Properties of excited baryons in a deformed oscillator quark model

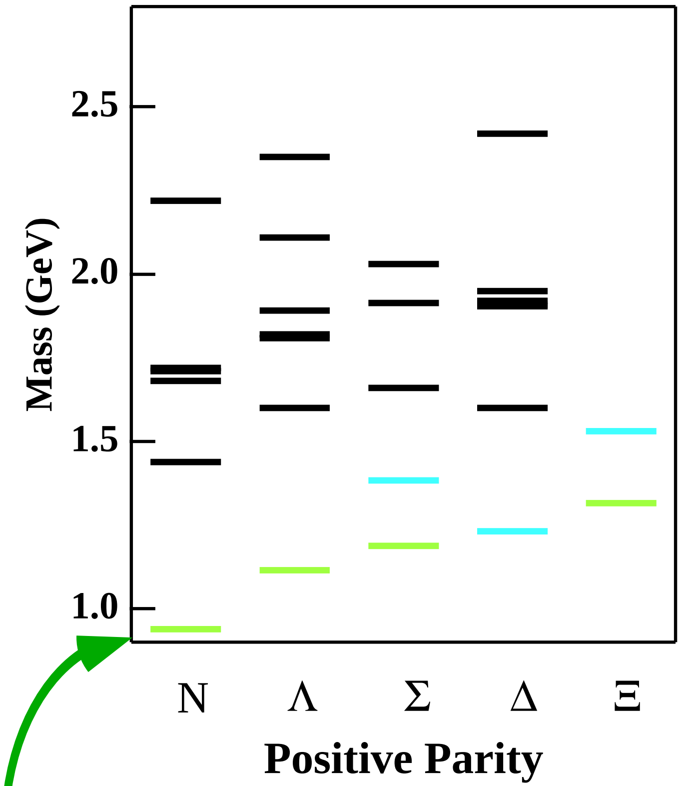
Miho KOMA(TAKAYAMA)
Atsushi HOSAKA
Hiroshi TOKI
(RCNP, Osaka Univ. Japan)

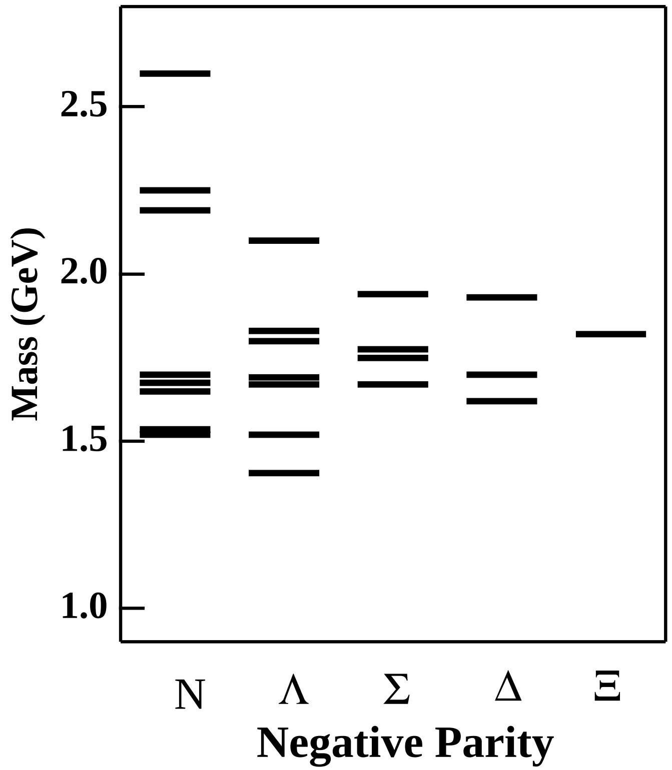
Contents

- Systematics in the masses of excited baryons
- Deformed Oscillator Quark Model (DOQ)
- Intra-band transitions
- Summary

Systematics in the mass spectra

- Observed mass spectra [PDG (2000)]





Ground states

$$m_N = m_0 + 3m_u$$

$$m_\Sigma = m_0 + 2m_u + m_s$$

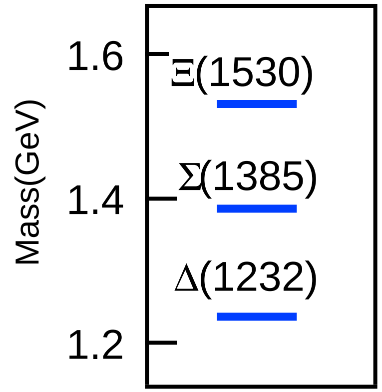
$$m_\Lambda = m_0 + 2m_u + m_s$$

$$m_\Xi = m_0 + 2m_s + m_u$$

- Gell-Mann-Okubo Mass Formula

$$\frac{m_\Sigma + 3m_\Lambda}{2} = m_\Xi + m_N$$

- Equal Mass Splittings of Baryon Decuplet

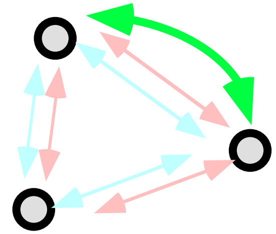


Consitituent Quark Model

- Consitituent Quark model

Baryon → **three quark system**

quark → **m ~ 300 MeV ,**
Spin, Flavor, and Color degrees of freedom



$$|(Baryon)\rangle = |(Orbit)\rangle|(Spin)\rangle|(Flavor)\rangle|(Color)\rangle$$

The wave functions must be Totally anti-symmetric;
change overall sign under every single interchange of any quark pair.

For the ground state ----

Orbit → Symmetric
Color → Anti-symmetirc

Spin → 1/2 (2) Mixed symmetric
3/2 (4) Symmetric

Flavor → (8) Mixed symmetric
(10) Symmetric
(1) Anti-symmetirc

Symmetry of the product of two wave functions

S * S --- S
A * A --- S
S * A --- A

S * MS --- MS
A * MS --- MS
MS * MS --- S, A, MS

^{2}8 and ^{4}10 are allowed

Ground states are assigned to ...

^{2}8 --- **N, Λ, Σ(1189), Ξ(1317)**

^{4}10 -- **Δ, Σ(1385), Ξ(1530), Ω**

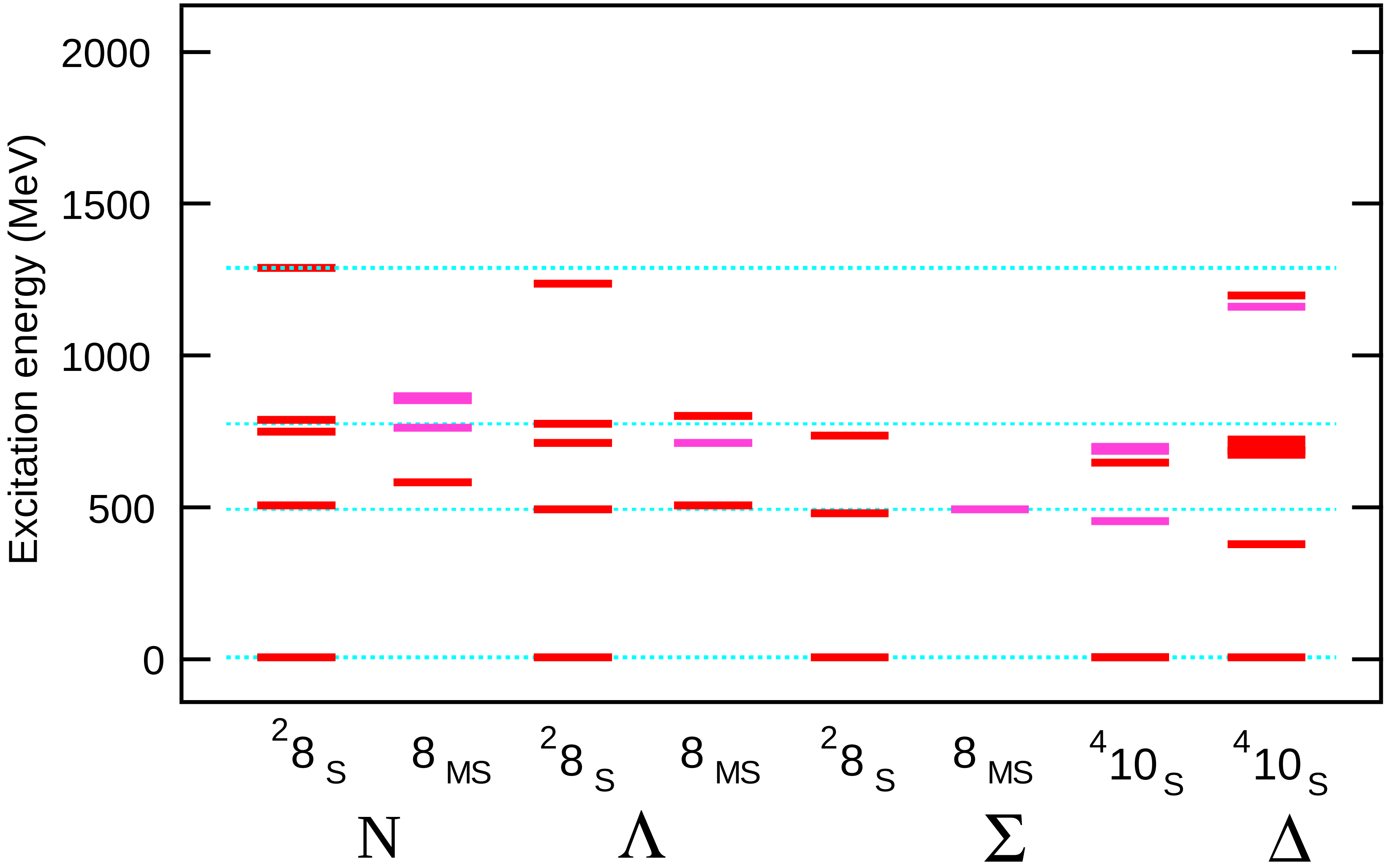
Naming Scheme for Baryons

Isospin	Strangeness			
	$S=0$	$S=-1$	$S=-2$	$S=-3$
$I=0$	-	Λ	-	Ω
$I=1/2$	N	-	Ξ	-
$I=1$	-	Σ	-	-
$I=3/2$	Δ	-	-	-

