Many-body resonances in He isotopes and those mirror nuclei

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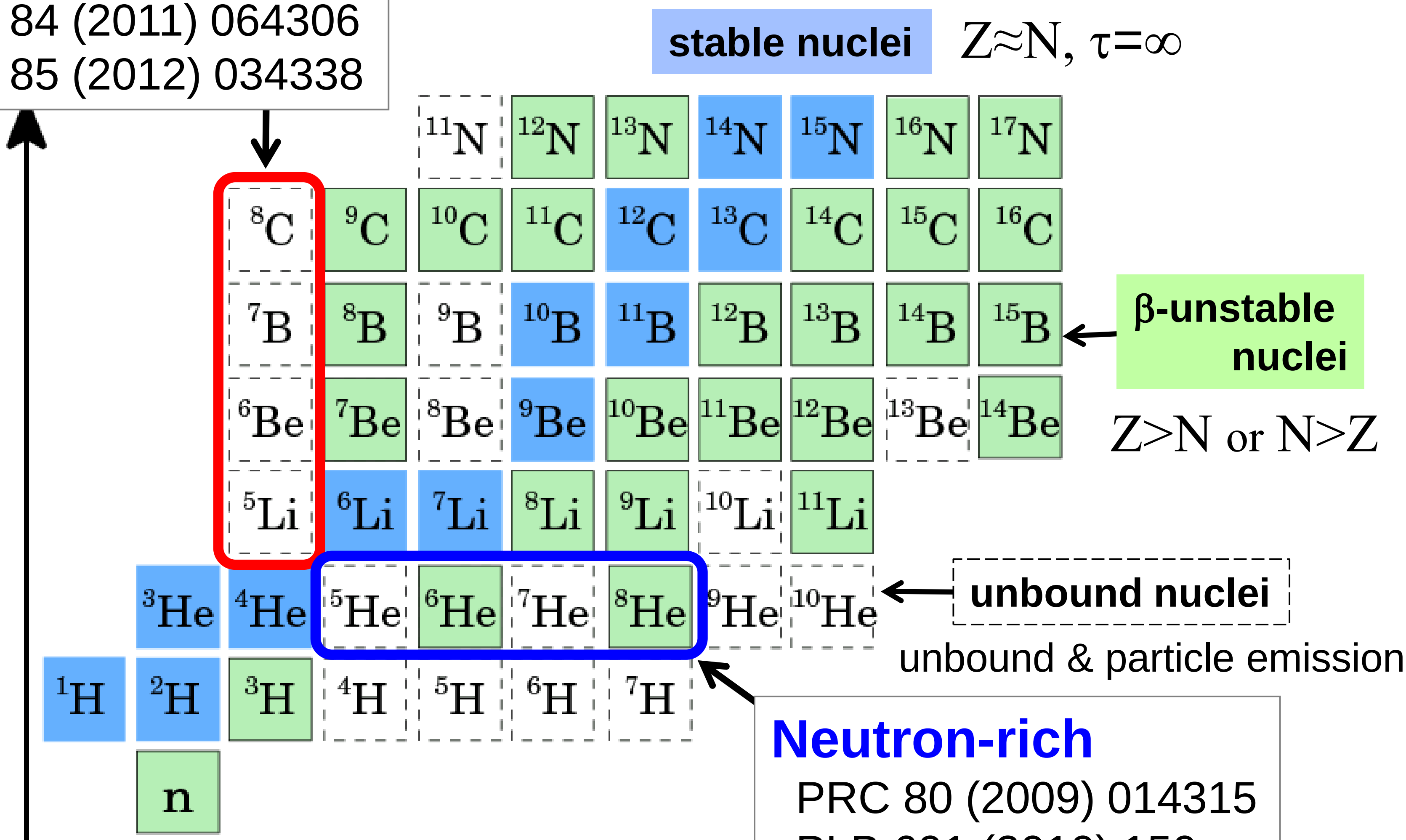
Outline

- Structure of Light Unstable Nuclei
  - He isotopes (neutron-rich)
  - Mirror nuclei (proton-rich)
- Cluster Orbital Shell Model (**COSM**)
  - Core nuclei + valence protons / neutrons
- Complex Scaling Method (**CSM**)
  - Many-body resonances & continuum states
  - Give continuum level density, Green's function
  - Strength functions

Nuclear Chart

Proton-rich
PRC 84 (2011) 064306
PRC 85 (2012) 034338

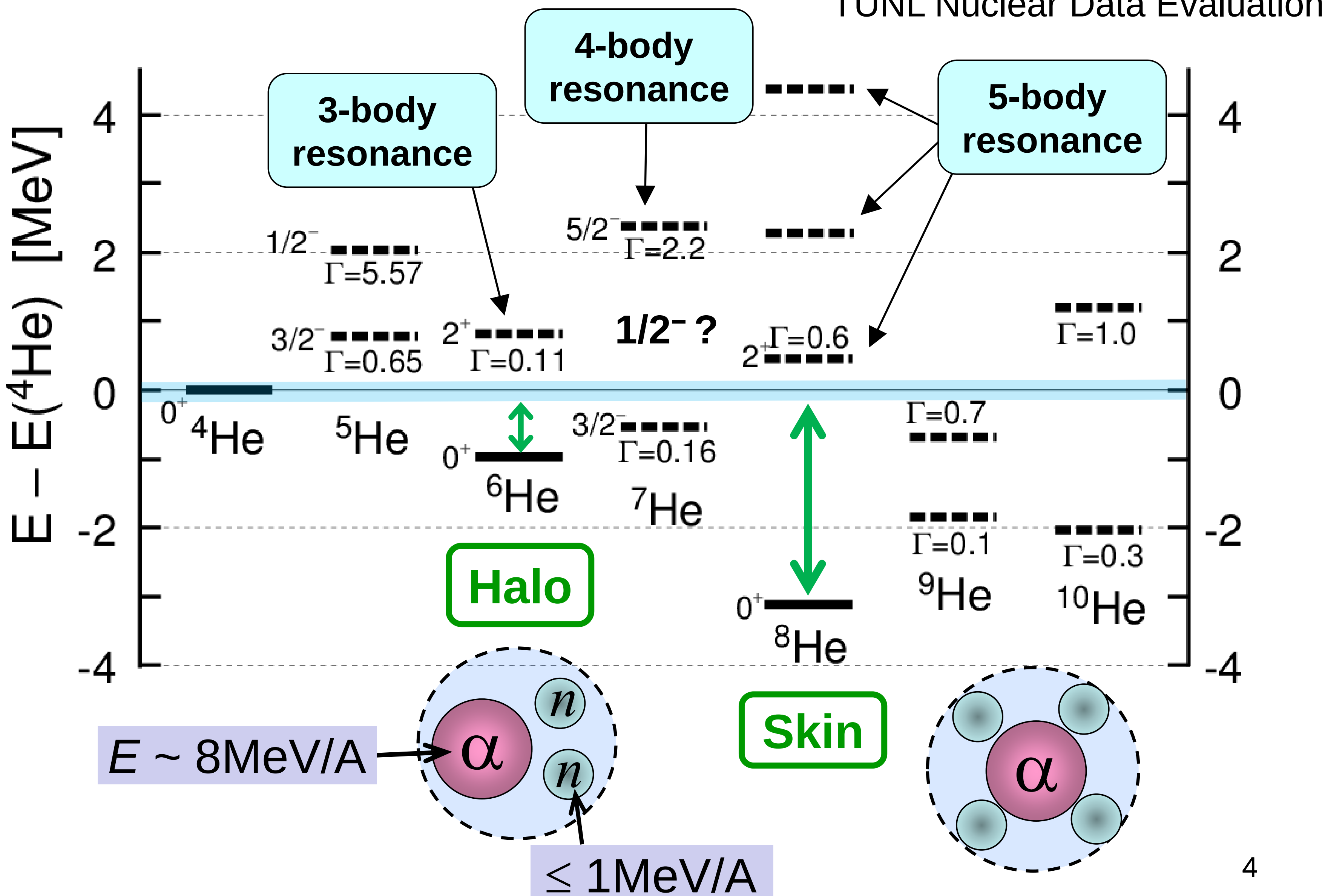


Neutron-rich
PRC 80 (2009) 014315
PLB 691 (2010) 150

Mirror symmetry between **proton-rich** & **neuron-rich**
(with Coulomb)

Neutron-rich He isotopes : experiment

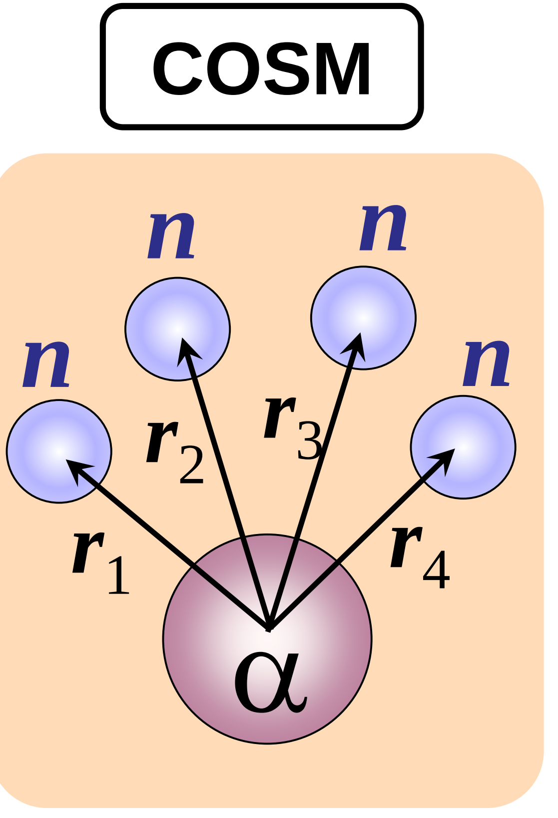
TUNL Nuclear Data Evaluation



Method

- Cluster Orbital Shell Model (COSM)
  - Open channel effect is treated.

