Coupled channel hadronic interactions from Lattice QCD

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for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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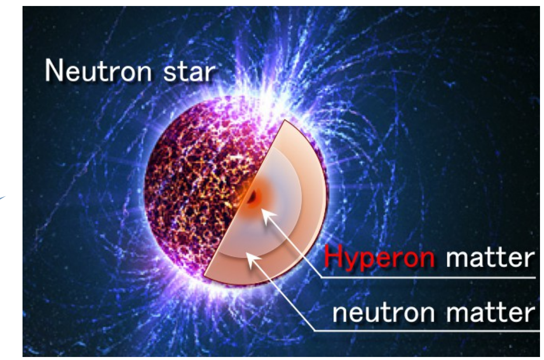
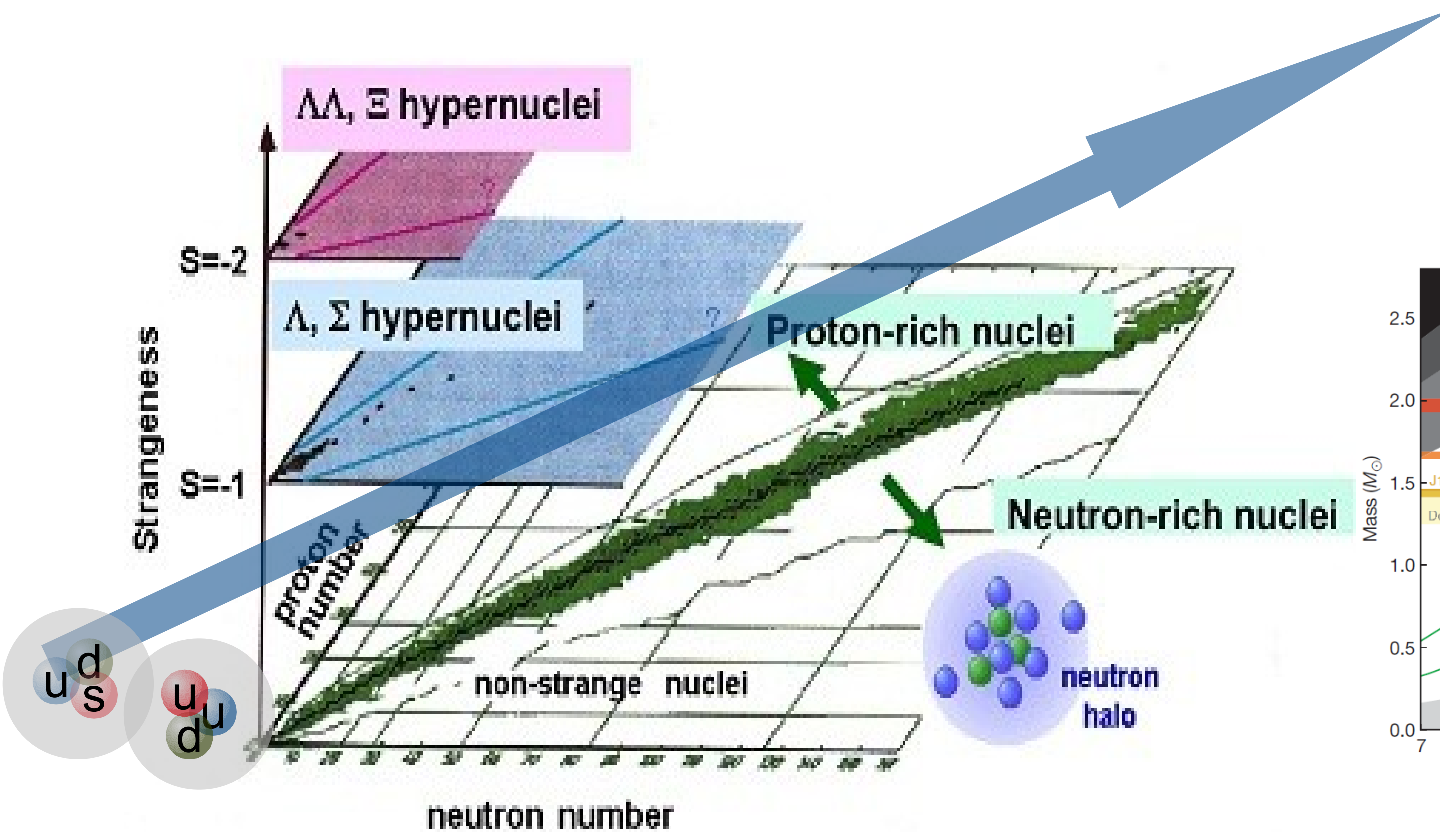
Introduction

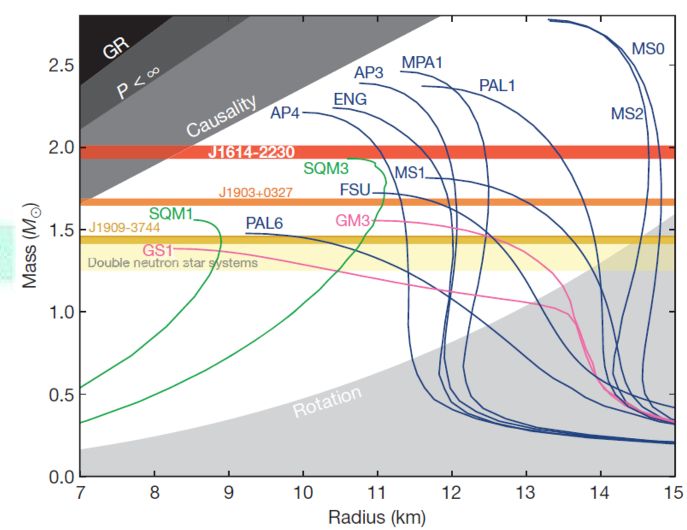
Introduction

<u>BB interactions are inputs for nuclear structure, astrophysical phenomena</u>

Once we obtain a “proper” nuclear potential,
 we apply them to the structure of (hyper-) nucleus
 and neutron star calcration.