They are the ground states of each Spin-Flavor Multiplet

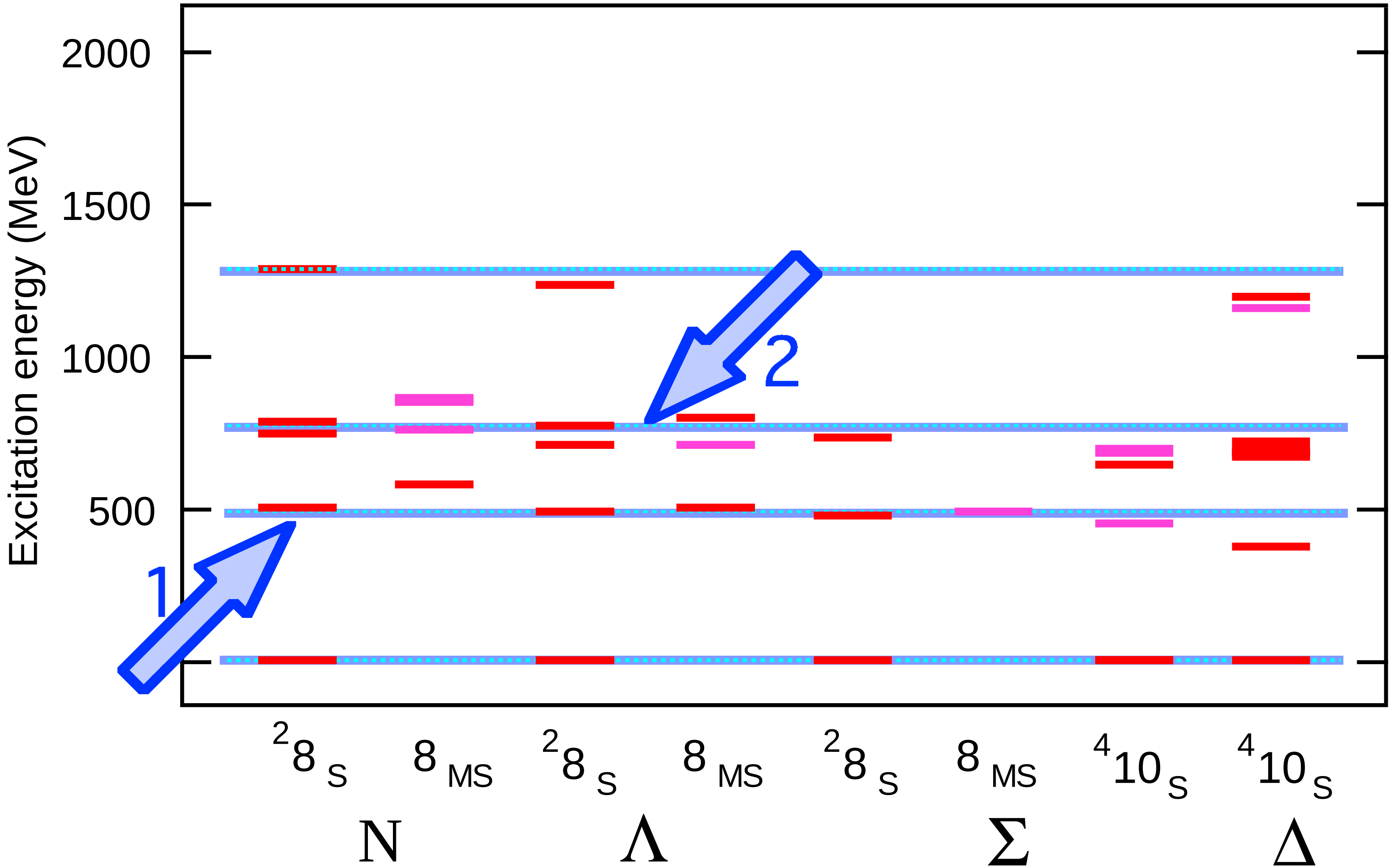
⇨ **Measure the Masses of Excited Baryons from these Ground States**

- After classification ...



Positive Parity

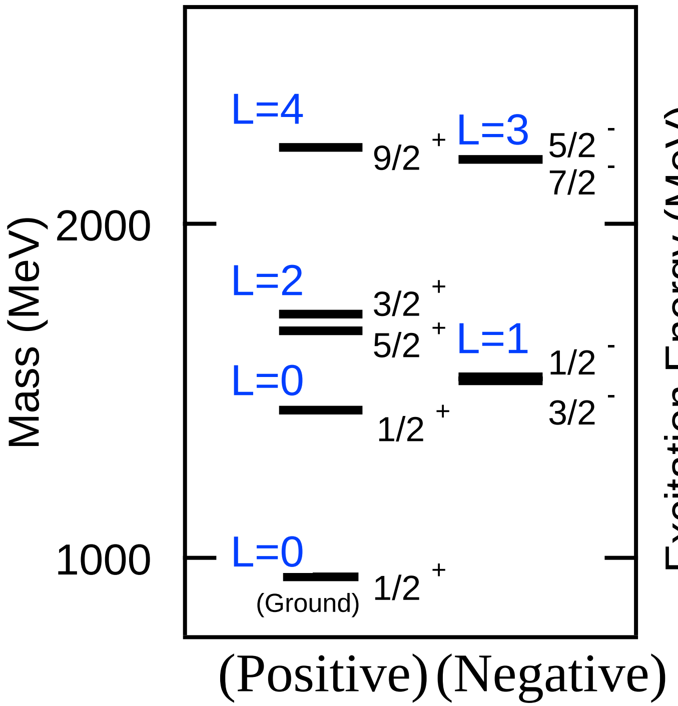
- After classification ...



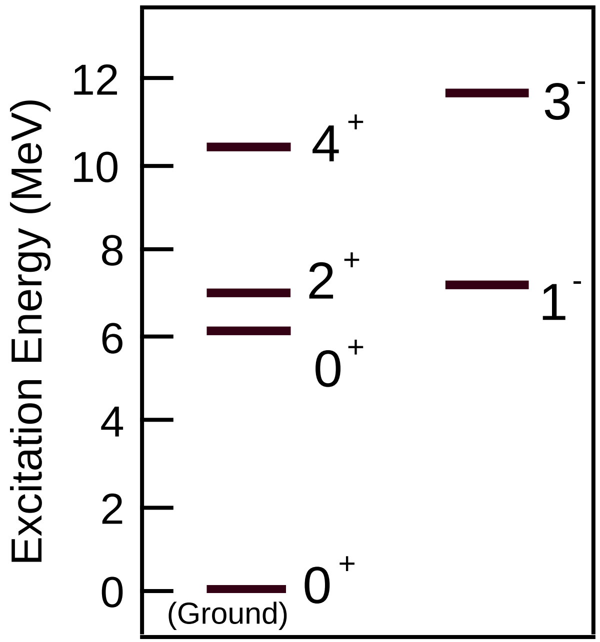
Remarkable Systematics!

1. First Excited States --- 500 MeV
2. Second Excited States --- 700 MeV, Degeneracy of Spin States

- Nucleon spectra -- Excitation energy --



Nucleon



Oxygen

★Feature of the spectra

= Rotational Band Structur

- **Level Spacing (Positive parity < Negative parity) (L(L+1) structure)**
- **Degeneracy of Spin state J=L+S**
- **The Roper state (1/2+) is Identified with the Band Head**

Analogy to Energy Spectrum of Deformed Nuclei

⇓

Are Excited Baryons Spatially Deformed ?

Deformed Oscillator Quark Model (DOQ)

- Hamiltonian

$$H_{DOQ} = \sum_{i=1}^{3}\left(\frac{\mathbf{P}_i^2}{2m} + \frac{1}{2}m(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2)\right) - H_{c.m.}$$

Murthy et al., PR**D30** ('84) 152
Hosaka et al., MPLA **13** ('98) 1699

$$E_{int} = (N_x + 1)\omega_x + (N_y + 1)\omega_y + (N_z + 1)\omega_z$$

$$N_x = n_{\rho_x} + n_{\lambda_x}$$

- Intrinsic state

- **Axial symmetry (for N = $N_x+N_y+N_z$ = 0,1,2) --> $N_x = N_y$**

- **Energy Minimization (Volume Conservation Condition)**

$$\frac{\partial E}{\partial \omega_x} = 0 \quad \frac{\partial E}{\partial \omega_z} = 0 \quad \Leftarrow \quad \omega_x \omega_y \omega_z = \omega^3 = \text{const.}$$

- Energy Minimization (Volume Conservation Condition)