    ^{8}He : ^{7}He+n, ^{6}He+n+n, ^{5}He+n+n+n, ...



- Complex Scaling Method

$$\mathbf{r} \to \mathbf{r}e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

  - Resonances with correct boundary condition as Gamow states

    $E = E_r - i\Gamma/2$

  - Give continuum level density (resonance+continuum)
  - Beyond drip-lines, α-cluster states

A. T. Kruppa, R. G. Lovas, B. Gyarmati, PRC37(1988) 383 (**^{8}Be**)
S. Aoyama, TM, K. Kato, K. Ikeda, PTP116(2006) 1 (**CSM review**)
C. Kurokawa , K. Kato, PRC71 (2005) 021301 (**^{12}C as 3α**)

Cluster Orbital Shell Model (n-rich)

- System is obtained based on RGM equation

$$H(^A\mathrm{He}) = H(^4\mathrm{He}) + H_{\mathrm{rel}}(N_V n) \qquad \Phi(^A\mathrm{He}) = \mathcal{A}\left\{\psi(^4\mathrm{He})\cdot\sum_{i=1}^{N} C_i \cdot \chi_i(N_V n)\right\}$$

valence neutron number

i : configuration

$\psi(^4\mathrm{He})$: $(0\mathrm{s})^4$ ← No explicit tensor correlation

$\chi_i(N_V n) = \mathcal{A}\{\varphi_{i1}\varphi_{i2}\varphi_{i3}\cdots\}$ $\qquad \varphi_i$: $L \le 2$ Relative motion with Gaussian expansion

- Orthogonarity Condition Model (OCM) is applied.

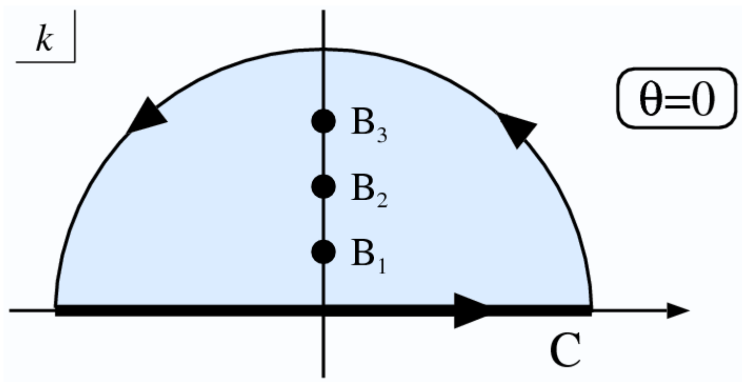
$$\sum_{i=1}^{N}\left\langle \chi_j \left| \sum_{k}^{4}\left(T_k + V_k^{cn}\right) + \sum_{k<l}^{N_V}\left(V_{kl}^{nn} + \frac{\vec{p}_i\cdot\vec{p}_j}{A_c m}\right)\right| \chi_i \right\rangle C_i = (E - E_{4\mathrm{He}})\, C_j$$

$\langle \varphi_i | \phi_{\mathrm{PF}} \rangle = 0$: Remove Pauli Forbidden states (PF)

Y. Suzuki, K. Ikeda, PRC38(1988)410, H. Masui, K. Kato, K. Ikeda, PRC73(2006)034318

Complex Scaling for 2-body case

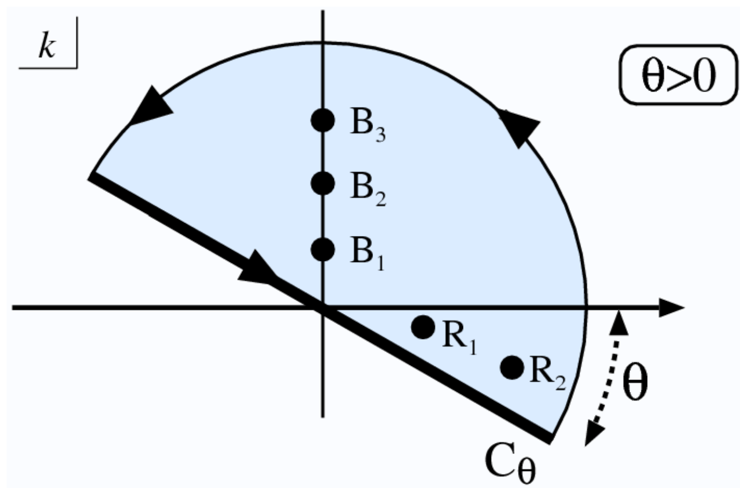
$$U(\theta)\ :\ \mathbf{r}\to\mathbf{r}\cdot\exp(i\theta)\ ,\qquad \mathbf{k}\to\mathbf{k}\cdot\exp(-i\theta)\ ,\qquad \theta\in\mathbb{R}$$



Completeness relation

$$1=\sum_B|\varphi_B\rangle\langle\tilde{\varphi}_B|+\int_C dk|\varphi_k\rangle\langle\tilde{\varphi}_k|$$

T. Berggren, NPA109('68)265.



$$1=\sum_B|\varphi_B\rangle\langle\tilde{\varphi}_B| + \sum_R|\varphi_R\rangle\langle\tilde{\varphi}_R| + \int_{C_\theta} dk_\theta\left|\varphi_{k_\theta}\right\rangle\left\langle\tilde{\varphi}_{k_\theta}\right|$$

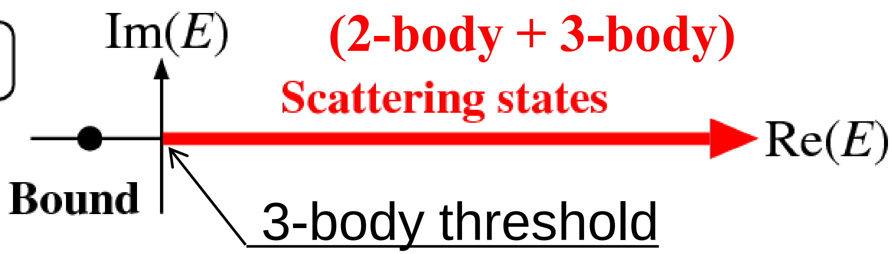
J.Aguilar and J.M.Combes, Commun. Math. Phys.,22('71)269.
E.Balslev and J.M.Combes, Commun. Math. Phys.,22('71)280.

B.G.Giraud, K.Kato, A.Ohnishi
J. Phys. A **37** ('04)11575

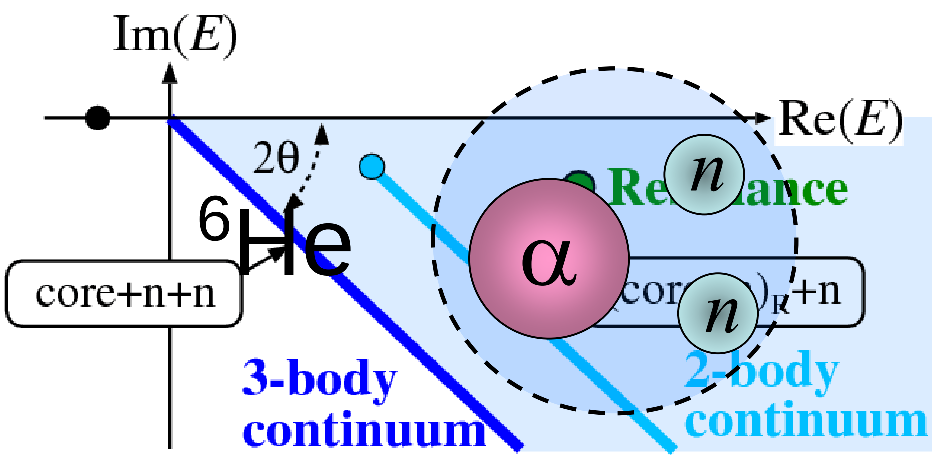
Complex Scaling for 3-body case

$$U(\theta)\ :\ \mathbf{r}\to\mathbf{r}\cdot\exp(i\theta)\ ,\qquad \mathbf{k}\to\mathbf{k}\cdot\exp(-i\theta)\ ,\qquad \theta\in\mathbb{R}$$

