

# Halo formation in $^{11}\text{Li}$ : The Role of Pairing and Tensor Correlations nuclei

Takayuki MYO 明孝之  
Osaka Institute of Technology  
大阪工業大学

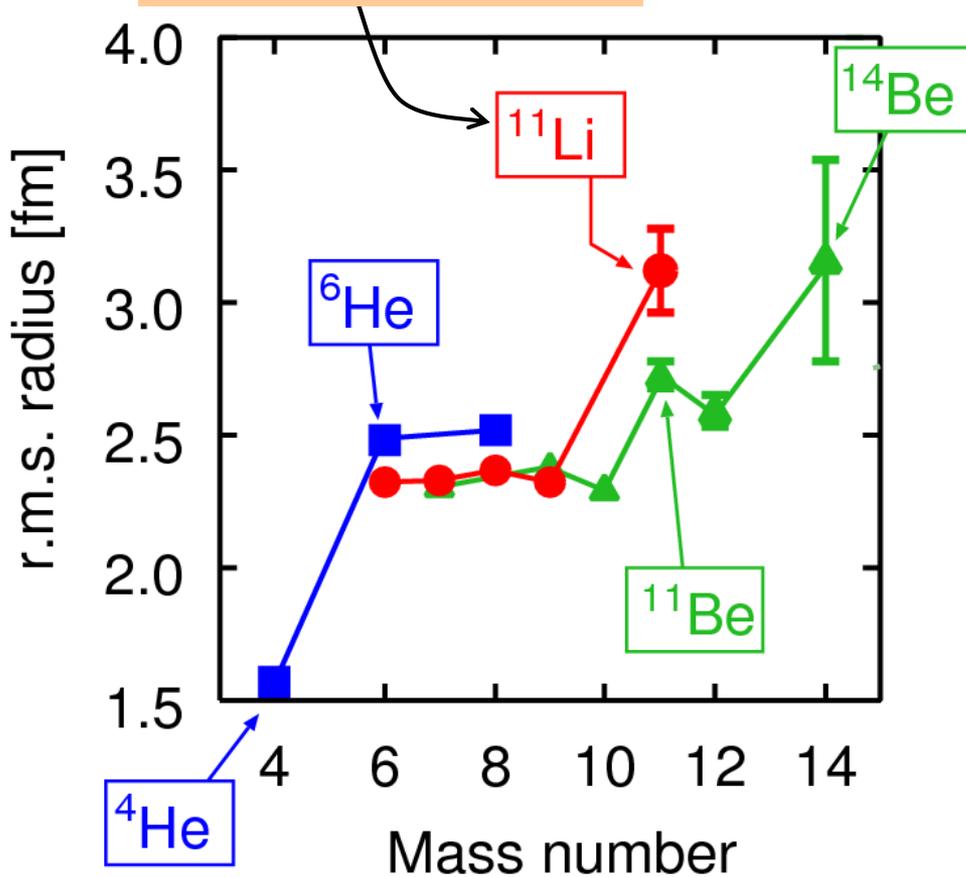


# Outline

- **Role of  $V_{\text{tensor}}$**  in the nuclear structure **by describing strong tensor correlation explicitly.**
- Tensor Optimized Shell Model (**TOSM**) to describe tensor correlation.
- Halo formation in  $^{11}\text{Li}$ 
  - Coexistence of tensor and pairing correlations
- Unitary Correlation Operator Method (**UCOM**) to describe short-range correlation.
- **TOSM+UCOM** to He & Li isotopes with  $V_{\text{bare}}$

# Characteristics of Li-isotopes

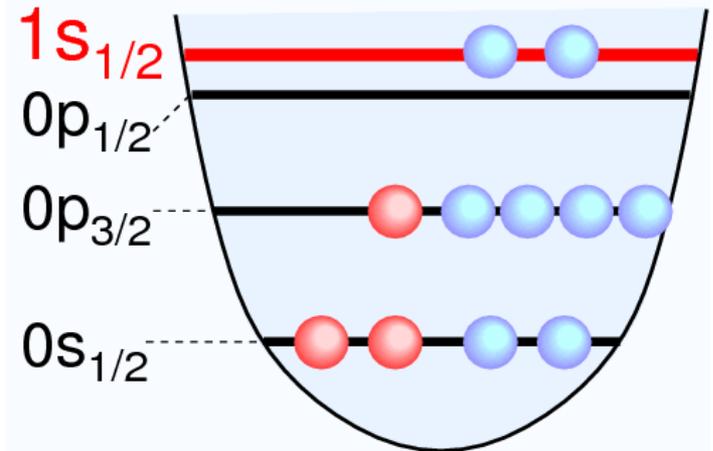
## Halo structure



Tanihata et al., PRL55(1985)2676.  
PLB206(1998)592.

## ✓ Breaking of magicity N=8

- $^{10-11}\text{Li}$ ,  $^{11-12}\text{Be}$
- $^{11}\text{Li}$  ...  $(1s)^2 \sim 50\%$ .  
(Expt by Simon et al., PRL83)
- **Mechanism is unclear**



$^{11}\text{Li}$

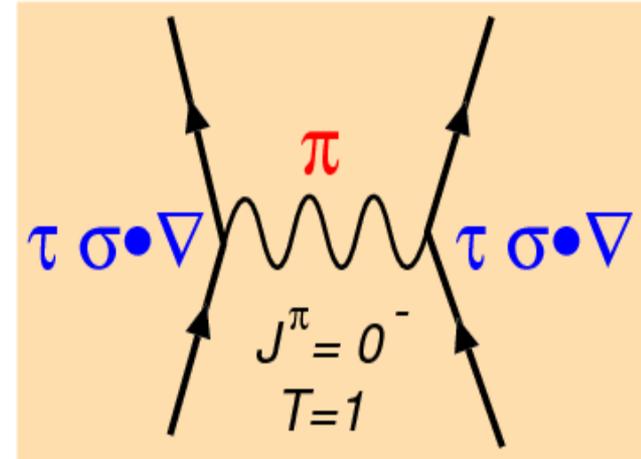
# Importance of tensor force

- Tensor force ( $V_{\text{tensor}}$ ) plays a significant role in the nuclear structure.

– In  ${}^4\text{He}$ ,  $\langle V_{\text{tensor}} \rangle \sim \langle V_{\text{central}} \rangle$

$P(D) \sim 15\%$  (AV18, GFMC)

R.B. Wiringa, S.C. Pieper, J. Carlson,  
V.R. Pandharipande, PRC62(2001)

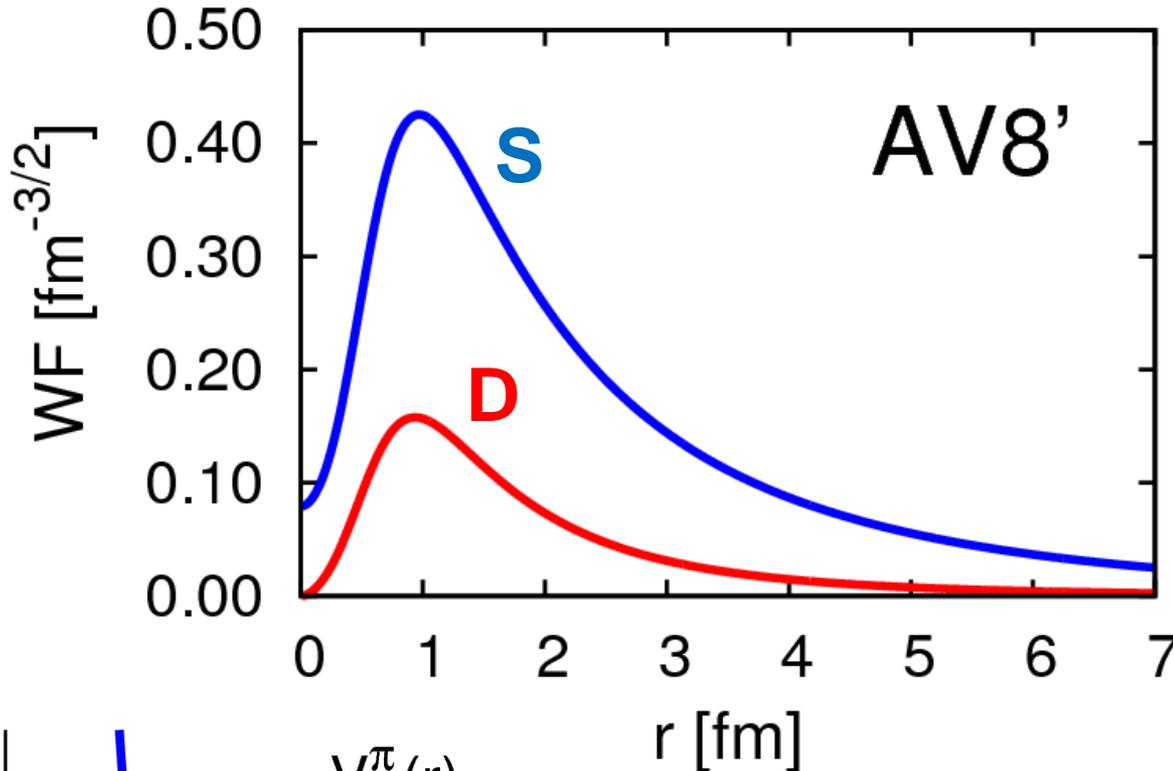


- We would like to understand **role of  $V_{\text{tensor}}$**  in the nuclear structure **by describing tensor correlation explicitly.**

- ✓ wave function : shell model, cluster model
- ✓  $LS$  splitting, halo formation, level inversion, etc,...

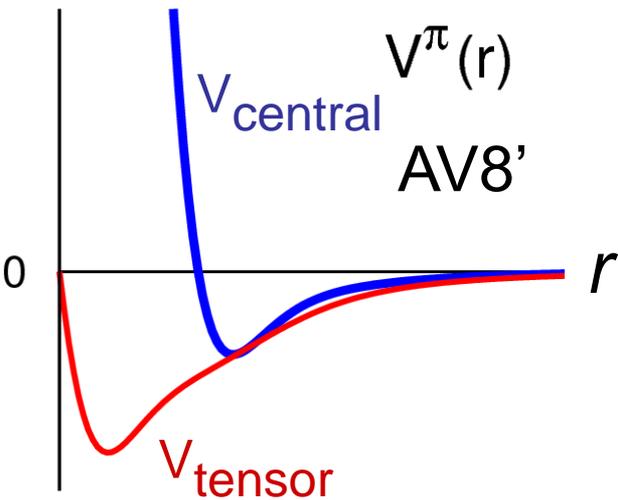
**Li isotopes**

# Deuteron properties & tensor force



Energy	-2.24 MeV
Kinetic	19.88
Central	-4.46
<b>Tensor</b>	<b>-16.64</b>
LS	-1.02
P(L=2)	5.77%
Radius	1.96 fm

