ADVANCES IN THE THEORY OF $0\nu\beta\beta$ DECAY

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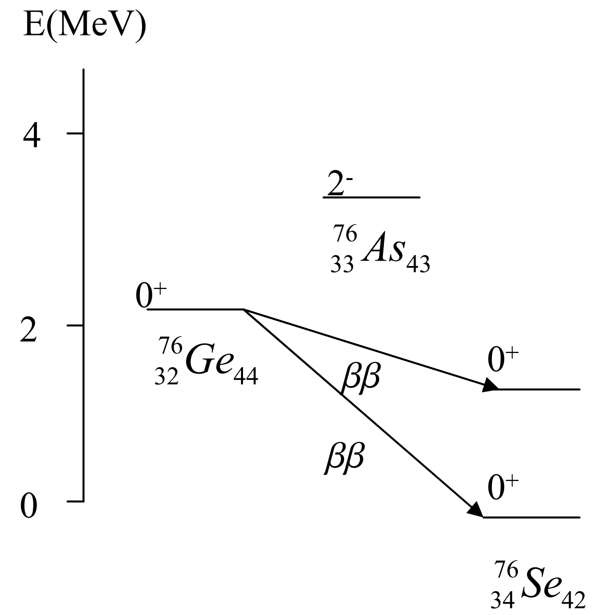
NDM12 (Neutrino and Dark Matter)

INTRODUCTION

Fundamental Process $0\nu\beta\beta$:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-} $$



Half-life for the process:

$$ \left[T_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+})\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^{2}\left|f_{b}(m_{i},U_{ei})\right|^{2} $$

Phase-space factor
(Atomic physics)

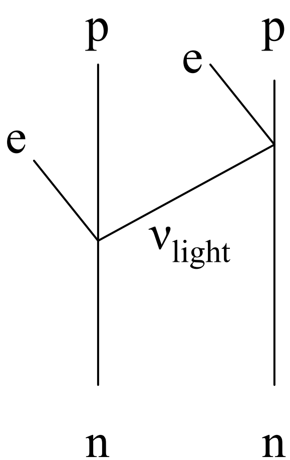
Matrix elements
(Nuclear physics)

Beyond the standard model
(Particle physics)

1. ADVANCES IN PARTICLE PHYSICS

The transition operator *T(p)* depends on the model of 0νββ decay. Three scenarios have been considered [¶,§].

1



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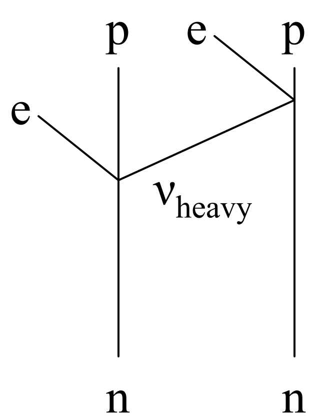

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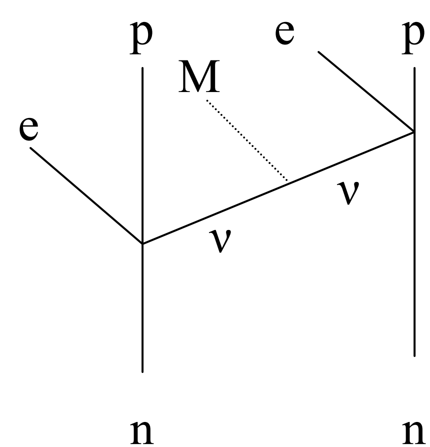


$m_{\nu light} \ll 1\text{MeV}$ $\quad$ $m_{\nu heavy} \gg 1\text{GeV}$

¶ T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

§ F.Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

After the discovery of neutrino oscillations, attention has been focused on scenario 1.

Brief theory of T(p). Scenario 1: Light neutrinos

Weak interaction Hamiltonian

$$H^{\beta} = \frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma_{\mu}\left(1-\gamma_5\right)\nu_{eL}\right]J_L^{\mu\dagger} + h.c.$$

All recent calculations use the formulation of Šimkovic *et al.* § for the nucleon current, thus making comparison between different nuclear models easier

$$J_L^{\mu\dagger} = \bar{\Psi}\tau^{+}\left[\underbrace{g_V(q^2)\gamma^{\mu}}_{\text{vector}} \underbrace{- ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p}q_{\nu}}_{\text{weak-magnetism}} \underbrace{- g_A(q^2)\gamma^{\mu}\gamma_5}_{\text{axial vector}} \underbrace{+ g_P(q^2)q^{\mu}\gamma_5}_{\text{induced pseudo-scalar}}\right]\Psi$$

HOC

q^{μ} = momentum transferred from hadrons to leptons

§ F. Šimkovic *et al.*, loc.cit.

From the weak interaction Hamiltonian, H^β, and the weak nucleon current, J^μ , one finds the transition operator, $T(p)$, which can be written as

$$T(p) = H(p)\frac{\langle m_\nu \rangle}{m_e} \qquad\qquad f_b = \frac{\langle m_\nu \rangle}{m_e}$$

with $p = |\vec{q}|$ and

$$\boxed{\langle m_\nu \rangle = \sum_{k=ligth} (U_{ek})^2 m_k}$$

Advance: The average light neutrino mass is now well constrained by atmospheric, solar, reactor and accelerator neutrino oscillation experiments [§]

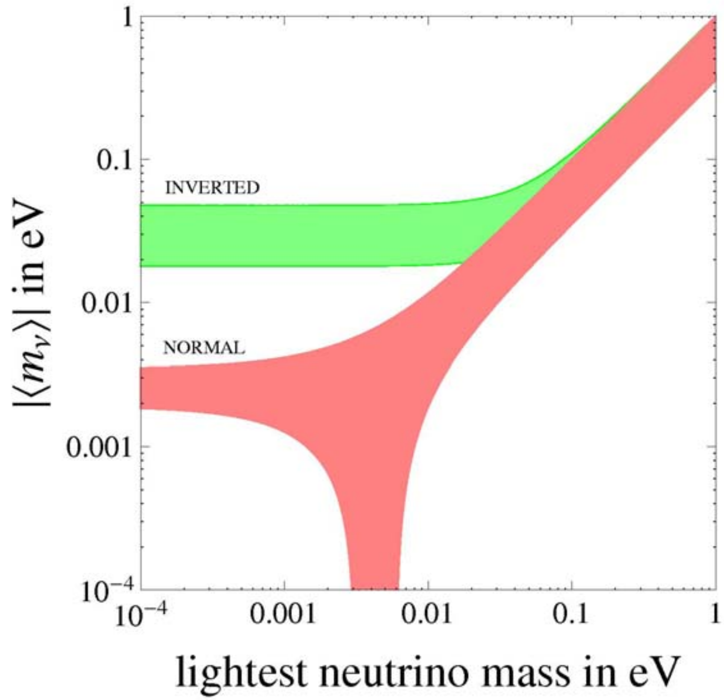
$$\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3} \right|$$

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \varphi_{2,3} = [0, 2\pi]$$

$$\left(m_1^2, m_2^2, m_3^2\right) = \frac{m_1^2 + m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

$$\sin^2\theta_{12} = 0.312, \sin^2\theta_{13} = 0.016, \sin^2\theta_{23} = 0.466$$

$$\delta m^2 = 7.67\times10^{-5} eV^2, \Delta m^2 = 2.39\times10^{-3} eV^2$$



[§] G.L. Fogli *et al.*, Phys. Rev. D75, 053001(2007); D78, 033010 (2008).

Brief theory of H(p). Scenario 1

To lowest order and in momentum space, *H(p)*, can be written as

$$H(p) = \sum_{n,n'} \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p)\vec{\sigma}_n \bullet \vec{\sigma}_{n'}]$$

Higher order corrections (HOC) induce a tensor term, and modify the Fermi and Gamow-Teller terms, producing an operator [§]

$$H(p) = \sum_{n,n'} \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p)\vec{\sigma}_n \bullet \vec{\sigma}_{n'} + h^T(p) S_{nn'}^p]$$

[The general formulation of Tomoda [¶] includes more terms, nine in all, 3GT, 3F, 1T, one pseudoscalar (P) and one recoil (R). This formulation is no longer used but it will have to be revisited if a very accurate description of 0νββ is needed.]