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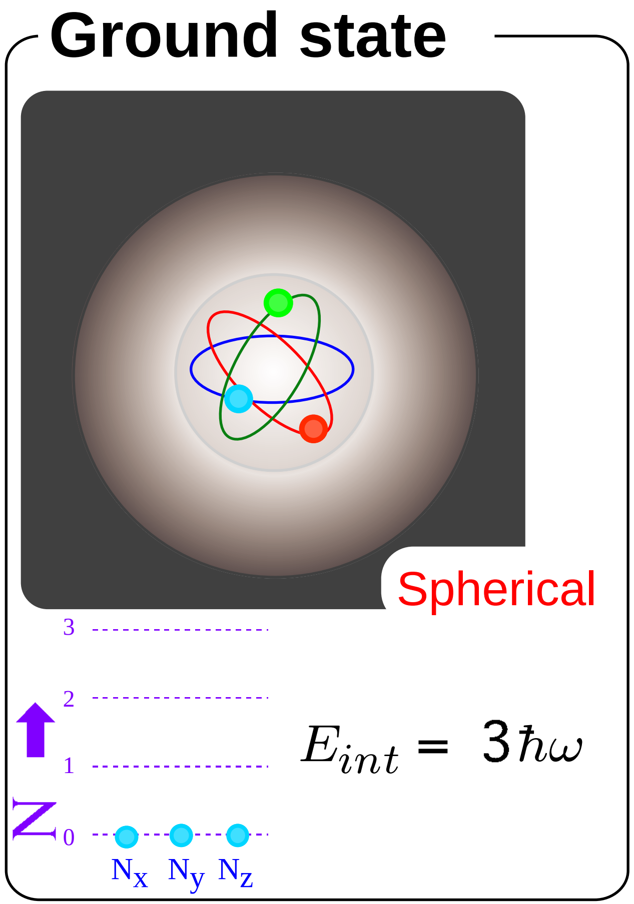
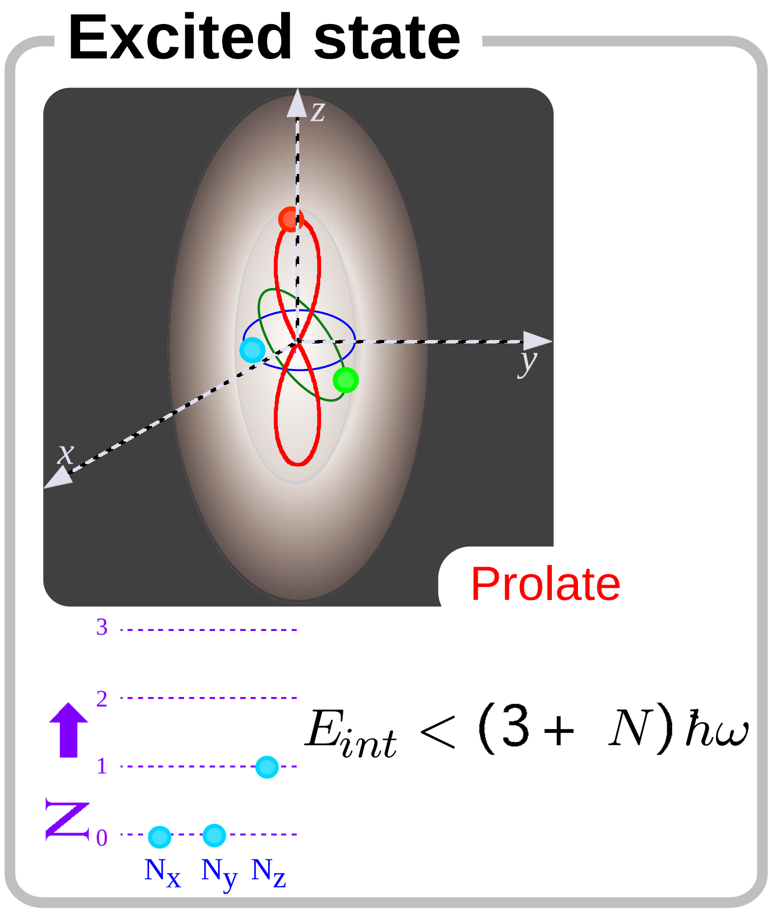
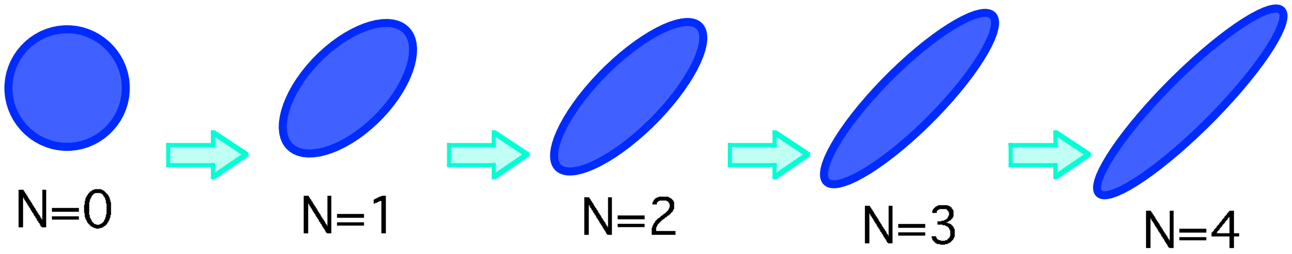




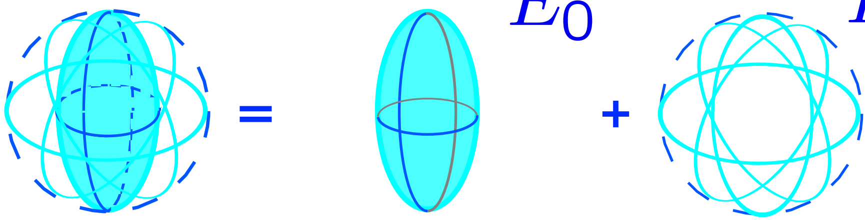
Prolate states have lower energy than oblate states.



$$d \equiv \frac{\omega_x}{\omega_z} \longrightarrow N + 1$$

Total Energy

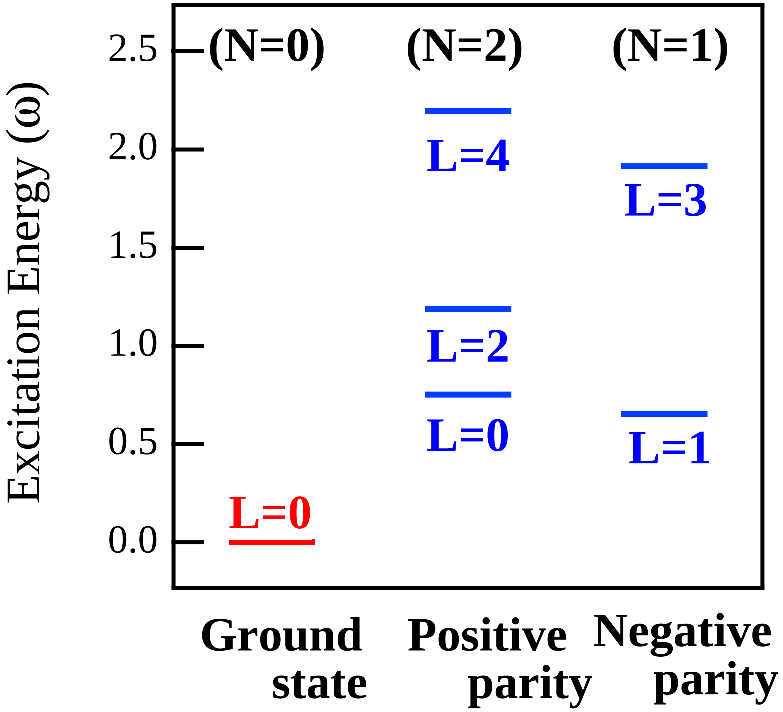
$$E_{total}^{N,L} = \underbrace{E_{int}^{N} - \frac{\langle L^2 \rangle}{2\mathcal{I}}}_{E_0} + \underbrace{\frac{L(L+1)}{2\mathcal{I}}}_{E_{rot}}$$



$\mathcal{I}$:Morment of Inertia
(Cranking Model)

$\langle L^2 \rangle$:Expectation Value of L^2

Moment of Inertia and the expectation value of L^2 can be obtained as a function of N, ω.

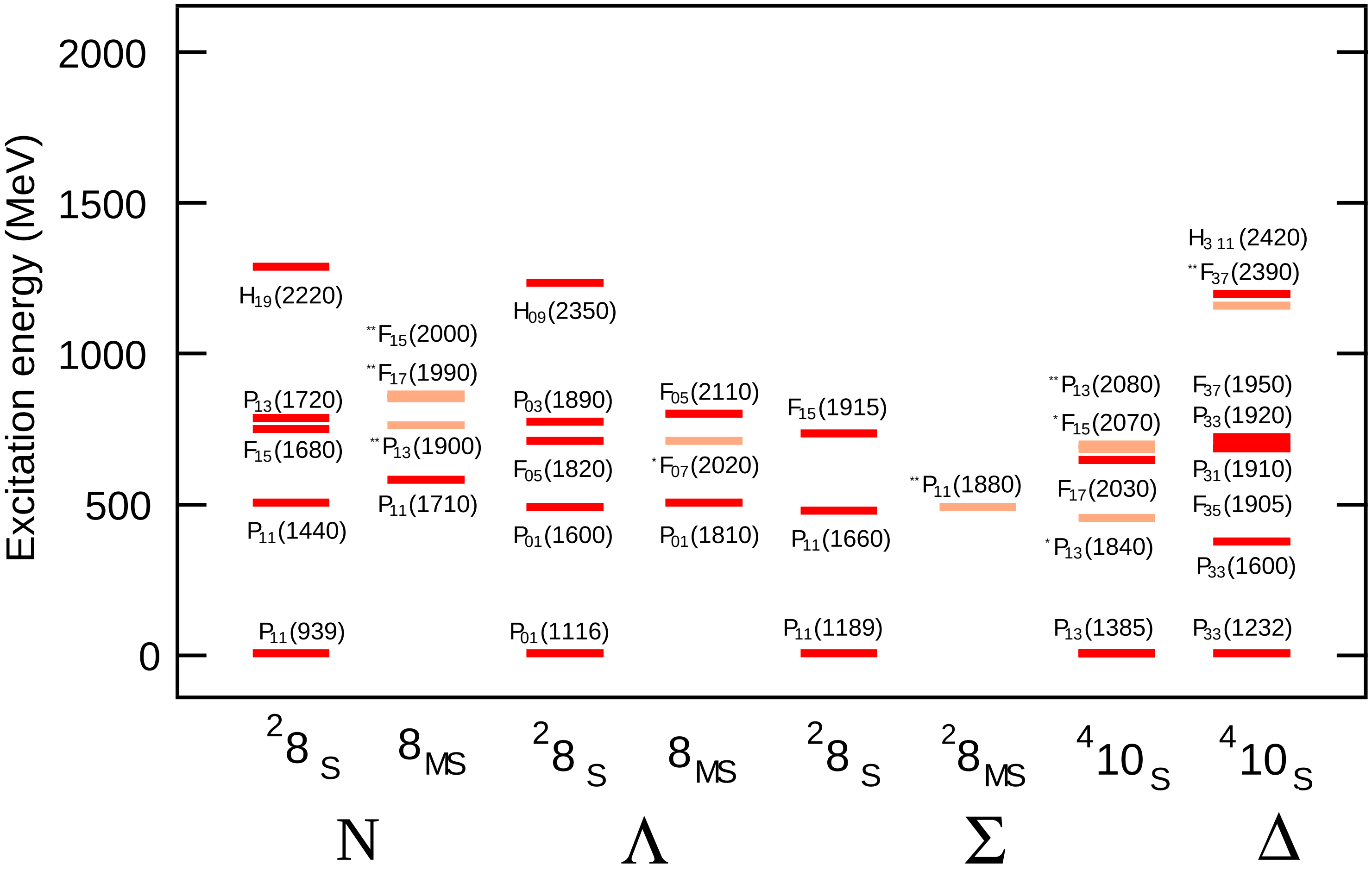


Energy of each state is written in units of ω

ω is our only one parameter!

