Modification of nucleon spectral function in the nuclear matter from QCD sum rules

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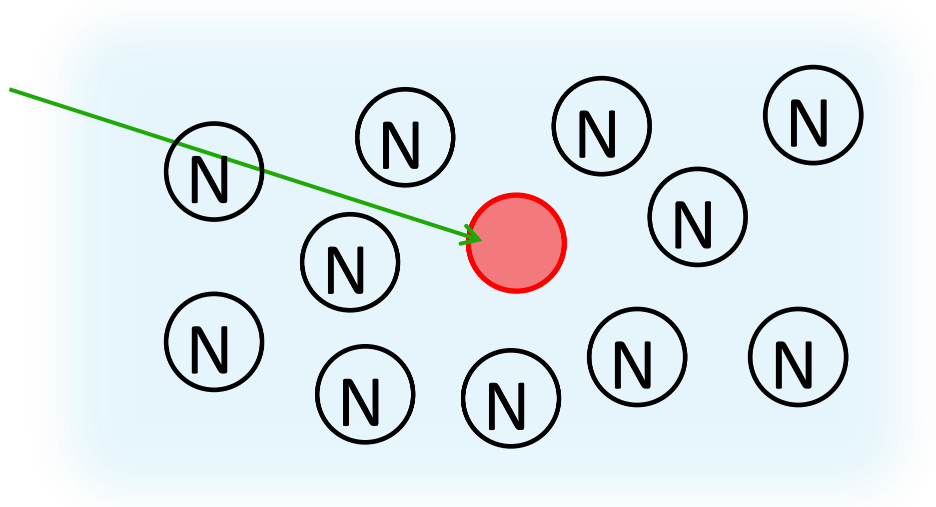
Outline

- Introduction
- QCD sum rules
- Nucleon QCD sum rule in vacuum
- Nucleon QCD sum rule in nuclear matter
- Summary

Introduction

- Hadron properties in the nuclear medium

Probe hadron



The properties of the hadron are modified in the medium.

Partial restoration of the chiral symmetry

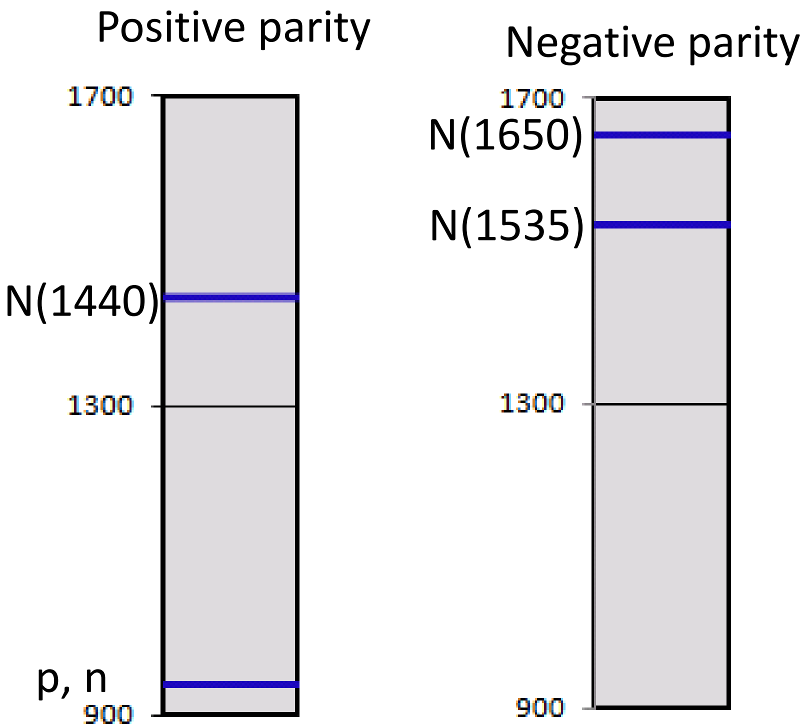
Interaction with the nucleons in the nuclear matter

Nuclear matter

We focus on the nucleon ground state and its negative parity excited state.

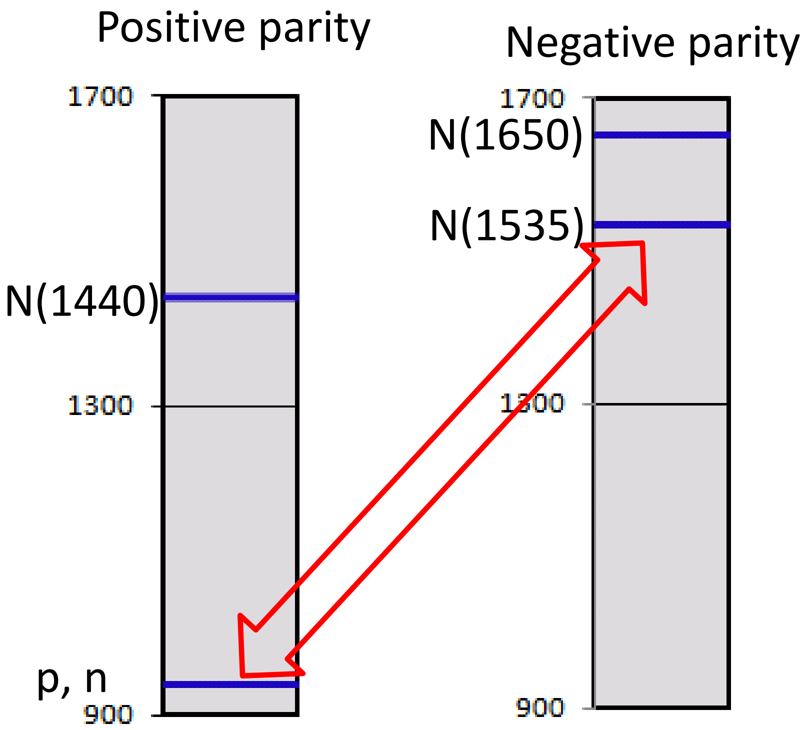
Introduction

Mass spectrum of the nucleons



Introduction

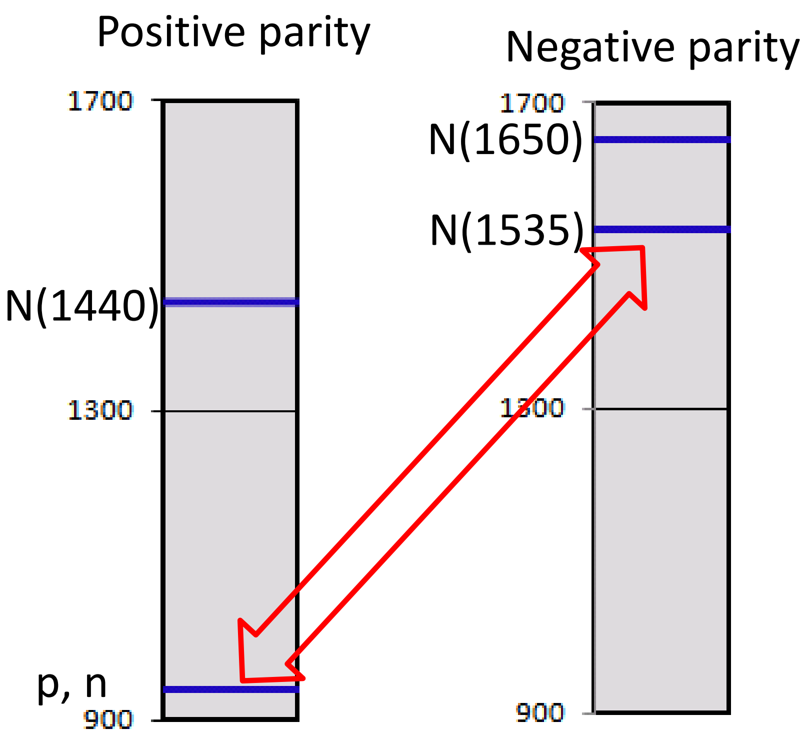
Mass spectrum of the nucleons



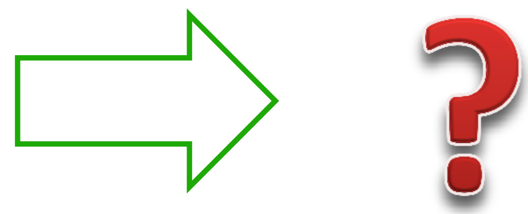
- The mass difference between nucleon ground state and N(1535) is about 600 MeV.
- It is predicted that Chiral symmetry breaking cause these difference.

Introduction

Mass spectrum of the nucleons



In the nuclear matter



When chiral symmetry is restored, the mass spectrum will change.

To investigate these properties from QCD, non perturbative method is needed.

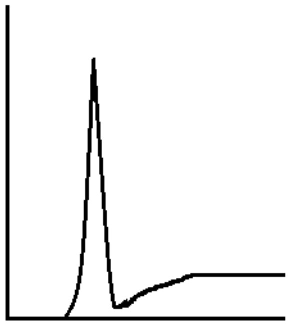
⇒ Analysis of QCD sum rule in nuclear matter

QCD sum rules

$$\Pi(q) \equiv i \int e^{iqx} \langle 0|T[\eta(x)\overline{\eta}(0)]|0\rangle d^4x$$

$$= \int_0^\infty \frac{1}{\pi} \frac{\mathrm{Im}\Pi(t)}{t-q^2} dt = \int_0^\infty \frac{\rho(t)}{t-q^2} dt$$

Hadronic spectral function



is calculated by the operator product expansion (OPE)

Non perturbative contributions are expressed by some Condensates.

