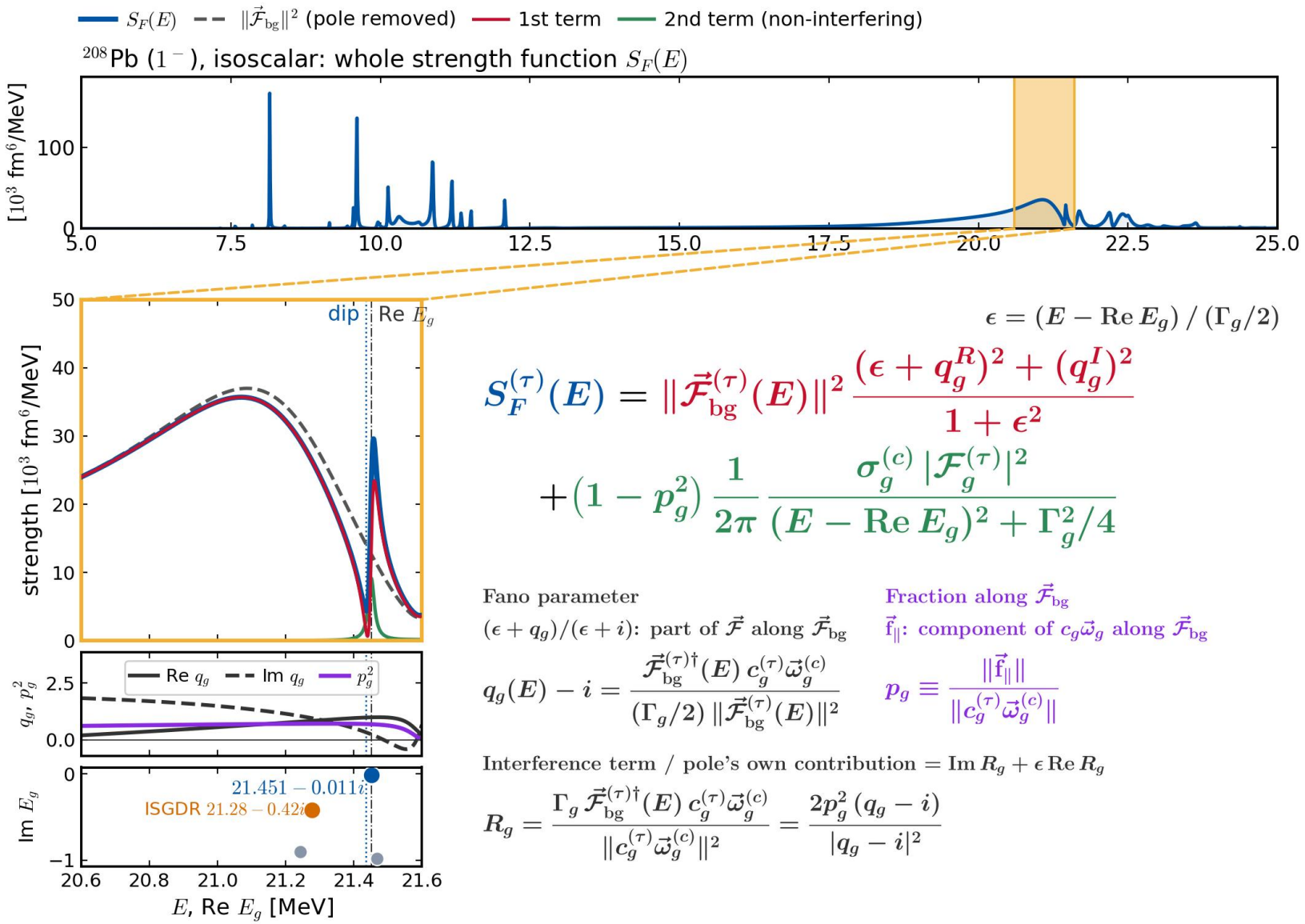
Peaks and the 36 S-Matrix Poles E_g; $\sigma_g^{(c)}/\Gamma_g \approx 1$ for All Poles below 13 MeV Including the IVGDR, 1.97 for the ISGDR





5-d. Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Dip at 21.44 MeV

Narrow Pole (Γ_g = 22 keV) above the ISGDR: $|R_g| > 1$, Re $R_g > 0$ → Dip below Re E_g, Peak above



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.451 - 0.011i$ MeV

Collectivity (IS): non-collective

C_g^2	0.0364
$N_{\rm tot}\eta_g$	8.80
$C_g^2 N_{\rm tot}\eta_g$	0.32

At E = Re E_g ($\epsilon = 0$)

q_g	$0.99 + 0.24i$
p_g^2	0.68
R_g	$0.87 - 0.67i$
$\lvert R_g\rvert$	1.10 > 1

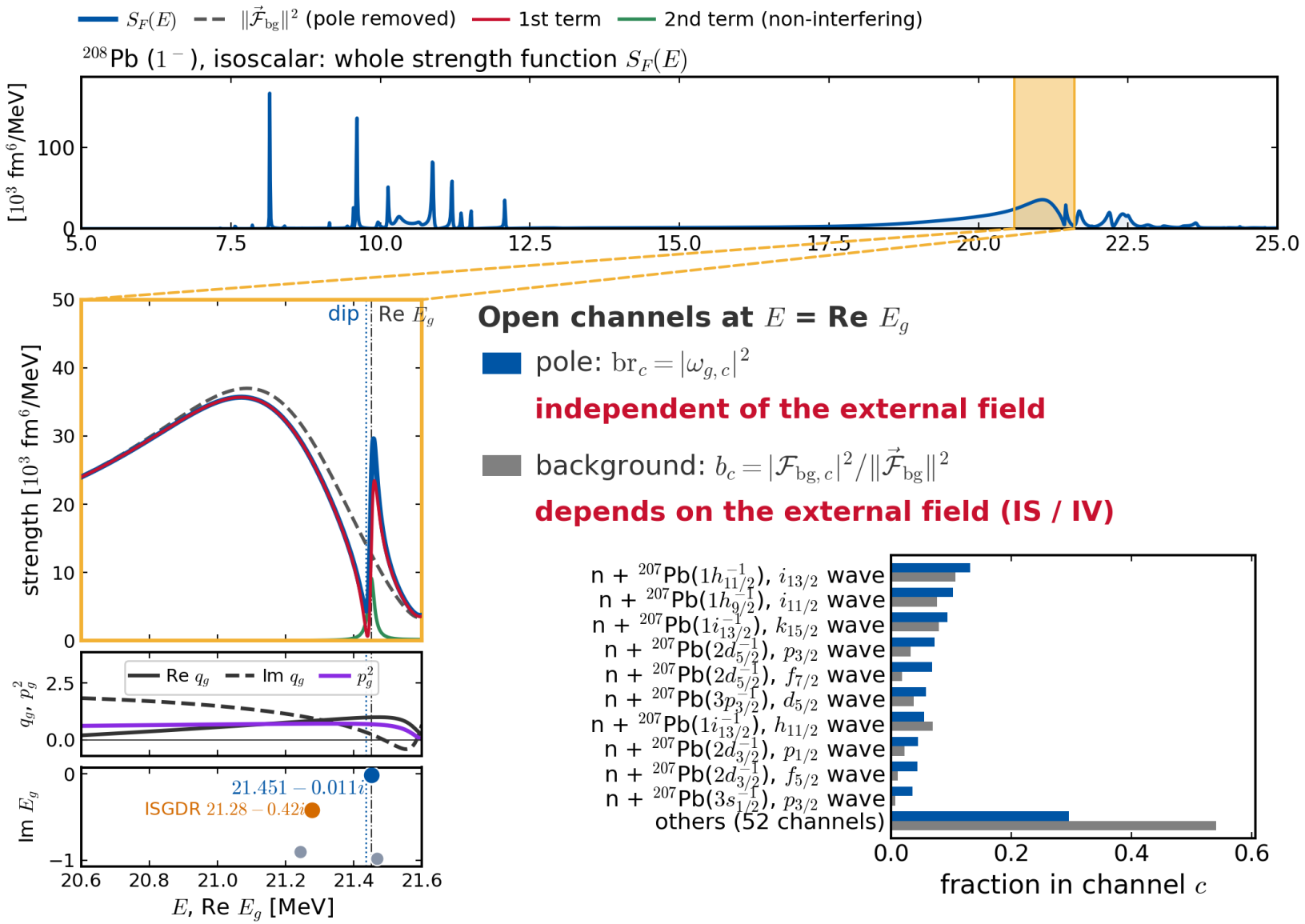
strong interference

Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **dip and peak**

5-d. Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Dip at 21.44 MeV

The Pole Decays into Many Channels; the IS Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.451 - 0.011i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.05

Pole decays into (br_c)

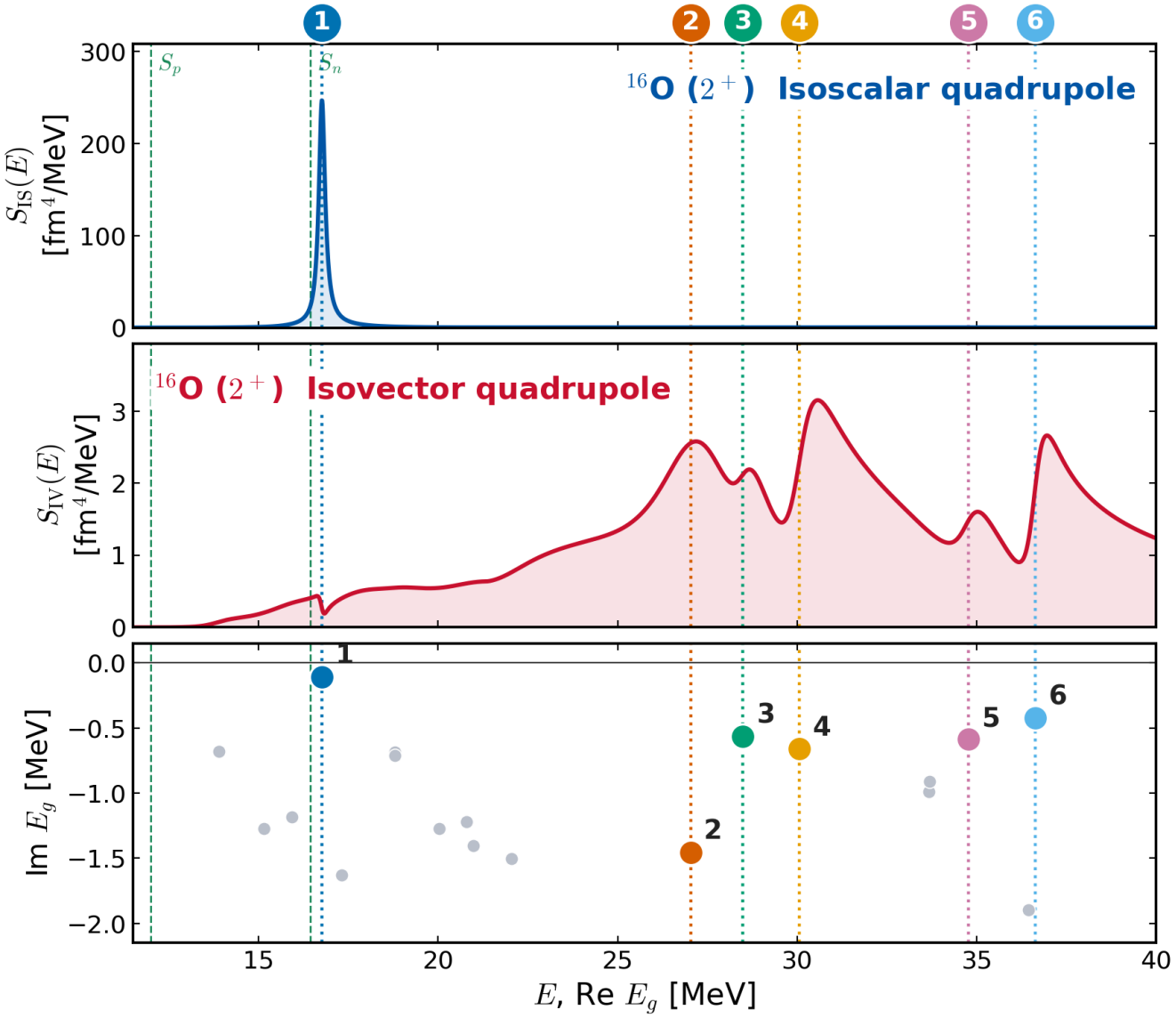
n + ^{207}Pb($1h_{11/2}^{-1}$), $i_{13/2}$ wave	0.13
n + ^{207}Pb($1h_{9/2}^{-1}$), $i_{11/2}$ wave	0.10

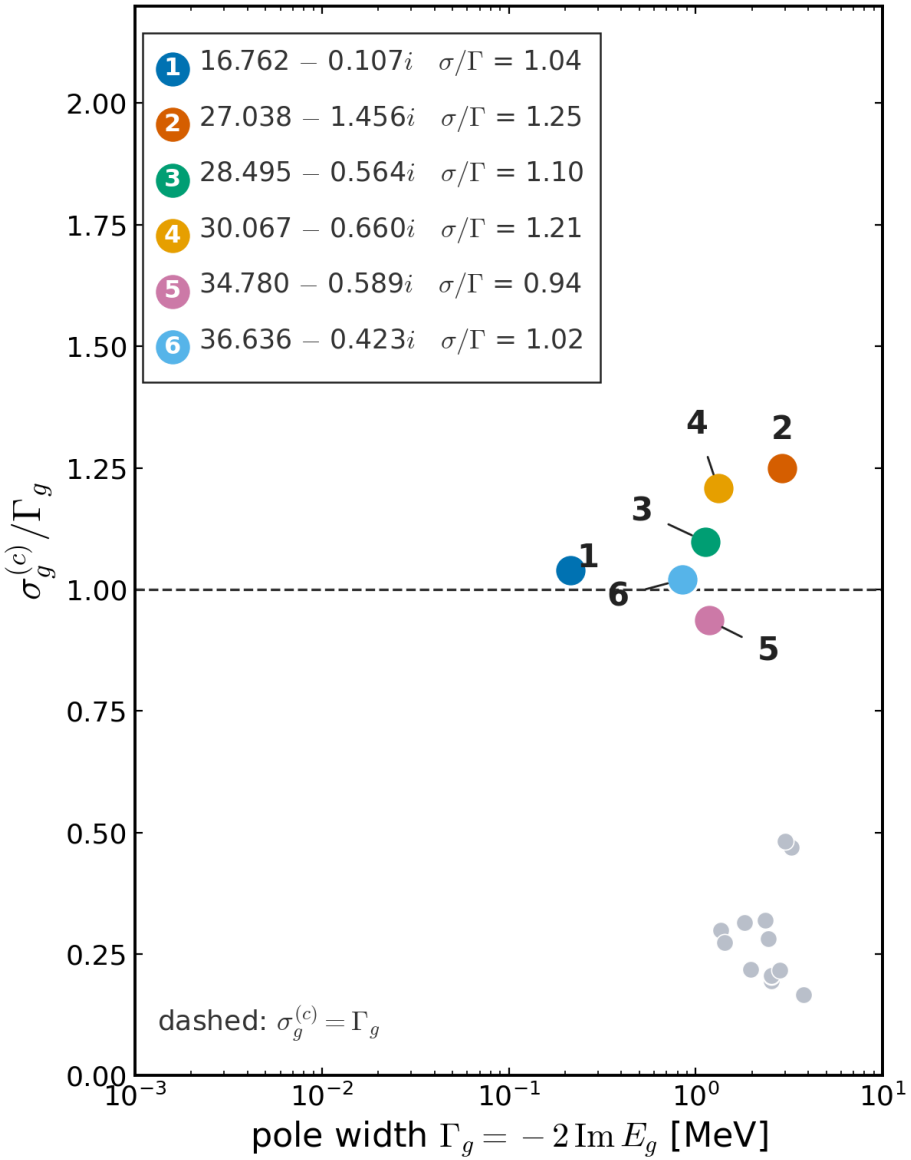
Background at Re E_g (b_c)

p + ^{207}Tl($2d_{5/2}^{-1}$), $f_{7/2}$ wave	0.12
n + ^{207}Pb($1h_{11/2}^{-1}$), $i_{13/2}$ wave	0.11

5-b. Numerical Results: ^{16}O ($J^\pi = 2^+$) Strength Functions & Resonance Poles

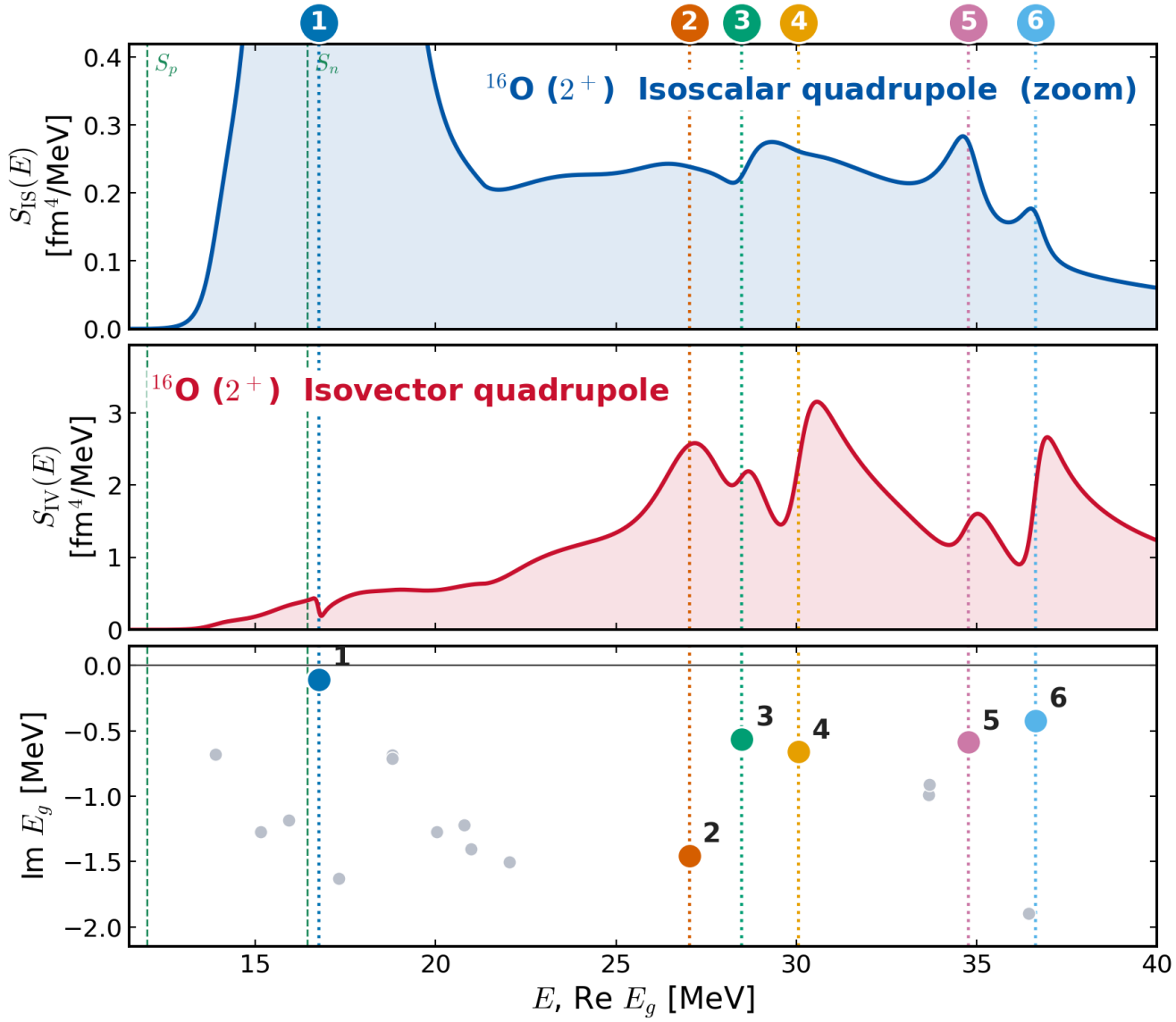
Peaks and the 20 S-Matrix Poles E_g; $\sigma_g^{(c)}/\Gamma_g \approx 1$ Only for the ISGQR and 36.64 – 0.42i MeV, 0.17–1.25 for the Others

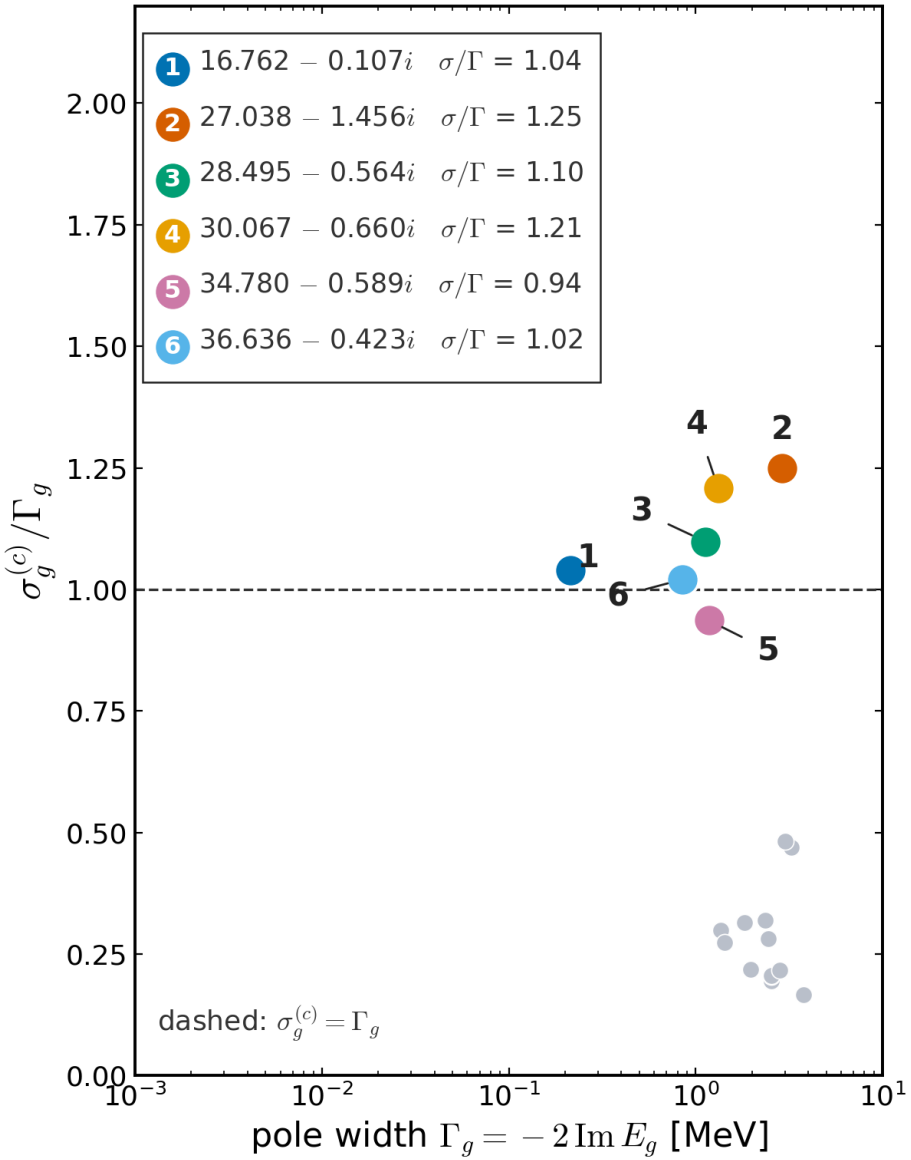




5-b. Numerical Results: ^{16}O ($J^\pi = 2^+$) Continuum Zoom & Dip Structure

Isoscalar Dip at 28.2–28.3 MeV and the Pole 28.49 – 0.56i MeV with Re $\tilde{B}_g < 0$ ($\sigma_g^{(c)}/\Gamma_g = 1.10$)





2. Jost function and Riemann sheet

$$\lim_{r\to\infty} \phi_{lj}^{(r)}(r) \to \frac{1}{2} J_{lj}^{(-)}(k) e^{+ikr}$$

$$= \begin{cases} e^{-\kappa_b r} & \\ & k_i^{res}<0 \\ e^{-ik_r^{res} r} e^{k_i^{res} r} & \end{cases}$$

$$\Psi(t) = e^{-iEt/\hbar}$$

$$= \begin{cases} e^{-iE_B t/\hbar} & \\ & \\ e^{-iE_R^r t/\hbar} e^{-\Gamma t/2\hbar} & \end{cases}$$

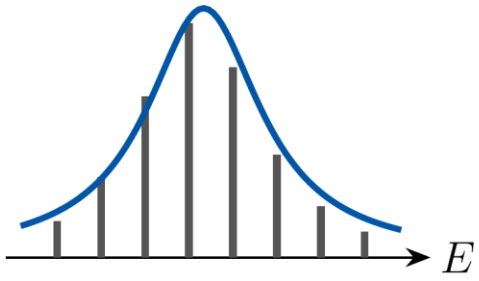
Bound state: converges to zero at r→∞ / stationary oscillation in time.

Resonance: diverges at r→∞ / converges to zero in finite time (life time).

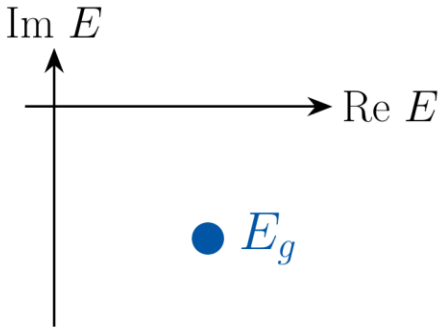
Motivation: What Is Missing for Resonances in the Continuum

The Pole Gives Only Its Total Strength $\tilde{B}_g$; This Work Obtains the Channel Amplitudes of Each Pole

A resonance in the continuum is a pole of the S-matrix $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$



Discretized continuum
split into pseudostates; amplitudes averaged over an energy region



The pole
gives its complex strength
$\widetilde{B}_g = \mathrm{Res}[R_F, E_g]$
one number, not the channel amplitudes

This work: channel amplitudes of each resonance pole (Takagi factorization of the S-matrix residue)

(a) collectivity of the pole: coherence and participation, as for discrete states
(b) decay widths and branching ratios of the same pole
(c) interference of the pole with the background: line shape of the strength function

Fano Formula within Jost-RPA

Interference of a Pole with the Background in the Open-Channel Amplitude Space

1. Pole Term of the Amplitude Vector, Fano Parameter q_g & Fraction p_g

Strength function $S_F^{(\tau)}(E) = \vec{\mathcal{F}}^{(\tau)\dagger}(E^*)\,\vec{\mathcal{F}}^{(\tau)}(E)$

Open-channel amplitude vector
$$\vec{\mathcal{F}}^{(\tau)}(E) \equiv \sqrt{\frac{2m}{\pi\hbar^2}}\,\boldsymbol{\mathcal{S}}_{cc}(E) \times \sum_q \int dr\, \boldsymbol{\Psi}_{F,c}^{(+)*\mathsf{T}}(r;E^*)\,\boldsymbol{\xi}_q(r)\, f_q^{(\tau)}(r)$$

Pole term $\vec{\mathcal{F}}^{(\tau)}(E) = \vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E) + \dfrac{c_g^{(\tau)}\,\vec{\boldsymbol{\omega}}_g^{(c)}}{E - E_g}$

Pole coefficient $c_g^{(\tau)} = \sqrt{\dfrac{i\sigma_g^{(c)}}{2\pi}}\,\mathcal{F}_g^{(\tau)}$

Reduced energy $\epsilon = \dfrac{E - \mathrm{Re}\,E_g}{\Gamma_g/2}$

Fano parameter (complex, E-dependent)
$$\frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\,\vec{\mathcal{F}}^{(\tau)}(E)}{\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2} = \frac{\epsilon + q_g(E)}{\epsilon + i}$$
$$\Rightarrow\; q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\,c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction of the pole term along $\vec{\mathcal{F}}_{\rm bg}$
$$p_g(E) = \frac{|\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\,c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}|}{\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|\,\|c_g^{(\tau)}\vec{\boldsymbol{\omega}}_g^{(c)}\|} \in [0,1]$$

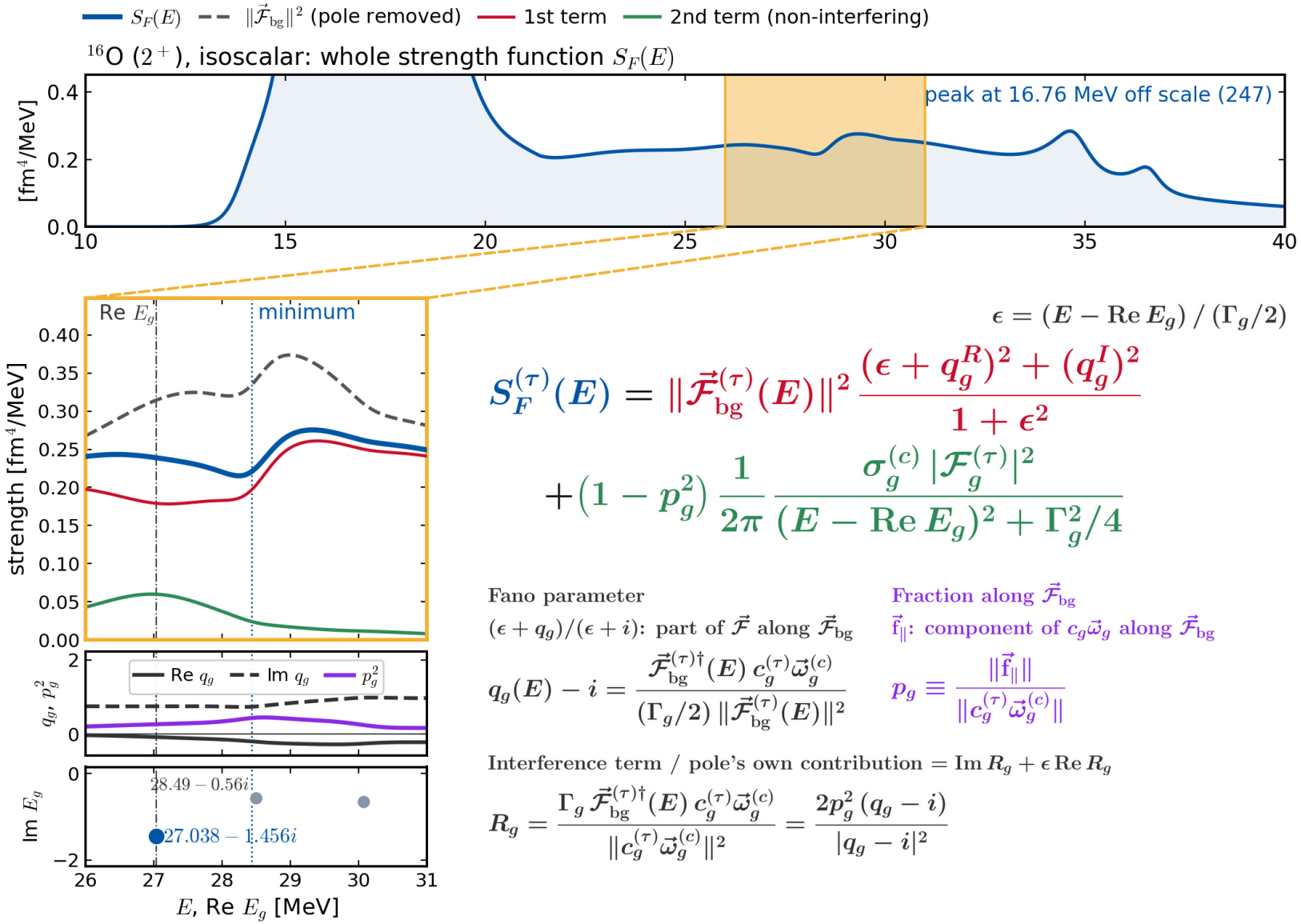
2. Fano Formula: Parallel Part and Non-Interfering ($1 - p_g^2$) Part

$$S_F^{(\tau)}(E) = \underbrace{\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2\,\frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2}}_{\text{background + interference + parallel pole part}} + \underbrace{\left(1 - p_g^2\right)\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}}_{\text{non-interfering: Breit–Wigner}}$$

No interference: $p_g = 0 \iff q_g = i$: $S_F^{(\tau)} = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}\|^2 + \text{Breit–Wigner}$

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 27.04 – 1.46i MeV in the Isoscalar Strength

Broad Non-Collective Pole (Γ_g = 2.9 MeV): $|R_g| > 1$ and Im $R_g < -1$ → S_F below the Background over the Width



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter
$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$
$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	27.038 − 1.456i MeV

Collectivity (IS): non-collective

C_g^2	0.0054
$N_{\rm tot}\eta_g$	4.35
$C_g^2 N_{\rm tot}\eta_g$	0.023

At E = Re E_g ($\epsilon = 0$)

q_g	-0.08 + 0.75i
p_g^2	0.27
R_g	-0.62 − 1.93i
$\|R_g\|$	2.03 > 1

strong interference

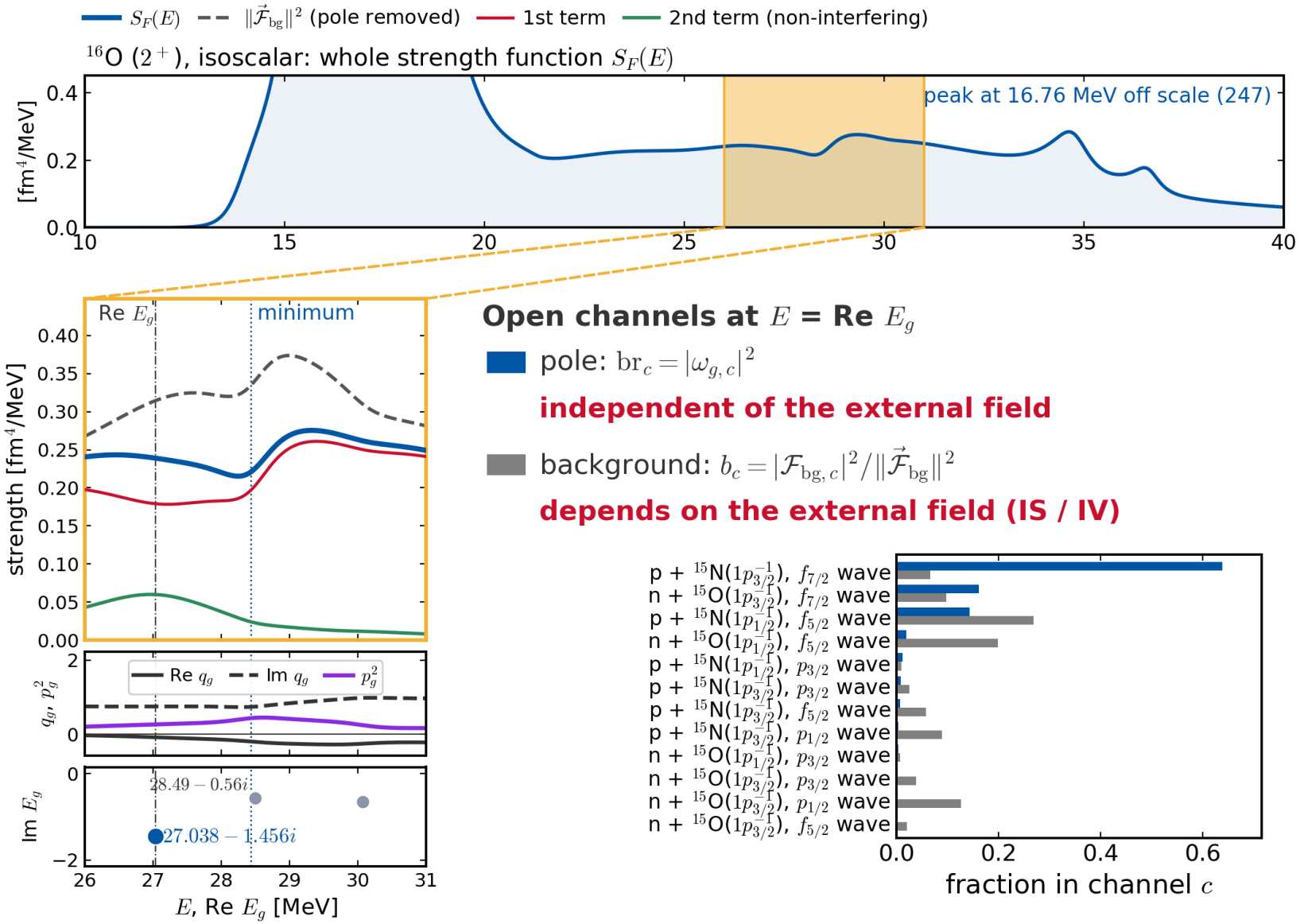
Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ S_F **below the background**

minimum at 28.43 MeV

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 27.04 – 1.46i MeV in the Isoscalar Strength