- Comparison ---Data v.s. DOQ---

★**All existing data with 3 or 4 stars — and some with 1 or 2 stars — are shown.**



Positive Parity

- Comparison ---Data v.s. DOQ---

Takayama et al., PTP **101**('99)

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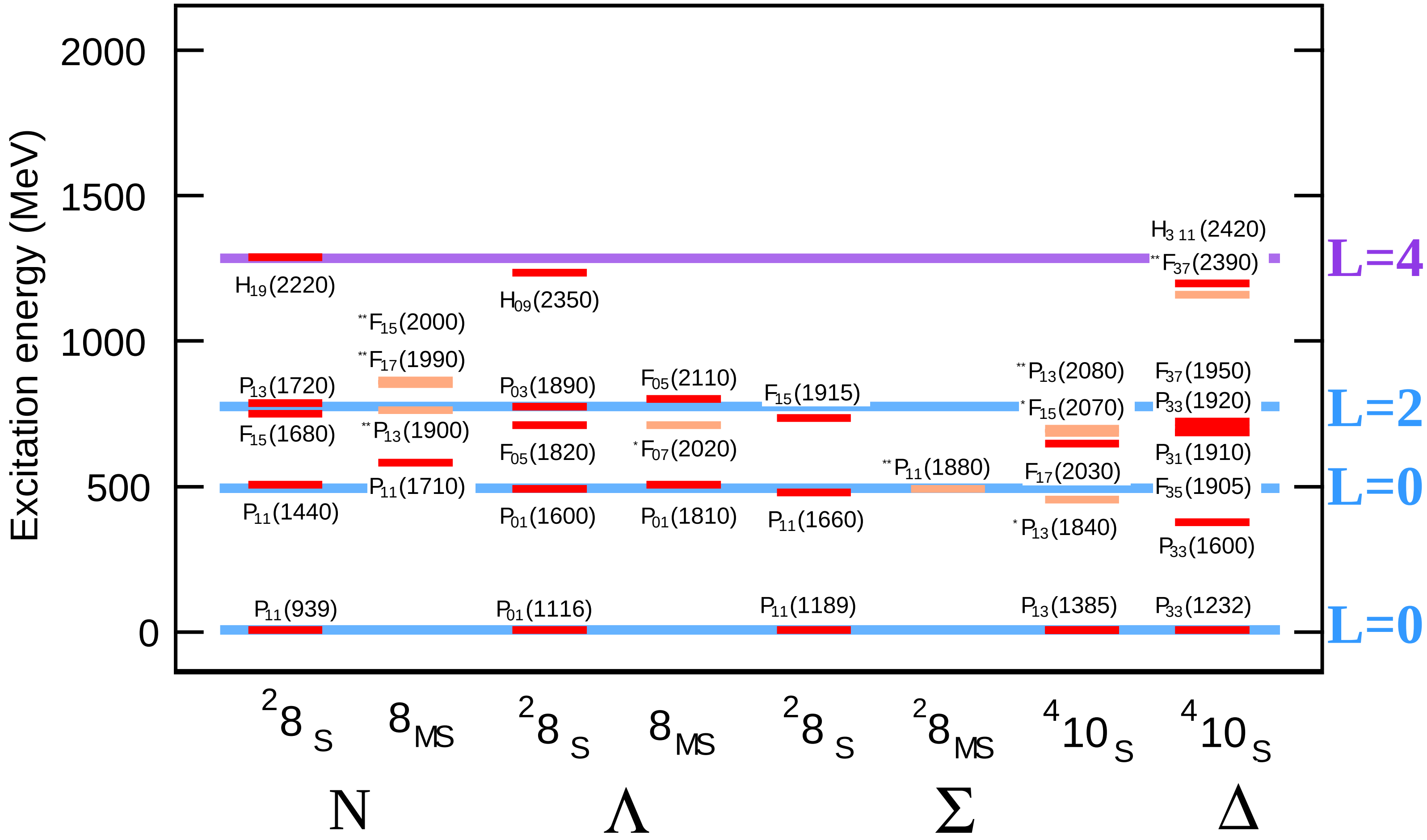
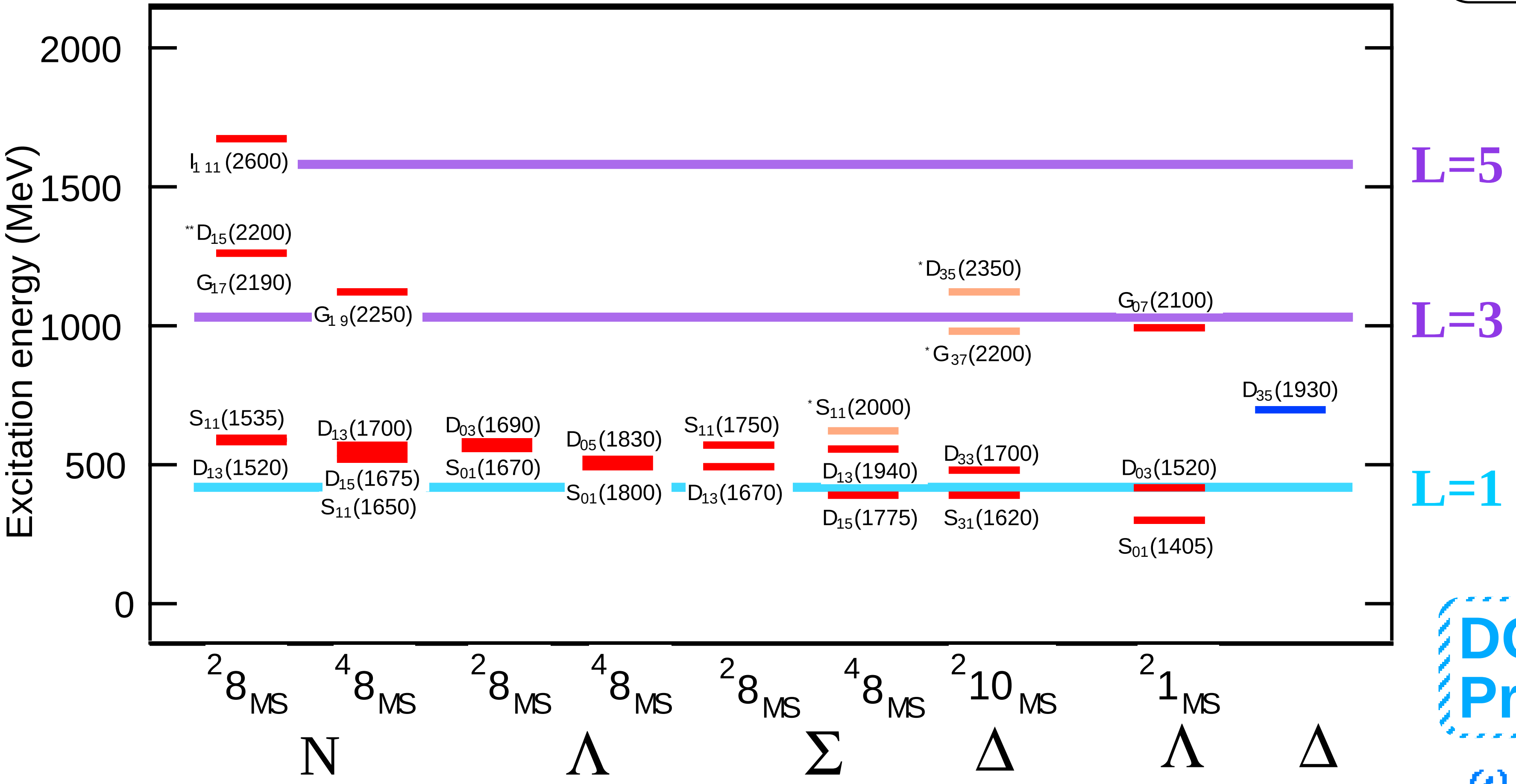


Positive Parity

DOQ Prediction

ω=644MeV

Takayama et al.,
PTP **101**('99)



DOQ Prediction

ω=644MeV

Negative Parity

cf. Regge trajectory (M^2 *vs.* J)

Total Spin J or Angular momentum L

Mass or Excitation Energy

Summary

- We have studied systematically all existing data of flavor SU(3) baryons in the scheme of the DOQ.

- Numbers of States

Almost all states fall into the DOQ systematics.