$\Rightarrow$ $\langle \overline{q}q \rangle$, $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ $\cdots$

An order parameter of chiral symmetry

$\Downarrow$

Application of the analyses in the nuclear matter.

QCD sum rules

Vacuum

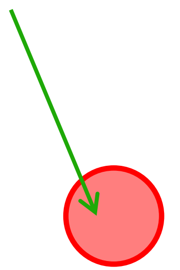
$$\Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle d^4x$$

Nuclear matter

$$i \int d^4x e^{iqx} \langle \Psi_0 | T[\eta(x)\overline{\eta}(0)] | \Psi_0 \rangle$$

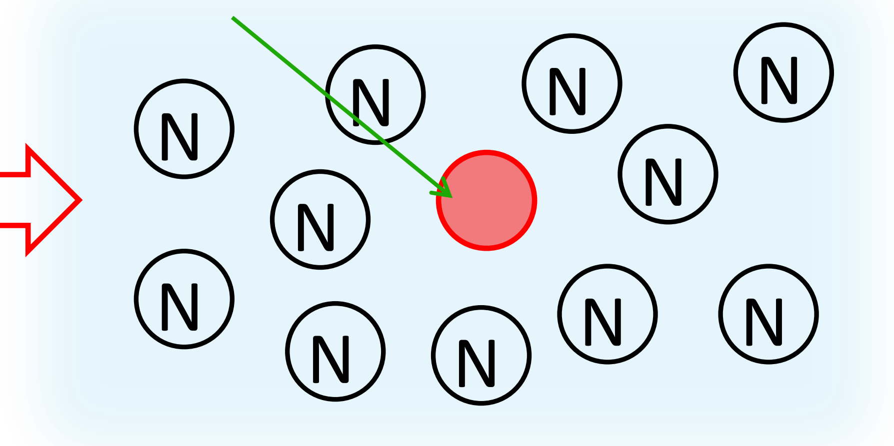
Ψ_0 : Ground state of nuclear matter

Probe hadron



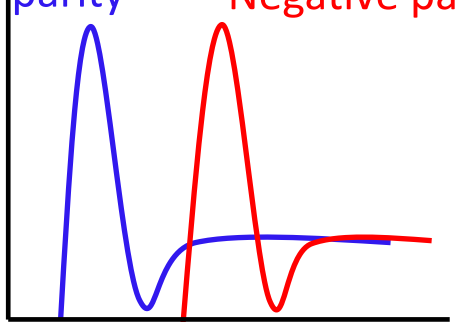
In vacuum

Probe hadron



In nuclear matter

Spectral function:



QCD sum rules

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Nuclear matter

$$i\int d^4x e^{iqx}\langle \Psi_0|T[\eta(x)\overline{\eta}(0)]|\Psi_0\rangle$$

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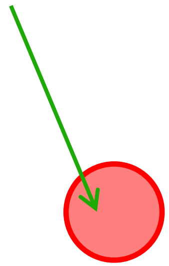
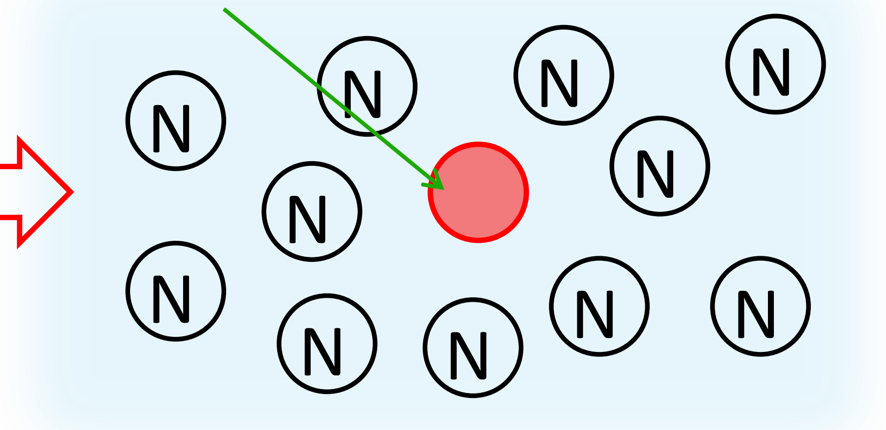


In vacuum

Probe hadron

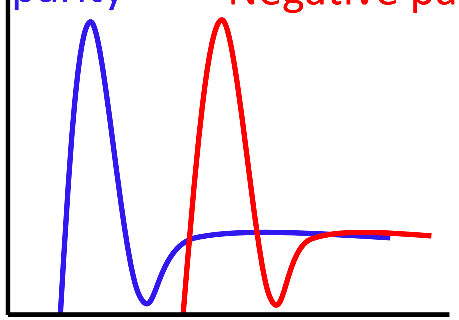


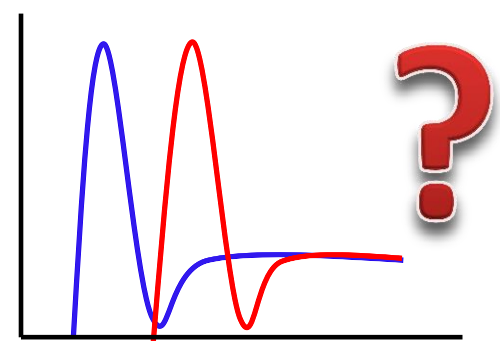
In nuclear matter

Spectral function:

Positive parity

Negative parity





QCD sum rules

Vacuum

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Nuclear matter

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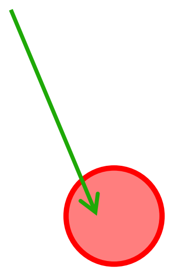
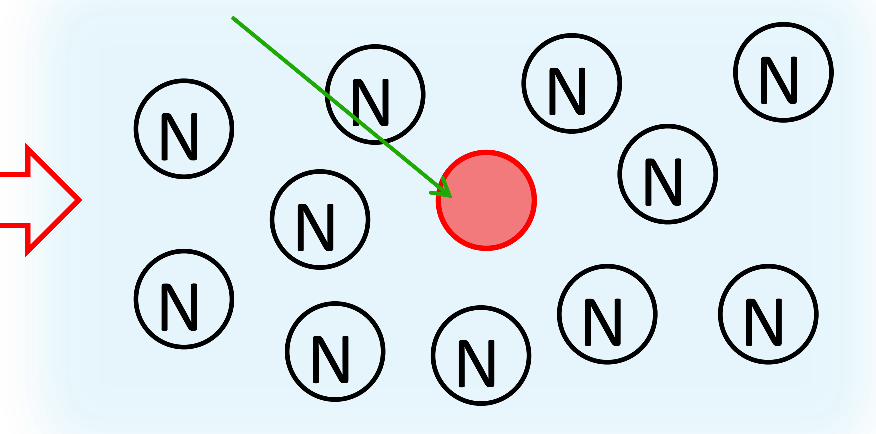


In vacuum

Probe hadron



In nuclear matter

OPE side:

Modification: $\langle 0 | O_i | 0 \rangle \Rightarrow \langle \Psi_0 | O_i | \Psi_0 \rangle \equiv \langle O_i \rangle_\rho$

Chiral condensate: $\langle \overline{q} q \rangle_0 \Rightarrow \langle \overline{q} q \rangle_\rho = \langle \overline{q} q \rangle_0 + \frac{\sigma_N}{2m_q} \rho + \cdots$

New condensate: $\Rightarrow \langle q^\dagger q \rangle_\rho = \frac{3}{2} \rho$

QCD sum rules

Vacuum

$$\Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle d^4x$$

Nuclear matter

$$i \int d^4x e^{iqx} \langle \Psi_0 | T[\eta(x)\overline{\eta}(0)] | \Psi_0 \rangle$$

$$\Pi(q) = \int_0^\infty \frac{\rho(t)}{t - q^2}$$

is calculated by OPE

Transformation

Gaussian sum rule

$$G(s,\tau) = \int_0^\infty \rho(\omega) \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{(\omega^2 - s)^2}{4\tau}\right) d\omega$$

OPE side

τ、s: parameter

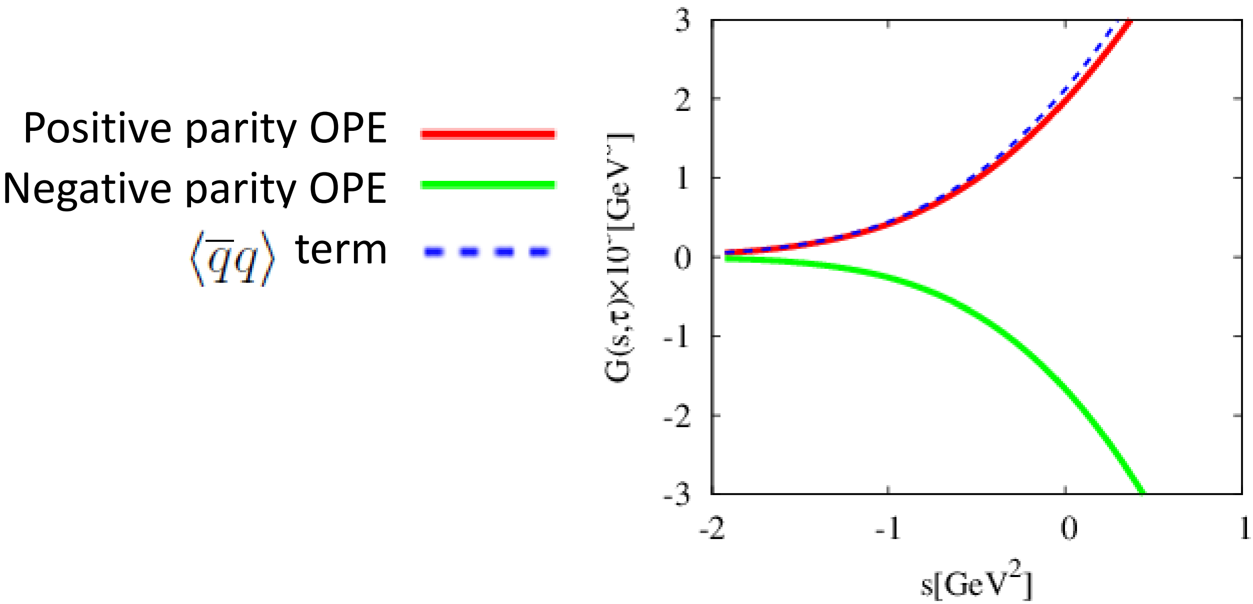
We extract the information on the spectral function with maximum entropy method (MEM).

