

## Pion photo- and electroproduction and the chiral MAID interface

1

**Marius Hilt, Björn C. Lehnhart, Stefan Scherer, Lothar Tiator**  
**NSTAR 2015, Osaka, Japan, May 25 – 28, 2015**

---

<sup>1</sup>Phys. Rev. C **87**, 045204 (2013), Phys. Rev. C **88**, 055207 (2013)

**1. Introduction**

**2. Renormalization and power counting**

**3. Application to pion photo- and electroproduction**

**4. Summary and outlook**

## 1. Introduction

### Effective field theory

... if one writes down the **most general possible Lagrangian**, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian **to any given order of perturbation theory**, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. ... <sup>2</sup>

---

<sup>2</sup>S. Weinberg, Physica A 96, 327 (1979)

**... if we include in the Lagrangian all of the infinite number of interactions allowed by symmetries, then there will be a counterterm available to cancel every ultraviolet divergence. ...<sup>3</sup>**

---

<sup>3</sup>S. Weinberg, *The Quantum Theory of Fields*, Vol. I, 1995, Chap. 12

Perturbative calculations in effective field theory require **two main ingredients**

## 1. Knowledge of the **most general effective Lagrangian**

(a) Mesonic ChPT  $[\text{SU}(3) \times \text{SU}(3)]^4 (\pi, K, \eta)$

$$\underbrace{2}_{\mathcal{O}(q^2)} + \underbrace{10+2}_{\mathcal{O}(q^4)} + \underbrace{90+4+23}_{\mathcal{O}(q^6)} + \dots$$

- $q$ : Small quantity such as a pion mass
- Even powers
- Two-loop level

---

<sup>4</sup>Gasser, Leutwyler (1985), Fearing, Scherer (1996), Bijnens, Colangelo, Ecker (1999), Ebertshäuser, Fearing, Scherer (2002) Bijnens, Girlanda, Talavera (2002)

(b) Baryonic ChPT  $[\text{SU}(2) \times \text{SU}(2) \times \text{U}(1)]^5 (\pi, N)$

$$\underbrace{\mathcal{O}(q)}_2 + \underbrace{\mathcal{O}(q^2)}_7 + \underbrace{\mathcal{O}(q^3)}_{23} + \underbrace{\mathcal{O}(q^4)}_{118} + \dots$$

- Odd and even powers (spin)
- One-loop level

Each term comes with an independent low-energy constant  
**(LEC)**

Lowest-order Lagrangians:  $F, M^2 = 2B\hat{m}, m, g_A$

Higher-order Lagrangians:  $l_i, c_i, d_i, e_i, \dots$

---

<sup>5</sup>Gasser, Sainio, Švarc (1988), Bernard, Kaiser, Meißner (1995), Ecker, Mojžiš (1996), Fettes, Meißner, Mojžiš, Steininger (2000)

## 2. Consistent **expansion scheme** for observables

- (a) Tree-level diagrams, loop diagrams  $\rightsquigarrow$  ultraviolet divergences, regularization (of infinities)
- (b) Renormalization condition
- (c) Power counting scheme for renormalized diagrams
- (d) Remove regularization

**ChPT: Momentum and quark mass expansion at fixed ratio**

$$m_{\text{quark}}/q^2$$
<sup>6</sup>

---

<sup>6</sup>J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984)

## 2. Renormalization and power counting

- Most general Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \dots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \dots$$

### Basic Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left( i\gamma_\mu \partial^\mu - \boxed{m} \right) \Psi - \frac{1}{2} \boxed{\frac{g_A}{F}} \bar{\Psi} \gamma_\mu \gamma_5 \tau^a \partial^\mu \pi^a \Psi + \dots$$

$m$ ,  $g_A$ , and  $F$  denote the chiral limit of the physical nucleon mass, the axial-vector coupling constant, and the pion-decay constant, respectively

- **Power counting:** Associate chiral order  $D$  with a diagram

- Square of the lowest-order pion mass:

$$M^2 = B(m_u + m_d) \sim \mathcal{O}(q^2)$$

- Nucleon mass in the chiral limit  $m \sim \mathcal{O}(q^0)$

- Loop integration in  $n$  dimensions  $\sim \mathcal{O}(q^n)$

- Vertex from  $\mathcal{L}_\pi^{(2k)} \sim \mathcal{O}(q^{2k})$

- Vertex from  $\mathcal{L}_{\pi N}^{(k)} \sim \mathcal{O}(q^k)$

- Nucleon propagator  $\sim \mathcal{O}(q^{-1})$

- Pion propagator  $\sim \mathcal{O}(q^{-2})$

- Renormalization

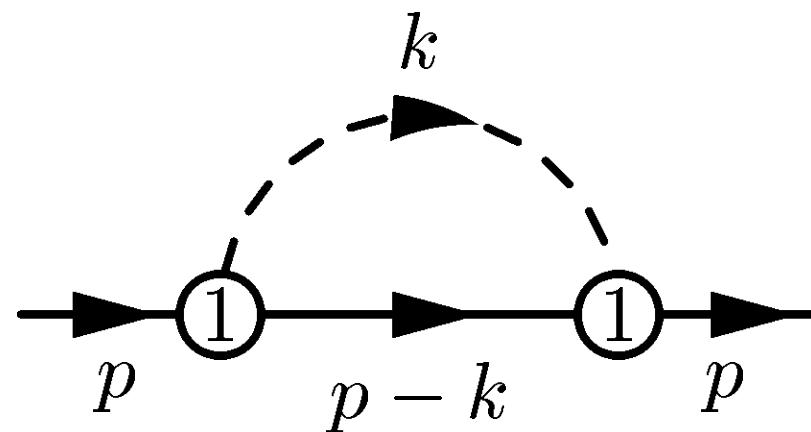
- Regularize (typically dimensional regularization)

$$\begin{aligned} I(M^2, \mu^2, n) &= \mu^{4-n} \int \frac{d^n k}{(2\pi)^n} \frac{i}{k^2 - M^2 + i0^+} \\ &= \frac{M^2}{16\pi^2} \left[ R + \ln \left( \frac{M^2}{\mu^2} \right) \right] + O(n-4), \end{aligned}$$

$$R = \frac{2}{n-4} - [\ln(4\pi) + \Gamma'(1)] - 1 \rightarrow \infty$$

- Adjust counterterms such that they absorb all the divergences occurring in the calculation of loop diagrams
- Renormalization prescription: Adjust finite pieces such that renormalized diagrams satisfy a given power counting

- Example: Contribution to nucleon mass



**Goal:**  $D = n \cdot 1 - 2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 = n - 1$

$$\Sigma = -\frac{3g_{A0}^2}{4F_0^2} \left[ (\not{p} + m) I_N + M^2 (\not{p} + m) I_{N\pi}(-p, 0) + \dots \right]$$

Apply  $\widetilde{\text{MS}}$  renormalization scheme

$$\begin{aligned} \Sigma_r &= -\frac{3g_{Ar}^2}{4F_r^2} [M^2 (\not{p} + m) \underbrace{I_{N\pi}^r(-p, 0)}_1 + \dots] = \boxed{\mathcal{O}(q^2)} \\ &= -\frac{1}{16\pi^2} + \dots \end{aligned}$$

GSS<sup>7</sup>: It turns out that loops have a much more complicated low-energy structure if baryons are included. Because the nucleon mass  $m_N$  does not vanish in the chiral limit, the mass scale  $m$  (nucleon mass in the chiral limit) occurs in the effective Lagrangian  $\mathcal{L}_{\pi N}^{(1)}$  .... This complicates life a lot.