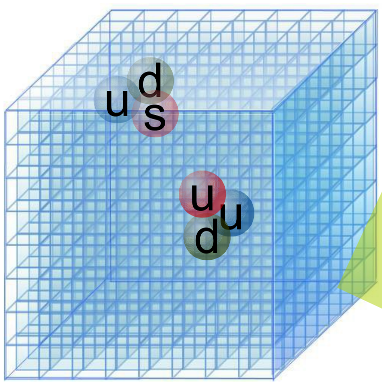


Can we derive hadronic interactions from QCD?

Derivation of hadronic interaction from QCD

Start with the fundamental theory,QCD, to obtain a “proper” interaction

Lattice QCD simulation

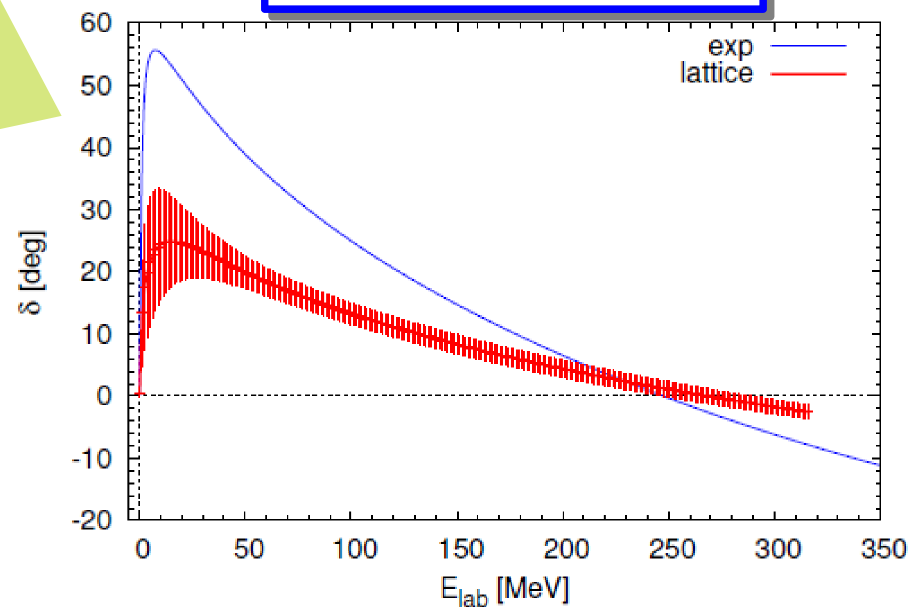


Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

1. Measure the discrete energy spectrum, **E**
2. Put the E into the formula which connects E and **δ**

Scattering phase shift



HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

1. Measure the NBS wave function, **Ψ**
2. Calculate potential, **V**, through Schrödinger eq.
3. Calculate observables by scattering theory

Lüscher's method

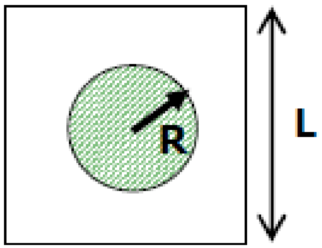
Lüscher's finite volume formula

Lattice calculation is performed in a finite volume.
→ we have discrete energy spectrum even in the scattering state.

Lüscher's formula describes a fixed relation
between the scattering phase shift in the continuum
and an energy level in a finite volume L^3

- Assumptions
  - Two particles scatter from a finite range interaction.
  - The Helmholtz wave function is in the asymptotic region: R<r<L/2, i.e., two particles are well separated

$$\frac{1}{\tan\delta_0(k)} = \frac{4\pi}{k}\cdot\frac{1}{L^3}\sum_p \frac{1}{p^2-k^2}$$



S-matrix has only one parameter, δ,
and it is related to an energy level in a one-to-one correspondence.

Lüscher's finite volume formula

Extension of Lüscher's method to the multi-channel situation

Parametrization of 2x2 S-matrix

$$S(E)=\begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

Two-channel S-matrix has 3-parameters

$$\delta_1(E),\quad \delta_2(E),\quad \eta(E)$$

These are related to the energy E by an eigenvalue equation (s-wave)

$$\cos\left(\Delta_1+\Delta_2-\delta_1^0-\delta_2^0\right)=\eta\cos\left(\Delta_1-\Delta_2-\delta_1^0+\delta_2^0\right)$$

$$\frac{1}{\tan\Delta_i}=\frac{4\pi}{k_i}\cdot\frac{1}{L^3}\sum_p\frac{1}{p^2-k_i^2}$$

Unlike the single channel case,

the number of equations is less than the number of parameters in S-matrix.

Several prescriptions are proposed.