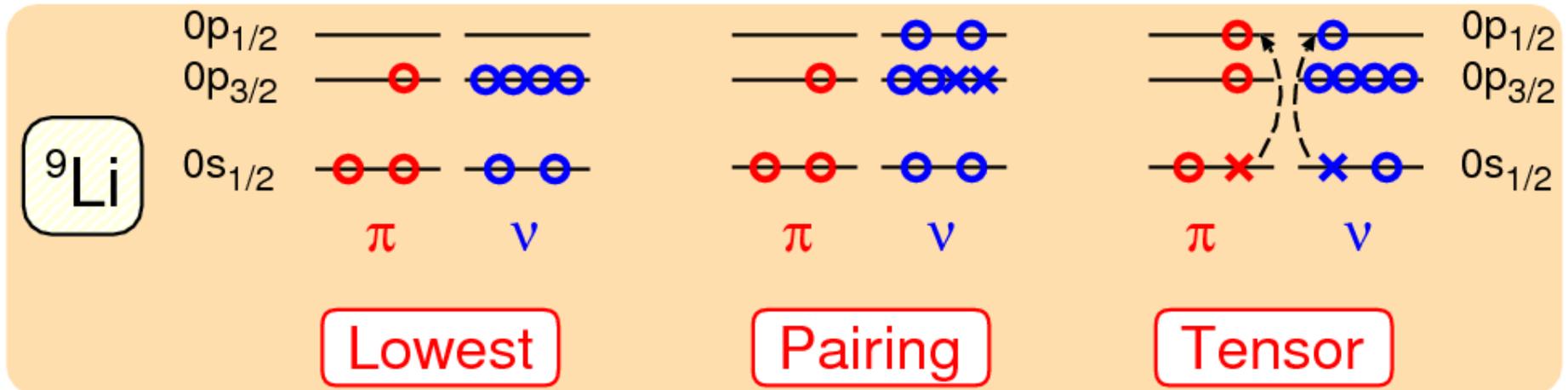
$R_m(s) = 2.00$  fm  
 $R_m(d) = 1.22$  fm



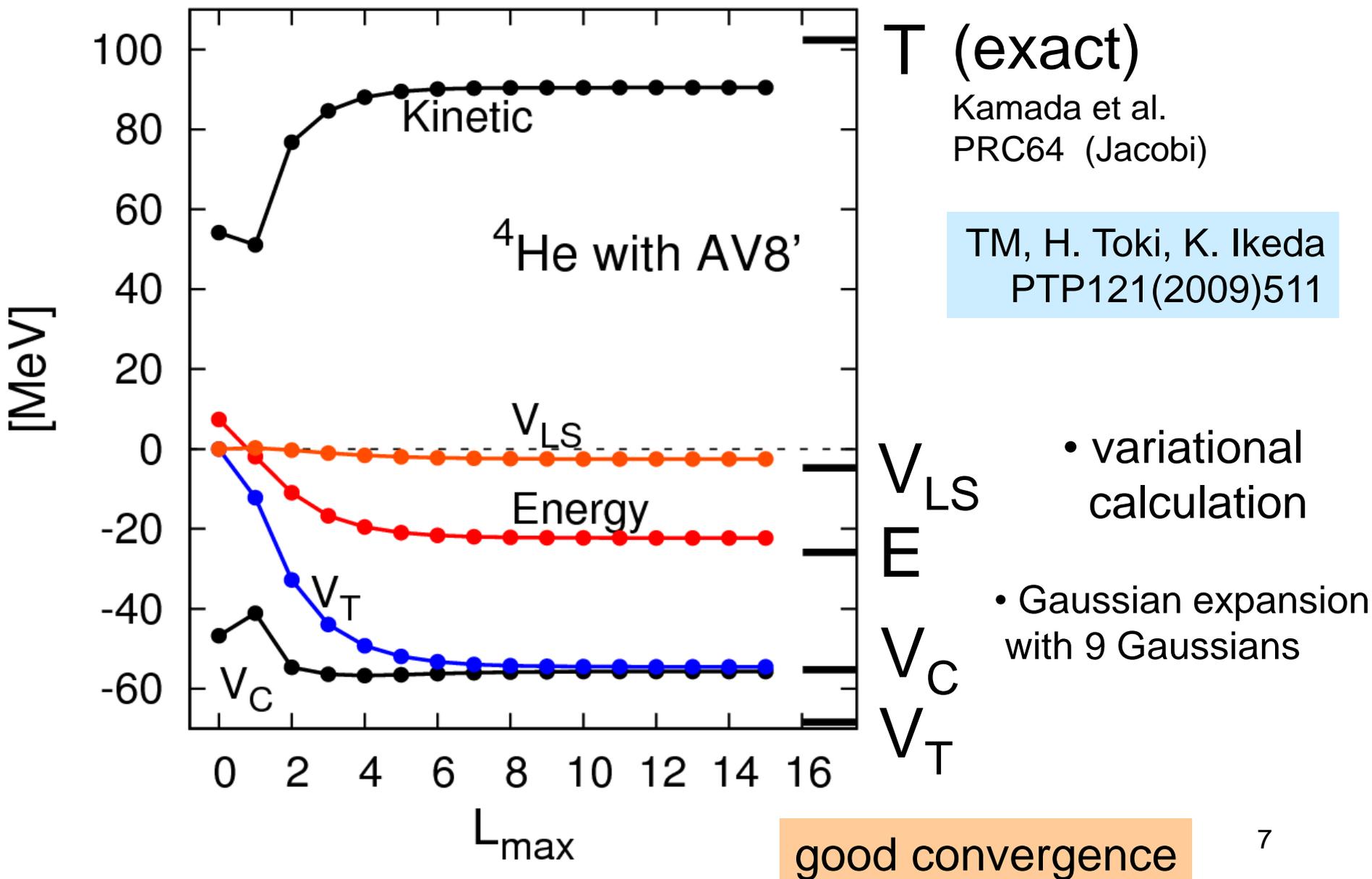
*d*-wave is  
**“spatially compact”**  
 (high momentum)

# $^9\text{Li}$ in TOSM

- **Tensor-optimized shell model**  
TM et al., PTP121(2009), PTP117(2007).
- $0s+0p+1s0d$  within  $2p2h$  excitations,  $G$ -matrix (Akaishi)
- Length parameters  $b_{0s}$ ,  $b_{0p1/2}$ ,  $b_{0p3/2}$ , ... are determined **independently** and **variationally** (Gaussian expansion).
  - Describe **high momentum component** from  $V_{\text{tensor}}$   
cf. CPP-HF by Sugimoto et al., NPA740 / Akaishi NPA738



# $^4\text{He}$ in TOSM + short-range UCOM



# Configurations of ${}^4\text{He}$ with AV8'

$(0s_{1/2})^4$	83.0 %
$(0s_{1/2})^{-2}_{JT}(p_{1/2})^2_{JT}$ <b><math>JT=10</math></b>	2.6
$JT=01$	0.1
$(0s_{1/2})^{-2}_{10}(1s_{1/2})(d_{3/2})_{10}$	2.3
$(0s_{1/2})^{-2}_{10}(p_{3/2})(f_{5/2})_{10}$	1.9
Radius [fm]	1.54

TM, H. Toki, K. Ikeda  
PTP121(2009)511

• deuteron correlation  
with  $(J, T)=(1, 0)$

Cf. R.Schiavilla et al. (VMC)  
PRL98(2007)132501  
R. Subedi et al. (JLab)  
Science320(2008)1476

${}^{12}\text{C}(e, e' pN)$

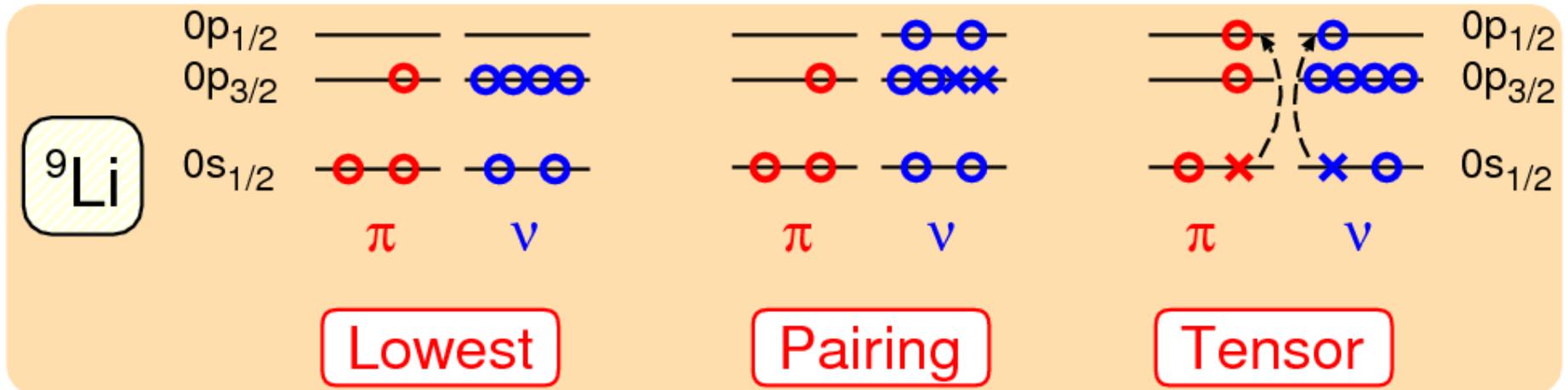
S.C.Simpson, J.A.Tostevin  
PRC83(2011)014605

${}^{12}\text{C} \rightarrow {}^{10}\text{B} + pn$

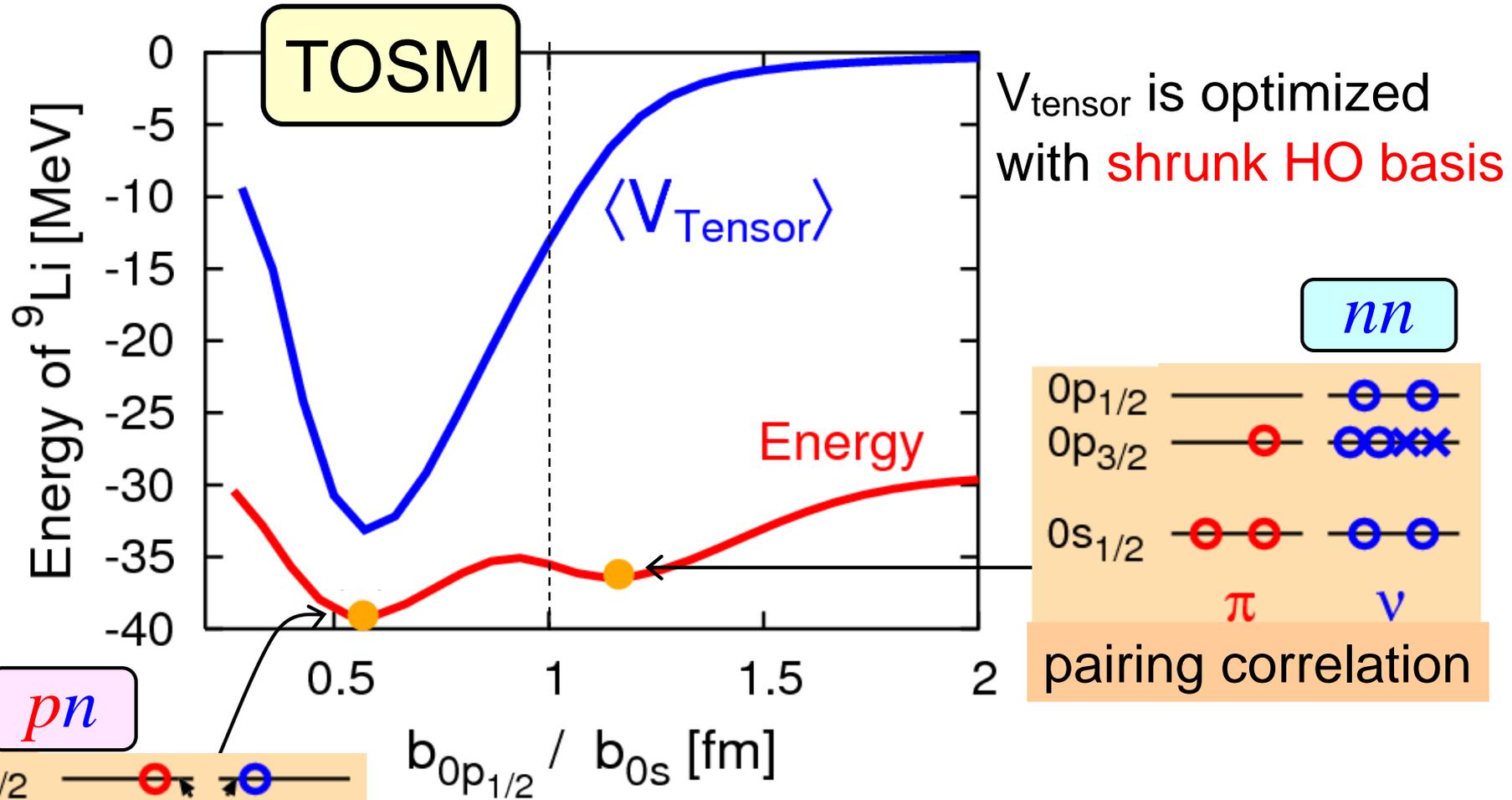
- ${}^4\text{He}$  contains  $p_{1/2}$  of “ **$pn$ -pair**”  
– Same feature in  ${}^5\text{He}$ - ${}^8\text{He}$  ground state