The Pole Decays Mainly into p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ Wave; the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$27.038 - 1.456i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.25

Pole decays into (br_c)

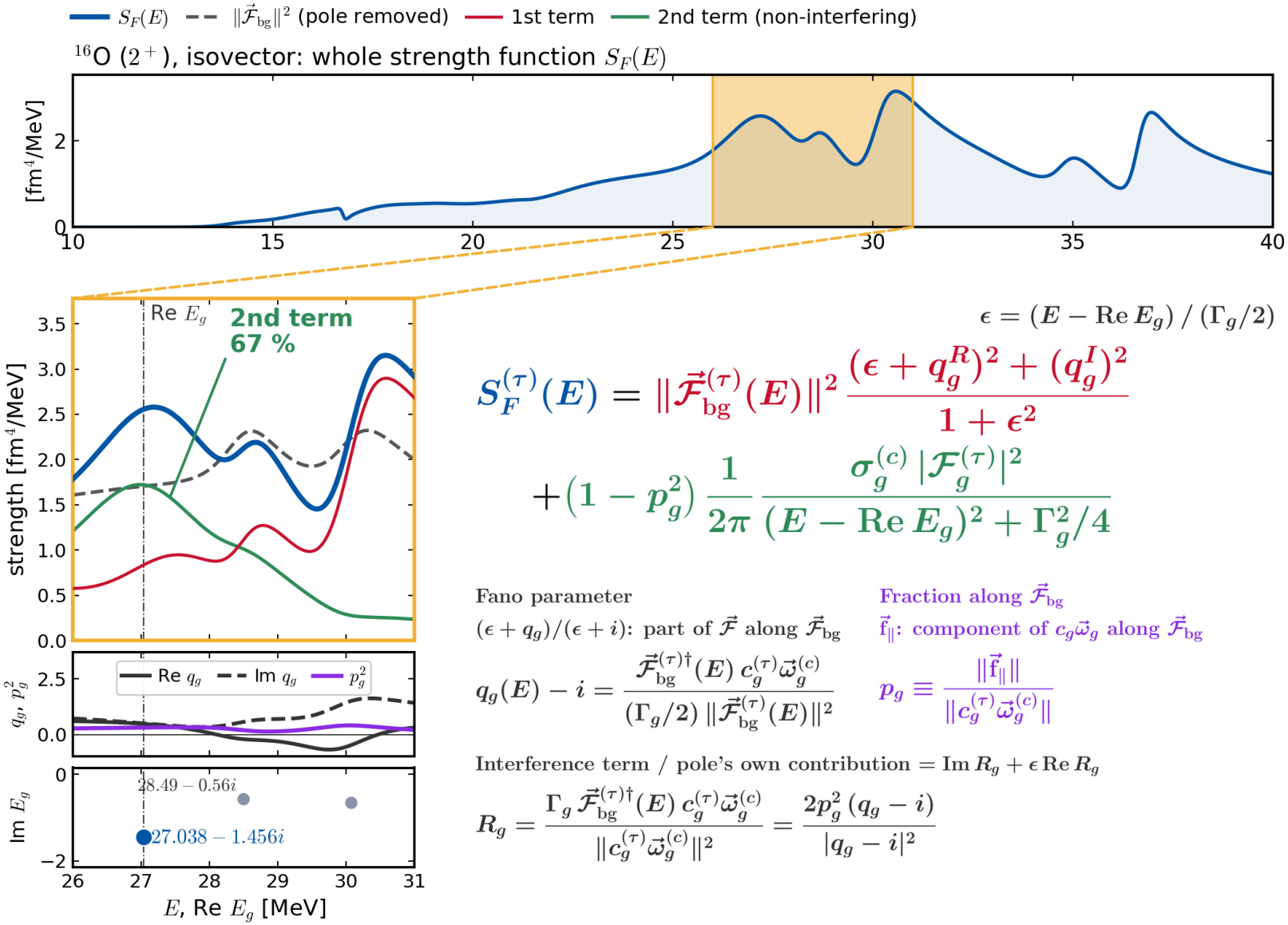
p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.64
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.16

Background at Re E_g (b_c)

p + ^{15}N($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.27
n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.20

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 27.04 – 1.46i MeV in the Isovector Strength

Isovector Field (Non-Collective): $|R_g| \sim 1$, Interference Comparable to the Pole's Own; Broad Peak at 27.2 MeV



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\frac{1}{2\pi}\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$27.038 - 1.456i$ MeV

Collectivity (IV): non-collective

C_g^2	0.1679
$N_{\rm tot}\eta_g$	4.35
$C_g^2 N_{\rm tot}\eta_g$	0.73

At E = Re E_g ($\epsilon = 0$)

q_g	$0.48 + 0.51i$
p_g^2	0.32
R_g	$0.65 - 0.66i$
$\lvert R_g\rvert$	$0.93 \sim 1$

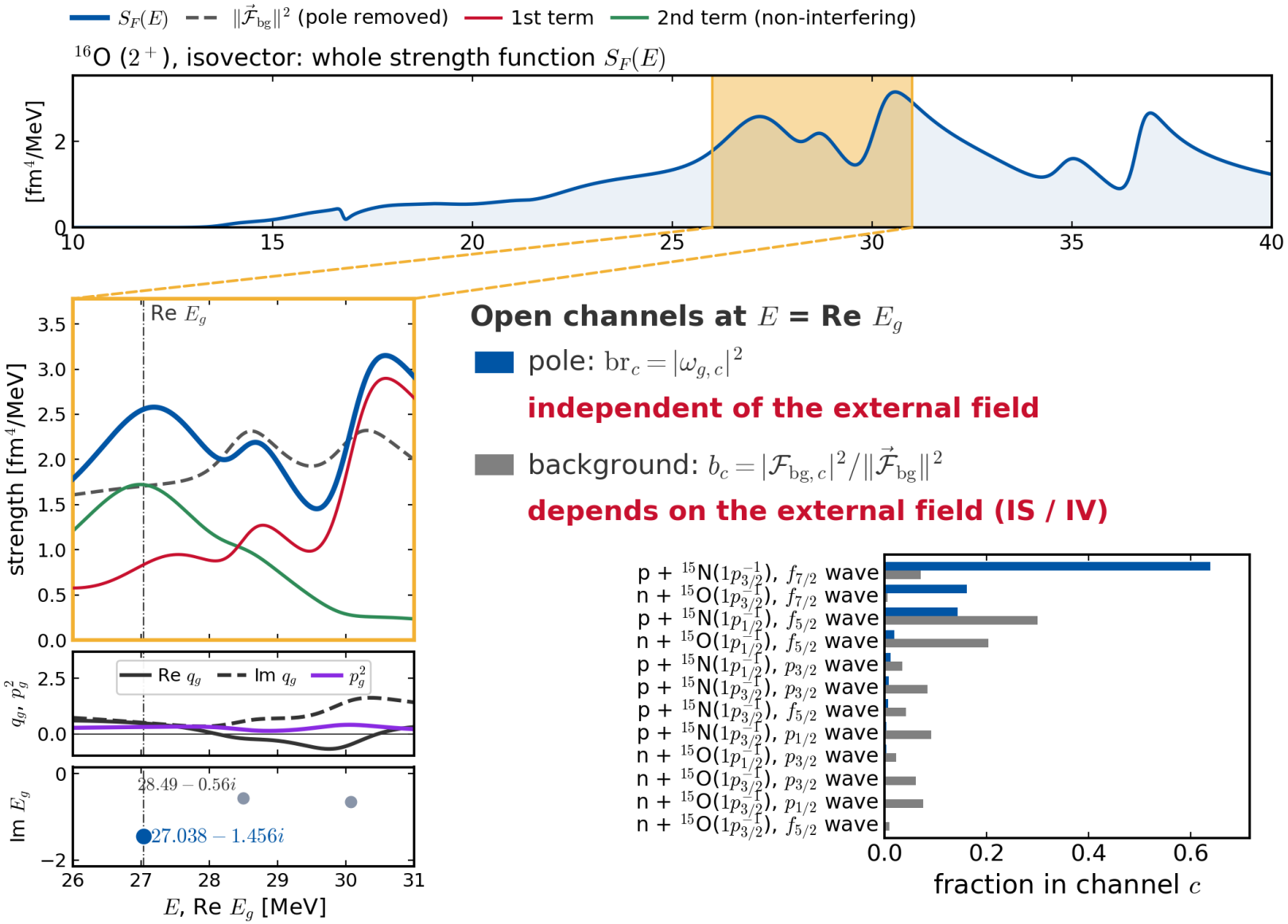
interference comparable

Im R_g < 0: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **broad peak**

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 27.04 – 1.46i MeV in the Isovector Strength

Same br$_c$ as in the IS Field; the IV Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$27.038 - 1.456i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.25

Pole decays into (br_c)

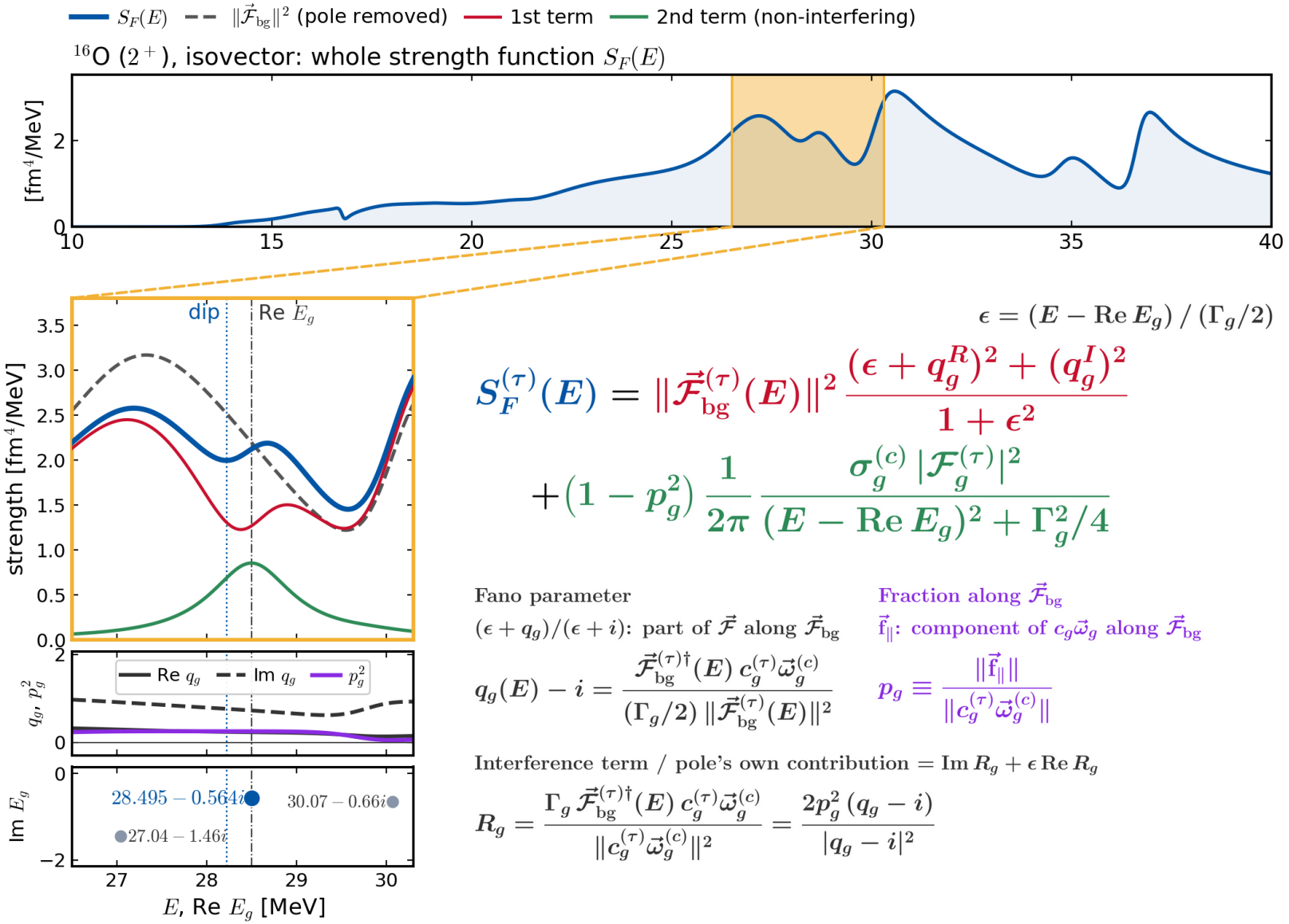
p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.64
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.16

Background at Re E_g (b_c)

p + ^{15}N($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.30
n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.20

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 28.49 – 0.56i MeV in the Isovector Strength

Same Pole, Isovector Field (Non-Collective): $|R_g| > 1$, Re $R_g > 0$ → Dip below Re E_g and Peak at 28.67 MeV



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\frac{1}{2\pi}\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$28.495 - 0.564i$ MeV

Collectivity (IV): non-collective

C_g^2	0.0630
$N_{\rm tot}\eta_g$	5.68
$C_g^2 N_{\rm tot}\eta_g$	0.36

At E = Re E_g ($\epsilon = 0$)

q_g	$0.23 + 0.72i$
p_g^2	0.25
R_g	$0.89 - 1.07i$
$\lvert R_g\rvert$	1.39 > 1

strong interference

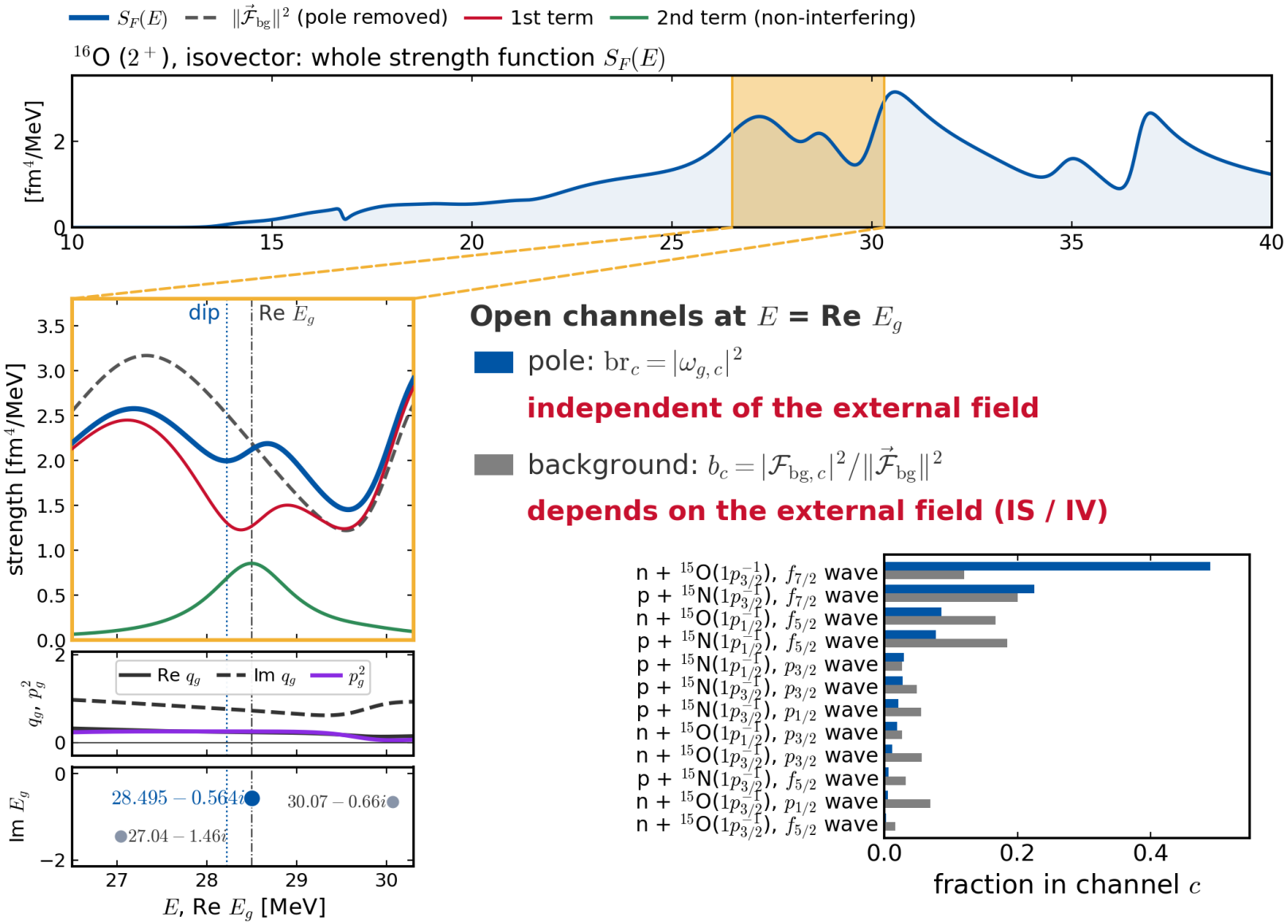
Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ **dip and peak**

dip at 28.22 MeV

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 28.49 – 0.56i MeV in the Isovector Strength

Same br$_c$ as in the IS Field; the IV Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$28.495 - 0.564i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.10

Pole decays into (br_c)

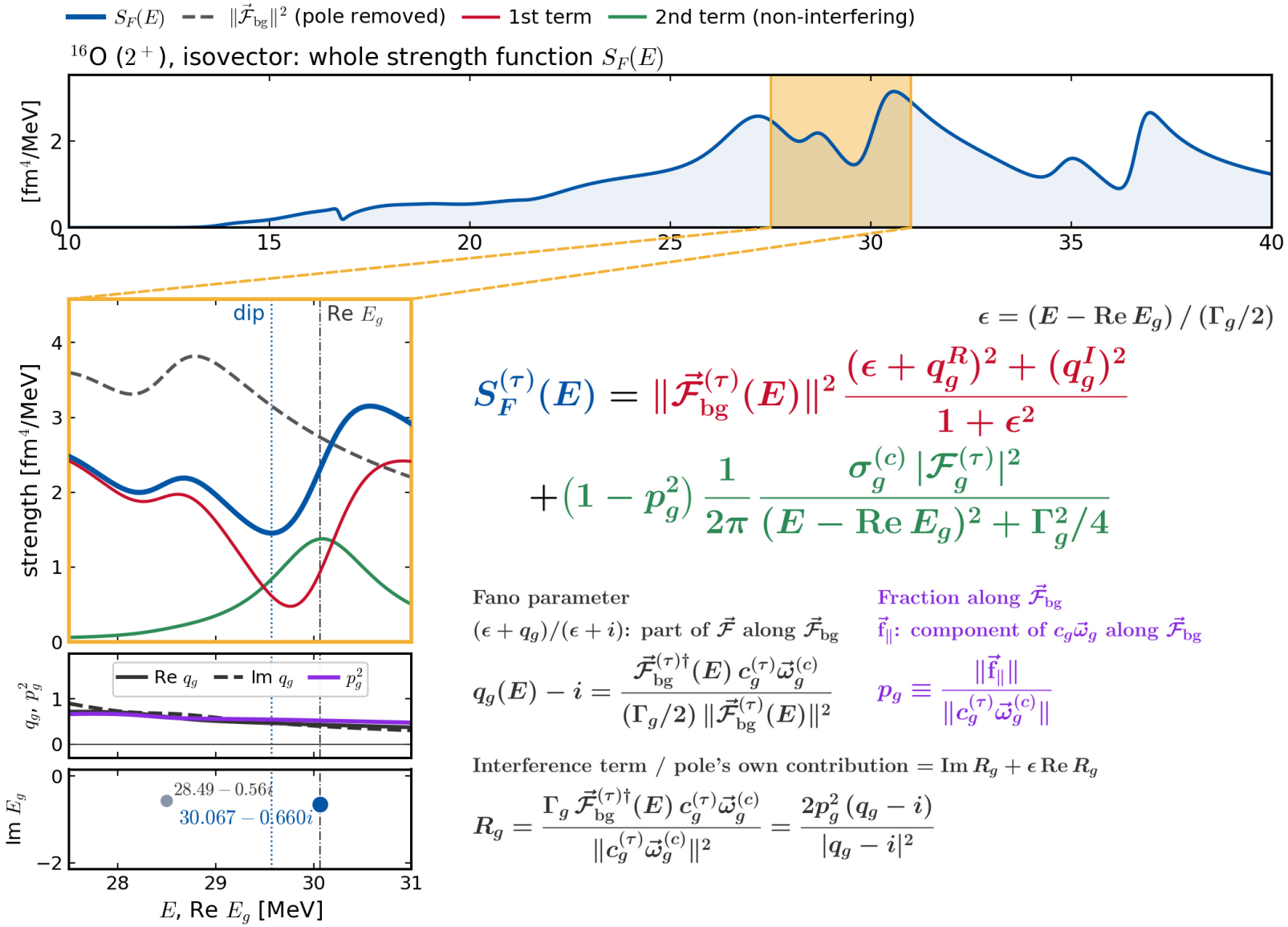
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.49
p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.23

Background at Re E_g (b_c)

p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.20
p + ^{15}N($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.18

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 30.07 − 0.66i MeV in the Isovector Strength

Isovector Field (Collective): $|R_g| > 1$, Re $R_g > 0$ → Dip below Re E_g and Peak at 30.58 MeV



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter
$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$
$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$30.067 - 0.660i$ MeV

Collectivity (IV): collective

C_g^2	0.1561
$N_{\rm tot}\eta_g$	7.06
$C_g^2 N_{\rm tot}\eta_g$	1.10

At E = Re E_g ($\epsilon = 0$)

q_g	$0.43 + 0.40i$
p_g^2	0.52
R_g	$0.82 - 1.15i$
$\|R_g\|$	1.41 > 1

strong interference

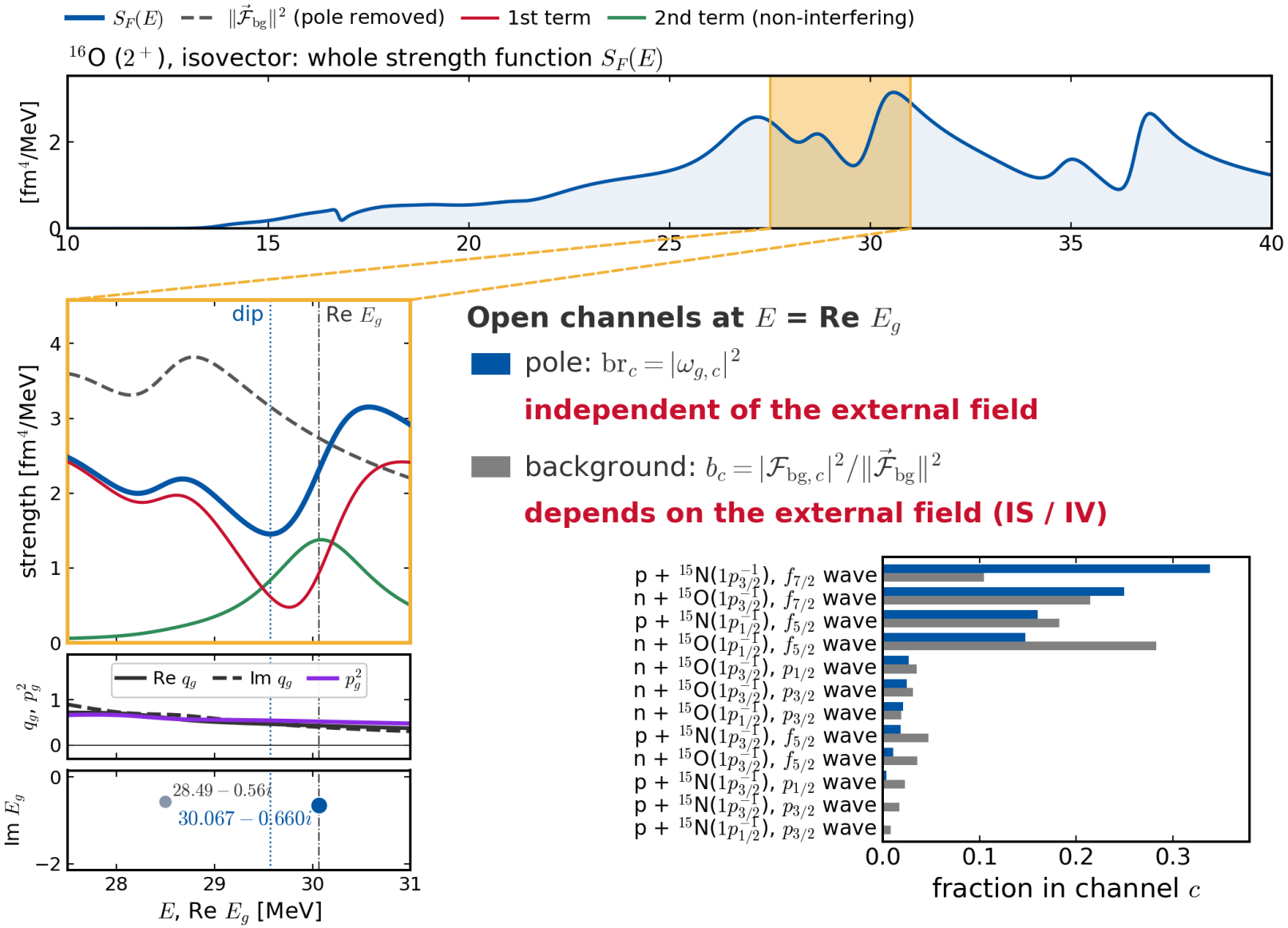
Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ **dip and peak**

dip at 29.57 MeV

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 30.07 – 0.66i MeV in the Isovector Strength

The Pole Decays into Several Channels (Largest: p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ Wave); the IV Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$30.067 - 0.660i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.21

Pole decays into (br_c)

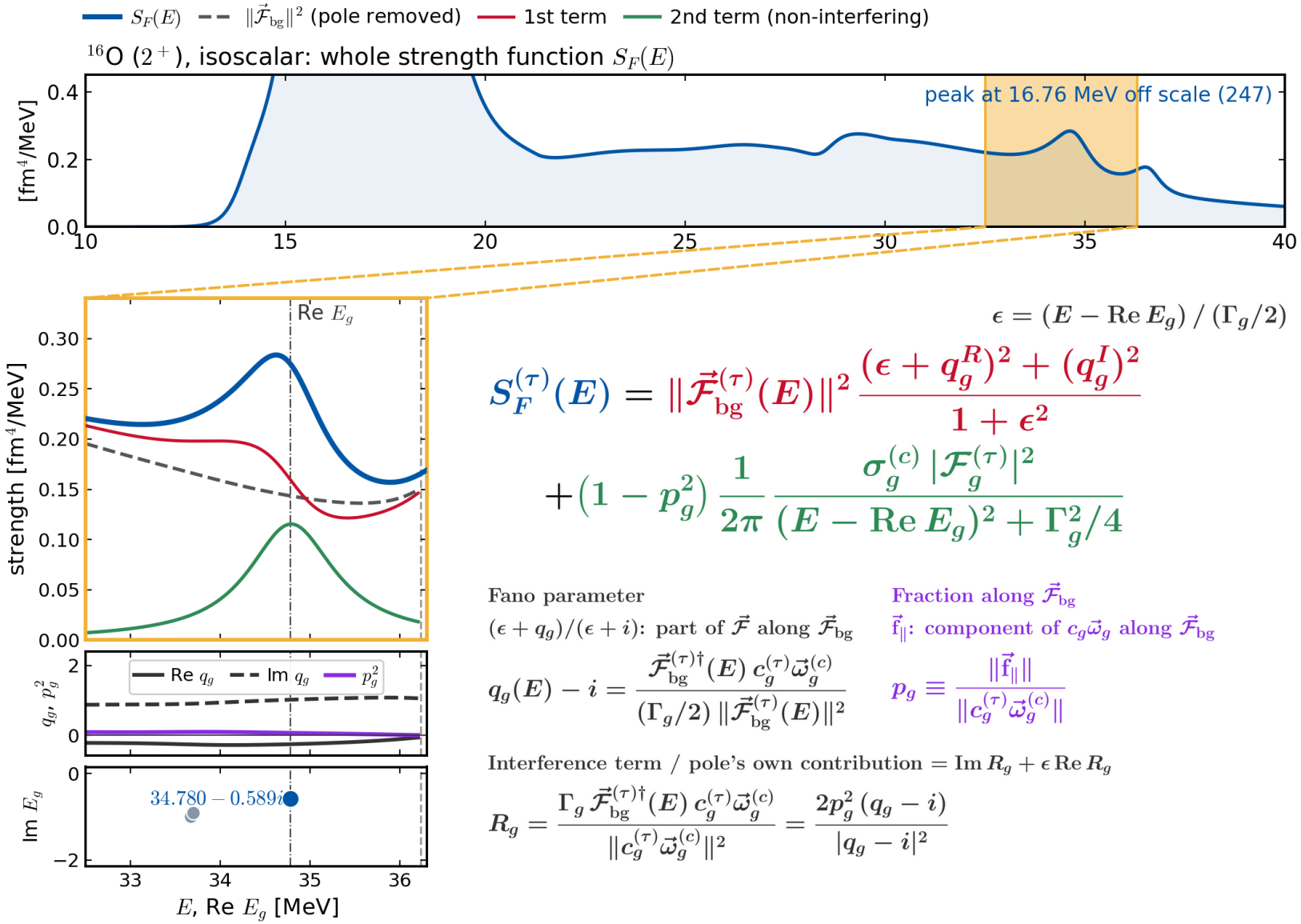
p + ^{15}N($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.34
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.25

Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.28
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.21

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 34.78 – 0.59i MeV in the Isoscalar Strength

Non-Collective Pole: $|R_g| < 1/\sqrt{2}$, Peak at 34.6 MeV Mainly the Pole's Own Contribution, below Re E_g (Re $R_g < 0$)



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter
$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$
$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	34.780 − 0.589i MeV

Collectivity (IS): non-collective

C_g^2	0.0284
$N_{\rm tot}\eta_g$	5.37
$C_g^2 N_{\rm tot}\eta_g$	0.15

At E = Re E_g ($\epsilon = 0$)

q_g	-0.25 + 1.02i
p_g^2	0.07
R_g	-0.58 + 0.06i
$\lvert R_g\rvert$	0.58 < $1/\sqrt{2}$

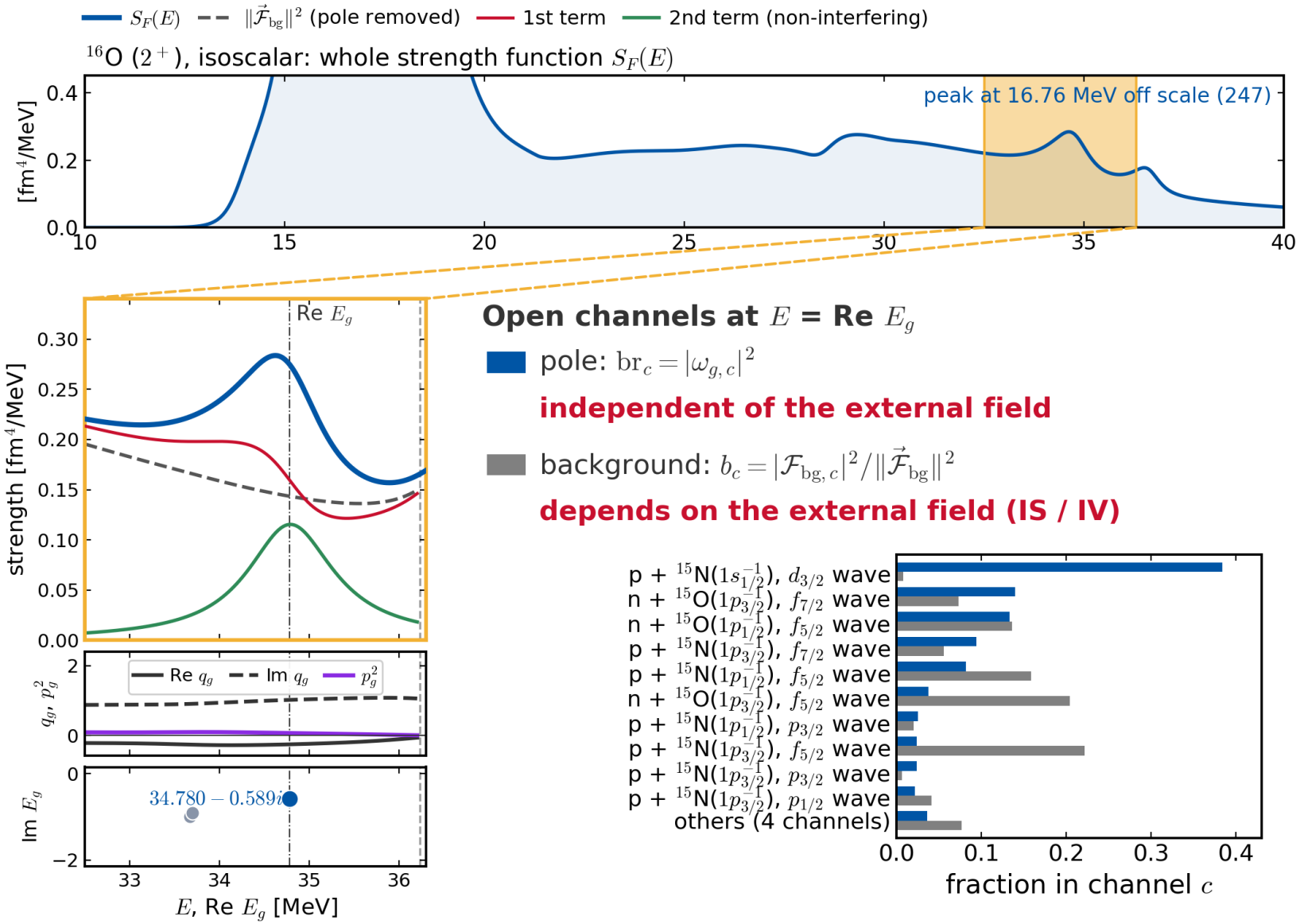
pole's own contribution dominates

Im R_g > 0: raises S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak on the background**

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 34.78 – 0.59i MeV in the Isoscalar Strength

The Pole Decays into Several Channels (Largest: p + ^{15}N($1s_{1/2}^{-1}$), $d_{3/2}$ Wave); the IS Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$34.780 - 0.589i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	0.94

Pole decays into (br_c)