	(*** , ****)	(* , **)
known baryons	50	31
DOQ	49	13

Single Parameter!
ω=644MeV

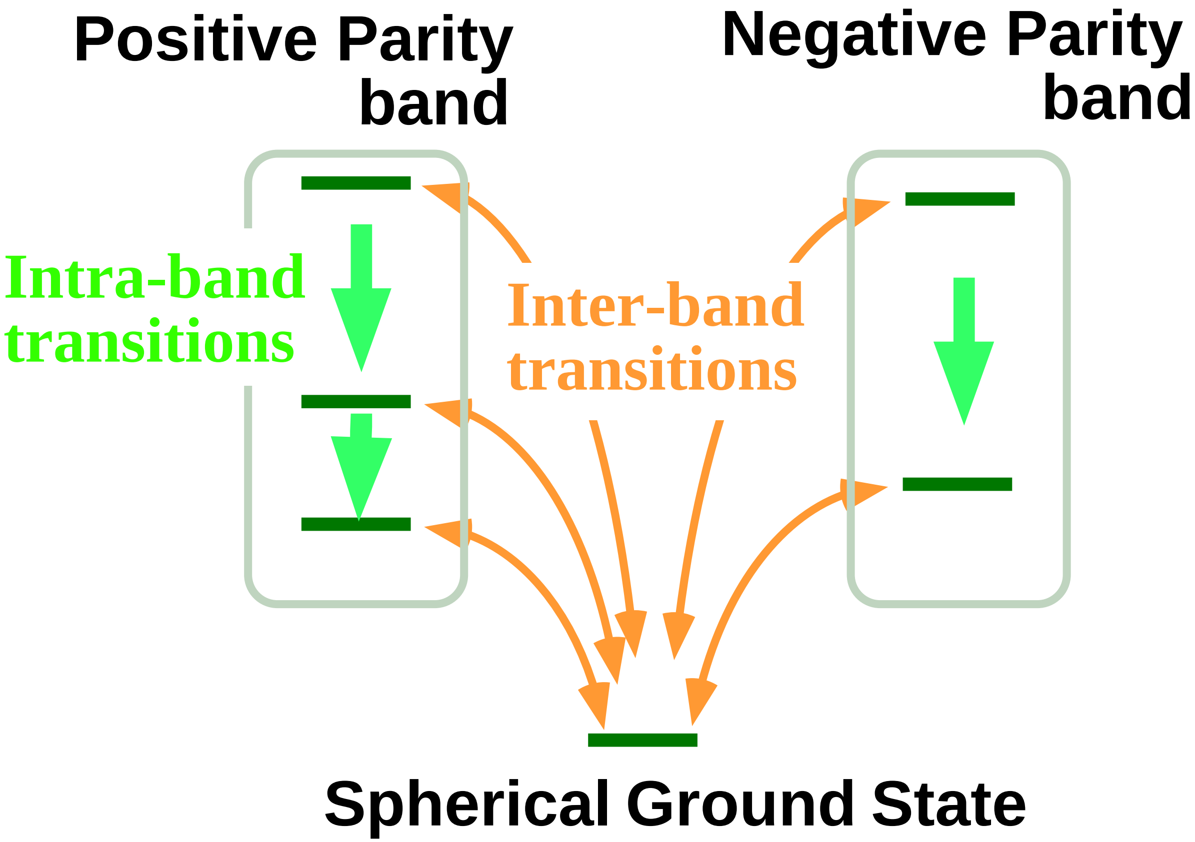
There is a very simple, flavor independent systematics in the masses of flavor SU(3) baryons.

The structure of the spectrum can be described as rotational bands of deformed excited state.

For further study ...

Is there any effect of deformation in the transition properties ?

- Inter–band transitions
- Intra–band transitions



It is difficult to observe Intra-band transitions with a photon

–– Transitions with a pion

Intra-Band transitions in the DOQ Model

We calculate **the decay width** of $N^{*\prime} \to N^{*}\pi$ with **DOQ wave functions** and **Non-relativistic πqq interaction Hamiltonian**

Decay Width:

$$W_{f,i} = 2\pi\delta(E_{P_f} + \omega_k - E_{P_i})\,|\mathcal{M}|^2$$

Transition Amplitude:

$$\mathcal{M} = \langle P_f; J_f, M_f | H | P_i; J_i, M_i \rangle$$

Momentum, spin and its z compoment: $P_{i(f)}; J_{i(f)}, M_{i(f)}$

Energy and momentum of emitted pion: $\vec{k}, \omega_k$

Integrate (Average) over...

Spin z-component of Initial and final states
Momentum and charge of emitted pion

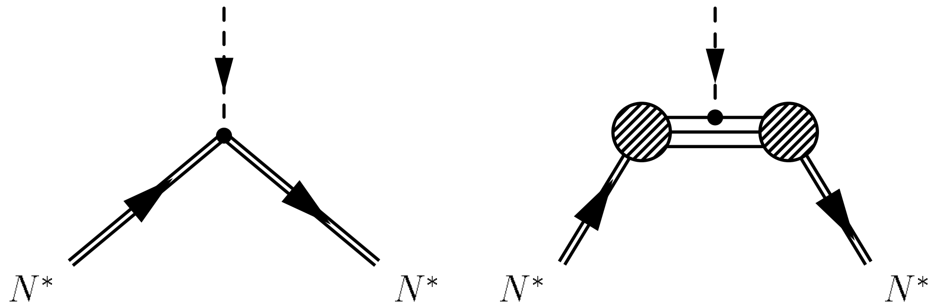
$$W = 6\pi \int \frac{d^3k}{(2\pi)^3}\delta(E_{P_f} + \omega_k - E_{P_i})\frac{1}{2J_i + 1}\sum_{M_i, M_j}|\mathcal{M}|^2$$

Non-relativistic πqq interaction Hamiltonian

$$\mathcal{M} = \langle P_f; J_f, M_f | H | P_i; J_i, M_i \rangle$$

$$\rightarrow 3 \int \Psi^*(P_f; J_f, M_f) H_{\pi qq} \Psi(P_i; J_i, M_i)$$

DOQ wave function



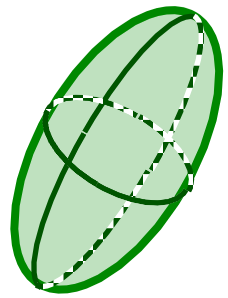
Permutation symmetry of quarks in the baryons

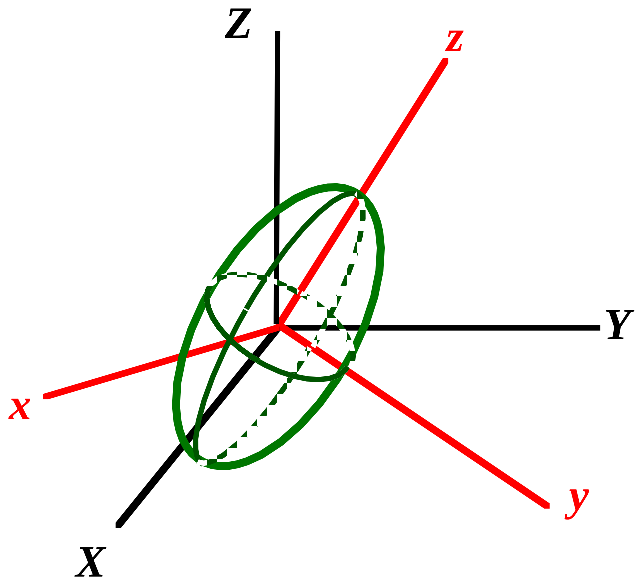
$$H_{\pi qq} = -\frac{g}{2m} \frac{1}{\sqrt{2\omega_k}} \left(\vec{\sigma} \cdot \vec{\nabla} \exp(i\vec{k} \cdot \vec{x}) \right)$$

DOQ wave function --- Angular momentum projection ---

1) **Intrinsic wave functions** obtained by the variation are deformed.

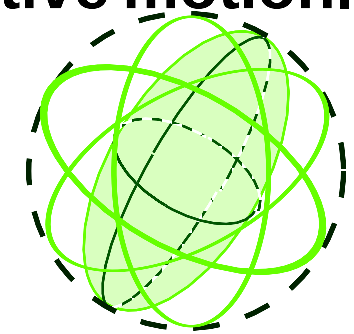
$$\langle\{\vec{x}\}|N^{(int)}\rangle \sim$$





2) Introduce the Eular angle $\Theta=(\alpha,\beta,\gamma)$ which connect **the body-fixed frame** and the Lab-frame.
Wave function can be written as a product of intrinsic wave function and collective motion.

$$D^{L}_{0L_z}(\vec{\Theta})\langle\{\vec{x}\}|N^{(int)}\rangle\rangle \sim$$



Spin-Orbital wave function with total spin J and its z-component M

$$\Psi(P;J,M)=\left[\sqrt{\frac{2L+1}{4\pi}}D^{L}_{0L_z}\langle\{\vec{x}_i\}|N^{(int)}\rangle\otimes|\chi_\mu\rangle\right]_{JM}$$

Transition amplitudes

1) Separate the contibution from the deformed intrinsic state and the collective motion

$$\mathcal{M} = \int d\Theta \langle [L_f, 1/2]^{J_f}_{M_f} | 3\mathcal{O}^{int}_N(\Theta) | [L_i, 1/2]^{J_i}_{M_i} \rangle$$

2) Integration over the body-fixed coordinate
= Operator in the collective coordinate

$$\mathcal{O}^{int}_N(\Theta) \equiv \int d\{\vec{x}_i\} \langle N^{(int)} | \{\vec{x}_i\} \rangle \langle \{\vec{x}_i\} | \hat{R}^\dagger(\Theta) H_{\pi qq} \hat{R}(\Theta) | \{\vec{x}_i\} \rangle \langle \{\vec{x}_i\} | N^{(int)} \rangle$$

$$\mathcal{O}^{int}_N(\Theta, \vec{k}) = -\frac{(4\pi)^{3/2} g}{2m\sqrt{2\omega_k}} \sum_{l,m} i^l k Y^*_{lm}(\hat{k})$$

$$\times \left[\sqrt{\frac{l+1}{(2l+1)(2l+3)}} [Y_{l+1}\sigma]^l_m Q^{(l+1)}_N(k) + \sqrt{\frac{l}{(2l+1)(2l-1)}} [Y_{l-1}\sigma]^l_m Q^{(l-1)}_N(k) \right]$$

l-moment: $Q^{(l)}_N(k) \equiv \langle N^{(int)} | j_l(kx) Y_{l0}(\hat{x}) | N^{(int)} \rangle$

The *l*-moment contain all information of the deformed intrinsic states.

