Resonance extraction from the finite Volume

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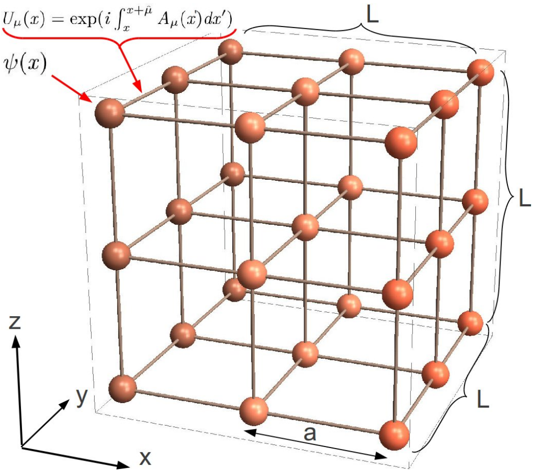
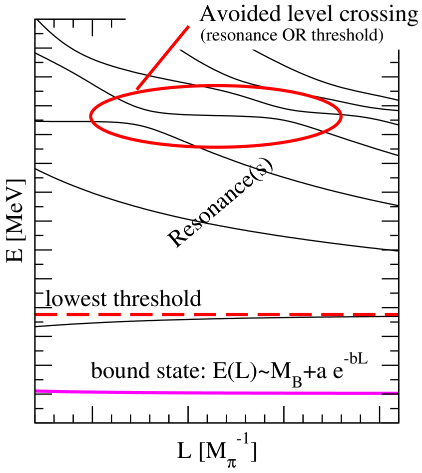
The cubic lattice



- Side length L, periodic boundary conditions $\Psi(\vec{x}) \stackrel{!}{=} \Psi(\vec{x} + \hat{\mathbf{e}}_i\, L)$
  $\rightarrow$ finite volume effects
  $\rightarrow$ Infinite volume $L \rightarrow \infty$ extrapolation
- Lattice spacing a
  $\rightarrow$ finite size effects
  Modern lattice calculations: $a \simeq 0.07$ fm $\rightarrow p \sim 2.8$ GeV
  $\rightarrow$ (much) larger than typical hadronic scales;

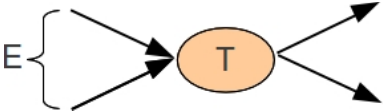
  not considered here.
- Unphysically large quark/hadron masses
  $\rightarrow$ (chiral) extrapolation required.

Eigenvalues in the finite volume



- Unitarity of the scattering matrix S: $SS^\dagger = \mathbb{1}$ $\quad [S = \mathbb{1} - i\,\frac{p}{4\pi E}\,T]$.

$$\mathrm{Im}\,T^{-1}(E) = \sigma \equiv \frac{p}{8\pi\,E}$$



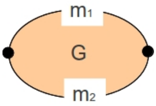
- $\rightarrow$ Generic (Lippman-Schwinger) equation for unitarizing the T-matrix:

$$T = V + V\,G\,T \qquad \mathrm{Im}\,G = -\sigma$$

V: (Pseudo)potential, σ: phase space.

- G: Green's function:

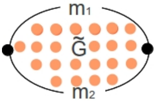
$$G = \int \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2 + i\epsilon},$$
$$\omega_{1,2}^2 = m_{1,2}^2 + \vec{q}^{\;2}$$

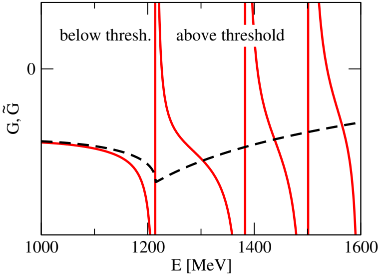


$$\Psi(\vec{x}) \overset{!}{=} \Psi(\vec{x} + \hat{\mathbf{e}}_i\, L) = \exp\left(i\, L\, q_i\right) \Psi(\vec{x}) \implies q_i = \frac{2\pi}{L}\, n_i, \quad n_i \in \mathbb{Z}, \quad i = 1, 2, 3$$

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\, g(|\vec{q}\,|^2) \to \frac{1}{L^3} \sum_{\vec{n}} g(|\vec{q}|^2), \quad \vec{q} = \frac{2\pi}{L}\, \vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

$$G \to \tilde{G} = \frac{1}{L^3} \sum_{\vec{q}} \frac{f(|\vec{q}|)}{E^2 - (\omega_1 + \omega_2)^2}$$





- $E > m_1 + m_2$: $\tilde{G}$ has poles at *free energies in the box*, $E = \omega_1 + \omega_2$
- $E < m_1 + m_2$: $\tilde{G} \to G$ exponentially with L (regular summation theorem).

Finite $\to$ infinite volume: the Lüscher equation

Warning: Very crude re-derivation.

- Measured eigenvalues of the Hamiltonian (tower of *lattice levels* $E(L)$)
  $\to$ Poles of scattering equation $\tilde{T}$ in the finite volume $\to$ determines V:

$$\tilde{T} = (1 - V\tilde{G})^{-1} V \;\to\; \quad V^{-1} - \tilde{G} \overset{!}{=} 0 \;\to\; V^{-1} = \tilde{G}$$

- The interaction V determines the T-matrix in the infinite volume limit:

$$T = \left(V^{-1} - G\right)^{-1} = \left(\tilde{G} - G\right)^{-1}$$

- Re-derivation of Lüscher's equation (T determines the phase shift δ):

$$\boxed{p \cot \delta(p) = -8\pi\sqrt{s}\left(\tilde{G}(E) - \mathrm{Re}\, G(E)\right)}$$

  - V and dependence on renormalization have disappeared (!)
  - p: c.m. momentum
  - E: scattering energy
  - $\tilde{G} - \mathrm{Re}G$: known kinematical function
    ($\simeq \mathcal{Z}_{00}$ up to exponentially suppressed contributions)
  - One phase at one energy.

Energy interpolations to relate eigenvalues:

Unitarized CHPT in one- and two-channel scattering.

Energy interpolation through unitarized ChPT

[M.D., Meißner, JHEP (2012)] using IAM [Oller, Oset, Peláez, PRC (1999)]

Unitary extension of ChPT, can be matched to ChPT order-by-order:

$$T_\ell = V_\ell^{\mathrm{IAM}} + V_\ell^{\mathrm{IAM}}\, G\, T_\ell \qquad V_\ell^{\mathrm{IAM}} = \left(1 - V_\ell^{[4]}(V_\ell^{[2]})^{-1}\right)^{-1} V_\ell^{[2]}$$

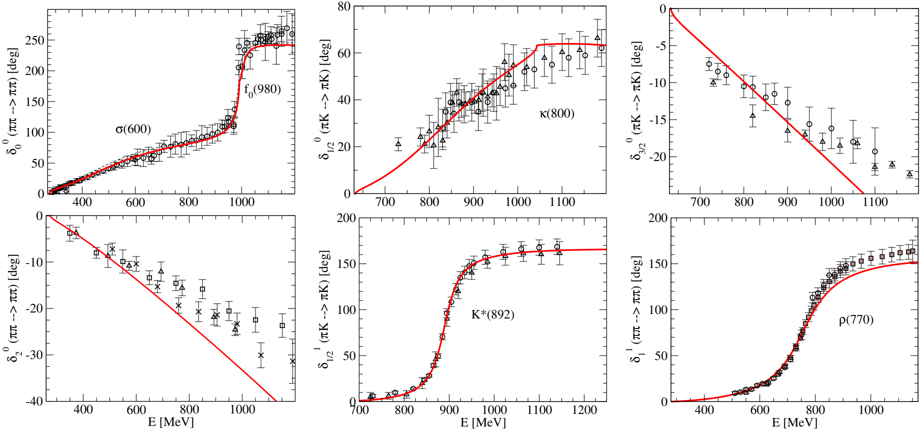
$$V_\ell^{[2]} \equiv V_\ell^{[2]}(p, M_\pi,\, M_K,\, M_{\eta_8},\, f), \qquad V_\ell^{[4]} \equiv V_\ell^{[4]}(p, M_\pi,\, M_K,\, M_{\eta_8},\, f,\, L_i)$$