# $^9\text{Li}$ in TOSM

- **Tensor-optimized shell model**  
TM et al., PTP121(2009), PTP117(2007).
- $0s+0p+1s0d$  within  $2p2h$  excitations,  $G$ -matrix (Akaishi)
- Length parameters  $b_{0s}$ ,  $b_{0p1/2}$ ,  $b_{0p3/2}$ , ... are determined **independently** and **variationally** (Gaussian expansion).
  - Describe **high momentum component** from  $V_{\text{tensor}}$   
cf. CPP-HF by Sugimoto et al., NPA740 / Akaishi NPA738



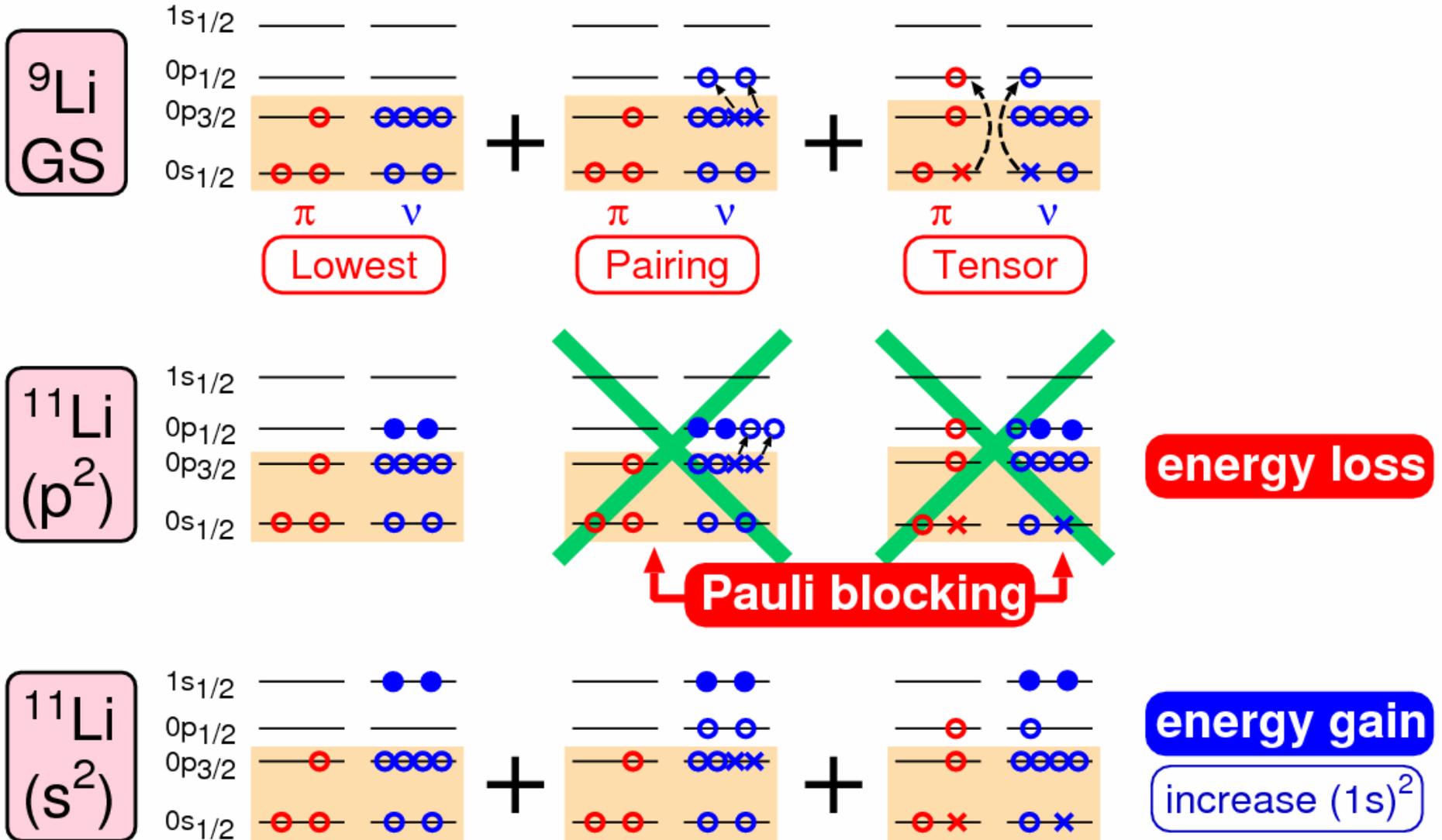
# Energy surface for $b$ -parameter in ${}^9\text{Li}$



Dominant part of the tensor correlation

cf. 1<sup>st</sup> order (residual interaction): T. Otsuka et al.  
PRL95(2005)232502.

# Expected effects of pairing and tensor correlations in $^{11}\text{Li}$



Pairing-blocking :

K.Kato, T.Yamada, K.Ikeda, PTP101('99)119, Masui, S.Aoyama, TM, K.Kato, K.Ikeda, NPA673('00)207.  
 TM, S.Aoyama, K.Kato, K.Ikeda, PTP108('02)133, H.Sagawa, B.A.Brown, H.Esbensen, PLB309('93)1.

# $^{11}\text{Li}$ in coupled $^9\text{Li}+n+n$ model

- System is solved based on RGM

$$H(^{11}\text{Li}) = H(^9\text{Li}) + H_{nn} \quad \Phi(^{11}\text{Li}) = \mathcal{A} \left\{ \sum_{i=1}^N \psi_i(^9\text{Li}) \cdot \chi_i(nn) \right\}$$

$$\sum_{i=1}^N \left\langle \psi_j(^9\text{Li}) \left| H(^{11}\text{Li}) - E \right| \mathcal{A} \left\{ \psi_i(^9\text{Li}) \cdot \chi_i(nn) \right\} \right\rangle = 0$$

$\psi_i(^9\text{Li})$ : **shell model type configuration**  $\rightarrow$  **TOSM**

- Orthogonality Condition Model (OCM) is applied.

$$\sum_{i=1}^N \left[ H_{ij} (^9\text{Li}) + (T_1 + T_2 + V_{c1} + V_{c2} + V_{12}) \cdot \delta_{ij} \right] \chi_j(nn) = E \chi_i(nn)$$

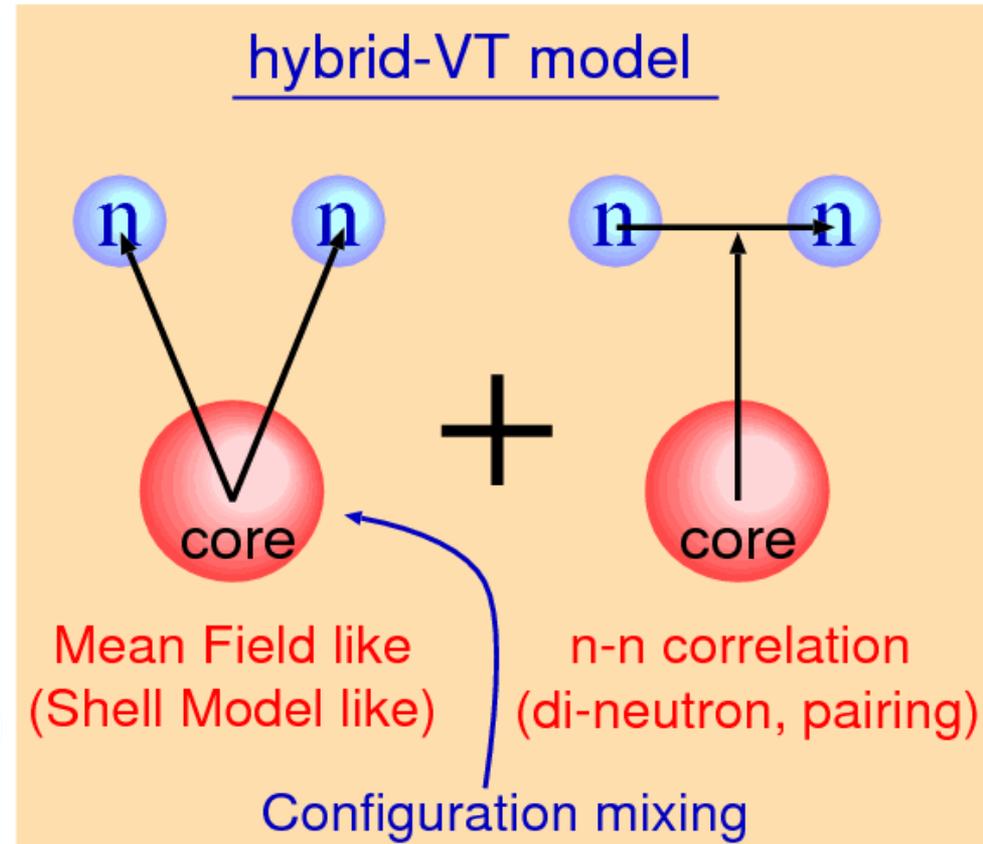
$$H_{ij} (^9\text{Li}) = \langle \psi_i | H(^9\text{Li}) | \psi_j \rangle : \text{Hamiltonian for } ^9\text{Li}$$

$$\chi(nn) = \mathcal{A} \{ \phi_1 \phi_2 \} : \text{hybrid-TVmodel with Gaussian expansion}$$

$$\langle \phi_i | \phi_\alpha \rangle = 0, \{ \phi_\alpha \in ^9\text{Li} \} : \text{Orthogonality to the Pauli-forbidden states}^{12}$$