3) Integrate over the collective coordinate

$$\mathcal{M}=\frac{3(4\pi)g}{2m\sqrt{2\omega_k}}\sum_{l,m} i^l k Y^*_{lm}(\hat{k})(-)^m \begin{pmatrix} J_f & l & J_i \\ -M_f & m & M_i \end{pmatrix} \frac{\hat{J}_f \hat{J}_i (-1)^{J_f+L_f+1/2}}{\hat{l}}$$

$$\times \begin{pmatrix} J_i & J_f & l \\ 1/2 & -1/2 & 0 \end{pmatrix} \left[\frac{(L_i-J_i)(2J_i+1)+(L_f-J_f)(2J_f+1)+l+1}{\widehat{l+1}} Q_N^{(l+1)}(k) + \frac{(L_i-J_i)(2J_i+1)+(L_f-J_f)(2J_f+1)-l}{\widehat{l-1}} Q_N^{(l-1)}(k) \right]$$

4) Substitute into the decay width (Integral over the momentum of π)

$$W=\frac{27g^2}{2m^2}\tilde{k}\sum_l \left| F^{(N)}_{l;L_iJ_iL_fJ_f}(\tilde{k}) \right|^2$$

$$\tilde{k}=\sqrt{(E_i-E_f)^2-m_\pi^2}$$

$$F^{(N)}_{l;L_iJ_iL_fJ_f}(k) \equiv i^l k \frac{\hat{J}_f(-1)^{J_f+L_f+1/2}}{(2l+1)} \begin{pmatrix} J_i & J_f & l \\ 1/2 & -1/2 & 0 \end{pmatrix}$$

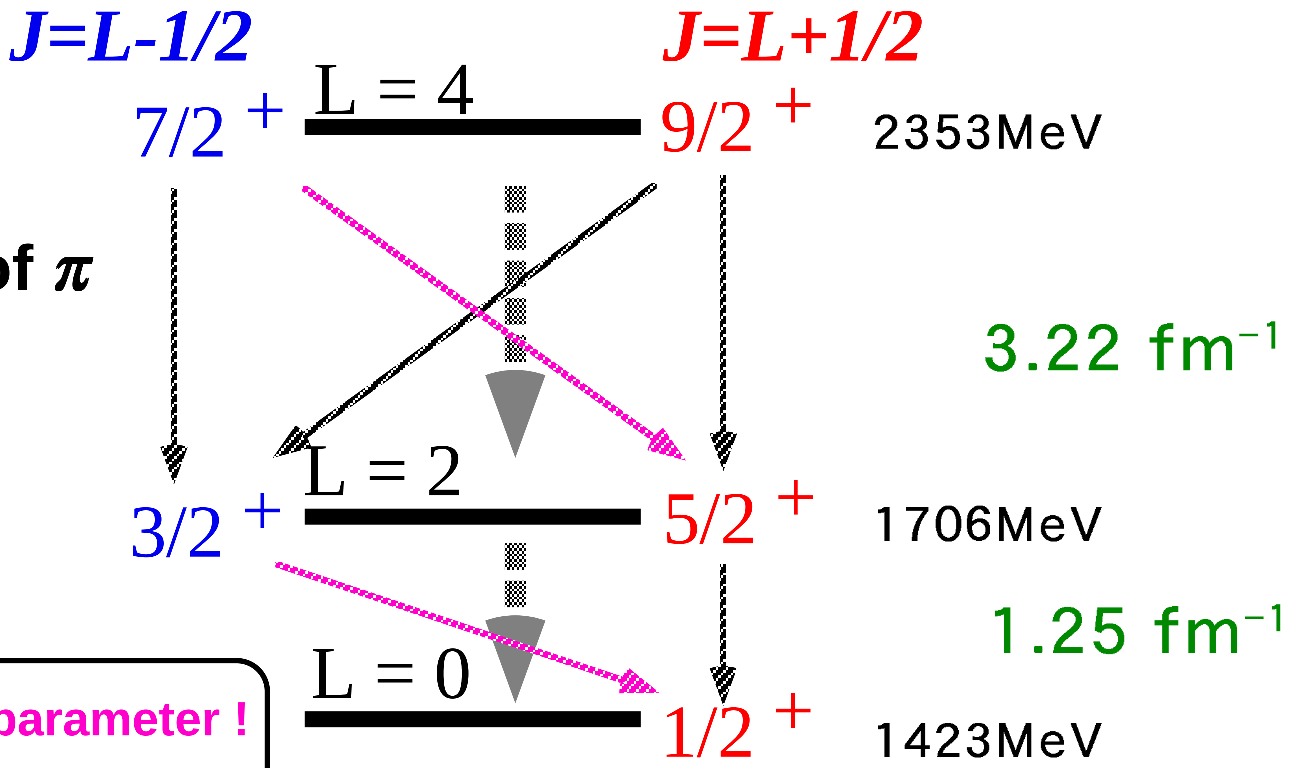
$$\times \left[\frac{(L_i-J_i)(2J_i+1)+(L_f-J_f)(2J_f+1)+l+1}{\widehat{l+1}} Q_N^{(l+1)}(k) + \frac{(L_i-J_i)(2J_i+1)+(L_f-J_f)(2J_f+1)-l}{l-1} Q_N^{(l-1)}(k) \right]$$

(Geometrical factor)

×(*l*-th moment)

Selection rule for N=2, Positive parity band

Angular momentum of π
$l = 1, 3$
(Parity conservation)



Parameters: No fit parameter !

Pion mass :138 MeV
Quark mass:300MeV
DOQ parameter: ω=644MeV
Deformation parameter for N=2: d=3
πqq **coupling constant** g=7.14

Momentum transfer from DOQ spectrum

Decay Width

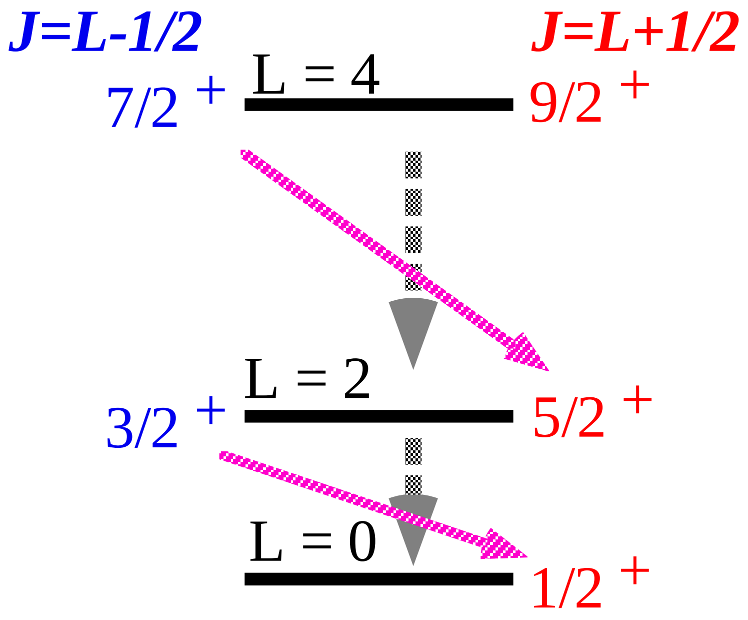
$L = 4 \to 2$	$9/2 \to 5/2$	$9/2 \to 3/2$	$7/2 \to 5/2$	$7/2 \to 3/2$
$\Gamma(\mathrm{MeV})$	**18.9**	**35.3**	**115**	**0.5**
$L = 2 \to 0$	$5/2 \to 1/2$	-	$3/2 \to 1/2$	-
$\Gamma(\mathrm{MeV})$	**0.2**	-	**0.5**	-

***L* = *2* → *0* transitions are almost forbidden**

7/2 → 5/2 transition has large decay width
(a pion with *l* = 1 can contribute in this channel)

Long wave length limit $\tilde{k} \to 0$

Compare 7/2 → 5/2, 3/2 → 1/2 transitions



$$\frac{W(L=4\to 2)}{W(L=2\to 0)} = \frac{9}{7}$$

Contribution of *l = 1* pion is dominant for both

$$W = \frac{27g^2}{2m^2}\tilde{k}\left(\tilde{k}^2 \frac{L-1}{2L-1}\frac{1}{5}(Q_N^{(2)}(\tilde{k}))^2\right) \sim \tilde{k}^7$$

Transitions with low momentum transfer are strongly suppressed.

Ratio of the width is determined only by geometrical factor with the same momentum transfer $\tilde{k}$

$$\frac{W(L+2\to L)}{W(L\to L-2)} = \frac{(L-3)(2L-1)}{(2L-3)(L-1)}$$

(Alaga rule for Baryons)

l-th moment

$$Q^{(l)}_{N=2,S}(k) \equiv \langle (2S)| j_l(kx) Y_{l\,0}(\hat{x}) |(2S)\rangle$$

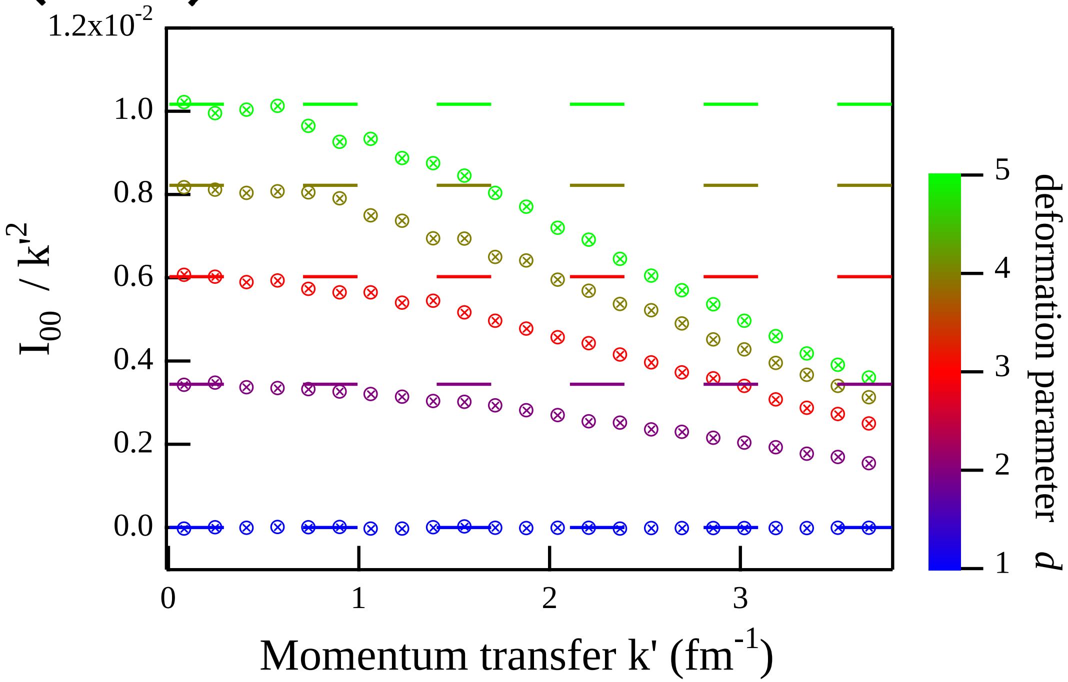
$$= \int d^3\rho d^3\lambda \Psi^{2S}(\vec{\rho},\vec{\lambda}) j_l(k'\lambda) Y_{l\,0}(\hat{\lambda}) \Psi^{2S}(\vec{\rho},\vec{\lambda})$$

