Pulsar Kick and Relativistic Mean-Field Theory for Neutrino-Hadron

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§ 1. Introduction

High Density Matter Study

⇒ Exotic Phases : Strange Matter, Ferromagnetism

Meson Condensation, Quark matter

They may exist inside Neutron Stars

Observable Information ・・・・Neutrino Emissions

Neutrino Scattering
& Absorption
in Surface Region

S.Reddy, M.Prakash and J.M. Lattimer, P.R.D58 #013009 (1998)

Influence from Hyperons Λ, Σ

Magnetar 10^{15}G in surface $10^{17\text{-}19}$G inside (?)

Magnetic Field → Large Asymmetry ?

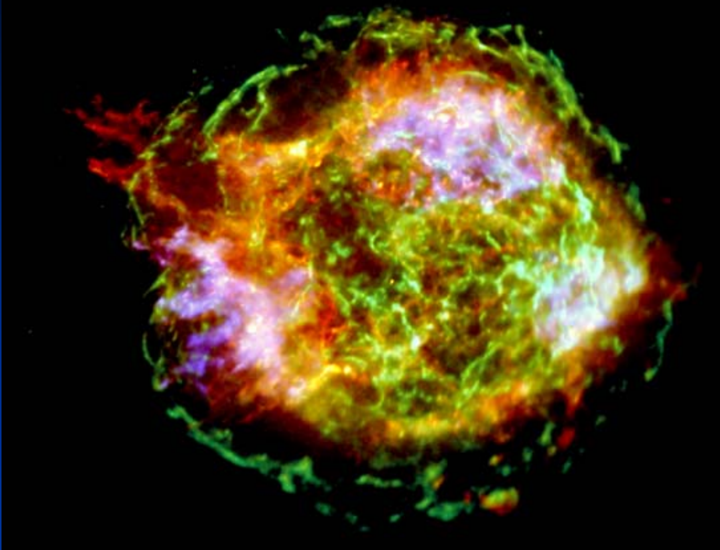
P. Arras and D. Lai, P.R.D60, #043001 (1999)

S. Ando, P.R.D68 #063002 (2003)

Pulsar Kick

A.G.Lyne, D.R.Lomier, Nature 369, 127 (94)

CasA



http://chandra.harvard.edu/photo/2004/casa/casa_xray.jpg

Asymmetry of Supernova Explosion
kick and translate Pulsar with

Kick Velocity: Average … 400km/s,
Highest … 1500km/s

Explosion Energy ~ 10^{53} erg

(almost Neutrino Emissions)

***1% Asymmetry* are sufficient to explain the Pulsar Kick**

Present Work ‥ Neutrino Scattering and Absorption
In Hot and Dense Neutron-Star-Matter

Estimating Kick Velocity

§ 2. Formulation

Magnetic Field : $\vec{B} = B\hat{z}.$

$$\text{Lagrangian}: \mathcal{L} = \mathcal{L}_{RMF} + \mathcal{L}_{lep.} + \mathcal{L}_{mag} + \mathcal{L}_{int}$$

Baryon **Lepton** B & L – Mag.

1. **Proto-Nuetron-Star (PNS) Matter without Mag. Field**
2. **Baryon Wave Function under Mag. Field in Perturbative Way**
3. **Cross-Sections for ν reactions**

Weak Interaction

$$\mathcal{L}_{int} = G_F\{\bar{\psi}_{l'}\gamma_\mu(1-\gamma_5)\psi_l\}\{\bar{\psi}_{B'}\gamma^\mu(c_V - c_A\gamma_5)\psi_B\}$$

$\nu_e + B \longrightarrow \nu_e + B$ **: scattering**

$\nu_e + B \longrightarrow e^- + B'$ **: absorption**

S.Reddy, M.Prakash and J.M. Lattimer, P.R.D58 #013009 (1998)