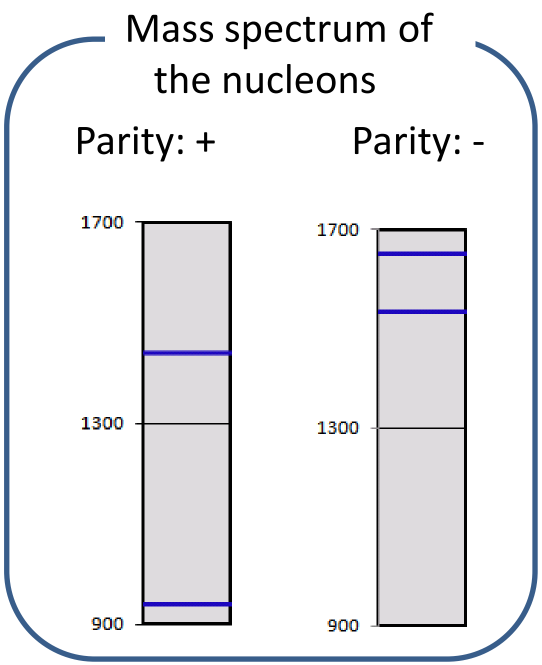
Nucleon QCD sum rule in vacuum

The behavior of the OPE data in the vacuum

$$G^{\pm}(s,\tau) = G_1(s,\tau) \pm G_2(s,\tau)$$

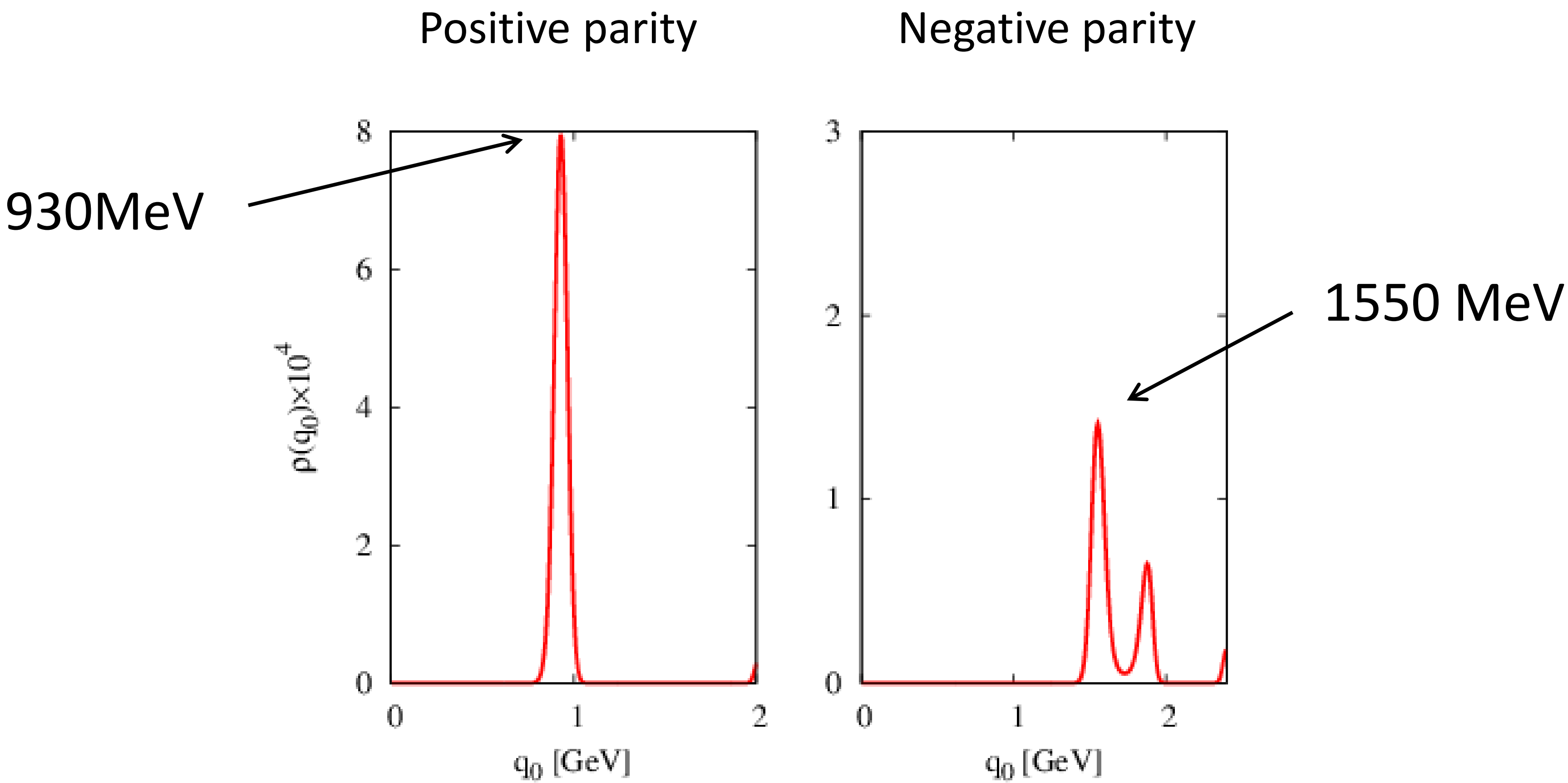
contains the $\langle \overline{q}q \rangle$ term





The difference between positive parity and negative parity is mainly caused by chiral condensate term.

Nucleon QCD sum rule in vacuum



In both positive and negative parity, the peaks are found.

In the negative parity state, the peak correspond to the N(1535) or (and) N(1650).

Nucleon QCD sum rule in nuclear matter

The behavior of the OPE data in the nuclear matter

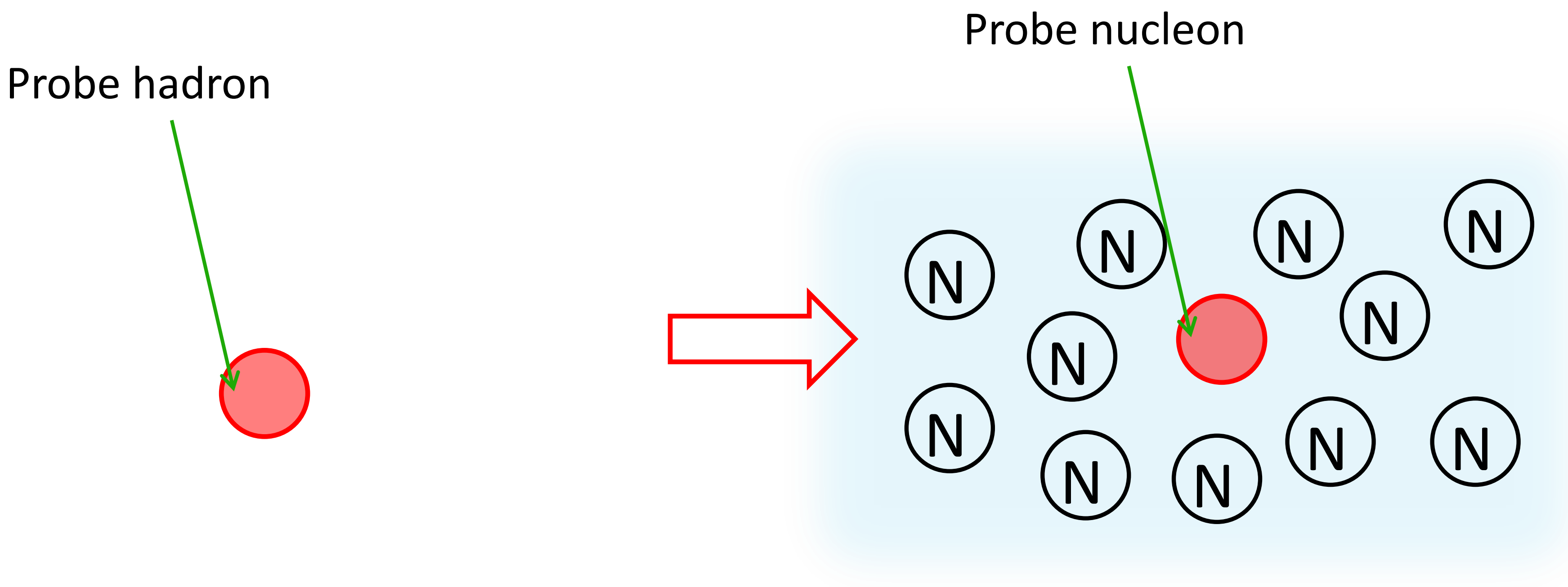
Ψ_0 : Ground state of nuclear matter

Vacuum

$$\Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle d^4x$$

Nuclear matter

$$i \int d^4x e^{iqx} \langle \Psi_0 | T[\eta(x)\overline{\eta}(0)] | \Psi_0 \rangle$$