---

<sup>7</sup>J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. **B307**, 779 (1988)

One possible solution: Extended on-mass-shell (EOMS) scheme<sup>8</sup>

Main idea: Perform additional subtractions such that renormalized diagrams satisfy the power counting

Motivation for this approach<sup>9</sup>

Terms violating the power counting are analytic in small quantities (and can thus be absorbed in a renormalization of counterterms)

- Example (chiral limit)

$$H(p^2, m^2; n) = - \int \frac{d^n k}{(2\pi)^n} \frac{i}{[(k-p)^2 - m^2 + i0^+][k^2 + i0^+]}$$

---

<sup>8</sup>T. Fuchs, J. Gegelia, G. Japaridze, S. Scherer, Phys. Rev. D **68**, 056005 (2003)

<sup>9</sup>J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999)

**Small quantity**

$$\Delta = \frac{p^2 - m^2}{m^2} = \mathcal{O}(q)$$

We want the **renormalized integral** to be of order

$$D = n - 1 - 2 = n - 3$$

**Result of integration**

$$H \sim F(n, \Delta) + \Delta^{n-3} G(n, \Delta)$$

- $F$  and  $G$  are hypergeometric functions
- analytic in  $\Delta$  for arbitrary  $n$

## Observation<sup>10</sup>

$F$  corresponds to **first** expanding the integrand in small quantities and **then** performing the integration

⇒ **Algorithm:** Expand integrand in small quantities and subtract those (integrated) terms whose order is **smaller** than suggested by the power counting

---

<sup>10</sup> J. Gegelia, G. Japaridze, K. S. Turashvili, Theor. Math. Phys. **101**, 1313 (1994)

**Here:**

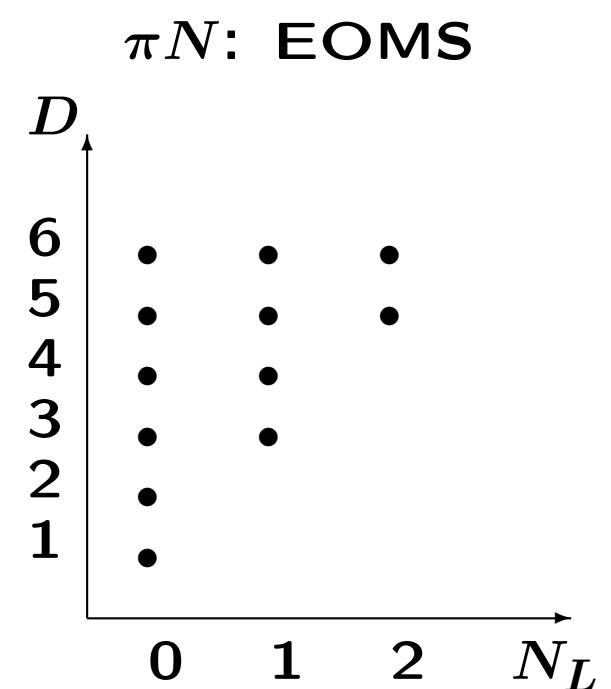
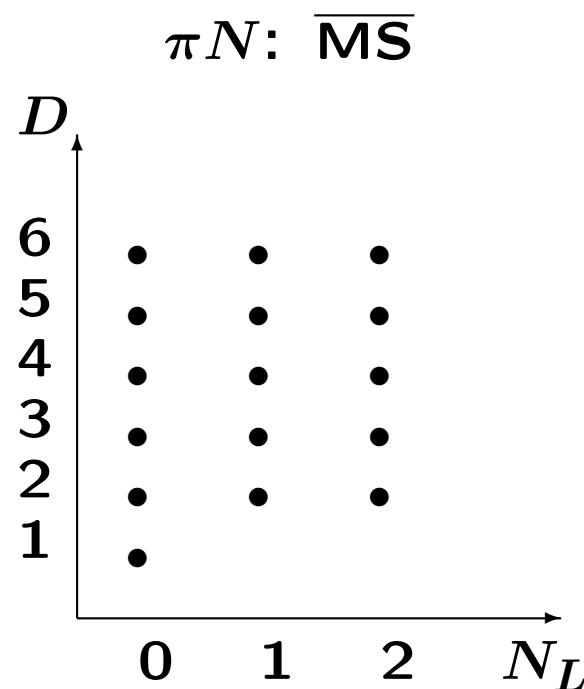
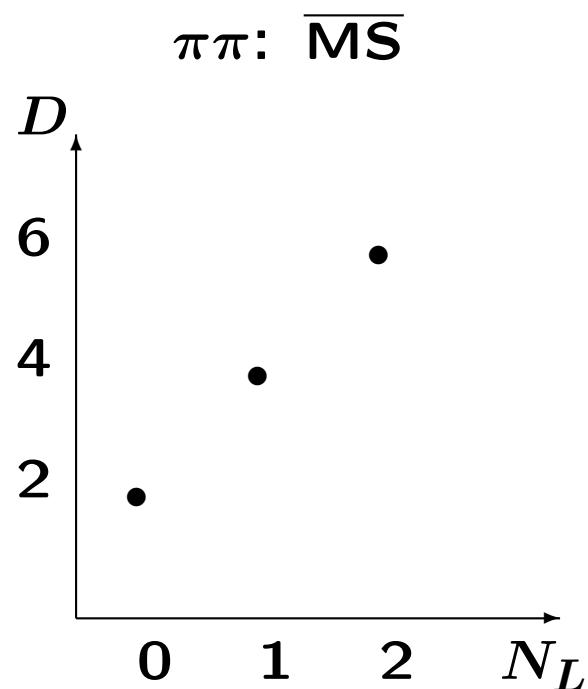
$$\begin{aligned} H^{\text{subtr}} &= - \int \frac{d^n k}{(2\pi)^n} \frac{i}{(k^2 - 2k \cdot p + i0^+)(k^2 + i0^+)} \Big|_{p^2=m^2} \\ &= -2\bar{\lambda} + \frac{1}{16\pi^2} + O(n-4) \end{aligned}$$

**where**

$$\bar{\lambda} = \frac{m^{n-4}}{(4\pi)^2} \left\{ \frac{1}{n-4} - \frac{1}{2} [\ln(4\pi) + \Gamma'(1) + 1] \right\}$$

$$H^R = H - H^{\text{subtr}} = \mathcal{O}(q^{n-3})$$

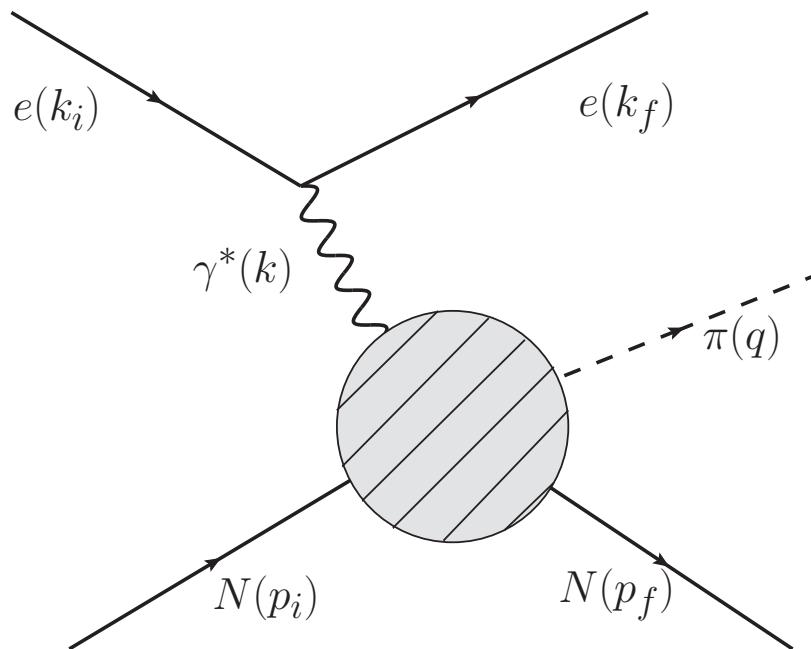
## Chiral versus loop expansion



### 3. Application to pion photo- and electroproduction

$$e(k_i) + N(p_i) \rightarrow e(k_f) + N(p_f) + \pi(q)$$

One-photon-exchange approximation



## Invariant amplitude

$$\mathcal{M} = \text{leptonic vertex} \times i \text{ propagator} \times \text{hadronic vertex} = \epsilon_\mu \mathcal{M}^\mu$$

$$\epsilon_\mu = e \frac{\bar{u}(k_f) \gamma_\mu u(k_i)}{k^2}, \quad \mathcal{M}^\mu = -ie \langle N(p_f), \pi(q) | J^\mu(0) | N(p_i) \rangle.$$

## Current conservation

$$k_\mu \mathcal{M}^\mu = 0$$

## Parameterization in terms of **six** invariant amplitudes

$$\mathcal{M}^\mu = \bar{u}(p_f) \left( \sum_{i=1}^6 \boxed{A_i(s, t, u)} M_i^\mu \right) u(p_i), \quad u(p): \text{Dirac spinor}$$

## Mandelstam variables

$$s + t + u = 2m_N^2 + M_\pi^2 - Q^2, \quad Q^2 = -k^2$$

$$M_1^\mu = -\frac{i}{2} \gamma_5 (\gamma^\mu k - k \gamma^\mu), \quad \dots$$

cm frame

$$\mathcal{M} = \frac{4\pi W}{m_N} \chi_f^\dagger \mathcal{F} \chi_i, \quad \chi: \text{Pauli spinor}$$

six CGLN amplitudes

$$\mathcal{F} = i\vec{\sigma} \cdot \vec{a}_\perp \boxed{\mathcal{F}_1(W, \Theta_\pi, Q^2)} + \dots$$