# Hamiltonian of $^{11}\text{Li}$

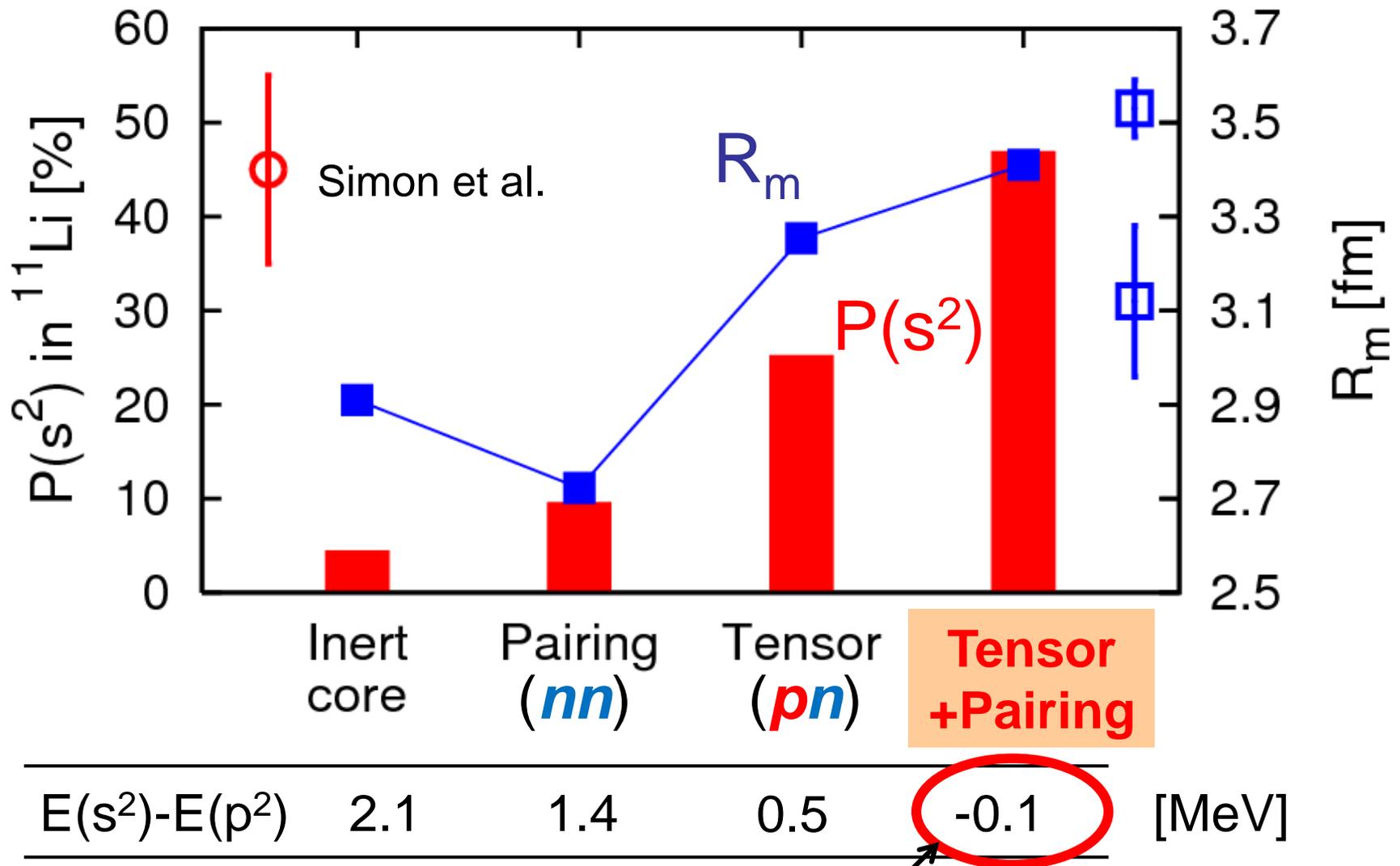
- $V_{\text{core-n}}$  : folding potential
  - Same strength for s- and p-waves
  - Adjust to reproduce  $S_{2n}=0.31$  MeV
- $V_{nn}$  : Argonne (AV8')
- $2n$  : Gaussian expansion



TM, K.Kato, H.Toki, K.Ikeda, PRC76('07)024305.

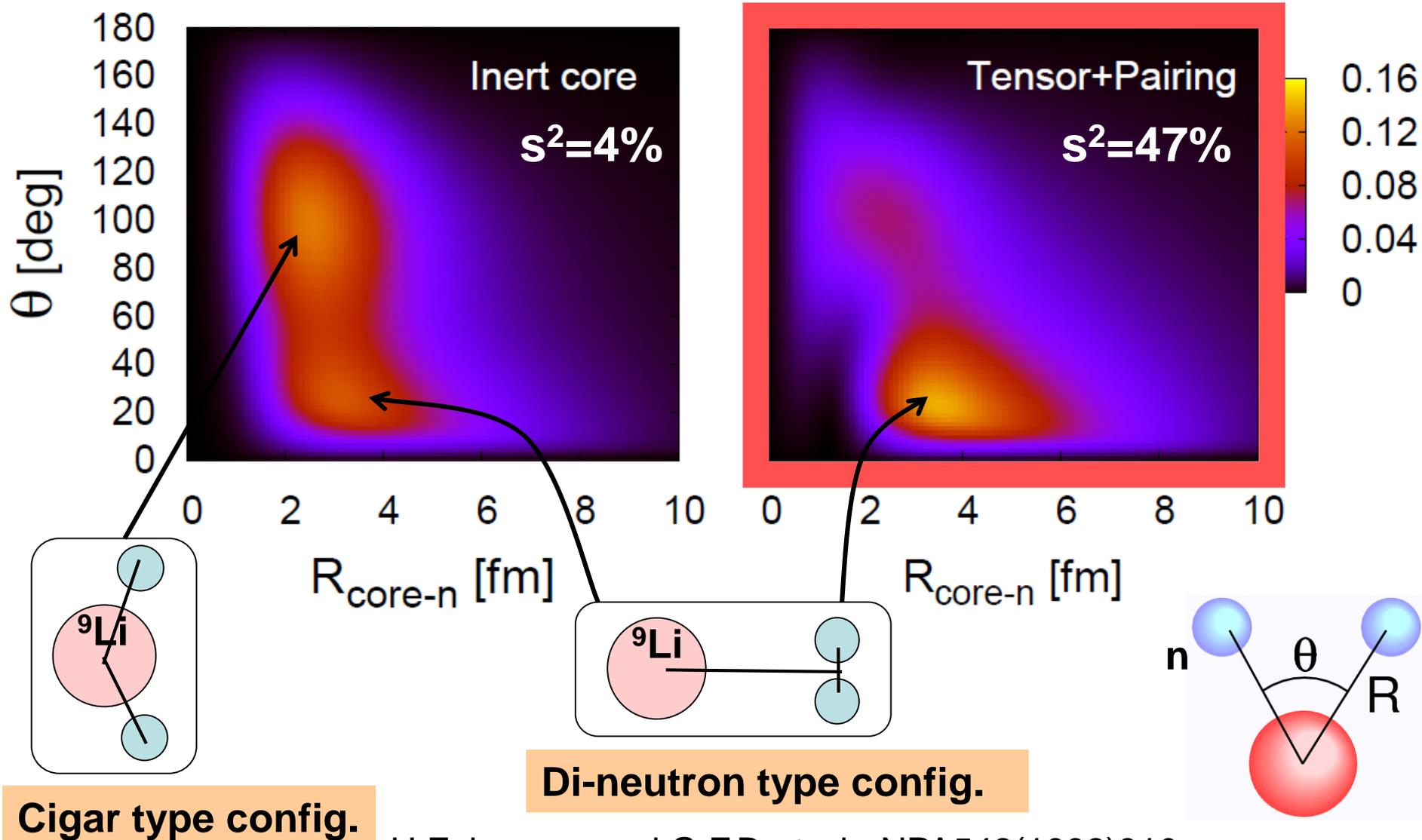
TM. Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119('08)561.

# $^{11}\text{Li}$ G.S. properties ( $S_{2n}=0.31$ MeV)



Pairing correlation couples  $(0p)^2$  and  $(1s)^2$  for last  $2n$

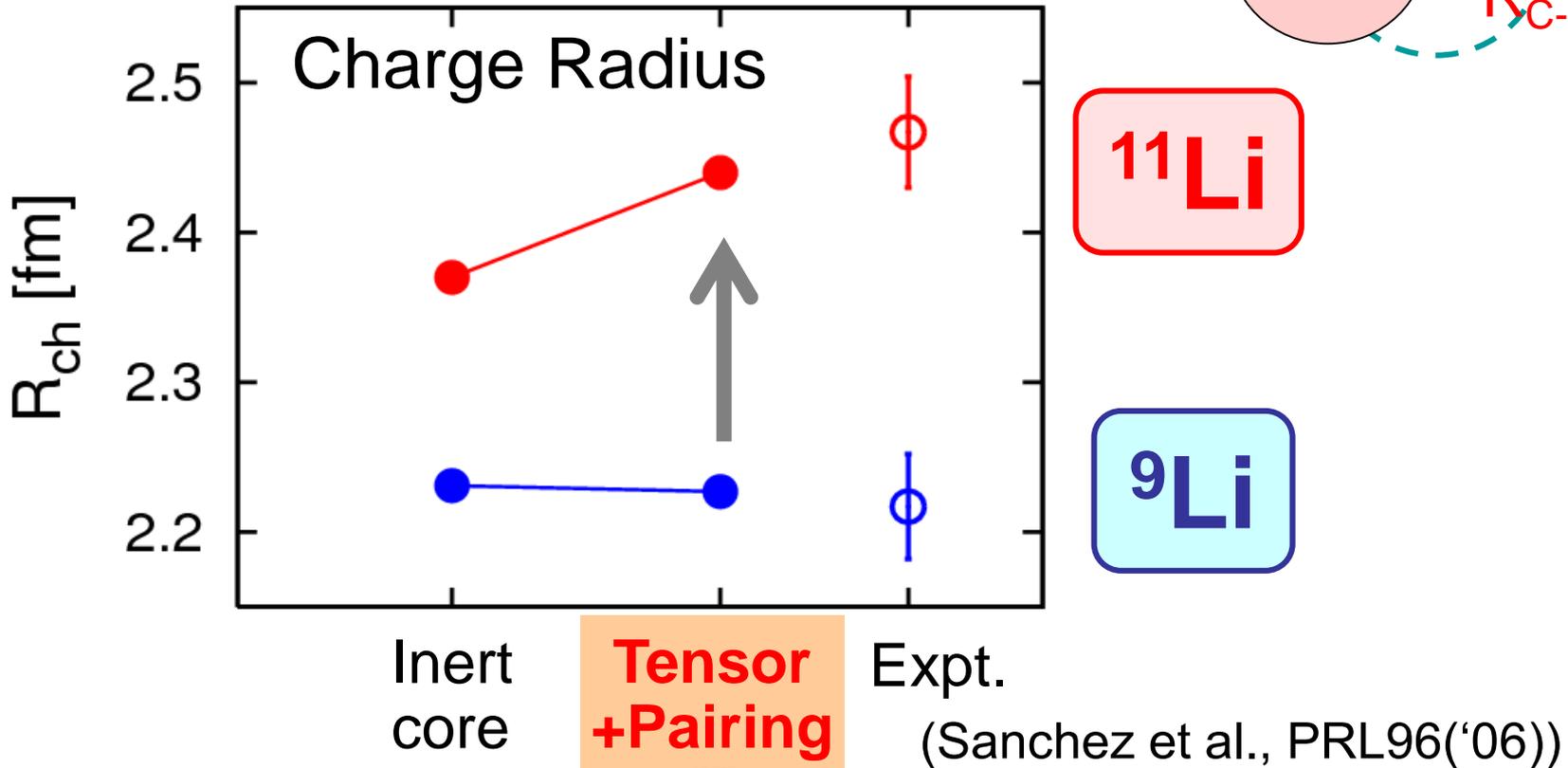
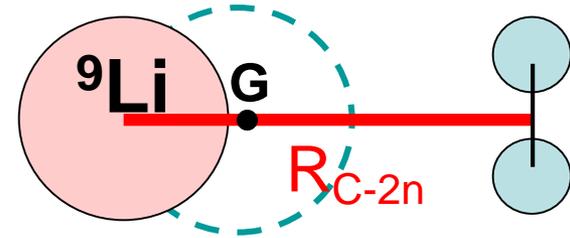
# 2n correlation density in $^{11}\text{Li}$