[§] F. Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999). [¶] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

The form factors $h^{F,GT,T}(p)$ are given by:

$$h^{F,GT,T}(p) = v(p)\tilde{h}^{F,GT,T}(p)$$

with

$$\boxed{v(p) = \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}}$$

Ã=closure energy=$1.12A^{1/2}$(MeV)

called neutrino "potential", and $\tilde{h}(p)$ listed by Šimkovic *et al.*

The finite nucleon size (FNS) is taken into account by taking the coupling constants, g_V and g_A, momentum dependent

$$g_V(p^2) = g_V \frac{1}{\left(1+\frac{p^2}{M_V^2}\right)^2}$$

$$g_V = 1; M_V^2 = 0.71\left(GeV/c^2\right)^2$$

$$g_A(p^2) = g_A \frac{1}{\left(1+\frac{p^2}{M_A^2}\right)^2}$$

$$g_A = 1.25; M_A^2 = 1.09\left(GeV/c^2\right)^2$$

Short range correlations (SRC) are taken into account by convoluting the “potential” *v(p)* with the Jastrow function *j(p)* parametrized in various forms (Miller-Spencer, MS/ Argonne/CD Bonn) or by other methods (UCOM)

$$u(p) = \int v(p - p')\, j(p')\, dp'$$

Brief theory of $T_h(p)$. Scenario 2: Heavy neutrinos

During the last year, scenario 2 has again become of interest. The transition operator for this scenario is the same as for 1, but with

$$T_h(p) = H_h(p) m_p \left\langle m_{\nu_h}^{-1} \right\rangle \qquad f_b = m_p \left\langle \frac{1}{m_{\nu_h}} \right\rangle$$

$$\boxed{\left\langle m_{\nu_h}^{-1} \right\rangle = \sum_{k=heavy} \left(U_{ek_h}\right)^2 \frac{1}{m_{k_h}}}$$

Brief theory of $H_h(p)$. Scenario 2

Same as for scenario 1 but with neutrino “potential”

$$\boxed{v(p) = \frac{2}{\pi} \frac{1}{m_p m_e}}$$

Constraints on the average inverse heavy neutrino mass are model dependent.
Advance: V. Tello *et al.*, Phys. Rev. Lett. 106, 151801 (2011) have recently worked out constraints from lepton flavor violating processes and (potentially LHC experiments). In this model,

$$f_b \equiv \eta = \frac{M_W^4}{M_{WR}^4} \sum_{k=heavy} \left(V_{ek_h}\right)^2 \frac{m_p}{m_{k_h}}$$

$$M_W = 80.41 \pm 0.10 GeV$$
$$M_{WR} = 3.5 TeV$$

[Also, a fourth scenario is currently being discussed, namely the mixing of two more "sterile" neutrinos, 4 and 5, with mass ~1eV.]

2. ADVANCES IN NUCLEAR PHYSICS

Calculation of the "nuclear matrix elements" $M_{0\nu}$

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Calculations up to 2008:

1. Quasi-particle random phase approximation (QRPA).
   Results depend on fine-tuning of the interaction, especially near the spherical-deformed transition, for example ^{150}Nd.
2. Shell model (SM).
   Cannot address nuclei with many particles in the valence shells, for example ^{150}Nd, due to the exploding size of the Hamiltonian matrices ($>10^9$).

Recent advances >2009:

3. Development of a program to compute $0\nu\beta\beta$ (and $2\nu\beta\beta$) nuclear matrix elements in the closure approximation within the framework of the microscopic Interacting Boson Model (IBM-2).

EVALUATION OF MATRIX ELEMENTS IN IBM-2 ¶

All matrix elements, F, GT and T, can be calculated at once using the compact expression:

$$V_{s_1,s_2}^{(\lambda)} = \frac{1}{2}\sum_{n,n'} \tau_n^+ \tau_{n'}^+ \left[\Sigma_n^{(s_1)} \times \Sigma_{n'}^{(s_2)}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

$$\lambda = 0, s_1 = s_2 = 0 (F)$$

$$\lambda = 0, s_1 = s_2 = 1 (GT)$$

$$\lambda = 2, s_1 = s_2 = 1 (T)$$

In second quantized form:

$$V_{s_1,s_2}^{(\lambda)} = -\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2}\sum_J (-1)^J \sqrt{1+(-1)^J \delta_{j_1 j_2}} \sqrt{1+(-1)^J \delta_{j'_1 j'_2}}$$

$$\times G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1\, j'_2; J) \left[\left(\pi_{j_1}^\dagger \times \pi_{j_2}^\dagger\right)^{(J)} \cdot \left(\tilde{\nu}_{j'_1} \times \tilde{\nu}_{j'_2}\right)^{(J)} \right]$$

Creates a pair of protons with angular momentum J

Annihilates a pair of neutrons with angular momentum J

¶ J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).

The fermion operator V is then mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)^{(2)}_M \mapsto B_\pi(j,j') d_{\pi,M}^\dagger$$

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2}\sum_{j_1}\sum_{j'_1} G_{s_1 s_2}^{(\lambda)}\left(j_1 j_1 j'_1\, j'_1;0\right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

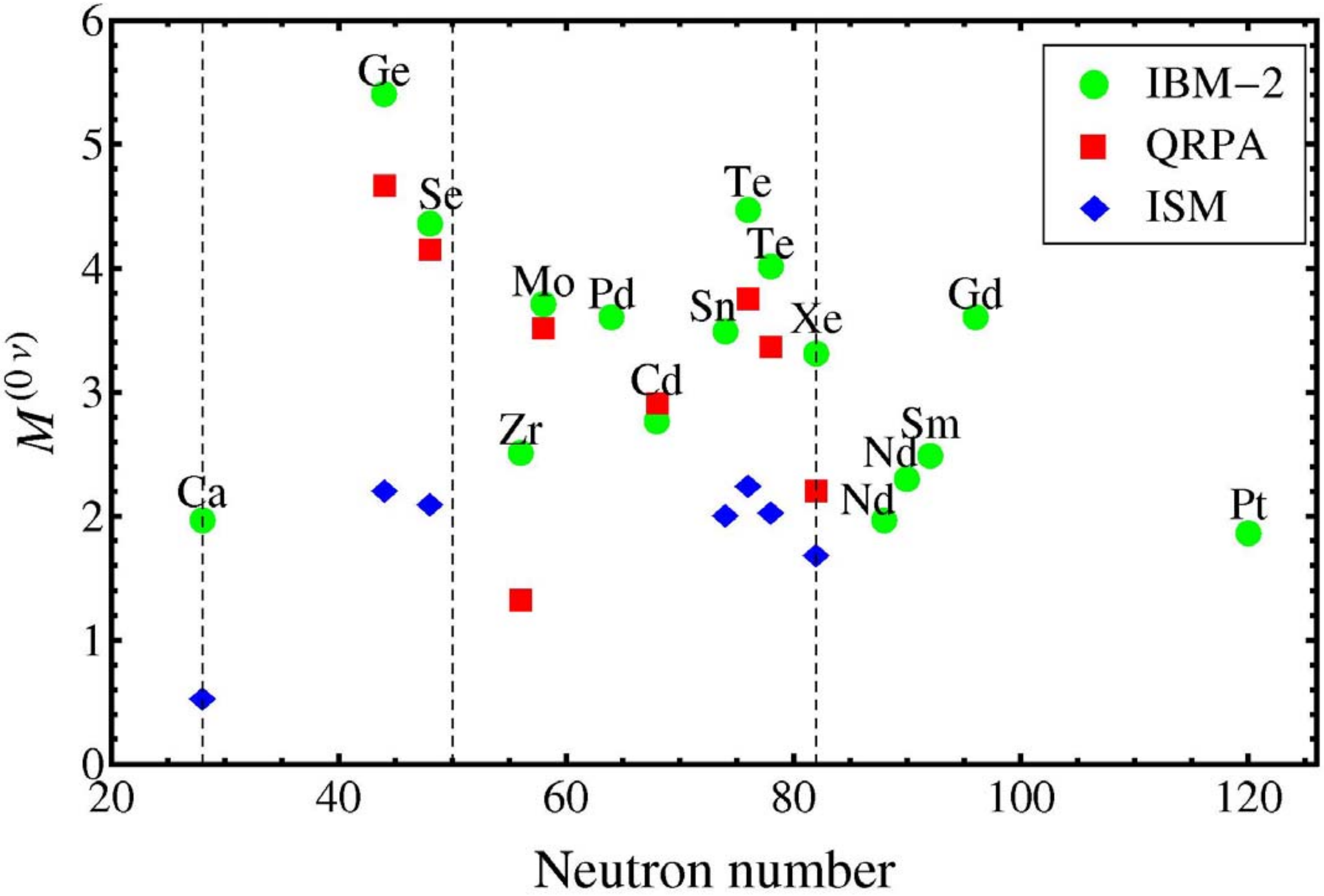
$$-\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2}\sqrt{1+\delta_{j_1 j_2}}\sqrt{1+\delta_{j'_1 j'_2}}\, G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1\, j'_2;2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