S. He, et al., JHEP0507 (2005) 011

V. Bernard et al., JHEP1101 (2011) 019

Jia-Jun Wu et al., PRC90 (2014) 055206

M. Lage et al., PLB681 (2009) 439.

M. Doring et al., EPJA 48 (2012) 114

HAL QCD method

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0|H_1^{\alpha}(t,\vec{x}+\vec{r})H_2^{\alpha}(t,\vec{x})|E\rangle$$

E : Total energy of the system

Local composite interpolating operators

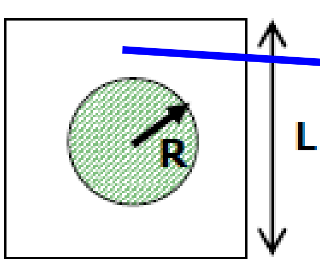
$$B_{\alpha}=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_{c\alpha} \qquad D_{\mu\alpha}=\epsilon^{abc}(q_a^T C\gamma_{\mu} q_b)q_{c\alpha}$$

$$M=(\bar{q}_a\gamma_5 q_a)$$ Etc.....

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- Using the reduction formula,

$$\Psi^{\alpha}(E,\vec{r}) = \sqrt{Z_{H_1}}\sqrt{Z_{H_2}}\left(e^{i\vec{p}\cdot\vec{r}}+\int\frac{d^3q}{2E_q}\frac{T(q,p)}{4E_p(E_q-E_p-i\epsilon)}e^{i\vec{q}\cdot\vec{r}}\right)$$

C.-J.D.Lin et al.,NPB619 (2001) 467.



$$\Psi(E,\vec{r})\simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Phase shift is defined as

$$S\equiv e^{i\delta}$$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

Potential in HAL QCD method

We define potentials which satisfy Schrödinger equation

$$\left(p^2+\nabla^2\right)\Psi^{\alpha}(E,\vec{r}) \equiv \int d^3y\, \underline{U^{\alpha}_{\alpha}(\vec{x},\vec{y})}\, \Psi^{\alpha}(E,\vec{y})$$

Energy independent potential

$$\left(p^2+\nabla^2\right)\Psi^{\alpha}(E,\vec{r}) = K^{\alpha}(E,\vec{r})$$

$$\begin{aligned} K^{\alpha}(E,\vec{r}) &\equiv \int dE'\, K^{\alpha}(E',\vec{x}) \underline{\int d^3y\, \tilde{\Psi}^{\alpha}(E',\vec{y})\Psi^{\alpha}(E,\vec{y})} \\ &= \int d^3y \left[\underline{\int dE'\, K^{\alpha}(E',\vec{x})\,\tilde{\Psi}^{\alpha}(E',\vec{y})}\right]\Psi^{\alpha}(E,\vec{y}) \\ &= \int d^3y\, U^{\alpha}_{\alpha}(\vec{x},\vec{y})\,\Psi^{\alpha}(E,\vec{y}) \end{aligned}$$

We can define an energy independent potential but it is fully non-local.

This potential automatically reproduce the scattering phase shift

N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent method

Let's start with the normalized four-point correlator.

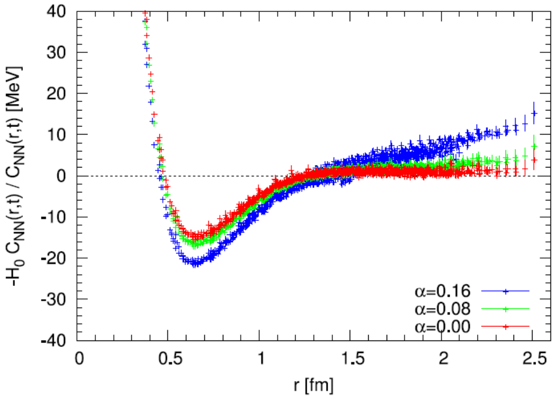
$$R_I^{B_1 B_2}(t,\vec{r}) = F_{B_1 B_2}(t,\vec{r})\, e^{(m_1+m_2)t}$$

$$= A_0 \Psi(\vec{r}, E_0)\, e^{-(E_0 - m_1 - m_2)t} + A_1 \Psi(\vec{r}, E_1)\, e^{-(E_1 - m_1 - m_2)t} + \cdots$$

Each wave functions satisfy Schrödinger eq. with proper energy

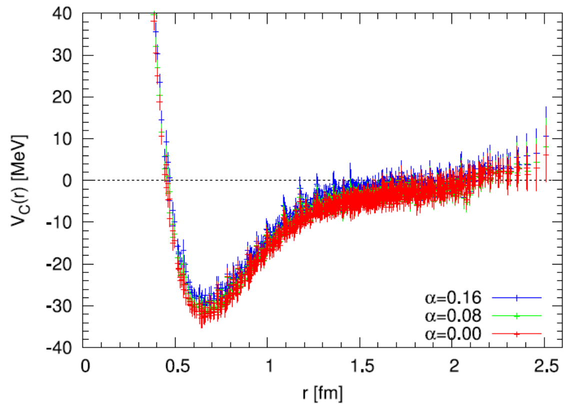
$$\left(\frac{p_0^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi(\vec{r}, E_0) = \int U(\vec{r}, \vec{r}\,')\Psi(\vec{r}\,', E_0)\, d^3 r'$$

$$\left(\frac{p_1^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi(\vec{r}, E_1) = \int U(\vec{r}, \vec{r}\,')\Psi(\vec{r}\,', E_1)\, d^3 r'$$



$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R_I^{B_1 B_2}(t,\vec{r}) = \int U(\vec{r}, \vec{r}\,')\, R_I^{B_1 B_2}(t,\vec{r})\, d^3 r'$$



A single state saturation is not required!!

BB interaction from NBS wave function

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r\,'$$

Derivative (velocity) expansion of U is performed to deal with its non-locality.

