In ordinary space there are only five regular and perfect symmetries, those of the Platonic solids from tetrahedrons to icosahedrons (20 sides). In four dimensions there are six others. This paper focuses upon one of these, an adaptation of Hilbert’s 16-Zell. Offering 64 faces to one hoping to map quarks in a perfectly symmetrical array, it is here made quite visualizable by projection, as with the hypercube, into 3-space. On it the mid-plane vertices (A, B, E, & F) seat the zero-charge baryons while D & H seat -1, and C & G +1 particles. Since all baryons have three quarks, one looks for intersections of three faces; thus Georgi first tried a cube, balancing it on one tip and generating the octet by projecting it downward. But this would not do, for the faces, being square, commit whatever quark it is assigned to four vertices. Better to resort to the inscribed tetrahedron, which commits its quarks to only three corners. In fact, two 16-Zells inscribe, as mirror images, into the hypercube, since it has twice as many vertices as the 16-Zell -- which seats baryons on one tetrahedron and antibaryons on the other; and one can then map mesons, as stars of David, wherever a face from the former parallels and projects upon a face of the latter. Furthermore, one can plot leptons nicely, as points not corners, on the Zell’s eight vertices with neutrinos and electron-like leptons in equal numbers, since the neutral plane has four vertices and the charged ones (H, D, C, & G) also number four. And finally, because a cube inscribes into a dodecahedron in five different orientations, one is presented with ten 64-faced figures -- reminiscent of the ten dimensions of late-century physics.

Unlike the standard octet and decuplet, on which baryons are mapped only as wholes, the 16-Zell lends itself to the plotting of each quark individually and to the assignment, shown here on tables, of one of the seven quarks to each of its 64 faces as decreed by the quantum numbers of that locus. Wherever three such faces meet at a vertex, a baryon is generated, except when a charge mismatch occurs. The color rule that a baryon must be white now simply reflects the fact that such faces can define a vertex only by sloping at three different slants. Clearly, the 16-Zell should supplant the standard model, for 1) by arraying quarks on its tetrahedrons in two different ways, it can plot 1/2-spin or 3/2-spin particles; 2) by questioning that model’s mapping of doubly-charmed baryons, it can replace them with an entire new suit of btb baryons (b’ = the 7th quark); and 3) by offering home seats for the 25 known basic baryons such that these knowns comprise just three-eighths of the 67 baryons so mapped, it avoids being wildly theoretical by rooting the model very firmly in experimental fact.

The Zell also bans any suit of scb baryons noting that the faces to which one assigns c-quarks play a special role, as Hilbert shows, in that they frame-in or encase its 14 nested tetrahedrons instead of being of them. Even certain fine details intrude themselves automatically and surprisingly into these plots: such as 1) there are charmed udc, sdc, & usc baryons, and 2) that, while the multiplet of 1/2-spin baryons includes two uds’s, that of the 3/2-spin baryons has only one uds - exactly as experiment shows.

These successes astonishingly won in the face of extreme constraints -- especially that, for every surface where two tetrahedrons interface, each must bear the same quark -- suggest the baring of nature’s deepest structure.