The coefficients A, B are obtained by means of the so-called OAI mapping procedure [§]

Matrix elements of the mapped operators are then evaluated with realistic wave functions of the initial and final nuclei taken from the literature. They fit all experimental data for excitation energies, B(E2) values and quadrupole moments, B(M1) values and magnetic moments, etc., very well.

[§] T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309, 1 (1978).

IBM-2 RESULTS (JAN 2012)
LIGHT NEUTRINO EXCHANGE



IBM-2 from J. Barea and F. Iachello, Phys. Rev. C 79, 044301 (2009) and to be published.
QRPA from F. Šimkovic *et al.*, Phys. Rev. C 77, 045503 (2008).
ISM from E. Caurier *et al.*, Phys. Rev. Lett. 100, 052503 (2008).

MS-SRC

SENSITIVITY ANALYSIS (IBM-2)

Estimated sensitivity to input parameter changes:

1. Single-particle energies [¶,§] 10%
2. Strength of surface delta interaction 5%
3. Oscillator parameter 5%
4. Closure energy 5%

Estimated sensitivity to model assumptions:

1. Truncation to S, D space 1% (spherical)-10% (deformed)
2. Isospin purity 1%(GT)-20%(F)-1%(T)

Estimated sensitivity to operator assumptions:

1. Form of the operator 5%
2. Finite nuclear size (FNS) 2%
3. Short range correlations (SRC) [#] 5%

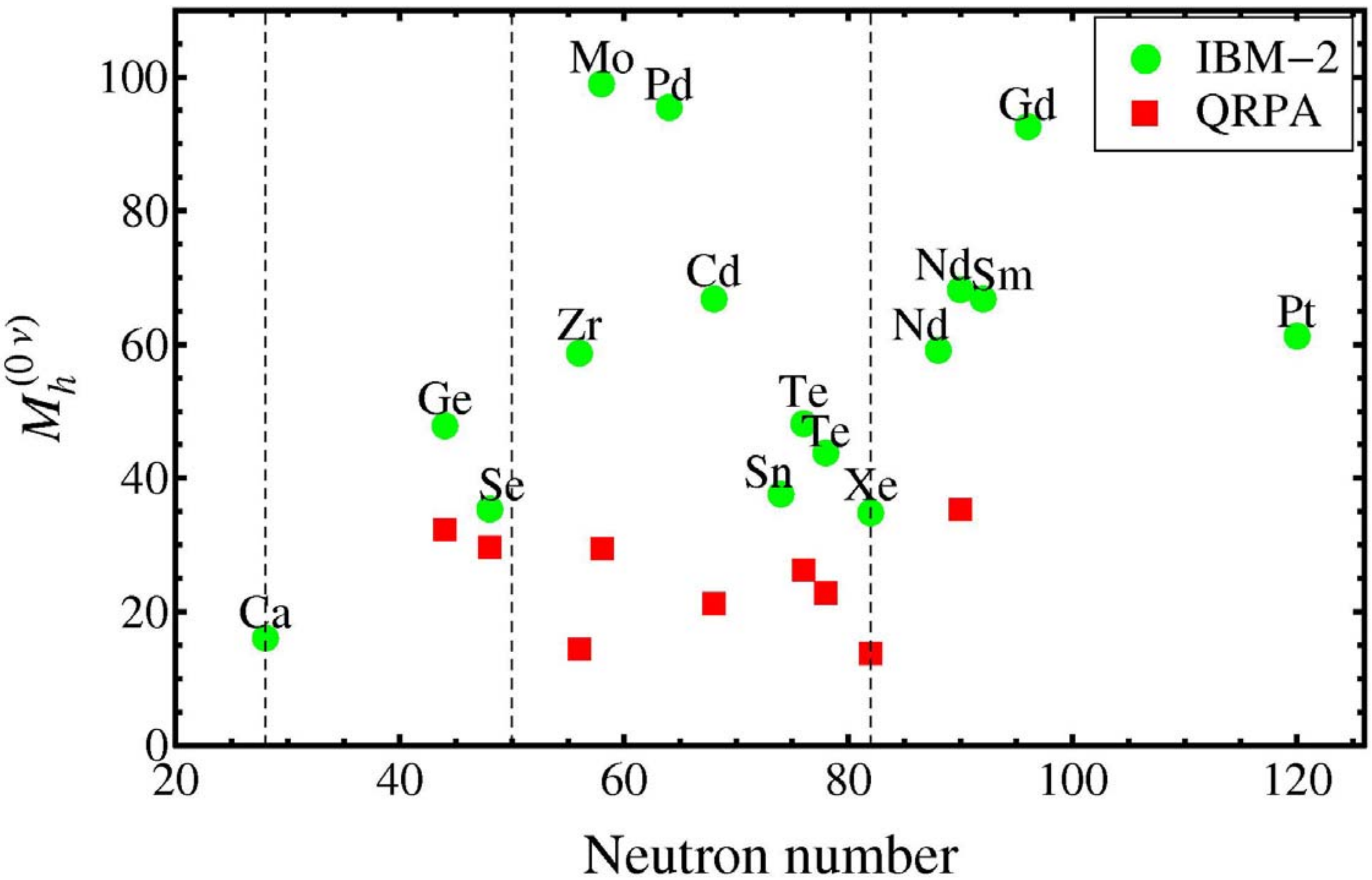
Total: 44%-55% (addition) or 16%-19% (quadrature).

¶ This point has been emphasized by J. Suhonen and O. Civitarese, Phys. Lett. B668, 277 (2008).

§ New experiments are being done to check the single particle levels in Ge, Se and Te,
J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008).

\# This point is discussed in many articles, for example, M. Kortelainen and J. Suhonen, Phys. Rev. C 75, 051303 (R) (2007).

IBM-2 RESULTS (JAN 2012)
HEAVY NEUTRINO EXCHANGE



IBM-2 from J. Barea, J. Kotila and F. Iachello, to be published (2012). MS-SRC.
QRPA from Šimkovic *et al.*, Phys. Rev. C 60, 055502 (1999). MS-SRC.

3. ADVANCES IN ATOMIC PHYSICS

For an extraction of the neutrino mass and for estimates of the half-life one also needs the phase-space factor $G_{0\nu}$. A general formulation was given by Doi *et al.* [*] and by Tomoda [¶], who tabulated results for selected cases. Also, a calculation of phase-space factors is reported in the book of Boehm and Vogel [§] where a complete tabulation is given. These calculations use an approximate expression for the electron wave functions at the nucleus.

Advance: We have done a novel calculation [**] with exact Dirac electron wave functions and including screening by the electron cloud.

[*] M. Doi, T. Kotani, N. Nishiura, K. Okuda and E. Takasugi, Prog. Theor. Phys. 66 (1981) 1739.