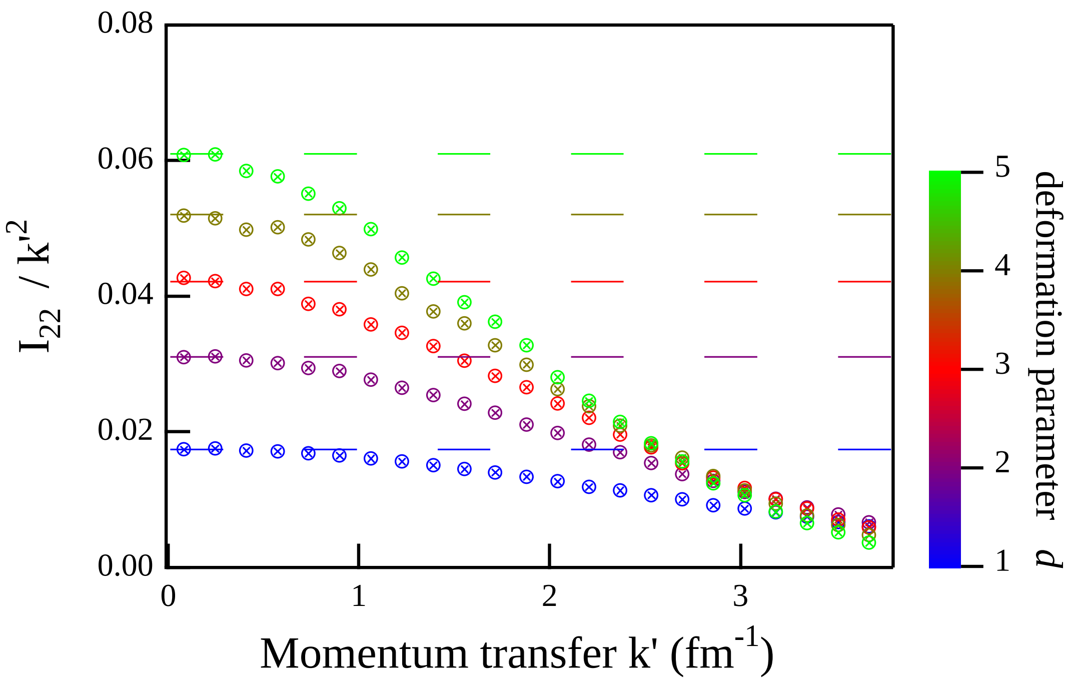
$$= \frac{1}{2}\int d^3\lambda \left[\left(\psi_{000}(\vec{\lambda}) j_l(k'\lambda) Y_{l\,0}(\hat{\lambda})\psi_{000}(\vec{\lambda})\right) + \left(\psi_{002}(\vec{\lambda}) j_l(k'\lambda) Y_{l\,0}(\hat{\lambda})\psi_{002}(\vec{\lambda})\right)\right]$$

$$\equiv \frac{1}{2}\left[I^{(l)}_{00} + I^{(l)}_{22}\right],$$

Symmetrized DOQ wave function in the Jacobi coordinate

(l = 2)

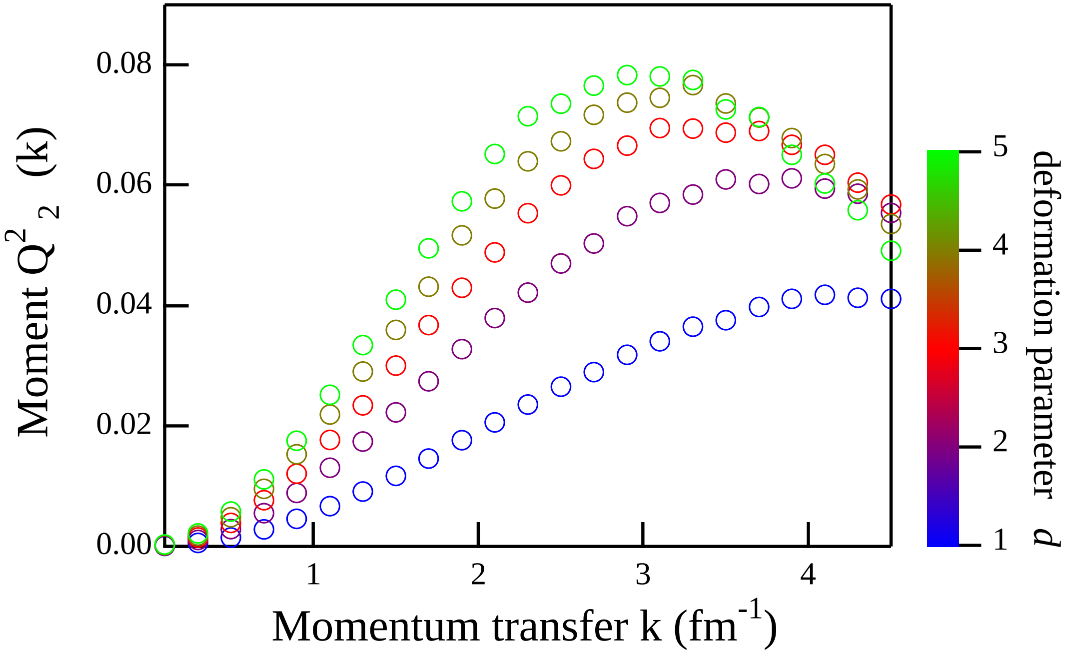




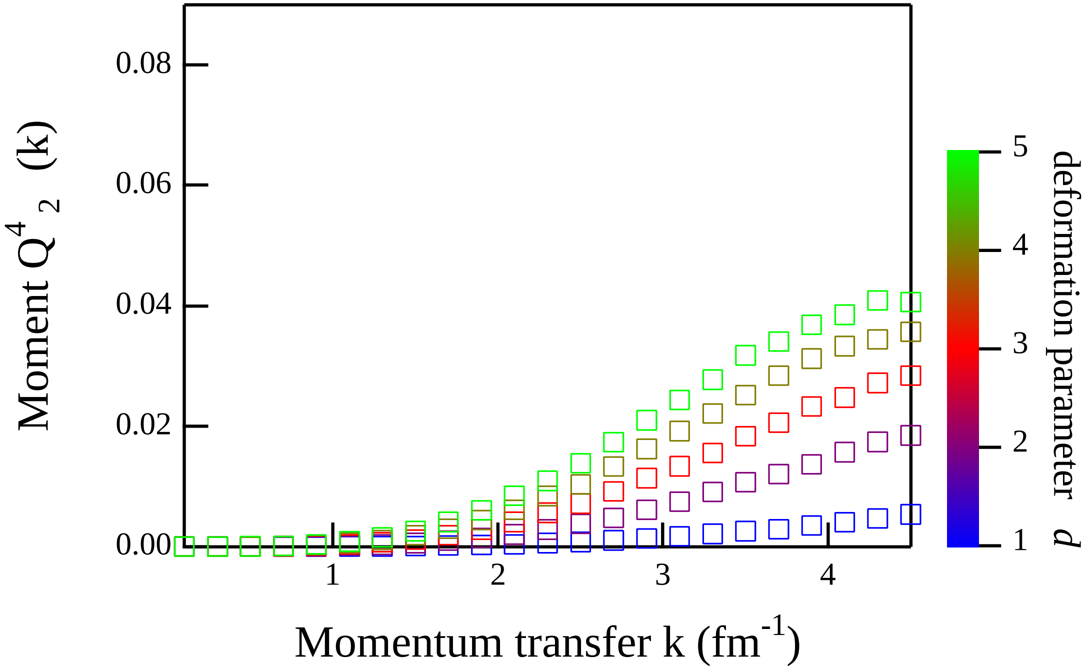
Dashed line -- long wave length limit

l-th moment

(*l* = 2)



(*l* = 4)

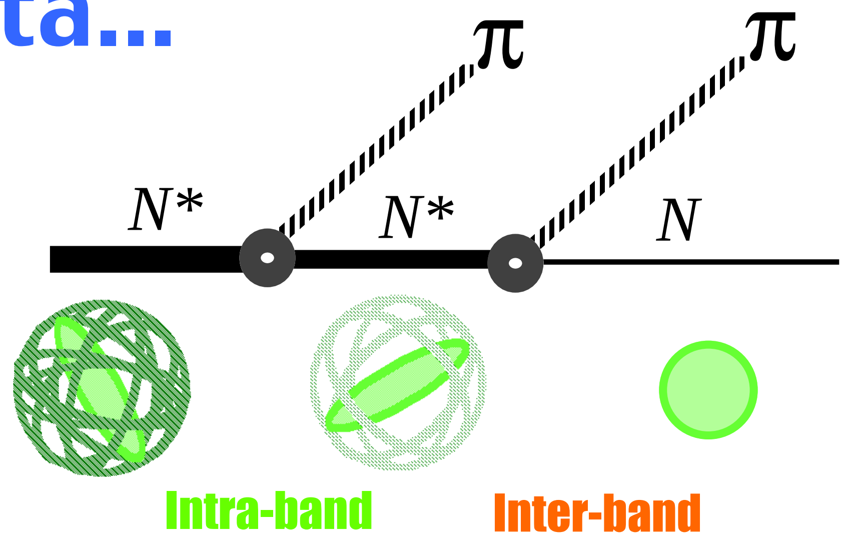


To compare with the data...