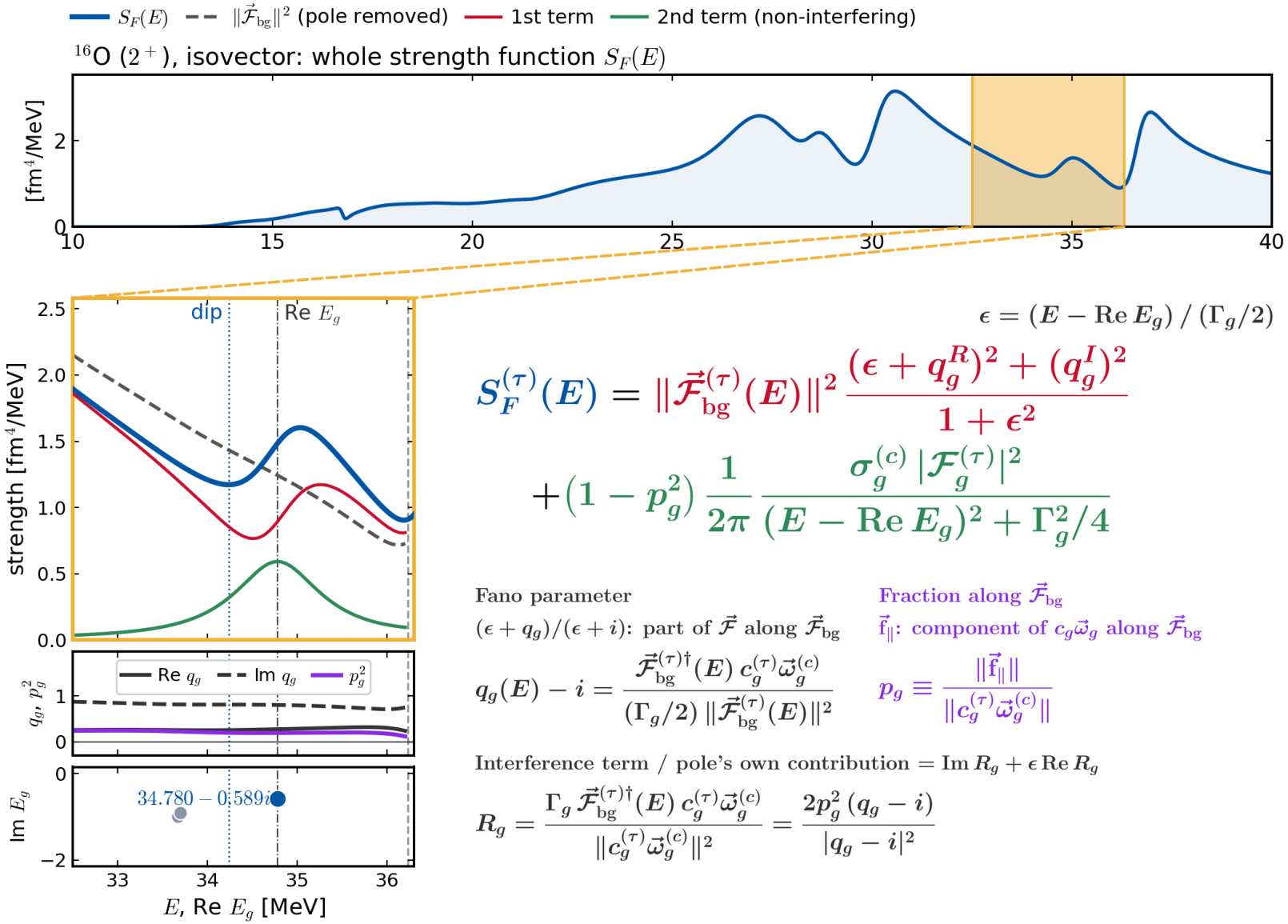
p + ^{15}N($1s_{1/2}^{-1}$), $d_{3/2}$ wave	0.38
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.14

Background at Re E_g (b_c)

p + ^{15}N($1p_{3/2}^{-1}$), $f_{5/2}$ wave	0.22
n + ^{15}O($1p_{3/2}^{-1}$), $f_{5/2}$ wave	0.20

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 34.78 – 0.59i MeV in the Isovector Strength

S



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	34.780 − 0.589i MeV

Collectivity (IV): non-collective

C_g^2	0.1674
$N_{\rm tot}\eta_g$	5.37
$C_g^2 N_{\rm tot}\eta_g$	0.90

At E = Re E_g ($\epsilon = 0$)

q_g	0.27 + 0.80i
p_g^2	0.19
R_g	0.93 − 0.67i
$\lvert R_g\rvert$	1.15 > 1

strong interference

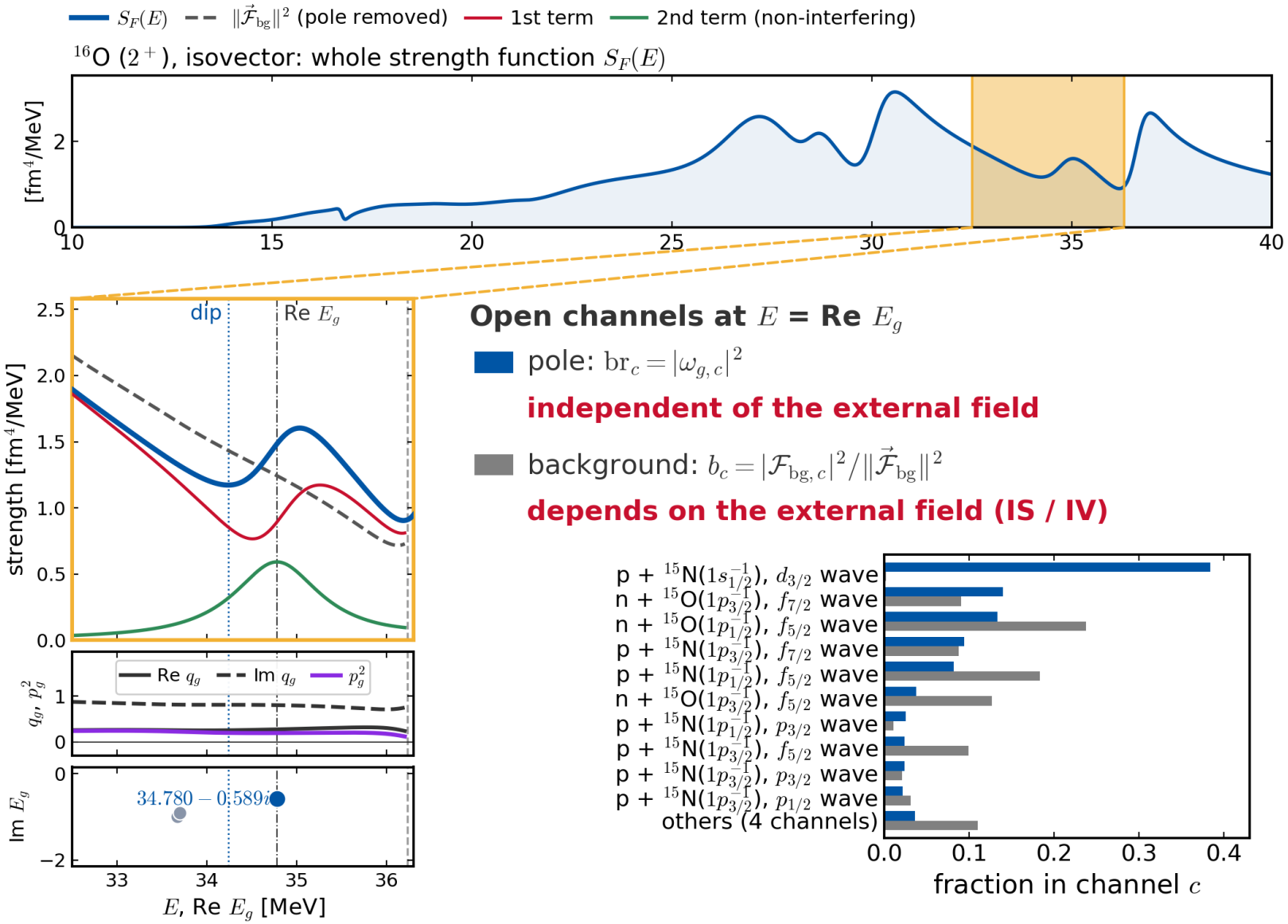
Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **dip and peak**

dip at 34.24 MeV

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 34.78 – 0.59i MeV in the Isovector Strength

Same br$_c$ as in the IS Field; the IV Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$34.780 - 0.589i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	0.94

Pole decays into (br_c)

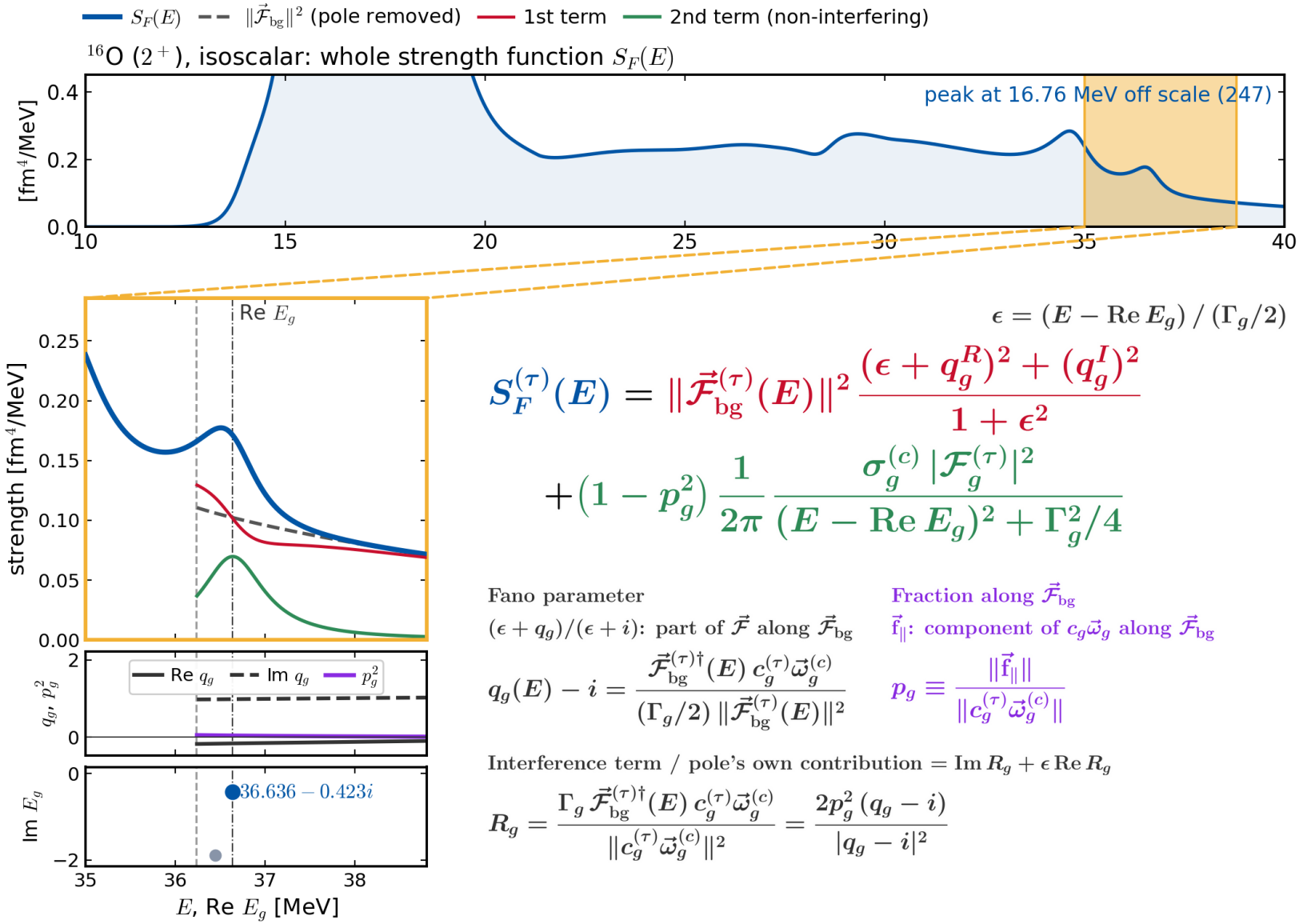
p + ^{15}N($1s_{1/2}^{-1}$), $d_{3/2}$ wave	0.38
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.14

Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.24
p + ^{15}N($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.18

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 36.64 – 0.42i MeV in the Isoscalar Strength

Non-Collective Pole: $|R_g| < 1/\sqrt{2}$, Peak at 36.5 MeV Mainly the Pole's Own Contribution, below Re E_g (Re $R_g < 0$)



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\mathrm{f}}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\mathrm{f}}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$36.636 - 0.423i$ MeV

Collectivity (IS): non-collective

C_g^2	0.0129
$N_{\rm tot}\eta_g$	5.19
$C_g^2 N_{\rm tot}\eta_g$	0.067

At E = Re E_g ($\epsilon = 0$)

q_g	-0.17 + 0.98i
p_g^2	0.04
R_g	-0.47 − 0.05i
$\lvert R_g\rvert$	0.47 < $1/\sqrt{2}$

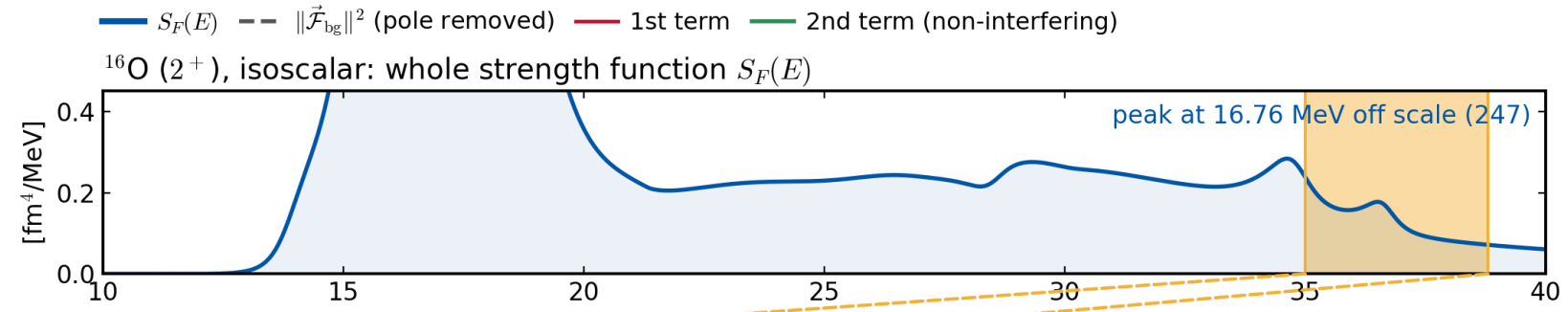
pole's own contribution dominates

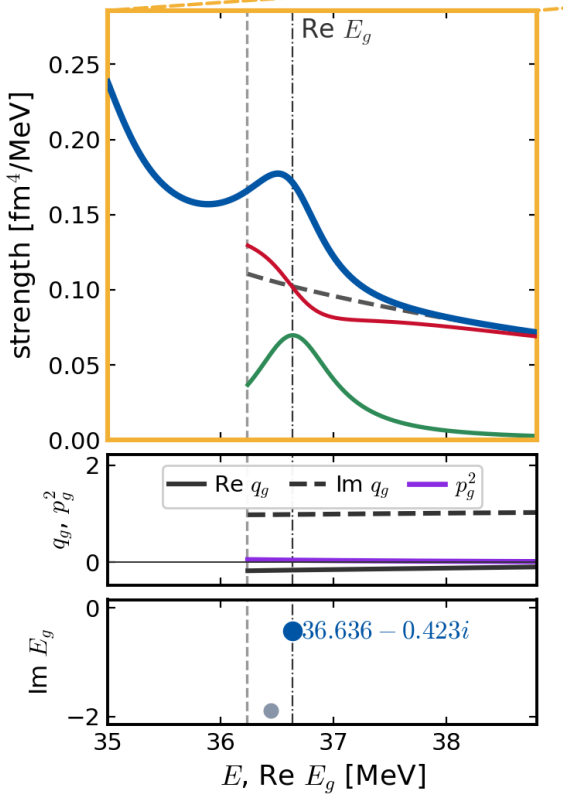
Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak on the background**

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 36.64 – 0.42i MeV in the Isoscalar Strength

The Pole Decays into Many Channels; the IS Background Is Spread over Many Channels





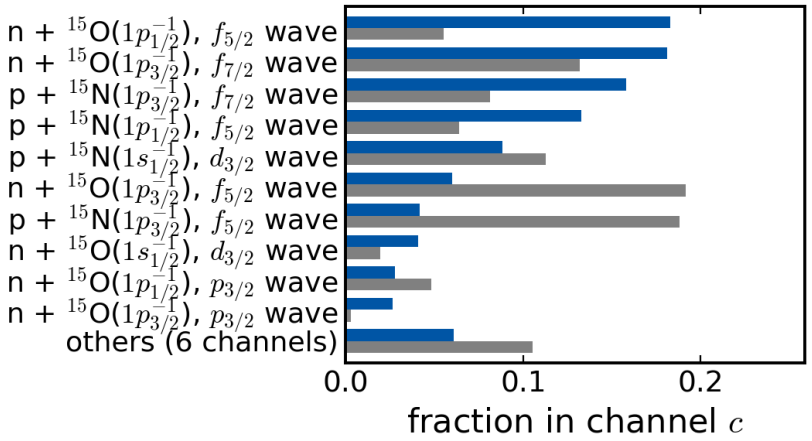
Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)



Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$36.636 - 0.423i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.02

Pole decays into (br_c)

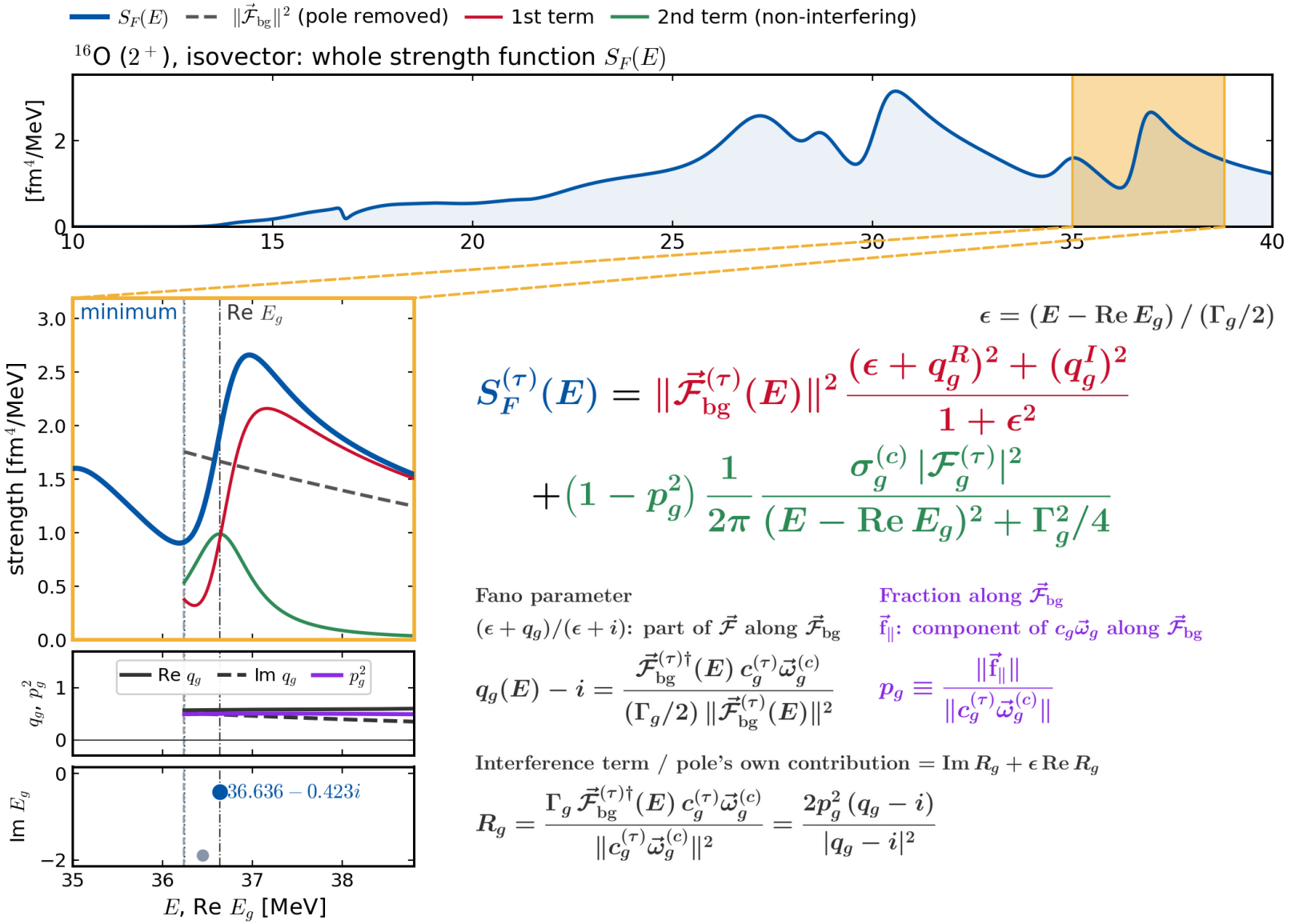
n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.18
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.18

Background at Re E_g (b_c)

n + ^{15}O($1p_{3/2}^{-1}$), $f_{5/2}$ wave	0.19
p + ^{15}N($1p_{3/2}^{-1}$), $f_{5/2}$ wave	0.19

Fano Analysis: ^{16}O ($J^\pi = 2^+$) Pole 36.64 – 0.42i MeV in the Isovector Strength

Most Collective IV Pole: $|R_g| > 1$, Re $R_g > 0$ → S_F Lowered below Re E_g, Peak at 36.96 MeV



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\, E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$36.636 - 0.423i$ MeV

Collectivity (IV): collective

C_g^2	0.3515
$N_{\rm tot}\eta_g$	5.19
$C_g^2 N_{\rm tot}\eta_g$	1.82

At E = Re E_g ($\epsilon = 0$)

q_g	$0.58 + 0.49i$
p_g^2	0.50
R_g	$0.97 - 0.86i$
$\|R_g\|$	1.30 > 1

strong interference

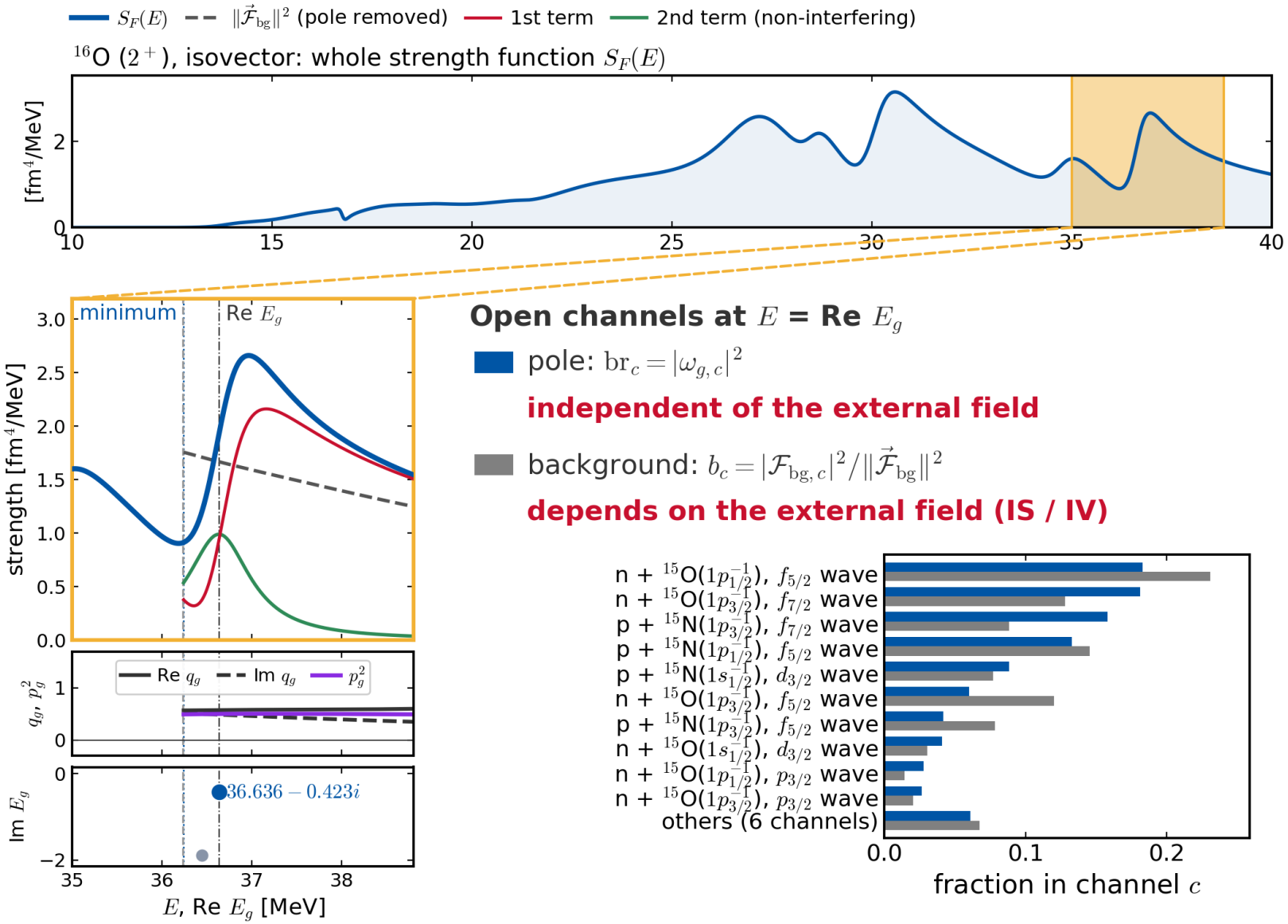
Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **minimum at the threshold and peak**

minimum at 36.24 MeV

Branching Ratios: ^{16}O ($J^\pi = 2^+$) Pole 36.64 – 0.42i MeV in the Isovector Strength

Same br$_c$ as in the IS Field; the IV Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$36.636 - 0.423i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.02

Pole decays into (br_c)

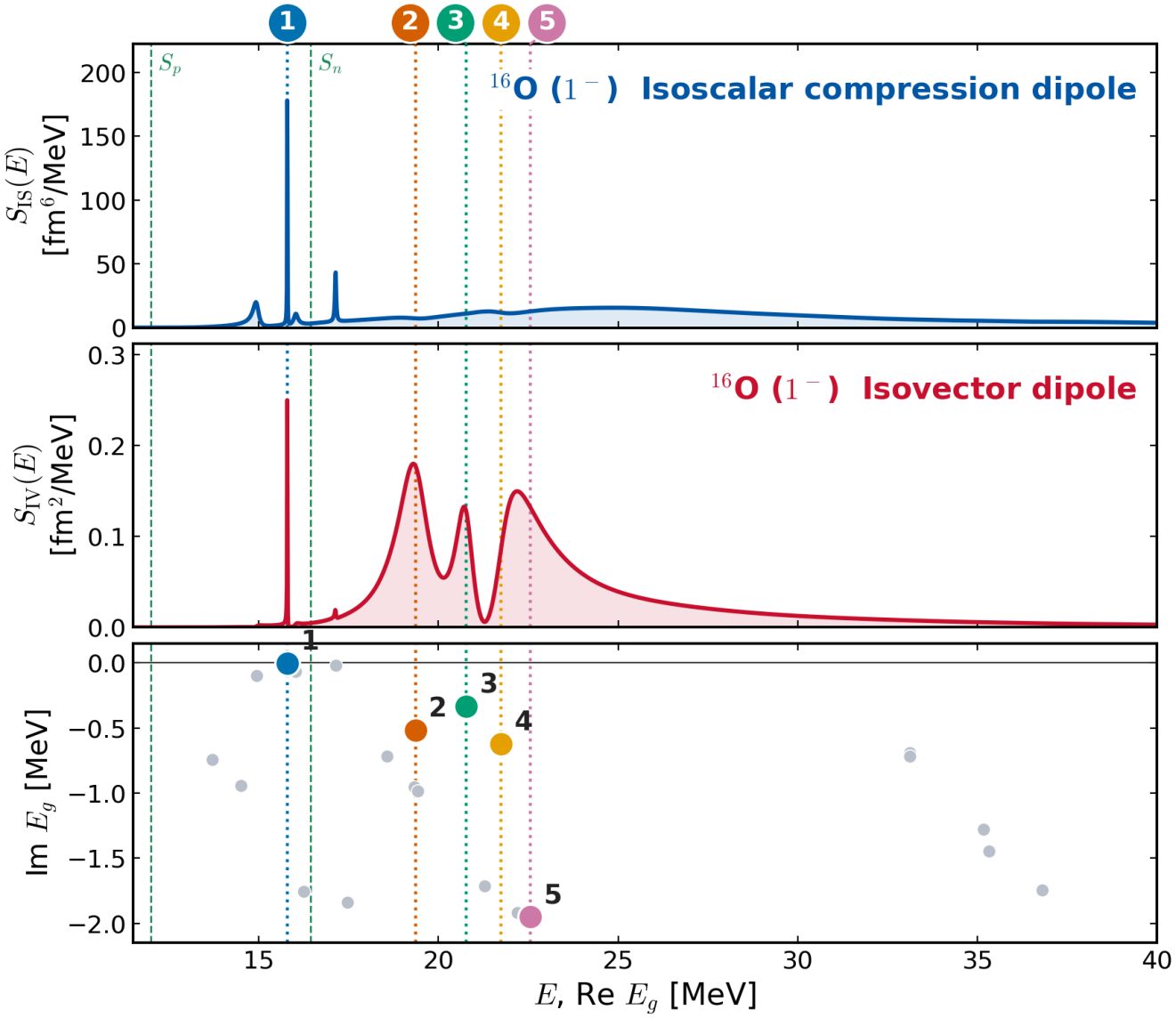
n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.18
n + ^{15}O($1p_{3/2}^{-1}$), $f_{7/2}$ wave	0.18

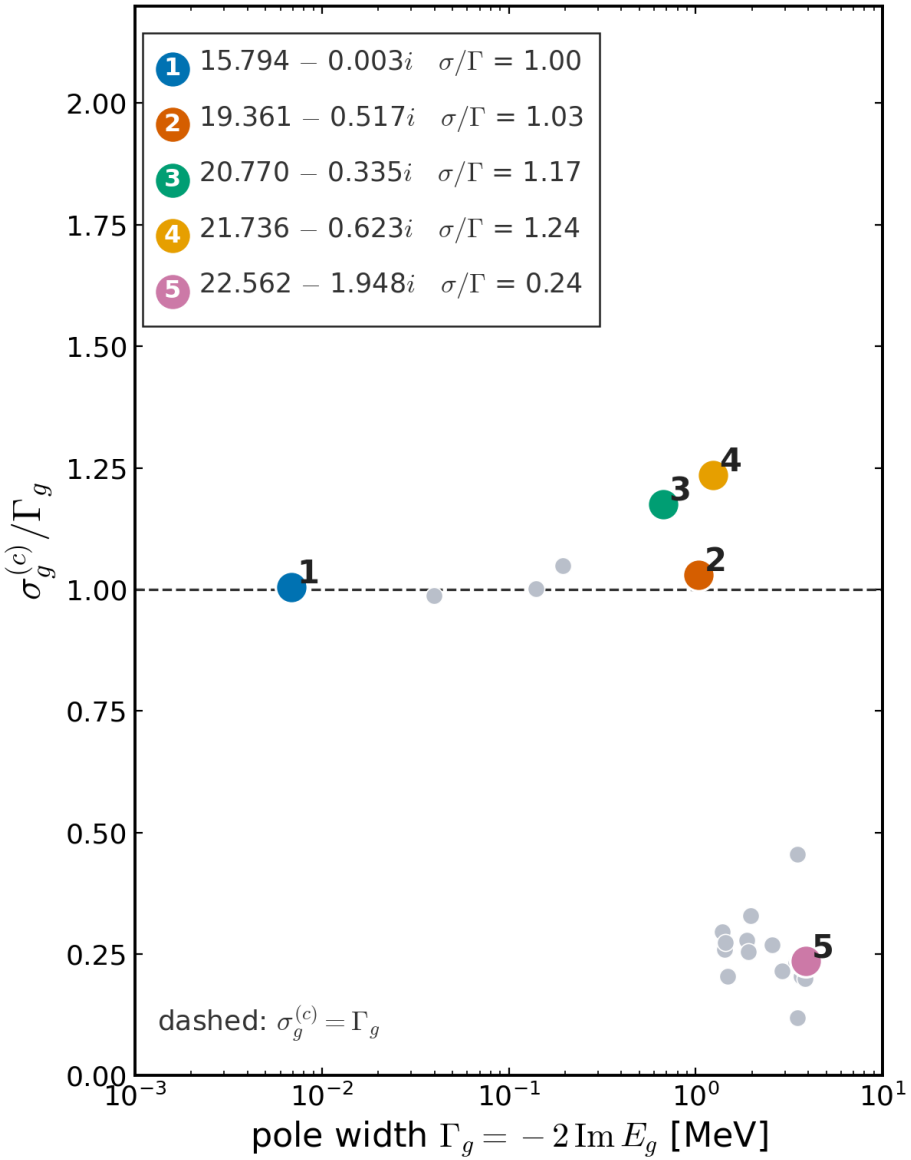
Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.23
p + ^{15}N($1p_{1/2}^{-1}$), $f_{5/2}$ wave	0.15

Numerical Results: ^{16}O ($J^\pi = 1^-$) Strength Functions & Resonance Poles

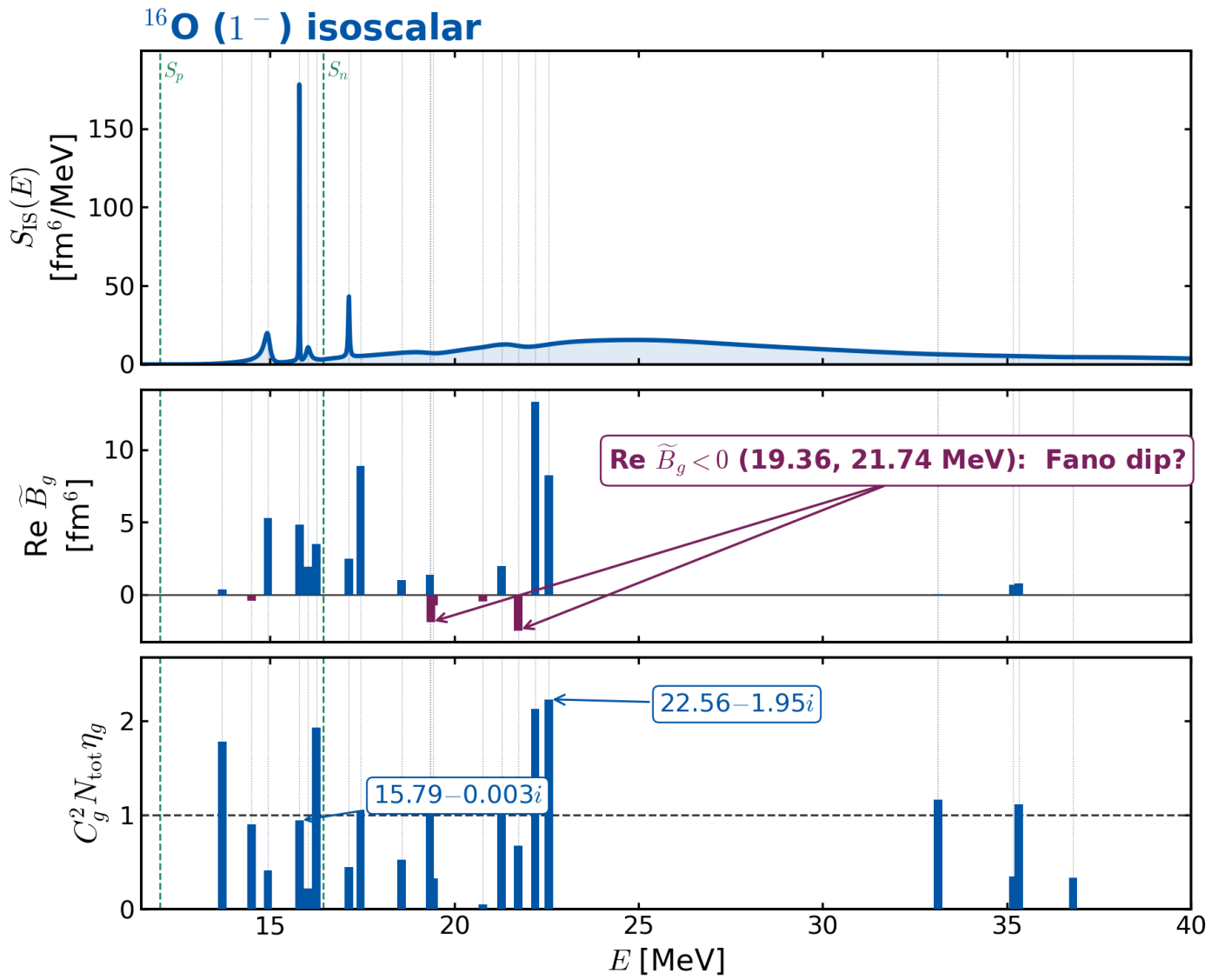
Peaks and the 22 S-Matrix Poles E_g; $\sigma_g^{(c)}/\Gamma_g \approx 1$ for the Narrow Poles and 19.36 MeV, 1.24 for the IVGDR, 0.24 for 22.56 – 1.95i MeV





Numerical Results: ^{16}O ($J^\pi = 1^-$) Collectivity Decomposition: Isoscalar

Tallest IS Peak 15.79 – 0.003i MeV Is Single-Channel-Like ($C_g^2 N_{tot} \eta_g = 0.94$); Most Collective IS Pole 22.56 – 1.95i MeV (2.2)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^2$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE \left(-\frac{1}{\pi}\,\mathrm{Im}\,\frac{\widetilde{B}_g}{E - E_g}\right)$$

strength of the pole term on the real axis
< 0: the pole lowers S_F – interference with the background (Fano dip?)