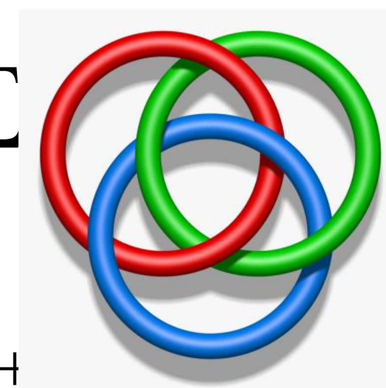
θ=0



$$1=\sum_B|\varphi_B\rangle\langle\tilde{\varphi}_B|+\int_C dE|\varphi_E\rangle\langle\tilde{\varphi}_E|$$



$$1=\sum_B \ldots + \ldots + \ldots |\varphi_{E_\theta}\rangle\langle\tilde{\varphi}_{E_\theta}|$$



Borromean rings

Halo nuclei : "Core nuclei+n+n" with Borromean condition

^{6}He=^{4}He+n+n, ^{11}Li=^{9}Li+n+n, ^{14}Be=^{12}Be+n+n, …

Schrödinger Eq. and Wave Func. in CSM

$$U(\theta)HU^{-1}(\theta)=H_\theta=T_\theta+V_\theta \qquad T_\theta=e^{-2i\theta}\cdot T, \quad V=V(\mathbf{r}e^{i\theta})$$

$$H\Phi=E\Phi \;\rightarrow\; H_\theta\Phi_\theta=E\Phi_\theta, \qquad \Phi_\theta(\mathbf{r})=e^{i3/2\cdot\theta}\cdot\Phi(\mathbf{r}e^{i\theta})$$

Asymptotic Condition in CSM ($\boldsymbol{r}\rightarrow\infty$)

	No Scaling	Scaling
Bound	$\Phi\rightarrow 0$	$\Phi_\theta\rightarrow 0$
Resonance	$\Phi\rightarrow\infty$	$\Phi_\theta\rightarrow 0$
Continuum	$\Phi\rightarrow e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$	$\Phi_\theta\rightarrow e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$

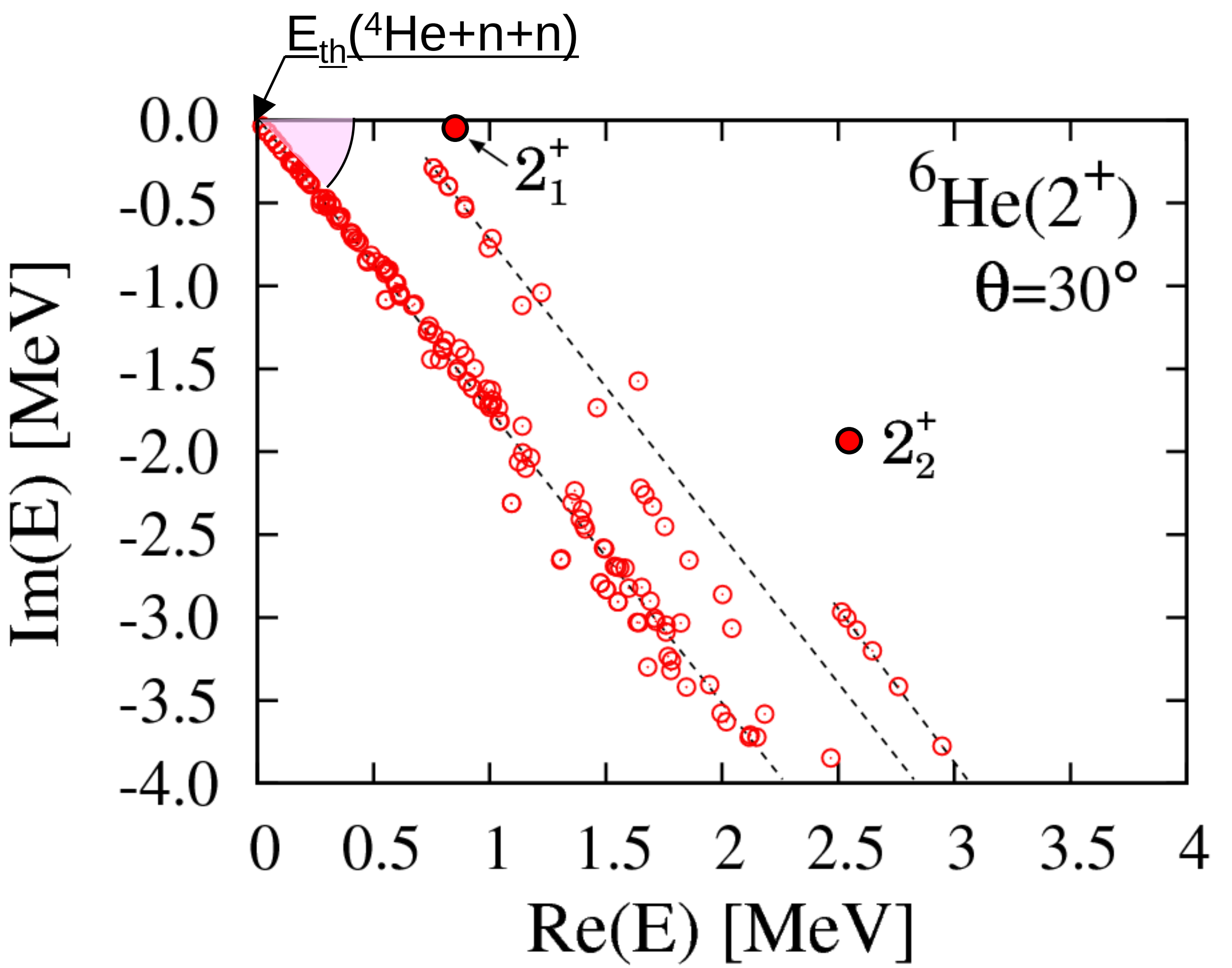
damping condition

$$\Phi^{res}\sim\exp(ik_r r)\;=\exp(i\mathrm{k}_r e^{-i\theta_r}r) \qquad k_r=\mathrm{k}_r\cdot e^{-i\theta_r}, \quad \theta_r>0$$

$$\Phi_\theta^{res}\sim\exp(ik_r r_\theta)=\exp(i\mathrm{k}_r e^{i(\theta-\theta_r)}r)$$

$$=\exp\left[i\mathrm{k}_r r\cos(\theta-\theta_r)\right]\cdot\exp\left[-\mathrm{k}_r r\sin(\theta-\theta_r)\right]$$

Spectrum of ^{6}He with ^{4}He+n+n model



^{6}He$^{(*)}$
^{5}He+n
^{4}He+n+n

Continuum states are discretized using **Gaussian basis functions**

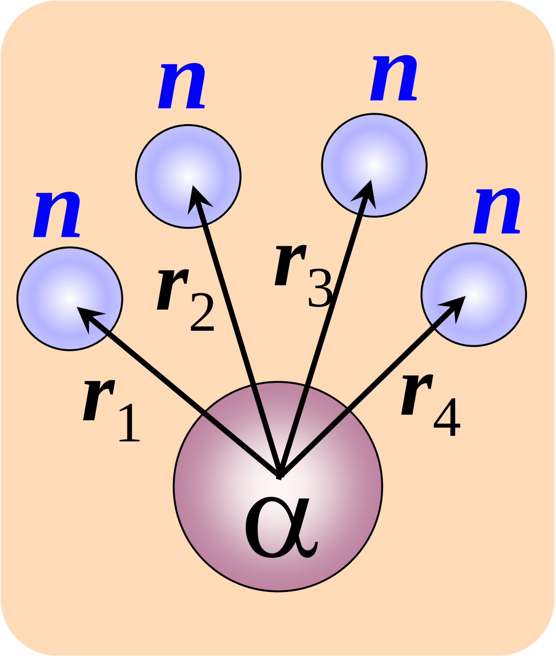
$$\phi_\ell(\mathbf{r}) = \sum_n C_n \cdot r^\ell e^{-\left(r/b_n\right)^2} Y_\ell(\hat{\mathbf{r}})$$

A. Csoto, PRC49 ('94) 3035,
S. Aoyama et al. PTP94('95)343, T. Myo et al. PRC63('01)054313

Hamiltonian

- $V_{\alpha\text{-n}}$: microscopic KKNN potential
  - s,p,d,f-waves of α-n scattering
- V_{nn} : Minnesota potential with slightly strengthened (+ Coulomb for *p*-rich nuclei)