[¶] T. Tomoda, *loc.cit.*

[§] F. Bohm and P. Vogel, *Physics of massive neutrinos*, Cambridge University Press, 1987.

[**] J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

The wave functions are obtained by solving numerically [¶] the Dirac equation with potential

$$V(r) = \begin{bmatrix} -\dfrac{Z_d}{r} \\ -Z_d\left(\dfrac{3-(r/R)^2}{2R}\right) \end{bmatrix} \varphi(r) \qquad \begin{matrix} r>R \\ \\ r<R \end{matrix}$$

The function φ(r) is obtained numerically [§] by solving the Thomas-Fermi equation

$$\frac{d^2\varphi}{dx^2} = \frac{\varphi^{3/2}}{\sqrt{x}} \qquad x = r/b \qquad b \simeq 0.885 a_0 Z_d^{-1/3}$$

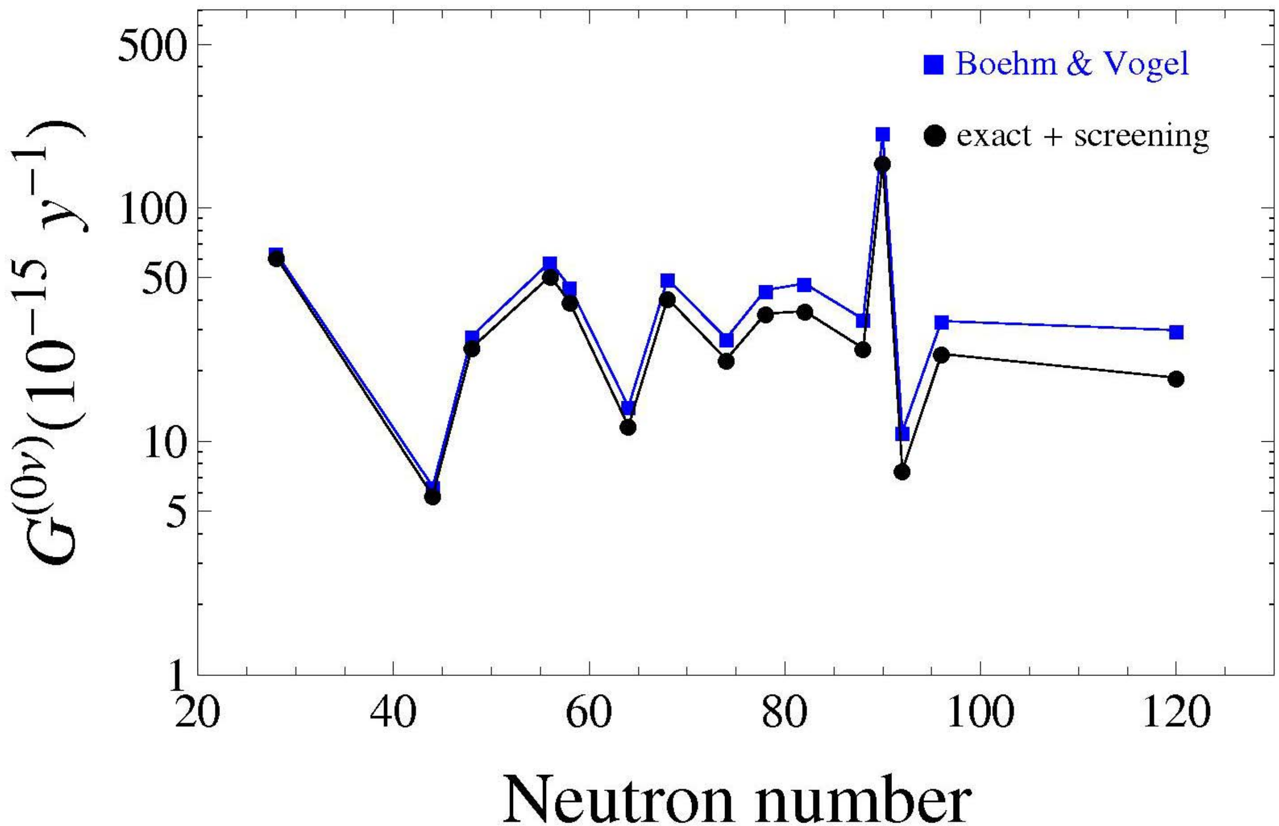
with boundary conditions (final nucleus positive ion with charge +2)

$$\varphi(0) = 1 \qquad \varphi(\infty) = \frac{2}{Z_d}$$

¶ F. Salvat, J.M. Fernadez-Varea, and W. Williamson Jr., Comp. Phys. Comm. 90 (1995) 151.

§ S. Esposito, Am. J. Phys. 70 (2002) 852.

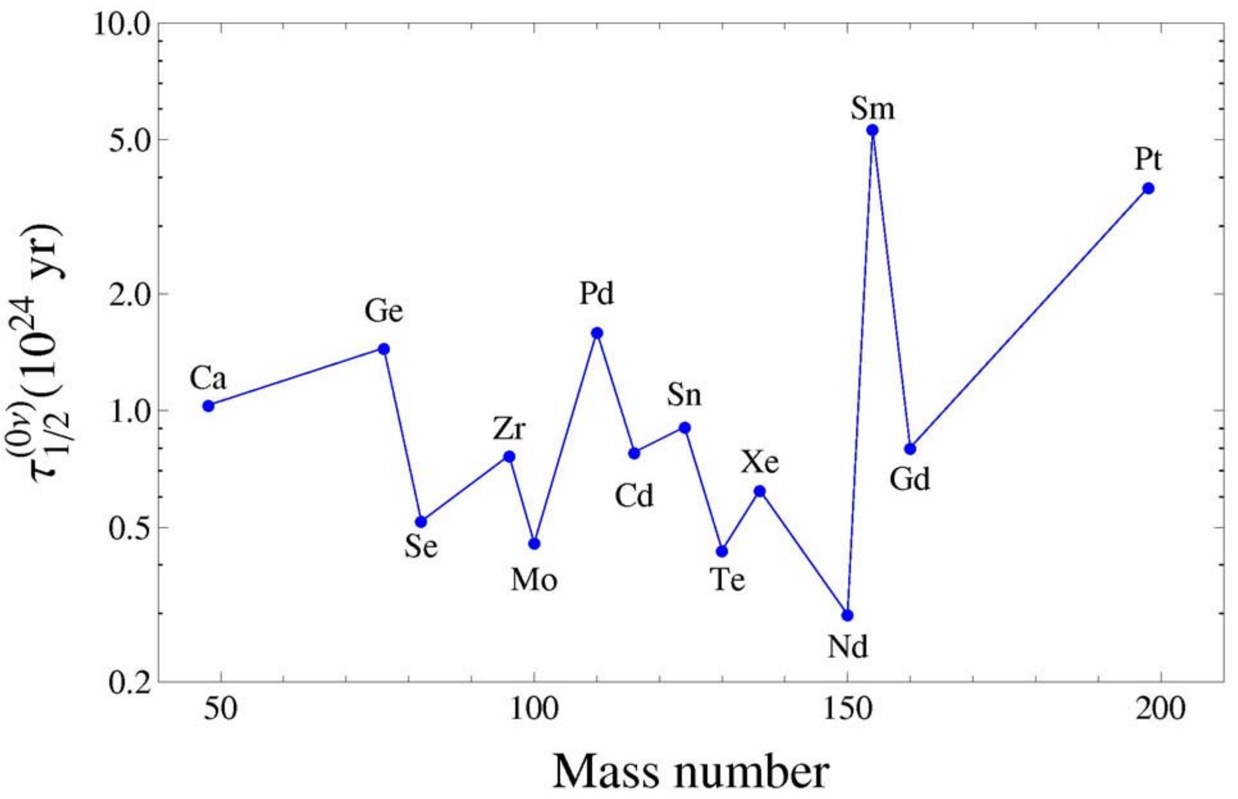
Comparison between approximate § and exact + screening ¶ phase space factors