Table : L_i [$\times 10^3$] fitted to meson-meson scattering data

L_1	L_2	L_3	L_4
$0.873^{+0.017}_{-0.028}$	$0.627^{+0.028}_{-0.014}$	-3.5 [fixed]	$-0.710^{+0.022}_{-0.026}$
L_5	$L_6 + L_8$	L_7	$q_{\max}$ [MeV]
$2.937^{+0.048}_{-0.094}$	$1.386^{+0.026}_{-0.050}$	$0.749^{+0.106}_{-0.074}$	981 [fixed]

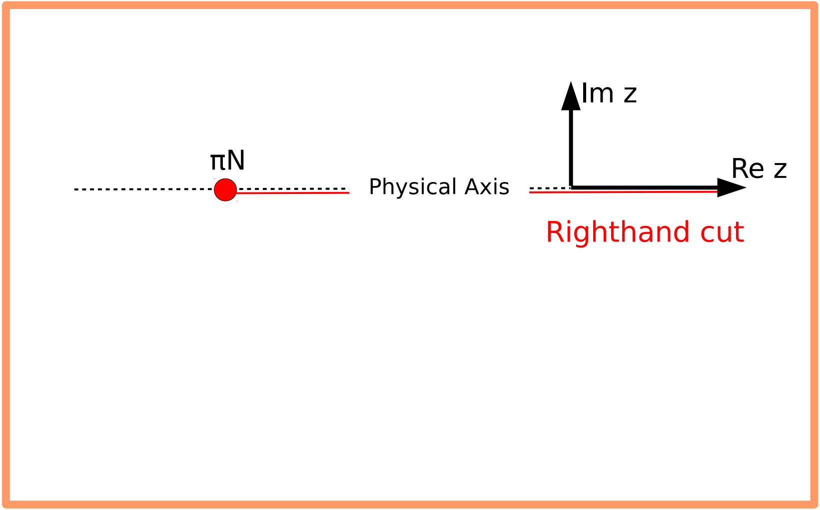
Fit to meson-meson PW data using unitary ChPT with NLO terms

[M.D., Meißner, JHEP (2012)] using IAM [Oller, Oset, Peláez, PRC (1999)]

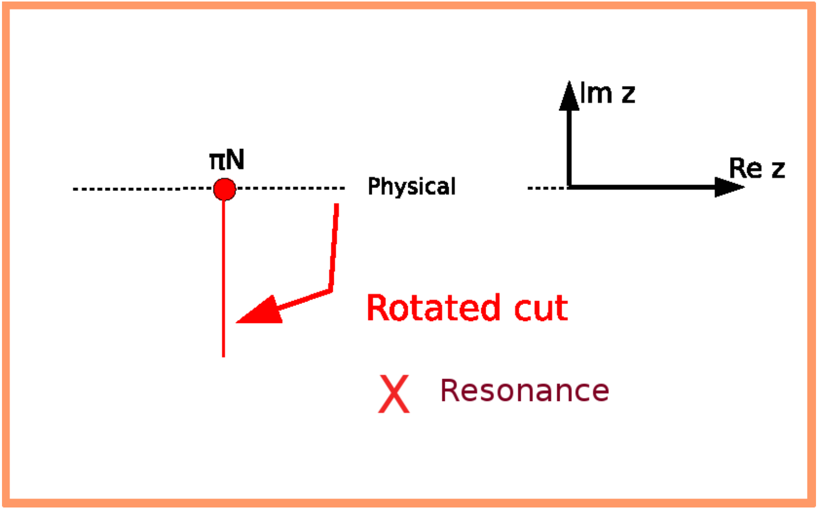


- A resonance is characterized by its (complex) pole position and residues, corresponding to resonance mass, width, and branching ratio.

$(z = E)$



$(z = E)$



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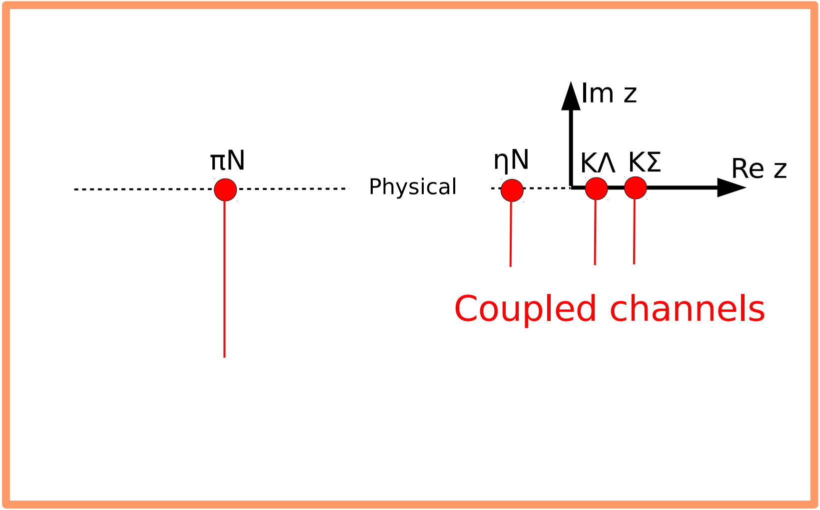
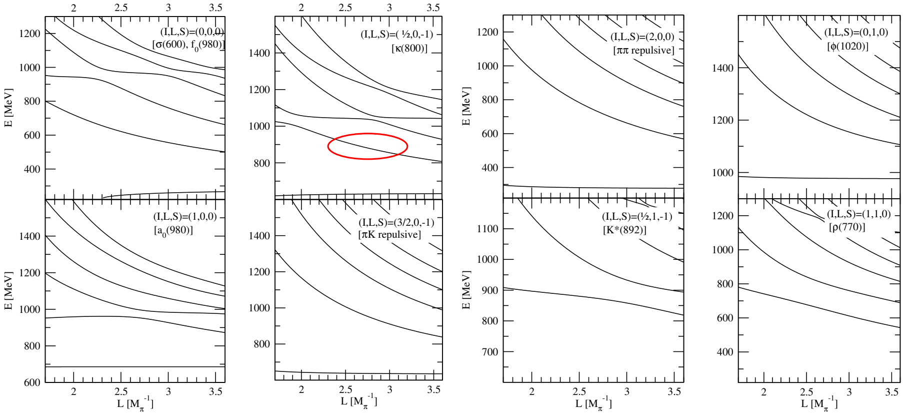


Resonance properties of scalar and vector resonances

I	L	S	Resonance	sheet	z_0 [MeV]	a_{-1} $[M_\pi]$	a_{-1} $[M_\pi]$
0	0	0	$\sigma(600)$	pu	$434+i\,261$	$-31-i\,19\,(\bar{K}K)$	$-30+i\,86\,(\pi\pi)$
0	0	0	$f_0(980)$	pu	$1003+i\,15$	$16-i\,79\,(\bar{K}K)$	$-12+i\,4\,(\pi\pi)$
1/2	0	-1	$\kappa(800)$	pu	$815+i\,226$	$-36+i\,39\,(\eta K)$	$-30+i\,57\,(\pi K)$
1	0	0	$a_0(980)$	pu	$1019-i\,4$	$-10-i\,107\,(\bar{K}K)$	$21-i\,31\,(\pi\eta)$
0	1	0	$\phi(1020)$	p	$976+i\,0$	$-2+i\,0\,(\bar{K}K)$	—
1/2	1	-1	$K^*(892)$	pu	$889+i\,25$	$-10+i\,0.1\,(\eta K)$	$14+i\,4\,(\pi K)$
1	1	0	$\rho(770)$	pu	$755+i\,95$	$-11+i\,2\,(\bar{K}K)$	$33+i\,17\,(\pi\pi)$

- Pole positions z_0 [MeV] ($\sim$ masses and widths)
- Residues $a_{-1}[M_\pi]$ ($\sim$ branching ratios)
- I, L, S: isospin, angular momentum, strangeness.