- For the case of oct-oct system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_T(r)\right]}_{\text{Leading order part}}+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

- For the case of dec-oct and dec-dec system,

$$((\vec{r}\cdot\vec{S_1})^2-\frac{\vec{r}^2}{3}\vec{S_1}^2+(\vec{r}\cdot\vec{S_2})^2-\frac{\vec{r}^2}{3}\vec{S_2}^2)V_{T^2}(r)$$

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_{T_1}(r)+S_{ii}V_{T_2}(r)+O(Spin\,op^3)\right]}_{\text{Leading order part}}+O(\nabla)$$

- For the case of ps-meson-ps-meson system,

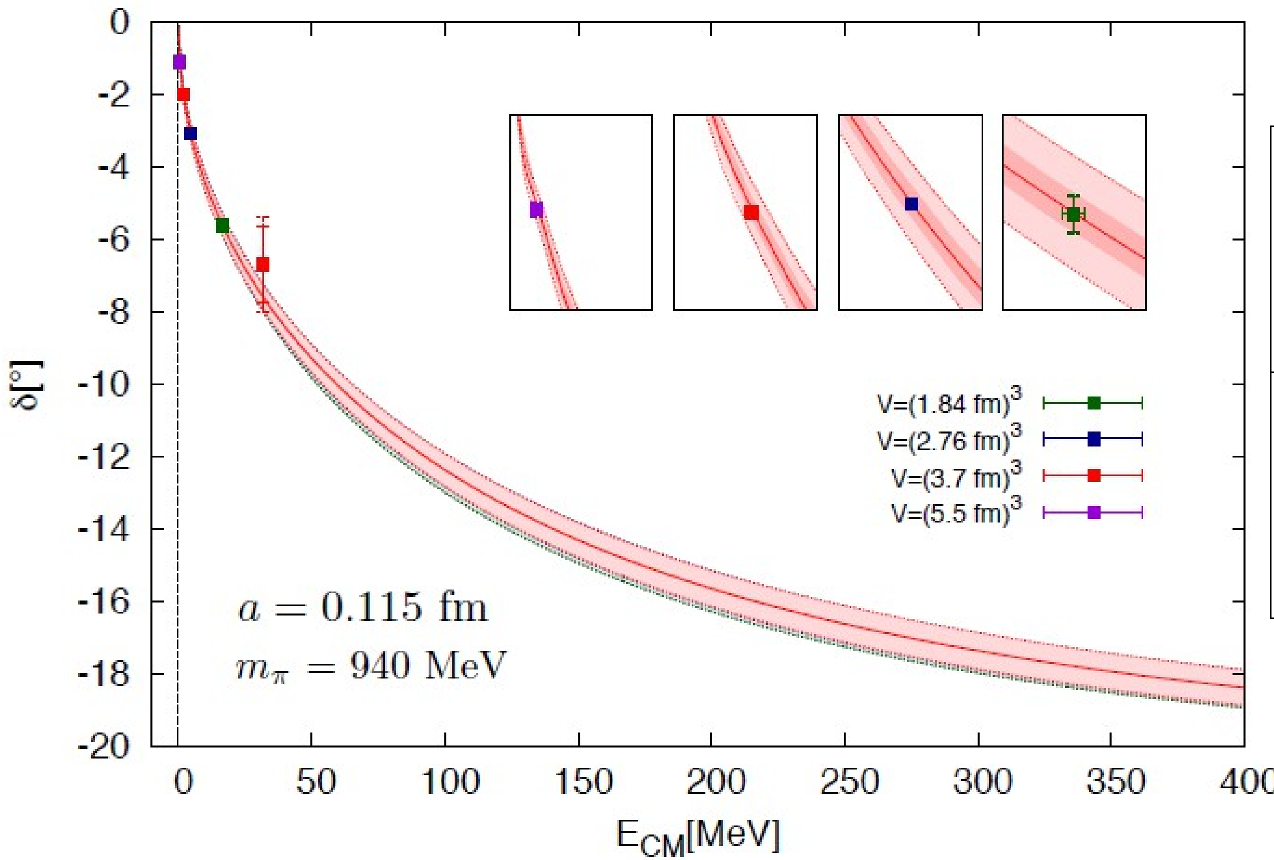
$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)\right]}_{\text{Leading order part}}+O(\nabla^2)$$

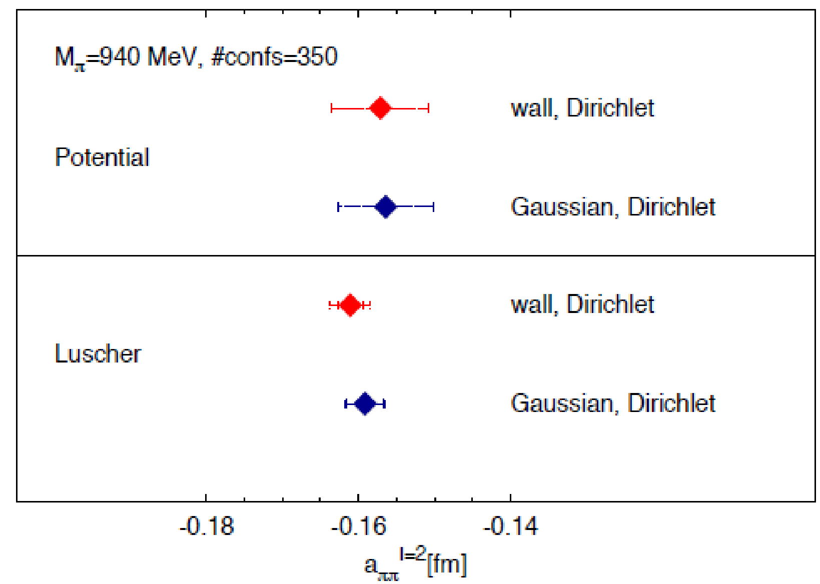
T. Kurth et al., JHEP 1312 (2013) 015

Phase shift from potential and FV method

Comparison between the potential method and Lüscher's method

π-π scattering with quench QCD





Resulting scattering phase shifts are consistent from both methods

Coupled channel Schrödinger equation

NBS wave function with ith energy eigen state

$$\Psi^{\alpha}(E_i,\vec{r}) = \langle 0|(B_1 B_2)^{\alpha}(\vec{r})\,|E_i\rangle$$

$$\Psi^{\beta}(E_i,\vec{r}) = \langle 0|(B_1 B_2)^{\beta}(\vec{r})\,|E_i\rangle$$