§ F. Böhm and P. Vogel, *loc. cit.*

¶ J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

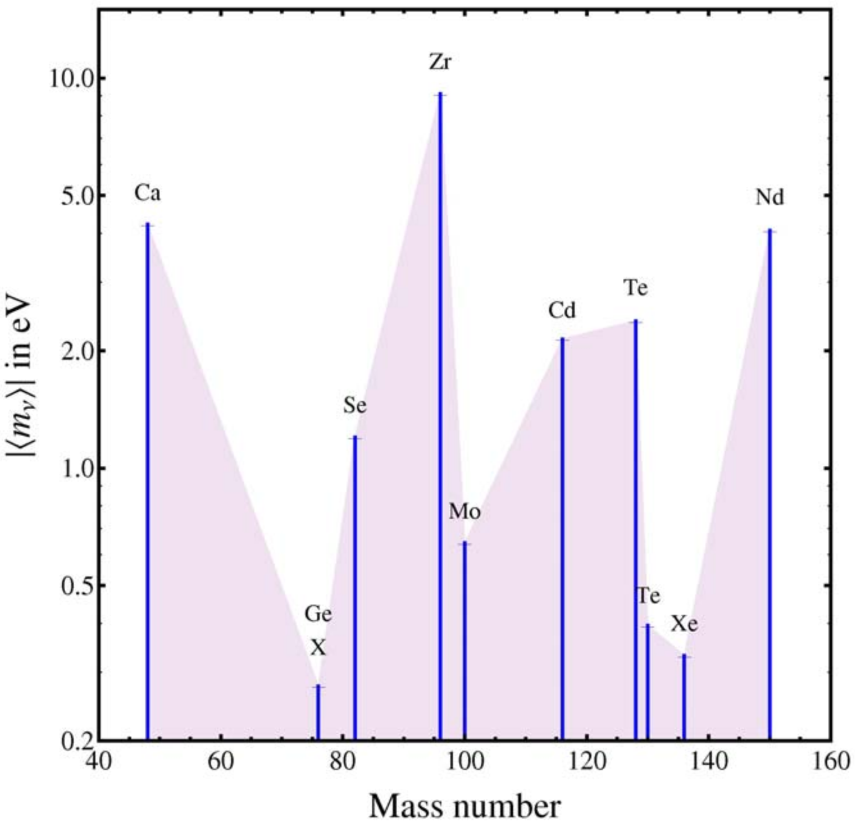
FINAL RESULTS FOR HALF-LIVES



$m_\nu = 1$ eV
$g_A = 1.269$

Nuclear matrix elements from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009) and to be published.
Phase space factors from J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

LIMITS ON NEUTRINO MASS (LIGHT NEUTRINOS)

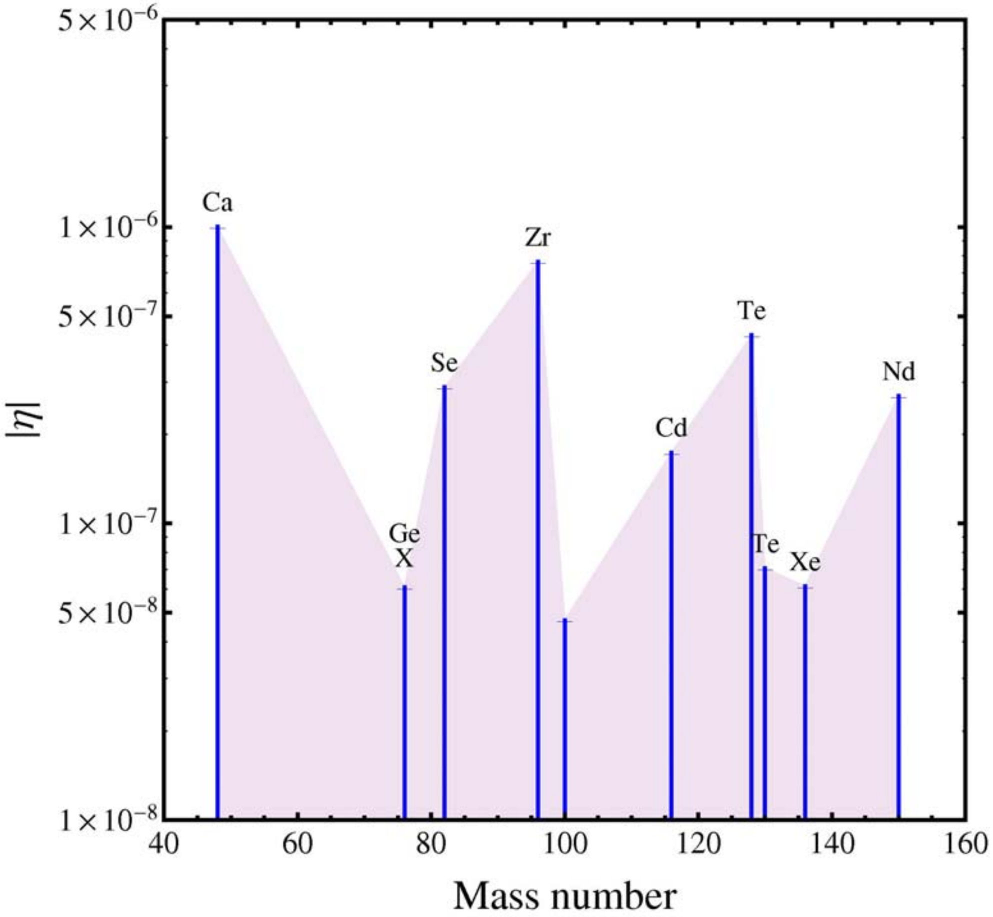


$$\left[\tau_{1/2}^{(0\nu)}\right]^{-1} = G_{0\nu} \left|M_{0\nu}\right|^2 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

$$M_{0\nu} = g_A^2 M^{(0\nu)}$$

Theory: Nuclear matrix elements from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009) and to be published.
Phase space factors from J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).
Experimental upper limits: from a compilation of A.S. Barabash, Phys. Atom. Nucl. 74, 603 (2011) and recent measurement for Xe from KamLAND-Zen, A. Gando *et al.*, arXiv:1201.4664vi [hep-ex] (2012).
× : from H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B586, 198 (2004).

LIMITS ON LEPTON VIOLATING PARAMETER

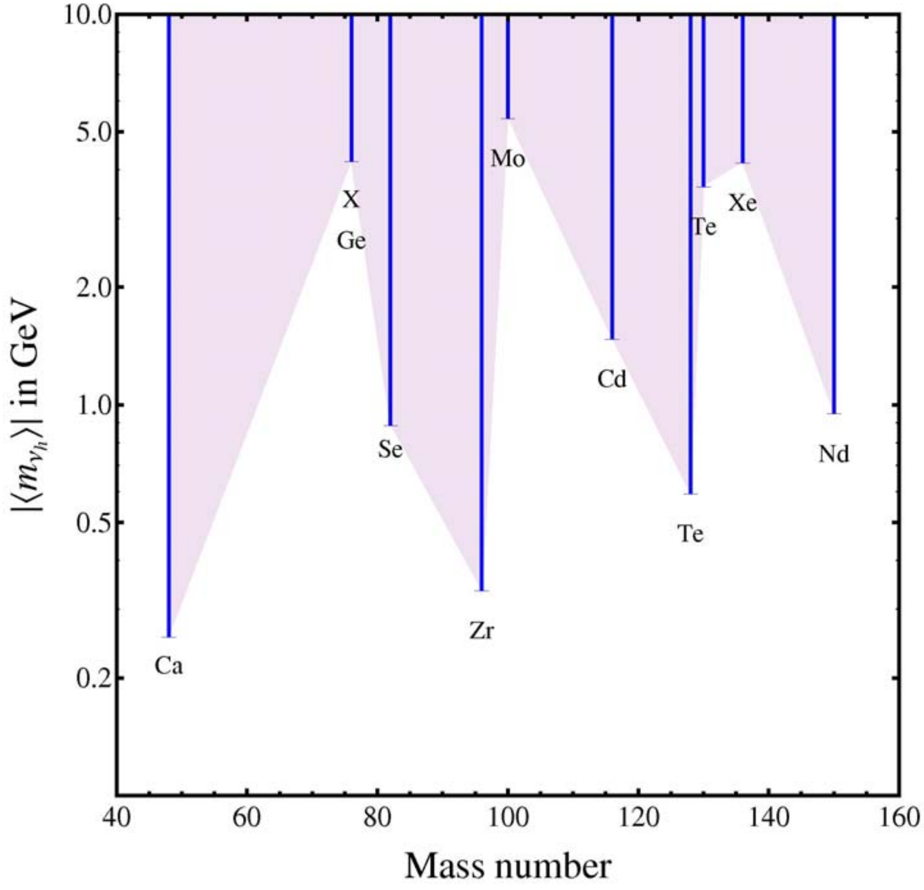


$$\left[\tau_{1/2}^{(0\nu_h)}\right]^{-1} = G_{0\nu}\left|M_{0\nu_h}\right|^2 |\eta|^2$$