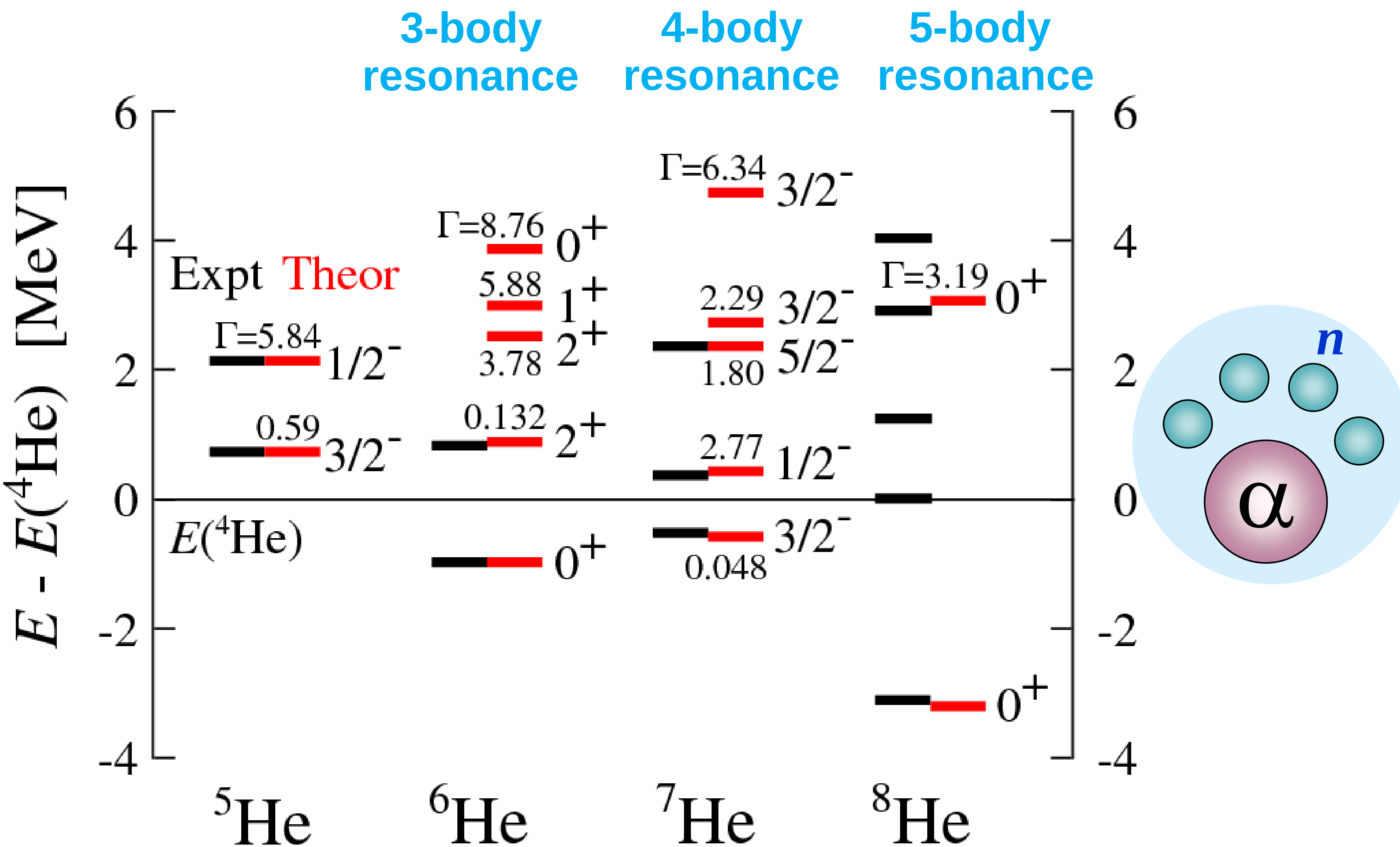
COSM



Fit energy of ^{6}He(0^+)

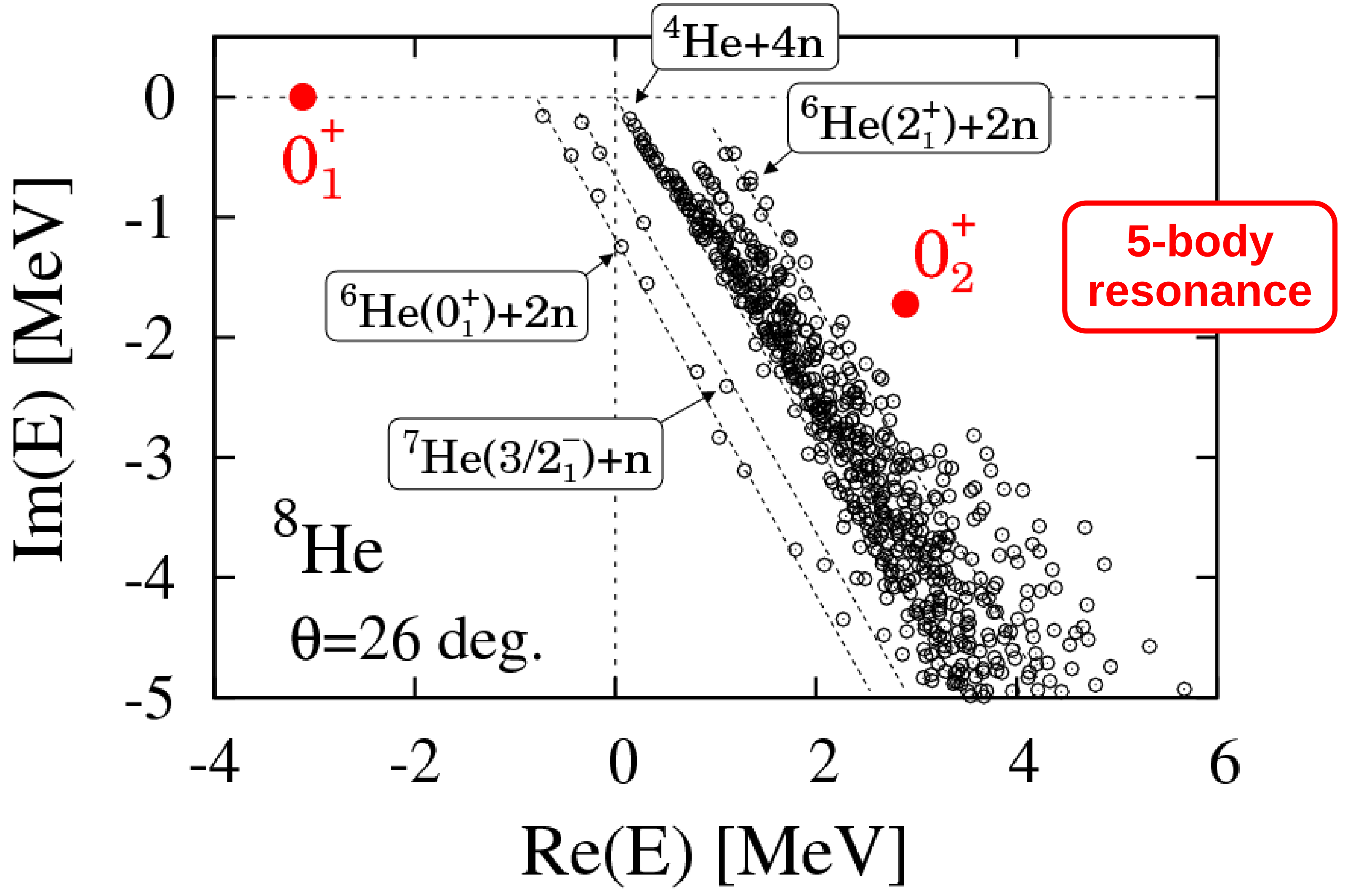
A. Csoto, PRC48(1993)165.
K. Arai, Y. Suzuki and R.G. Lovas, PRC59(1999)1432.
TM, S. Aoyama, K. Kato, K. Ikeda, PRC63(2001)054313.
TM et al. PTP113(2005)763.