Multipole expansion of  $\mathcal{F}_i$  in terms of Legendre polynomials and

$$\mathcal{F}_1 = \sum_{l=0}^{\infty} \left\{ [lM_{l+} + E_{l+}] P'_{l+1}(x) + [(l+1)M_{l-} + E_{l-}] P'_{l-1}(x) \right\}, \quad \dots$$

$$x = \cos \Theta_\pi = \hat{q} \cdot \hat{k}$$

$$E_{l\pm}, M_{l\pm}, L_{l\pm} : \text{functions of } W \text{ and } Q^2$$

## **Isospin decomposition: four physical channels**

$$A_i(\gamma^{(*)} p \rightarrow n\pi^+) = \sqrt{2} \left( A_i^{(-)} + A_i^{(0)} \right),$$

$$A_i(\gamma^{(*)} p \rightarrow p\pi^0) = A_i^{(+)} + A_i^{(0)},$$

$$A_i(\gamma^{(*)} n \rightarrow p\pi^-) = -\sqrt{2} \left( A_i^{(-)} - A_i^{(0)} \right),$$

$$A_i(\gamma^{(*)} n \rightarrow n\pi^0) = A_i^{(+)} - A_i^{(0)},$$

**expressed in terms of three isospin amplitudes (0), (+), and (-)**

## 1. Number of diagrams

- $\mathcal{O}(q^3)$ : 15 tree-level diagrams + 50 one-loop diagrams
- $\mathcal{O}(q^4)$ : 20 tree-level diagrams + 85 one-loop diagrams

## 2. Calculate loop contributions numerically using CAS MATHEMATICA with FeynCalc and LoopTools packages

## 3. Checks: Current conservation and crossing symmetry

## 4. LECs from other processes (mesonic and baryonic Lagrangians)

LEC	Source
$l_3$	$M_\pi = 134.977 \text{ MeV}$
$l_4, l_6$	<b>pion form factor</b>
$c_1$	<b>proton mass</b> $m_p = 938.272 \text{ MeV}$
$c_2, c_3, c_4$	<b>pion-nucleon scattering</b>
$c_6, c_7$	<b>magnetic moment of proton</b> ( $\mu_p = 2.793$ ) and neutron ( $\mu_n = -1.913$ )
$d_6, d_7,$ $e_{54}, e_{74}$	<b>world data for nucleon electromagnetic form factors</b> ( $Q^2 < 0.3 \text{ GeV}^2$ )
$d_{16}$	<b>axial-vector coupling constant</b> $g_A = 1.2695$
$d_{18}$	<b>pion-nucleon coupling</b>
$d_{22}$	<b>axial radius of the nucleon</b> $\langle r_A^2 \rangle = 12/M_A^2$ , $M_A = 1.026 \text{ GeV}$

$$l_i: \mathcal{L}_\pi^{(4)},$$

$$c_i: \mathcal{L}_{\pi N}^{(2)}, \quad d_i: \mathcal{L}_{\pi N}^{(3)}, \quad e_i: \mathcal{L}_{\pi N}^{(4)}$$

## 5. Analytic expressions for the contact diagrams

(a) 4 LECs at  $\mathcal{O}(q^3)$

isospin

$$\mathcal{L}_{\pi N}^{(3)} = \frac{\textcolor{red}{d_8}}{2m} \left( i\bar{\Psi} \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( \tilde{f}_{\mu\nu}^+ u_\alpha \right) D_\beta \Psi + \text{H.c.} \right) \quad (+)$$

$$+ \frac{\textcolor{red}{d_9}}{2m} \left( i\bar{\Psi} \epsilon^{\mu\nu\alpha\beta} \text{Tr} \left( f_{\mu\nu}^+ + 2v_{\mu\nu}^{(s)} \right) u_\alpha D_\beta \Psi + \text{H.c.} \right) \quad (0)$$

$$- \frac{\textcolor{red}{d_{20}}}{8m^2} \left( i\bar{\Psi} \gamma^\mu \gamma_5 \left[ \tilde{f}_{\mu\nu}^+, u_\lambda \right] D^{\lambda\nu} \Psi + \text{H.c.} \right) \quad (-)$$

$$+ i \frac{\textcolor{red}{d_{21}}}{2} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \tilde{f}_{\mu\nu}^+, u^\nu \right] \Psi \quad (-)$$

Structures contribute to **photoproduction**, no free parameters for **electroproduction**

(b) 15 LECs at  $\mathcal{O}(q^4)$

$$\begin{aligned}\mathcal{L}_{\pi N}^{(4)} = & -\frac{e_{48}}{4m} \left( i\bar{\Psi} \text{Tr} \left( f_{\lambda\mu}^+ + 2v_{\lambda\mu}^{(s)} \right) h_\nu^\lambda \gamma_5 \gamma^\mu D^\nu \Psi + \text{H.c.} \right) \\ & + \text{14 more terms}\end{aligned}$$

- photoproduction

isospin channel	(0)	(+)	(-)
# LECs	5	5	1

- electroproduction

isospin channel	(0)	(+)	(-)
# LECs	2	2	0

## 6. Web interface chiral MAID

[<http://www.kph.uni-mainz.de/MAID/chiralmaid/>]

# MAID

## Photo- and Electroproduction of Pions, Etas and Kaons on the Nucleon

Institut für Kernphysik, Universität Mainz

Mainz, Germany

**MAID2007**

[unitary isobar model for \(e,e'p\)](#)

**DMT2001**

[dynamical model for \(e,e'p\)](#)

**KAON-MAID**

[isobar model for \(e,e'K\)](#)

**ETA-MAID**

[isobar model for \(e,e'h\)](#)

[reggeized isobar model for \(q,h\)](#)

**ChiralMAID** NEW

[chiral perturbation theory approach for \(e,e'p\)](#)

**2-PION-MAID**

[isobar model for \(q,pp\)](#)

**archive**

[MAID2000](#) [MAID2003](#) [DMT2001original](#) [ETAprime2003](#)

[Back to Theory Group Homepage](#)



**M A X I D**

[ChiralMAID info and updates \(please read first\)](#)

**Pion Photo- and Electroproduction on the Nucleon in relativistic chiral perturbation theory**

---

[M. Hilt](#), [S. Scherer](#), [L. Tiator](#)

---

- [Electromagnetic Multipoles \( \$E\_H, M\_H, L\_H, S\_H\$ \)](#)
- [Amplitudes \( \$F\_1, \dots, F\_6, H\_1, \dots, H\_6, A\_1, \dots, A\_6\$ \)](#)
- [Differential Cross Sections \( \$ds\_T, ds\_L, ds\_{LT}, ds\_{TT}, \dots\$ \)](#)
- [5-fold Diff. Cross Section \( \$d^5s, G, ds^V = ds\_T + e ds\_L + e ds\_{TT} \cos 2f + \dots\$ \)](#)
- [Total Cross Sections \( \$s\_T, s\_L, s\_{LT}, s\_{TT}, \dots\$ \)](#)
- [Transverse Polarization Observables \( \$ds/dW, T, S, P, E, F, G, H, \dots\$ \)](#)

---

External services:

[MAID Homepage](#) [MAID2003](#) [DMT2001](#) [KAON-MAID](#) [ETA-MAID2000](#) [ETA-MAID2003](#) [ETA'-MAID](#)

---

[A1 kinematics calculator for electroproduction \(Java\)](#)

[SAID Partial-Wave Analyses](#)

---

[Back to Theory Group Homepage](#)

---

# Multipoles

The multipoles can be given in 4 unique sets of isospin or charge channels ([click here for a larger image](#)):

$$\begin{aligned} & \left( A_p^{1/2}, A_n^{1/2}, A^{3/2} \right), \left( A^{1/2}, A^0, A^{3/2} \right), \left( A^0, A^+, A^- \right), \left( A_{\pi^+ n}, A_{\pi^- p}, A_{\pi^0 p}, A_{\pi^0 n} \right) \\ & A_{\pi^+ n} = \sqrt{2} (A^- + A^0) = \sqrt{2} (A_p^{1/2} - \frac{1}{3} A^{3/2}) = \sqrt{2} (A^0 + \frac{1}{3} A^{1/2} - \frac{1}{3} A^{3/2}) \\ & A_{\pi^- p} = -\sqrt{2} (A^- - A^0) = \sqrt{2} (A_n^{1/2} + \frac{1}{3} A^{3/2}) = \sqrt{2} (A^0 - \frac{1}{3} A^{1/2} + \frac{1}{3} A^{3/2}) \\ & A_{\pi^0 p} = A^+ + A^0 = A_p^{1/2} + \frac{2}{3} A^{3/2} = A^0 + \frac{1}{3} A^{1/2} + \frac{2}{3} A^{3/2} \\ & A_{\pi^0 n} = A^+ - A^0 = -A_n^{1/2} + \frac{2}{3} A^{3/2} = -A^0 + \frac{1}{3} A^{1/2} + \frac{2}{3} A^{3/2} \end{aligned}$$