Two-channel coupling case

$$\int dr\,\tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\,\delta_{\beta}{}^{\gamma}$$

We define potentials which satisfy a coupled channel Schrodinger equation

$$\begin{pmatrix} (p_{\alpha}^2+\nabla^2)\Psi^{\alpha}(E_i,\vec{r}) \\ (p_{\beta}^2+\nabla^2)\Psi^{\beta}(E_i,\vec{r}) \end{pmatrix} = \int dr' \begin{pmatrix} U^{\alpha}{}_{\alpha}(\vec{r},\vec{r}\,') & U^{\alpha}{}_{\beta}(\vec{r},\vec{r}\,') \\ U^{\beta}{}_{\alpha}(\vec{r},\vec{r}\,') & U^{\beta}{}_{\beta}(\vec{r},\vec{r}\,') \end{pmatrix} \begin{pmatrix} \Psi^{\alpha}(E_i,\vec{r}\,') \\ \Psi^{\beta}(E_i,\vec{r}\,') \end{pmatrix}$$

Leading order of velocity expansion and time-derivative method

$$\begin{pmatrix} \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_{\alpha}}\right) R^{\alpha}_{E_0}(t,\vec{r}) \\ \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_{\beta}}\right) R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_0}(t,\vec{r}) \\ R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$\Delta^{\alpha}_{\beta} = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

Considering two different energy eigen states

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_{\alpha}}-\frac{\partial}{\partial t}\right) R^{\alpha}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_{\beta}}-\frac{\partial}{\partial t}\right) R^{\alpha}_{E1}(t,\vec{r}) \\ \left(\frac{\nabla^2}{2\mu_{\alpha}}-\frac{\partial}{\partial t}\right) R^{\beta}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_{\beta}}-\frac{\partial}{\partial t}\right) R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$

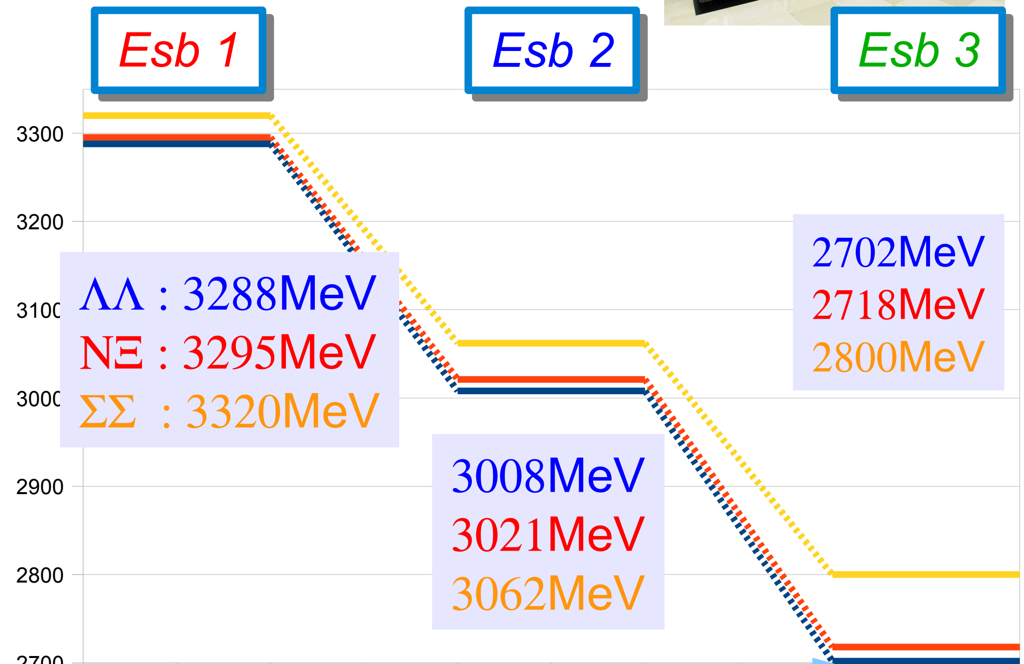
Numerical results

Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson quark action
  - β = 1.90, a^{-1} = 2.176 [*GeV*], 32^3x64 lattice, L = 2.902 [*fm*].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter

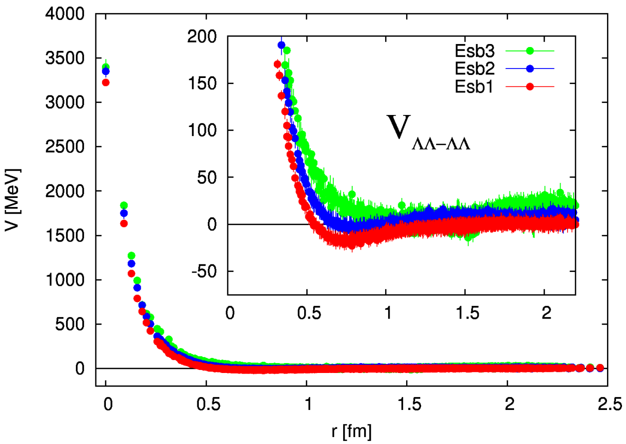


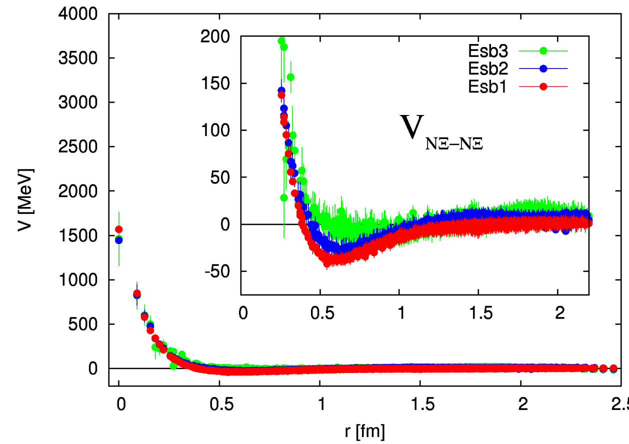
SU(3) breaking effects becomes larger

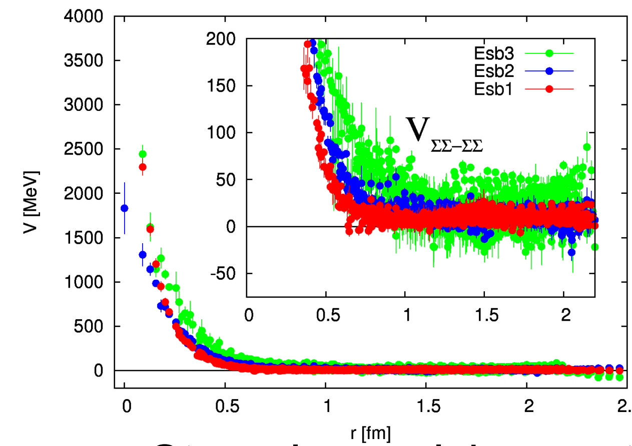
ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

Diagonal elements



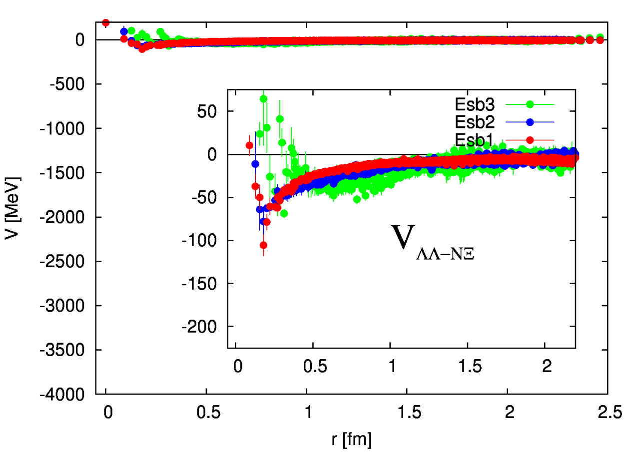


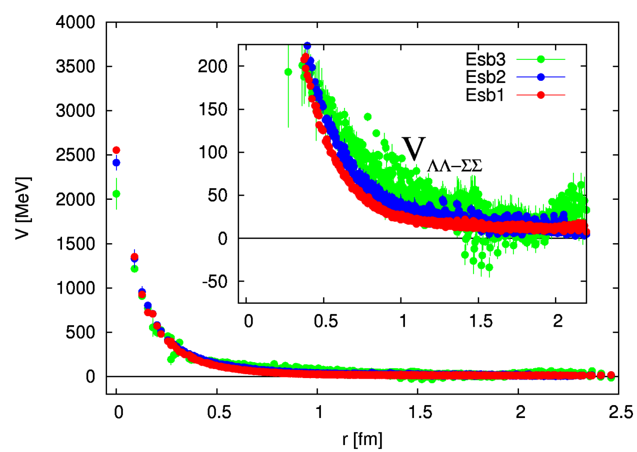


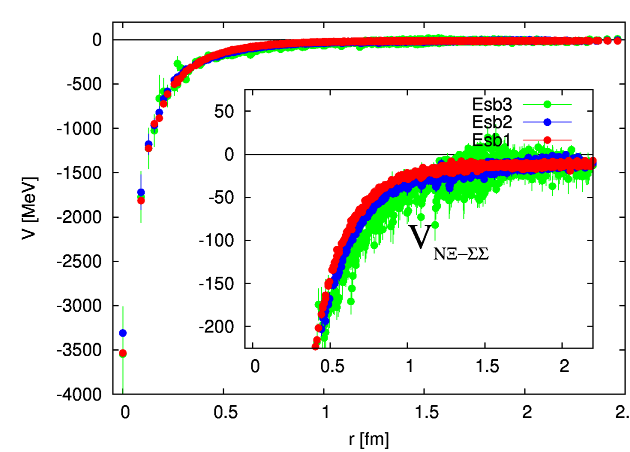
shallow attractive pocket — Deeper attractive pocket — Strongly repulsive

All channels have repulsive core

Off-diagonal elements







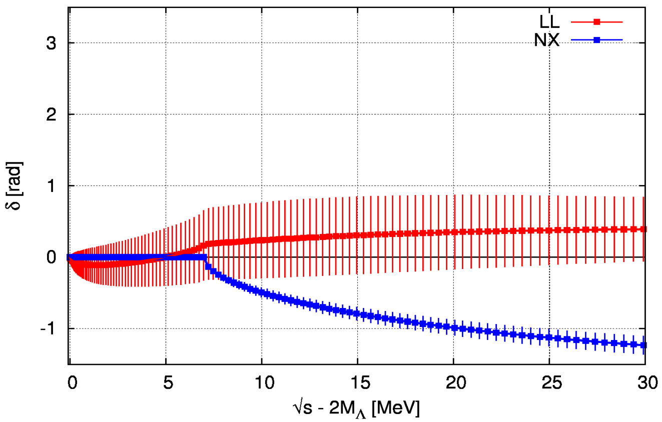
In this channel, our group found the "H-dibaryon" in the SU(3) limit.