§ 2-1 Proto Neutron-Star-Matter in RMF

N, Λ, σ, ω, ρ

RMF Lagrangian

$$\mathcal{L}_{RMF} = \bar{\psi}_N(i\not\partial - M_N)\psi_N + \bar{\psi}_\Lambda(i\not\partial - M_\Lambda)\psi_\Lambda + g_\sigma\bar{\psi}_N\psi_N\sigma + g_\sigma^\Lambda\bar{\psi}_\Lambda\psi_\Lambda\sigma$$
$$+g_\omega\bar{\psi}_N\gamma_\mu\psi_N\omega^\mu + g_\omega^\Lambda\bar{\psi}_\Lambda\gamma_\mu\psi_\Lambda\omega^\mu - \widetilde{U}[\sigma] + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

ψ_N (nucleon), ψ_Λ (Λ), σ and ω

$\widetilde{U}[\sigma]$: the self-energy potential of the scalar mean-field.

$$[\not p - M_N^* - U_0(N)\gamma_0]\, u_N(p) = 0 \qquad [\not p - M_\Lambda^* - U_0(\Lambda)\gamma_0]\, u_\Lambda(p) = 0$$

$$M_N^* = M_N - g_\sigma\sigma \qquad M_\Lambda^* = M_\Lambda - g_\sigma^\Lambda\sigma$$

$$U_0(N) = \frac{g_\omega}{m_\omega^2}(g_\omega\rho(N) + g_\omega^\Lambda\rho_\Lambda) \qquad U_0(\Lambda) = \frac{g_\omega^\Lambda}{m_\omega^2}(g_\omega\rho(N) + g_\omega^\Lambda\rho_\Lambda)$$

$$\frac{\partial}{\partial\sigma}\widetilde{U}[\sigma] = g_\sigma\rho_s(N) + g_\sigma^\Lambda\rho_s(\Lambda)$$