Experimentally, Intra-band transitions may be observed in the

$$N^* \to N\pi\,\pi$$

process



However, $L = 2 \to L = 0$ transitions, where some $N^* \to N\pi\,\pi$ processes are observed, are almost forbidden.

There is no analysis with the decay into $N\pi\pi$ for $L = 4$ baryons. Moreover, $J = 7/2$ baryon is not identified yet.

(due to the large decay width from intra-band transitions?!)

N(2220) DECAY MODES $(J = 2/9)$

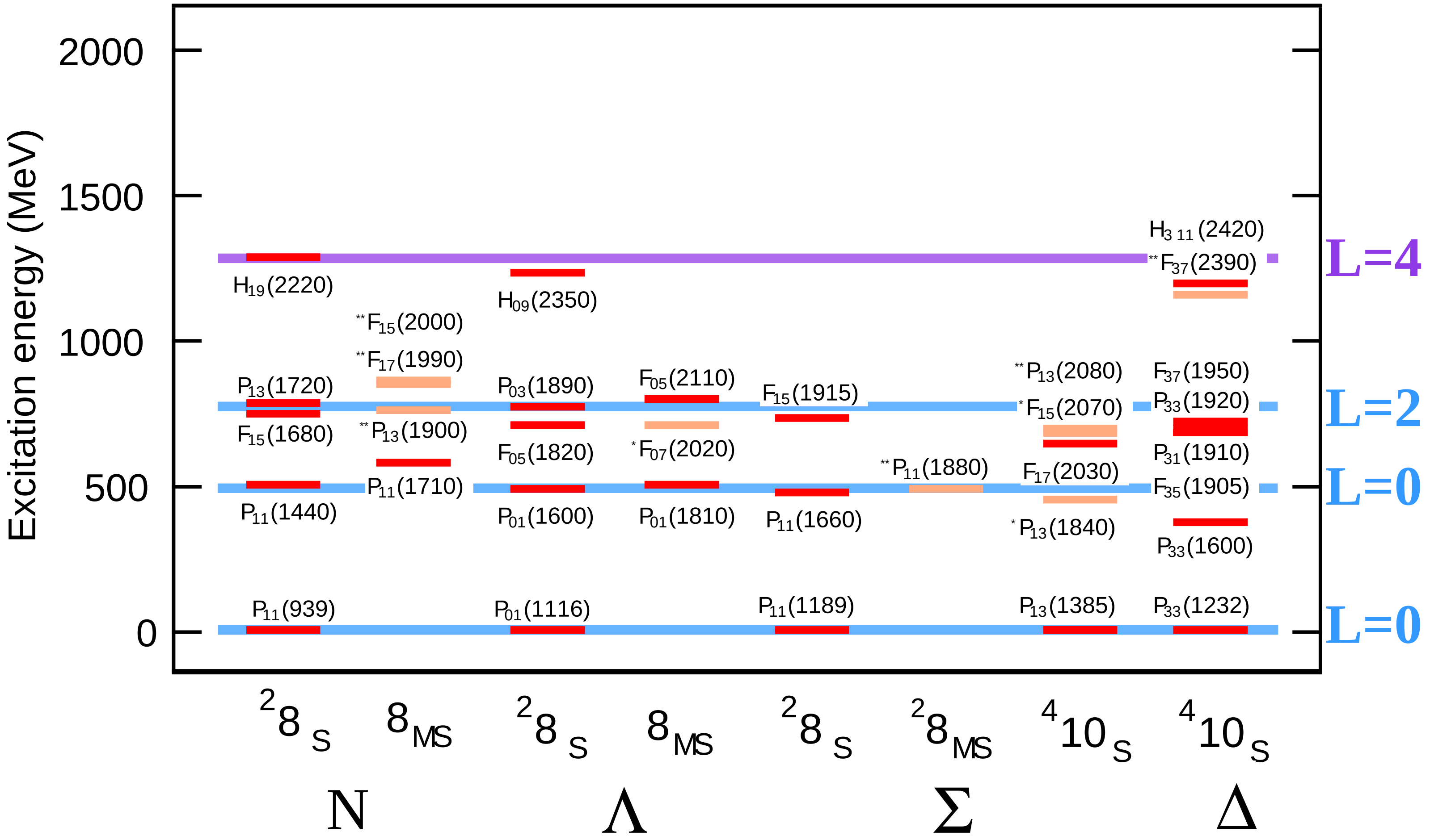
The following branching fractions are our estimates, not fits or averag

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	10–20 %
Γ_2	$N\eta$	
Γ_3	ΛK	

- Comparison ---Data v.s. DOQ---

Takayama et al., PTP **101**('99)

★**All existing data** with 3 or 4 stars — and some with 1 or 2 stars — are shown.



Positive Parity

DOQ Prediction

ω=644MeV

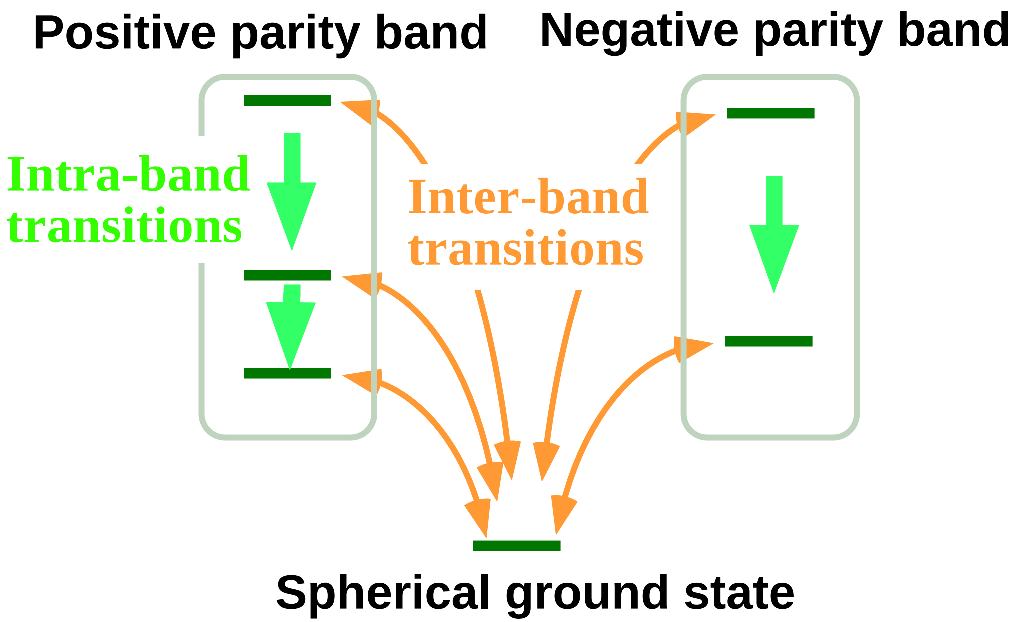
Summary and Future Plan

Mass spectrum of flavor SU(3) baryons ***Deformation!***

- **Systematics for all flavor multiplets**
- **Rotational bands formed by deformed excited states**

Is there any signs of the deformation in various transitions?

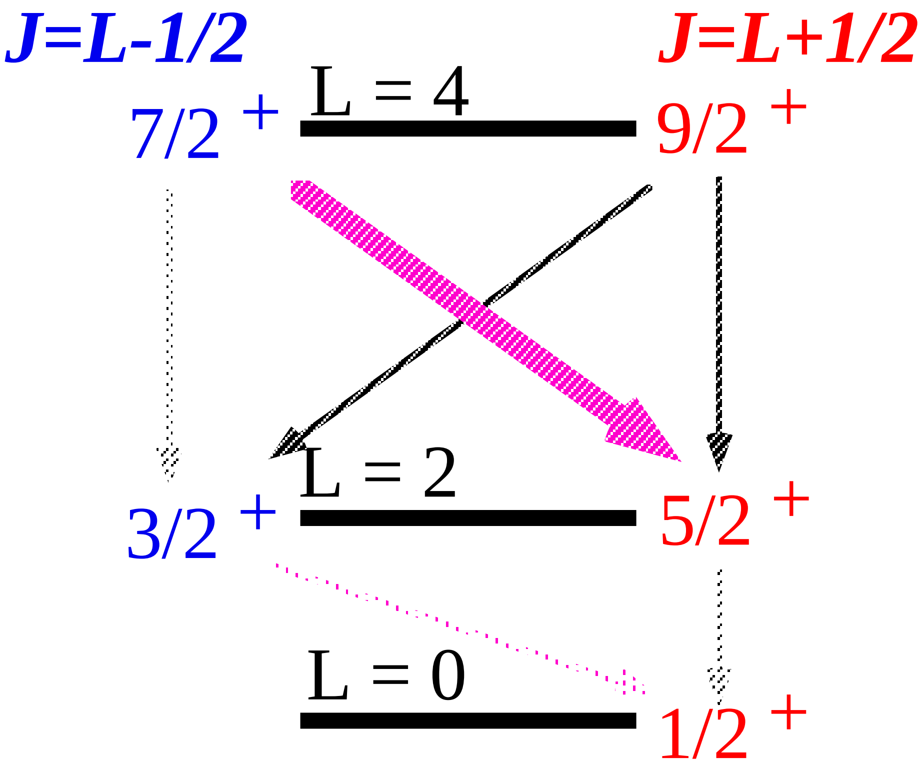
- **Inter-band transitions**
- **Intra-band transitions**



We have calculated the decay widths of intra-band transitions with pions in the DOQ model

Transitions from J = 5/2, 3/2
-- forbidden

Transitions from J = 7/2
-- Large decay width



Next, we will calculate ....

- **Negative parity band**
- **Inter-band transitions with pions**
- **Decay into** $\Delta(1232)\,\pi$

= the decay width of $N^* \rightarrow N\pi\pi$ **process in the DOQ model**

Angular momentum of π
l = 1, 3
(Parity conservation)