He isotopes : Expt vs. Complex Scaling



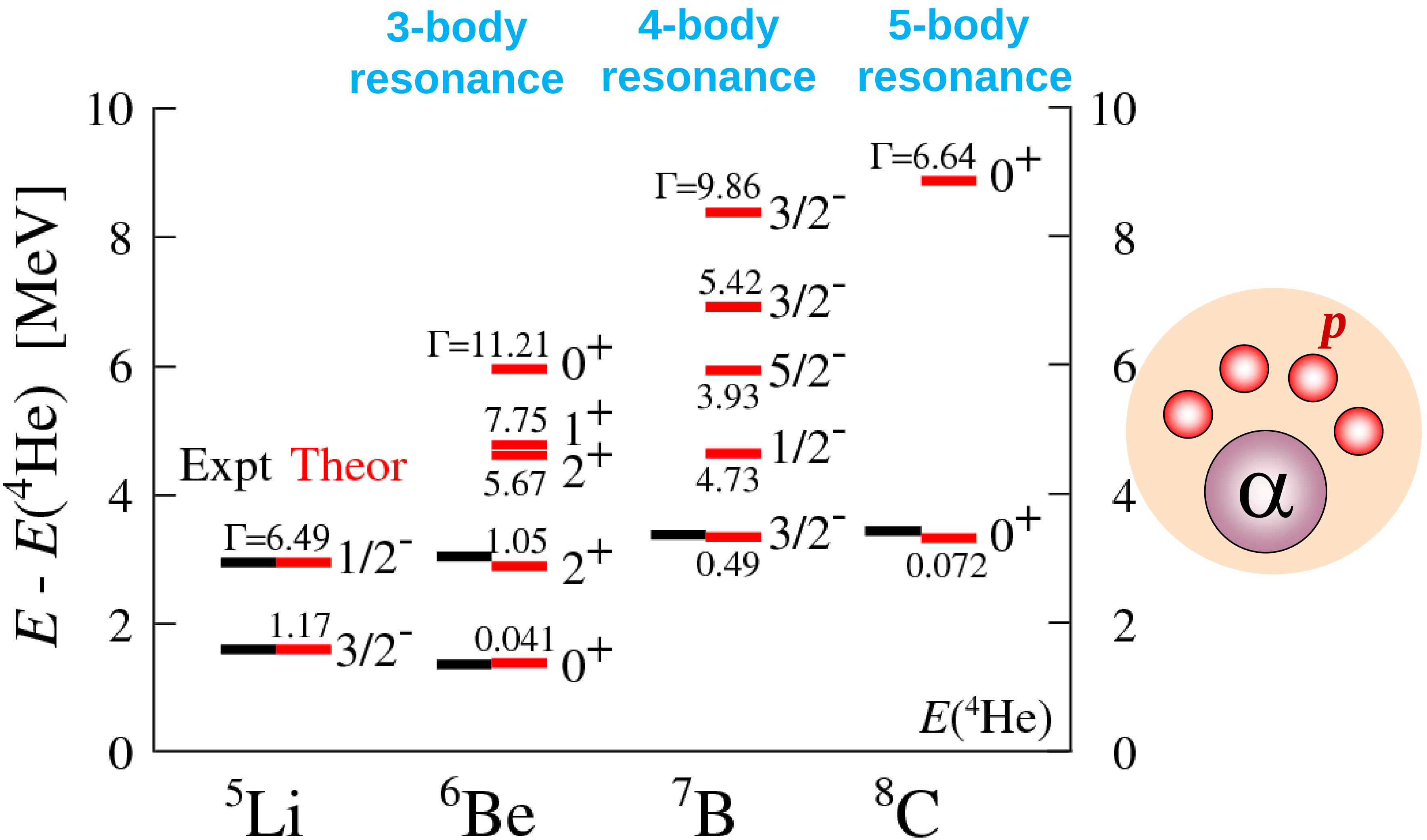
TM, K.Kato, K.Ikeda PRC76('07)054309
TM, R.Ando, K.Kato PRC80('09)014315
TM, R.Ando, K.Kato, PLB691('10)150
TUNL Nuclear Data Evaluation

Energy of ^{8}He with complex scaling



Eigenvalue problem with 32,000 dim.
Full diagonalization of complex matrix @ SX8R of NEC

Proton-rich side : ^{4}He+p+p+p+p



TM, Kikuchi, Kato PRC84 (2011) 064306
PRC85 (2012) 034338

Expt: Charity et al. PRC84(2011)014320

S-factor of ^{6}He-n component in ^{7}He

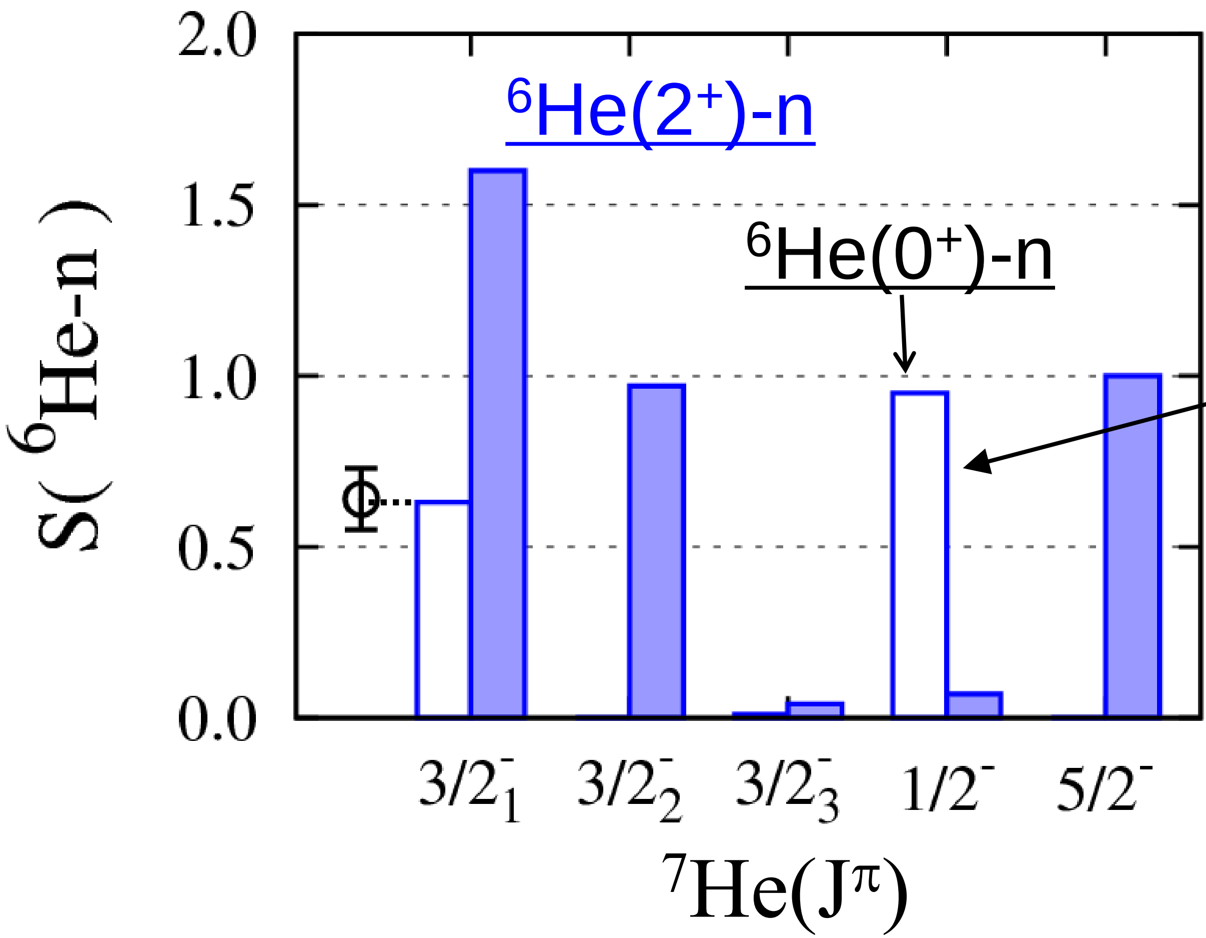
$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{He}(J') \middle| a_{nlj} \middle| {}^7\mathrm{He}(J) \right\rangle^2$$