Prediction of levels (also for $M_\pi \neq M_\pi^{\text{phys.}}$)



[M.D., Meißner, JHEP (2012)]

Loops in t- and u-channel (1-loop calculation): [Albaladejo, Oller, Oset, Rios, Roca, JHEP (2013)]

Reconstruction of the $\kappa(800)$ stabilized by ChPT

Fit potential

$$V^{\text{fit}} = \left(\frac{V_2 - V_4^{\text{fit}}}{V_2^2}\right)^{-1}, \quad V_4^{\text{fit}} = a + b(s-s_0) + c(s-s_0)^2 + d(s-s_0)^3 + \cdots$$

[$V_2 \equiv V_{\text{LO}}$ from f_π, f_K, f_η, M_π, M_K, M_η; $s \equiv E^2$]

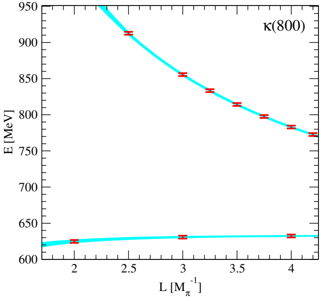


Figure : Pseudo lattice-data and $(s^0,\ s^1,\ s^2)$ fit to those data with uncertainties (bands).

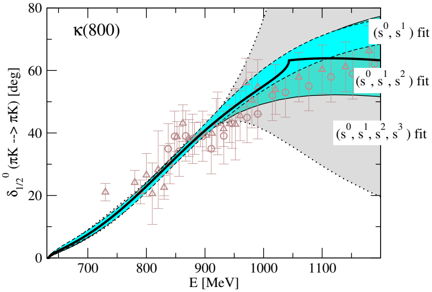
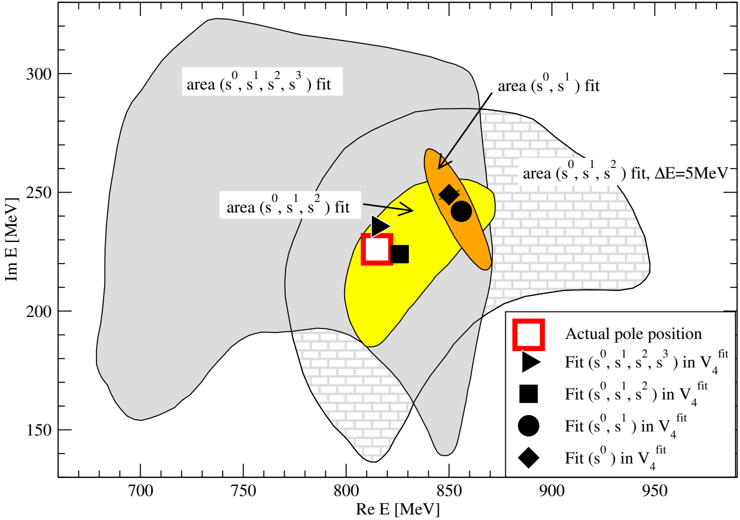


Figure : Solid line: Actual phase shift. Error bands of the $(s^0,\ s^1)$, $(s^0,\ s^1,\ s^2)$, and $(s^0,\ s^1,\ s^2,\ s^3)$ fits.

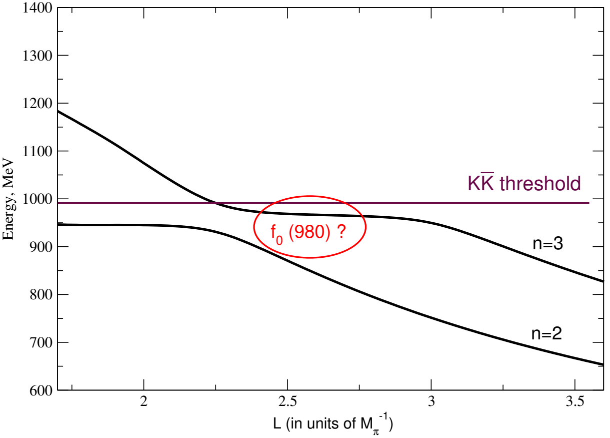
[see also: HadronSpectrum; Dudek, Wilson et al., 1406.4158]

The $\kappa(800)$ pole



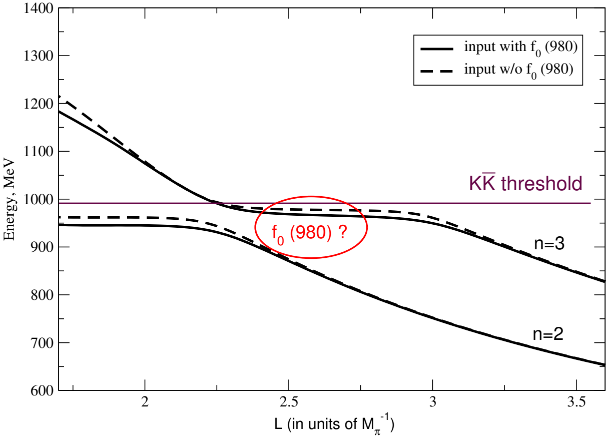
Convergence of results determines degrees of freedom (more formal: F-test).

Avoided level crossing in the energy levels



Using Unitarized Chiral Perturbation Theory to produce energy levels

[M. D., Meißner, Oset, Rusetsky, EPJA (2011)]

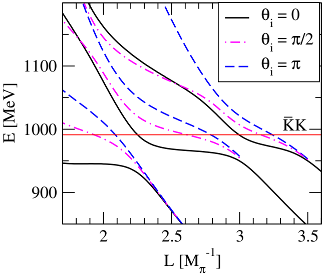


- Weaker coupling to the $K\bar{K}$ channel, $f_0(980)$ disappears
- Avoided level crossing still occurs at the same place: threshold!
- The threshold can be moved by using twisted b.c.!

Need for an interpolation in energy (coupled channels)

Twisting the boundary conditions [Bernard, Lage, Meißner, Rusetsky, JHEP (2011), M.D., Meißner, Oset, Rusetsky, EPJA (2011)]

- S-wave, coupled-channels $\pi\pi$, $\bar{K}K \to f_0(980)$.
- Three unknown transitions
  - $V(\pi\pi \to \pi\pi)$
  - $V(\pi\pi \to \bar{K}K)$
  - $V(\bar{K}K \to \bar{K}K)$



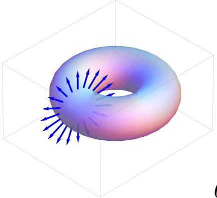
- Twisted B.C. for the s-quark:

$u(\vec{x} + \hat{\mathbf{e}}_i L) = u(\vec{x})$

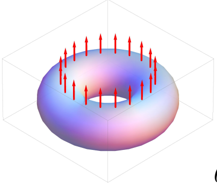
$d(\vec{x} + \hat{\mathbf{e}}_i L) = d(\vec{x})$

$s(\vec{x} + \hat{\mathbf{e}}_i L) = e^{i\theta_i} s(\vec{x})$

- Periodic B.C.: $\Psi(\vec{x} + \hat{\mathbf{e}}_i L) = \Psi(\vec{x})$
- Periodic in 2 dim.:

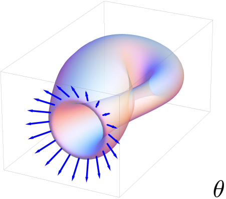


$\theta_1 = 0$

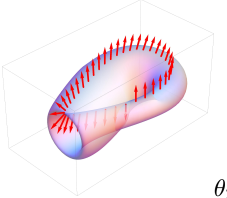


$\theta_2 = 0$

- Twisted B.C.: $\Psi(\vec{x} + \hat{\mathbf{e}}_i L) = e^{i\theta_i} \Psi(\vec{x})$
- Periodic/antiperiodic:



$\theta_1 = 0$



$\theta_2 = \pi$

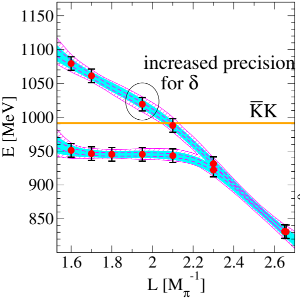
Coupled-channel systems with thresholds
[M.D., Meißner, Oset/Rusetsky, EPJA **47** (2011)]

- Need for an interpolation in energy ($\to$ Unitarized ChPT,...)
- Expand a two-channel transition V in energy (i, j: $\pi\pi$, $\bar{K}K$):

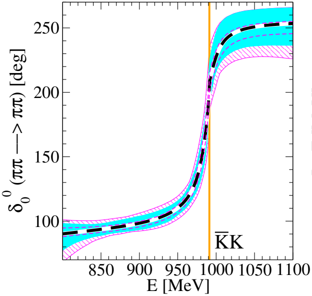
$$V_{ij}(E) = a_{ij} + b_{ij}(E^2 - 4M_K^2)$$

- Include model-independently known LO contribution in a, b.
- Or even NLO contributions (7 LECs: more fit parameters).