Further details can be found in D. Drechsel and L. Tiator, J. Phys. G 18 (1992) 449-497. ([scanned version](#))

Type of the multipoles:  (p(1/2), n(1/2), 3/2)  (1/2, 0, 3/2)  (0, +, -)  charge channels

Choose pion angular momentum l :  El+  El-  MI+  MI-  LI+  LI-  SI+  SI-

Reduced multipoles:

## Choose kinematical variables

choose an independent (running) variable:  Q<sup>2</sup>  W

choose values for Q<sup>2</sup>, W, step size and maximum value:

Q <sup>2</sup> (GeV/c) <sup>2</sup>	W (MeV)	increment	upper value	click here	
0	1074	1	1100	Calculate	Reset

## Change of model parameters:

**O( $q^3$ ) (all couplings in GeV $^{-2}$ )**

0		+		-	
d <sub>9</sub>		d <sub>8</sub>		d <sub>20</sub>	d <sub>21</sub>
-1.216		-1.092		4.337	-4.260

**O( $q^4$ ) (all couplings in GeV $^{-3}$ )**

## Isospin 0

e <sub>48</sub>	e <sub>49</sub>	e <sub>50</sub>	e <sub>51</sub>	e <sub>52</sub>	e <sub>53</sub>	e <sub>112</sub>
5.235	0.925	2.205	6.629	-4.103	-2.654	9.342

## Isospin +

e <sub>67</sub>	e <sub>68</sub>	e <sub>69</sub>	e <sub>71</sub>	e <sub>72</sub>	e <sub>73</sub>	e <sub>113</sub>
-8.269	-0.925	-1.035	-4.352	10.539	2.120	-13.745

## Isospin -

e <sub>70</sub>
3.910

[Back to Pion Electroproduction Main Page](#)

## Multipoles

---

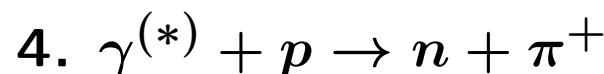
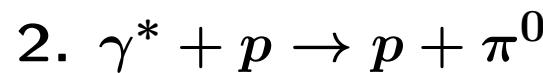
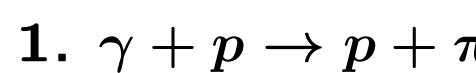
C h M A I D 2 0 1 2  
M. Hilt, S. Scherer, L. Tiator  
Institut fuer Kernphysik, Universitaet Mainz  
\*\*\*\*\*

Pion angular momentum l= 0

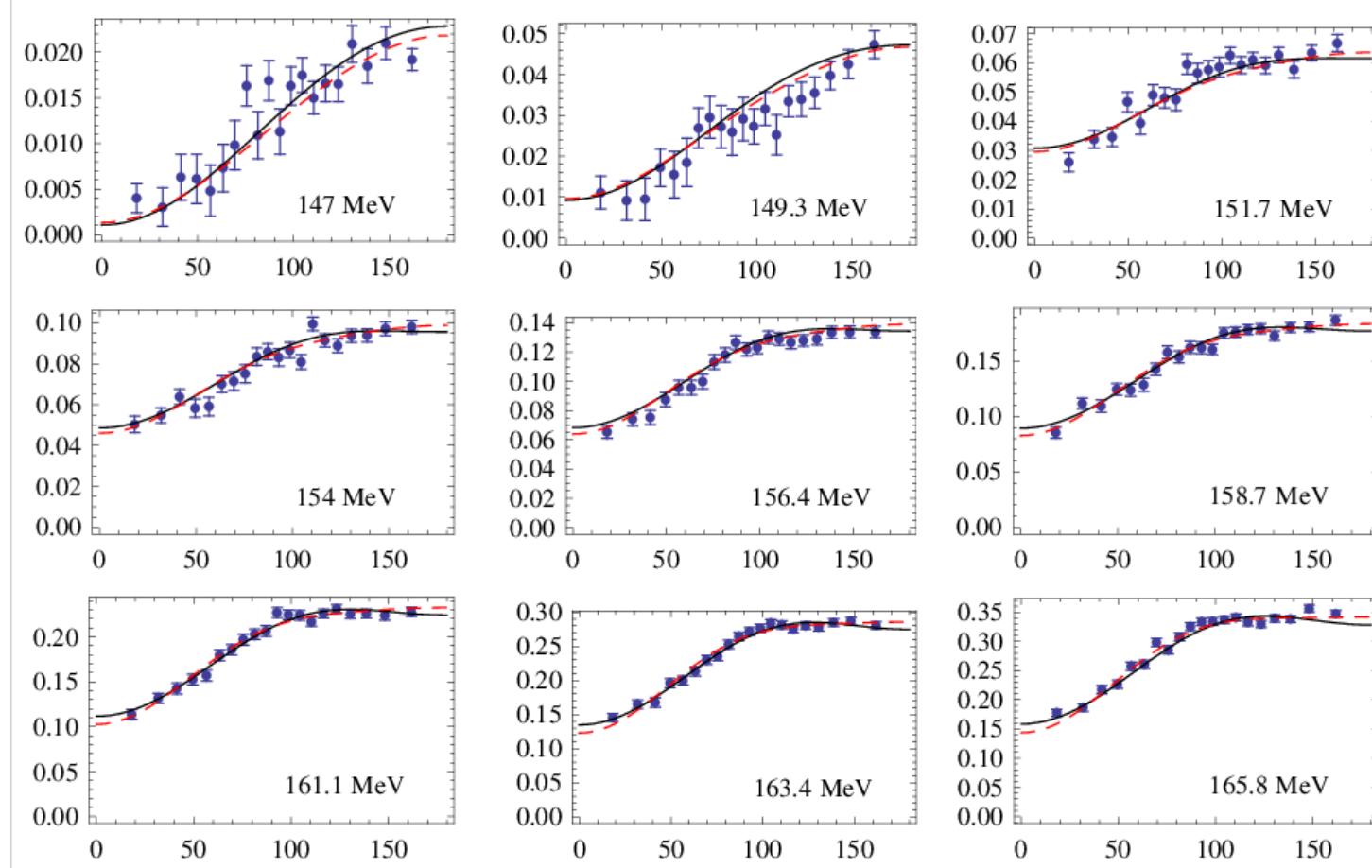
All multipoles are given in  $10^{-3}/M_{\pi^+}$

Q^2 = .000 (GeV/c)^2				e,gA,F[GeV],gpiN=gA*mp/F			
*****				d8,d9,d20,d21 [GeV^-2]			
.3028	1.2695	.0924	13.2100				
-1.0920	-1.2160	4.3370	-4.2600				
5.2350	.9250	2.2050	6.6290	-4.1030	-2.6540	e48,e49,e50,e51,e52,e53 [GeV^-3]	
-8.2690	-.9250	-1.0350	3.9100	-4.3520	10.5390	2.1200	e67,e68,e69,e70,e71,e72,e73 [GeV^-3]
9.3420	-13.7450						e112,e113 [GeV^-3]
W	E0+(pi0_p)	E0+(pi0_n)	E0+(pi+_n)	E0+(pi-_p)	E(lab)	q(cm)	
(MeV)	Re	Im	Re	Im	Re	Im	(MeV)
1074.00	-1.0608	.0000	2.8400	.0000	27.1931	.0000	145.54
1075.00	-.9960	.0000	2.8898	.0000	26.9933	.0000	146.69
1076.00	-.9210	.0000	2.9504	.0000	26.7940	.0000	147.84
1077.00	-.8301	.0000	3.0279	.0000	26.5939	.0000	148.98
1078.00	-.7093	.0000	3.1376	.0000	26.3903	.0000	150.13
1079.00	-.4769	.0000	3.3685	.0000	26.1676	.0000	151.28
1080.00	-.3758	.3249	3.4564	.3534	25.9705	-.0617	152.43
1081.00	-.3959	.4764	3.4121	.5183	25.7986	-.0924	153.58
1082.00	-.4162	.5891	3.3672	.6412	25.6292	-.1166	154.74
1083.00	-.4367	.6826	3.3218	.7433	25.4625	-.1378	155.89
1084.00	-.4573	.7641	3.2758	.8323	25.2982	-.1573	157.04
1085.00	-.4780	.8371	3.2293	.9121	25.1364	-.1757	158.20
1086.00	-.4989	.9035	3.1822	.9849	24.9770	-.1933	159.36
1087.00	-.5199	.9649	3.1346	1.0523	24.8200	-.2104	160.52
1088.00	-.5410	1.0221	3.0864	1.1151	24.6654	-.2270	161.67
1089.00	-.5623	1.0758	3.0377	1.1741	24.5131	-.2433	162.83
1090.00	-.5837	1.1264	2.9884	1.2299	24.3630	-.2593	164.00
1091.00	-.6053	1.1745	2.9384	1.2828	24.2152	-.2752	165.16
1092.00	-.6271	1.2202	2.8880	1.3333	24.0695	-.2909	166.32