neutron removal

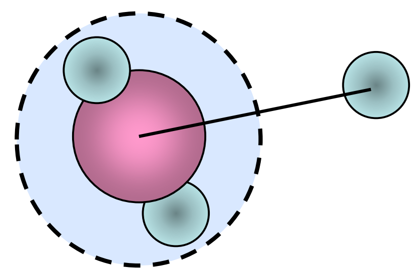
Bi-orthogonal relation

T. Berggren, NPA109(1968)265

TM, K.Kato, K.Ikeda, PRC76(2007)054309

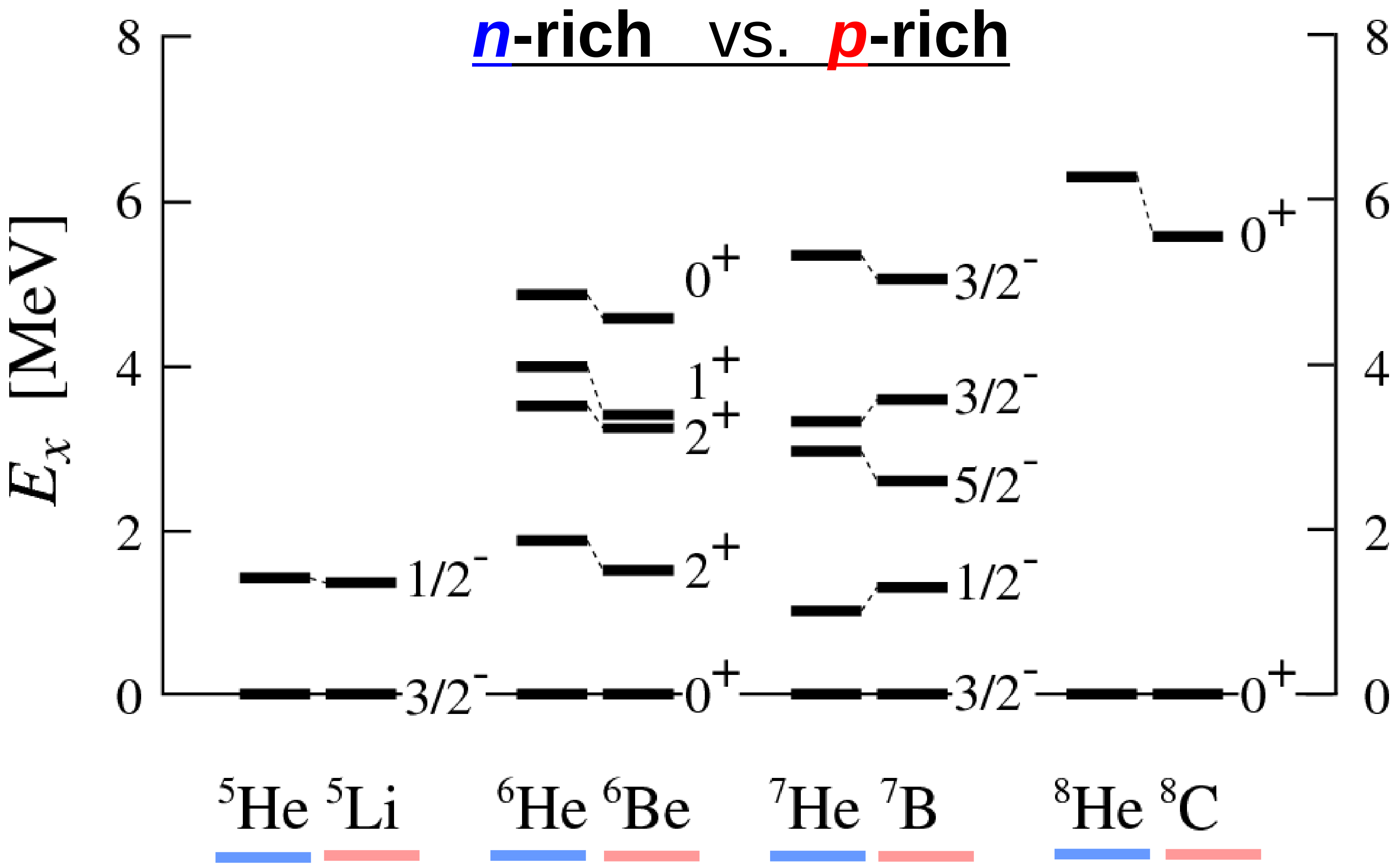


weak coupling of ^{6}He+*n*



^{6}He(halo)

Mirror Symmetry

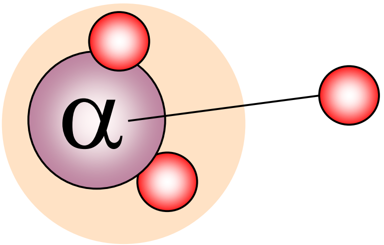


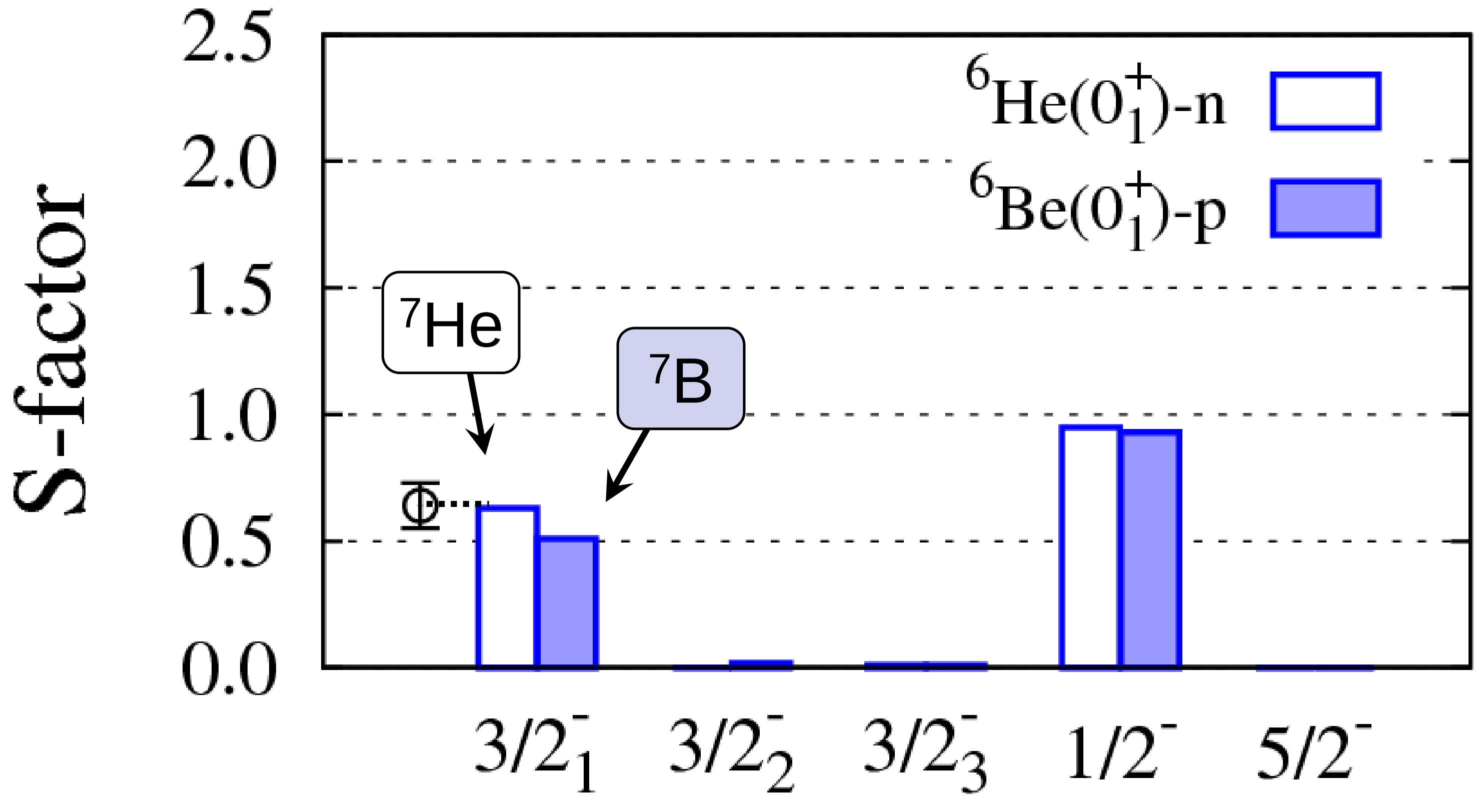
TM, Kikuchi, Kato PRC84 (2011) 064306 , 85 (2012) 034338

S-factors of ^{7}He & ^{7}B

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(\underline{0^+}) \middle| a_{nlj} \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$