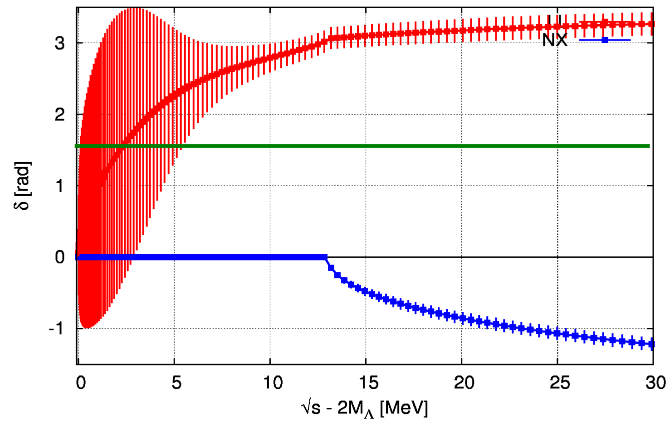
$\Lambda\Lambda$ and $N\Xi$ phase shifts

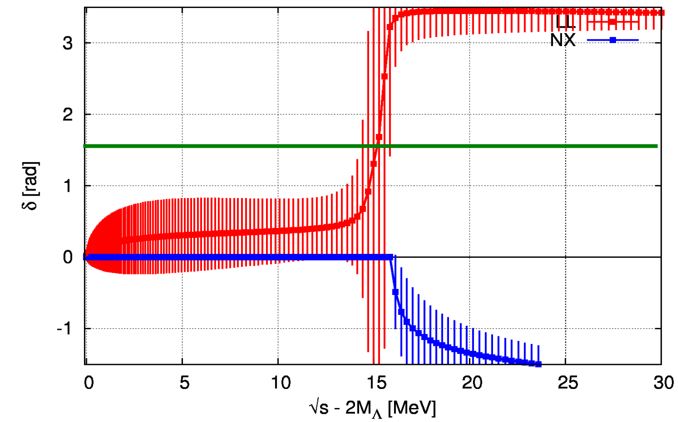
▶ N_f = 2+1 full QCD with L = 2.9fm

Preliminary!

m_π = 700 MeV | **m_π = 570 MeV** | **m_π = 410 MeV**

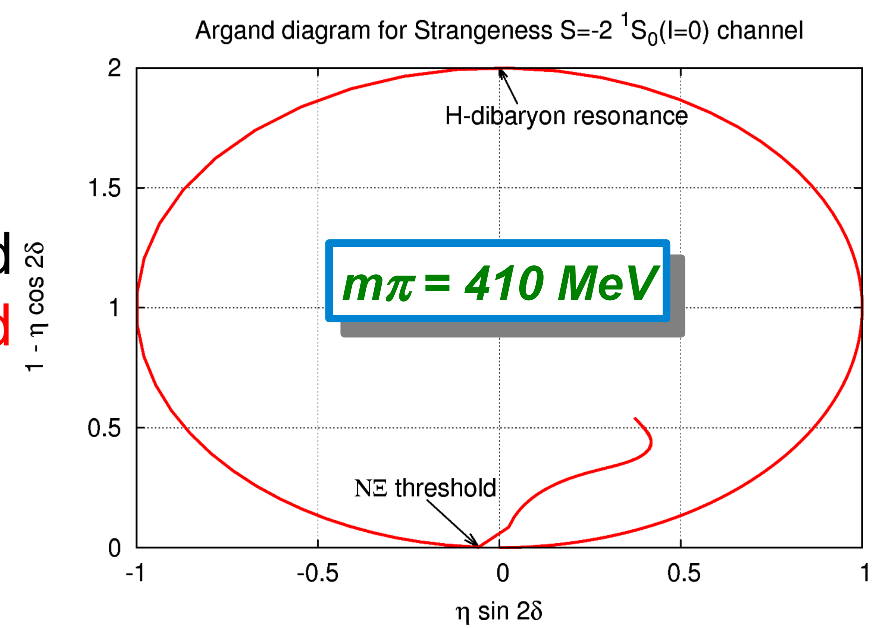






- **m_π = 700 MeV :** bound state
- **m_π = 570 MeV** : resonance near $\Lambda\Lambda$ threshold
- **m_π = 410 MeV** : resonance near $N\Xi$ threshold

H-dibaryon is unlikely bound state



Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

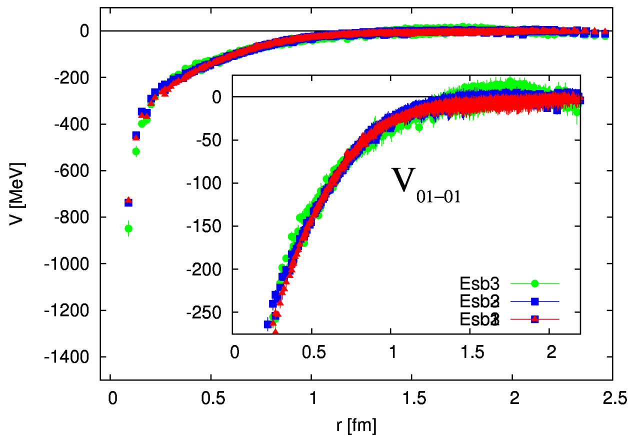
In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