## Fits to available experimental data



# Differential cross sections $d\sigma/d\Omega_\pi$ in $\mu\text{b}/\text{sr}$ for $\gamma + p \rightarrow p + \pi^0$ <sup>11</sup>

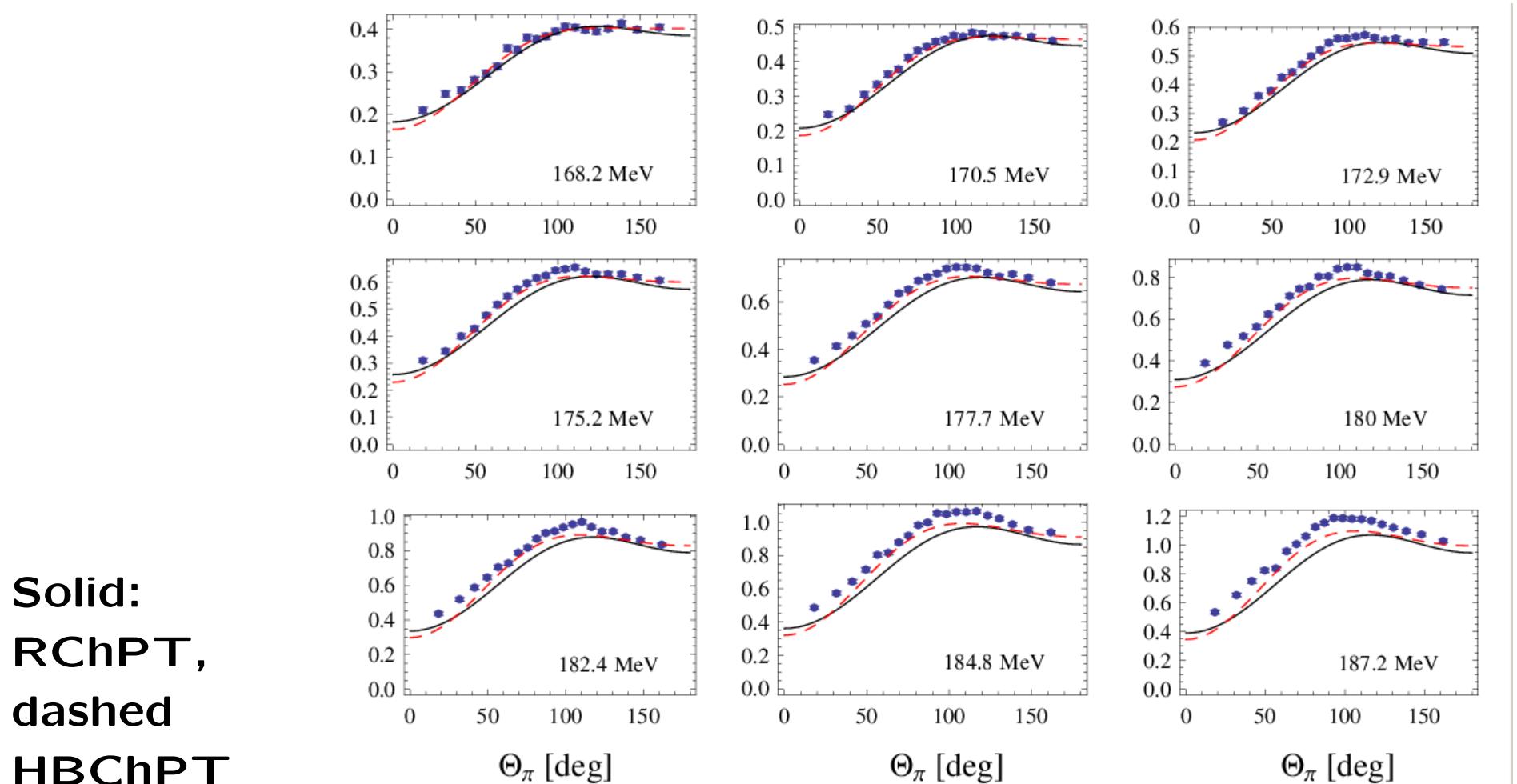


**Solid:**  
**RChPT,**  
**dashed**  
**HBChPT**

---

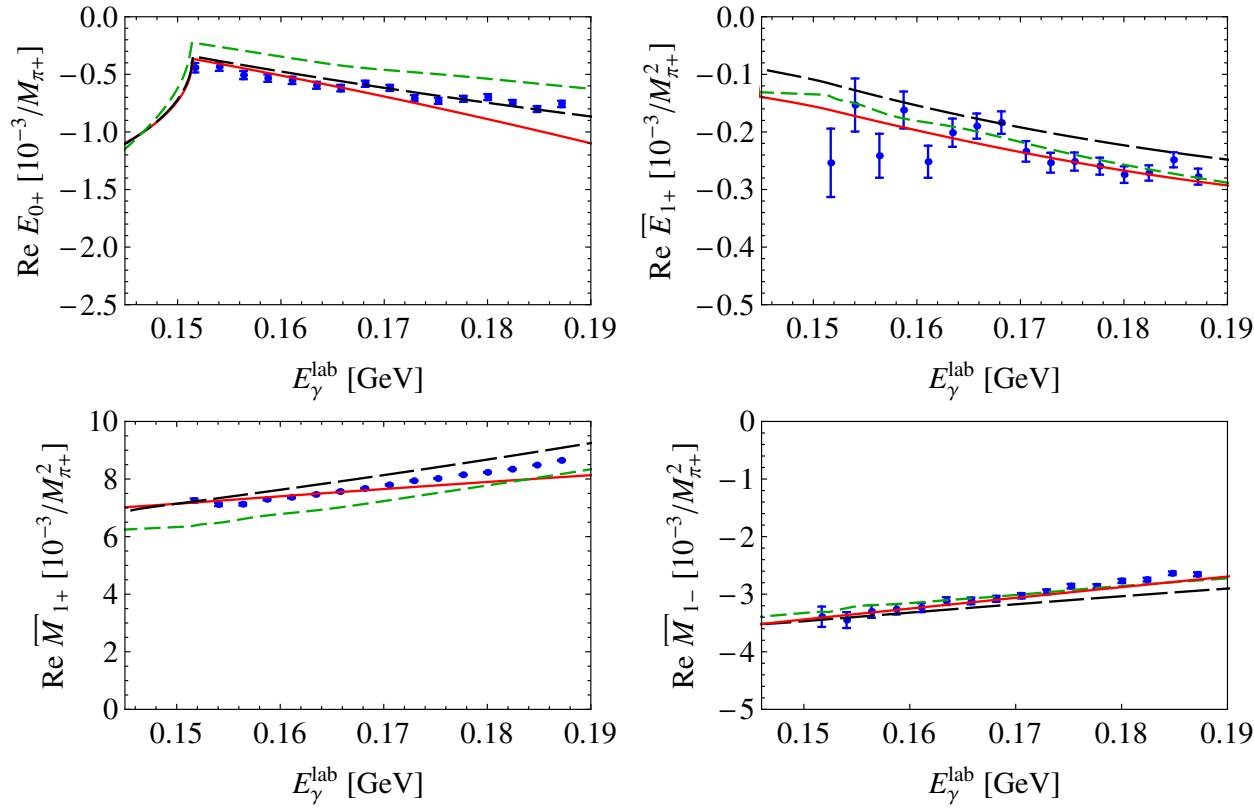
<sup>11</sup>Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

# Differential cross sections $d\sigma/d\Omega_\pi$ in $\mu\text{b}/\text{sr}$ for $\gamma + p \rightarrow p + \pi^0$ <sup>12</sup>