Bottom panel: collectivity measure

$$C_g^2 N_{\rm tot}\eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

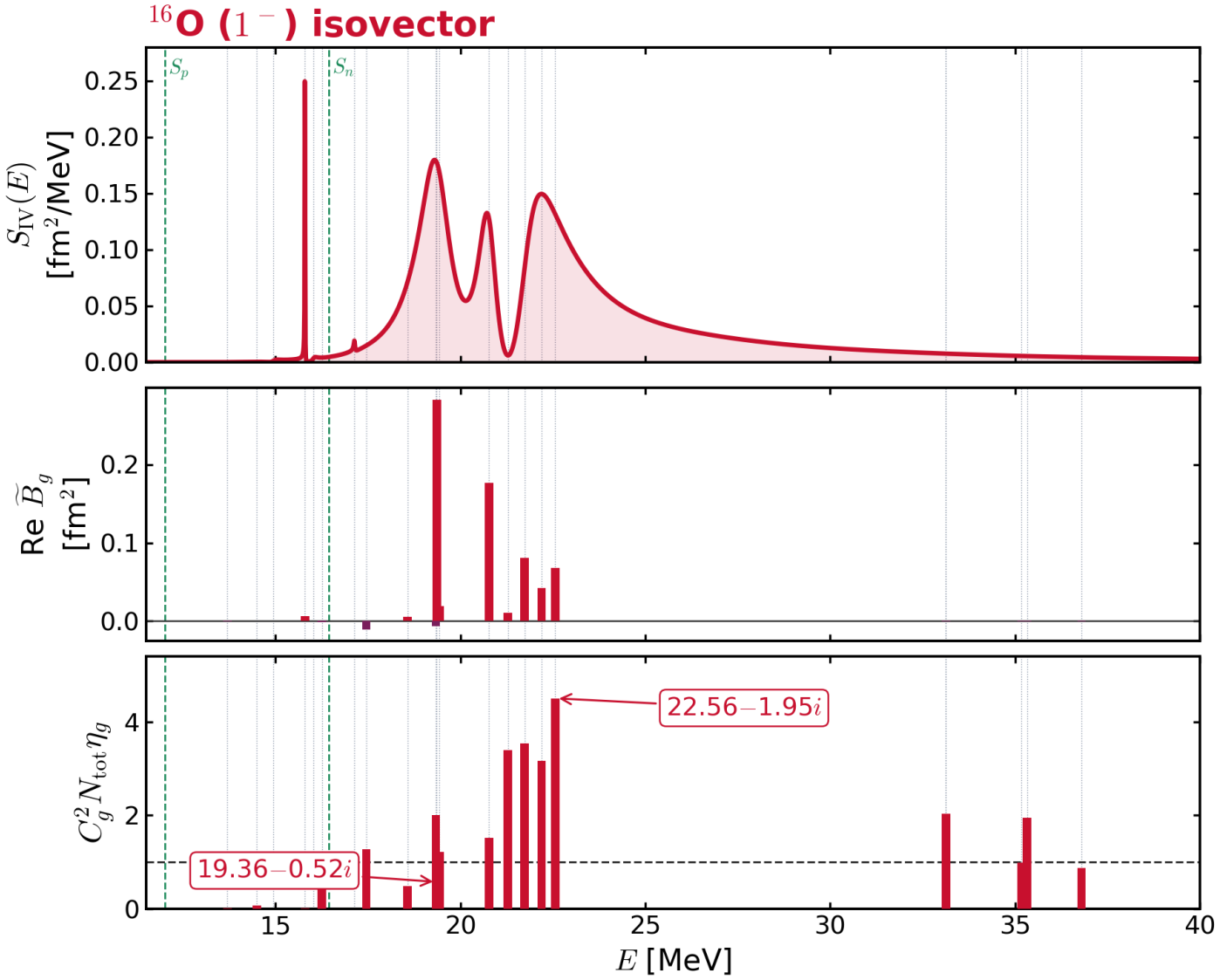
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\rm tot}\eta_g \sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\rm tot}\eta_g$

Numerical Results: ^{16}O ($J^\pi = 1^-$) Collectivity Decomposition: Isovector

Largest IV Strength 19.36 – 0.52i MeV Is Not Collective ($C_g^2 N_{tot} \eta_g = 0.58$); Most Collective IV Pole 22.56 – 1.95i MeV (4.5)



Complex transition probability of the pole

$$\widetilde{B}_g = \mathrm{Res}\left[R_F, E_g\right] = \mathcal{F}_g^2$$

Middle panel: its real part

$$\mathrm{Re}\,\widetilde{B}_g = \int dE \left(-\frac{1}{\pi}\,\mathrm{Im}\,\frac{\widetilde{B}_g}{E - E_g}\right)$$

strength of the pole term on the real axis
< 0: the pole lowers S_F – interference with the background (Fano dip?)

Bottom panel: collectivity measure

$$C_g^2 N_{\rm tot} \eta_g = \frac{\left|\sum_i F_{g,i}\right|^2}{\sum_i |F_{g,i}|^2}$$

$= 1$: one channel (dashed line)

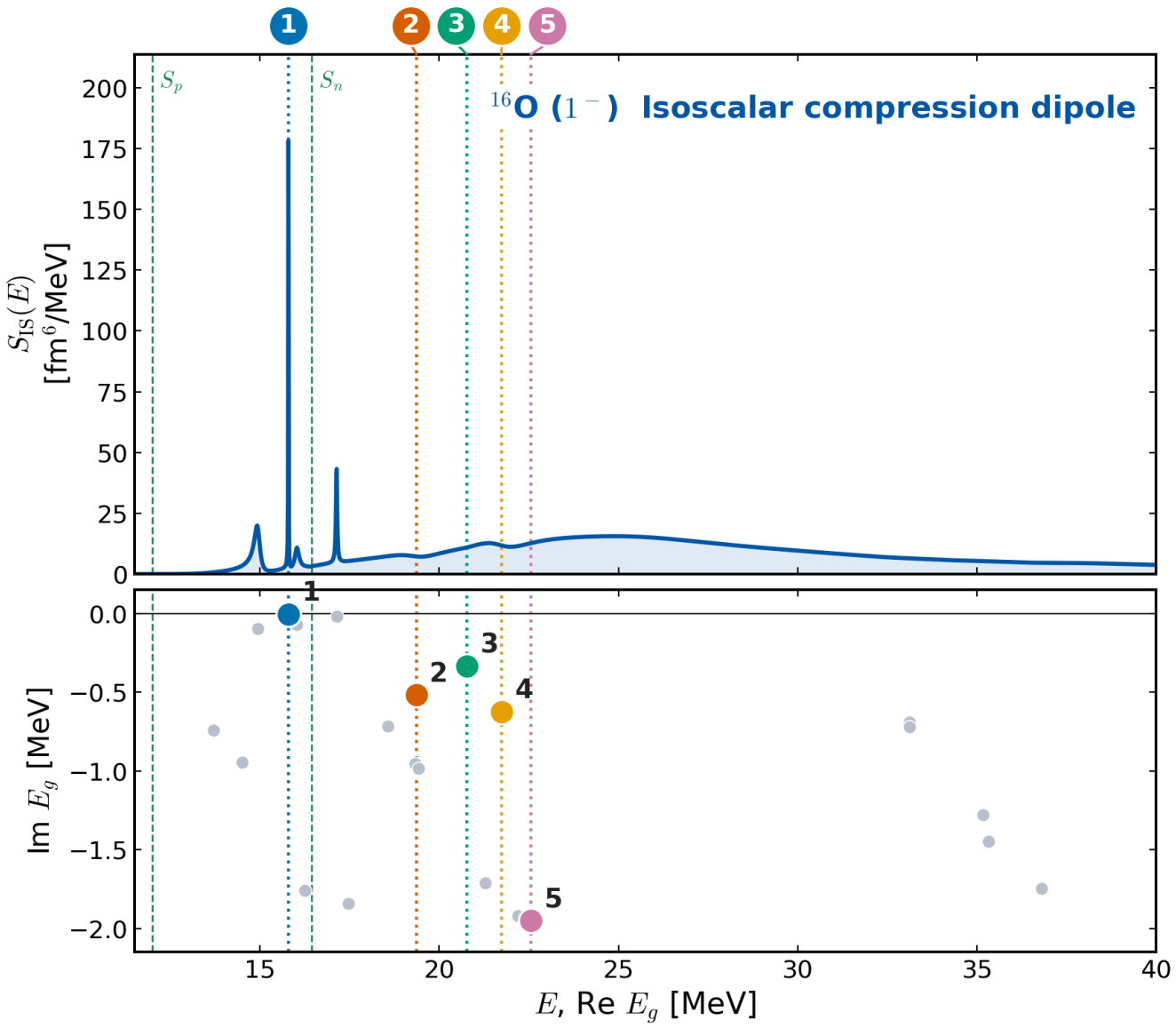
Linked through the phase $\Theta_g = \arg \mathcal{F}_g$

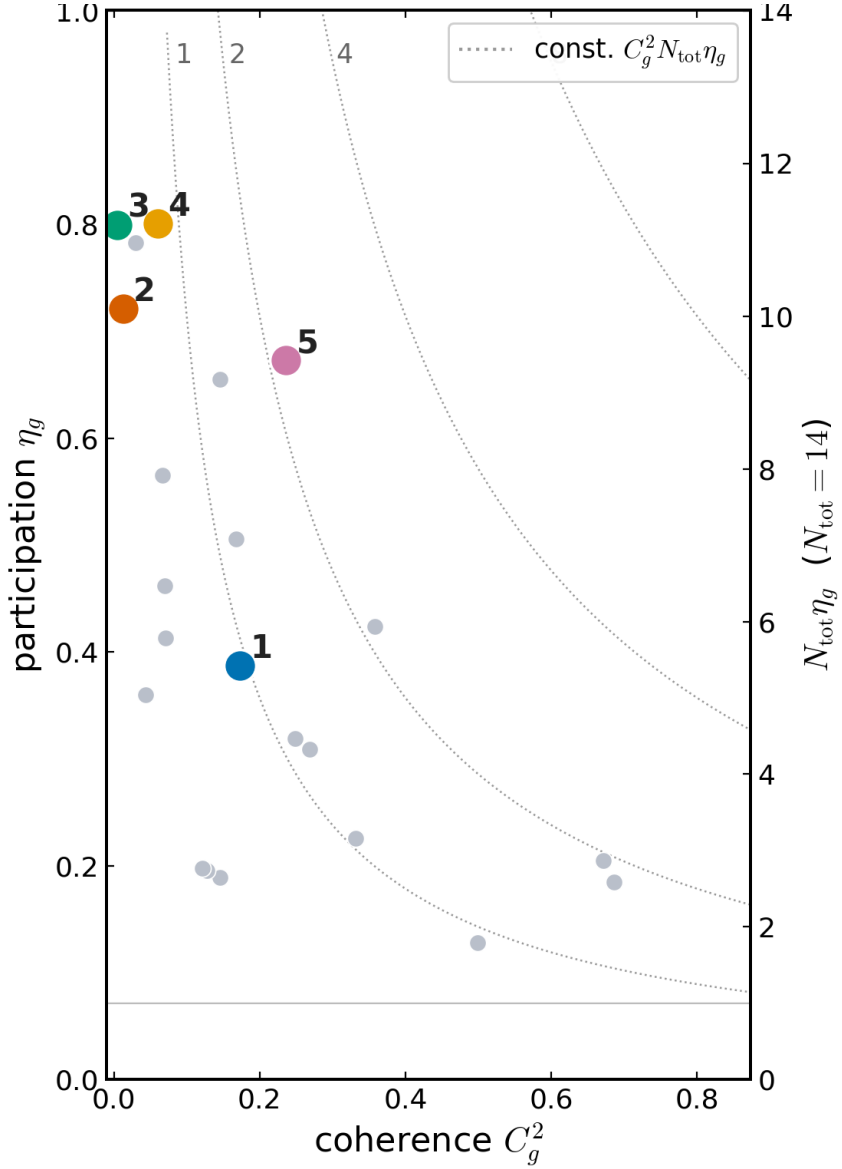
$$\mathrm{Re}\,\widetilde{B}_g = C_g^2 N_{\rm tot} \eta_g \sum_i |F_{g,i}|^2 \cos 2\Theta_g$$

$\cos 2\Theta_g < 0$: $\mathrm{Re}\,\widetilde{B}_g < 0$ even for a large $C_g^2 N_{\rm tot} \eta_g$

Collectivity Indicator Correlation: ^{16}O ($J^\pi = 1^-$) Isoscalar (IS)

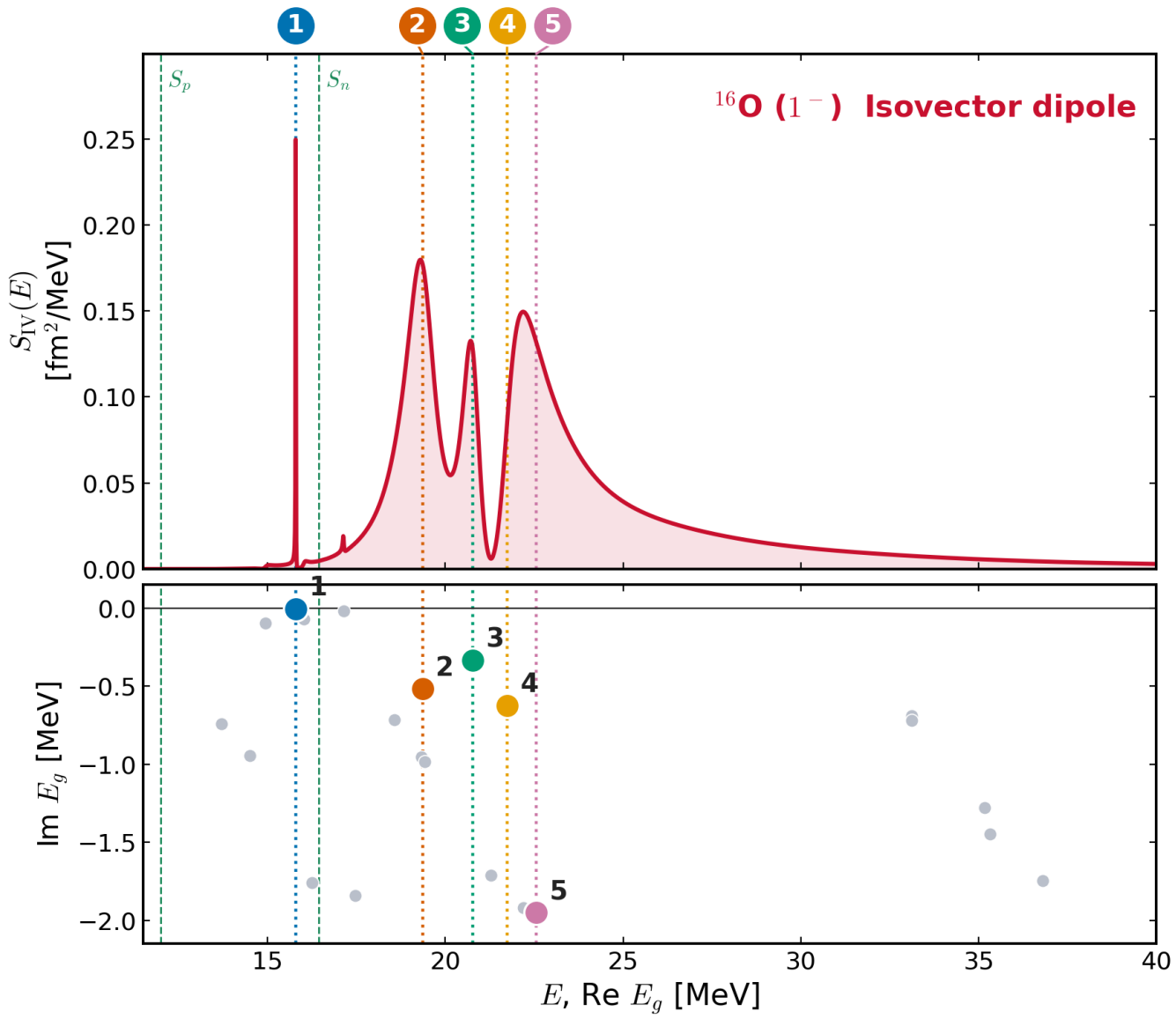
Participation η_{IS} vs. Coherence C_{IS}^2 (N_{tot} = 14); Dotted: Constant $C^2N_{tot}\,\eta$

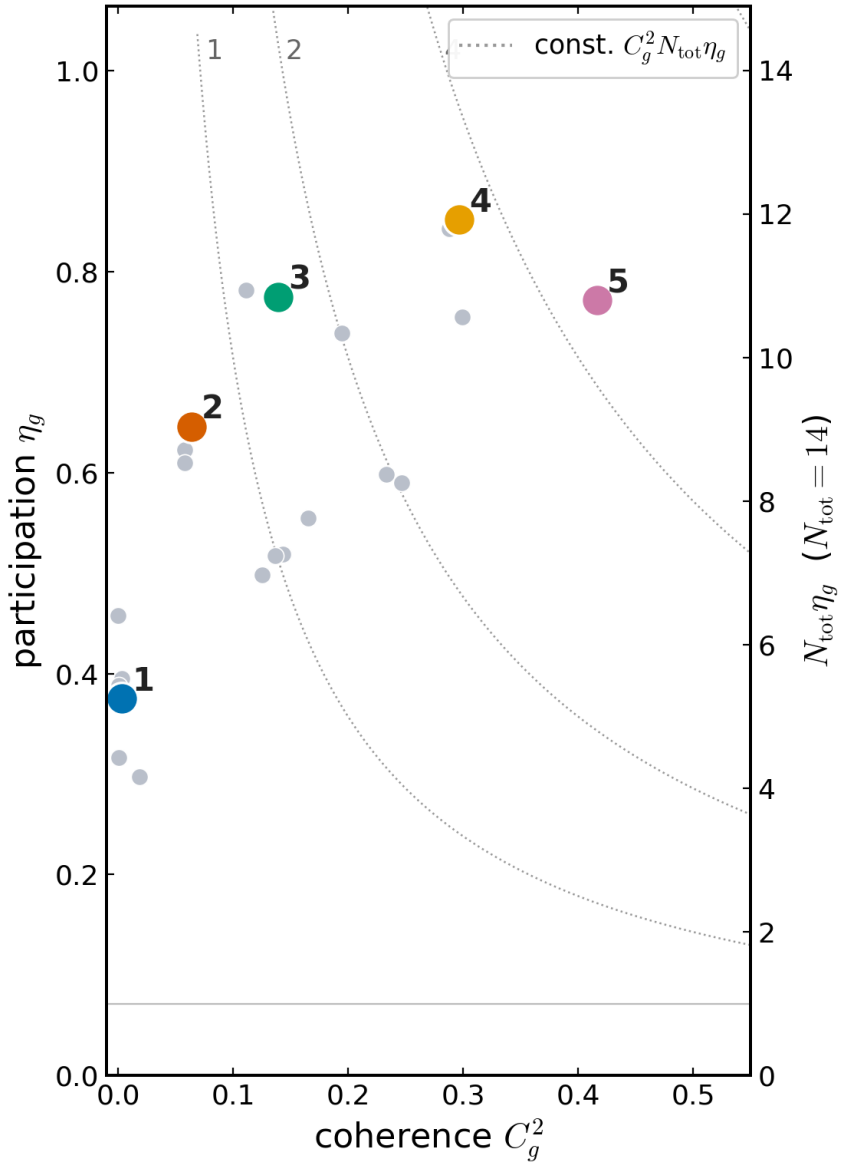




Collectivity Indicator Correlation: ^{16}O ($J^\pi = 1^-$) Isovector (IV)

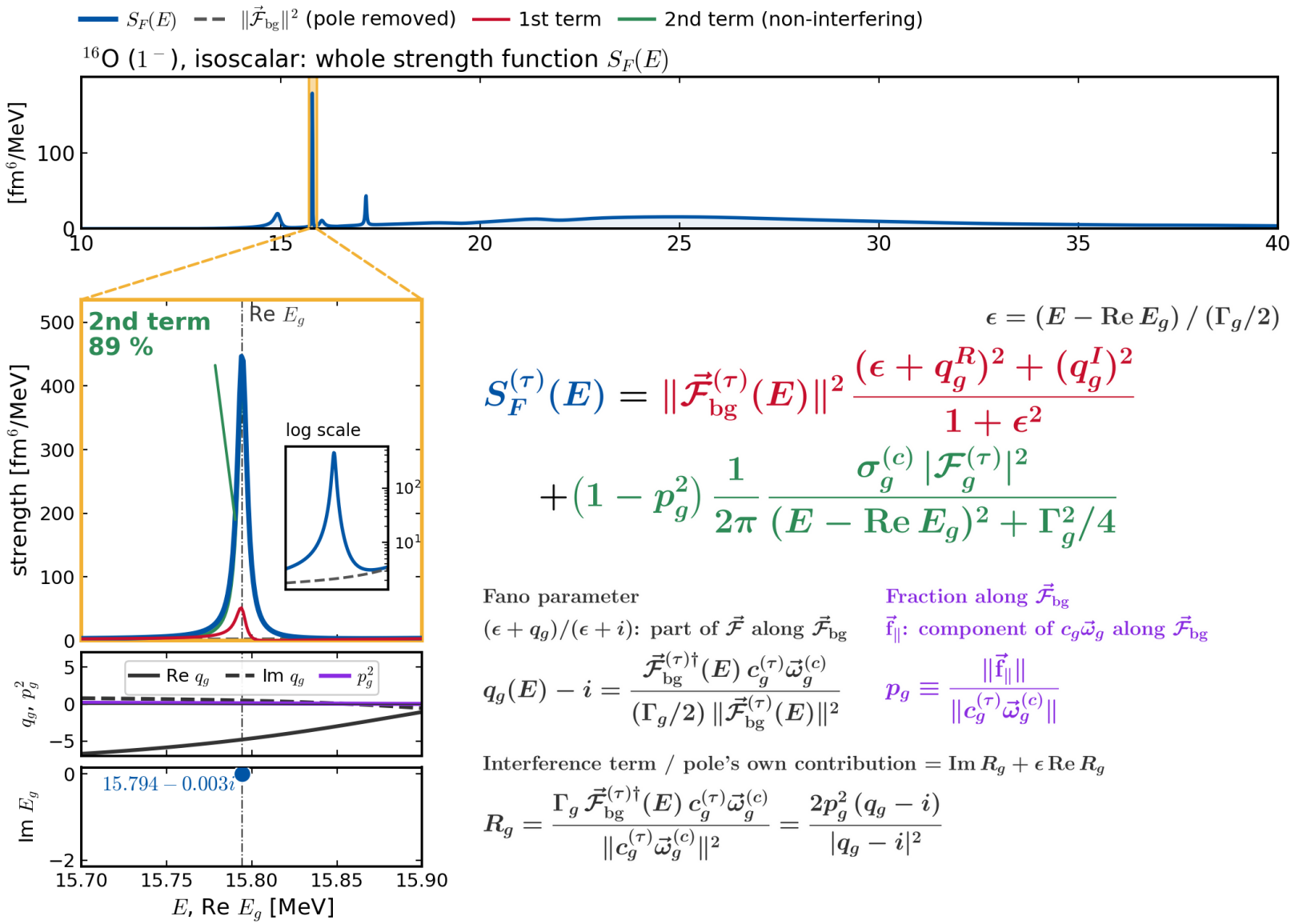
Participation η_{IV} vs. Coherence C_{IV}^2 ($N_{tot} = 14$); Dotted: Constant $C^2 N_{tot}\,\eta$





Fano Analysis: ^{16}O ($J^\pi = 1^-$) Isoscalar Peak at 15.79 MeV

Narrow Non-Collective Pole (Γ_g = 6.9 keV): $|R_g| \ll 1$, the 2nd (Breit–Wigner) Term Dominates S_F at Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$15.794 - 0.003i$ MeV

Collectivity (IS): non-collective

C_g^2	0.1734
$N_{\rm tot}\eta_g$	5.42
$C_g^2 N_{\rm tot}\eta_g$	0.94

At E = Re E_g ($\epsilon = 0$)

q_g	-4.78 + 0.49i
p_g^2	0.11
R_g	-0.045 − 0.005i
$\lvert R_g\rvert$	0.045 $\ll$ 1

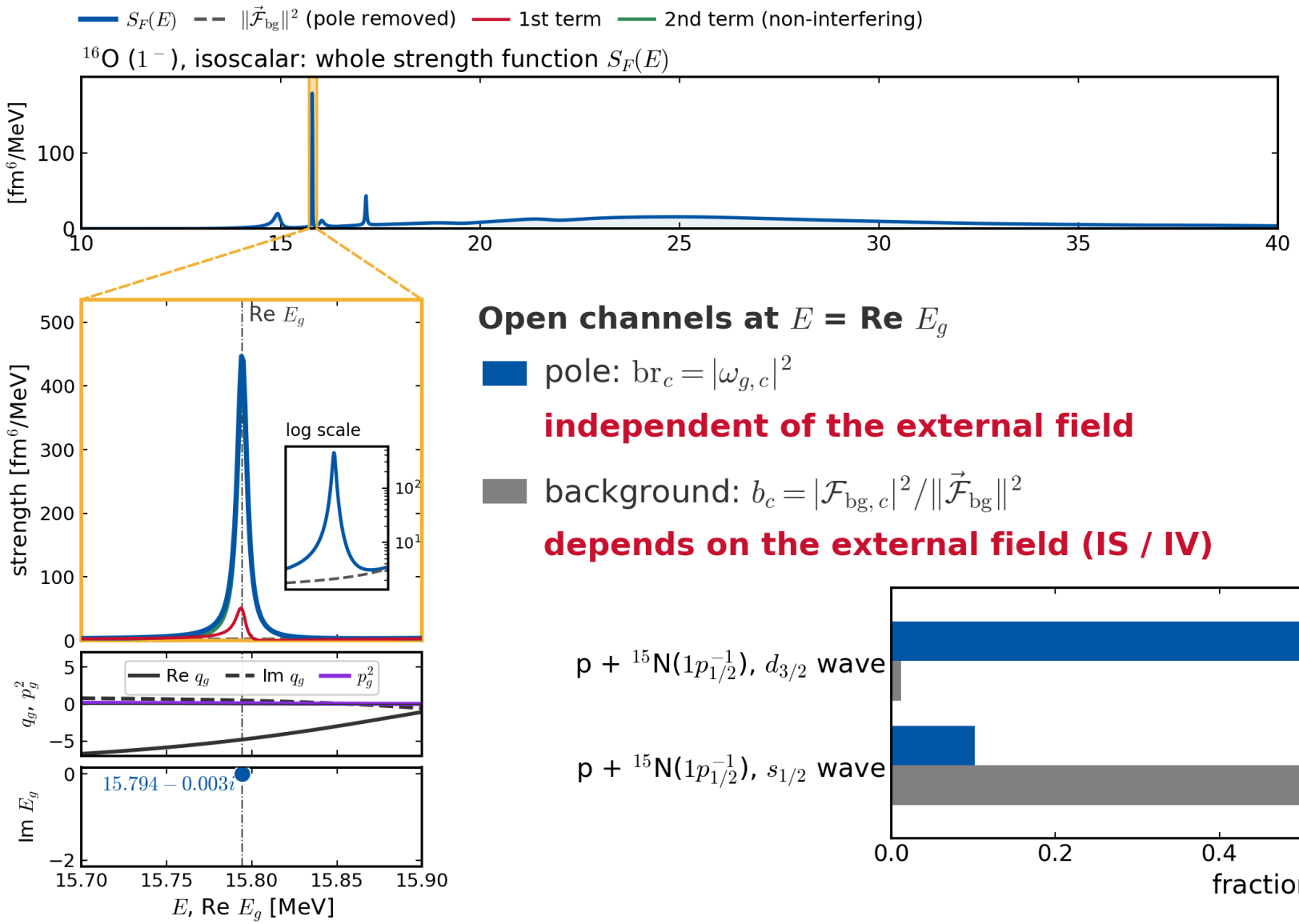
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak**

Branching Ratios: ^{16}O ($J^\pi = 1^-$) Isoscalar Peak at 15.79 MeV

The Pole Decays Mainly into p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ Wave; the IS Background Lies Mainly in p + ^{15}N($1p_{1/2}^{-1}$), $s_{1/2}$ Wave



Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$15.794 - 0.003i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.00

Pole decays into (br_c)

p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.90
p + ^{15}N($1p_{1/2}^{-1}$), $s_{1/2}$ wave	0.10

Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

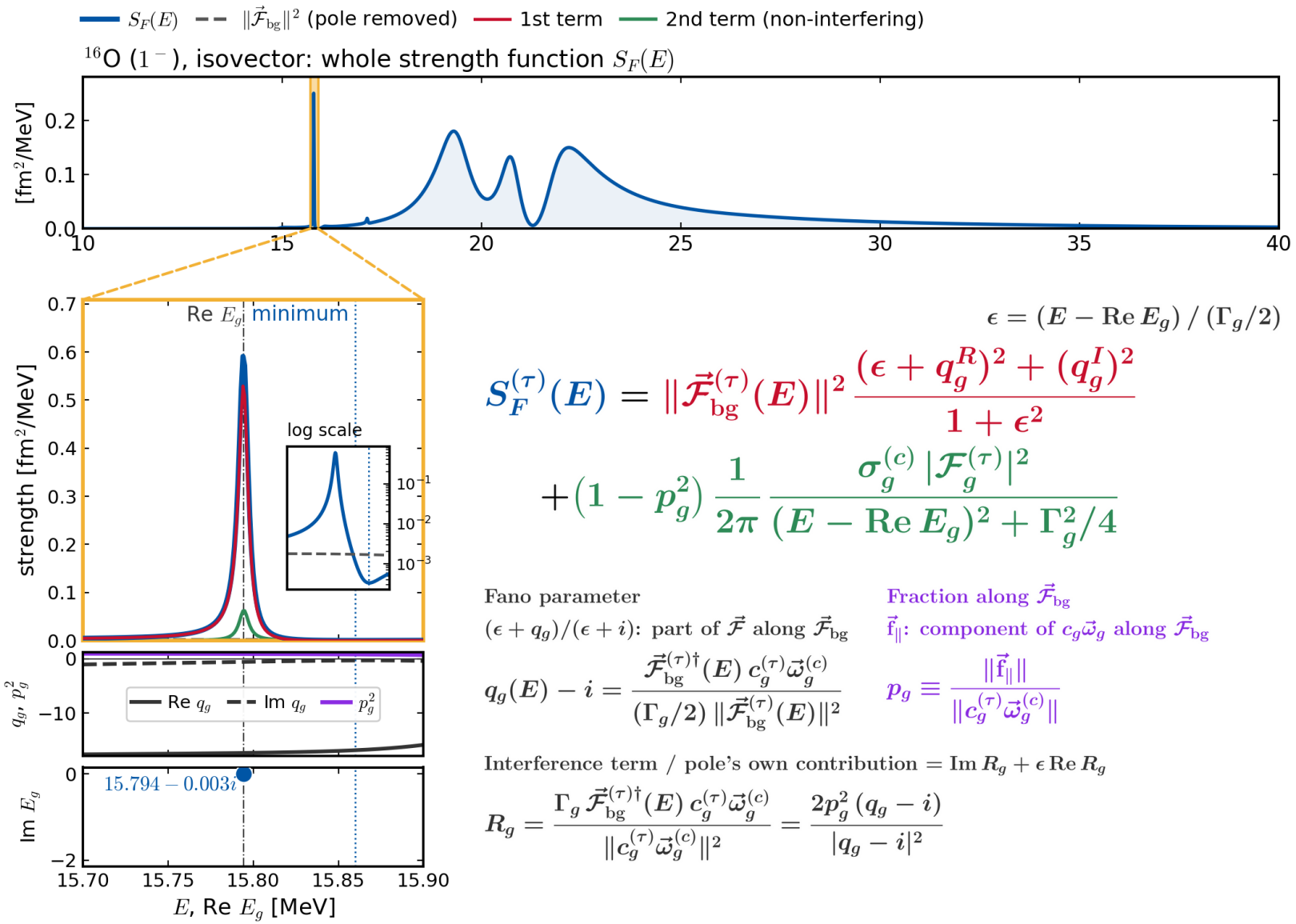
depends on the external field (IS / IV)

Background at Re E_g (b_c)

p + ^{15}N($1p_{1/2}^{-1}$), $s_{1/2}$ wave	0.99
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.01

Fano Analysis: ^{16}O ($J^\pi = 1^-$) Breit–Wigner Peak with a Large $|q_g|$

Narrow Non-Collective Pole (Γ_g = 6.9 keV): $|R_g| < 1/\sqrt{2}$, Breit–Wigner Peak; Minimum above It



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\frac{1}{2\pi}\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$15.794 - 0.003i$ MeV