$$\eta \equiv m_p \left\langle m_{\nu_h}^{-1} \right\rangle = \sum_{k=heavy} \left(U_{e\nu_h}\right)^2 \frac{m_p}{m_{k_h}}$$

Theory: Nuclear matrix elements from J. Barea, J. Kotila and F. Iachello, to be published.
Phase space factors from J. Kotila and F. Iachello, Phys. Rev. C 85, 034316 (2012).

LIMITS ON HEAVY NEUTRINO MASS



In the model of Tello *et al.*

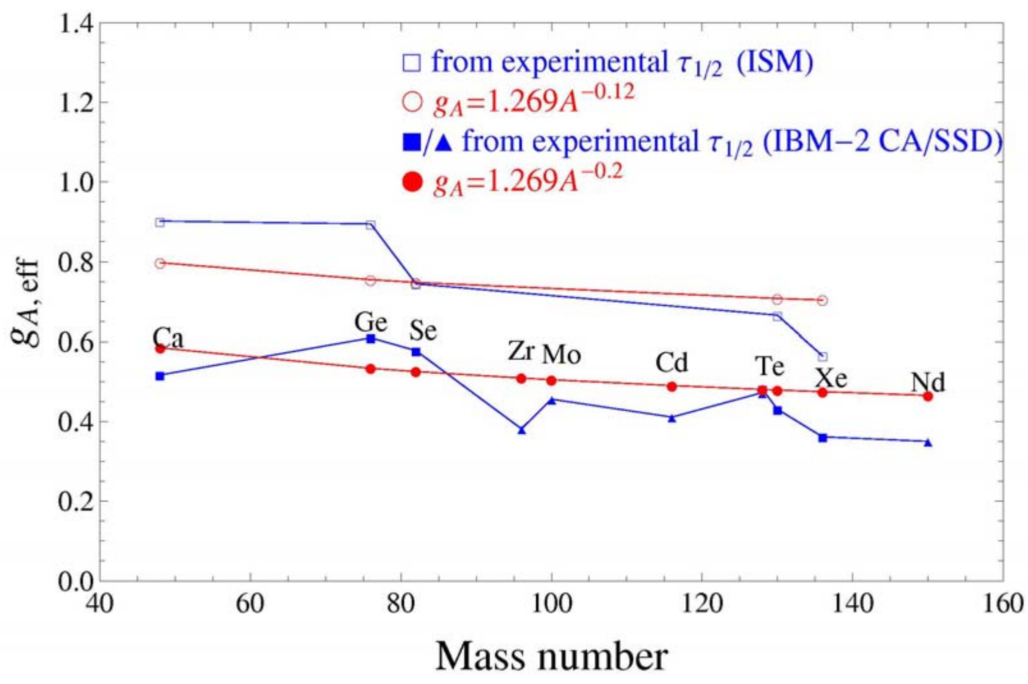
$$\eta = \frac{M_W^4}{M_{WR}^4} \frac{m_p}{\langle m_{\nu_h} \rangle}$$

If both mechanisms contribute

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} \left[\left| M_{0\nu} \right|^2 \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 + \left| M_{0\nu_h} \right|^2 \left| \eta \right|^2 \right]$$

RENORMALIZATION OF g_A: 2νββ DECAY

A simultaneous calculation of NME in 2νββ decay in the closure approximation has been completed, as well as a novel calculation of the 2νββ phase space factors. This calculation is of importance for 0νββ decay in order to estimate the renormalization of g_A which appears to the fourth power in the half-life. The values of $g_{A,eff}$ extracted from 2νββ are ~0.8 in the ISM and ~0.6 in IBM-2. Similar values are obtained by Ejiri, Part. Nucl. Phys. 64, 249 (2010).



From J. Barea, J. Kotila and F. Iachello, to be published.

The calculation of 2νββ in the closure approximation is similar to that of 0νββ except that the neutrino "potential" is different

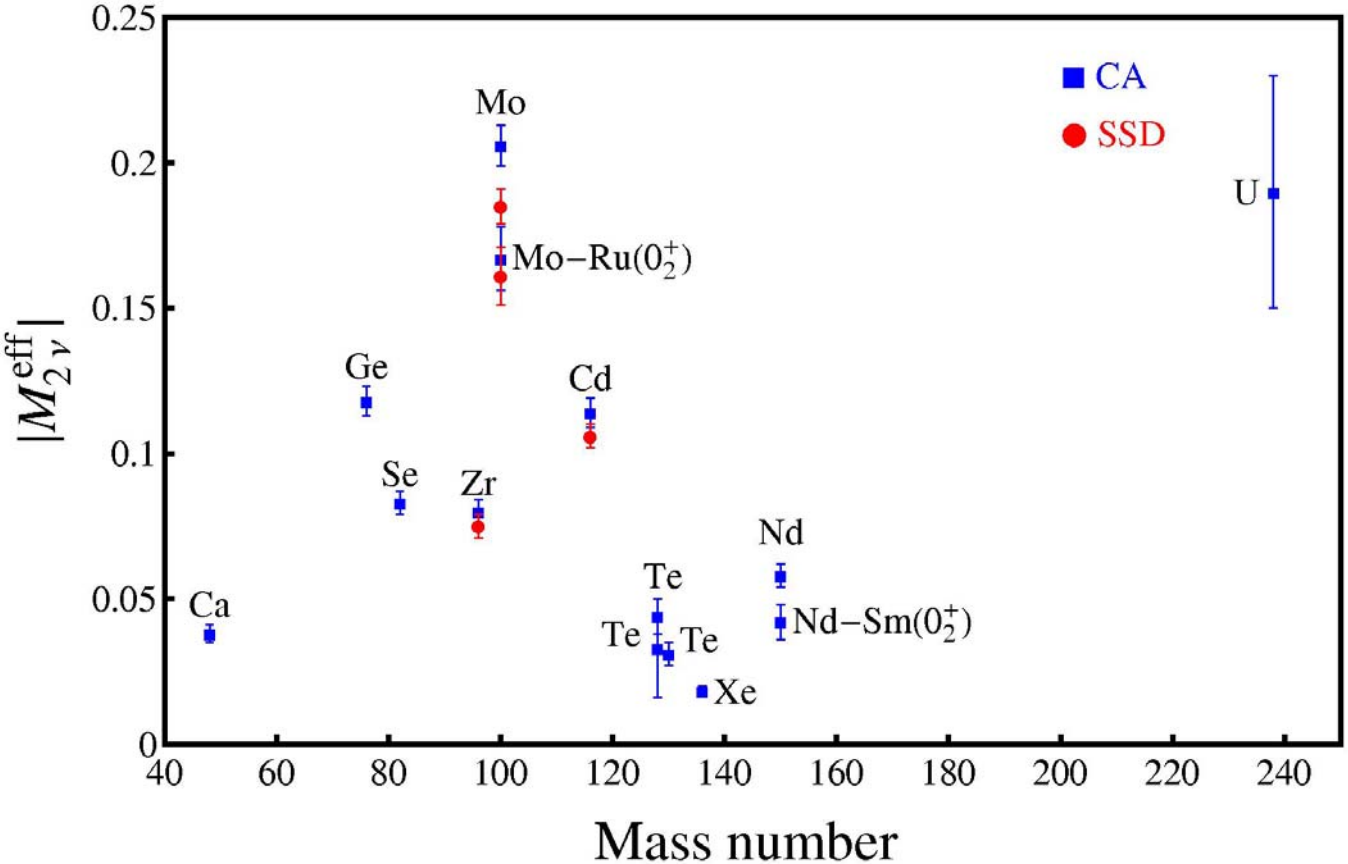
$$\boxed{v^{(2\nu)}(p) = \frac{\delta(p)}{p^2}}$$

In the IBM-2 approach in momentum space all calculations, 0ν-light-neutrino, heavy-neutrino, (Majoron, sterile neutrinos, …) and 2ν can be done at the same time, by changing the neutrino "potential", and thus eliminating possible sources of systematic and accidental error.