<sup>12</sup>Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

# *S- and reduced P-wave multipoles for $\gamma + p \rightarrow p + \pi^0$*



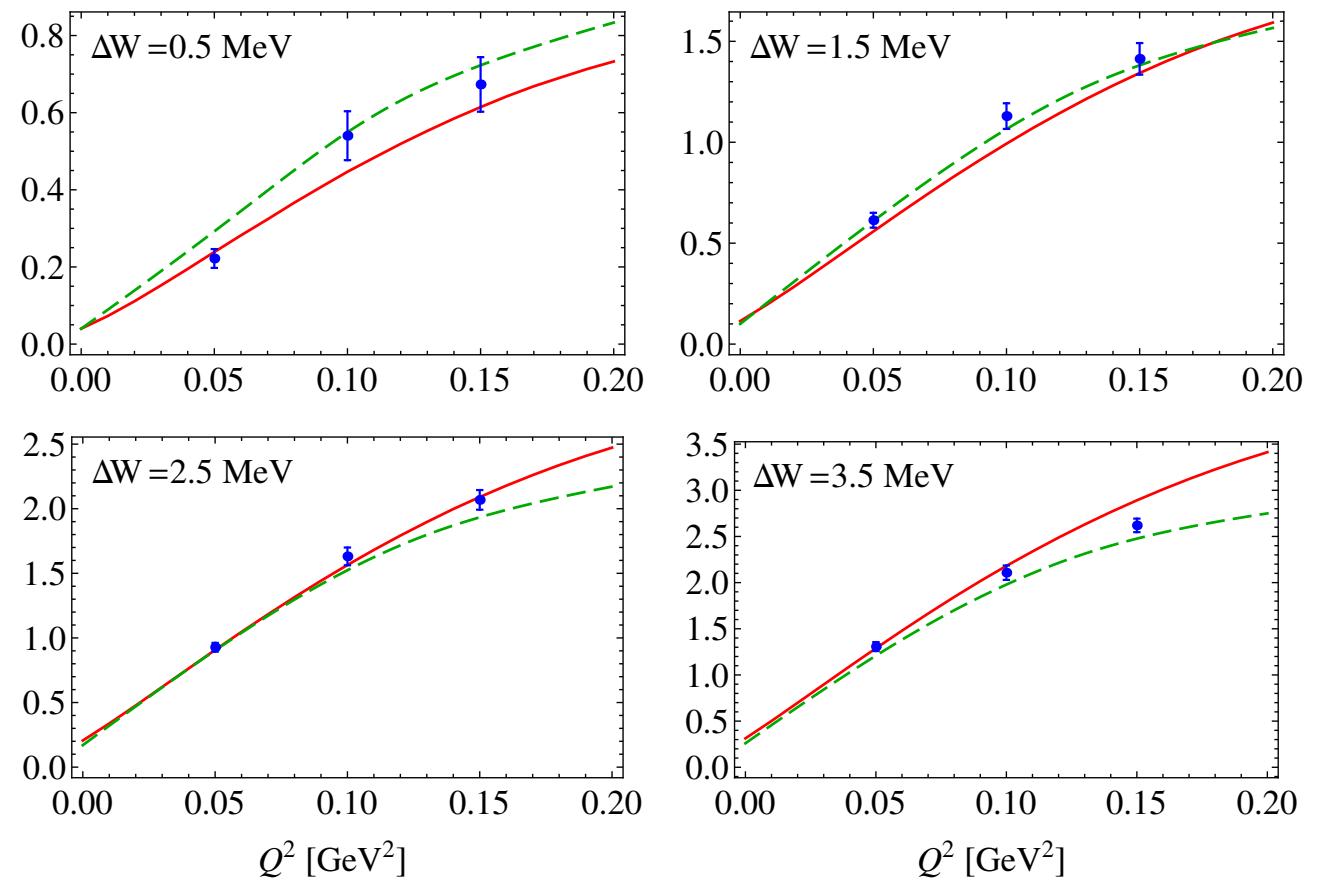
**Red RChPT; green DMT model<sup>13</sup>; black Gasparyan & Lutz<sup>14</sup>; data from Hornidge et al. (2013)**

---

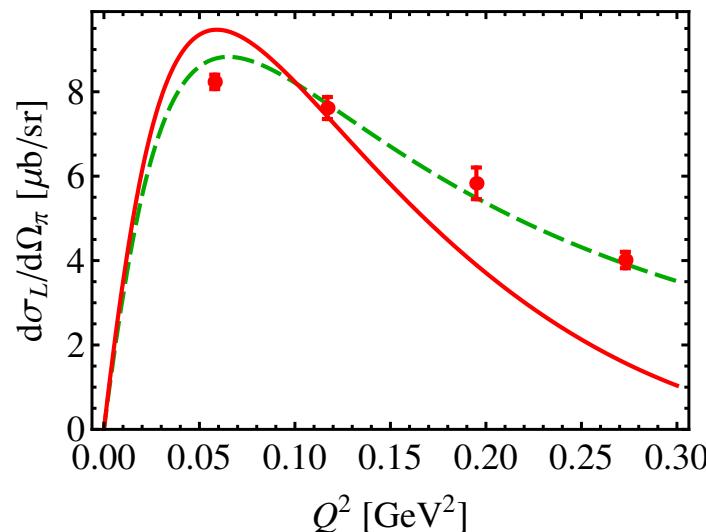
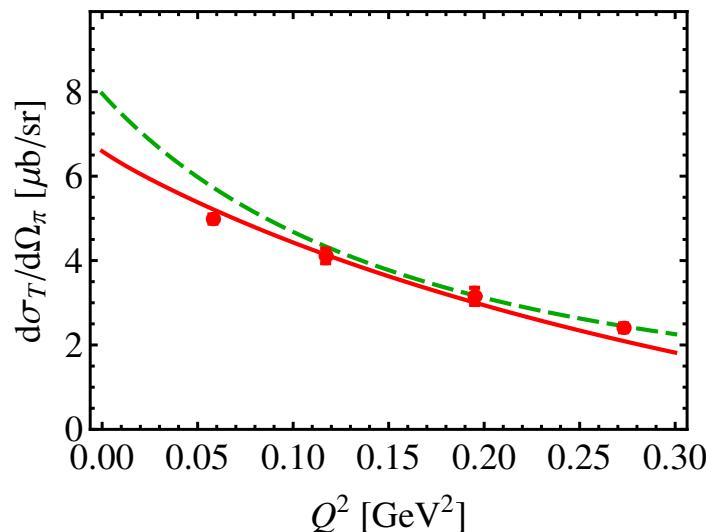
<sup>13</sup>S. S. Kamalov et al., Phys. Rev. C **64**, 032201 (2001)

<sup>14</sup>A. Gasparyan and M. F. M. Lutz, Nucl. Phys. **A848**, 126 (2010)

## Total cross sections for $\gamma^* + p \rightarrow p + \pi^0$ in $\mu\text{b}$



**Differential cross sections as a function of  $Q^2$  for  $\gamma^* + p \rightarrow n + \pi^+$  at  $W = 1125$  MeV and  $\Theta_\pi = 0^\circ$ .**



**red RChPT; green DMT model;**  
**data from Baumann (PhD thesis, JGU, 2005)**

Isospin channel	LEC	Value
0	$d_9$ [GeV $^{-2}$ ]	$-1.22 \pm 0.12$
0	$e_{48}$ [GeV $^{-3}$ ]	$5.2 \pm 1.4$
0	$e_{49}$ [GeV $^{-3}$ ]	$0.9 \pm 2.6$
0	$e_{50}$ [GeV $^{-3}$ ]	$2.2 \pm 0.8$
0	$e_{51}$ [GeV $^{-3}$ ]	$6.6 \pm 3.6$
0	$e_{52}^*$ [GeV $^{-3}$ ]	$-4.1$
0	$e_{53}^*$ [GeV $^{-3}$ ]	$-2.7$
0	$e_{112}$ [GeV $^{-3}$ ]	$9.3 \pm 1.6$
<hr/>		
from fits with all data		
+	$d_8$ [GeV $^{-2}$ ]	$-1.09 \pm 0.12$
+	$e_{67}$ [GeV $^{-3}$ ]	$-8.3 \pm 1.5$
+	$e_{68}$ [GeV $^{-3}$ ]	$-0.9 \pm 2.6$
+	$e_{69}$ [GeV $^{-3}$ ]	$-1.0 \pm 2.2$
+	$e_{71}$ [GeV $^{-3}$ ]	$-4.4 \pm 3.7$
+	$e_{72}^*$ [GeV $^{-3}$ ]	$10.5$
+	$e_{73}^*$ [GeV $^{-3}$ ]	$2.1$
+	$e_{113}$ [GeV $^{-3}$ ]	$-13.7 \pm 2.6$
<hr/>		
-	$d_{20}$ [GeV $^{-2}$ ]	$4.34 \pm 0.08$
-	$d_{21}$ [GeV $^{-2}$ ]	$-3.1 \pm 0.1$
-	$e_{70}$ [GeV $^{-3}$ ]	$3.9 \pm 0.3$

## **4. Summary and outlook**

- Baryonic ChPT: Renormalization condition  $\leftrightarrow$  consistent power counting
- Example: EOMS renormalization (manifestly Lorentz-invariant)
- Application to pion photo- and electroproduction
- 20 tree-level diagrams + 85 one-loop diagrams
- Chiral MAID interface