Collectivity (IV): non-collective

C_g^2	0.0033
$N_{\rm tot}\eta_g$	5.25
$C_g^2 N_{\rm tot}\eta_g$	0.018

At E = Re E_g ($\epsilon = 0$)

q_g	$-17.34 - 0.58i$
p_g^2	0.89
R_g	$-0.10 - 0.01i$
$\lvert R_g\rvert$	$0.10 < 1/\sqrt{2}$

pole's own contribution dominates

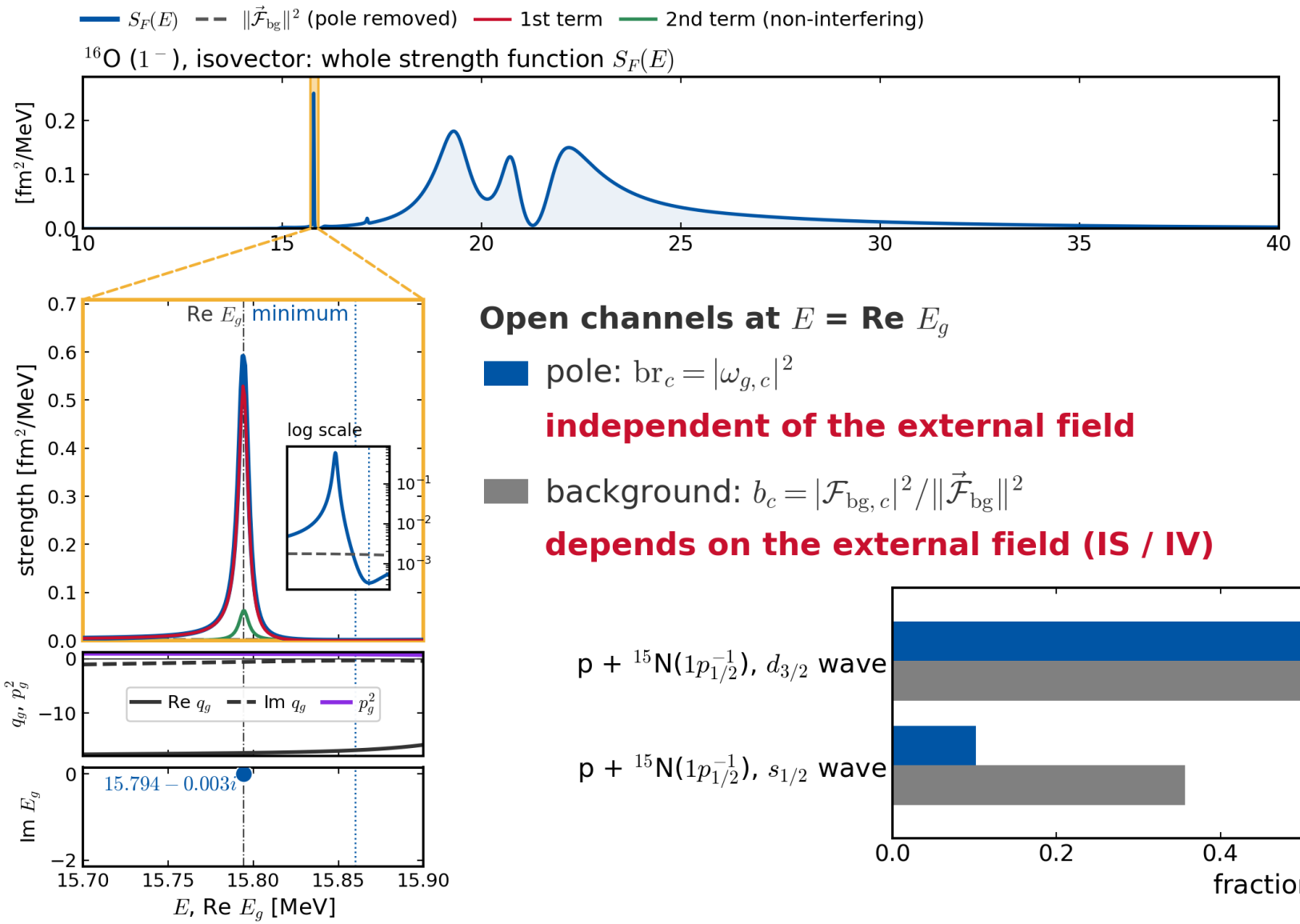
Im $R_g < 0$: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak with a large $|q_g|$**

minimum at 15.86 MeV

Branching Ratios: ^{16}O ($J^\pi = 1^-$) Breit–Wigner Peak with a Large $|q_g|$

Same br_c as in the IS Field; the IV Background Lies Mainly in p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ Wave, the Main Decay Channel of the Pole



Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$15.794 - 0.003i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.00

Pole decays into (br_c)

p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.90
p + ^{15}N($1p_{1/2}^{-1}$), $s_{1/2}$ wave	0.10

Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

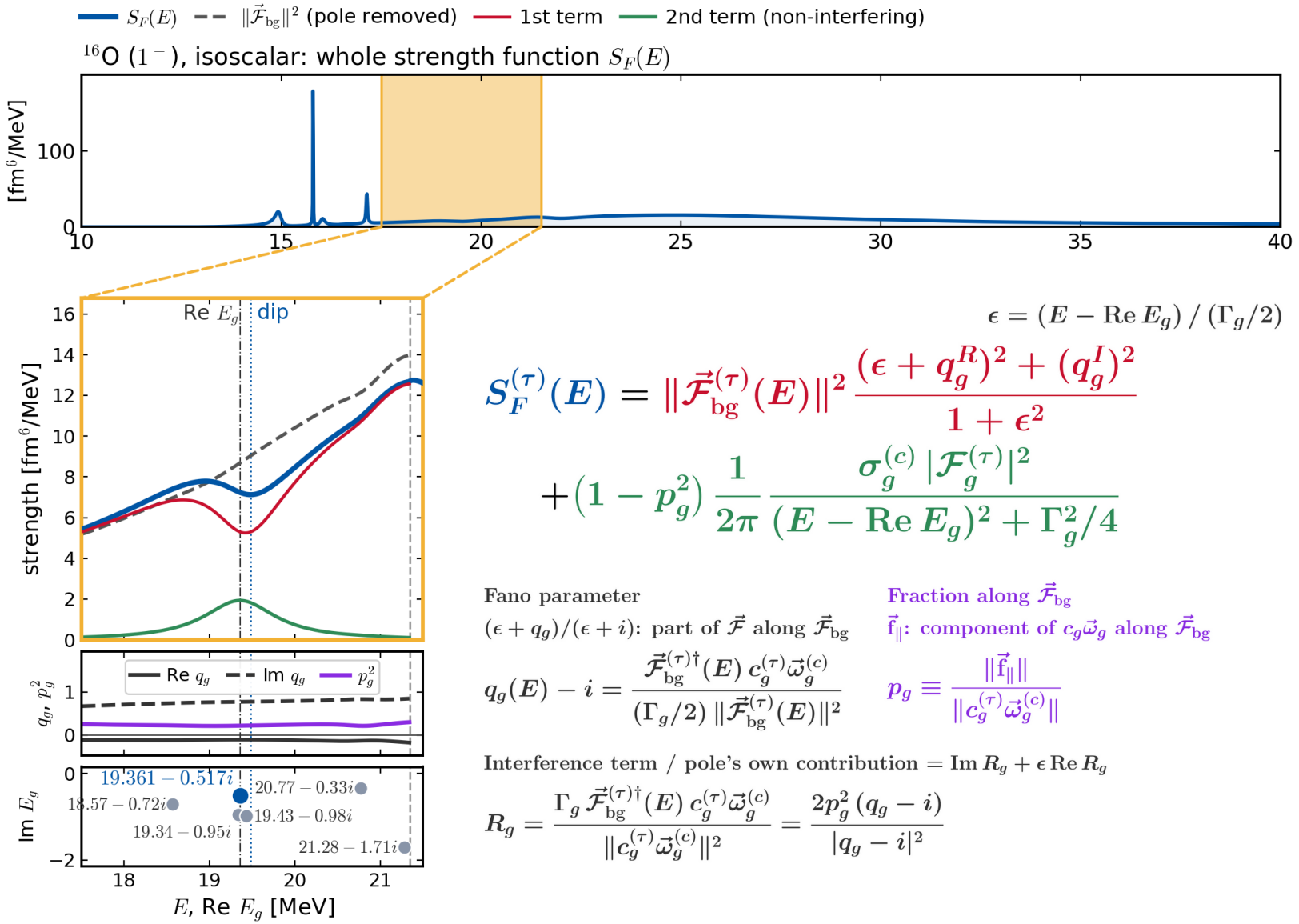
depends on the external field (IS / IV)

Background at Re E_g (b_c)

p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.64
p + ^{15}N($1p_{1/2}^{-1}$), $s_{1/2}$ wave	0.36

Fano Analysis: ^{16}O ($J^\pi = 1^-$) Pole 19.36 – 0.52i MeV in the Isoscalar Strength

Non-Collective Pole (Γ_g = 1.0 MeV): $|R_g| > 1$ and Im $R_g < -1$ → S_F below the Background, Shallow Dip



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	19.361 − 0.517i MeV

Collectivity (IS): non-collective

C_g^2	0.0130
$N_{\rm tot}\eta_g$	10.10
$C_g^2 N_{\rm tot}\eta_g$	0.13

At E = Re E_g ($\epsilon = 0$)

q_g	-0.10 + 0.77i
p_g^2	0.22
R_g	-0.73 − 1.59i
$\|R_g\|$	1.75 > 1

strong interference

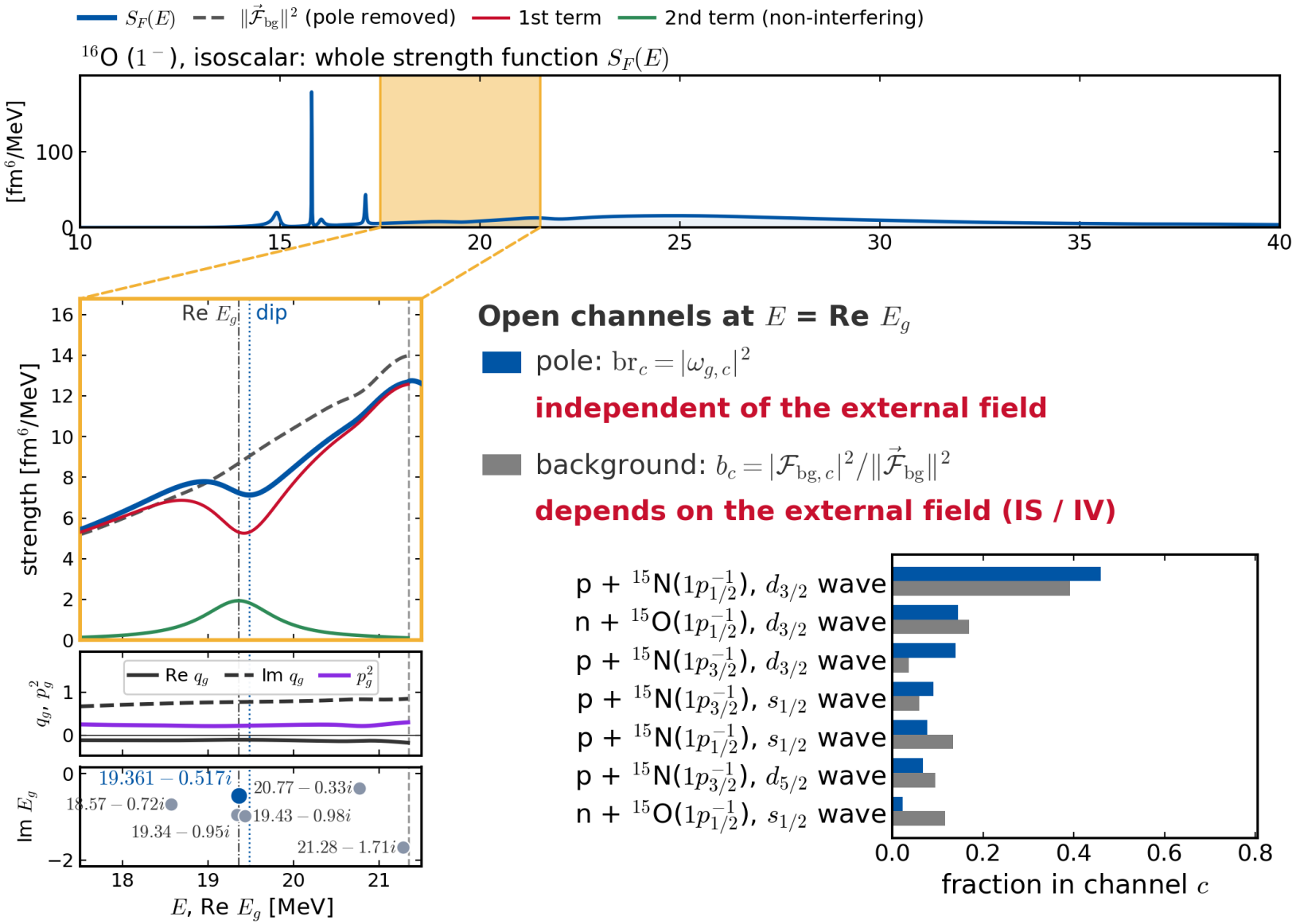
Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ **shallow dip**

dip at 19.48 MeV

Branching Ratios: ^{16}O ($J^\pi = 1^-$) Pole 19.36 – 0.52i MeV in the Isoscalar Strength

The Pole Decays into Several Channels (Largest: p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ Wave); the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$19.361 - 0.517i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.03

Pole decays into (br_c)

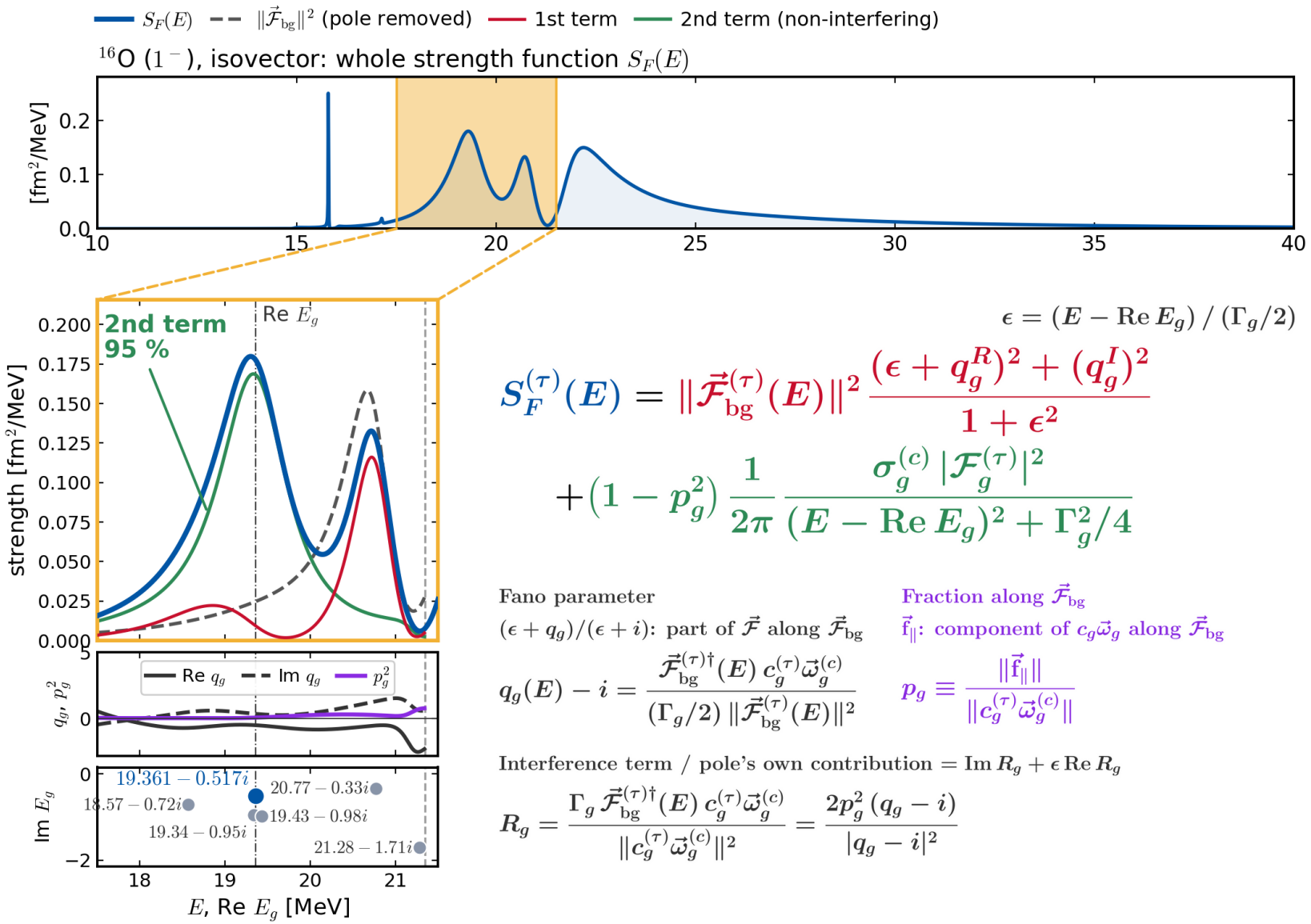
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.46
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.14

Background at Re E_g (b_c)

p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.39
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.17

Fano Analysis: ^{16}O ($J^\pi = 1^-$) Isovector Peak at 19.30 MeV

Non-Collective Pole: $|R_g| < 1/\sqrt{2}$, the Non-Interfering Breit–Wigner Term Dominates S_F at Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	19.361 − 0.517i MeV

Collectivity (IV): non-collective

C_g^2	0.0638
$N_{\rm tot}\eta_g$	9.04
$C_g^2 N_{\rm tot}\eta_g$	0.58

At E = Re E_g ($\epsilon = 0$)

q_g	-0.49 + 0.37i
p_g^2	0.09
R_g	-0.13 − 0.17i
$\lvert R_g\rvert$	0.22 < $1/\sqrt{2}$

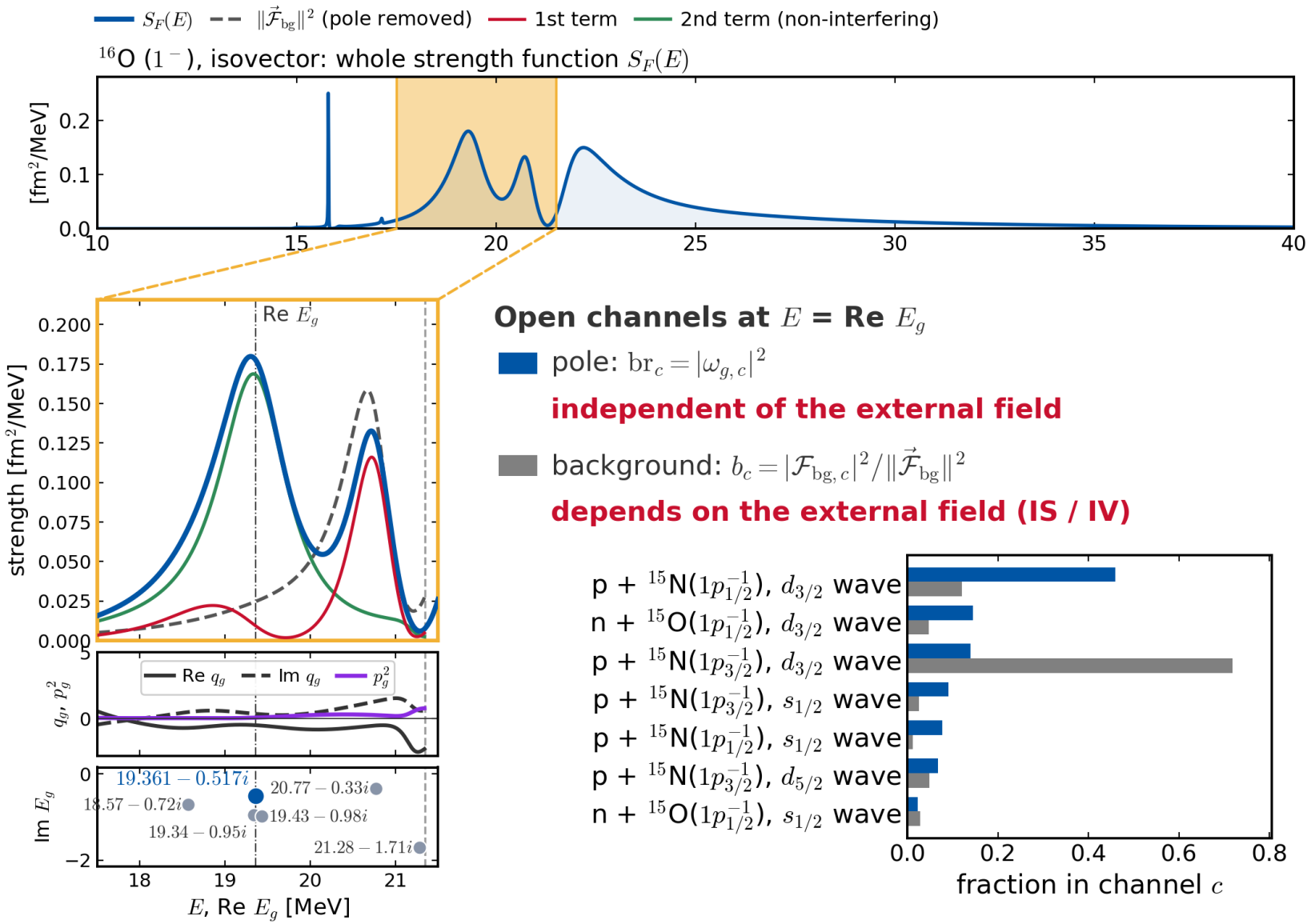
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak**

Branching Ratios: ^{16}O ($J^\pi = 1^-$) Isovector Peak at 19.30 MeV

Same br$_c$ as in the IS Field; the IV Background Lies Mainly in p + ^{15}N(1p$_{3/2}^{-1}$), d$_{3/2}$ Wave



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$19.361 - 0.517i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.03

Pole decays into (br_c)

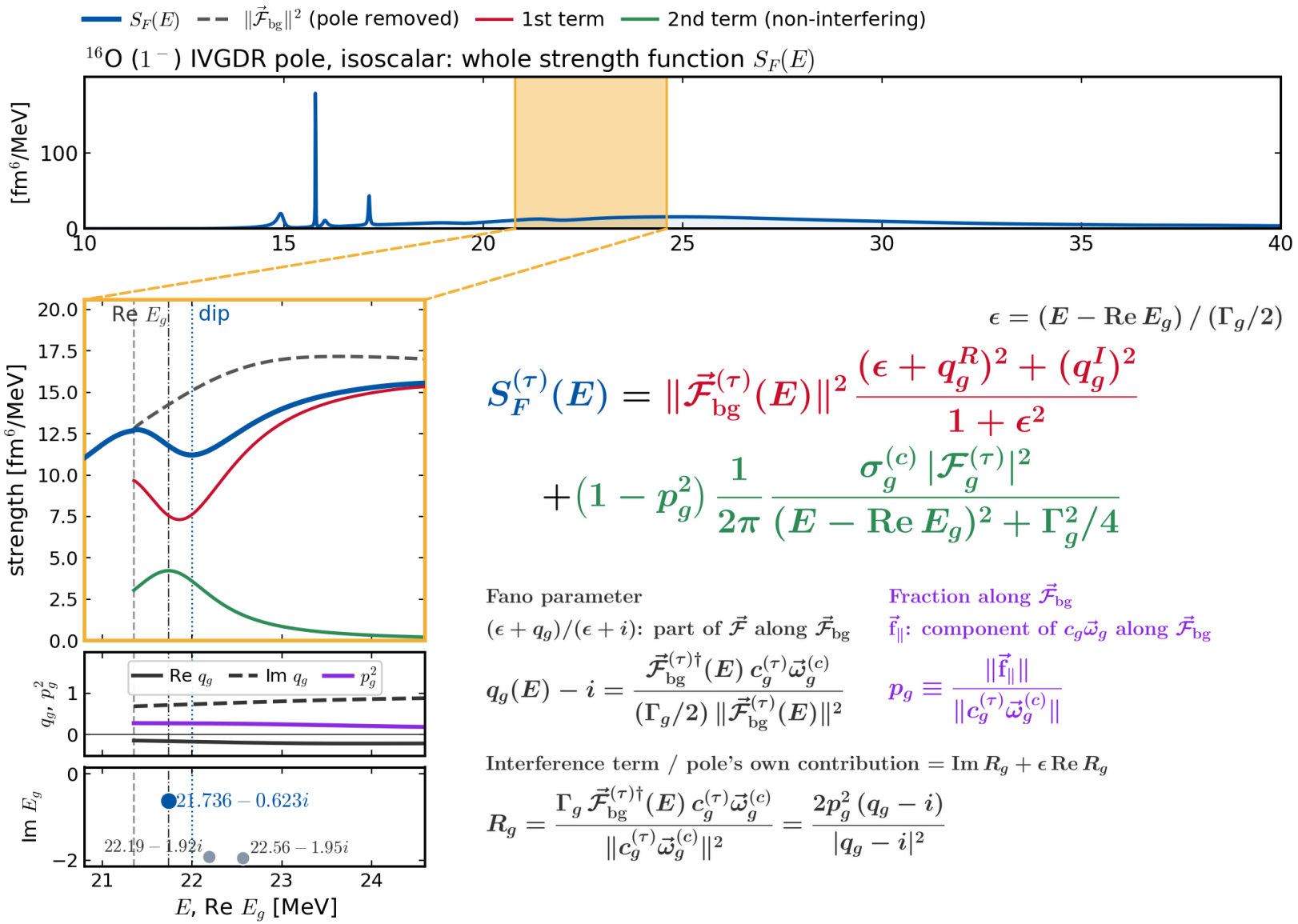
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.46
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.14

Background at Re E_g (b_c)

p + ^{15}N($1p_{3/2}^{-1}$), $d_{3/2}$ wave	0.72
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.12

Fano Analysis: ^{16}O ($J^\pi = 1^-$) IVGDR Pole in the Isoscalar Strength

Isoscalar Field (Non-Collective): $|R_g| > 1$ and Im $R_g < -1$ → S_F below the Background, Shallow Dip



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\frac{1}{2\pi}\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.736 - 0.623i$ MeV

Collectivity (IS): non-collective

C_g^2	0.0604
$N_{\rm tot}\eta_g$	11.21
$C_g^2 N_{\rm tot}\eta_g$	0.68

At E = Re E_g ($\epsilon = 0$)

q_g	-0.16 + 0.71i
p_g^2	0.27
R_g	-0.79 − 1.43i
$\|R_g\|$	1.63 > 1

strong interference

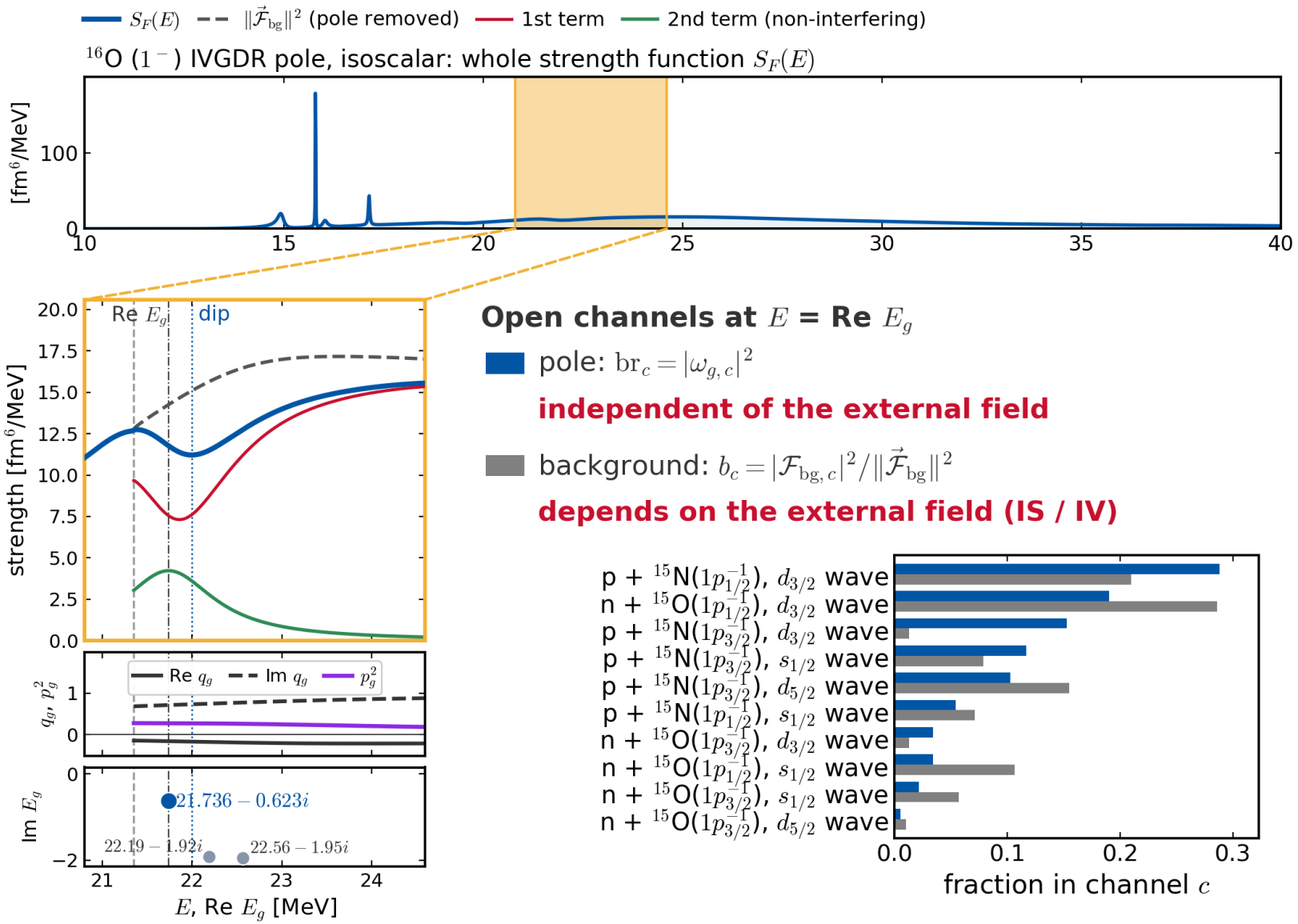
Im $R_g < -1$: S_F below the background at $\epsilon = 0$

⇒ **shallow dip**

dip at 22.00 MeV

Branching Ratios: ^{16}O ($J^\pi = 1^-$) IVGDR Pole in the Isoscalar Strength

The Pole Decays into Several Channels (Largest: p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ Wave); the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.736 - 0.623i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.24

Pole decays into (br_c)

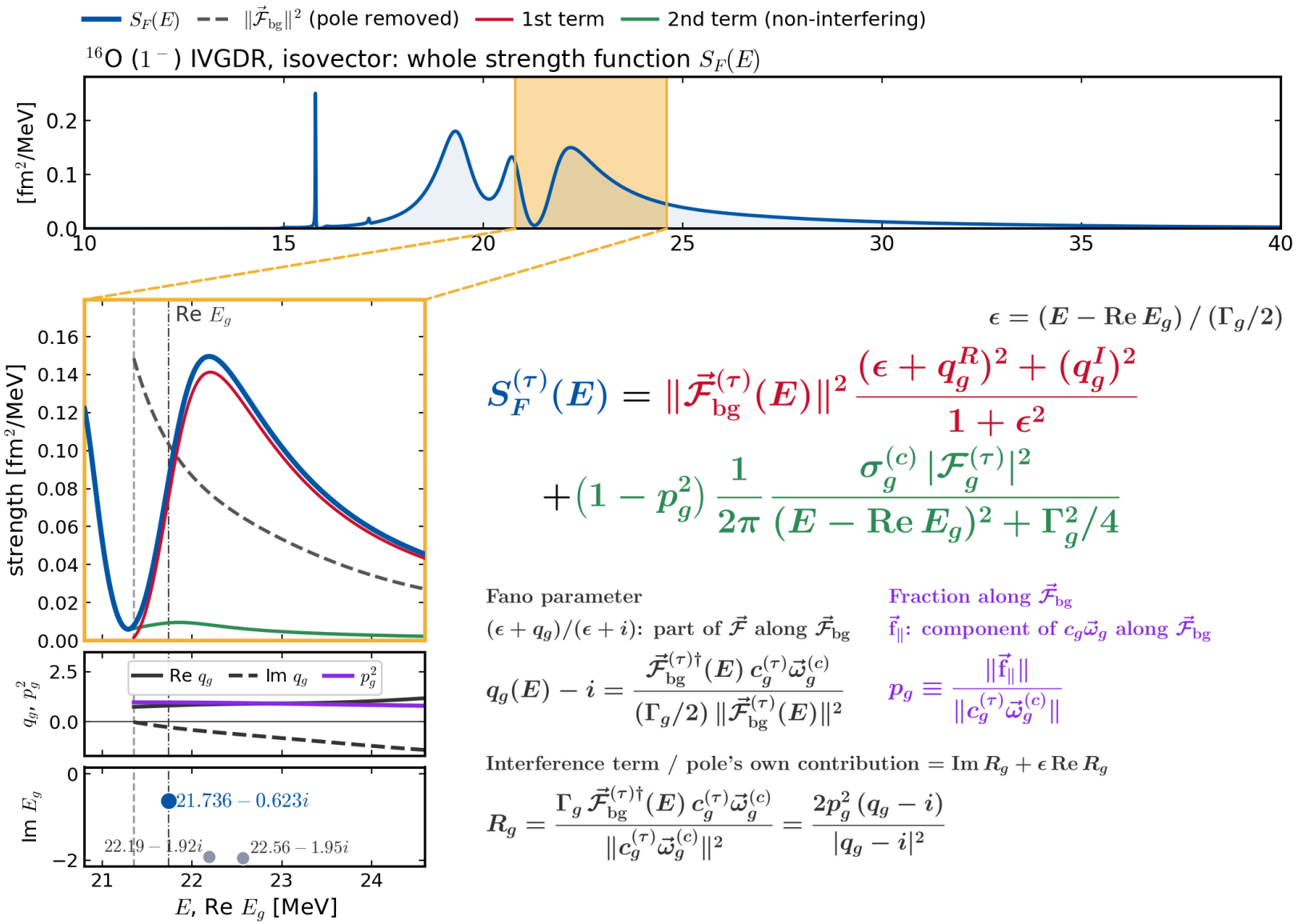
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.29
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.19

Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.29
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.21

Fano Analysis: ^{16}O ($J^\pi = 1^-$) Isovector Giant Dipole Resonance

$|R_g| > 1$: Interference Exceeds the Pole's Own Contribution; S_F below the Background at Re E_g, Peak at 22.19 MeV



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.736 - 0.623i$ MeV

Collectivity (IV): collective

C_g^2	0.2967
$N_{\rm tot}\eta_g$	11.92
$C_g^2 N_{\rm tot}\eta_g$	**3.54**

At E = Re E_g ($\epsilon = 0$)

q_g	$0.80 - 0.28i$
p_g^2	0.96
R_g	$0.68 - 1.08i$
$\|R_g\|$	1.28 > 1

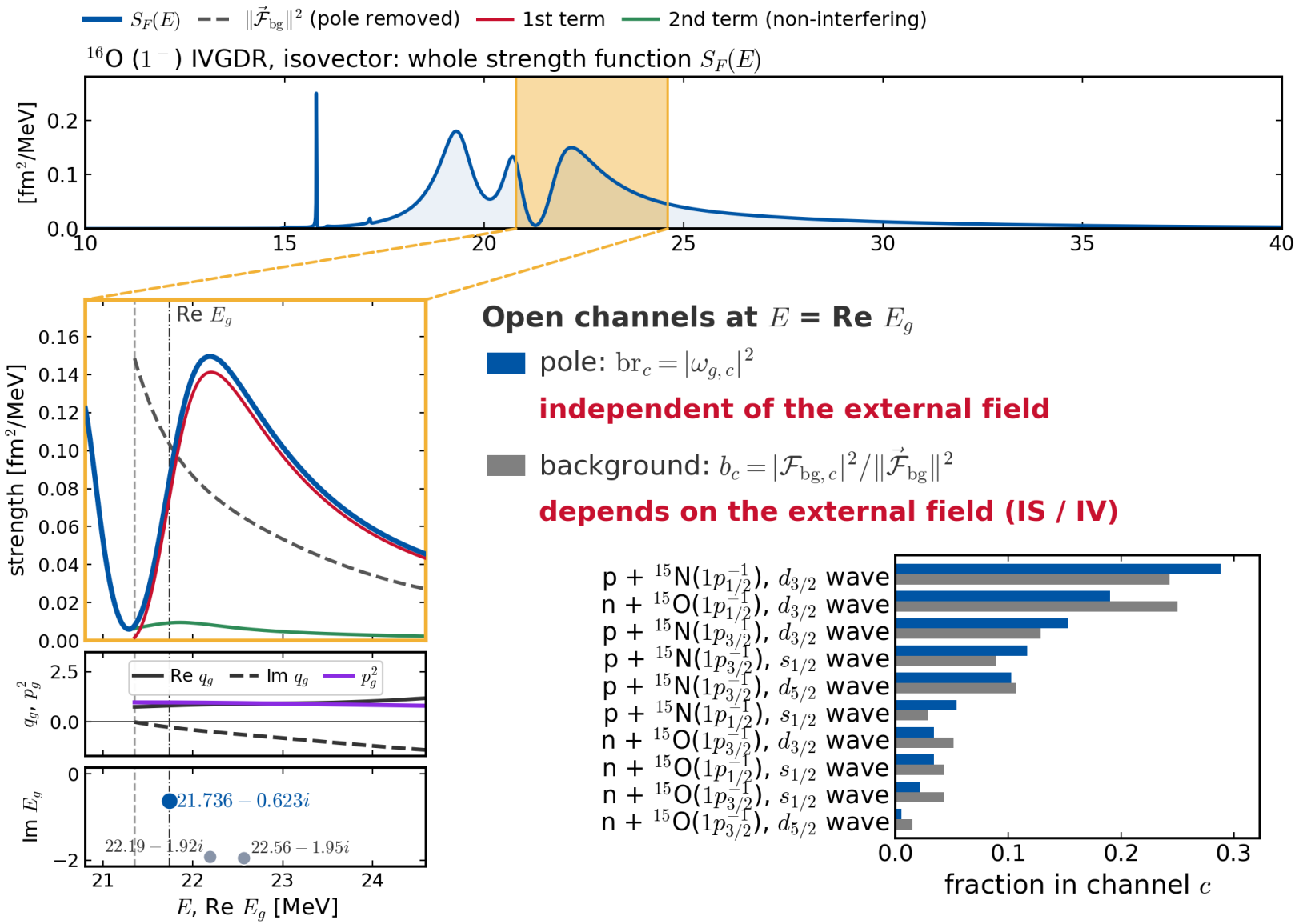
strong interference

Im $R_g < -1$: S_F below the background at $\epsilon = 0$

$\Rightarrow$ **asymmetric Fano profile**

Branching Ratios: ^{16}O ($J^\pi = 1^-$) Isovector Giant Dipole Resonance

Same br$_c$ as in the IS Field; the IV Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.736 - 0.623i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.24