EOS of Proto Neutron-Star-Matter (PNS) in RMF

N, Λ, σ, ω, ρ

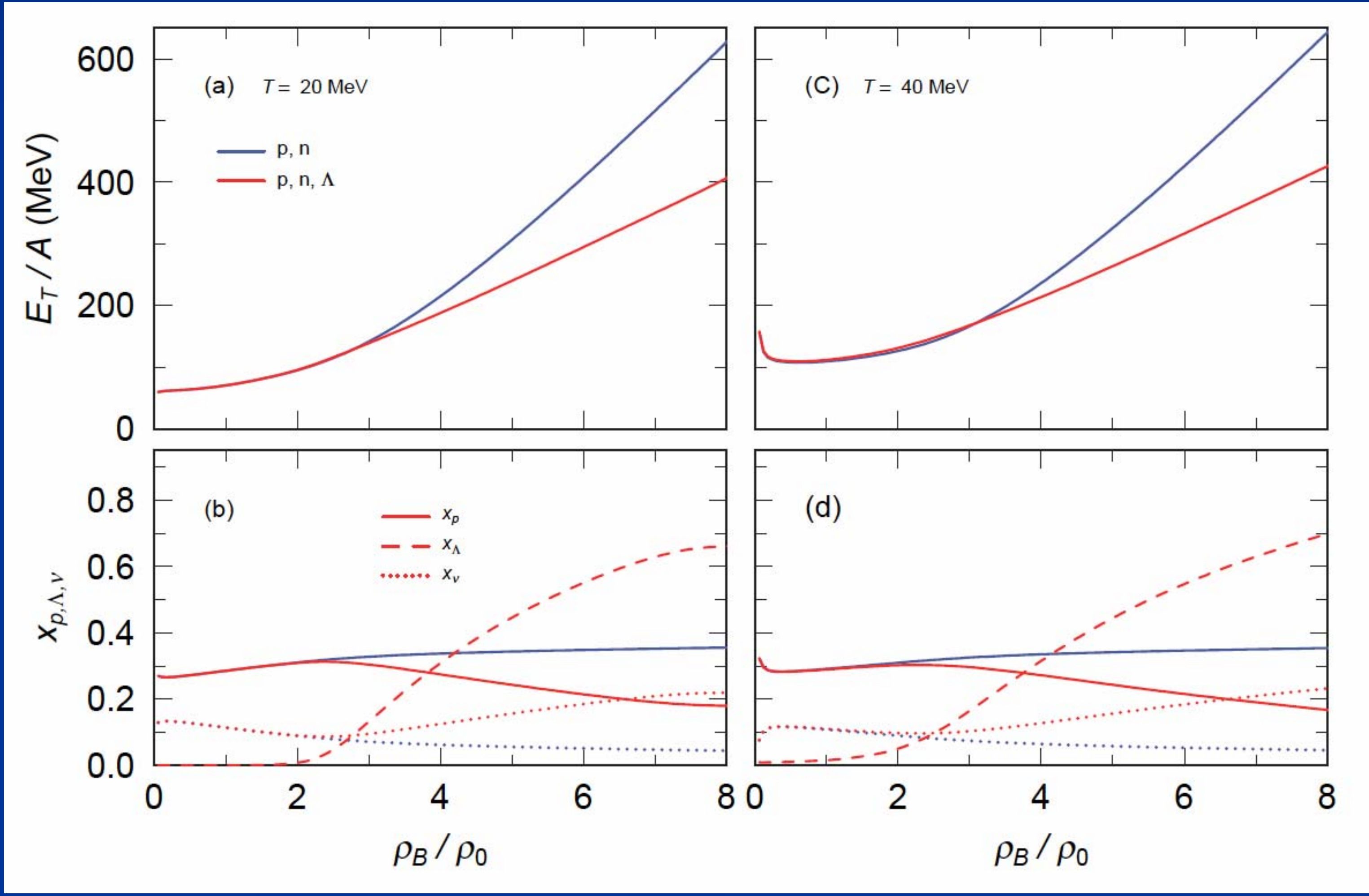
PM1-L1

$BE = 16\,\mathrm{MeV},\ M_N^*/M_N = 0.7,\ K = 200\,\mathrm{MeV}$ at $\rho_0 = 0.17\,\mathrm{fm}^{-3}$

$g_{\sigma,\omega}^{\Lambda} = \frac{2}{3} g_{\sigma,\omega}$ **SU(3)**

T.M, et al. PTP. 102, p809 (1999)

Charge Neutral ($\rho_p = \rho_e$) & Lepton Fraction : Y_L = 0.4



§ 2-2 Dirac Equation under Magnetic Fields

$\mu_N B$ << ε_F (Chem. Pot) → B can be treated perturbatively

~ 10^{17} G Landau Level can be ignored

Magnetic Part of Lagrangian

$$\mathcal{L}_{mag} = \sum_n \mu_n \bar{\psi}_n \sigma_{\mu\nu} \psi_n F^{\mu\nu} = -\sum_n \mu_n B \bar{\psi}_n \sigma_Z \psi_n$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad (1)$$

Dirac Eq. $\hat{K}(p)u(p) = [\gamma_\mu p^\mu - M^* - \mu B \sigma_z]\, u(p) = 0$

$$\text{Single Particle Energy}: \det \hat{K}(p) = (p_0^2 - e^2(\boldsymbol{p}, +1))(p_0^2 - e^2(\boldsymbol{p}, -1))$$

$$\text{Green Function } (S(p)): \hat{K}(p)S(p) = 1$$

$$\hat{K}(p) = \not{p} - M^* - \mu B \sigma_z$$

$$= \begin{pmatrix} p_0 - M^* - \mu B & 0 & -p_z & -p_x \\ 0 & p_0 - M^* + \mu B & -p_x & p_z \\ p_z & p_x & -p_0 - M^* - \mu B & 0 \\ p_x & -p_z & 0 & -p_0 - M^* + \mu B \end{pmatrix}$$

When $\mu B << 1$,

$$S(p) = \sum_{s=\pm 1} \left\{ \frac{u(\boldsymbol{p}, s)\bar{u}(\boldsymbol{p}, s)}{p_0 - e(\boldsymbol{p}, s) + i\delta} + \frac{v(-\boldsymbol{p}, s)\bar{v}(-\boldsymbol{p}, s)}{p_0 + e(\boldsymbol{p}, s) - i\delta} \right\} = \left[\hat{K}(p) \right]^{-1}$$

$$u(\boldsymbol{p}, s)\bar{u}(\boldsymbol{p}, s) = [(p_0 - e(\boldsymbol{p}, s))S(p))] \; (p_0 = e(\boldsymbol{p}, s))$$
$$\approx \boxed{\frac{(\not{p} + M)(1 + \gamma_5 \not{a}(p) s)}{4E_p^*}} + \boxed{\frac{\mu B p_z}{4E_p^{*\,3}} (\sigma \cdot \boldsymbol{p} - M^* \gamma_5 \gamma_0)} .$$

negligibly small

$$\boxed{a_z = \frac{E_p^*}{\sqrt{p_T^2 + M^{*2}}} \qquad \boldsymbol{a}_T = 0 \qquad a_0 = \frac{p_z}{\sqrt{p_T^2 + M^{*2}}}}$$