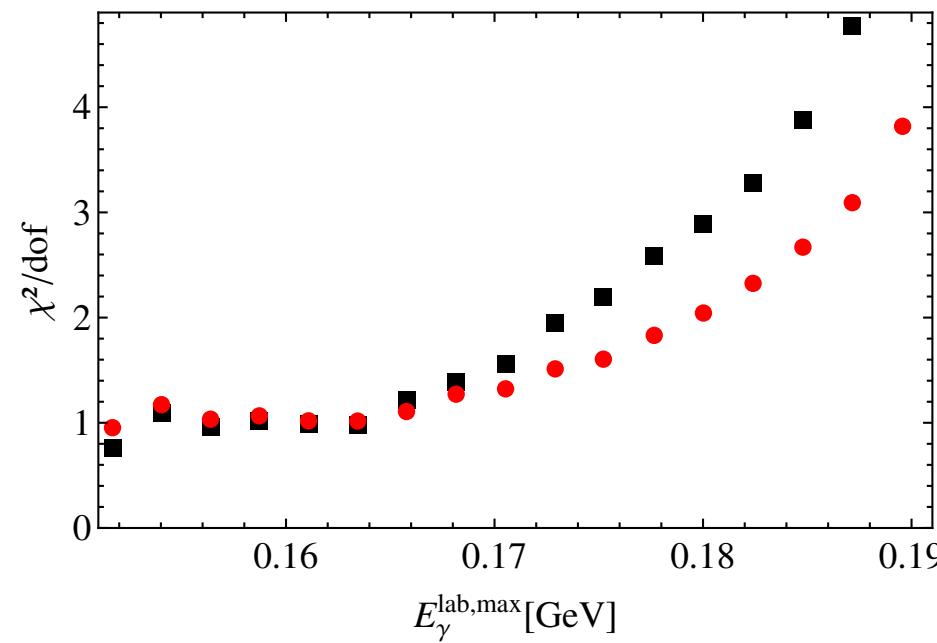
- Inclusion of heavy degrees of freedom (vector mesons, axial vector mesons,  $\Delta$ <sup>15</sup>)
- New data<sup>16</sup> ↵ reanalysis of LECs

---

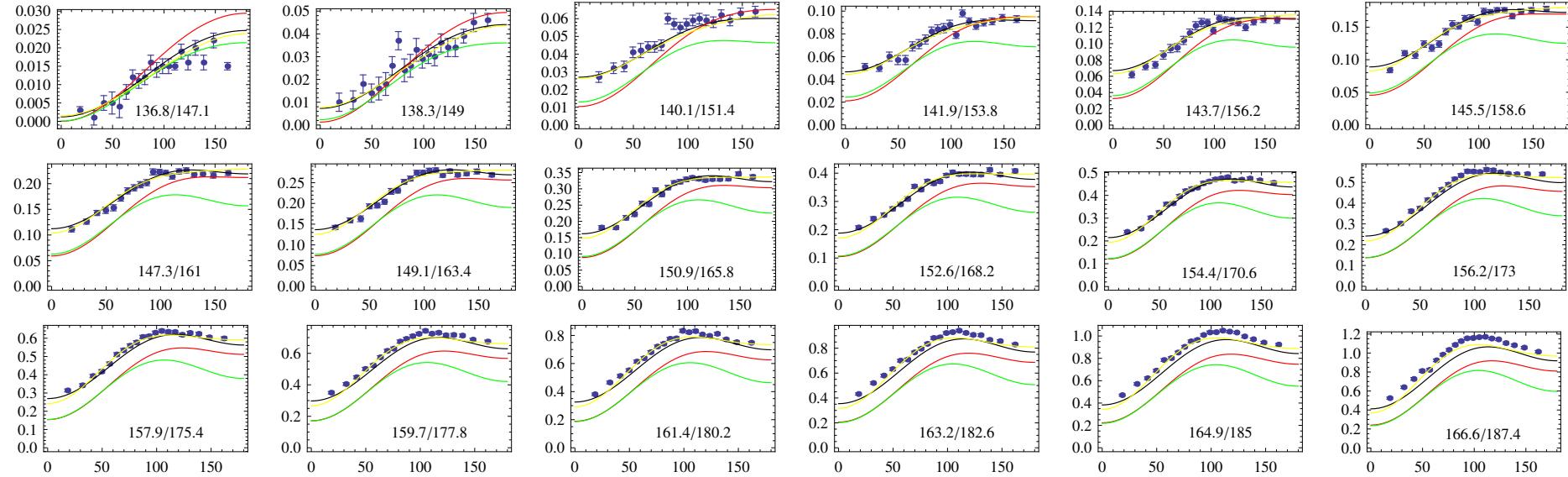
<sup>15</sup>A. N. H. Blin, T. Ledwig and M. J. V. Vacas, arXiv:1412.4083 [hep-ph]

<sup>16</sup>K. Chirapatpimol *et al.*,  $p(e, e'p)\pi^0$ , Phys. Rev. Lett. **114**, 192503 (2015), I. Fricic,  $p(e, e'\pi^+)n$ , PhD thesis, University of Zagreb, 2015

$\chi^2_{\text{red}}$  as a function of the fitted energy range: RBChPT vs.  
HBChPT



# Differential cross sections $d\sigma/d\Omega_\pi$ in $\mu\text{b}/\text{sr}$ for $\gamma + p \rightarrow p + \pi^0$

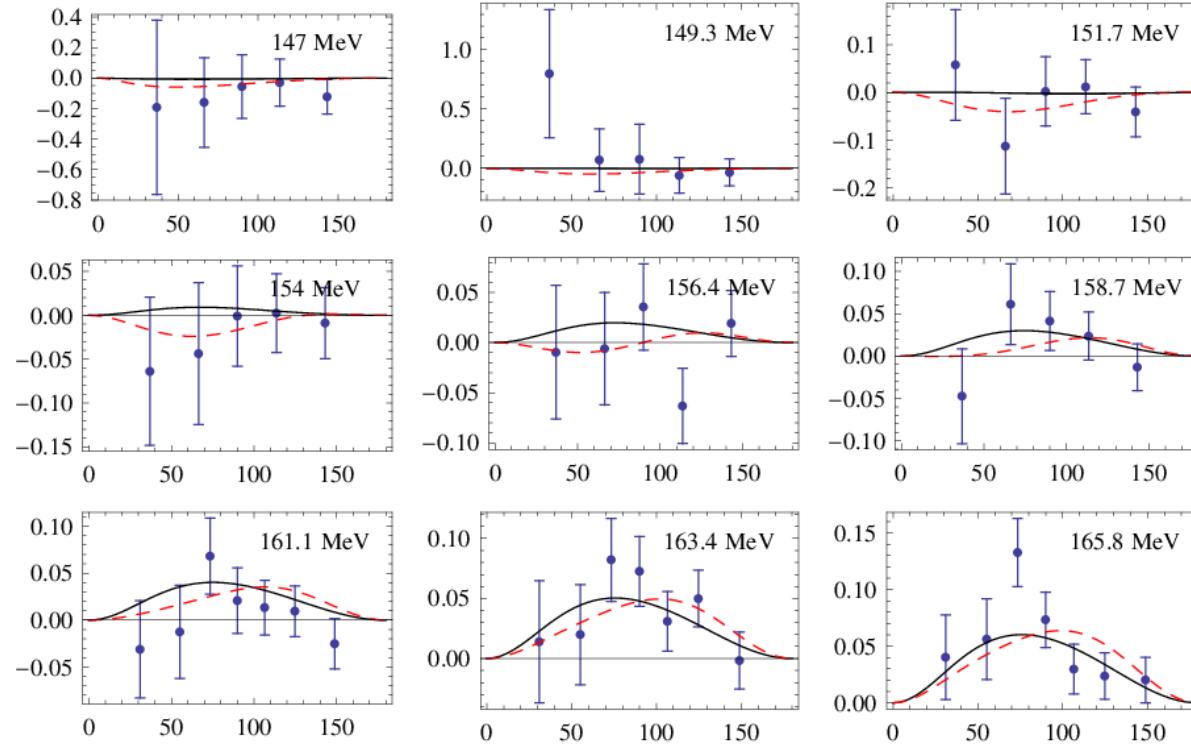


**Black:** RChPT  $\mathcal{O}(q^4)$ , **red** RChPT  $\mathcal{O}(q^3)$ , **yellow** HChPT  $\mathcal{O}(q^4)$ ,  
**green** RChPT + vector mesons  $\mathcal{O}(q^3)$

