Gluonic Higgs Scalar, Abelianization and Monopoles in QCD

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In Euclidean QCD, the maximally abelian (MA) gauge is defined so as to minimize the total amount of the off-diagonal gluons

$$R_{\rm off}[A_\mu(\cdot)] \equiv \int d^4x \ {\rm tr}\left\{[\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger\right\} \tag{1}$$

by the SU(N_c) gauge transformation, from which the local MA gauge condition is easily derived as $[\vec{H}, [\hat{D}_\mu, [\hat{D}_\mu, \vec{H}]]] = 0$.
We formulate a gauge invariant description of the MA projection in QCD by introducing the "gluonic Higgs scalar field" $\vec{\phi}(x) \equiv \Omega(x)\vec{H}\Omega^\dagger(x)$ with $\Omega(x) \in {\rm SU}(N_c)$ so as to minimize

$$R[\vec{\phi}(\cdot)] \equiv \int d^4x \ {\rm tr}\left\{[\hat{D}_\mu, \vec{\phi}(x)][\hat{D}_\mu, \vec{\phi}(x)]^\dagger\right\} \tag{2}$$

for an arbitrary given gluon field $\{A_\mu(x)\}$.
The gluonic Higgs scalar $\vec{\phi}(x)$ physically corresponds to a "color-direction" of the non-abelian gauge connection $\hat{D}_\mu$ averaged over μ at each x. The gluonic Higgs scalar $\vec{\phi}(x)$ is diagonalized in the MA gauge, and the monopoles appear at the hedgehog singularities [1,2] of $\vec{\phi}(x)$ in the SU(2) Landau gauge as shown in Fig.1 [3].
Through the projection along $\vec{\phi}(x)$, we can extract the abelian ${\rm U}(1)^{N_c-1}$ sub-gauge-manifold which is most close to the original SU(N_c) gauge manifold. This projection is manifestly gauge invariant, and is mathematically equivalent to the ordinary MA projection. In fact, the infrared-relevant gluonic mode can be extracted by the projection along the color-direction $\vec{\phi}(x)$.

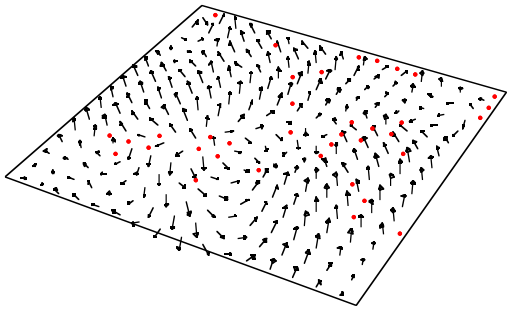

Figure 1: The correlation between the gluonic Higgs scalar field $\phi(x) = \phi^a(x)\frac{\tau^a}{2}$ and monopoles (dots) in the SU(2) Landau gauge in SU(2) lattice QCD with $\beta = 2.4$ and 16^4. The arrow denotes the SU(2) color direction of $(\phi^1(x), \phi^2(x), \phi^3(x))$. The monopole appears at the hedgehog singularity of the gluonic Higgs scalar $\phi(x)$.

References

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