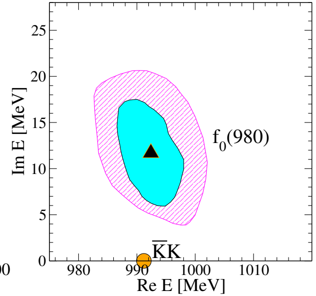
lattice data & fit



extracted phase shift



$f_0(980)$ pole position



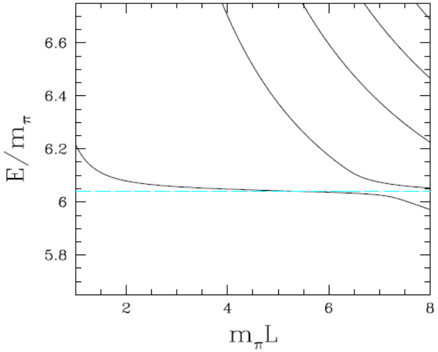
Moving frames and coupled channels:

Scanning resonance lineshapes.

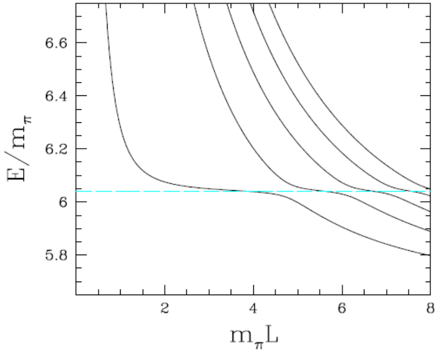
Moving frames to get more levels ($\Sigma^* \to \pi\Lambda$)

- Resonance scan needed because Lüscher method is modulus π.

[picture: courtesy of G. Schierholz]



$\boldsymbol{P} = 0$

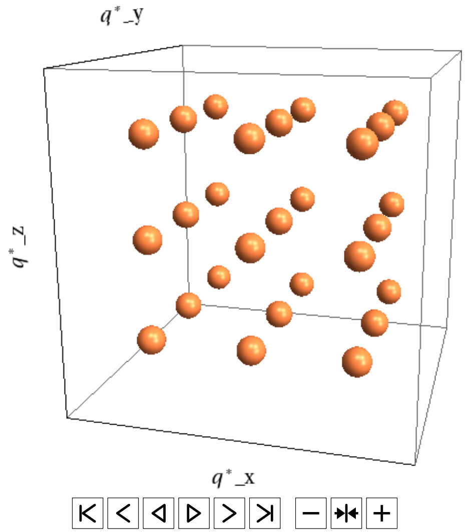


$\boldsymbol{P} = \frac{2\pi}{L}(0,0,1)$

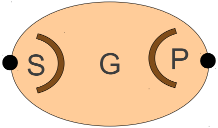
- Operators with non-zero momentum of the center-of-mass:
  $\vec{P} = \vec{p}_1 + \vec{p}_2 \neq 0$ Rummukainen, Gottlieb, NPB (1995)

Breaking of cubic symmetry through boost

Example: Lattice points $\vec{q}^*$ boosted with $P = (0,0,0) \to \frac{2\pi}{L}(0,0,2)$:



- Infinite volume limit: Rotational symmetry



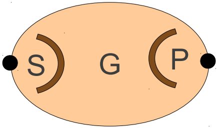
$$\int \frac{d^3\vec{q}}{(2\pi)^3}\, g(|\vec{q}|)\; Y_{\ell m}(\theta,\phi)\, Y^*_{\ell' m'}(\theta,\phi) \sim \delta_{\ell\ell'}\delta_{mm'}\,.$$

S→S	0	0	0
0	$P_{-1} \to P_{-1}$	0	0
0	0	$P_0 \to P_0$	0
0	0	0	$P_1 \to P_1$

Mixing of partial waves

Example: S- and P-waves

- Infinite volume limit: Rotational symmetry



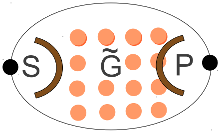
$$\int \frac{d^3\vec{q}}{(2\pi)^3}\, g(|\vec{q}|)\; Y_{\ell m}(\theta,\phi)\, Y^*_{\ell' m'}(\theta,\phi) \sim \delta_{\ell\ell'}\delta_{mm'}\,.$$

- Wigner-Eckart theorem:

S→S	0	0	0
0	P_{-1} → [illegible]	0	0
0	0	[illegible]	0
0	0	0	[illegible] → P_1

Equal

- Finite volume: Rotational symmetry $\to$ Cubic symmetry



$$\frac{1}{L^3}\sum_{\vec{n}} g(|\vec{q}|)\, Y_{\ell m}(\theta,\phi)\, Y^*_{\ell' m'}(\theta,\phi) \sim A_{\ell\ell' m m'}\,.$$

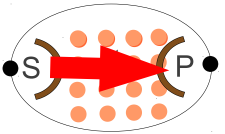
- $S-G$-wave mixing, but $S-P$ waves still orthogonal:

S→S	0	0	0
0	$P_{-1} \to P_{-1}$	0	0
0	0	$P_0 \to P_0$	0
0	0	0	$P_1 \to P_1$

Mixing of partial waves

Example: S- and P-waves

- Finite volume & boost: Cubic symmetry $\rightarrow$ subgroups of cubic symmetry



$$\frac{1}{L^3}\sum_{\vec{n}} g(|\vec{q}|)\, Y_{\ell m}(\theta,\phi)\, Y^*_{\ell' m'}(\theta,\phi) \sim A_{\ell\ell' m m'}.$$

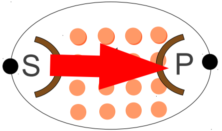
- For boost $P = \frac{2\pi}{L}$ (0,1,1):

S$\rightarrow$S	0	S$\rightarrow$P$_0$	0
0	P$_{-1}$ $\rightarrow$ P$_{-1}$	0	P$_{-1}$ $\rightarrow$ P$_1$
P$_0$$\rightarrow$S	0	P$_0$ $\rightarrow$ P$_0$	0
0	P$_1$ $\rightarrow$ P$_{-1}$	0	P$_1$ $\rightarrow$ P$_1$

Mixing of partial waves

Example: S- and P-waves

- Finite volume & boost: Cubic symmetry $\rightarrow$ subgroups of cubic symmetry



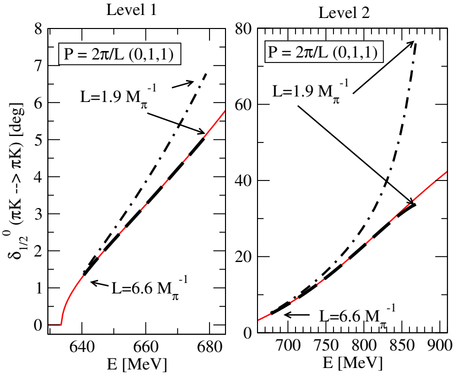
$$\frac{1}{L^3}\sum_{\vec{n}} g(|\vec{q}|)\, Y_{\ell m}(\theta,\phi)\, Y^*_{\ell' m'}(\theta,\phi) \sim A_{\ell\ell' m m'}.$$

- More complicated boosts:

S→S	S→P$_{-1}$	S→P$_0$	S→P$_1$
P$_{-1}$ → S	P$_{-1}$ → P$_{-1}$	P$_{-1}$ → P$_0$	P$_{-1}$ → P$_1$
P$_0$→S	P$_0$ → P$_{-1}$	P$_0$ → P$_0$	P$_0$ → P$_1$
P$_1$ → S	P$_1$ → P$_{-1}$	P$_1$ → P$_0$	P$_1$ → P$_1$

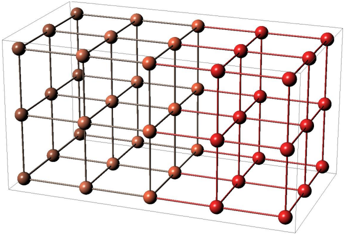
Knowledge of P-wave (from separate analysis of lattice data) allows to disentangle the S-wave:

$$p\,\cot\delta_S = -8\pi\,E\,\frac{p\,\cot\delta_P\,\hat{G}_{SS} - 8\pi E(\hat{G}_{SP}^2 - \hat{G}_{SS}\hat{G}_{PP})}{p\,\cot\delta_P + 8\pi\,E\,\hat{G}_{PP}}$$

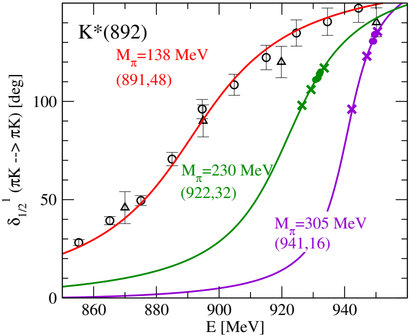


- $\delta_S \equiv \delta^0_{1/2}(\pi K \to \pi K)$
- Red solid: Actual S-wave phase shift.
- Dash-dotted: Reconstructed S-wave phase shift, PW-mixing ignored.
- Dashed: Reconstructed S-wave phase shift, PW-mixing disentangled.
- small p-wave: Level shift $\Delta E \simeq -\dfrac{6\pi E_S \delta_P}{L^3 p\omega_1\omega_2}$

See also: Rummukainen, Gottlieb, NPB (1995); Kim, Sachrajda, Sharpe, NPB (2005); Davoudi, Savage, PRD (2011), Z. Fu, PRD (2012); Leskovec, Prelovsek, PRD (2012); Dudek, Edwards, Thomas, PRD (2012); Hansen, Sharpe, PRD 86 (2012); Briceño, Davoudi, PRD 88 (2013); Göckeler, Horsley, Lage, Meißner, Rakow, Rusetsky, Schierholz, Zanotti, PRD (2012)



- $L_x = x\,L, L_y = L, L_z = L$
  $x = 1,\ 1.26,\ 2.04$
- $\frac{L}{2\pi}\,\vec{P} = (1,1,0), x = 1:$
  irreps A_1, B_1, B_2
- $\frac{L}{2\pi}\,\vec{P} = (1,1,0), x \neq 1:$
  irreps A_1, B_1 (!) pure p-wave level survives!



filled circles: $\frac{L}{2\pi}\,\vec{P} = (1,0,0)$

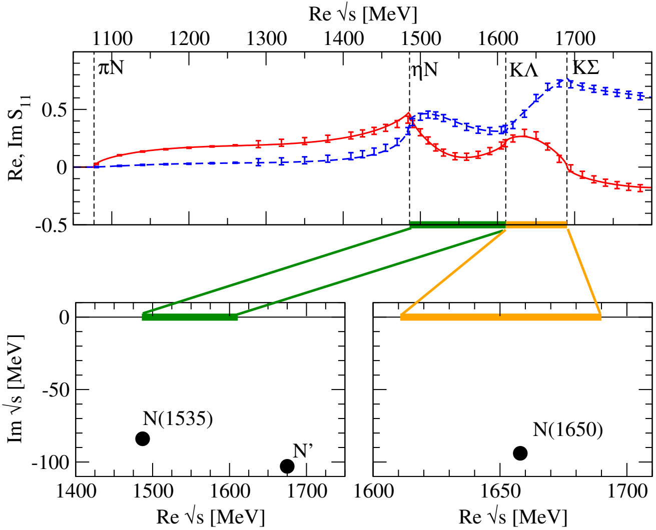
crosses: $\frac{L}{2\pi}\,\vec{P} = (1,1,0)$

$\rightarrow$ Find even more asymmetric geometries.

Chiral extrapolations and coupled channels:

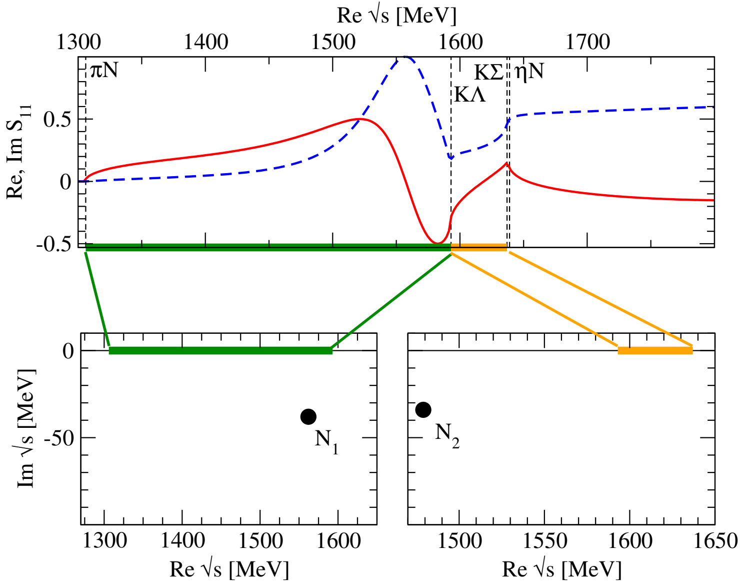
Baryons.

- Unitarized chiral interaction with NLO contact terms in BSE

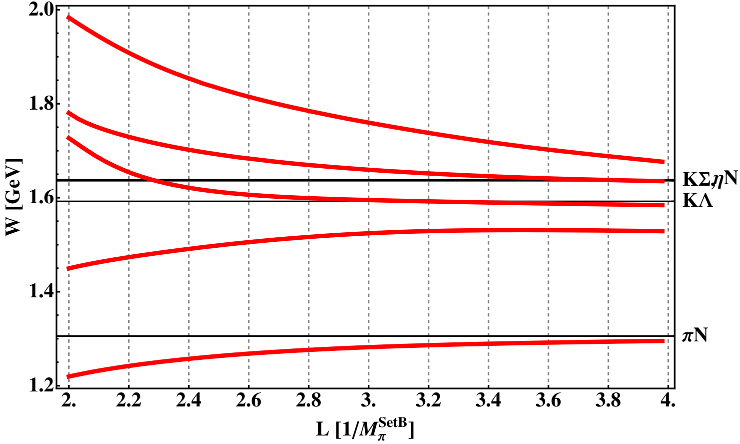


Data: SAID (2006); Extension of SAID analysis framework itself to finite volume in progress.

Chiral extrapolation to a QCDSF lattice setup



Prediction of the lattice spectrum



- No one-to-one mapping of levels to resonances → coupled channel analysis needed; hidden poles appear.