Pole decays into (br_c)

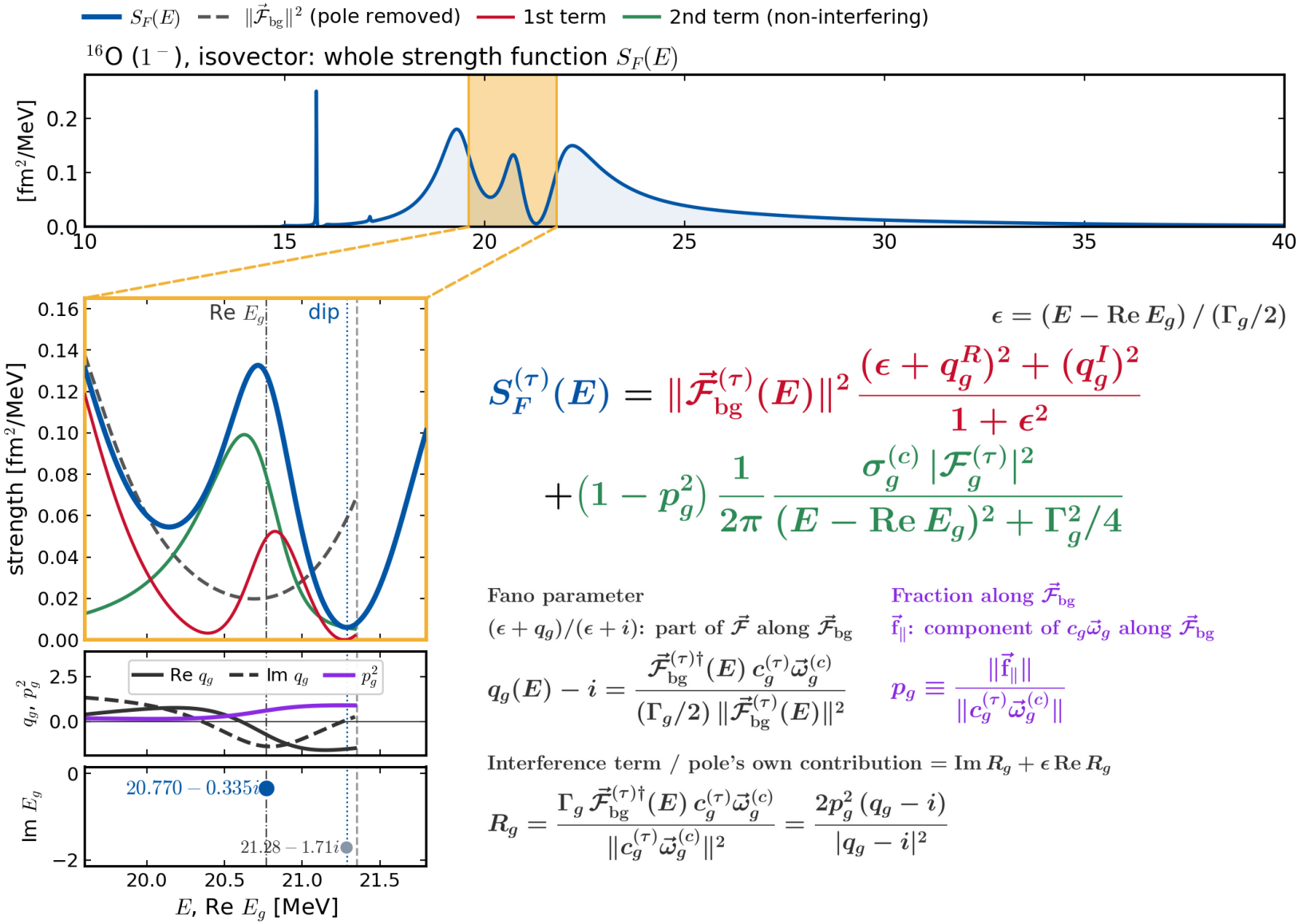
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.29
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.19

Background at Re E_g (b_c)

n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.25
p + ^{15}N($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.24

Fano Analysis: ^{16}O ($J^\pi = 1^-$) Isovector Peak at 20.71 MeV & Dip at 21.29 MeV

Collective Pole: $|R_g| < 1/\sqrt{2}$, Peak Mainly the Pole's Own Contribution; Dip above It



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\frac{1}{2\pi}\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$20.770 - 0.335i$ MeV

Collectivity (IV): collective

C_g^2	0.1393
$N_{\rm tot}\eta_g$	10.85
$C_g^2 N_{\rm tot}\eta_g$	1.51

At E = Re E_g ($\epsilon = 0$)

q_g	$-0.73 - 1.38i$
p_g^2	0.61
R_g	$-0.15 - 0.47i$
$\lvert R_g\rvert$	$0.49 < 1/\sqrt{2}$

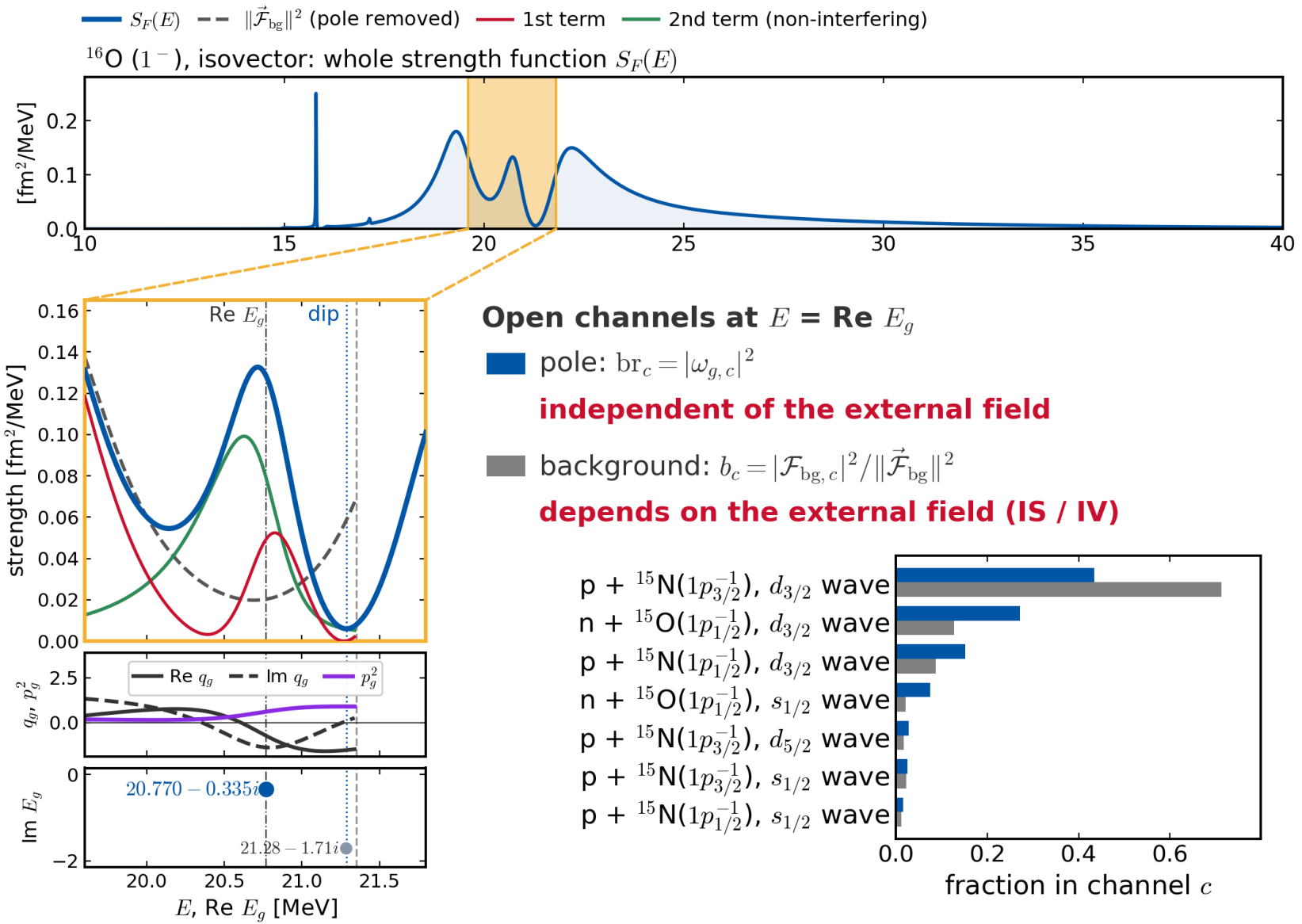
pole's own contribution dominates

Im $R_g < 0$: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **peak and dip**

Branching Ratios: ^{16}O ($J^\pi = 1^-$) Isovector Peak at 20.71 MeV & Dip at 21.29 MeV

The Pole Decays into Several Channels; the IV Background Lies Mainly in p + ^{15}N($1p_{3/2}^{-1}$), $d_{3/2}$ Wave, the Largest Decay Channel of the Pole



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$20.770 - 0.335i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.17

Pole decays into (br_c)

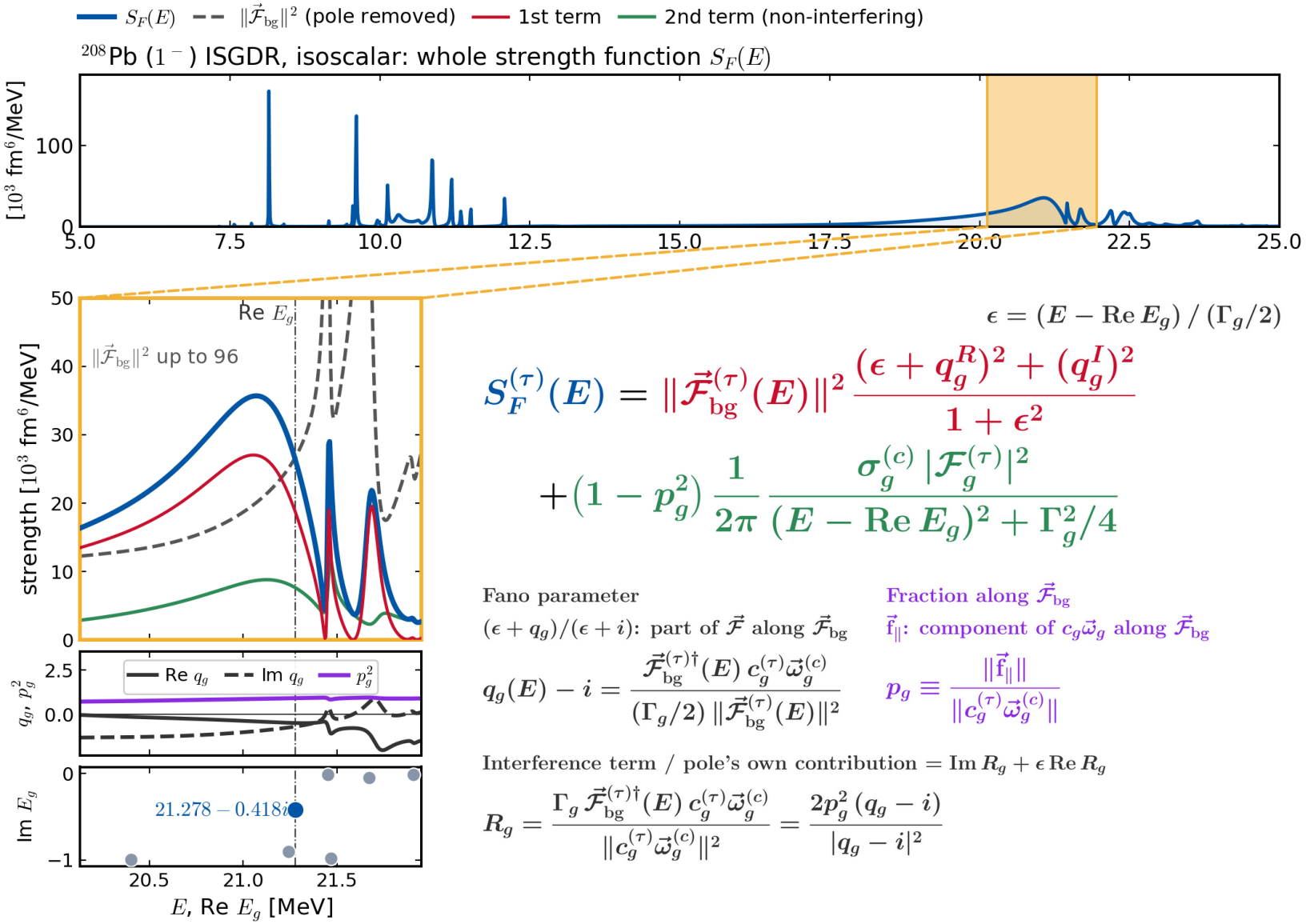
p + ^{15}N($1p_{3/2}^{-1}$), $d_{3/2}$ wave	0.43
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.27

Background at Re E_g (b_c)

p + ^{15}N($1p_{3/2}^{-1}$), $d_{3/2}$ wave	0.71
n + ^{15}O($1p_{1/2}^{-1}$), $d_{3/2}$ wave	0.13

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Giant Dipole Resonance

Most Collective IS Pole: $|R_g| \approx 1$, Interference as Large as the Pole's Own Contribution → $S_F \approx$ Background at Re E_g, Maximum below It



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\, E_g)^2 + \Gamma_g^2/4}$$

Fano parameter
$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$
$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.278 - 0.418i$ MeV

Collectivity (IS): collective

C_g^2	0.3744
$N_{\rm tot}\eta_g$	33.73
$C_g^2 N_{\rm tot}\eta_g$	**12.63**

At E = Re E_g ($\epsilon = 0$)

q_g	$-0.48 - 0.68i$
p_g^2	0.91
R_g	$-0.29 - 1.00i$
$\|R_g\|$	1.05 > 1

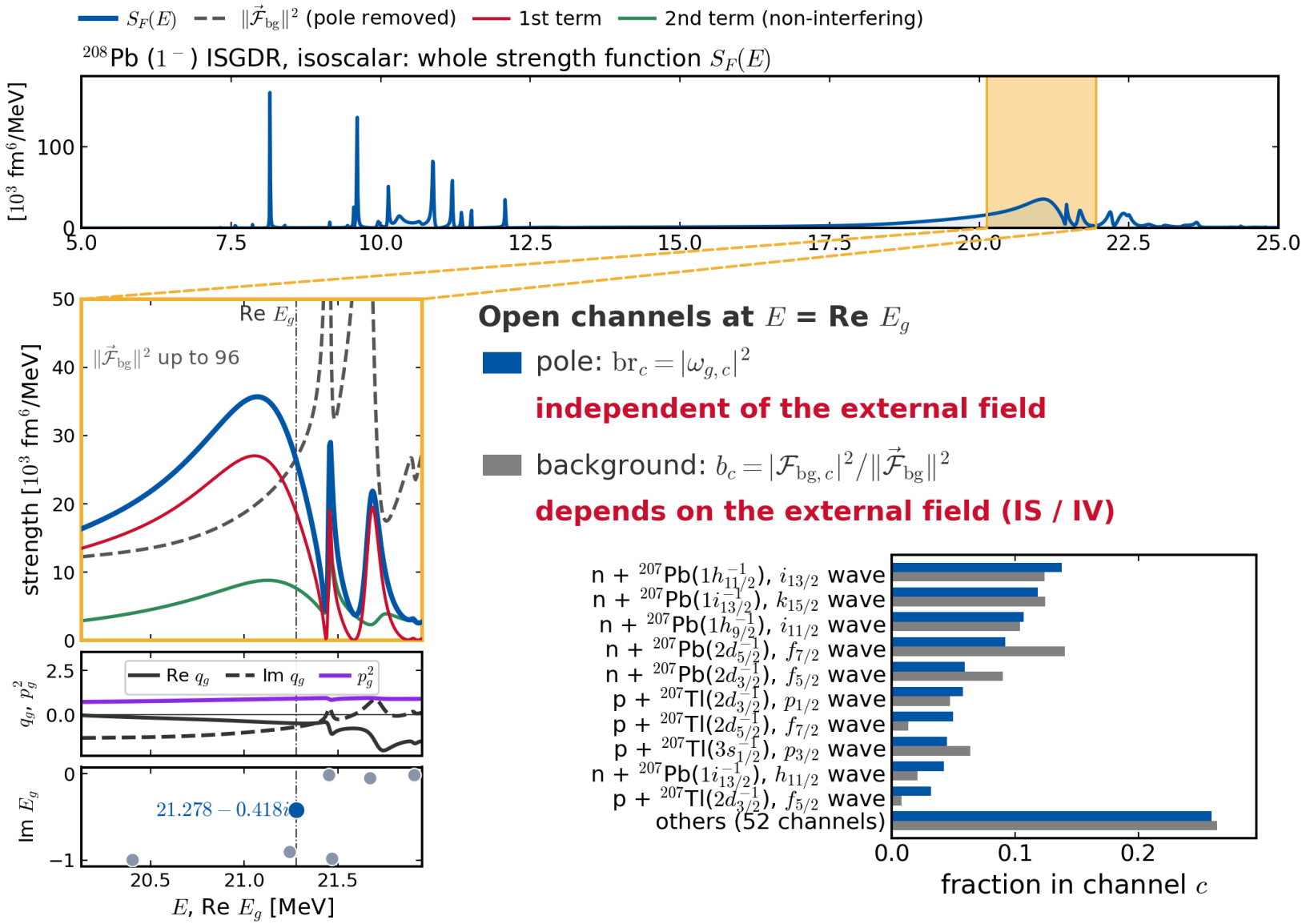
strong interference

Im $R_g \approx -1$: $S_F \approx$ background at $\epsilon = 0$

⇒ **Fano profile**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Giant Dipole Resonance

The Pole Decays into Many Channels; the IS Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.278 - 0.418i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.97

Pole decays into (br_c)

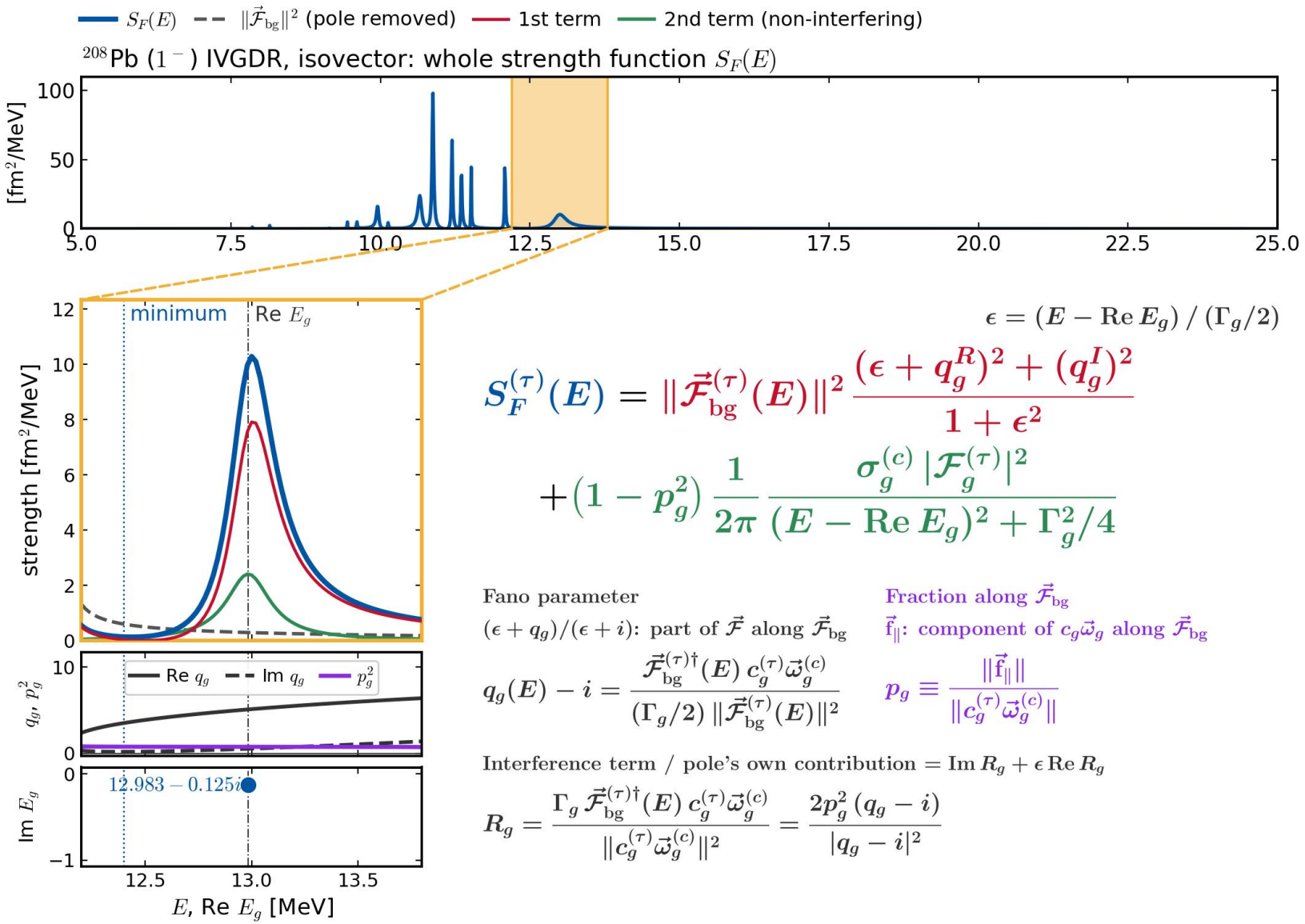
n + ^{207}Pb($1h_{11/2}^{-1}$), $i_{13/2}$ wave	0.14
n + ^{207}Pb($1i_{13/2}^{-1}$), $k_{15/2}$ wave	0.12

Background at Re E_g (b_c)

n + ^{207}Pb($2d_{5/2}^{-1}$), $f_{7/2}$ wave	0.14
n + ^{207}Pb($1i_{13/2}^{-1}$), $k_{15/2}$ wave	0.12

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isovector Giant Dipole Resonance

Most Collective IV Pole: $|R_g| < 1/\sqrt{2}$, Breit–Wigner Peak of the Pole; Minimum below It



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\, E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$12.983 - 0.125i$ MeV

Collectivity (IV): collective

C_g^2	0.4808
$N_{\rm tot}\eta_g$	16.77
$C_g^2 N_{\rm tot}\eta_g$	**8.06**

At E = Re E_g ($\epsilon = 0$)

q_g	$5.08 + 0.53i$
p_g^2	0.76
R_g	$0.30 - 0.03i$
$\lvert R_g\rvert$	$0.30 < 1/\sqrt{2}$

pole's own contribution dominates

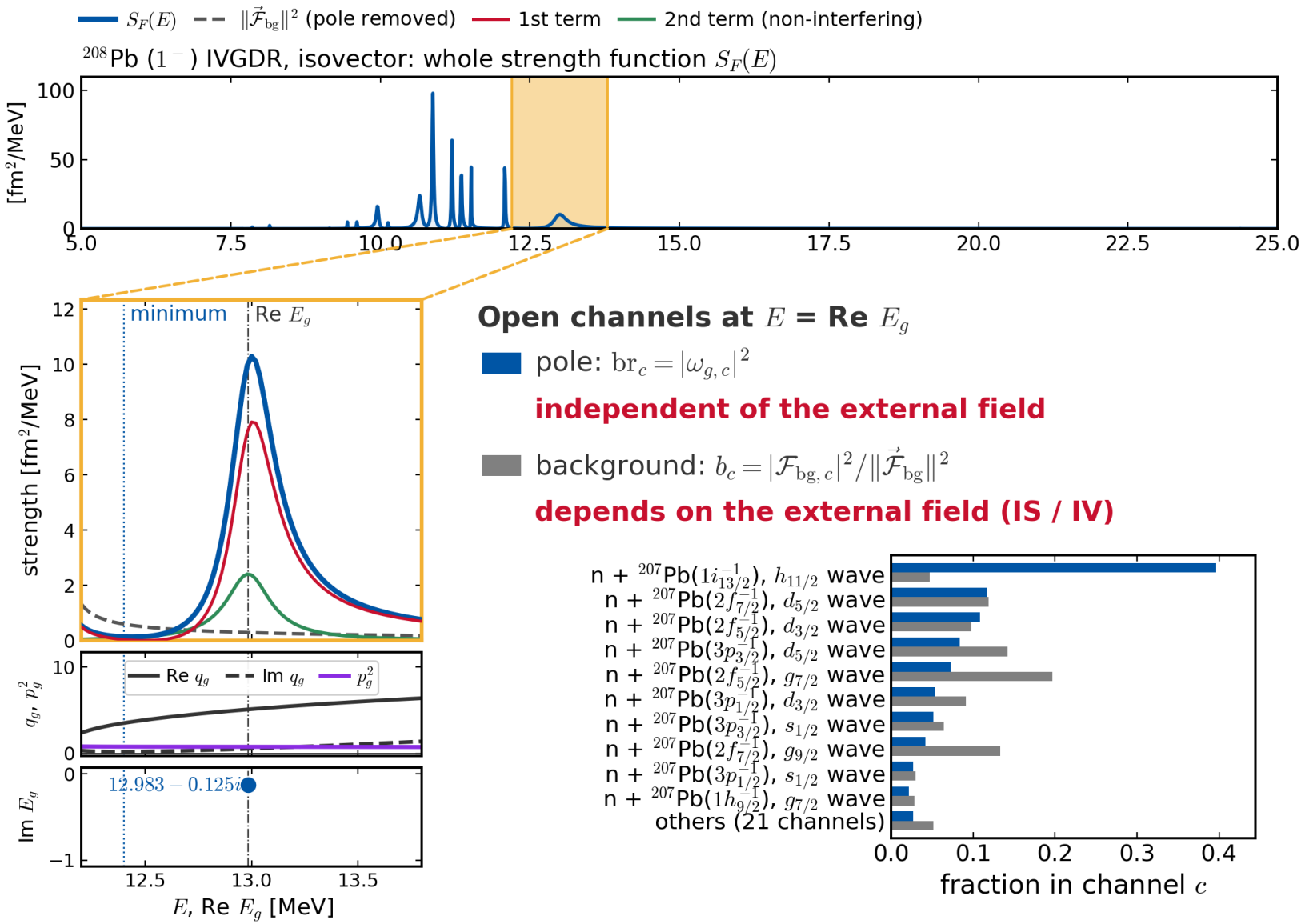
Im $R_g < 0$: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak with a large $|q_g|$**

minimum at 12.40 MeV

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isovector Giant Dipole Resonance

The Pole Decays into Several Channels (Largest: n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ Wave); the IV Background Is Spread over Many Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$12.983 - 0.125i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	0.97

Pole decays into (br_c)

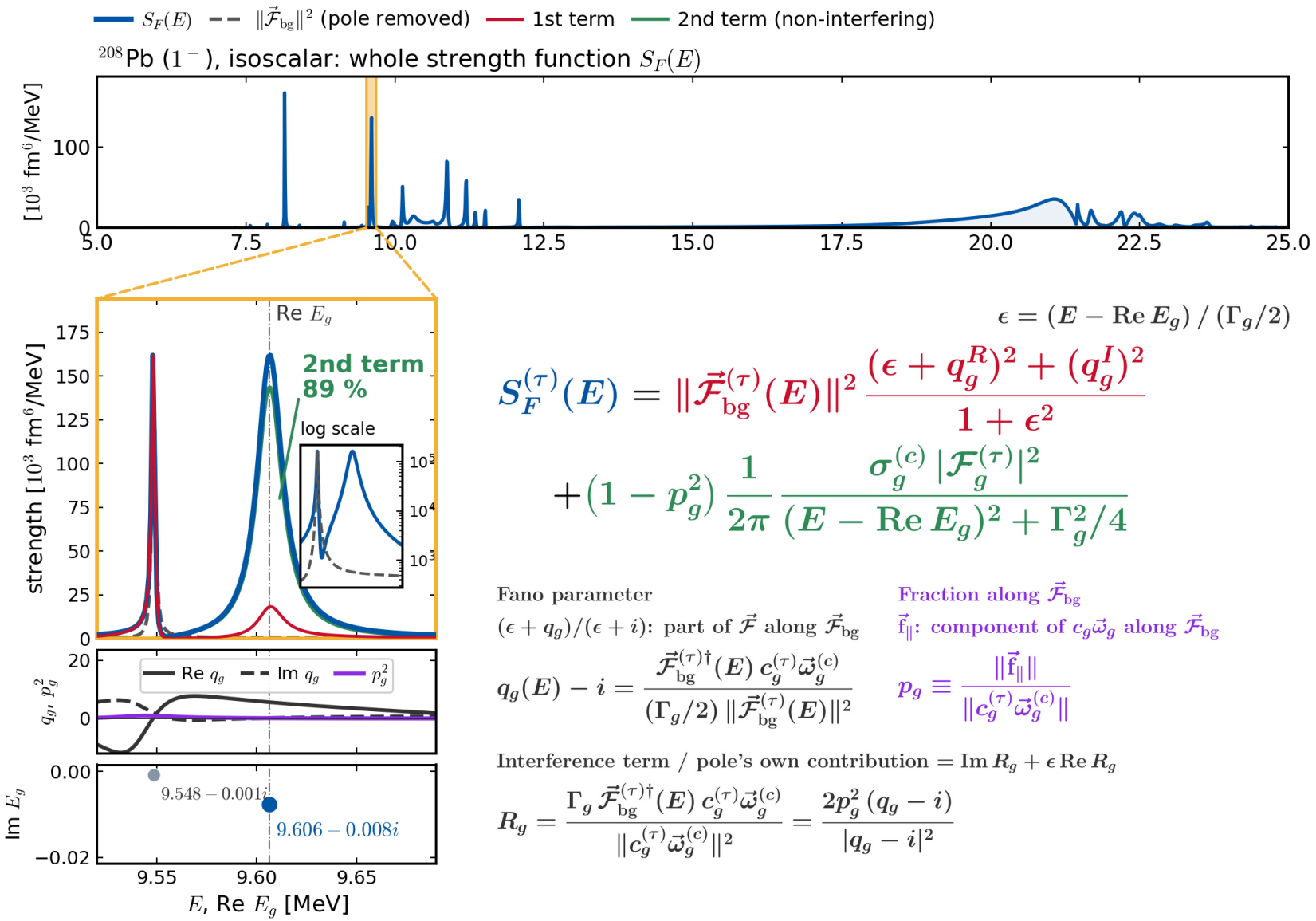
n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ wave	0.40
n + ^{207}Pb($2f_{7/2}^{-1}$), $d_{5/2}$ wave	0.12

Background at Re E_g (b_c)

n + ^{207}Pb($2f_{5/2}^{-1}$), $g_{7/2}$ wave	0.20
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.14

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 9.61 MeV

Collective Pole (Γ_g = 15 keV): $|R_g| \ll 1$, the 2nd (Breit–Wigner) Term Dominates S_F at Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter
$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$
$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$9.606 - 0.008i$ MeV

Collectivity (IS): collective

C_g^2	0.1914
$N_{\rm tot}\eta_g$	34.70
$C_g^2 N_{\rm tot}\eta_g$	**6.64**

At E = Re E_g ($\epsilon = 0$)

q_g	$5.59 + 0.18i$
p_g^2	0.11
R_g	$0.040 - 0.006i$
$\lvert R_g\rvert$	$0.040 \ll 1$

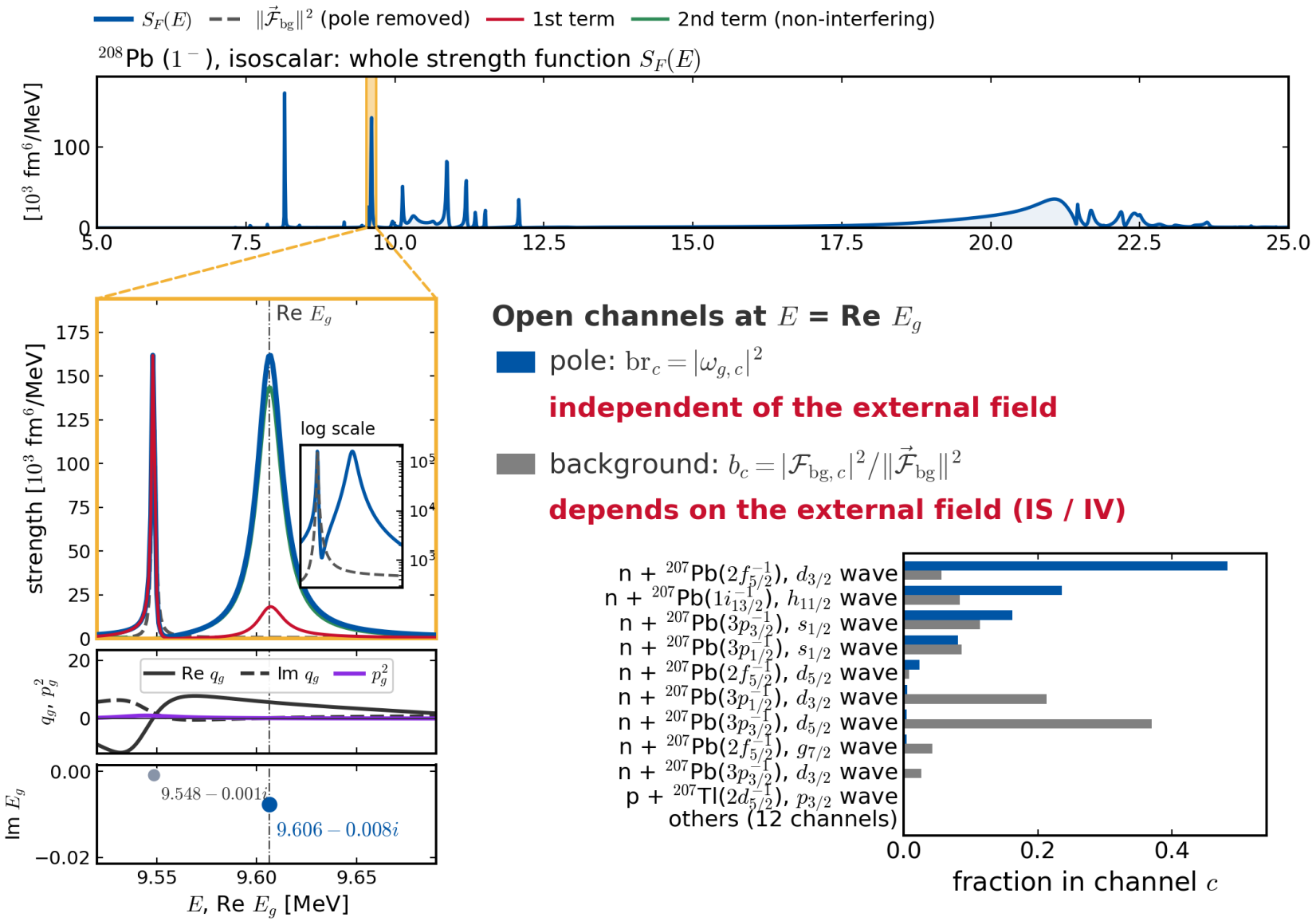
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 9.61 MeV