proton removal

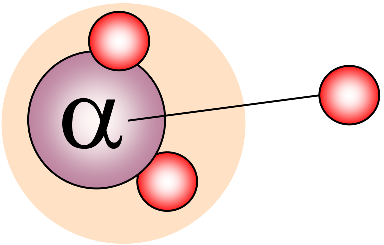


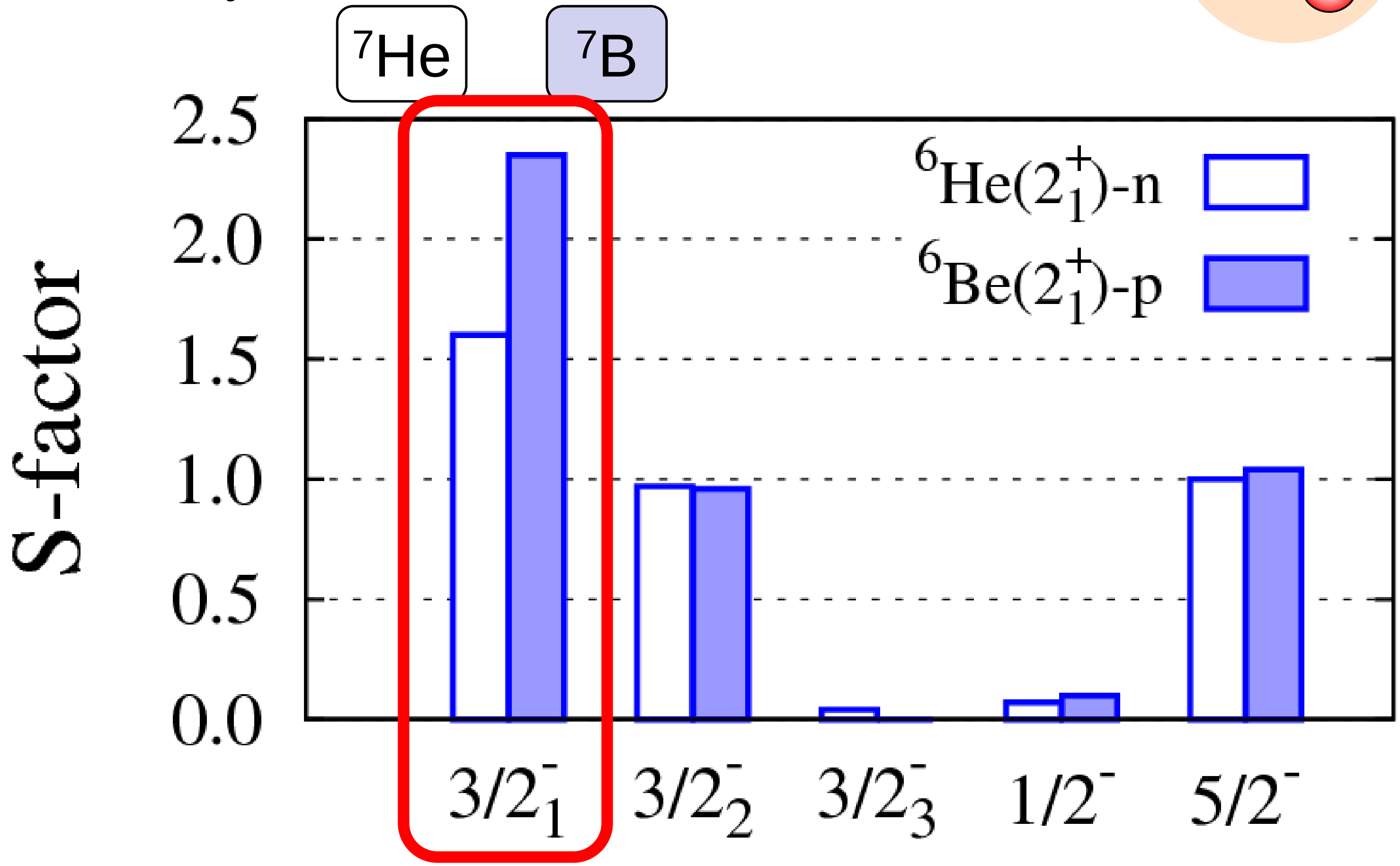


Expt. of ^{7}He(3/2$^-$) : F. Beck et al., Phys. Lett. B **645** (2007) 128

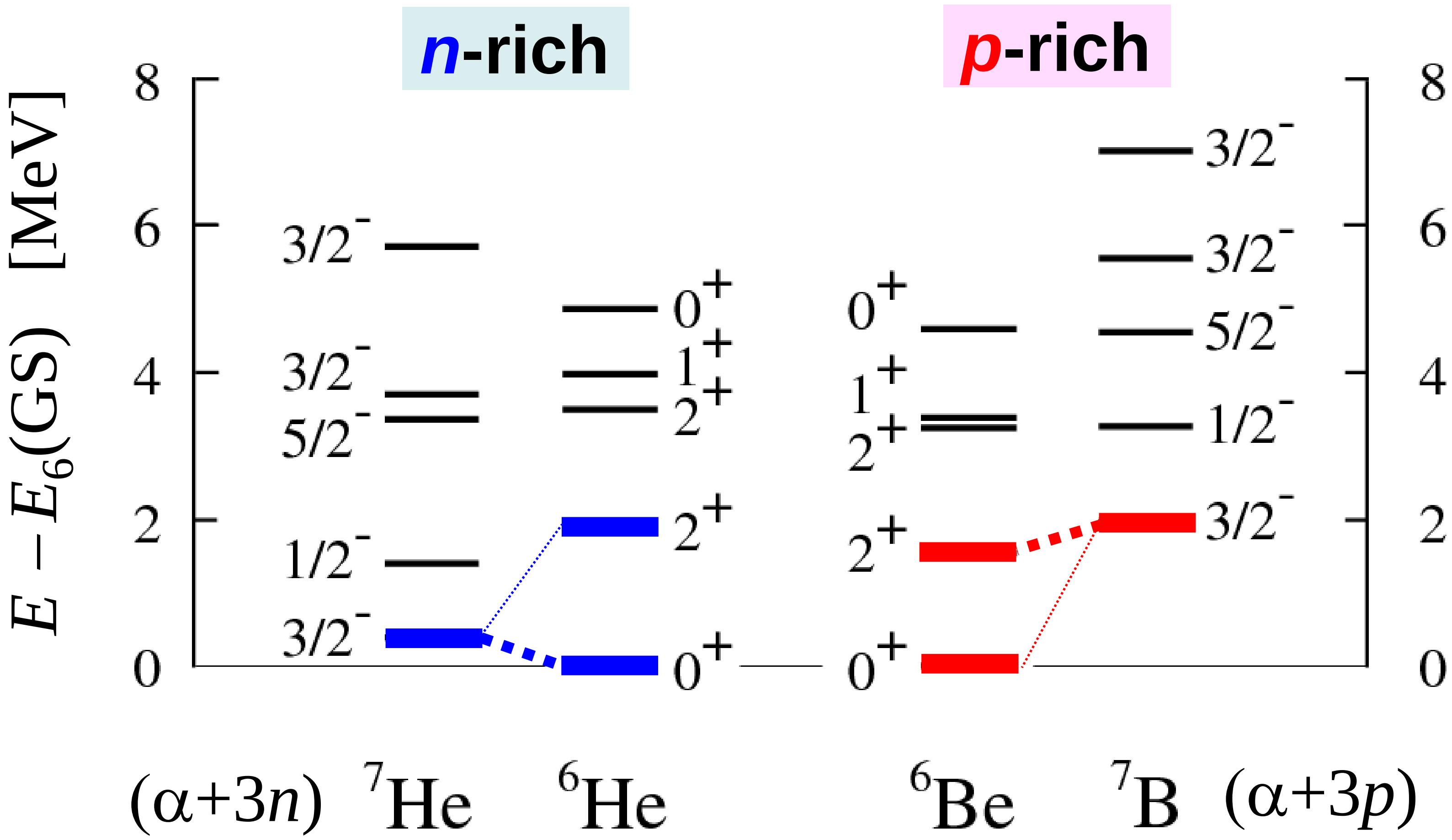
S-factors of ^{7}He & ^{7}B

$$S_{J',J} = \sum_{nlj} \left\langle {}^6\mathrm{Be}(\underline{2^+}) \middle| a_{nlj} \middle| {}^7\mathrm{B}(J^\pi) \right\rangle^2$$





Thresholds of [A=6]+N system



<u>Mirror symmetry breaking</u> due to the channel coupling effect caused by Coulomb force

Continuum Level Density (CLD) in CSM

$$\Delta E = -\frac{1}{\pi}\,\mathrm{Im}\left[\mathrm{Tr}\left[G(E)-G_0(E)\right]\right], \qquad G_{(0)} = \frac{1}{E-H_{(0)}},$$

$$\Delta E = \frac{1}{2i\pi}\,\mathrm{Tr}\left[S(E)^\dagger \frac{d}{dE} S(E)\right] \quad \rightarrow \quad \frac{1}{\pi}\frac{d\delta}{dE} \quad \text{(single channel case)}$$

S. Shlomo, NPA539('92)17

K. Arai and A. Kruppa, PRC60('99)064315

R. Suzuki, T. Myo and K. Kato, PTP113('05)1273.

CLD in CSM

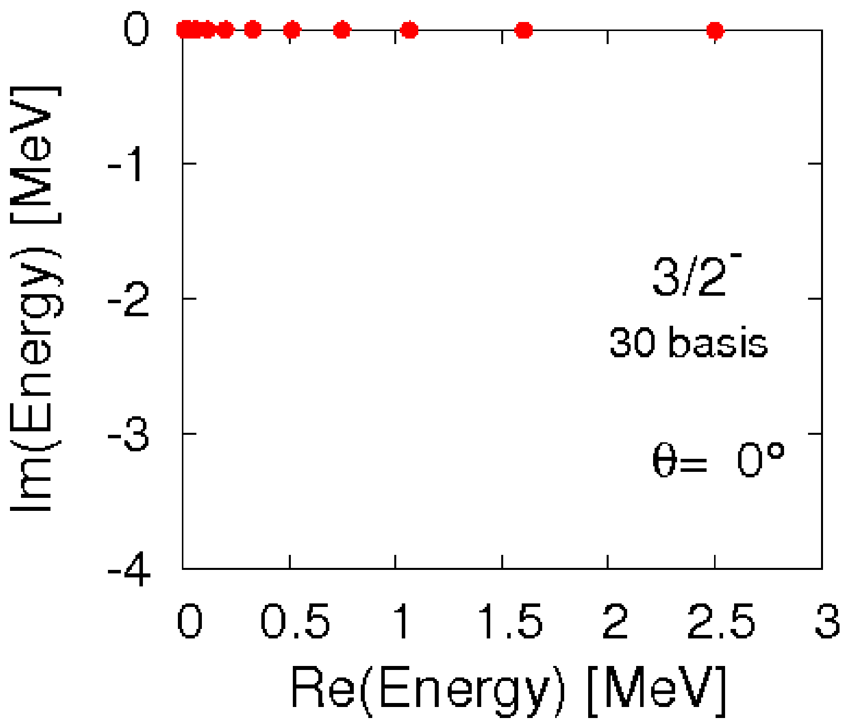
$$\Delta E = -\frac{1}{\pi}\,\mathrm{Im}\left[\mathrm{Tr}\left[G^\theta(E)-G_0{}^\theta(E)\right]\right]$$