H.Esbensen and G.F.Bertsch, NPA542(1992)310  
K. Hagino and H. Sagawa, PRC72(2005)044321

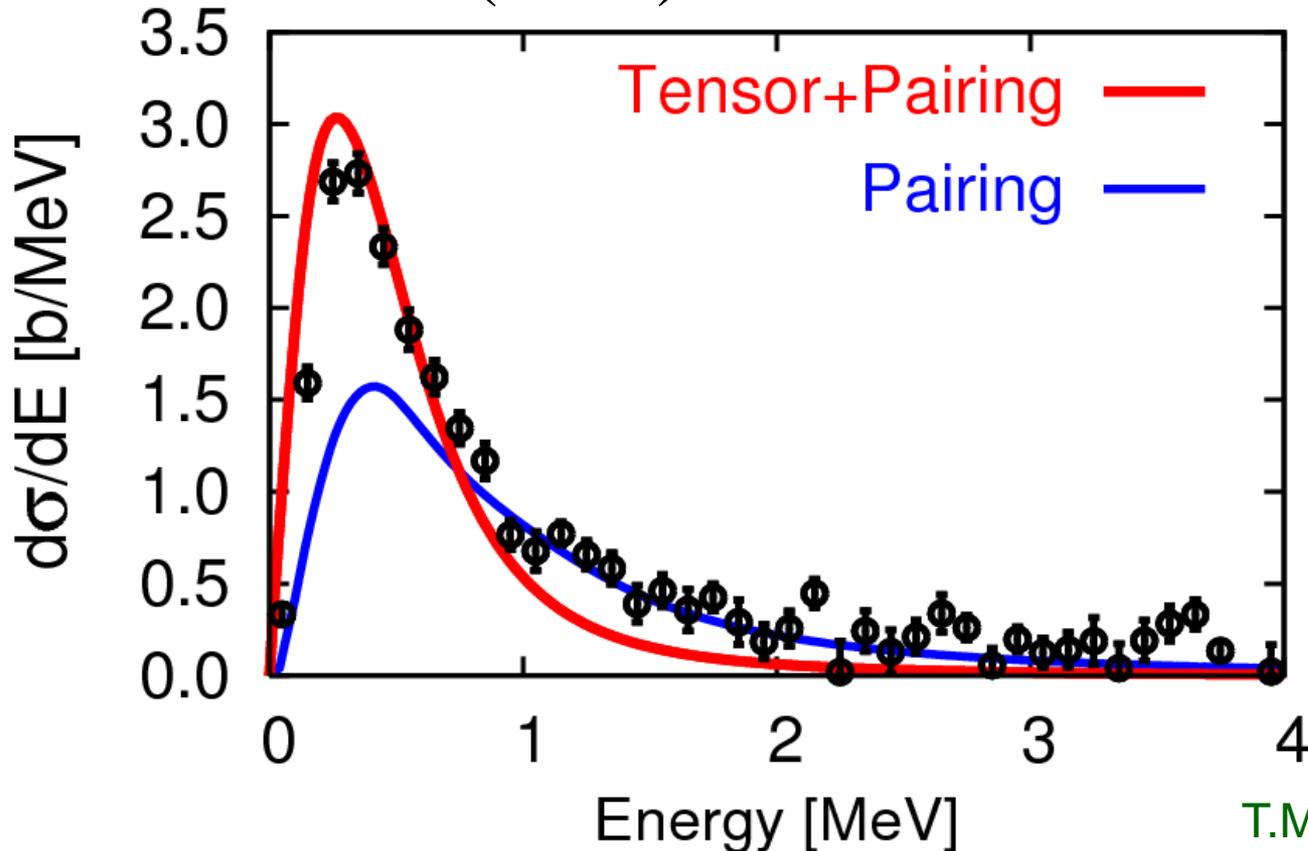
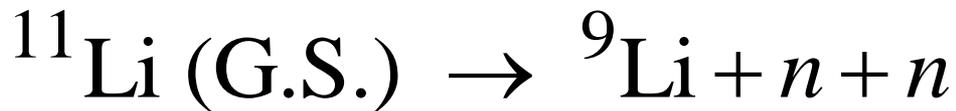
# Charge Radii of Li isotopes

$$R_{\text{proton}}^2(^{11}\text{Li}) = R_{\text{proton}}^2(^9\text{Li}) + \left(\frac{2}{11}\right)^2 R_{\text{C-2n}}^2$$



$R_{\text{C-2n}}$	4.67	<b>5.69</b>	[fm]
$P(s^2)$	4	47	%

# Coulomb breakup strength of $^{11}\text{Li}$



**No three-body resonance**

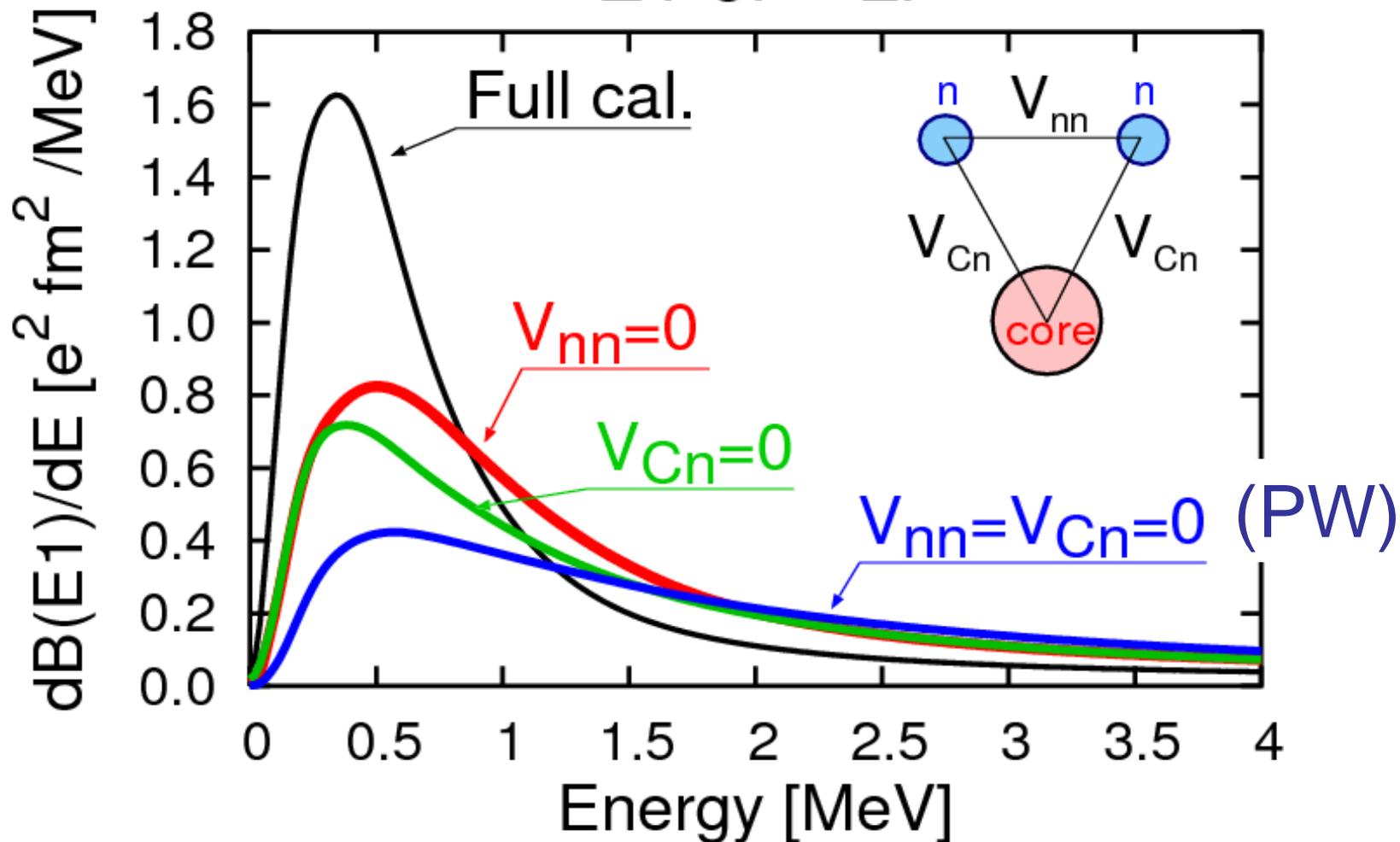
E1 strength by using the  
Green's function method  
+Complex scaling method  
+Equivalent photon method  
(TM et al., PRC63('01))

T.Myo, K.Kato, H.Toki, K.Ikeda  
PRC76(2007)024305

- Expt: T. Nakamura et al. , PRL96,252502(2006)
- Energy resolution with  $\sqrt{E} = 0.17$  MeV.