The Pole Decays into Several Channels (Largest: n + ^{207}Pb($2f_{5/2}^{-1}$), $d_{3/2}$ Wave); the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$9.606 - 0.008i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.00

Pole decays into (br_c)

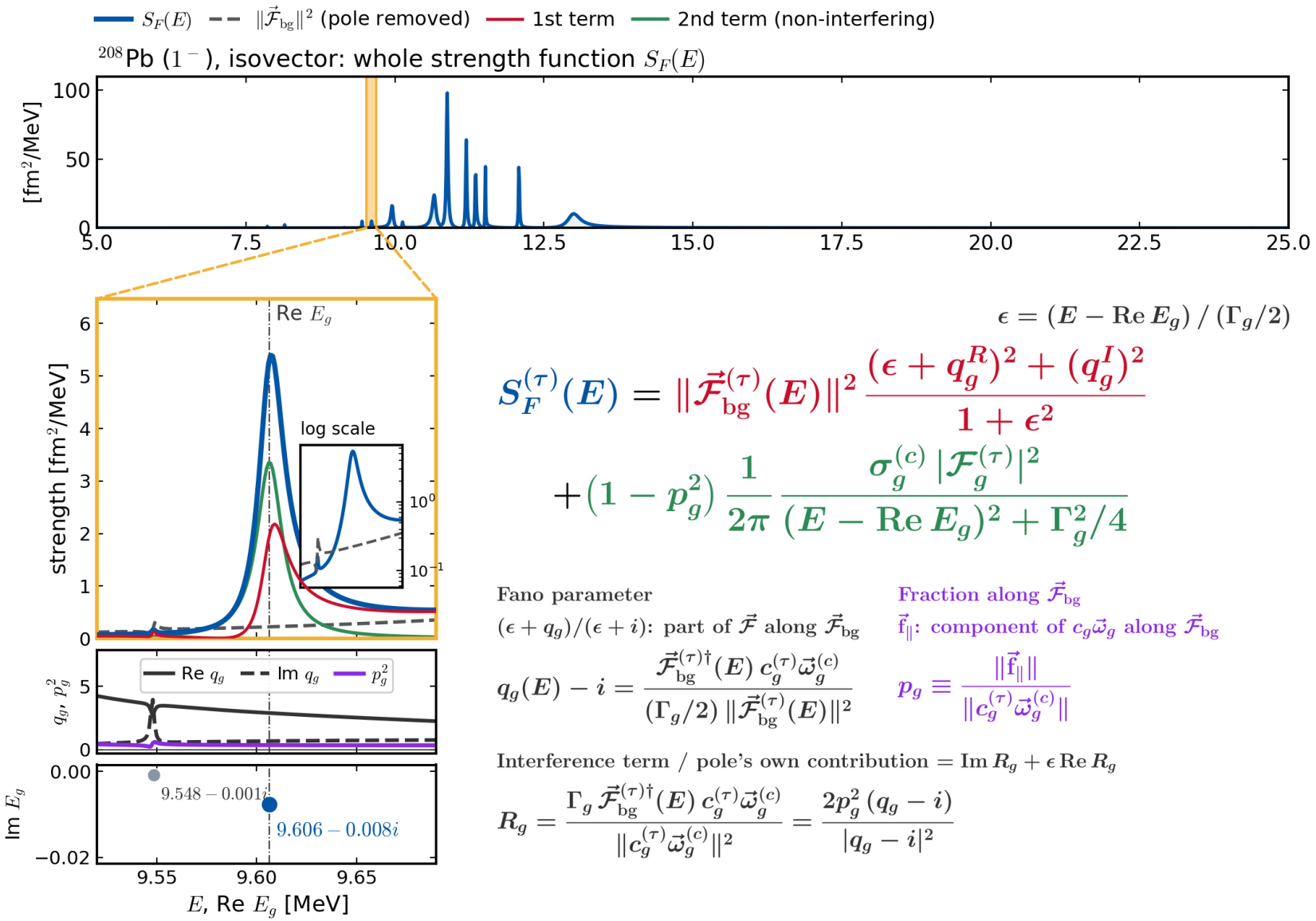
n + ^{207}Pb($2f_{5/2}^{-1}$), $d_{3/2}$ wave	0.48
n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ wave	0.24

Background at Re E_g (b_c)

n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.37
n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ wave	0.21

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 9.61 MeV

Isovector Field (Non-Collective): $|R_g| < 1/\sqrt{2}$; Both Terms Build the Peak (1st Term 37 %, 2nd Term 63 % of S_F at Re E_g)



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	9.606 − 0.008i MeV

Collectivity (IV): non-collective

C_g^2	0.0034
$N_{\rm tot}\eta_g$	29.79
$C_g^2 N_{\rm tot}\eta_g$	0.10

At E = Re E_g ($\epsilon = 0$)

q_g	2.89 + 0.68i
p_g^2	0.36
R_g	0.25 − 0.03i
$\|R_g\|$	0.25 < $1/\sqrt{2}$

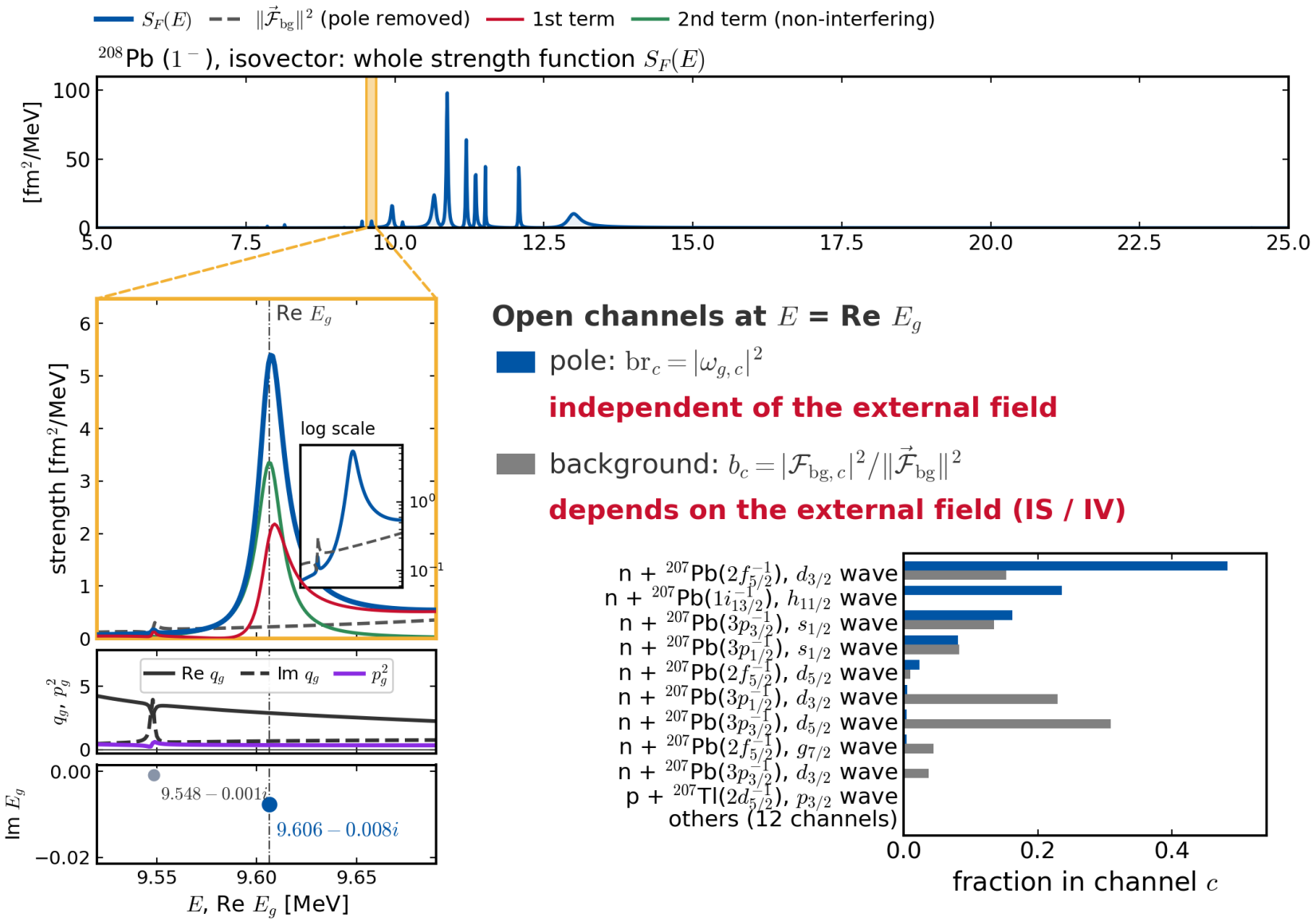
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **peak from both terms**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 9.61 MeV

Same br$_c$ as in the IS Field; the IV Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$9.606 - 0.008i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.00

Pole decays into (br_c)

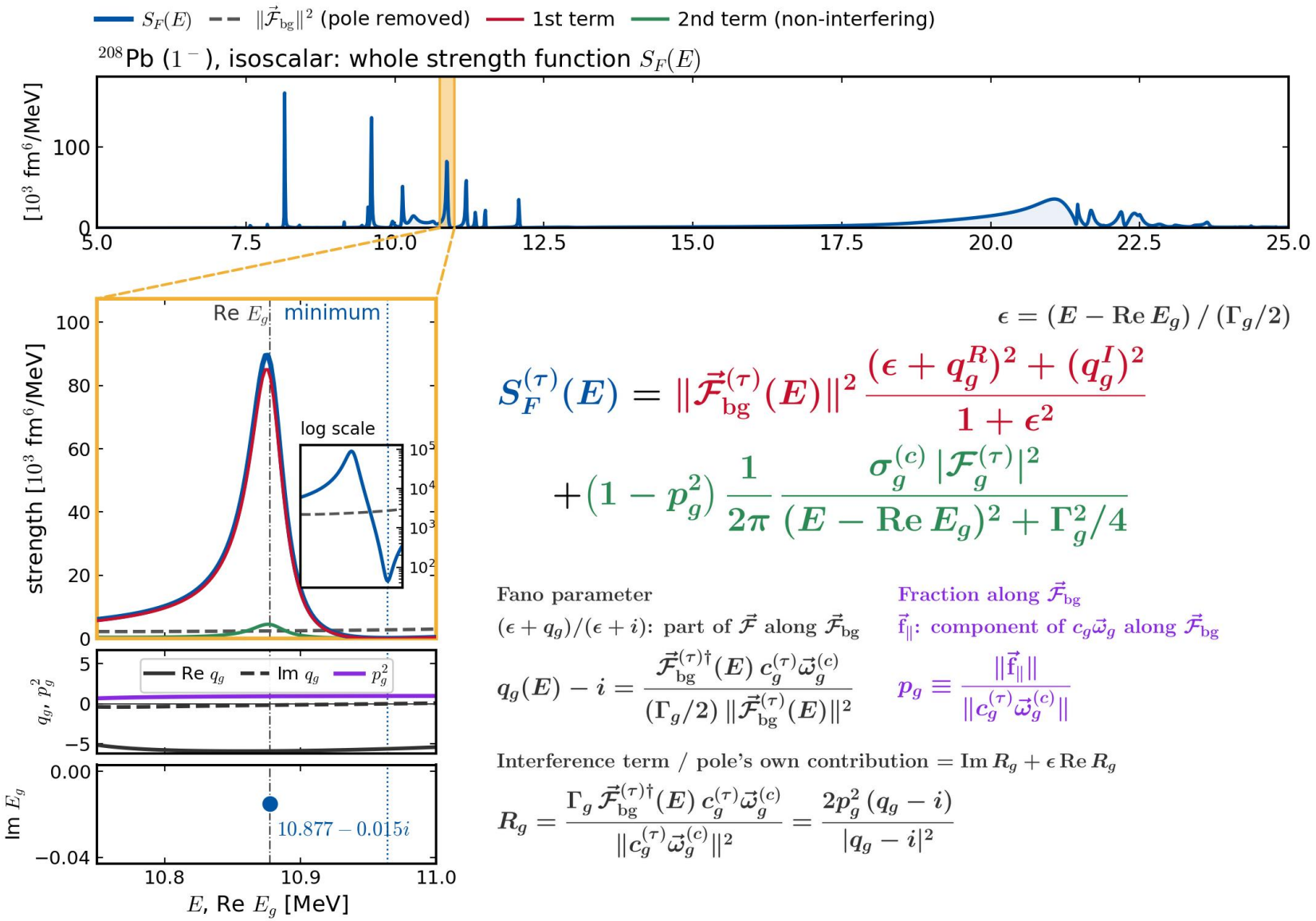
n + ^{207}Pb($2f_{5/2}^{-1}$), $d_{3/2}$ wave	0.48
n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ wave	0.24

Background at Re E_g (b_c)

n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.31
n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ wave	0.23

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 10.88 MeV

Narrow Collective Pole (Γ_g = 30 keV): $|R_g| < 1/\sqrt{2}$, Breit–Wigner Peak of the Pole; Minimum above It



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$10.877 - 0.015i$ MeV

Collectivity (IS): collective

C_g^2	0.0677
$N_{\rm tot}\eta_g$	39.44
$C_g^2 N_{\rm tot}\eta_g$	2.67

At E = Re E_g ($\epsilon = 0$)

q_g	$-5.88 - 0.16i$
p_g^2	0.95
R_g	$-0.31 - 0.06i$
$\lvert R_g\rvert$	$0.32 < 1/\sqrt{2}$

pole's own contribution dominates

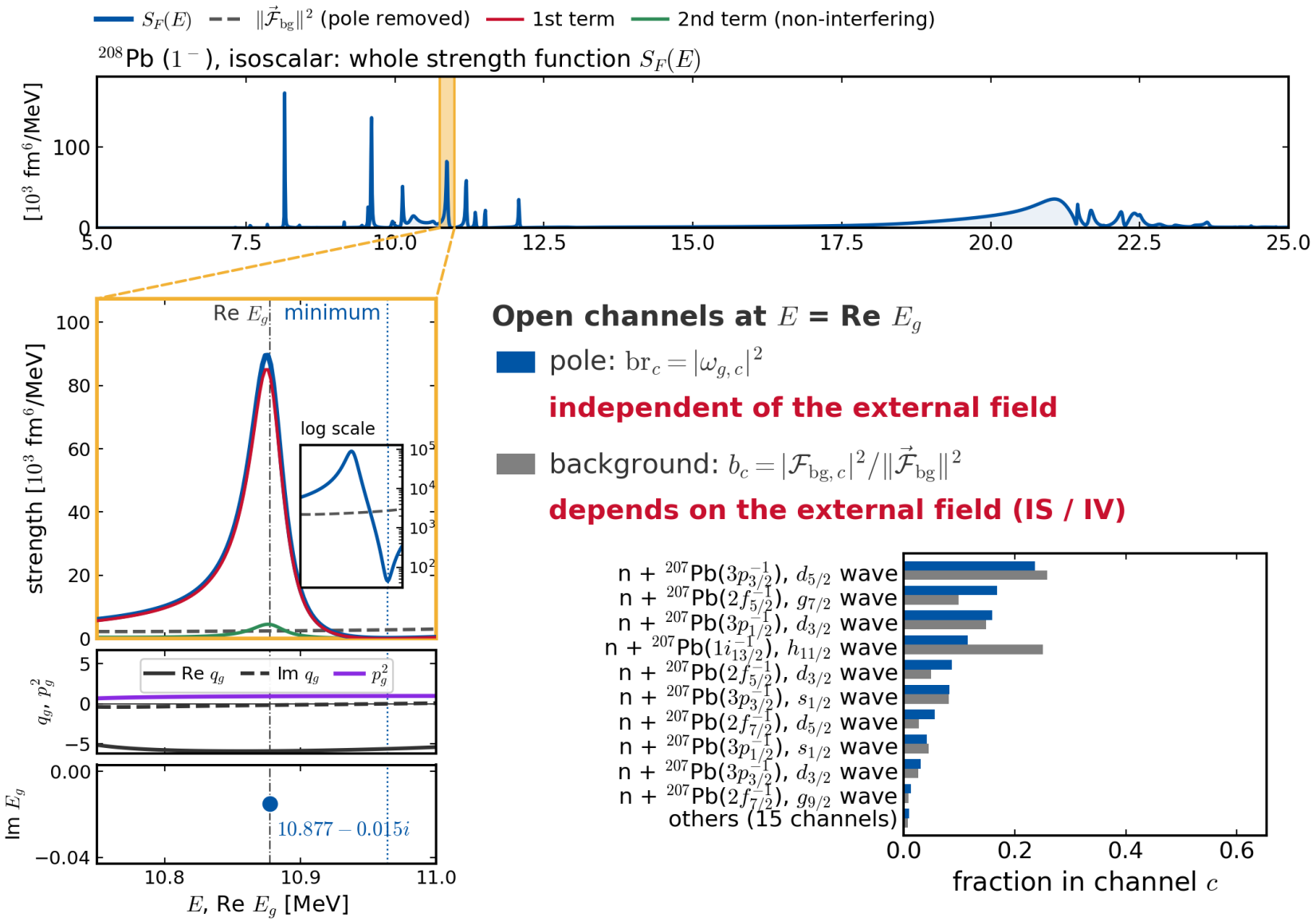
Im $R_g < 0$: lowers S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak with a large $|q_g|$**

minimum at 10.96 MeV

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 10.88 MeV

The Pole Decays into Many Channels; the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$10.877 - 0.015i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.01

Pole decays into (br_c)

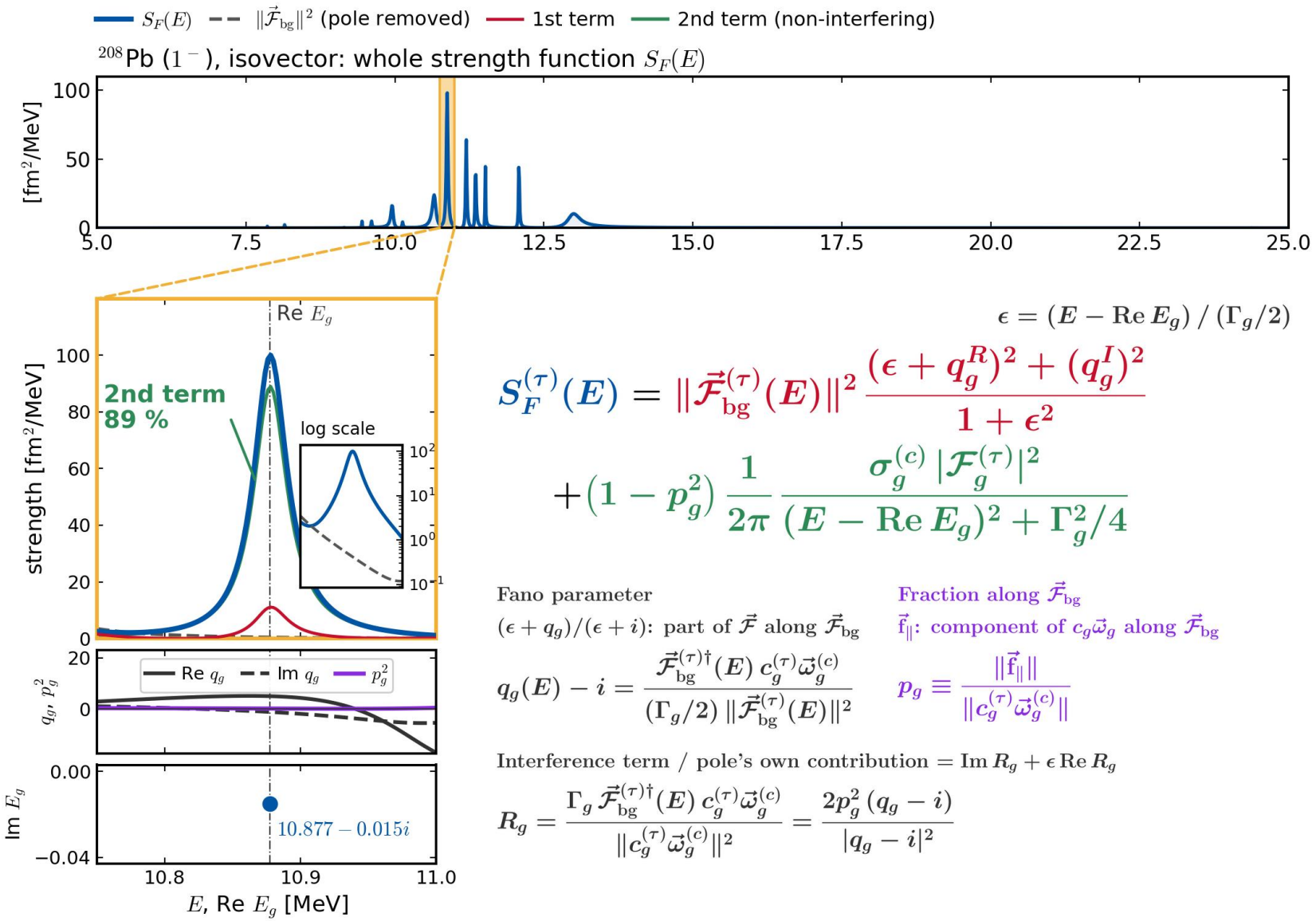
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.24
n + ^{207}Pb($2f_{5/2}^{-1}$), $g_{7/2}$ wave	0.17

Background at Re E_g (b_c)

n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.26
n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ wave	0.25

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 10.88 MeV

Same Pole, Isovector Field (Collective): $|R_g| \ll 1$, the Non-Interfering Breit–Wigner Term Dominates S_F at Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\, E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\mathrm{f}}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\mathrm{f}}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$10.877 - 0.015i$ MeV

Collectivity (IV): collective

C_g^2	0.1019
$N_{\rm tot}\eta_g$	29.13
$C_g^2 N_{\rm tot}\eta_g$	2.97

At E = Re E_g ($\epsilon = 0$)

q_g	$4.97 - 1.21i$
p_g^2	0.12
R_g	$0.041 - 0.018i$
$\lvert R_g\rvert$	$0.045 \ll 1$

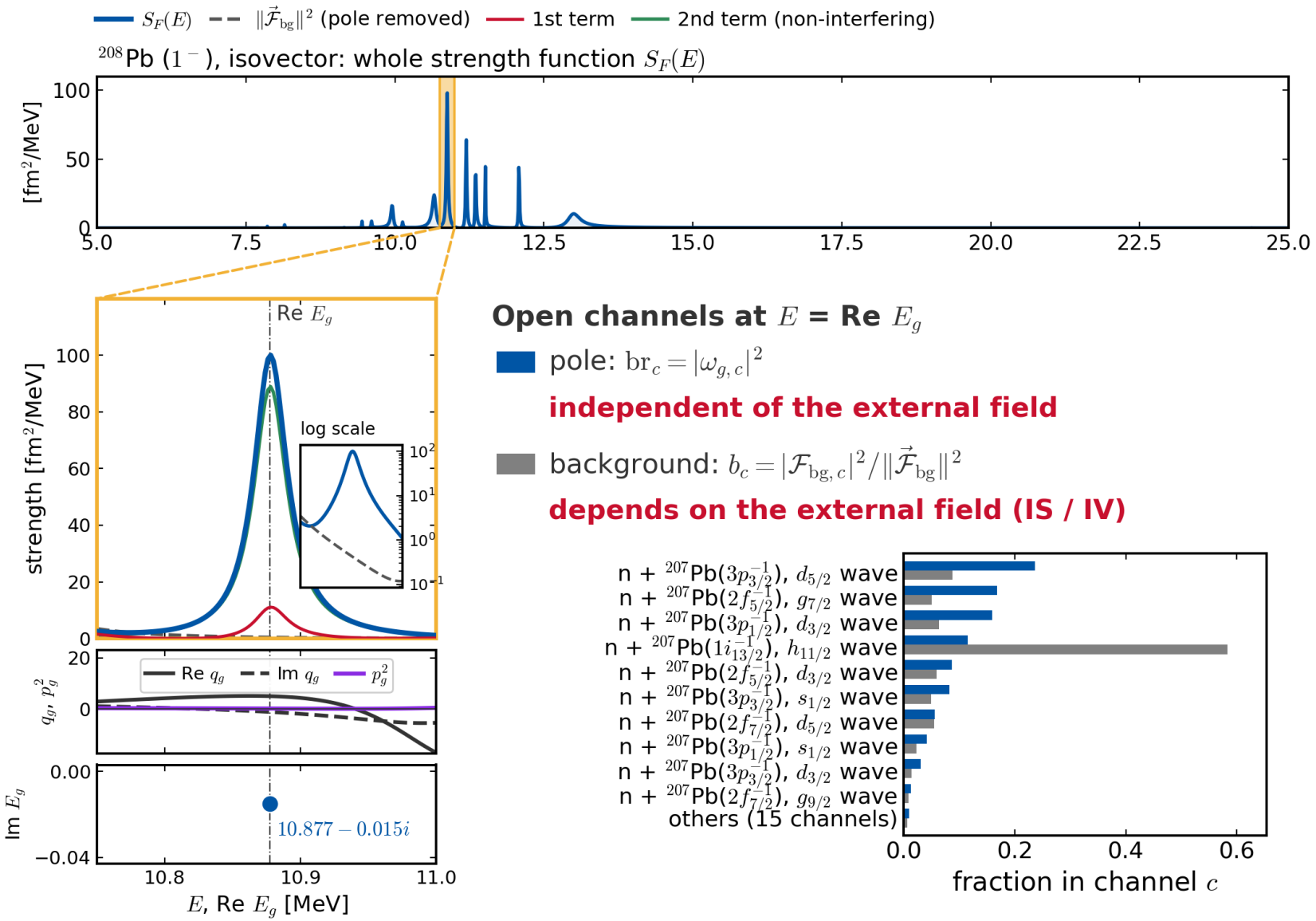
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 10.88 MeV

Same br$_c$ as in the IS Field; the IV Background Lies Mainly in n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ Wave



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$10.877 - 0.015i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.01

Pole decays into (br_c)

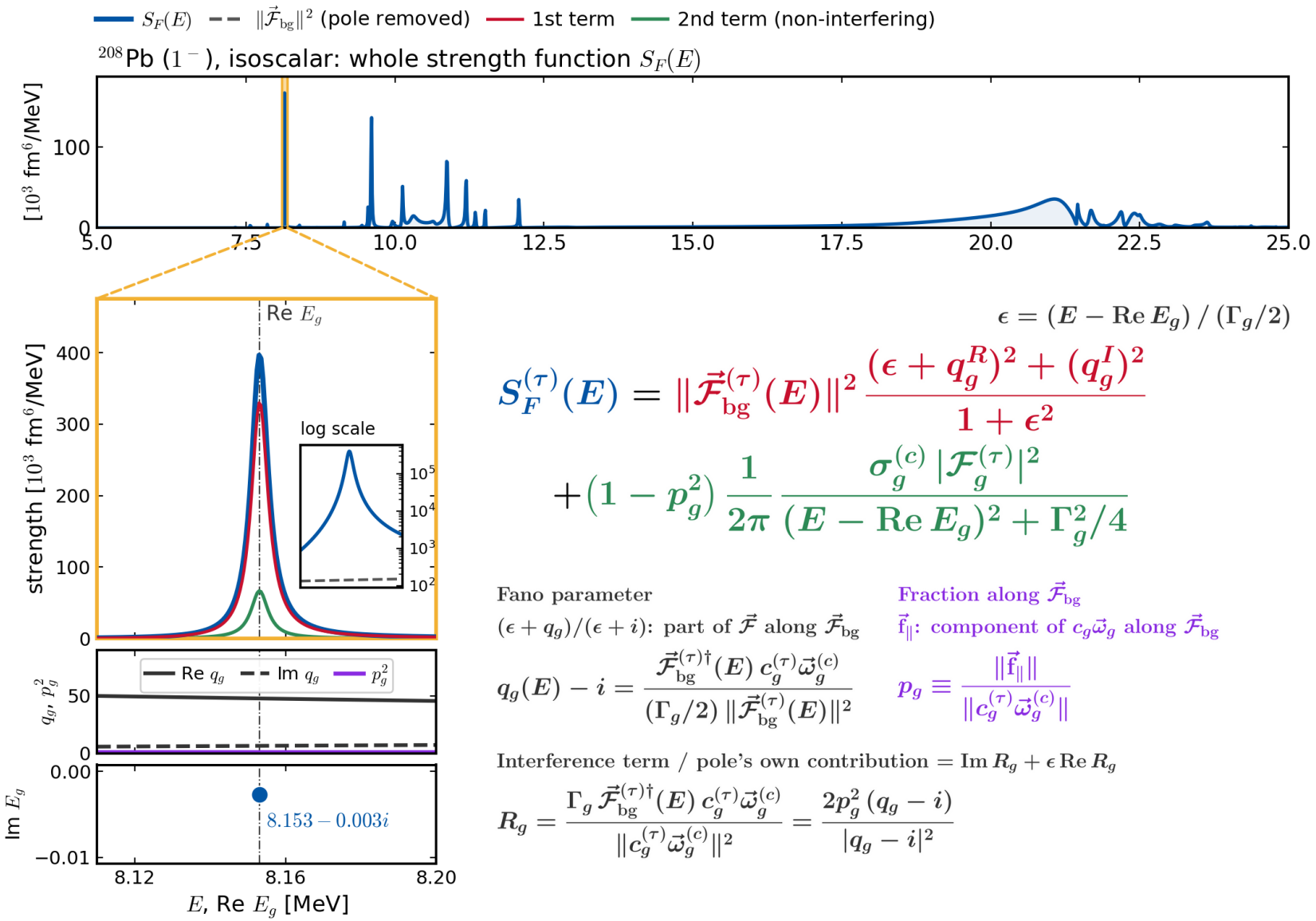
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.24
n + ^{207}Pb($2f_{5/2}^{-1}$), $g_{7/2}$ wave	0.17

Background at Re E_g (b_c)

n + ^{207}Pb($1i_{13/2}^{-1}$), $h_{11/2}$ wave	0.58
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.09

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 8.15 MeV

Tallest IS Peak: Non-Collective Pole (Γ_g = 5.5 keV), $|R_g| \ll 1$ → Breit–Wigner Peak of the Pole



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E - \mathrm{Re}\, E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$8.153 - 0.003i$ MeV

Collectivity (IS): non-collective

C_g^2	0.0315
$N_{\rm tot}\eta_g$	5.67
$C_g^2 N_{\rm tot}\eta_g$	0.18

At E = Re E_g ($\epsilon = 0$)

q_g	$47.73 + 6.42i$
p_g^2	0.83
R_g	$0.034 + 0.004i$
$\lvert R_g\rvert$	$0.035 \ll 1$

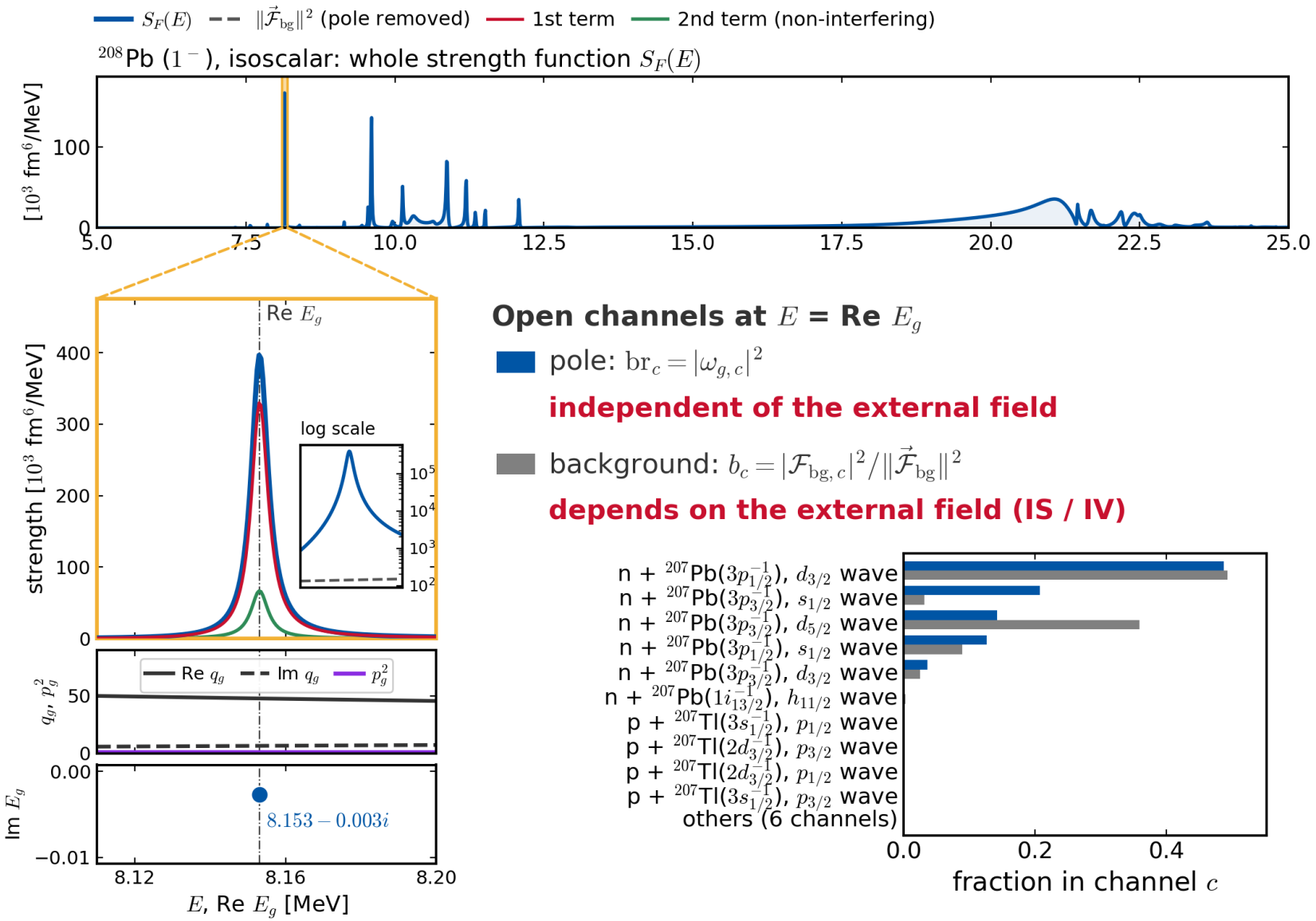
pole's own contribution dominates

Im R_g > 0: raises S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak with a large $|q_g|$**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 8.15 MeV

The Pole Decays into Several Channels (Largest: n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ Wave); the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$8.153 - 0.003i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.00

Pole decays into (br_c)