In vacuum

In nuclear matter

Nucleon QCD sum rule in nuclear matter

The behavior of the OPE data in the nuclear matter

Vacuum

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Nuclear matter

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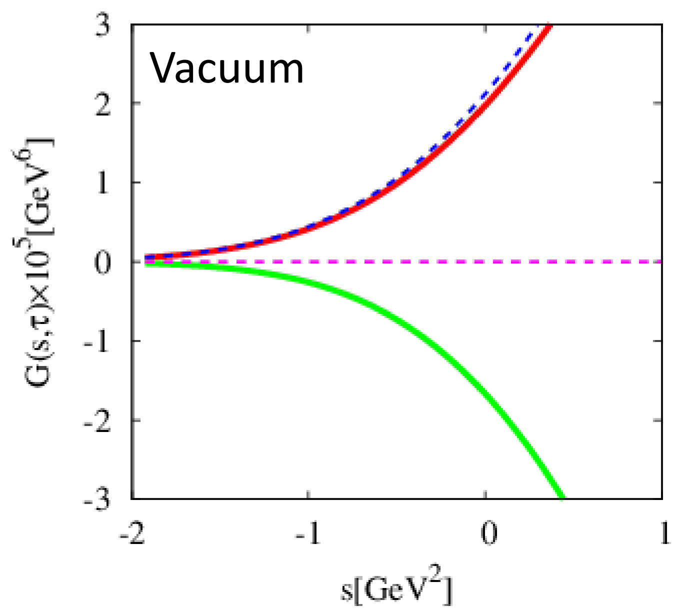
Positive parity OPE

Negative parity OPE

$\langle\overline{q}q\rangle_\rho$ term

$\langle q^\dagger q\rangle_\rho$ term

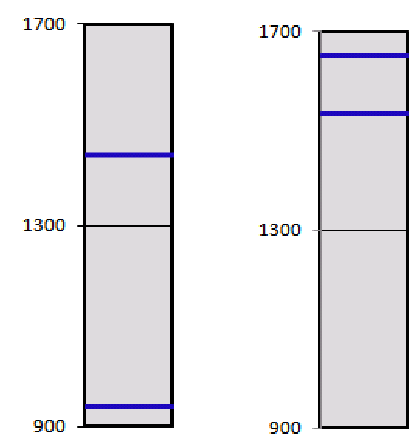
ρ_N: nuclear matter density



Mass spectrum of the nucleons

Parity: +

Parity: -



Nucleon QCD sum rule in nuclear matter

The behavior of the OPE data in the nuclear matter

$$G^{\pm}(s,\tau) = G_1(s,\tau) + G_u(s,\tau) \pm G_2(s,\tau)$$

contains the $\langle \overline{q}q \rangle_\rho$ term

contain the $\langle q^\dagger q \rangle_\rho$ terms

Positive parity OPE

Negative parity OPE

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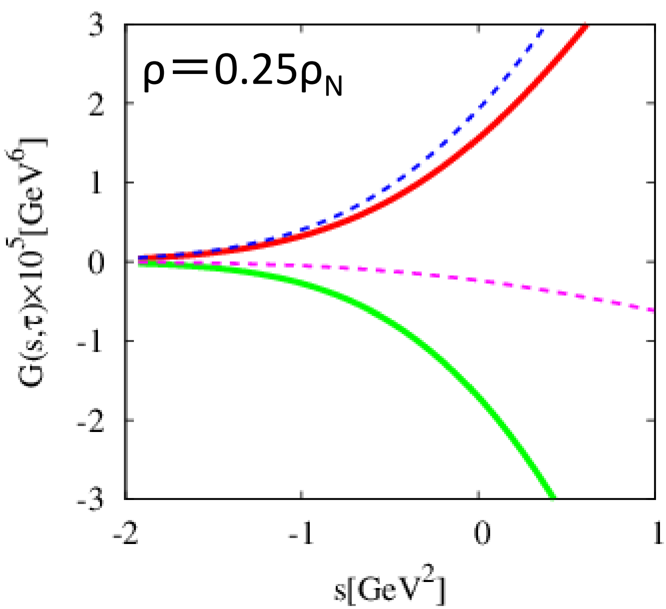

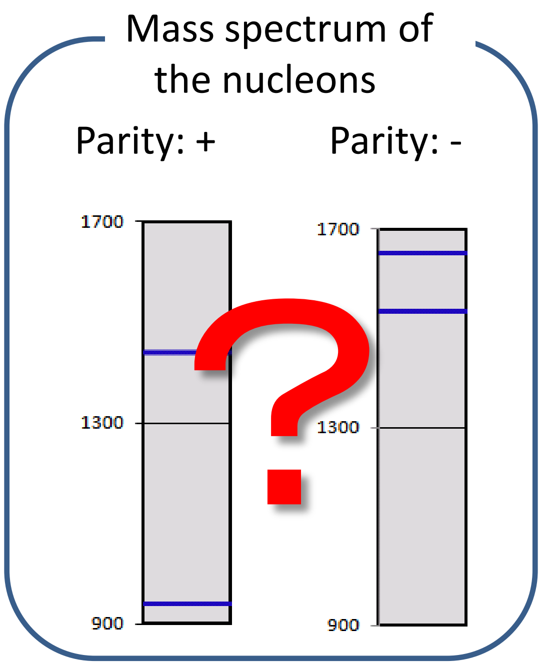

Positive parity: OPE data decreases.

Negative parity: OPE data slightly increases.

Nucleon QCD sum rule in nuclear matter

The behavior of the OPE data in the nuclear matter

$$G^{\pm}(s,\tau) = G_1(s,\tau) + G_u(s,\tau) \pm G_2(s,\tau)$$

contain the $\langle q^\dagger q \rangle_\rho$ terms (G_1, G_u)

contains the $\langle \overline{q} q \rangle_\rho$ term (G_2)

Positive parity OPE

Negative parity OPE

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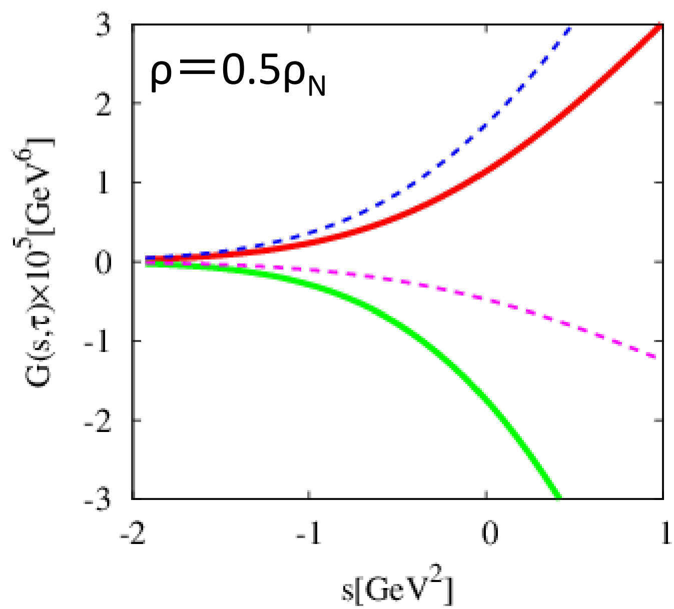

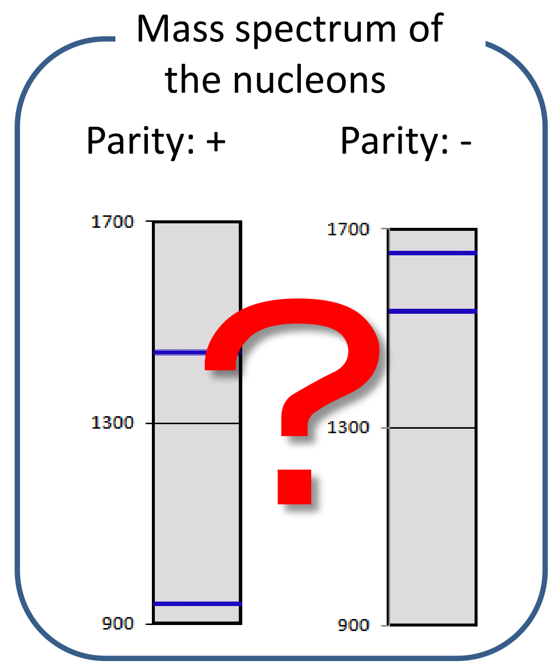

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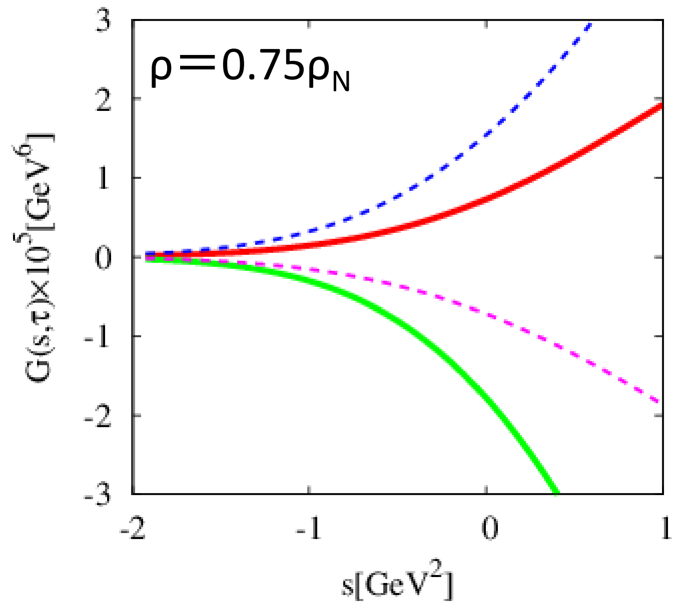