Spin Vector

Dirac Spinor

$$\text{Single Particle Energy}: \det \hat{K}(p) = (p_0^2 - e^2(\boldsymbol{p}, +1))(p_0^2 - e^2(\boldsymbol{p}, -1))$$

$$e(\boldsymbol{p}, s) = \left[\left(\sqrt{p_x^2 + M^{*2}} + s\mu B\right)^2 + p_z^2\right]^{\frac{1}{2}} \approx E_p^* + s\mu B \frac{\sqrt{p_T^2 + M^{*2}}}{E_p^*}$$

Fermi Distribution

$$n(e(\boldsymbol{p}), s) \approx n(\varepsilon(\boldsymbol{p}, s)) + n'(\varepsilon(\boldsymbol{p}, s)) \frac{\sqrt{p_T^2 + M^{*2}}}{E_p^*} \mu B s.$$

Momentum Distr. of Major Part is deformed as oblate

Relativistic Effects of Magnetic Contributions

1)Momentum Dependent of Spin Vector

2)Deformation of Fermi Distribution

The Cross-Section of ν -B

$$\frac{d^2\sigma}{dk' d\Omega'_k} = \frac{G_F^2}{8\pi^2}\boldsymbol{k}'^2 \sum_{s_i,s_f}\int \frac{d^3p}{(2\pi)^3}\tilde{W}_{BL}(2\pi)\delta(|\boldsymbol{k}|-|\boldsymbol{k}'|+e_i(\boldsymbol{p})-e_f(\boldsymbol{k}+\boldsymbol{p}-\boldsymbol{k}'))$$
$$\times[1-f_l'(\boldsymbol{k}')]n_B(e_i)[1-n_{B'}(e_f)]$$

$$\tilde{W}_{BL} = \mathrm{Tr}\left\{\frac{(\not{k}'+m_f)(1+\gamma_5\not{a}_{l'})}{4|\boldsymbol{k}'|}\gamma^\mu(1-\gamma_5)\frac{\not{k}'}{2|\boldsymbol{k}|}\gamma^\nu(1-\gamma_5)\right\}$$
$$\times \mathrm{Tr}\left\{\frac{(\not{p}'+M_f^*)(1+\gamma_5\not{a}_f(p'))}{4E_f^*(\boldsymbol{p}')}\gamma_\mu(c_V-c_A\gamma_5)\frac{(\not{p}+M_i^*)(1+\gamma_5\not{a}_i(p))}{4E_i^*(\boldsymbol{p})}\gamma_\nu(c_V-c_A\gamma_5)\right\}$$

$$m_f = 0 \quad \text{when } l_f = \nu \qquad m_f = m_e \quad \text{when } l_f = e$$

Fermi Distribution

$$n(e(\boldsymbol{p}),s) \approx n(\varepsilon(\boldsymbol{p},s)) + n'(\varepsilon(\boldsymbol{p},s))\frac{\sqrt{p_T^2+M^{*2}}}{E_p^*}\mu B s.$$