The half-lives for 2ν are calculated using

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu} \left| m_e c^2 M_{2\nu} \right|^2$$

The values of $g_{A,eff}$ can be extracted from the values of $|M_{2\nu}^{eff}|$ obtained from experimental half-lives



Using the values of $g_{A,eff}$ extracted from $2\nu\beta\beta$ one can make "absolute" predictions for $0\nu\beta\beta$ life-times.

Decay	$\tau_{1/2}^{0\nu}(10^{24}\text{yr})$ IBM-2 ren.	ISM ren.
^{48}Ca$\rightarrow^{48}$Ti	22.887	89.243
^{76}Ge$\rightarrow^{76}$Se	46.416	69.1217
^{82}Se$\rightarrow^{82}$Kr	17.627	18.4572
^{96}Zr$\rightarrow^{96}$Mo	29.546	
^{100}Mo$\rightarrow^{100}$Ru	18.137	
^{110}Pd$\rightarrow^{110}$Cd	68.640	
^{116}Cd$\rightarrow^{116}$Sn	35.050	
^{124}Sn$\rightarrow^{124}$Te	42.922	27.603
^{128}Te$\rightarrow^{128}$Xe	413.792	344.356
^{130}Te$\rightarrow^{130}$Xe	21.443	17.601
^{136}Xe$\rightarrow^{136}$Ba	31.785	25.260
^{148}Nd$\rightarrow^{148}$Sm	145.971	
^{150}Nd$\rightarrow^{150}$Sm	16.400	
^{154}Sm$\rightarrow^{154}$Gd	300.019	
^{160}Gd$\rightarrow^{160}$Dy	46.557	
^{198}Pt$\rightarrow^{198}$Hg	259.526	

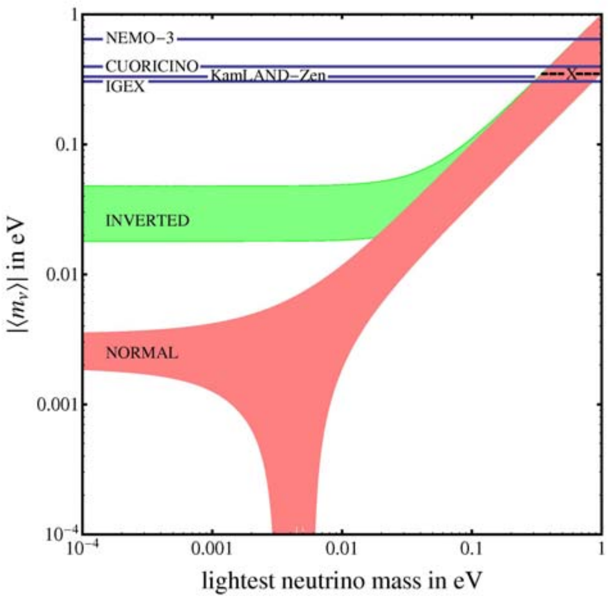
However, the question of whether or not g_A is renormalized in $0\nu\beta\beta$ as much as in $2\nu\beta\beta$ is a subject of debate.
In $2\nu\beta\beta$ only 1^+ and 0^+ intermediate states contribute.
In $0\nu\beta\beta$ all multipoles contribute.

CONCLUSIONS

• A new program (IBM-2) has been developed to calculate $0\nu\beta\beta$-light,-heavy, $0\nu\beta\beta M$ nuclear matrix elements in the closure approximation. The calculation for $0\nu\beta\beta$-light in all nuclei of interest has been completed. [Matrix elements to the first excited 0^+ states, 0_2, have also been calculated.]

• A novel calculation of the phase space factors has been completed.

• The current limits on neutrino mass using IBM-2 matrix elements and experimental results are given in the figure:



x H.V. Klapdor-Kleingrothaus *et al., loc.cit.*

- A calculation of NME in $0\nu\beta\beta$ decay for scenario 2 (heavy-neutrinos) has been completed (March 2012) and limits on the lepton violating parameter set.
- An analysis of $0\nu\beta\beta$ with $g_{A,eff}$ extracted from $2\nu\beta\beta$ has also been completed (March 2012) and renormalized half-lives calculated.

Progress has been made on all three fronts:
Particle physics, nuclear physics and atomic physics

APPENDIX A: DETAILS OF THE IBM-2 CALCULATION

The fermion operator V is mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)_M^{(2)} \mapsto B_\pi(j, j') d_{\pi,M}^\dagger$$

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2}\sum_{j_1}\sum_{j'_1} G_{s_1 s_2}^{(\lambda)}\left(j_1 j_1 j'_1\, j'_1; 0\right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

$$-\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2} \sqrt{1+\delta_{j_1 j_2}}\sqrt{1+\delta_{j'_1 j'_2}}\, G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1\, j'_2; 2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

The coefficients A, B are obtained by equating fermionic matrix elements in the Generalized Seniority (GS) basis with bosonic matrix elements, the so-called OAI mapping procedure ¶. The basis

$$\left(S^\dagger\right)^{\frac{n-\nu}{2}} \left(D^\dagger\right)^{\frac{\nu}{2}} |0\rangle$$

is constructed with operators:

$$S_\pi^\dagger = \sum_j \alpha_j \sqrt{\frac{j+\frac{1}{2}}{2}} \left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)}$$

$$D_\pi^\dagger = \sum_{j \le j'} \beta_{jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)^{(2)}$$

¶ T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309, 1 (1978).

The structure coefficients α_j, $\beta_{jj'}$ are obtained by diagonalizing the surface delta interaction (SDI). The strength of the interaction, A_T, is chosen as to reproduce the 0-2 separation in the two-particle system.

The fermion matrix elements are calculated using the commutator method of Frank and Van Isacker and Lipas *et al.* [¶,§].

¶ A. Frank and P. Van Isacker, Phys. Rev. C26, 1661 (1982).

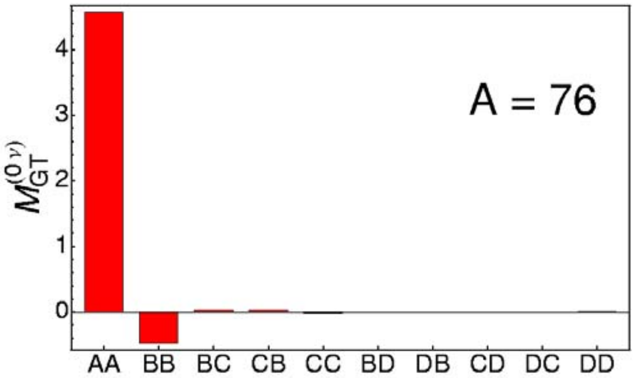
§ P.O. Lipas, M. Koskinen, H. Harter, R. Nojarov, and A. Faessler, Nucl. Phys. A508, 509 (1990).