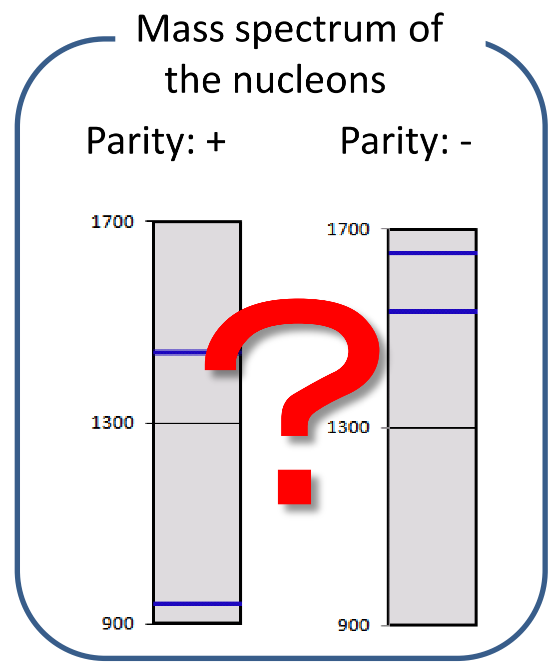

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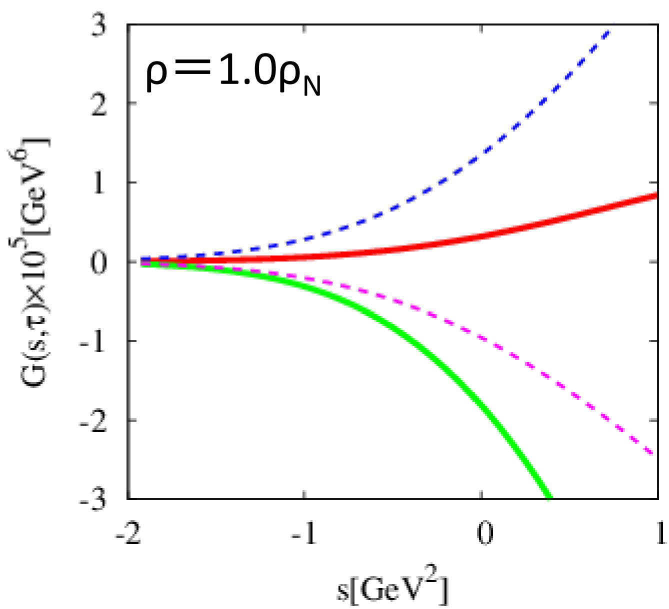
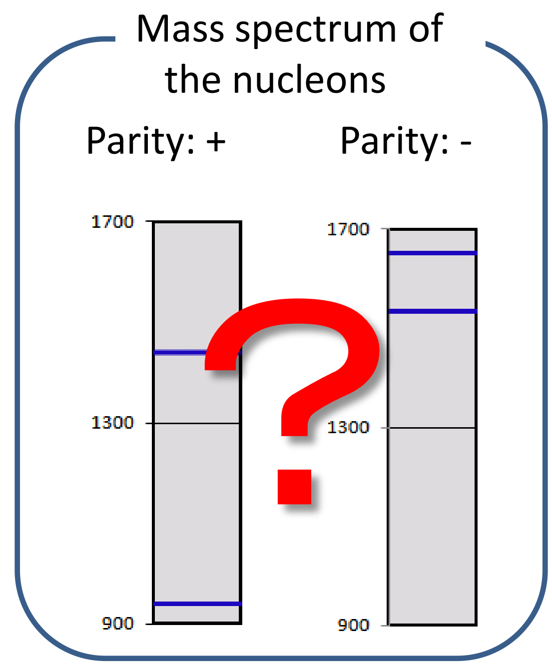
contain the $\langle q^\dagger q\rangle_\rho$ terms (G_1, G_u)

contains the $\langle \overline{q}q\rangle_\rho$ term (G_2)

- Positive parity OPE
- Negative parity OPE
- $\langle \overline{q}q\rangle_\rho$ term
- $\langle q^\dagger q\rangle_\rho$ term

ρ_N: nuclear matter density





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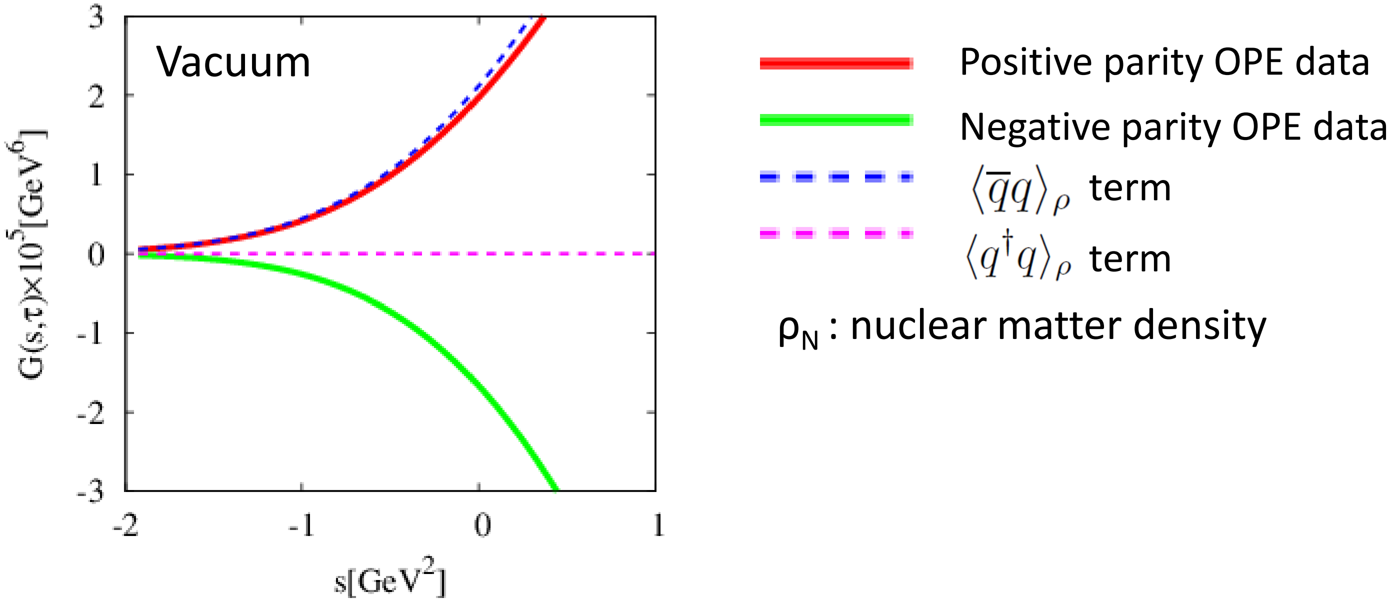
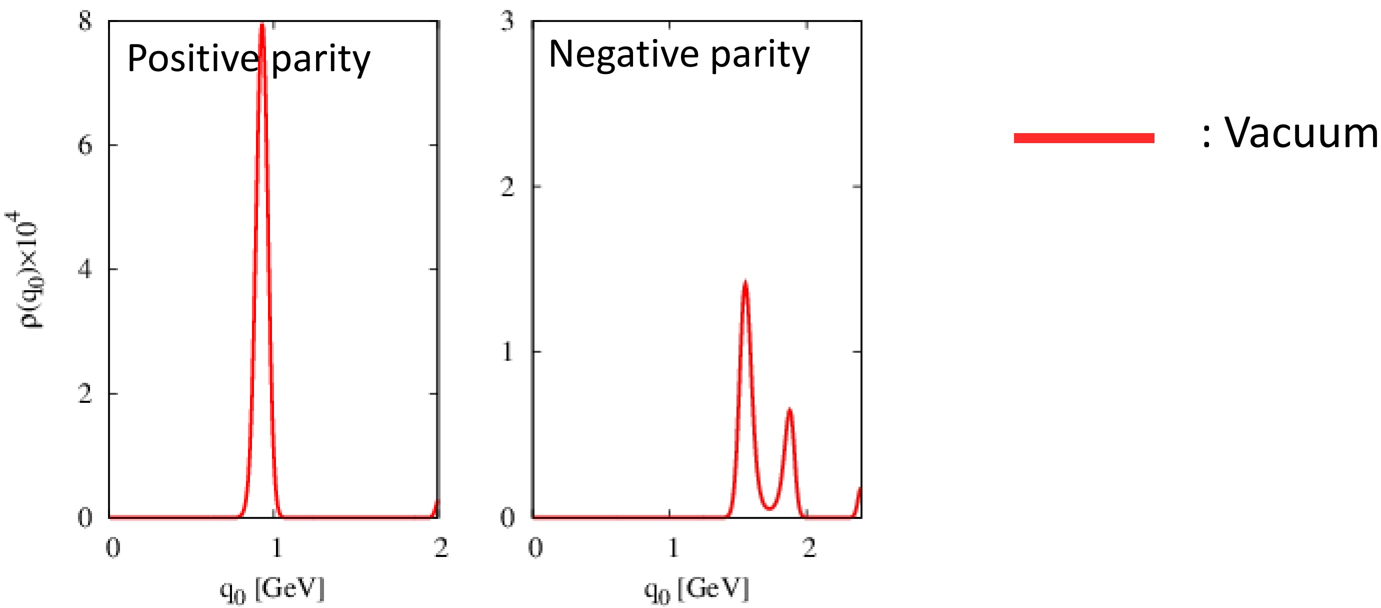
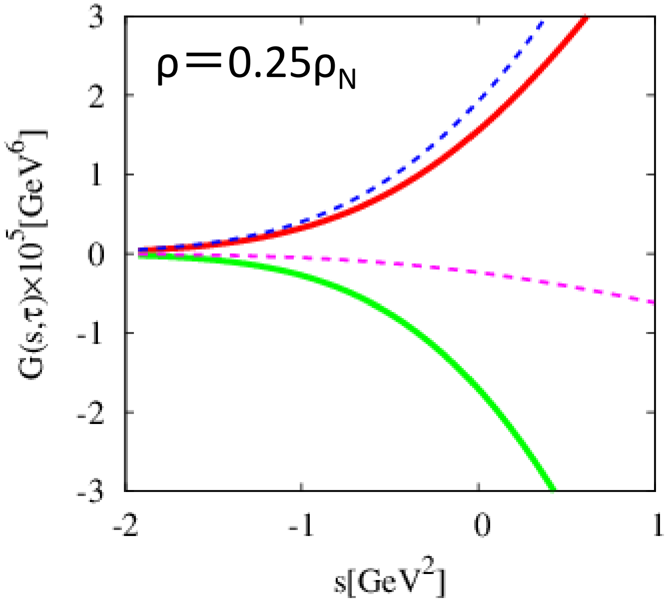
Nucleon QCD sum rule in nuclear matter