# Correlations in the final states of $^{11}\text{Li}$ breakup

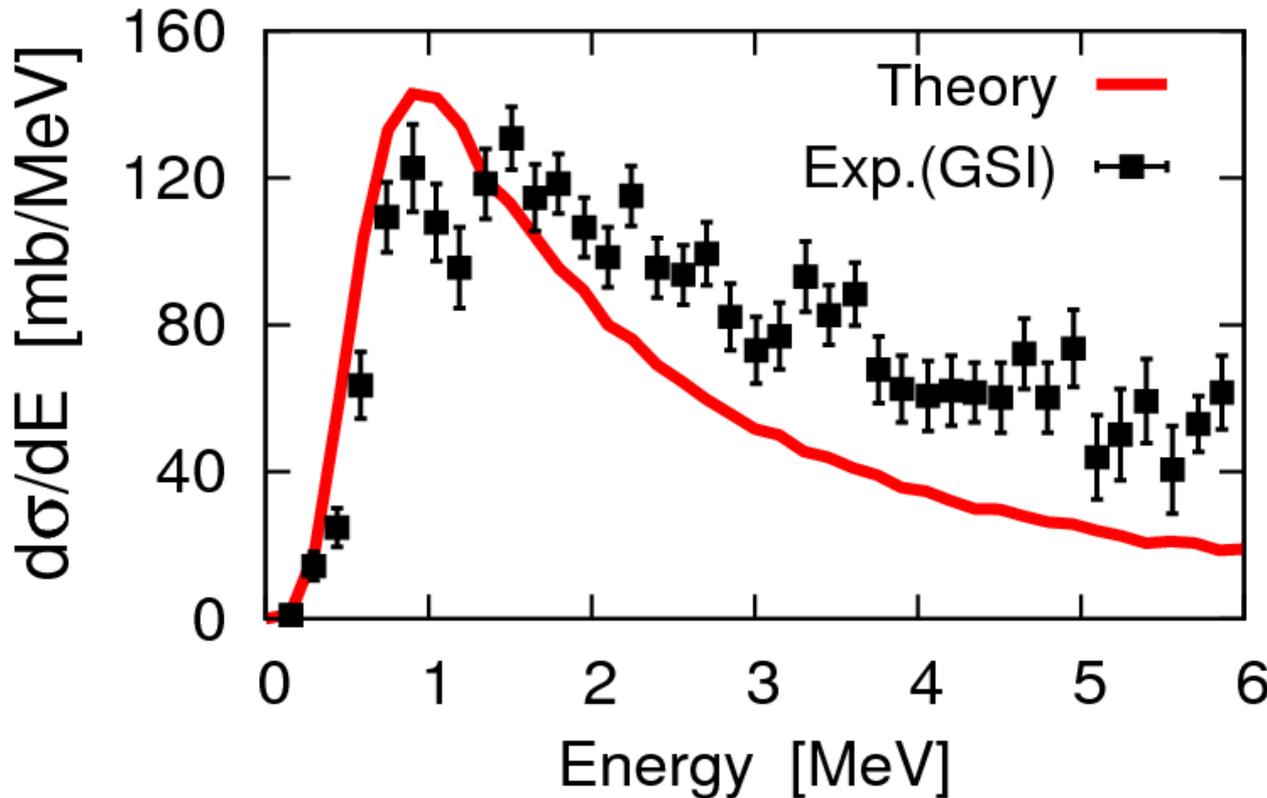
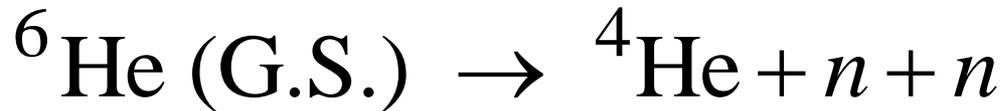
## E1 of $^{11}\text{Li}$



TM, K.Kato, H.Toki, K.Ikeda, PRC76('07)024305.

TM. Y.Kikuchi, K.Kato, H.Toki, K.Ikeda, PTP119('08)561.

# Coulomb breakup strength of ${}^6\text{He}$



E1+E2

Equivalent photon method

TM, K. Kato, S.

Aoyama and K. Ikeda

PRC63(2001)054313.

Kikuchi, TM, Takashina,

Kato, Ikeda

PTP122(2009)499

PRC81(2010)044308.

(+invariant mass of  
 $\alpha$ -n, and n-n)

${}^6\text{He}$  : 240MeV/A, Pb Target (T. Aumann et.al, PRC59(1999)1252)

# Virtual s-wave states in $^{10}\text{Li}$

- **$1s_{1/2}$  virtual state:**

$$(0p_{3/2})_{\pi}(1s_{1/2})_{\nu} \rightarrow 1^{-}, 2^{-}$$

$a_s$  : scattering length of  $^9\text{Li}+n$

$J^{\pi}$	Inert core	<b>Tensor + Pairing</b>
$1^{-}$	+1.4 fm	-5.6 fm
$2^{-}$	+0.8 fm	<b>-17.4 fm</b>

T.M. et al., PTP119('08)561  
arXiv:0803.0590

**Expt.** M. Thoennesen et al.,  
PRC59 (1999)111.  
M. Chartier et al.  
PLB510(2001)24.  
H.B. Jeppesen et al.  
PLB642(2006)449.

$$a_s = -10 \sim -25 \text{ fm}$$

cf.  $a_s(nn) : -18.5 \pm 0.5 \text{ fm}$

**Pauli-blocking  
naturally describes  
virtual s-state in  $^{10}\text{Li}$**

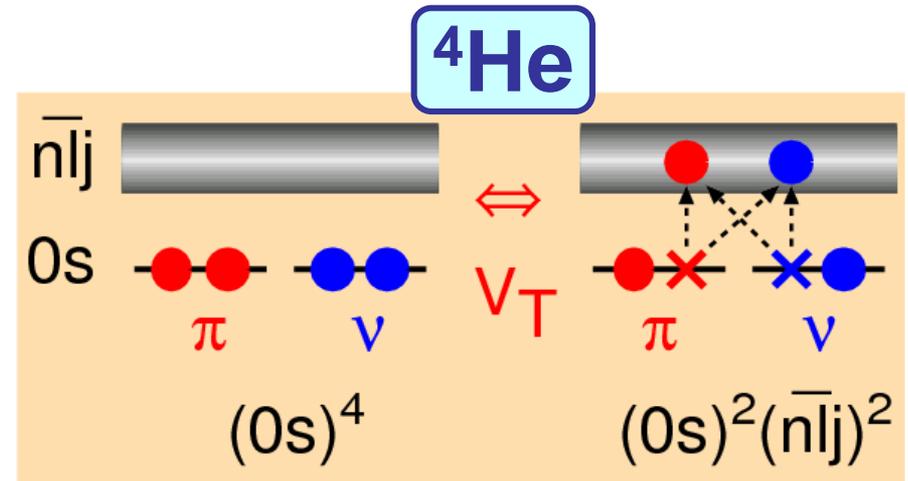
# Tensor-optimized shell model (TOSM)

TM, Sugimoto, Kato, Toki, Ikeda PTP117(2007)257

- Configuration mixing within **2p2h excitations** with high- $L$  orbits.

TM et al., PTP113(2005)

TM et al., PTP117(2007)



- Length parameters such as  $b_{0s}$ ,  $b_{0p}$ , ... are optimized **independently**, or **superposed by many Gaussian bases**.
  - **Spatial shrinkage** of **D-wave** as seen in deuteron
    - HF by Sugimoto et al.(NPA740) / Akaishi (NPA738)
    - RMF by Ogawa et al.(PRC73), AMD by Dote et al.(PTP115)
- Satisfy few-body results with Minnesota central force ( ${}^4, {}^6\text{He}$ )<sub>21</sub>

# Hamiltonian and variational equations in TOSM

$$H = \sum_{i=1}^A t_i - T_G + \sum_{i<j}^A v_{ij},$$

(0p0h+1p1h+2p2h)

$$\Phi(A) = \sum_k C_k \cdot \psi_k(A)$$

$\psi_k(A)$ : shell model type configuration with mass number  $A$

**Particle state**: Gaussian expansion for each orbit

$$\varphi_{lj}^n(\mathbf{r}) = \sum_{m=1}^N C_{lj,m}^n \cdot \phi_{lj,m}(\mathbf{r}) \quad \phi_{lj,m}(\mathbf{r}) = N_l(b_{lj,m}) \cdot r^l e^{-\left(r/b_{lj,m}\right)^2} \left[ Y_l(\hat{\mathbf{r}}), \chi_{1/2}^\sigma \right]_j$$

$$\langle \varphi_{lj}^n | \varphi_{lj}^{n'} \rangle = \delta_{n,n'}$$

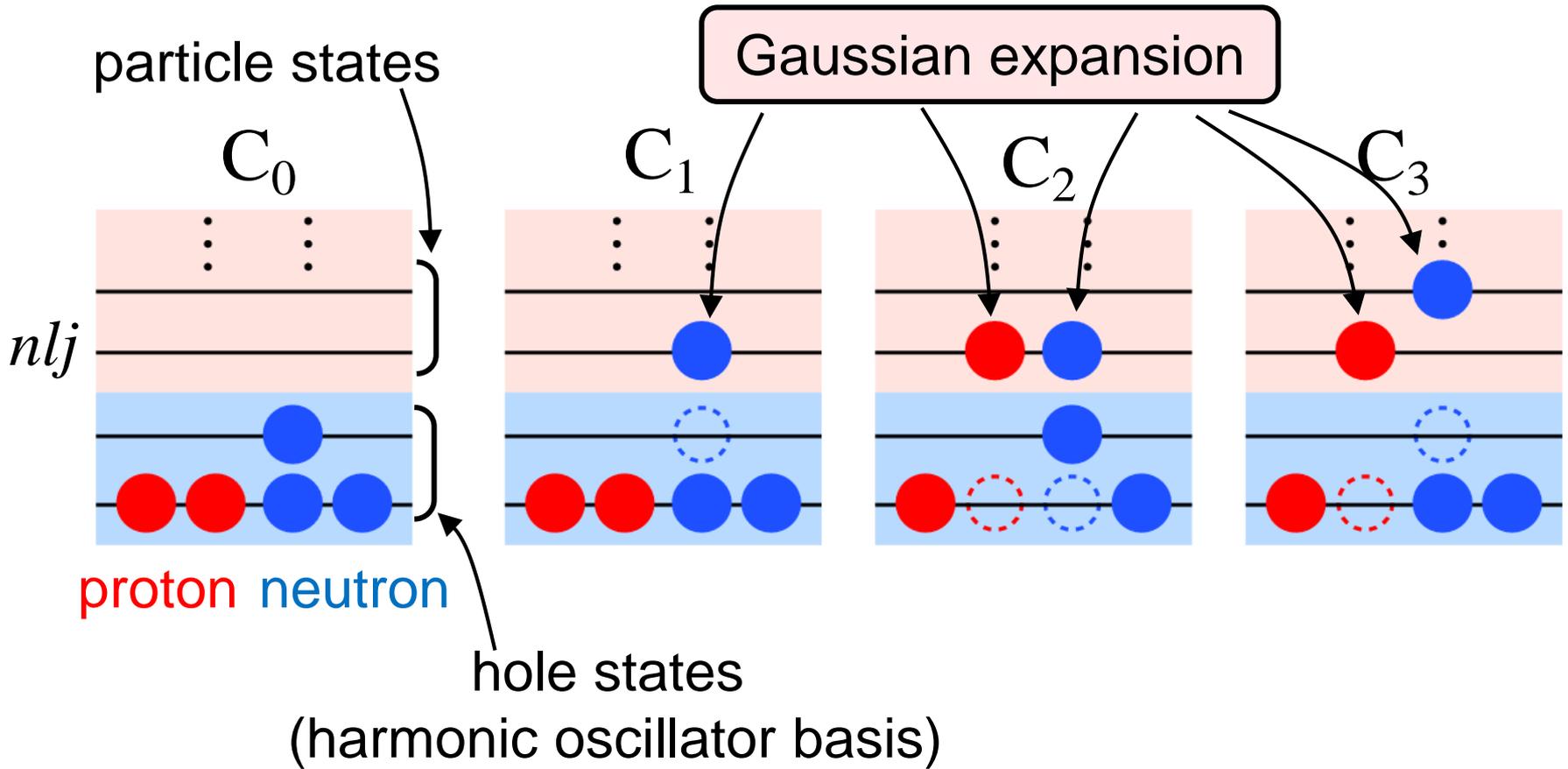
**Gaussian basis function**

$$\frac{\partial \langle H - E \rangle}{\partial C_k} = 0, \quad \frac{\partial \langle H - E \rangle}{\partial b_{lj,m}} = 0$$

TOSM code :  $p$ -shell region

c.m. excitation is excluded by Lawson's method

# Configurations in TOSM



Application to Hypernuclei by Umeya (NIT)  
to investigate  $\Lambda N$ - $\Sigma N$  coupling

# Unitary Correlation Operator Method

(short-range part)

$$\Psi_{\text{corr.}} = \mathbf{C} \cdot \Phi_{\text{uncorr.}}$$

TOSM

short-range correlator

$$\mathbf{C}^\dagger = \mathbf{C}^{-1} \quad (\text{Unitary trans.})$$

$$H\Psi = E\Psi \rightarrow \mathbf{C}^\dagger H\mathbf{C}\Phi \equiv H\Phi = E\Phi$$

Bare Hamiltonian

Shift operator depending on the relative distance

$$\mathbf{C} = \exp\left(-i \sum_{i < j} g_{ij}\right), \quad g_{ij} = \frac{1}{2} \left\{ \underline{p_r s(r_{ij})} + \underline{s(r_{ij}) p_r} \right\} \quad \vec{p} = \vec{p}_r + \vec{p}_\Omega$$

Amount of shift, variationally determined.