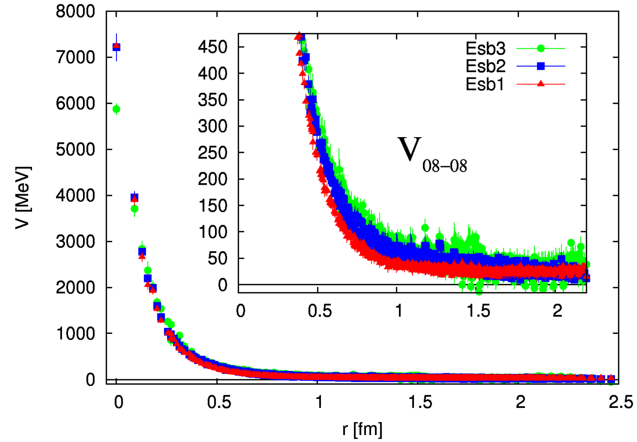
Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effectual measure of the SU(3) breaking effect.

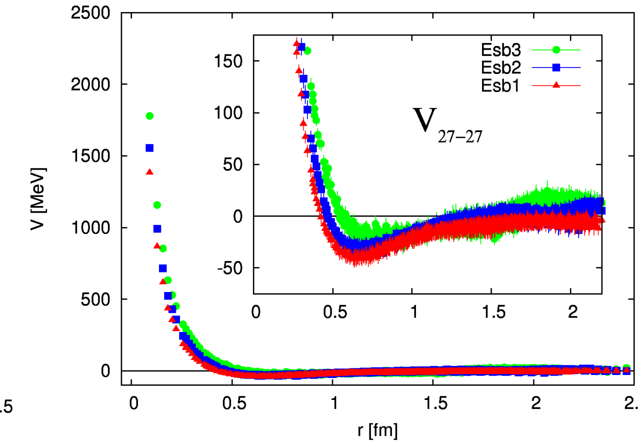
We will see how the SU(3) symmetry of potential will be broken
by changing the u,d quark masses lighter.

$1,\ 8_s,\ 27\ (I=0)\ {}^1S_0$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

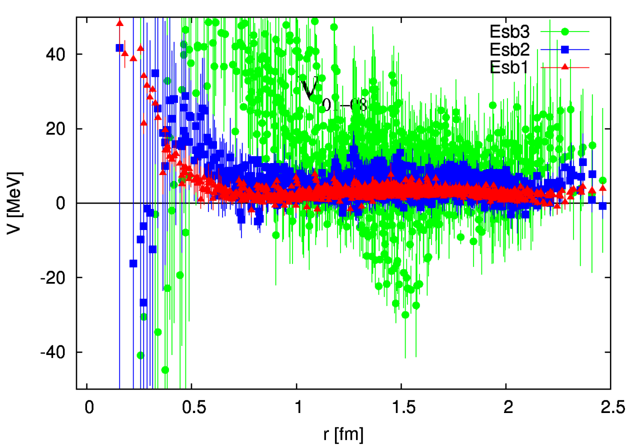


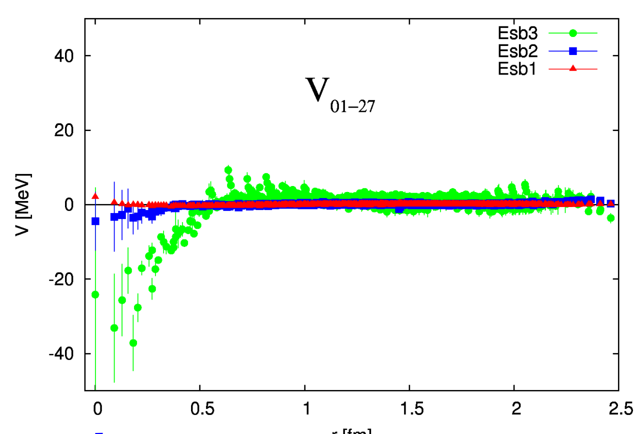


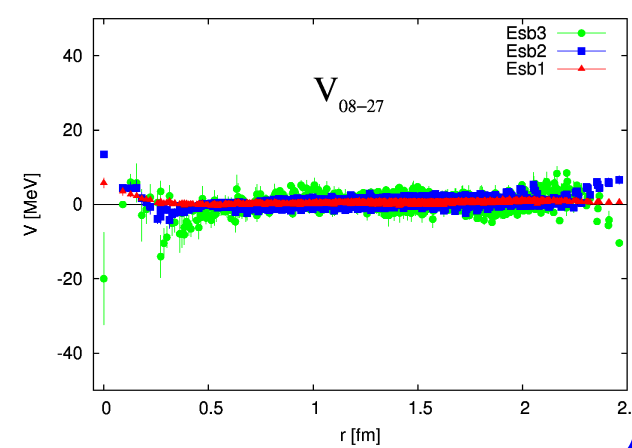


Strongly attractive
H-dibaryon channel

Pauli blocking effect







Mixture of singlet and octet
Is relatively larger than the others

27 plet does not mix so much to the other representations

Summary and outlook

- We introduced the coupled channel HAL QCD method.
- It allow us to tackle to several exotic systems such as H-dibaryon.
- Potentials are derived from NBS wave functions calculated with PACS-CS configurations
- H-dibaryon energy is going higher as a decreasing quark masses
- Small mixture between different SU(3) irreps can be seen as the flavor SU(3) breaking effect.
- The preliminary results at physical situation (m_π/m_K=0.28) coming soon.