Nucleon QCD sum rule in nuclear matter



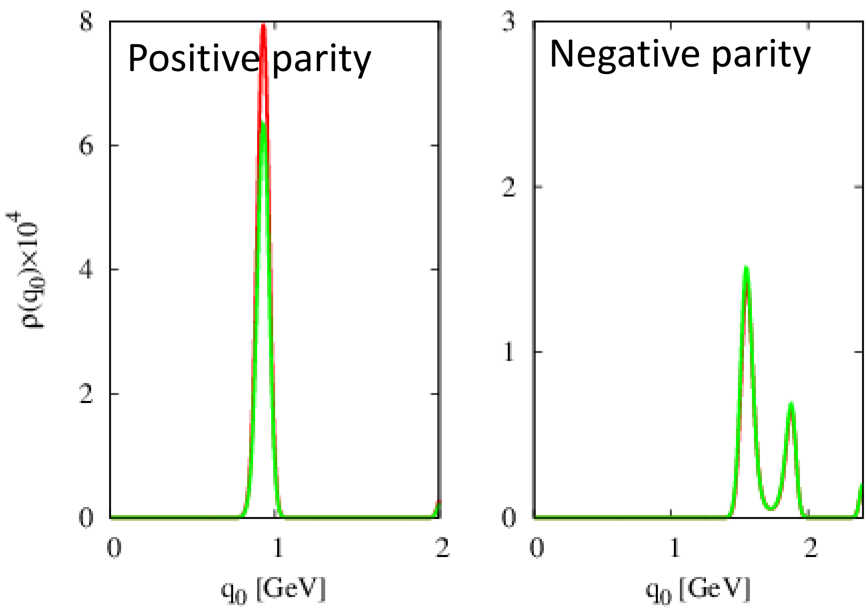
Positive parity OPE data

Negative parity OPE data

$\langle \overline{q}q \rangle_\rho$ term

$\langle q^\dagger q \rangle_\rho$ term

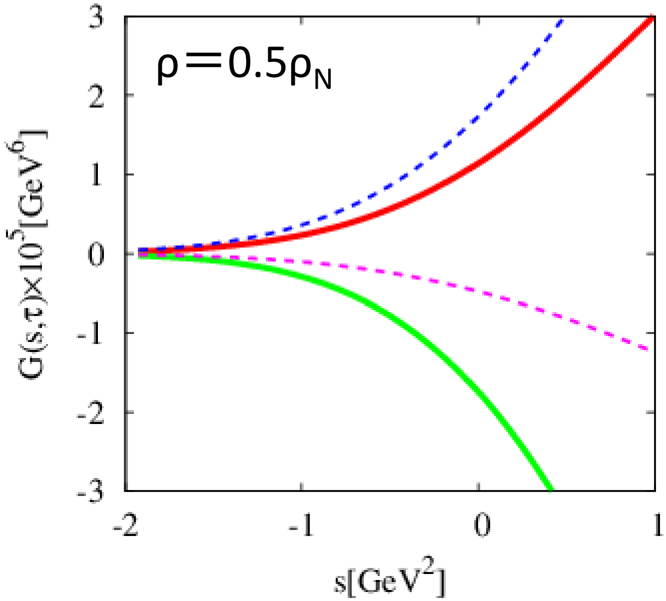
ρ_N : nuclear matter density



: Vacuum

: ρ=0.25ρ_N

Nucleon QCD sum rule in nuclear matter



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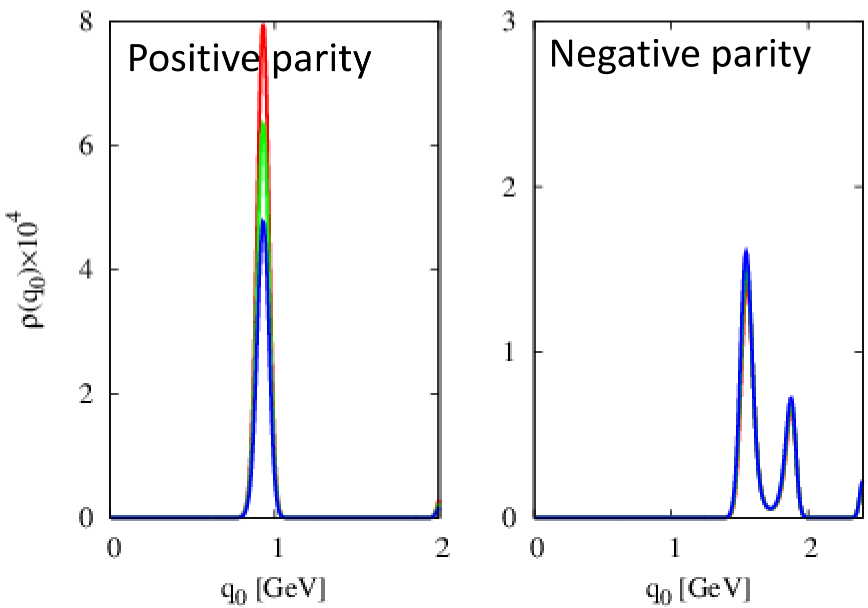
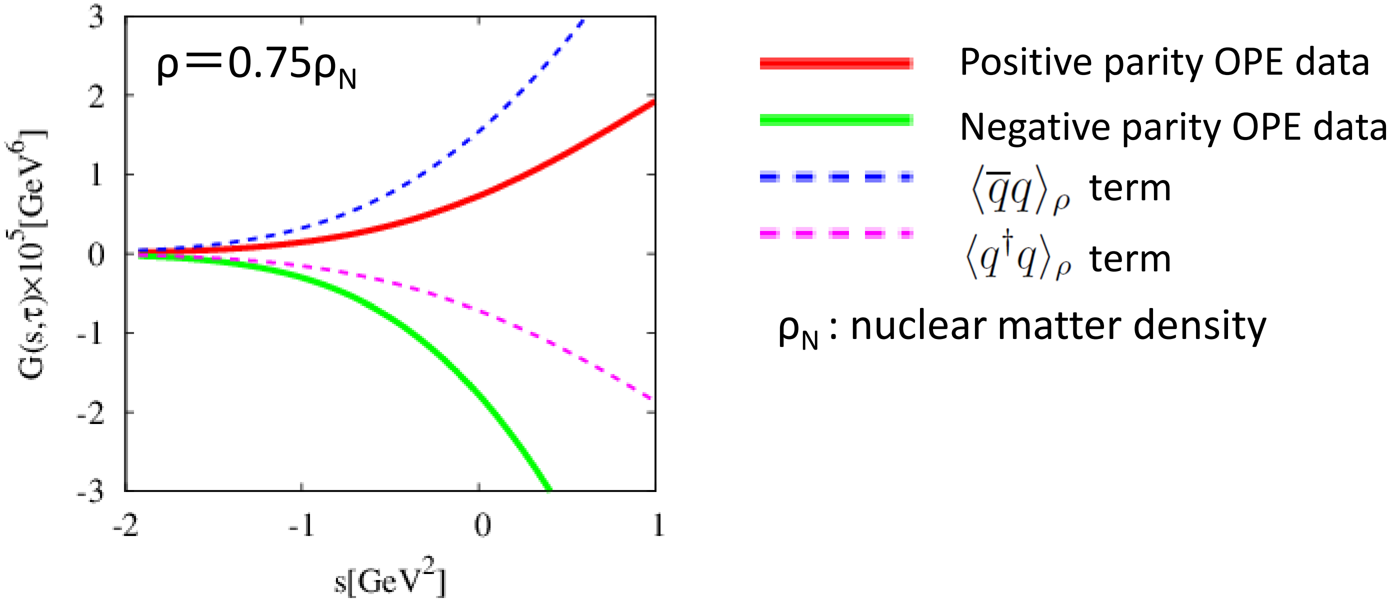