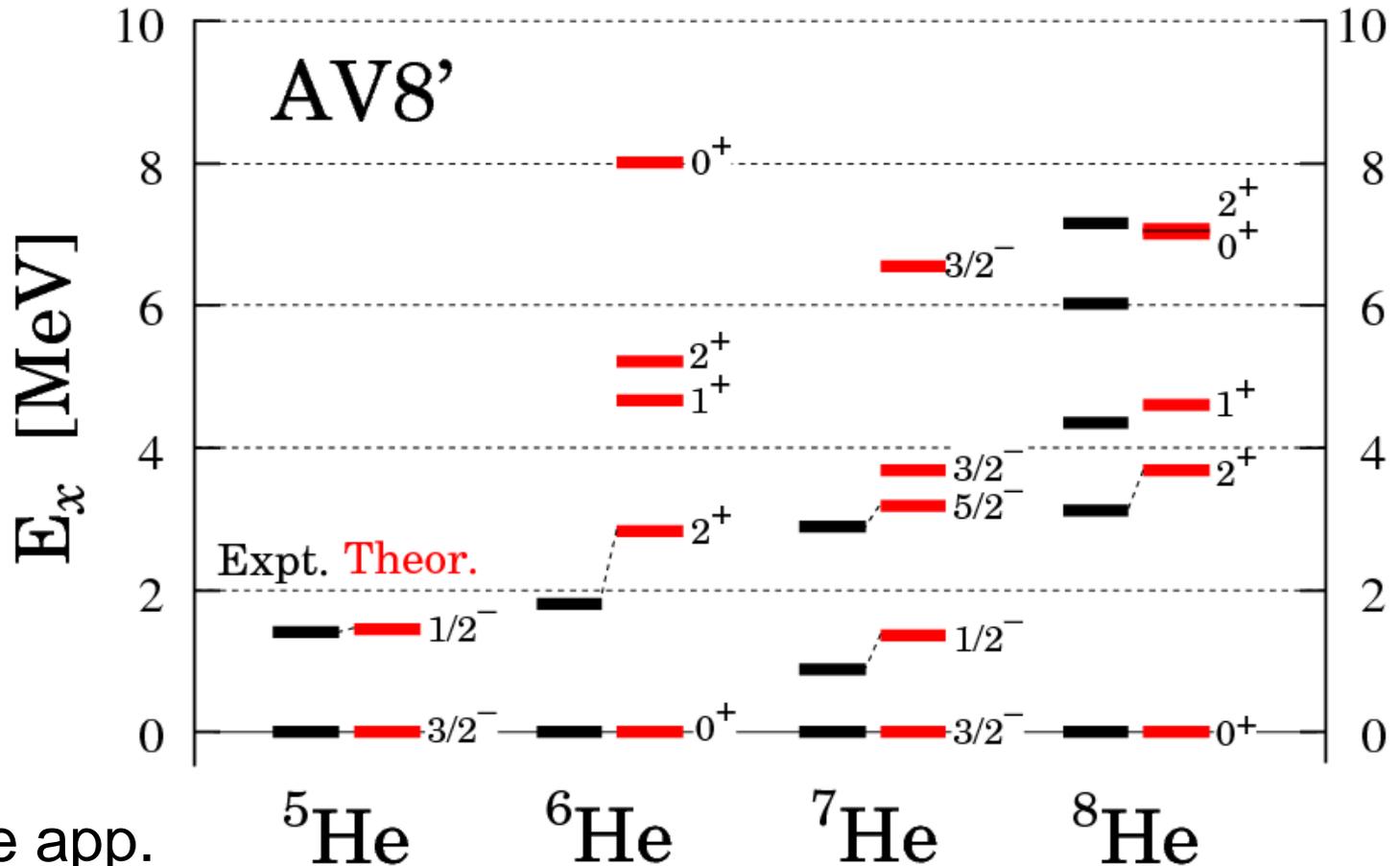
$$\mathbf{C}^\dagger r \mathbf{C} \simeq r + s(r) + \frac{1}{2} s(r) s'(r) \dots$$