Deformed Distribution

Perturbative Treatment $\Longrightarrow$

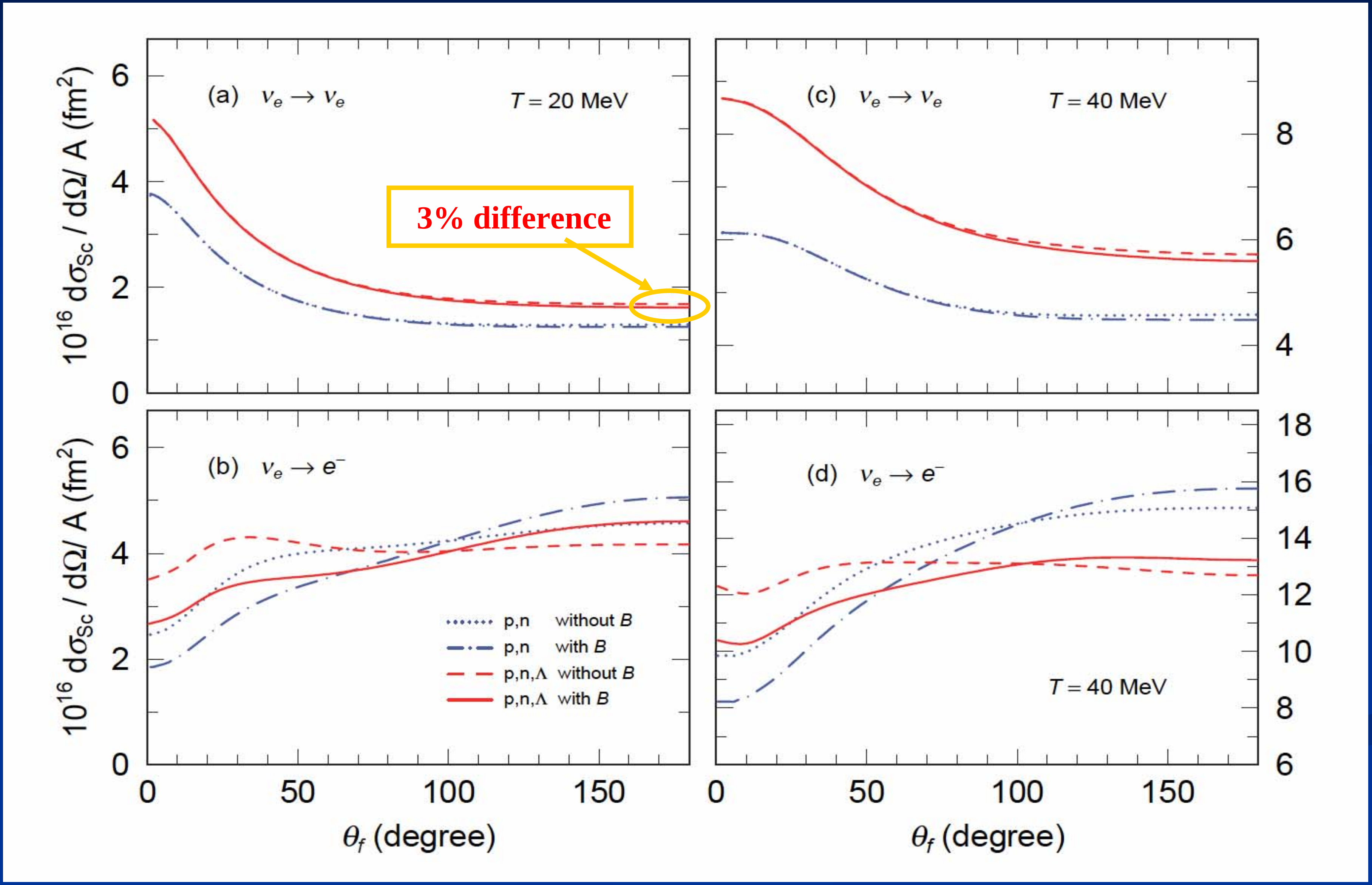
$$\sigma = \sigma_0 + \Delta\sigma \qquad \Delta\sigma \propto B$$