: Vacuum

: $\rho=0.25\rho_N$

: $\rho=0.5\rho_N$

Nucleon QCD sum rule in nuclear matter



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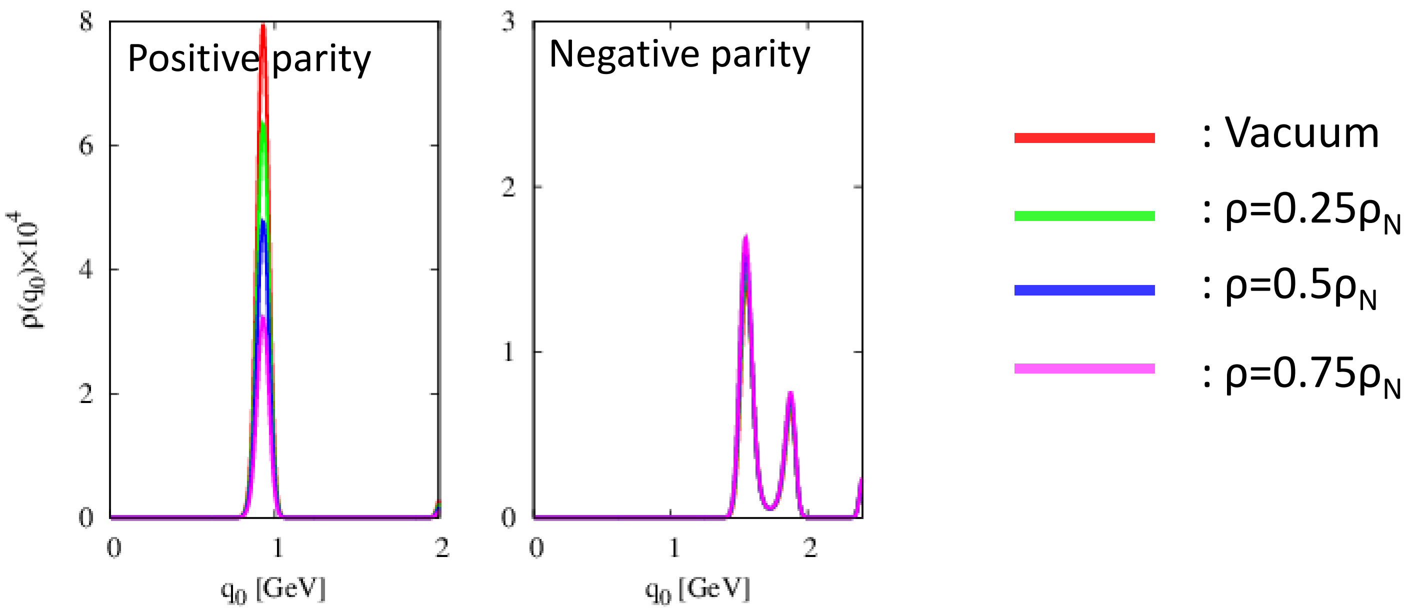
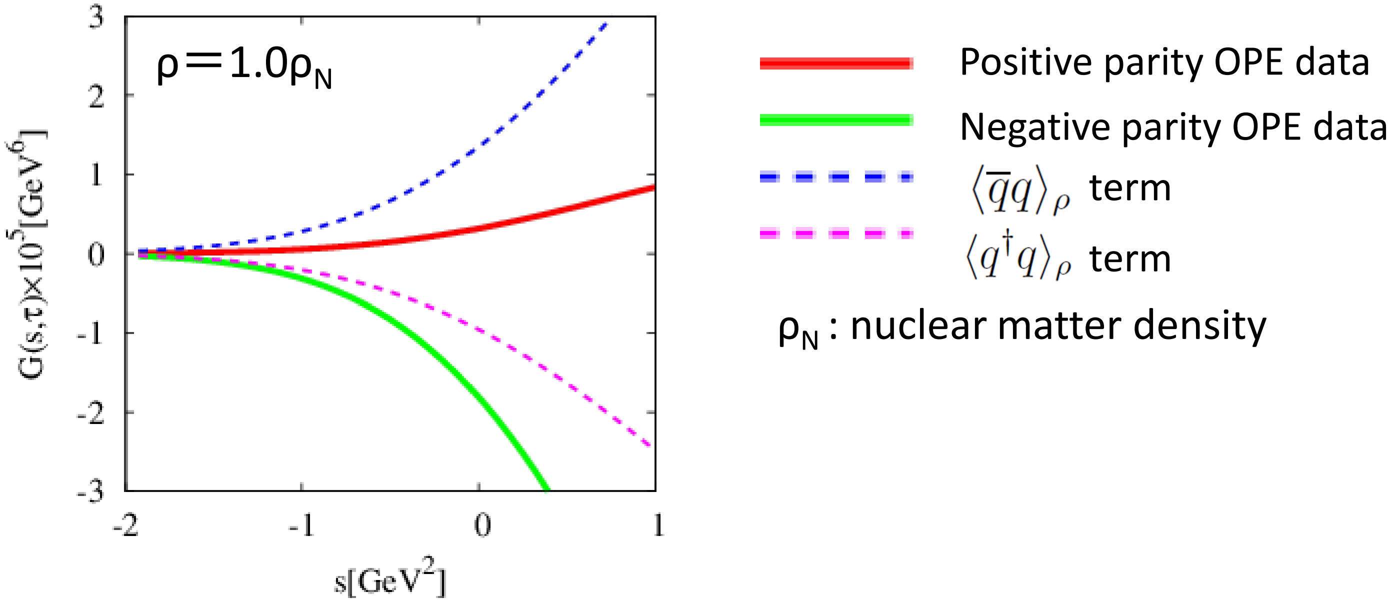
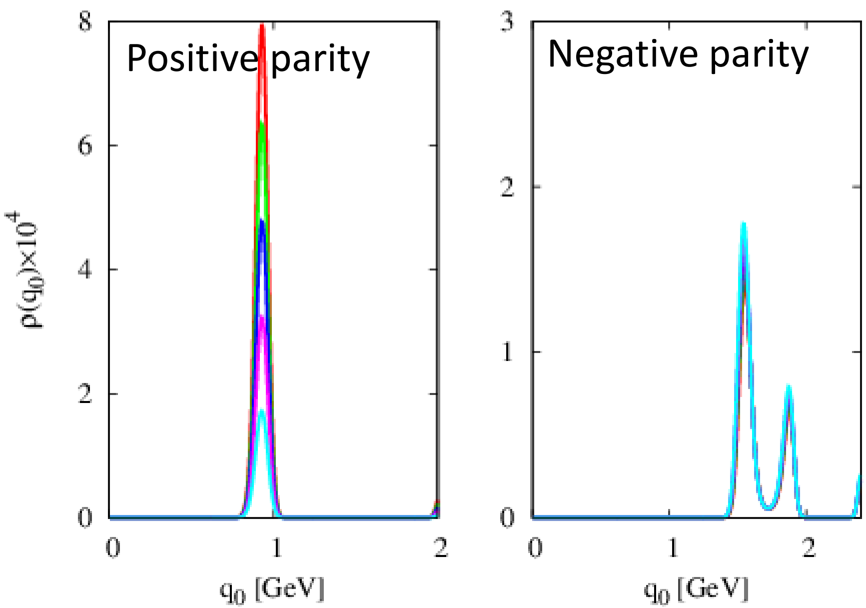
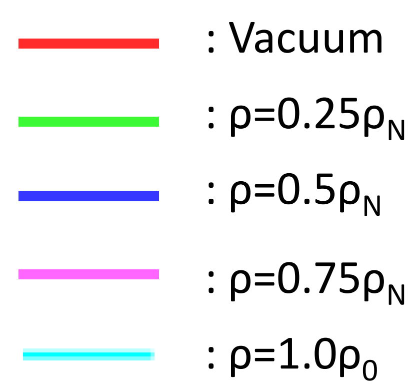


Nucleon QCD sum rule in nuclear matter







The values of the energies are obtained.