2-body cluster expansion

# $4\text{-}8\text{He}$ with TOSM+UCOM

- Excitation energies in MeV

TM, A. Umeya, H. Toki, K. Ikeda  
PRC84 (2011) 034315

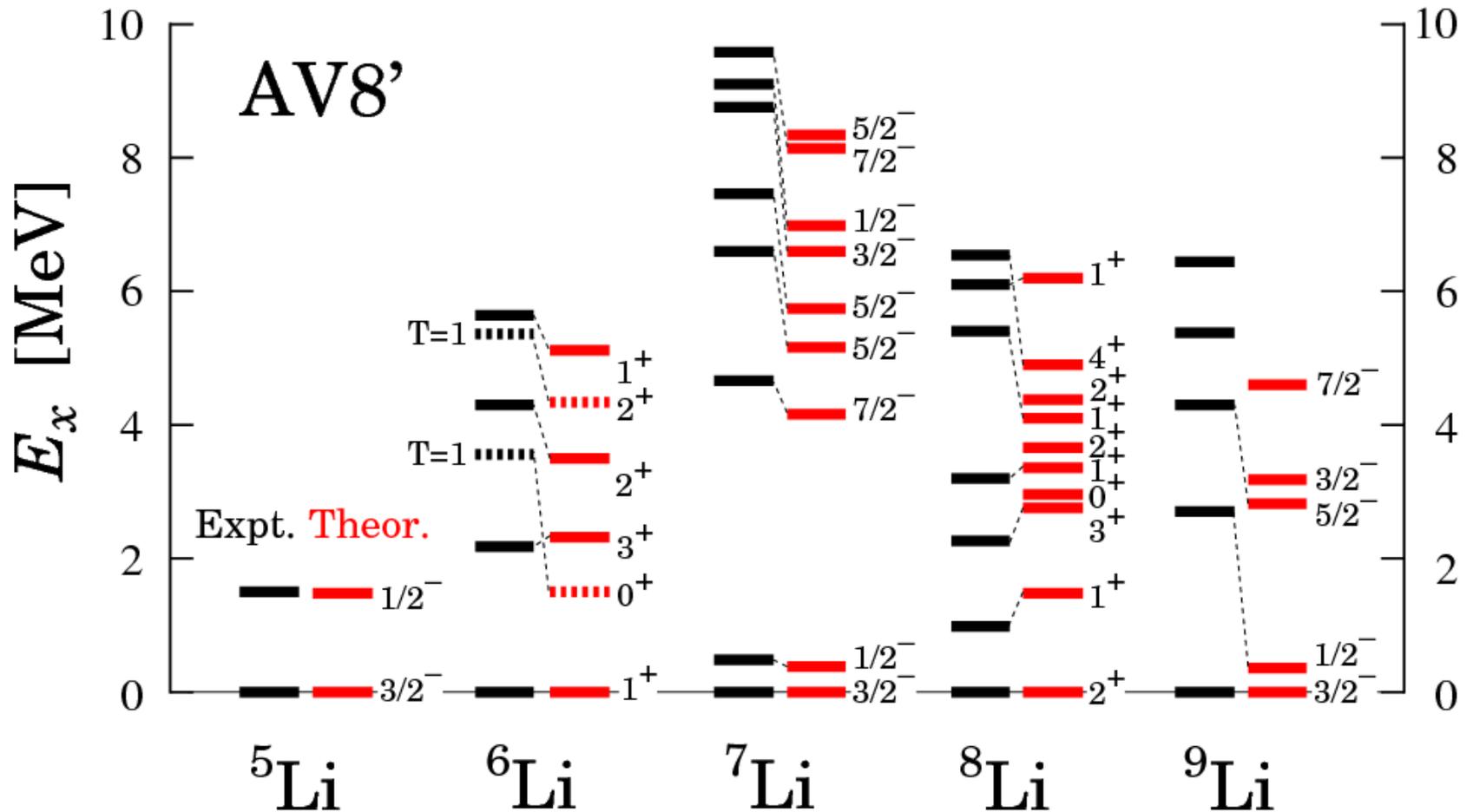


- Bound state app.
- No continuum
- No  $V_{\text{NNN}}$

Excitation energy spectra are reproduced well

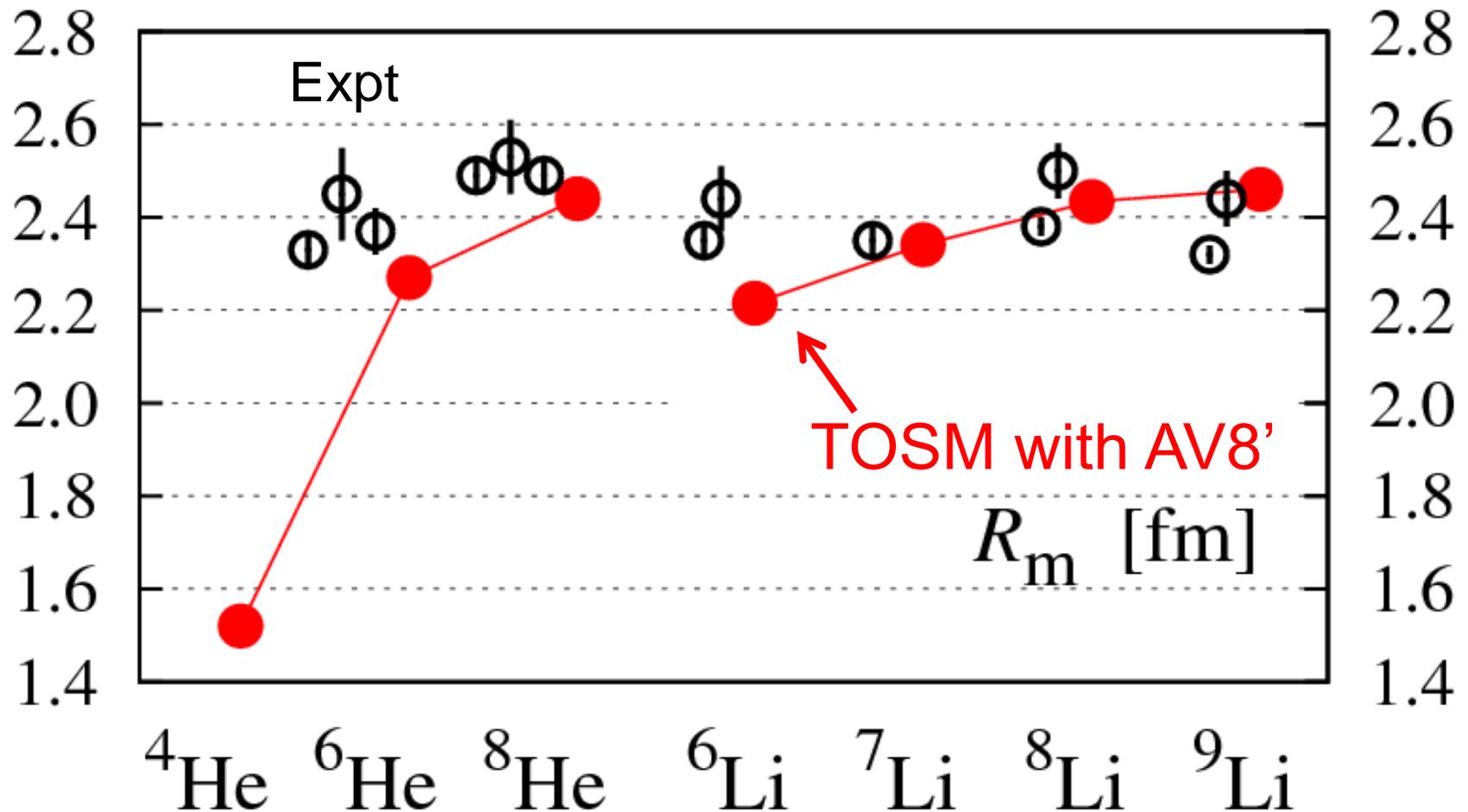
# ${}^5\text{-}9\text{Li}$ with TOSM+UCOM

- Excitation energies in MeV



- Excitation energy spectra are reproduced well

# Matter radius of He & Li isotopes



Halo

Skin

A. Dobrovolsky, NPA 766(2006)1

G. D. Alkhazov et al., PRL78('97)2313

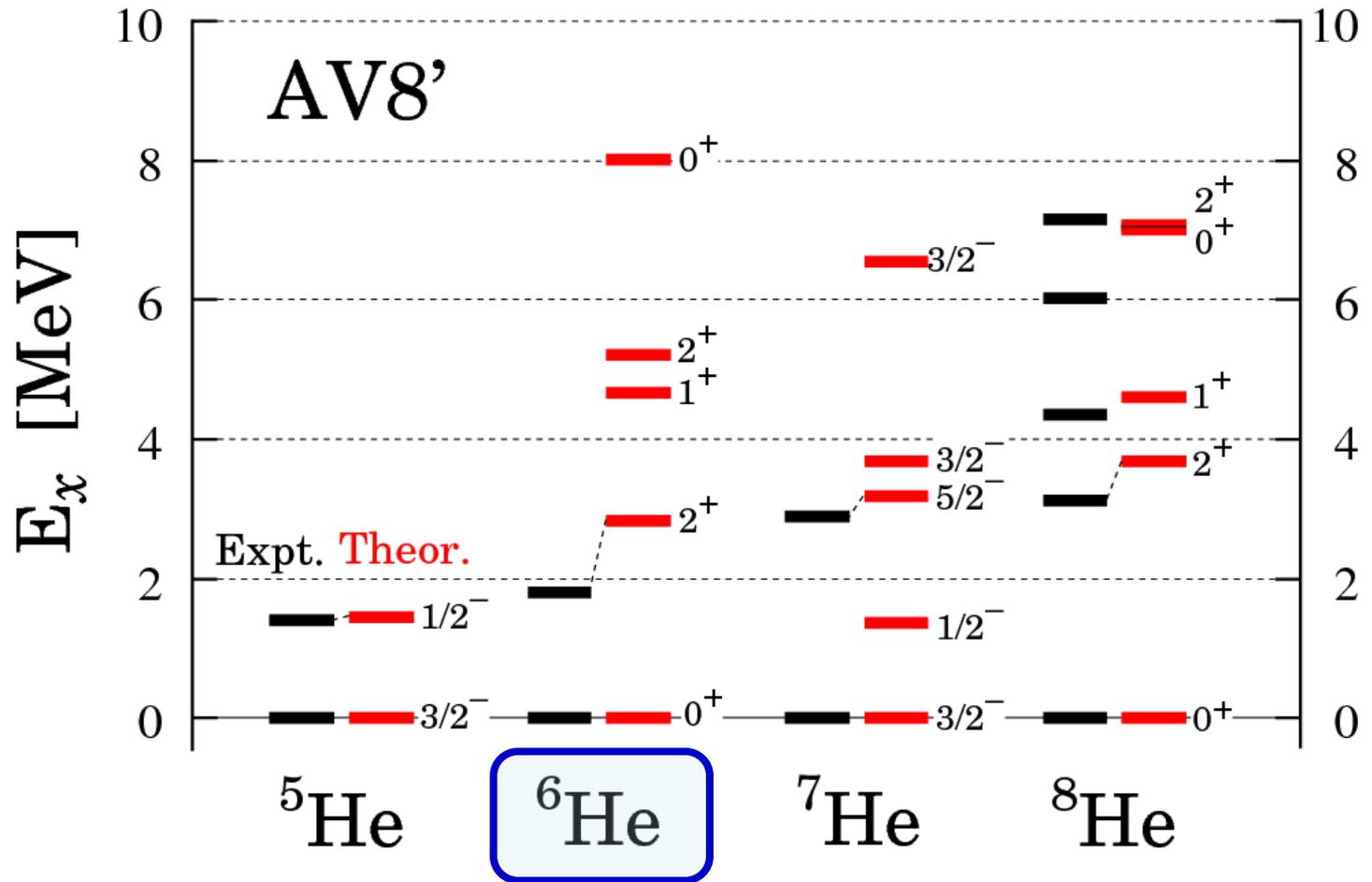
I. Tanihata et al., PLB289('92)261

P. Mueller et al., PRL99(2007)252501

O. A. Kiselev et al., EPJA 25, Suppl. 1('05)215.

# $^4\text{-}^8\text{He}$ with TOSM+UCOM

- Excitation energies in MeV

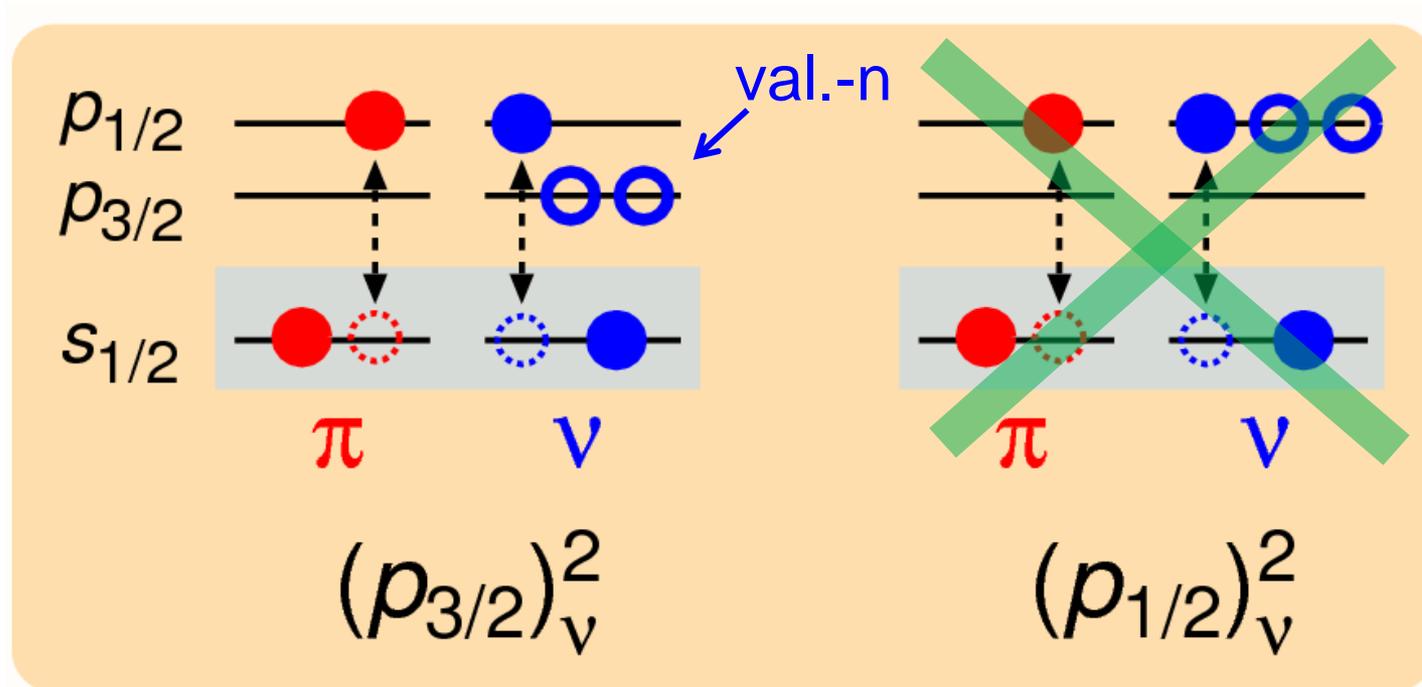


- No  $V_{\text{NNN}}$

- No continuum

- Excitation energy spectra are reproduced well

# Tensor correlation in ${}^6\text{He}$



Ground state

halo state ( $0^+$ )

Excited state

↑  
Tensor correlation is **suppressed**  
due to Pauli-Blocking

# ${}^6\text{He}$ : Hamiltonian component in TOSM

- Difference from  ${}^4\text{He}$  in MeV

${}^6\text{He}$	$0^+_1$	$0^+_2$
$n^2$ config	$(p_{3/2})^2$	$(p_{1/2})^2$

$$b_{\text{hole}} = 1.5 \text{ fm}$$

$$\hbar\omega = 18.4 \text{ MeV}$$

(hole)

same trend  
in  ${}^5\text{-}{}^8\text{He}$

LS splitting  
energy in  ${}^5\text{He}$

- Terasawa, Arima PTP23 ('60)
- Nagata, Sasakawa, Sawada, Tamagaki, PTP22('59)
- Myo, Kato, Ikeda, PTP113 ('05)

# Summary

- **Halo formation in  $^{11}\text{Li}$**  with tensor and pairing correlations.
  - Coexistence of tensor and pairing correlations
  - Pauli-blocking caused by halo neutrons
- $^4\text{He}$  contains “ **$pn$ -pair of  $p_{1/2}$** ” than  $p_{3/2}$ .
- **TOSM+UCOM** with bare nuclear force.
- **He isotopes with  $p_{3/2}$**  has large contributions of  $V_{\text{tensor}}$  & Kinetic energy.

**Review** Di-neutron clustering and deuteron-like tensor correlation in nuclear structure focusing on  $^{11}\text{Li}$

K. Ikeda, T. Myo, K. Kato and H. Toki

Springer, Lecture Notes in Physics 818 (2010)

“**Clusters in Nuclei**” Vol.1, 165-221.