Expansion to next to leading order (NLO)

$$(\pi_j^\dagger \times \pi_{j'}^\dagger)_M^{(2)} \mapsto B_\pi(j,j')\left(d_\pi^\dagger\right)_M + C_\pi(j,j')s_\pi^\dagger\left(s_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)} + D_\pi(j,j')s_\pi^\dagger\left(d_\pi^\dagger \tilde{d}_\pi\right)_M^{(2)}$$

Effect small <5%

Will be neglected henceforth.



Realistic IBM-2 wave functions are taken from the literature. They fit all experimental data for excitation energies, B(E2) values and quadrupole moments, B(M1) values and magnetic moments, etc., very well.

APPENDIX B: NUCLEAR MATRIX ELEMENTS TO 0_1

	$M^{(0\nu)}$ IBM-2§	QRPA¶	ISM*
$^{48}Ca \rightarrow {}^{48}Ti$	2.00		0.59
$^{76}Ge \rightarrow {}^{76}Se$	5.46	4.68	2.22
$^{82}Se \rightarrow {}^{82}Kr$	4.41	4.17	2.11
$^{96}Zr \rightarrow {}^{96}Mo$	2.53	1.34	
$^{100}Mo \rightarrow {}^{100}Ru$	3.73	3.53	
$^{110}Pd \rightarrow {}^{110}Cd$	3.62		
$^{116}Cd \rightarrow {}^{116}Sn$	2.78	2.93	
$^{124}Sn \rightarrow {}^{124}Te$	3.53		2.20
$^{128}Te \rightarrow {}^{128}Xe$	4.52	3.77	2.26
$^{130}Te \rightarrow {}^{130}Xe$	4.06	3.38	2.04
$^{136}Xe \rightarrow {}^{136}Ba$	3.35	2.22	1.70
$^{148}Nd \rightarrow {}^{148}Sm$	1.98		
$^{150}Nd \rightarrow {}^{150}Sm$	2.32		
$^{154}Sm \rightarrow {}^{154}Gd$	2.51		
$^{160}Gd \rightarrow {}^{160}Dy$	3.63		
$^{198}Pt \rightarrow {}^{198}Hg$	1.88		

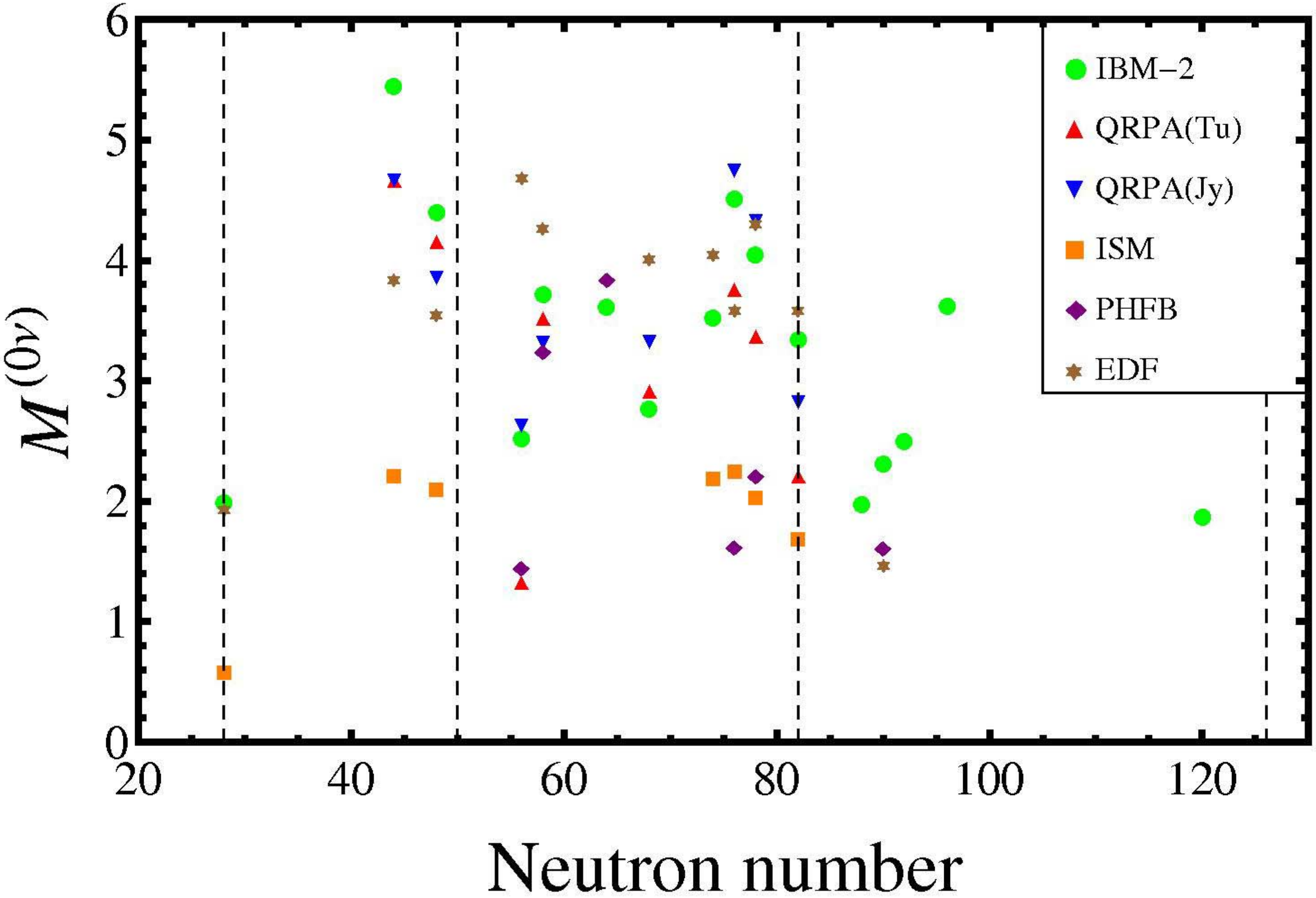
§ J. Barea and F. Iachello, Phys. Rev. C79 (2009) 044301 and to be published.
¶ F. Šimkovic, A. Faessler, V. Rodin, P. Vogel, and J. Engel, Phys. Rev. C77 (2008) 045503.
* E. Caurier, J. Menendez, F. Nowacki, and A. Poves, Phys. Rev. Lett. 100 (2008) 052503

NUCLEAR MATRIX ELEMENTS TO 0_2

	$M^{(0\nu)}$ IBM-2§	QRPA¶	ISM*
$^{48}Ca \to ^{48}Ti$	5.90		0.68
$^{76}Ge \to ^{76}Se$	2.48	1.28	1.49
$^{82}Se \to ^{82}Kr$	1.25	1.34	0.28
$^{96}Zr \to ^{96}Mo$	0.04		
$^{100}Mo \to ^{100}Ru$	0.42	1.27	
$^{110}Pd \to ^{110}Cd$	1.60		
$^{116}Cd \to ^{116}Sn$	1.05		
$^{124}Sn \to ^{124}Te$	2.72		0.80
$^{128}Te \to ^{128}Xe$	3.24		
$^{130}Te \to ^{130}Xe$	3.09		0.19
$^{136}Xe \to ^{136}Ba$	1.84	4.42	0.49
$^{148}Nd \to ^{148}Sm$	0.25		
$^{150}Nd \to ^{150}Sm$	0.39		
$^{154}Sm \to ^{154}Gd$	0.02		
$^{160}Gd \to ^{160}Dy$	0.75		
$^{198}Pt \to ^{198}Hg$	0.07		

§ J. Barea and F. Iachello, Phys. Rev. C79 (2009) 044301.
¶ F. Šimkovic, M. Nowak, W.A. Kaminsky, A.A. Raduta, and A. Faessler, Phys. Rev. C64 (2001) 035501(QRPA-RCM).
* J. Menendez, A. Poves, E. Caurier, F. Nowacki, Nucl. Phys. A818 (2009) 139.

APPENDIX C: SUMMARY OF MATRIX ELEMENTS-ALL



APPENDIX D: NUCLEAR MATRIX ELEMENTS LIGHT NEUTRINO EXCHANGE