Nπ (1/2⁻) channel

C.B. Lang and V. Verduci, Phys.Rev. D 87, 054502 (2013)

m_π=266 MeV; distillation method; variational analysis using a basis of N (3 quarks) and Nπ (5 quarks) interpolators;

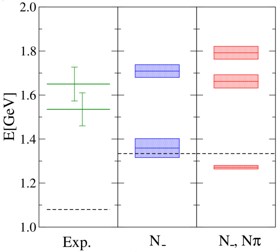
$$(N_\pm^{(i)})_\mu(\vec{p}=0) = \sum_{\vec{x}} \epsilon_{abc} \left(P_\pm \Gamma_1^{(i)} u_a(\vec{x})\right)_\mu \left(u_b^T(\vec{x})\, \Gamma_2^{(i)}\, d_c(\vec{x})\right)$$

$$\pi^+(\vec{p}=0) = \sum_{\vec{x}} \overline{d}_a(\vec{x})\gamma_5 u_a(\vec{x}) \, ,$$

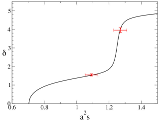
$$\pi^0(\vec{p}=0) = \sum_{\vec{x}} \frac{1}{\sqrt{2}} \left(\overline{u}_a(\vec{x})\gamma_5 u_a(\vec{x}) - \overline{d}_a(\vec{x})\gamma_5 d_a(\vec{x})\right)$$

$$O_{N\pi}(I=\frac{1}{2}, I_3=\frac{1}{2}) = p\pi^0 + \sqrt{2}\, n\pi^+$$

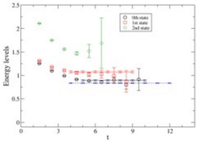
$$N\pi(\vec{p}=0) = \gamma_5 N_+(\vec{p}=0)\pi(\vec{p}=0)$$



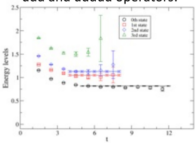
Lüscher relation
➛ phase shift:



only udd operators:



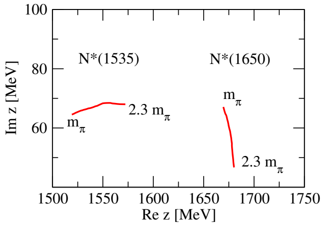
udd and uddud operators:

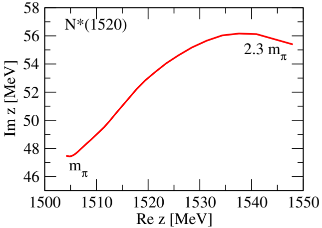


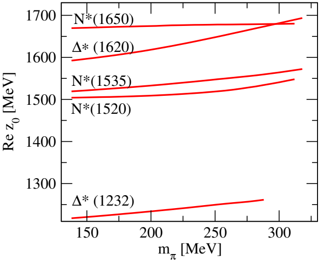
Assuming 2 elastic resonances with identical coupling we get m_1=1.678 GeV
m_2=1.873 GeV

Chiral extrapolation from dynamical coupled channels approaches
based on Jülich solution NPA 829, 170 (2009)

- Larger theoretical uncertainties.
- Quark mass dependence of effective $2\pi N$ channels is intricate.
- Finite-volume effects of 3-body channels unexplored.
- Accessing the Roper puzzle.





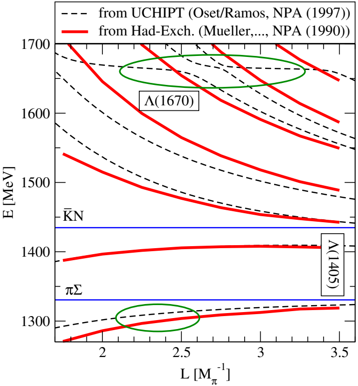


First prediction of the $\Lambda(1405)$ in the finite volume

[M. D./Haidenbauer/Meißner/Rusetsky, EPJA **47** (2011)]

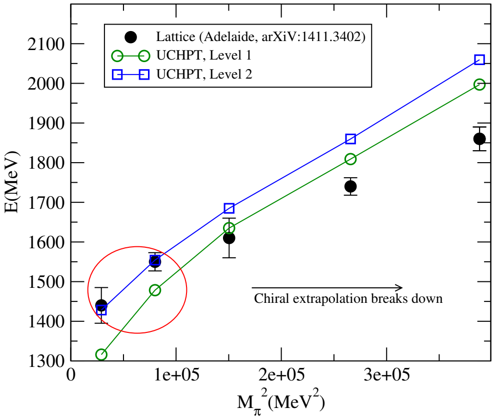
- (Non-factorizing/off-shell) Lippman-Schwinger equation in the finite volume,

$$T^{(\mathrm{P})}(q'',q') = V(q'',q') + \frac{2\pi^2}{L^3}\sum_{i=0}^{\infty} \vartheta^{(\mathrm{P})}(i)\,\frac{V(q'',q_i)\,T^{(\mathrm{P})}(q_i,q')}{\sqrt{s}-E_a(q_i)-E_b(q_i)}, \quad q_i = \frac{2\pi}{L}\sqrt{i}\;.$$



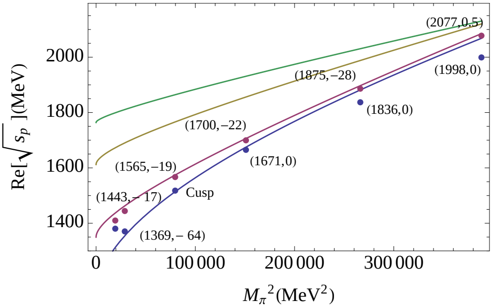
- Access to sub-$\bar{K}N$-threshold dynamics:
- Discrepancies of lowest levels: levels sensitive to different $\Lambda(1405)$ dynamics.
- One- or two-pole structure:
  - Will NOT lead to additional level.
  - but shifted threshold levels.

See also Martinez, Bayar, Jido, Oset, PRC 86 (2012).



Finite volume spectrum

- Coupled channels $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, $K\Xi$
- Unitarized LO χ potential V in $T = V + VGT$
- No freedom at this order $\rightarrow$ full prediction.
- UCHPT has two poles for the $\Lambda(1405)$.
- $\rightarrow$ new data not in conflict with two-pole structure of a molecular $\Lambda(1405)$ (but not yet a proof thereof).
- $\rightarrow$ needed: Statistical NLO analysis in combination with more accurate data.

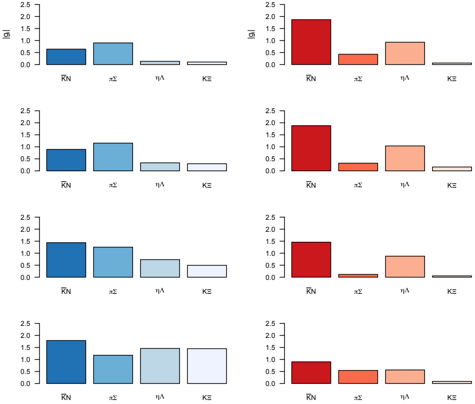


Two-pole structure in the infinite volume

- Coupled channels $\pi\Sigma,\ \bar{K}N,\ \eta\Lambda,\ K\Xi$
- Unitarized LO χ potential V in $T = V + VGT$
- No freedom at this order $\rightarrow$ full prediction.
- UCHPT has two poles for the $\Lambda(1405)$.
- $\rightarrow$ new data not in conflict with two-pole structure of a molecular $\Lambda(1405)$ (but not yet a proof thereof).
- $\rightarrow$ needed: Statistical NLO analysis in combination with more accurate data.

The $\Lambda(1405)$: Predictions from Unitarized CHPT

Raquel Molina, M.D., preliminary



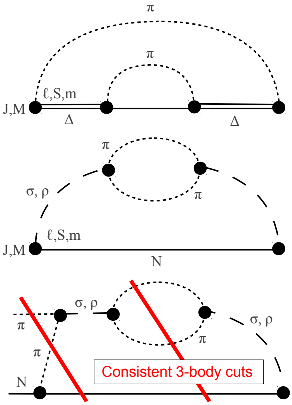
Residues at eigenvalues (Overlap)

- Coupled channels $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$
- Unitarized LO χ potential V in $T = V + VGT$
- No freedom at this order $\rightarrow$ full prediction.
- UCHPT has two poles for the $\Lambda(1405)$.
- $\rightarrow$ new data not in conflict with two-pole structure of a molecular $\Lambda(1405)$ (but not yet a proof thereof).
- $\rightarrow$ needed: Statistical NLO analysis in combination with more accurate data.

Three particles:

New finite volume methods.

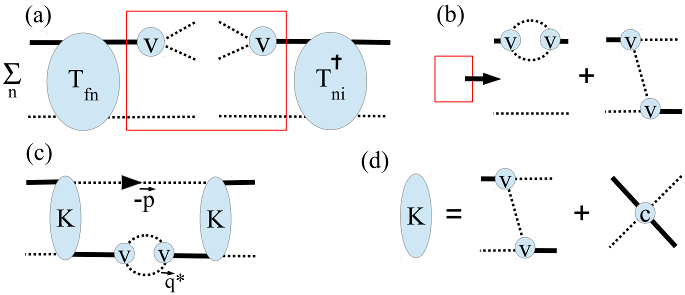
See also: Hansen/Sharpe (2014), Grießhammer/Kreuzer (2013), Roca/Oset (2013),...