Non-Magnetic Part

Magnetic Part

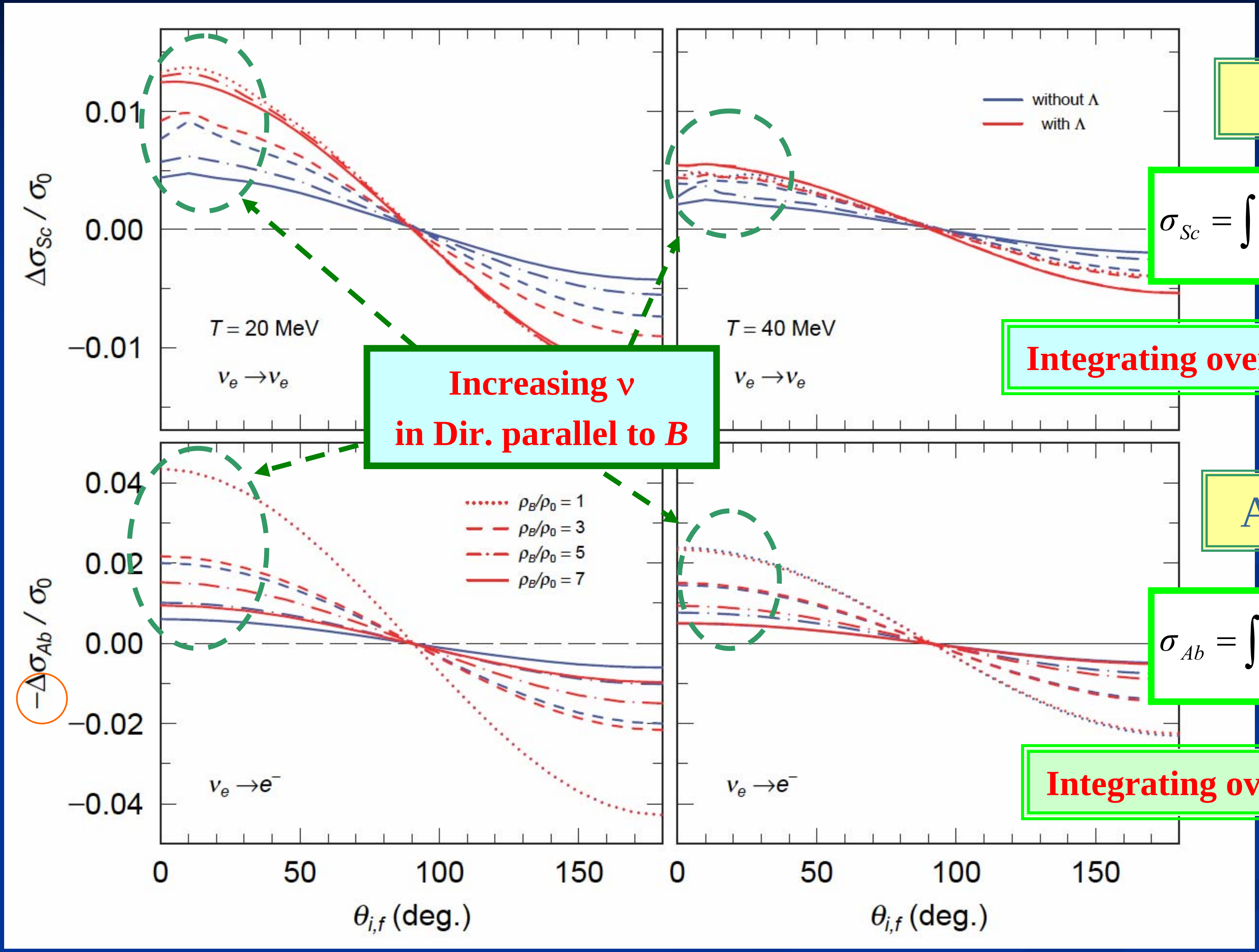
§ 3 Results

$k_i = \varepsilon_\nu$ (neutrino chem. pot.), $B = 2\times10^{17}$ G and $\theta_i = 0^\circ$



Magnetic parts of Cross-Sections

$$\sigma = \sigma_0 + \Delta\sigma \qquad \Delta\sigma \propto B$$



Scat.