---

<sup>17</sup>Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

# Photon asymmetries for $\gamma + p \rightarrow p + \pi^0$ <sup>18</sup>

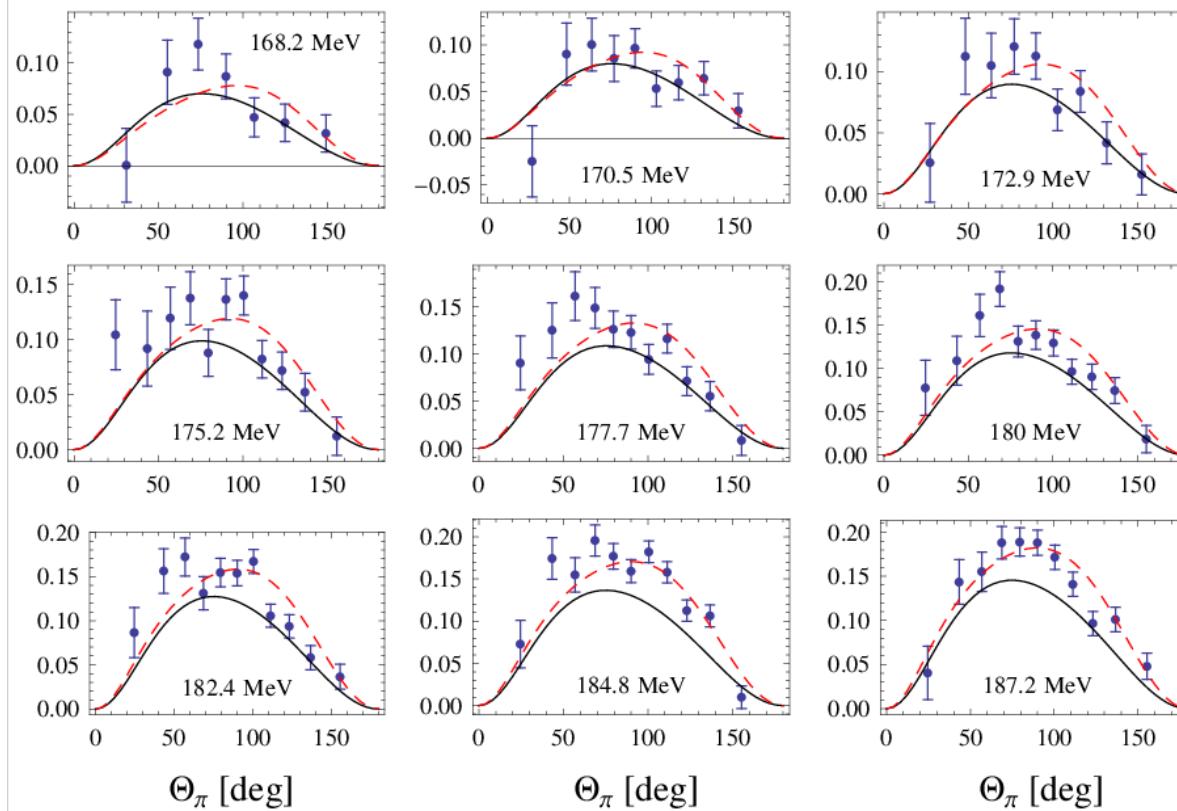


**Solid:**  
**RChPT,**  
**dashed**  
**HBChPT**

---

<sup>18</sup>Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

## Photon asymmetries for $\gamma + p \rightarrow p + \pi^0$ <sup>19</sup>



**Solid:**  
**RChPT,**  
**dashed**  
**HBChPT**

---

<sup>19</sup>Data taken from D. Hornidge et al., Phys. Rev. Lett. **111**, 062004 (2013)

# Multipoles

The multipoles can be given in 4 unique sets of isospin or charge channels ([click here for a larger image](#)):

$$\begin{aligned} & \left( A_p^{1/2}, A_n^{1/2}, A^{3/2} \right), \left( A^{1/2}, A^0, A^{3/2} \right), \left( A^0, A^+, A^- \right), \left( A_{\pi^+ n}, A_{\pi^- p}, A_{\pi^0 p}, A_{\pi^0 n} \right) \\ & A_{\pi^+ n} = \sqrt{2} (A^- + A^0) = \sqrt{2} (A_p^{1/2} - \frac{1}{3} A^{3/2}) = \sqrt{2} (A^0 + \frac{1}{3} A^{1/2} - \frac{1}{3} A^{3/2}) \\ & A_{\pi^- p} = -\sqrt{2} (A^- - A^0) = \sqrt{2} (A_n^{1/2} + \frac{1}{3} A^{3/2}) = \sqrt{2} (A^0 - \frac{1}{3} A^{1/2} + \frac{1}{3} A^{3/2}) \\ & A_{\pi^0 p} = A^+ + A^0 = A_p^{1/2} + \frac{2}{3} A^{3/2} = A^0 + \frac{1}{3} A^{1/2} + \frac{2}{3} A^{3/2} \\ & A_{\pi^0 n} = A^+ - A^0 = -A_n^{1/2} + \frac{2}{3} A^{3/2} = -A^0 + \frac{1}{3} A^{1/2} + \frac{2}{3} A^{3/2} \end{aligned}$$

Further details can be found in D. Drechsel and L. Tiator, J. Phys. G 18 (1992) 449-497. ([scanned version](#))

Type of the multipoles:  (p(1/2), n(1/2), 3/2)  (1/2, 0, 3/2)  (0, +, -)  charge channels

Choose pion angular momentum l :  1  El+  El-  MI+  MI-  Li+  Li-  SI+  SI-

Reduced multipoles:

Choose kinematical variables

choose an independent (running) variable:  Q<sup>2</sup>  W

choose values for Q<sup>2</sup>, W, step size and maximum value:

Q <sup>2</sup> (GeV/c) <sup>2</sup>	W (MeV)	increment	upper value	click here	
0	1080	0.01	0.1	Calculate	Reset

Change of model parameters:

## Multipoles

---

C h M A I D 2 0 1 2  
M. Hilt, S. Scherer, L. Tiator  
Institut fuer Kernphysik, Universitaet Mainz  
\*\*\*\*\*

Pion angular momentum l= 1

All multipoles are given in  $10^{-3}/M_{\pi^+}$

W = 1080.000 (MeV)							
*****							
.3028	1.2695	.0924	13.2100			e,gA,F[GeV],gpiN=gA*mp/F	
-1.0920	-1.2160	4.3370	-4.2600			d8,d9,d20,d21 [GeV^-2]	
5.2350	.9250	2.2050	6.6290	-4.1030	-2.6540	e48,e49,e50,e51,e52,e53 [GeV^-3]	
-8.2690	-.9250	-1.0350	3.9100	-4.3520	10.5390	e67,e68,e69,e70,e71,e72,e73 [GeV^-3]	
9.3420	-13.7450					e112,e113 [GeV^-3]	
Q^2	E1+(pi0_p)	E1+(pi0_n)	E1+(pi+_n)	E1+(pi-_p)	E(lab)	q(cm)	
(GeV/c)^2	Re	Im	Re	Im	Re	Im	(MeV)
.00	-.0479	-.0001	-.0177	-.0002	1.4327	.0001	-.1.4755 -.0001
.01	-.0583	-.0001	-.0171	-.0002	1.4017	.0001	-.1.4600 -.0001
.02	-.0673	-.0001	-.0149	-.0001	1.3364	.0001	-.1.4106 -.0001
.03	-.0754	-.0001	-.0115	-.0001	1.2631	.0001	-.1.3536 .0000
.04	-.0831	-.0001	-.0071	-.0001	1.1903	.0001	-.1.2977 .0000
.05	-.0903	-.0001	-.0020	-.0001	1.1208	.0001	-.1.2457 .0000
.06	-.0973	-.0001	.0039	-.0001	1.0555	.0001	-.1.1985 .0000
.07	-.1040	-.0001	.0104	-.0001	.9944	.0001	-.1.1561 .0000
.08	-.1106	-.0001	.0174	-.0001	.9372	.0001	-.1.1183 .0000
.09	-.1171	-.0001	.0250	-.0001	.8836	.0001	-.1.0845 .0000
.10	-.1234	-.0001	.0331	-.0001	.8332	.0001	-.1.0545 .0000
Q^2	L1+(pi0_p)	L1+(pi0_n)	L1+(pi+_n)	L1+(pi-_p)	E(lab)	q(cm)	
(GeV/c)^2	Re	Im	Re	Im	Re	Im	(MeV)
.00	-.0393	-.0001	-.0168	-.0001	.7782	.0000	-.8101 .0000
.01	-.0440	-.0001	-.0169	-.0001	.6088	.0000	-.6471 .0000
.02	-.0468	.0000	-.0162	-.0001	.4781	.0000	-.5214 .0000
.03	-.0486	.0000	-.0151	.0000	.3797	.0000	-.4271 .0000
.04	-.0496	.0000	-.0138	.0000	.3047	.0000	-.3554 .0000
.05	-.0501	.0000	-.0124	.0000	.2465	.0000	-.2998 .0000