- πN scattering: Known large inelasticities $\pi\pi N$ [$\pi\Delta$, σN, ρN,...]
- $\pi\pi/\pi N$ boosted subsystems.
- Is it enough to include (boosted) 2-particle subsystems in the propagator? **No.**
- Three-body s-channel dynamics requires particle exchange transitions. $\Rightarrow$ **Three-body unitarity**

[Aaron, Almado, Young, PR 174 (1968) 2022,
Aitchison, Brehm, PLB 84 (1979) 349, PRD 25 (1982) 3069,...]

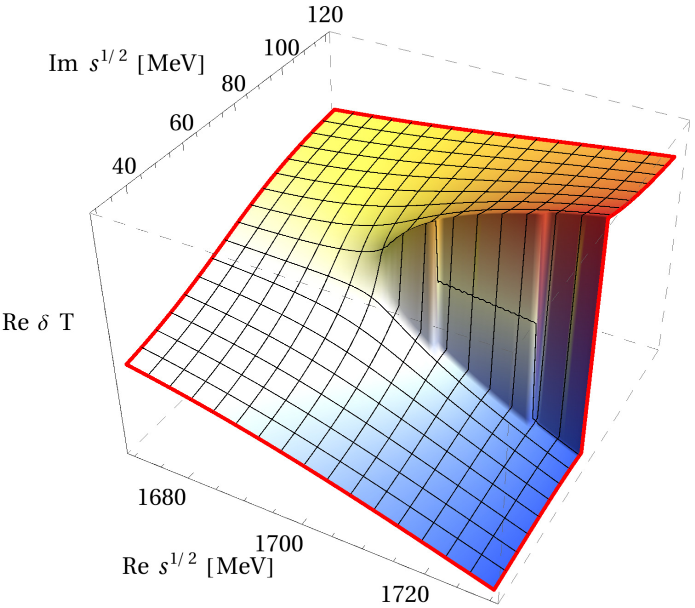
$$T_{fi} - T^{\dagger}_{fi} = i \sum_n d\Omega_n \, T_{fn} \, T^{\dagger}_{ni}$$



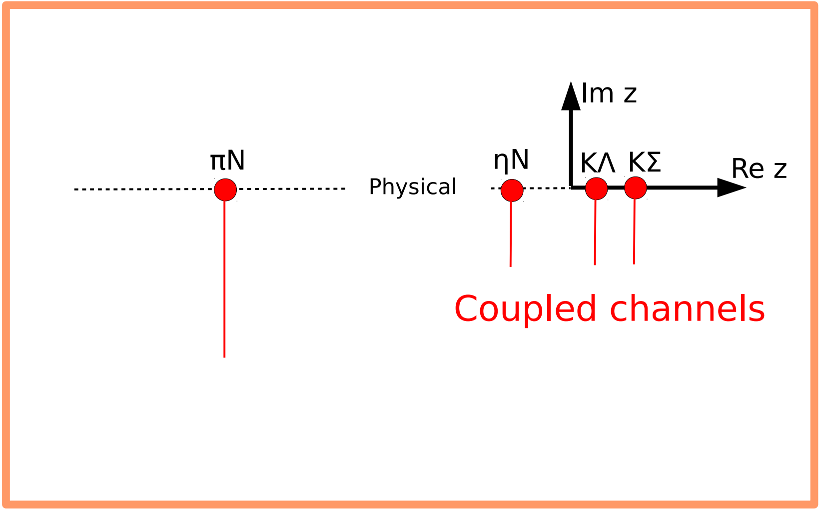
- (a, b) Unitarity relation and particle exchange.
- (c) Amplitude at one loop.
- (d) Contact interactions: unitarity preserved.

Consequence: Threshold openings in the complex plane

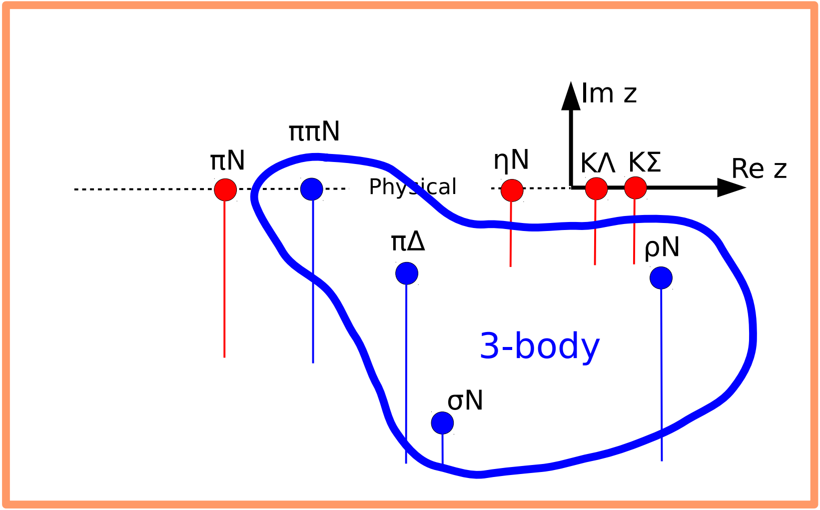
Existence shown model-independently in [S. Ceci, M.D., C. Hanhart *et. al.*, PRC 84 (2011)]



$(z = E)$

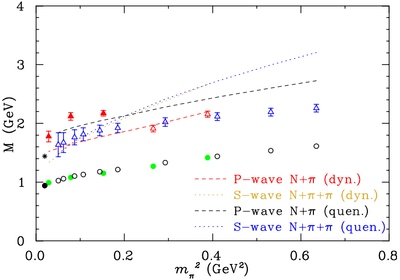


$(z = E)$



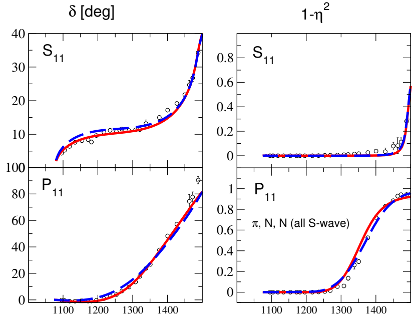
Applications to: Roper $N(1440)1/2^+$, $a_1(1260)$, $X(3872)$,...

The Roper puzzle



[Mahbub, Kamleh et al., PLB 693]

- open: quenched
- filled: full dynamical



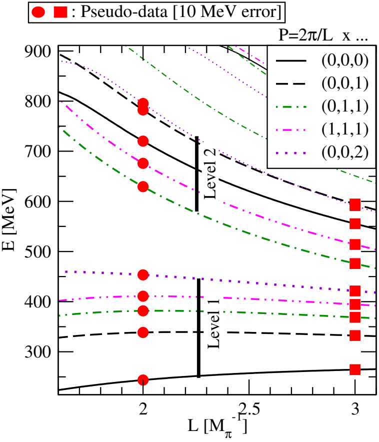
[Data: SAID 2006; Fit: Jülich 2012]

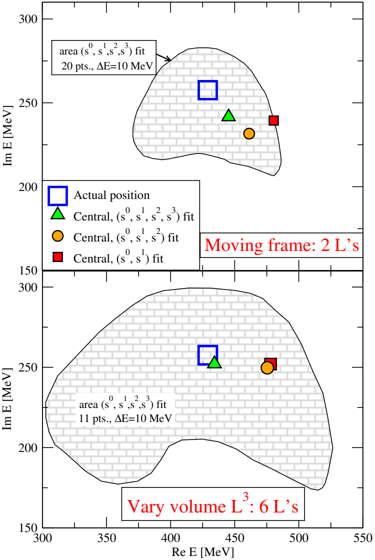
- Earliest onset of inelasticity into $\pi\pi N$ (all particles in S-wave)
- Entanglement of Roper pole and complex $\pi\Delta$ branch point
- Strong pion mass dependence expected
- Unknown, large 3-body finite-volume effects