$$\sigma_{Sc} = \int d\Omega_i \frac{d\sigma(\nu_e \to \nu_e)}{d\Omega_f}$$

Integrating over the initial angle

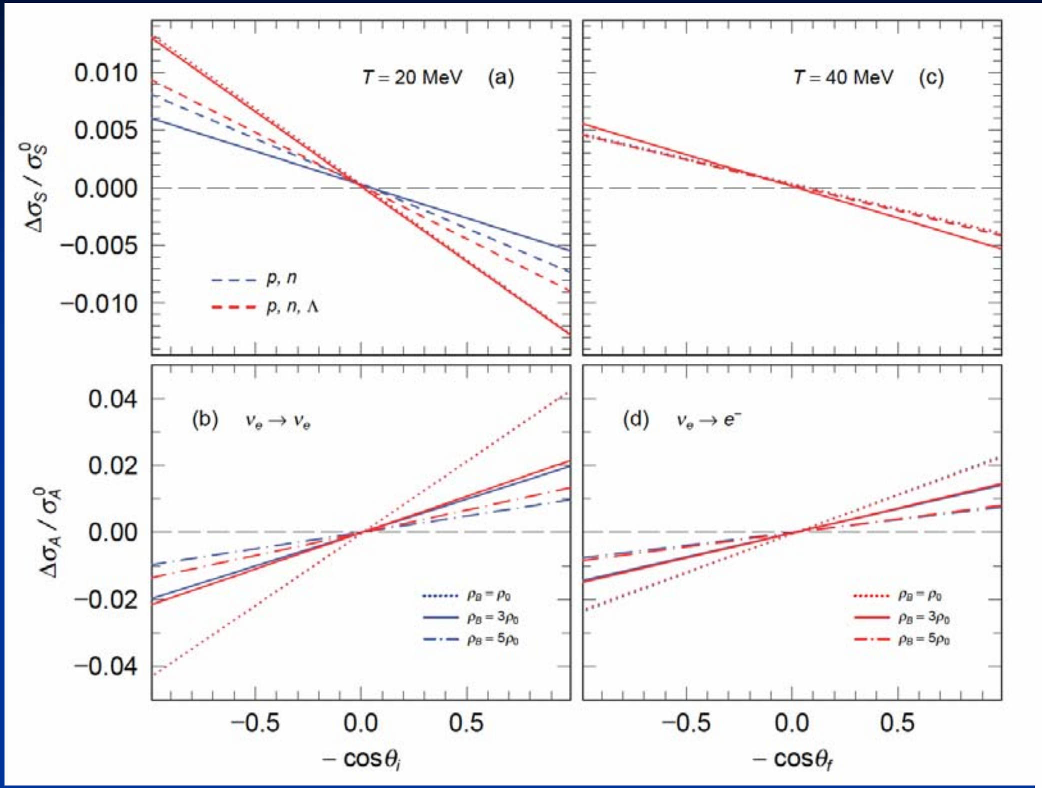
Absorp.

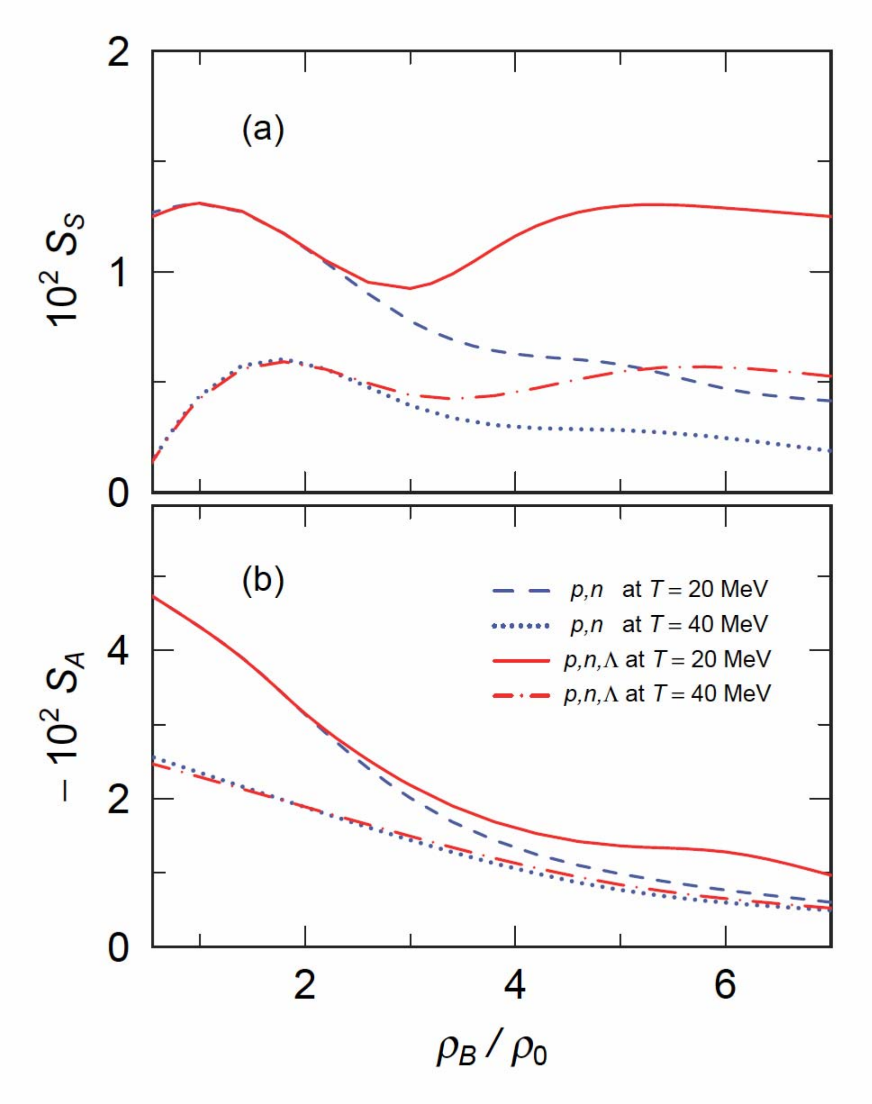
$$\sigma_{Ab} = \int d\Omega_f \frac{d\sigma(\nu_e \to e)}{d\Omega_f}$$