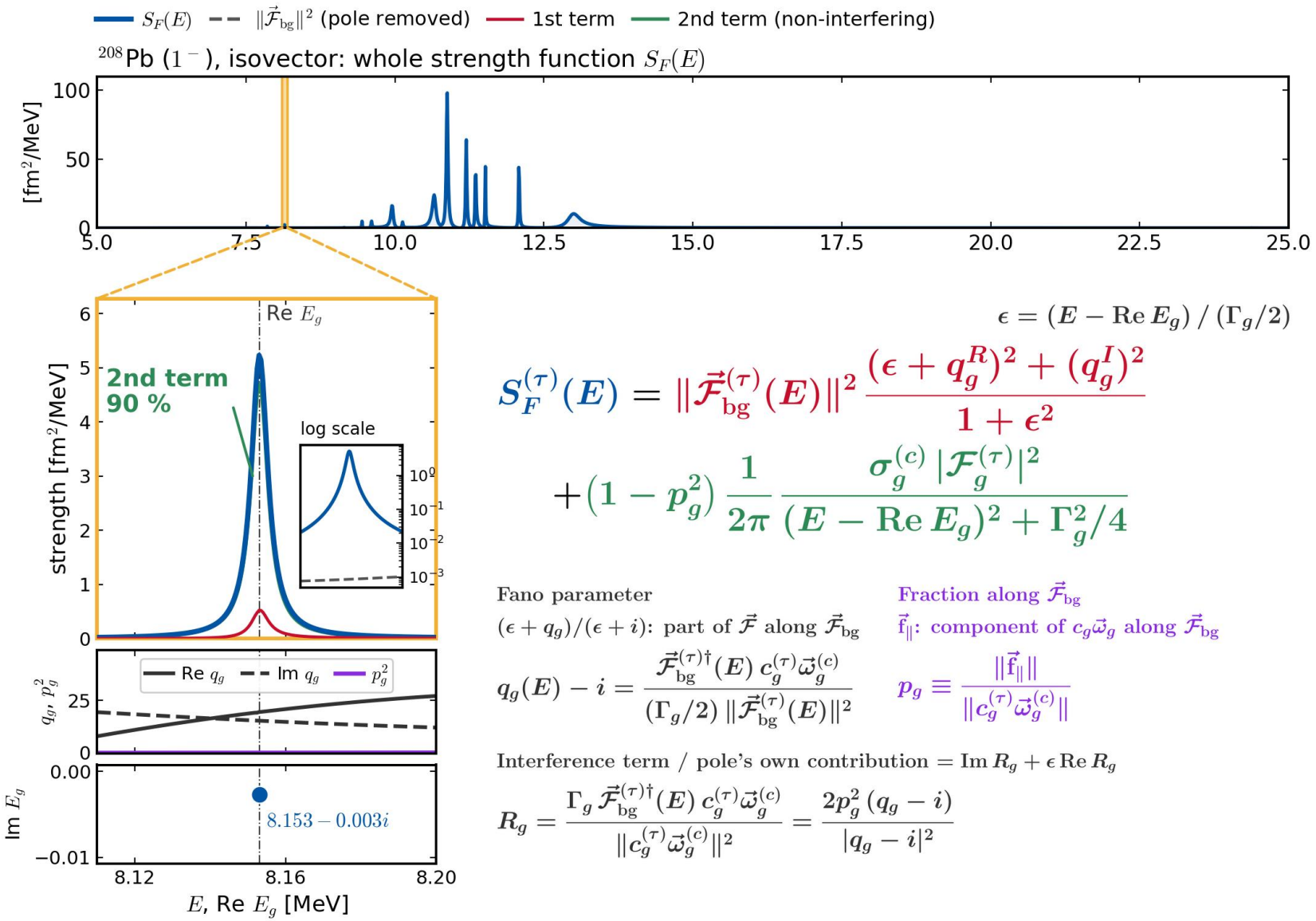
n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ wave	0.49
n + ^{207}Pb($3p_{3/2}^{-1}$), $s_{1/2}$ wave	0.21

Background at Re E_g (b_c)

n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ wave	0.49
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.36

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 8.15 MeV

Same Pole, Isovector Field (Non-Collective): $|R_g| \ll 1$, the Non-Interfering Breit–Wigner Term Dominates S_F at Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)}\vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$8.153 - 0.003i$ MeV

Collectivity (IV): non-collective

C_g^2	0.0002
$N_{\rm tot}\eta_g$	7.91
$C_g^2 N_{\rm tot}\eta_g$	0.002

At E = Re E_g ($\epsilon = 0$)

q_g	$19.45 + 15.28i$
p_g^2	0.10
R_g	$0.006 + 0.005i$
$\|R_g\|$	$0.008 \ll 1$

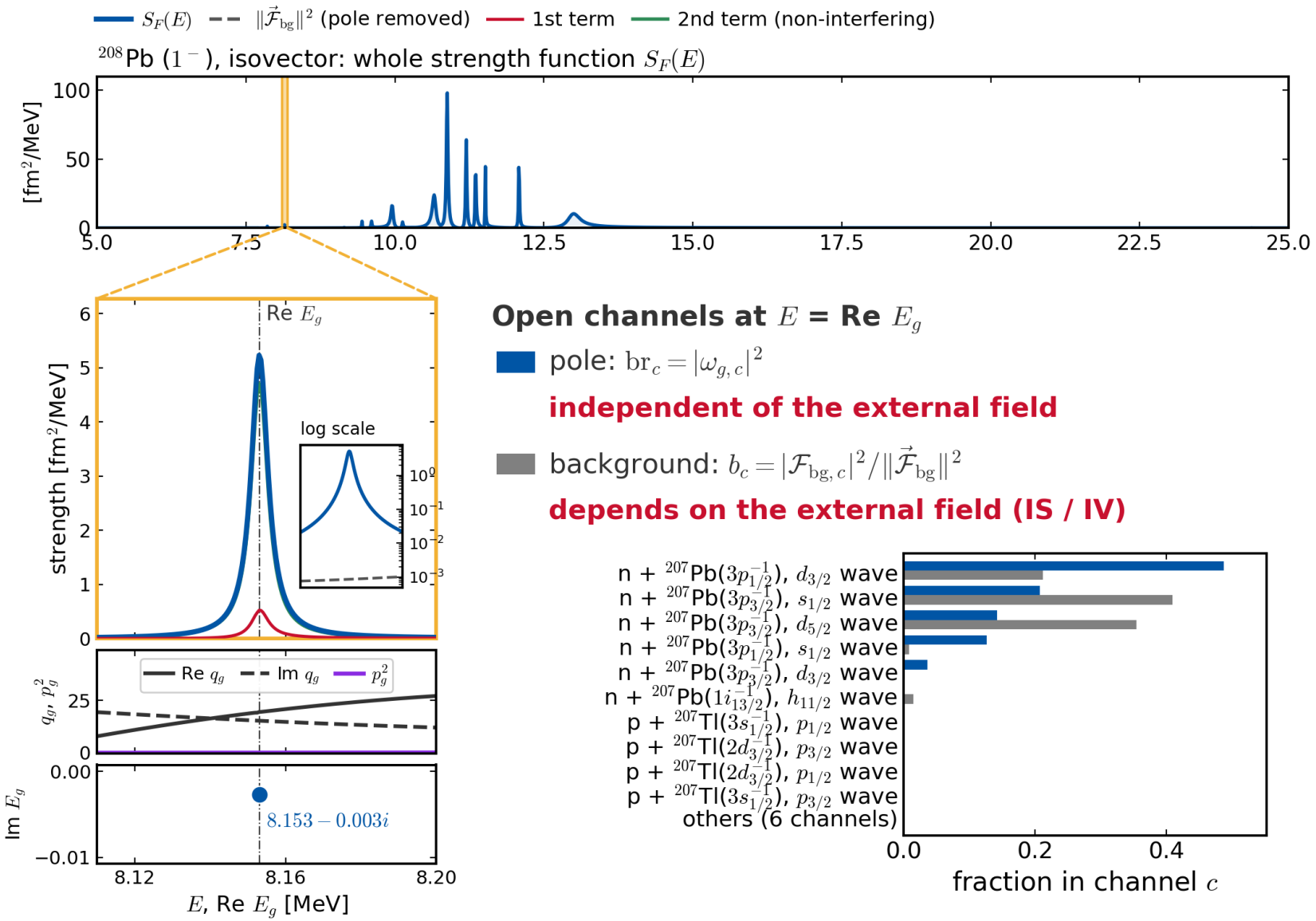
pole's own contribution dominates

Im $R_g > 0$: raises S_F at $\epsilon = 0$

$\Rightarrow$ **Breit–Wigner peak**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 8.15 MeV

Same br$_c$ as in the IS Field; the IV Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$8.153 - 0.003i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.00

Pole decays into (br$_c$)

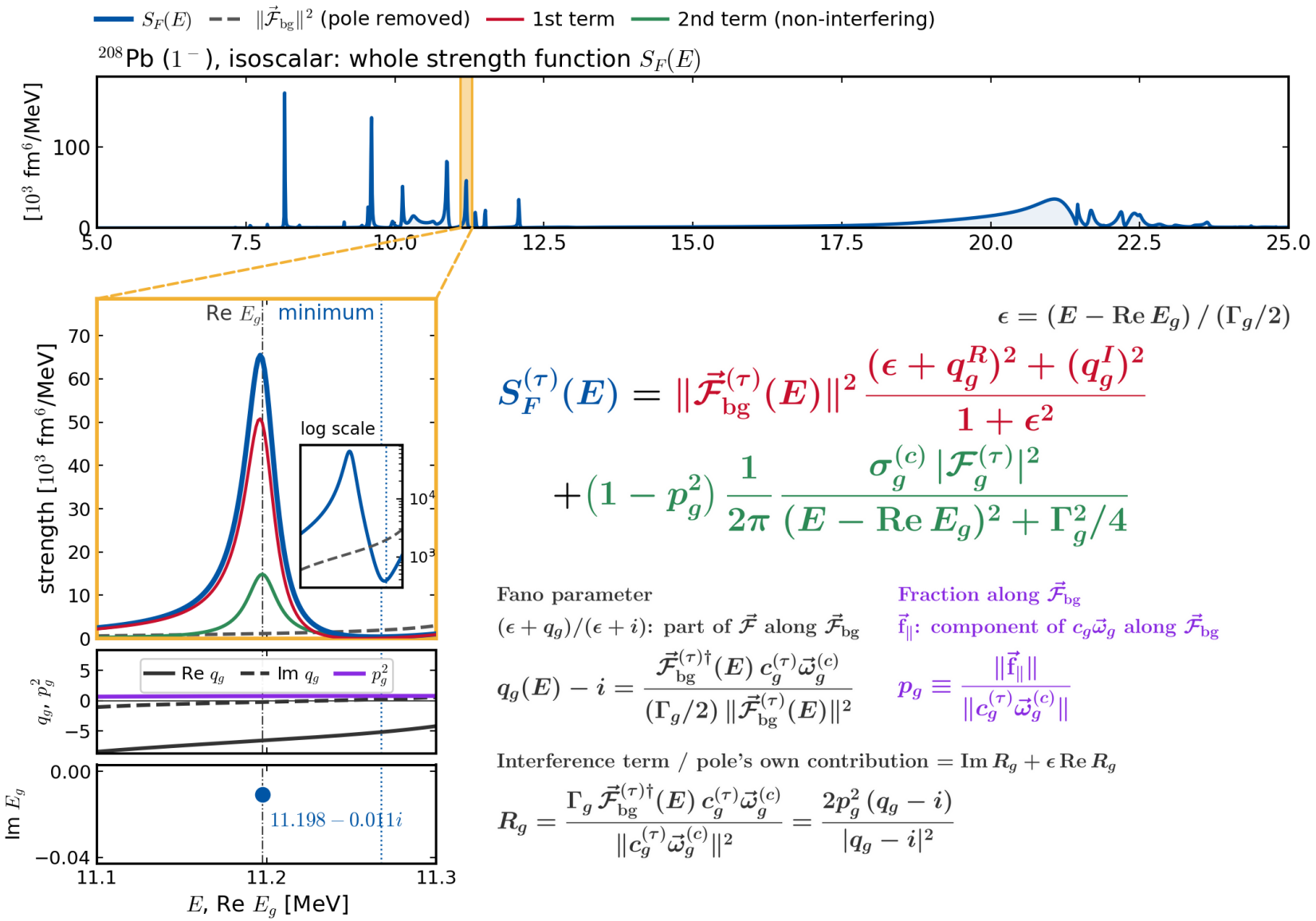
n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ wave	0.49
n + ^{207}Pb($3p_{3/2}^{-1}$), $s_{1/2}$ wave	0.21

Background at Re E_g (b_c)

n + ^{207}Pb($3p_{3/2}^{-1}$), $s_{1/2}$ wave	0.41
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.35

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 11.20 MeV

Narrow Collective Pole (Γ_g = 22 keV): $|R_g| < 1/\sqrt{2}$, Breit–Wigner Peak; Minimum above It



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$11.198 - 0.011i$ MeV

Collectivity (IS): collective

C_g^2	0.0875
$N_{\rm tot}\eta_g$	46.53
$C_g^2 N_{\rm tot}\eta_g$	**4.07**

At E = Re E_g ($\epsilon = 0$)

q_g	$-6.58 - 0.21i$
p_g^2	0.78
R_g	$-0.23 - 0.04i$
$\lvert R_g\rvert$	$0.23 < 1/\sqrt{2}$

pole's own contribution dominates

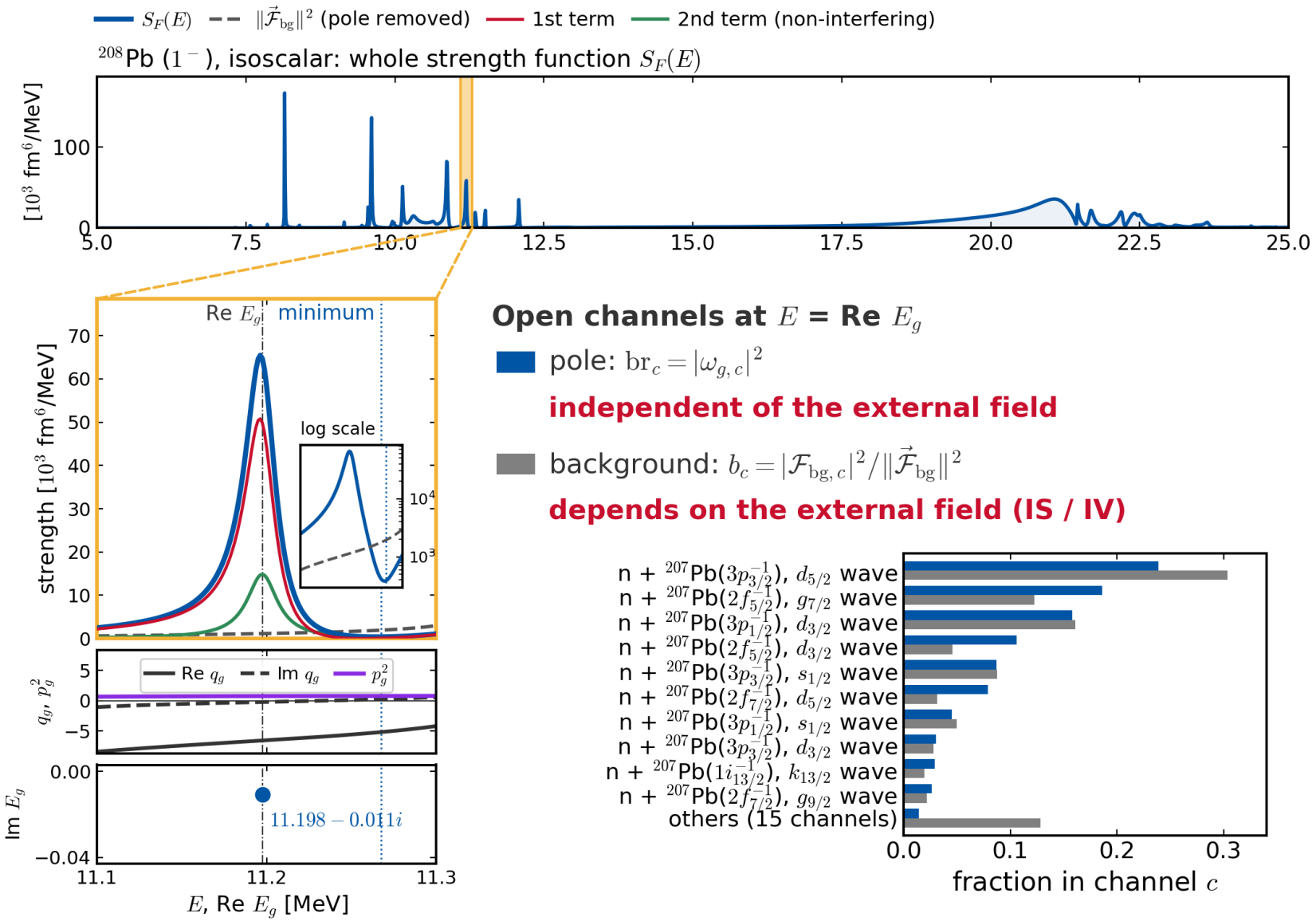
Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak with a large $|q_g|$**

minimum at 11.27 MeV

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Peak at 11.20 MeV

The Pole Decays into Many Channels; the IS Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$11.198 - 0.011i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.01

Pole decays into (br_c)

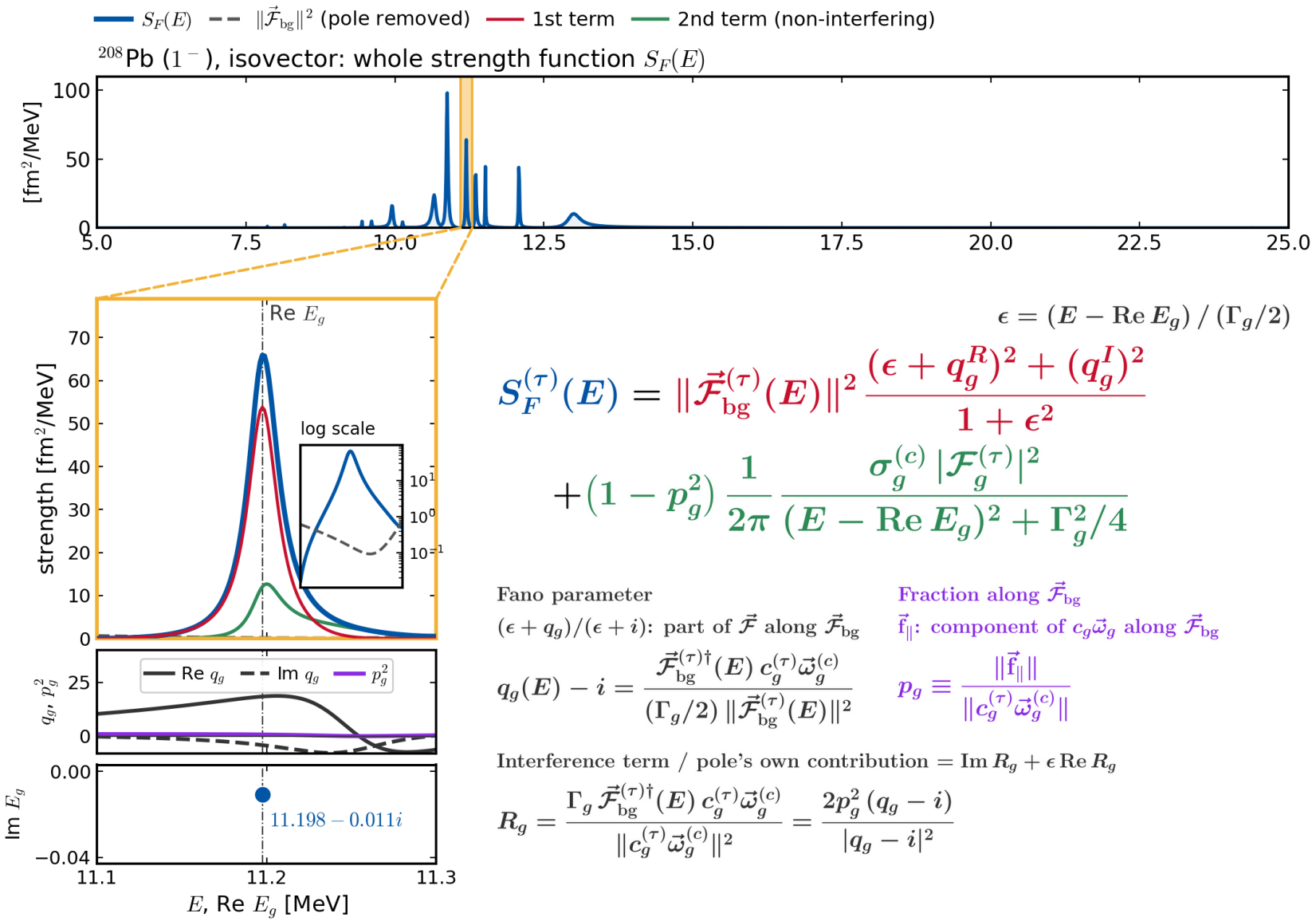
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.24
n + ^{207}Pb($2f_{5/2}^{-1}$), $g_{7/2}$ wave	0.19

Background at Re E_g (b_c)

n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.30
n + ^{207}Pb($3p_{1/2}^{-1}$), $d_{3/2}$ wave	0.16

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 11.20 MeV

Same Pole, Isovector Field (Collective): $|R_g| \ll 1$, Breit–Wigner Peak of the Pole



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\rm f}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\rm f}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$11.198 - 0.011i$ MeV

Collectivity (IV): collective

C_g^2	0.1192
$N_{\rm tot}\eta_g$	30.61
$C_g^2 N_{\rm tot}\eta_g$	**3.65**

At E = Re E_g ($\epsilon = 0$)

q_g	$18.45 - 4.07i$
p_g^2	0.82
R_g	$0.083 - 0.023i$
$\lvert R_g\rvert$	$0.086 \ll 1$

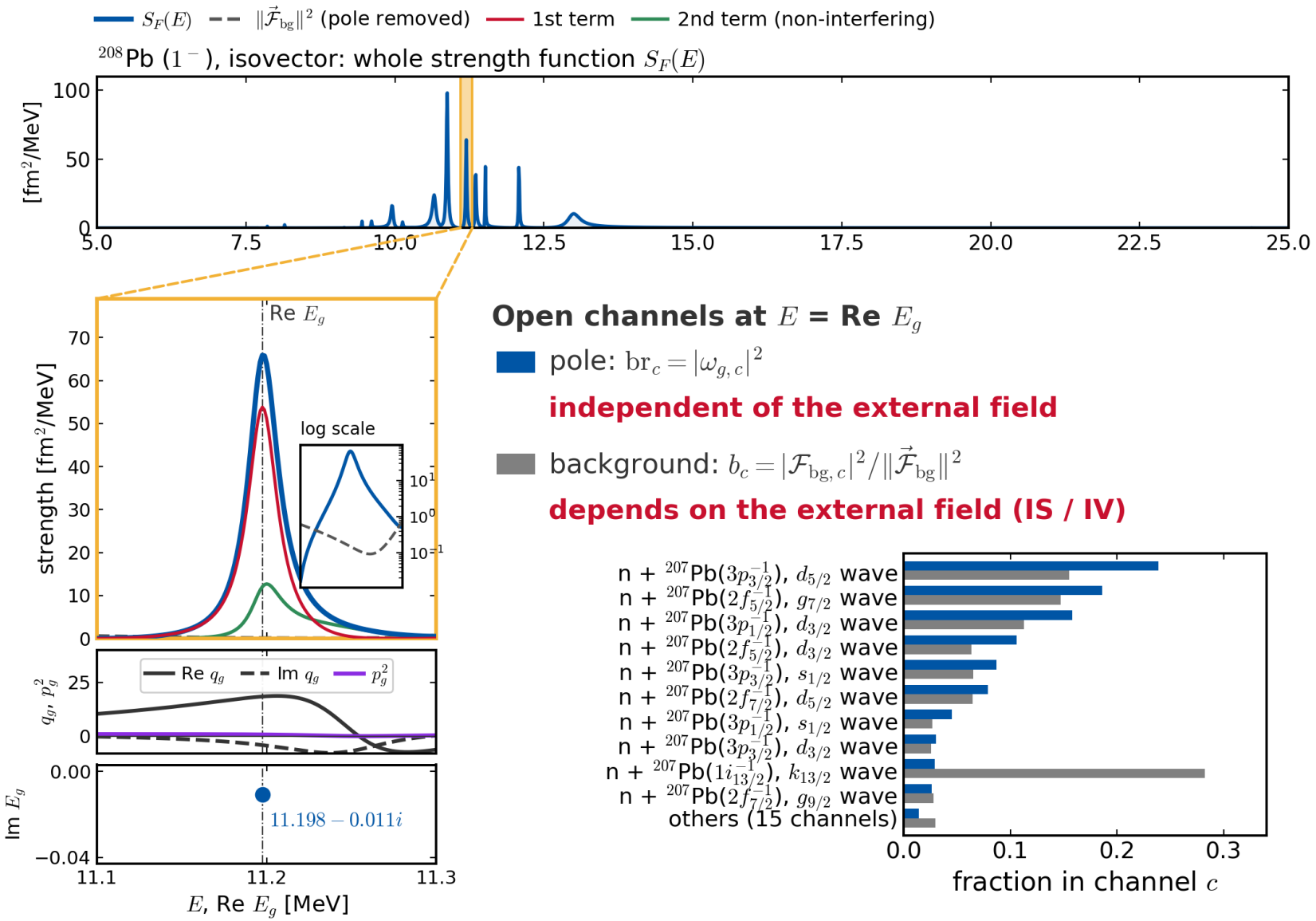
pole's own contribution dominates

Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **Breit–Wigner peak with a large $|q_g|$**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isovector Peak at 11.20 MeV

Same br$_c$ as in the IS Field; the IV Background Is Spread over Several Channels



Open channels at E = Re E_g

pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

independent of the external field

background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$11.198 - 0.011i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.01

Pole decays into (br_c)

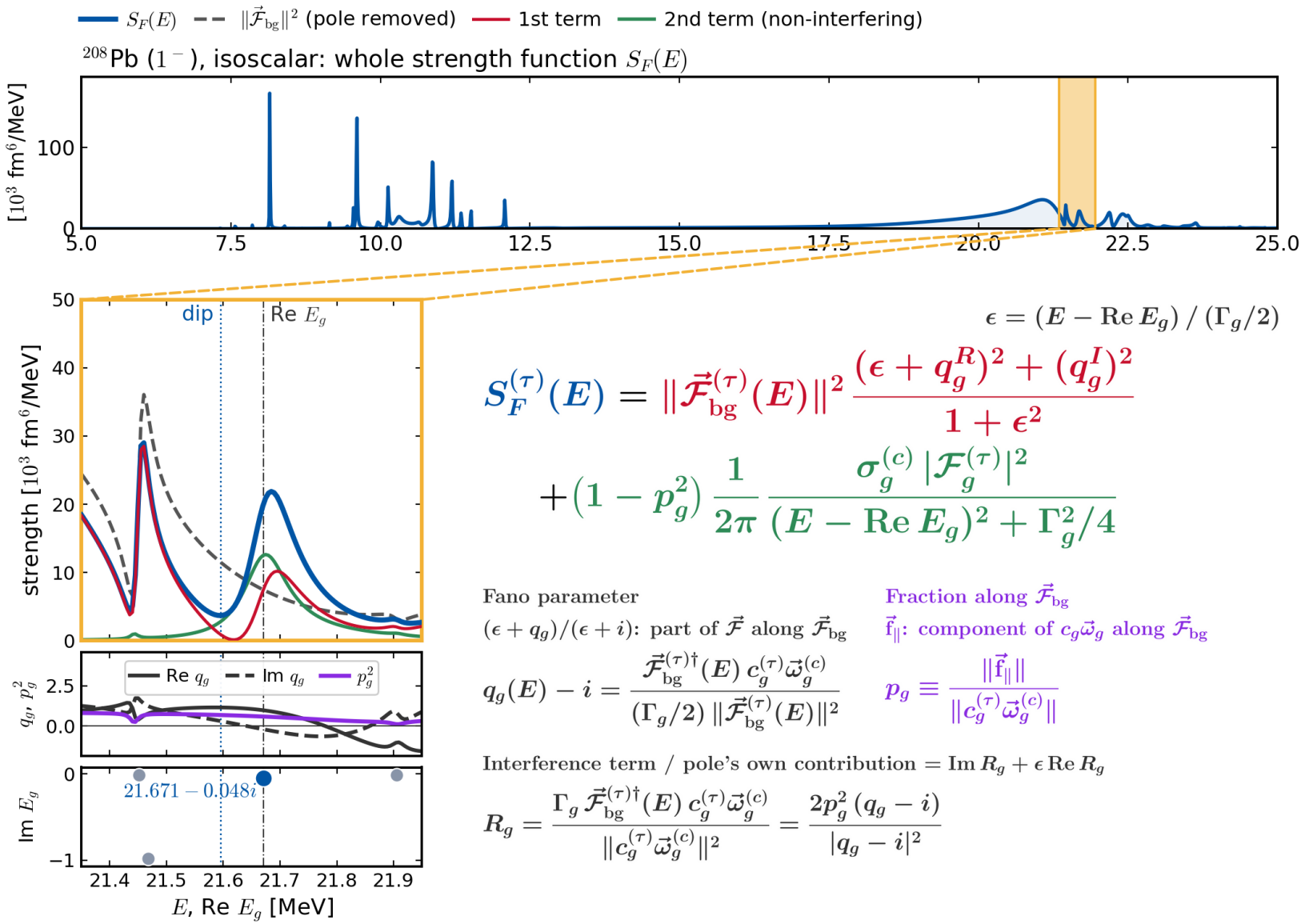
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.24
n + ^{207}Pb($2f_{5/2}^{-1}$), $g_{7/2}$ wave	0.19

Background at Re E_g (b_c)

n + ^{207}Pb($1i_{13/2}^{-1}$), $k_{13/2}$ wave	0.28
n + ^{207}Pb($3p_{3/2}^{-1}$), $d_{5/2}$ wave	0.16

Fano Analysis: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Dip at 21.60 MeV

Collective Pole: $|R_g| \sim 1$, Re $R_g > 0$ → Dip below Re E_g



$$\epsilon = (E - \mathrm{Re}\, E_g) \,/\, (\Gamma_g/2)$$

$$S_F^{(\tau)}(E) = \|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2 \frac{(\epsilon + q_g^R)^2 + (q_g^I)^2}{1+\epsilon^2} + (1-p_g^2)\,\frac{1}{2\pi}\,\frac{\sigma_g^{(c)}\,|\mathcal{F}_g^{(\tau)}|^2}{(E-\mathrm{Re}\,E_g)^2 + \Gamma_g^2/4}$$

Fano parameter

$(\epsilon + q_g)/(\epsilon + i)$: part of $\vec{\mathcal{F}}$ along $\vec{\mathcal{F}}_{\rm bg}$

$$q_g(E) - i = \frac{\vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{(\Gamma_g/2)\,\|\vec{\mathcal{F}}_{\rm bg}^{(\tau)}(E)\|^2}$$

Fraction along $\vec{\mathcal{F}}_{\rm bg}$

$\vec{\mathrm{f}}_\parallel$: component of $c_g\vec{\omega}_g$ along $\vec{\mathcal{F}}_{\rm bg}$

$$p_g \equiv \frac{\|\vec{\mathrm{f}}_\parallel\|}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|}$$

Interference term / pole's own contribution $= \mathrm{Im}\, R_g + \epsilon\, \mathrm{Re}\, R_g$

$$R_g = \frac{\Gamma_g\, \vec{\mathcal{F}}_{\rm bg}^{(\tau)\dagger}(E)\, c_g^{(\tau)} \vec{\omega}_g^{(c)}}{\|c_g^{(\tau)}\vec{\omega}_g^{(c)}\|^2} = \frac{2p_g^2\,(q_g - i)}{|q_g - i|^2}$$

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.671 - 0.048i$ MeV

Collectivity (IS): collective

C_g^2	0.1048
$N_{\rm tot}\eta_g$	20.95
$C_g^2 N_{\rm tot}\eta_g$	2.20

At E = Re E_g ($\epsilon = 0$)

q_g	$0.98 - 0.20i$
p_g^2	0.59
R_g	$0.48 - 0.59i$
$\|R_g\|$	$0.76 \sim 1$

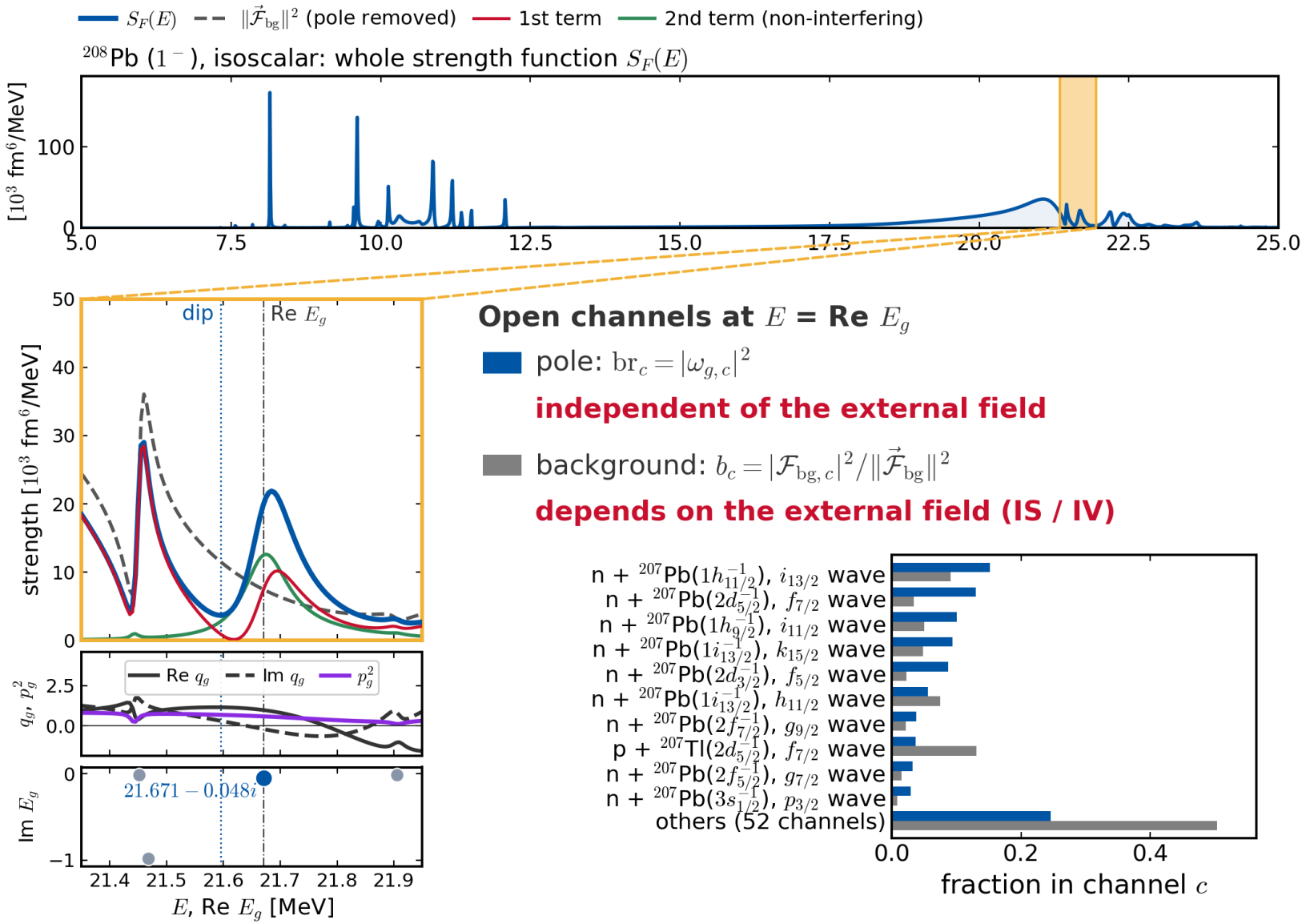
interference comparable

Im R_g < 0: lowers S_F at $\epsilon = 0$

⇒ **dip and peak**

Branching Ratios: ^{208}Pb ($J^\pi = 1^-$) Isoscalar Dip at 21.60 MeV

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pole: $\mathrm{br}_c = |\omega_{g,c}|^2$

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background: $b_c = |\mathcal{F}_{\mathrm{bg},c}|^2 / \|\vec{\mathcal{F}}_{\rm bg}\|^2$

depends on the external field (IS / IV)

Pole $E_g = \mathrm{Re}\, E_g - \frac{i}{2}\Gamma_g$

E_g	$21.671 - 0.048i$ MeV
$\sigma_g^{(c)}/\Gamma_g = \sum_c \Gamma_c/\Gamma_g$	1.28

Pole decays into (br_c)

n + ^{207}Pb($1h_{11/2}^{-1}$), $i_{13/2}$ wave	0.15
n + ^{207}Pb($2d_{5/2}^{-1}$), $f_{7/2}$ wave	0.13

Background at Re E_g (b_c)

p + ^{207}Tl($2d_{5/2}^{-1}$), $f_{7/2}$ wave	0.13
n + ^{207}Pb($1h_{11/2}^{-1}$), $i_{13/2}$ wave	0.09

Contact information

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