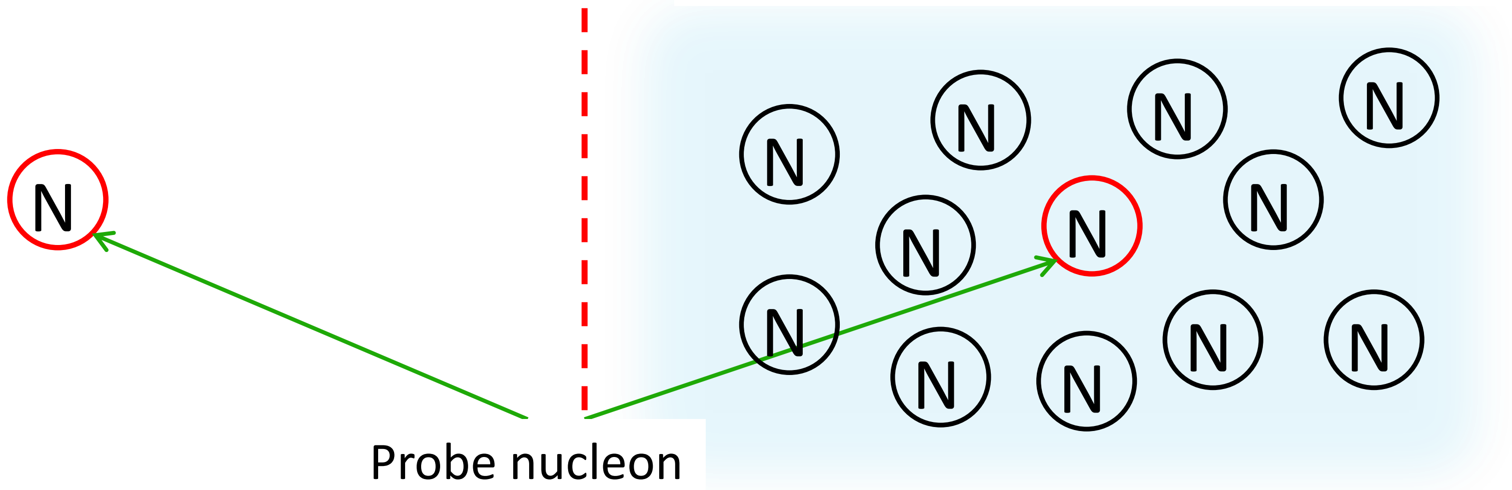
Nucleon QCD sum rule in nuclear matter

Vacuum

$$\Pi(q) = i\int d^4x e^{iqx}\langle \underline{0}|T[\eta(x)\overline{\eta}(0)]|\underline{0}\rangle$$

Nuclear matter

$$\Pi(q) = i\int d^4x e^{iqx}\langle \underline{\Psi_0}|T[\eta(x)\overline{\eta}(0)]|\underline{\Psi_0}\rangle$$



Propagator: $\dfrac{1}{\not{q} - M + i\epsilon}$

Propagator: $\dfrac{1}{\not{q} - M - \Sigma(q) + i\epsilon}$

$$\Rightarrow \quad \frac{\not{q} - \not{u}\Sigma_v + M^*}{(q_0 - E + i\epsilon)(q_0 + \overline{E} - i\epsilon)}$$

Pole of positive energy state: $E = \sqrt{M^{*2} + \vec{q}^2} + \Sigma_v$

Effective mass: $M^* = M + \Sigma_s$

Pole of negative energy state: $-\overline{E} = -\sqrt{M^{*2} + \vec{q}^2} + \Sigma_v$

Nucleon QCD sum rule in nuclear matter

Investigation of effective masses and vector self energies

$$E = \sqrt{\vec{q}^{\,2} + M^{*2}} + \Sigma^{v}$$

$$u = (1,0)$$

$$\Pi(q) = i \int d^4x e^{iqx} \underline{\theta(x_0)} \langle \Psi_0 | T[\eta(x)\overline{\eta}(0)] | \Psi_0 \rangle$$

$$= \sum_n |\lambda_{n+}^2| \left(\frac{(\sqrt{M^{*2} + \vec{q}^2}\gamma_0 + M^*)}{2\sqrt{M^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^+ + i\epsilon} + \frac{\gamma_i q^i}{2\sqrt{M^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^+ + i\epsilon} \right)$$

+ (contribution of negative parity states)

$$= \not{q}\Pi_1(q^2, q\cdot u) + \Pi_2(q^2, q\cdot u) + \not{u}\Pi_u(q^2, q\cdot u)$$

E^+ and $|\lambda_{n+}^2|$ are obtained from spectral function.

$$q_0\Pi_1 + \Pi_2 \Rightarrow \sum |\lambda_+|^2 \frac{E^+ + M_+^*}{2\sqrt{M_+^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2 \frac{E^- - M_-^*}{2\sqrt{M_-^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^- + i\epsilon},$$

$$q_0\Pi_1 - \Pi_2 \Rightarrow \sum |\lambda_+|^2 \frac{E^+ - M_+^*}{2\sqrt{M_+^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2 \frac{E^- + M_-^*}{2\sqrt{M_-^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^- + i\epsilon},$$

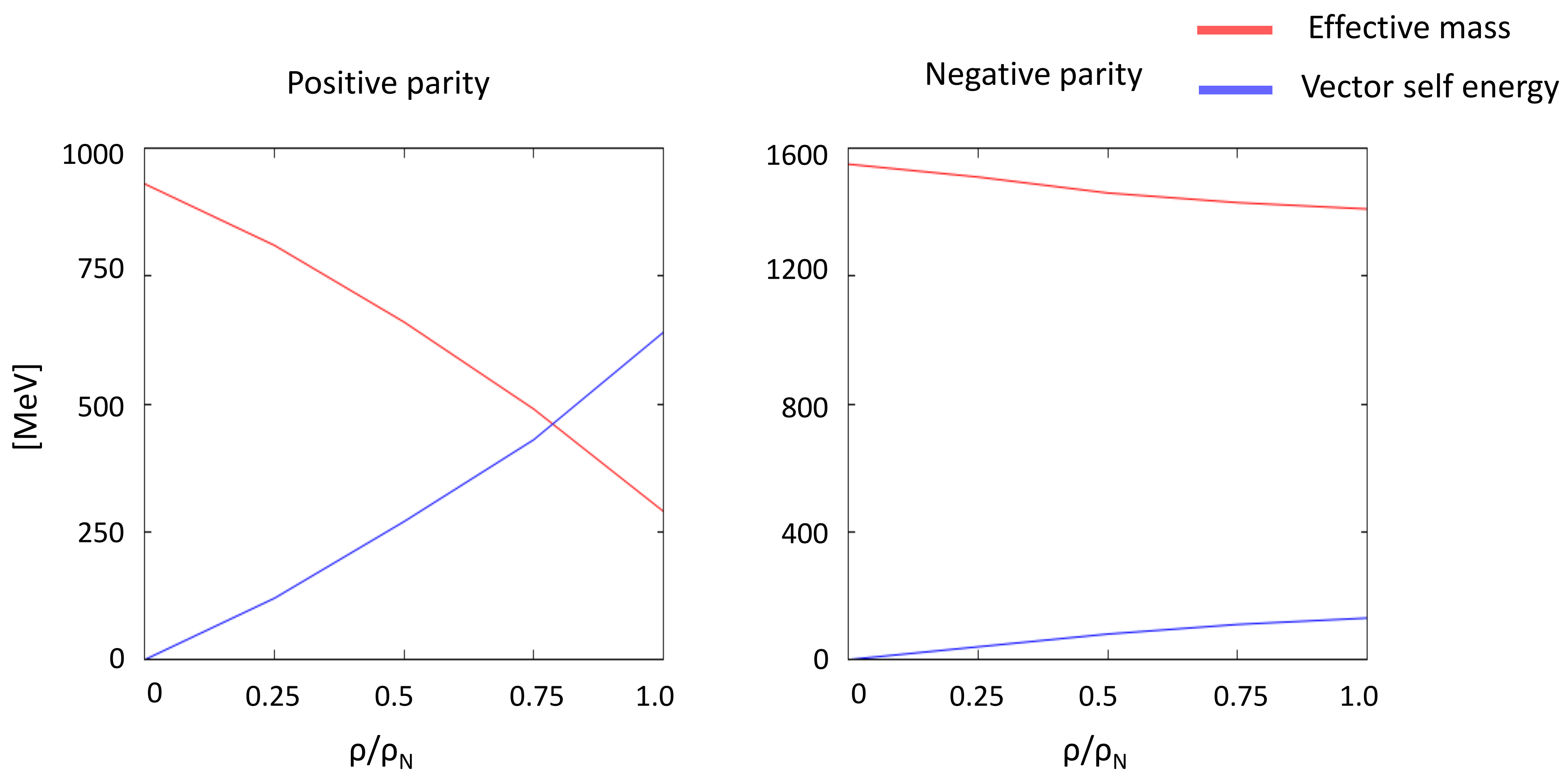
$$\Pi_u \Rightarrow \sum |\lambda_+|^2 \frac{-\Sigma_+^v}{2\sqrt{M_+^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^+ + i\epsilon} + |\lambda_-|^2 \frac{-\Sigma_-^v}{2\sqrt{M_-^{*2}+\vec{q}^2}} \frac{1}{q_0 - E^- + i\epsilon}.$$

↑ are calculated by OPE

↑ are expressed by the propagators in physical energy region

By fitting OPE side and the phenomenological side, we can investigate the self energies.

Nucleon QCD sum rule in nuclear matter



Summary

- We analyze the nucleon spectral function by using QCD sum rules with MEM

- We find that the difference between the positive and negative parity spectral function is mainly caused by the chiral condensate.

- The information of not only the ground state but also the negative parity excited state is extracted

- We apply this method to the analyses in nuclear medium and investigate the effective masses and the vector self-energies.

- As the density increases, the effective masses decrease and the vector self-energies increase.