Perspectives

- Rapid progress in the actual ab-initio calculations of resonances/phase shifts: $\rho(770), a_0(980), K^*$(s, p, d)$, [N(1535), \Lambda(1405)], \ldots$.
- Structure of $\Lambda(1405)$ from first principles comes into reach.
- Close to the physical point, finite volume effects dominate the spectrum.
- Use finite volume effects in your favor: Lüscher & extensions (coupled channels, moving frames, twisted boundary conditions,...)
- Energy interpolation needed in many aspects —Unitarized ChPT & coupled-channel approaches can provide a framework.
  - Prediction of levels & Chiral extrapolation
    $\rightarrow$ find suitable lattice setups to cover resonance region with eigenstates.
  - Provide lattice setups that are maximally sensitive to resonances.
  - Analysis of lattice data.
- Three-particle interaction bears new conceptual challenges and opportunities. [funded through NSF Career grant 2015-2020]

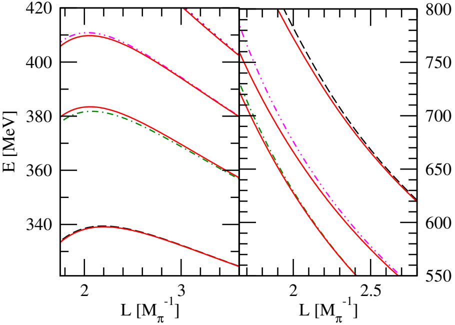
- The first two levels for the first five boosts:





Mixing of partial waves in boosted multiple channels: $\sigma(600)$

[M.D., E. Oset, A. Rusetsky, EPJA (2012)]



Solid: Levels from A_1^+.
Non-solid: Neglecting the D-wave.

- $\pi\pi$ & $\bar{K}K$ in S-wave, $\pi\pi$ in D-wave.
- Organization in Matrices (A_1^+), e.g. $\vec{P} = (2\pi/L)(0,0,1)$, $(2\pi/L)(1,1,1)$, and $(2\pi/L)(0,0,2)$:

$$V = \begin{pmatrix} V_S^{(11)} & V_S^{(12)} & 0 \\ V_S^{(21)} & V_S^{(22)} & 0 \\ 0 & 0 & V_D^{(22)} \end{pmatrix}$$

$$\tilde{G} = \begin{pmatrix} \tilde{G}_{00,00}^{R\,(1)} & 0 & 0 \\ 0 & \tilde{G}_{00,00}^{R\,(2)} & \tilde{G}_{00,20}^{R\,(2)} \\ 0 & \tilde{G}_{20,00}^{R\,(2)} & \tilde{G}_{20,20}^{R\,(2)} \end{pmatrix}$$

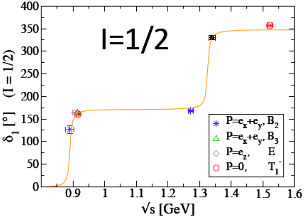
- Phase extraction: Expand and fit V_S, V_D *simultaneously* to different representations, as in case of multi-channels (reduction of error).

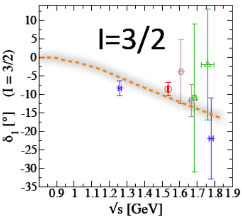
Kπ scattering and K* width in moving frames

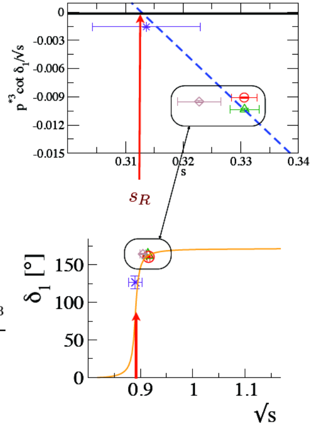
Prelovsek, Leskovec, Lang, Mohler,
this conf. and *arXiv: 1307.0736*

p-wave, coupled system of 5 $q\bar{q}$ and 3 Kπ operators, total momentum P=(000),(001),(011)

Representations B_2, B_3, E, T_1^-



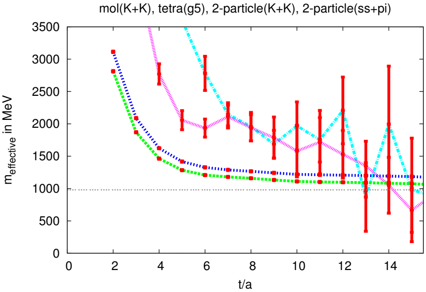


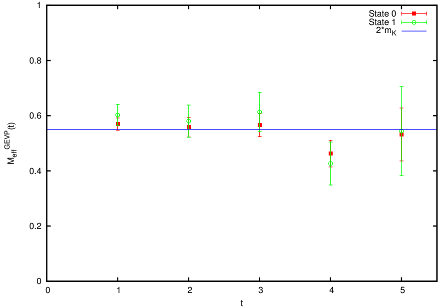


$$\frac{p^{*3}}{\sqrt{s}}\cot\delta_1(s) = \left[\sum_{K^*}\frac{g^2_{K^*_i}}{6\pi}\frac{1}{m^2_{K^*_i}-s}\right]^{-1} \qquad \Gamma[K^*\to K\pi] = \frac{g^2}{6\pi}\frac{p^{*3}}{s}$$

	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]	$m_{K^*(1410)}$ [GeV]	$g_{K^*(1410)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6	1.33 ± 0.02	input
exp	891.66 ± 0.26	5.72 ± 0.06	1.414 ± 0.0015	1.59 ± 0.03

[Wagner, Daldrop, Abdel-Rehim, Urbach *et. al.* [ETMC], JHEP (2013) & new results]



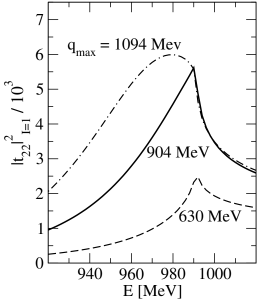


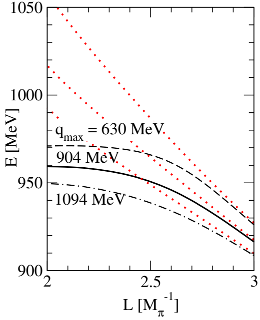
- $M_\pi \sim 300$ MeV, no singly disconnected diagrams.
- Operators: $\bar{K}K$ molecular, diquark-antidiquark, meson-meson.
- Two low-lying states, large overlap with meson-meson.

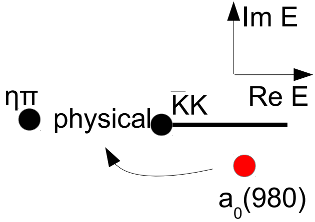
- $M_\pi \sim 300$ MeV, singly disconnected diagrams included.
- Operators: $q\bar{q}$, $\bar{K}K$ molecular.
- Again, two low lying states, no information on additional state.

The $a_0(980)$ in a multi-channel environment

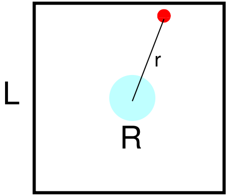
M.D., Meißner, Oset, Rusetsky, EPJA (2011); see also Lage, Meißner, Rusetsky, PLB (2009)



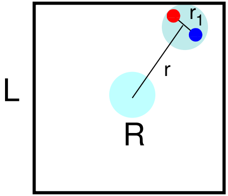




Three particles in a finite volume



2 particles



3 particles

- In case of 2 particles: $r \gg R$, when particles are near the walls
- In case of 3 particles: it may happen that $r \gg R, \quad r_1 \simeq R$, when the particles are near the walls

The problem with the disconnected contributions: is the finite-volume spectrum in the 3-particle case determined solely through the on-shell scattering matrix?

- Despite the presence of the disconnected contributions, the energy spectrum of the 3-particle system in a finite box is still determined by the on-shell scattering matrix elements in the infinite volume

[Polejaeva, Rusetski, EPJA (2012)]