$$G = \frac{1}{E-H^\theta}$$

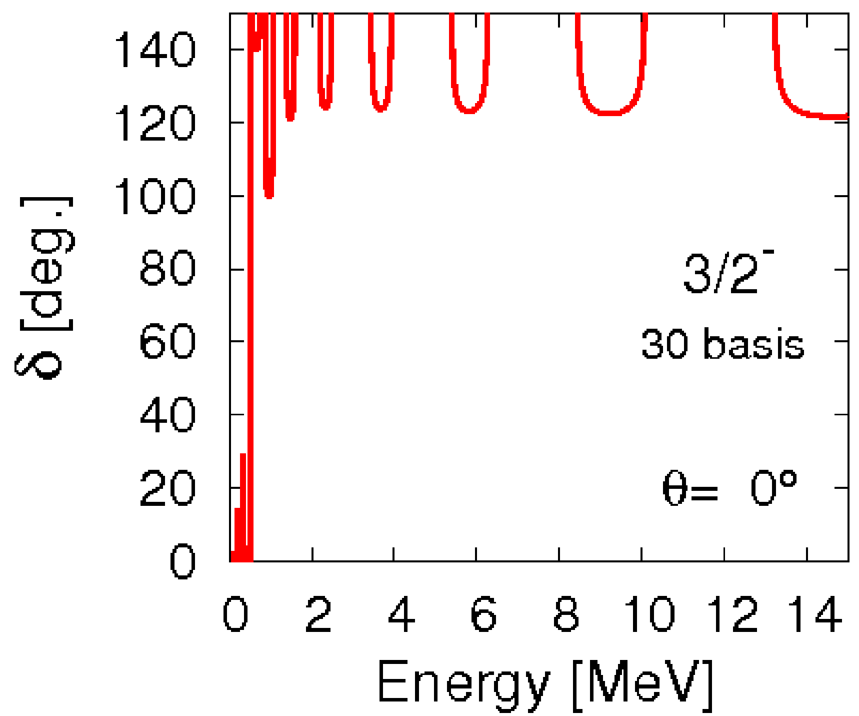
$$G_0 = \frac{1}{E-H_0^\theta} \quad \text{(asymptotic)}$$

$\alpha+n$ scattering with complex scaling using discretized continuum states

energy eigenvalues



$P_{3/2}$ scattering phase shift



30 Gaussian basis functions

Strength function $S(E)$ in CSM

- Strength function and response function

Bi-orthogonal relation

$$S(E) = \sum_i \langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle \cdot \delta(E - E_i)$$

$$= -\frac{1}{\pi} \mathrm{Im}\left[R(E) \right]$$

← initial state (Φ_0)

$$R(E) = \sum_i \frac{\langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_i \rangle \langle \tilde{\varphi}_i | \hat{O} | \Phi_0 \rangle}{E - E_i}$$

Response function

- Complex scaled-Green's function

complete set in CSM

$$G^\theta(E) = \frac{1}{E - H_\theta} = \sum_i \frac{|\varphi_i^\theta\rangle\langle\tilde{\varphi}_i^\theta|}{E - E_i^\theta}$$

Bound+Resonance+Continuum

Reaction theory

- LS-eq. (Kikuchi)
- CDCC (Matsumoto)
- Scatt. Amp. (Kruppa, Dote($K^{bar}N$))

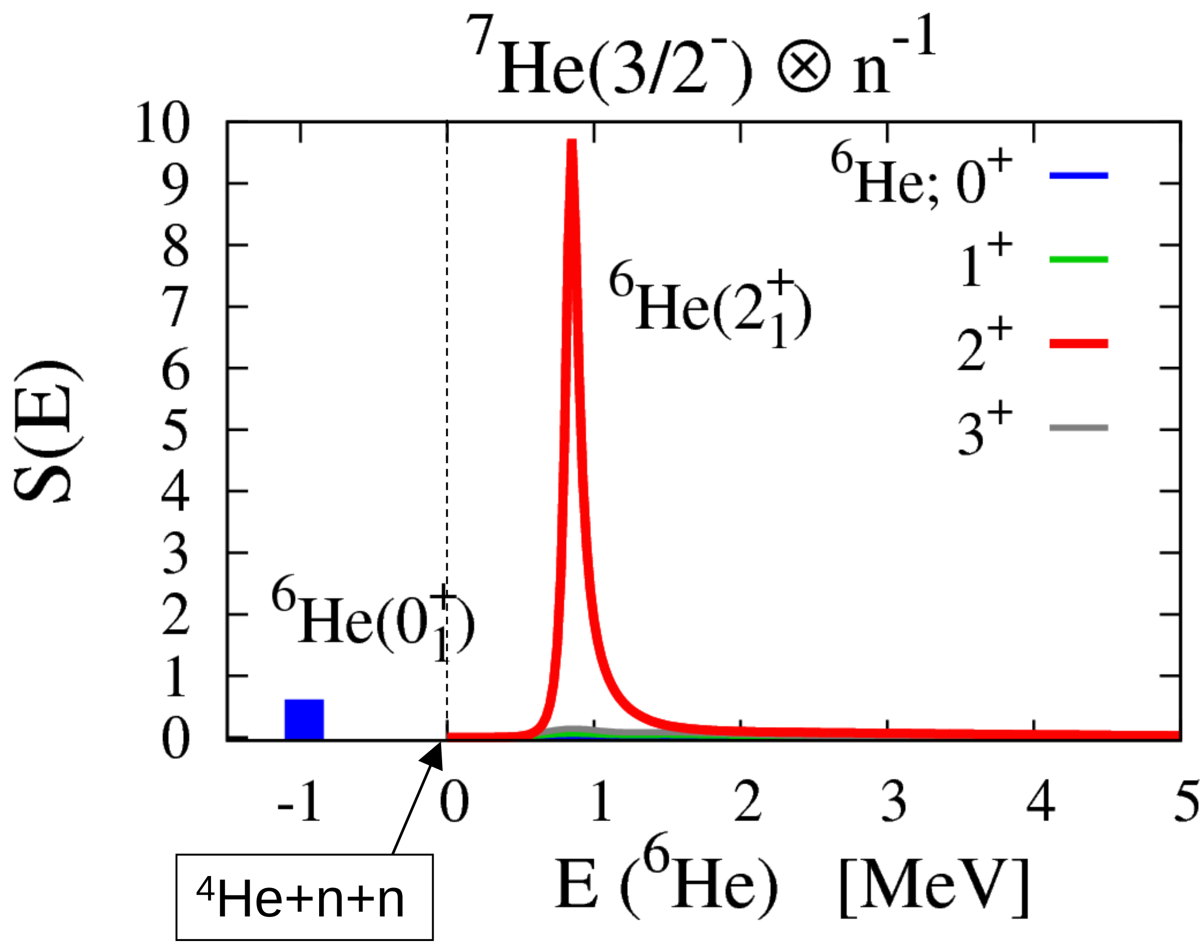
T. Berggren, NPA109('68)265, T. Myo, A. Ohnishi and K. Kato, PTP99('98)801

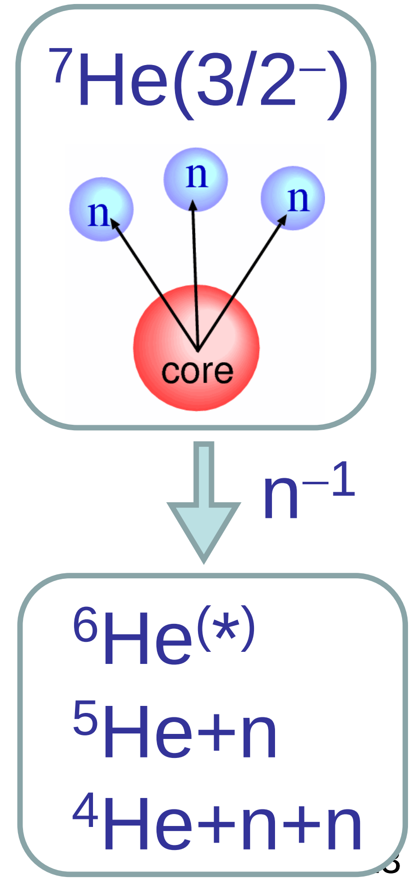
One-neutron removal strength of ^{7}He

$$S_{J',J}(E) = \sum_{nlj} \left| \left\langle {}^6\mathrm{He}^{J'}(E) \middle| a_{nlj} \middle| {}^7\mathrm{He}^{J} \right\rangle \right|^2$$

TM, Ando, Kato
PRC80(2009)014315

" ^{4}He+n+n" complete set using CSM





Summary

- **Light Unstable Nuclei**
  - He isotopes (***n*-rich**) & Mirror nuclei (***p*-rich**)
  - Mirror symmetry & Channel coupling (threshold)
- **Complex Scaling**
  - Resonance spectroscopy, Physical quantities
  - Continuum level density (resonance+continuum)
  - Strength functions using Green's function
  - Coulomb breakups, Nucleon removal, ...
  - Application to reaction theory (CDCC, LS eq.,…)