Integrating over the final angle

$$k_i = \varepsilon_\nu \text{ (neutrino chem. pot.)}, \;\; B = 2\times 10^{17}\,\text{G} \;\text{ and }\; \theta_i = 0^\circ$$

Magnetic Parts of Cross-Sections



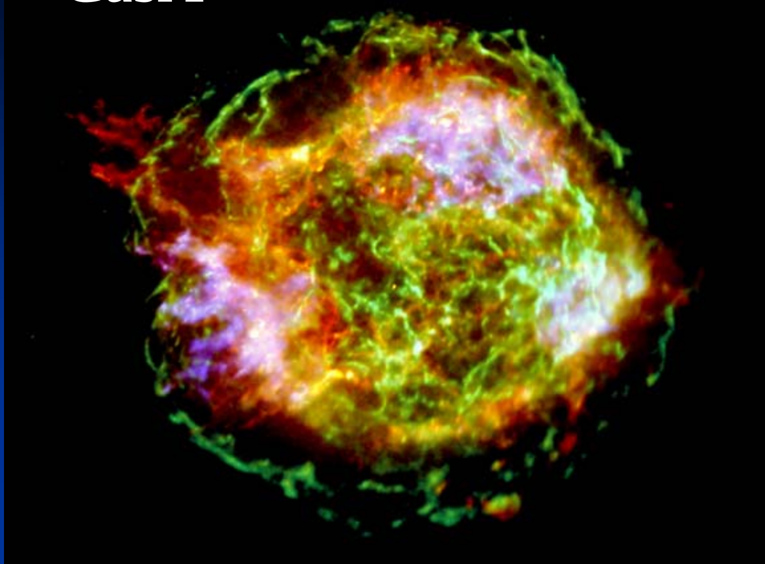


$$\sigma_{S,A} = \sigma_{S,A}^{0}\left(1 + S_{S,A}\cos\theta_{i,f}\right)$$

§ 4 Estimating Pulsar Kick Velocities of Proto-Neutron Star

A.G.Lyne, D.R.Lomier, Nature 369, 127 (94)

CasA



http://chandra.harvard.edu/photo/2004/casa/casa_xray.jpg

Asymmetry of Supernova Explosion

kick and translate Pulsar with

Kick Velocity: Average … 400km/s,

Highest … 1500km/s

Explosion Energy ~ 10^{53} erg

(almost Neutrino Emissions)

***1% Asymmetry* is sufficient to explain the Pulsar Kick**

D.Lai & Y.Z.Qian, Astrophys.J. 495 (1998) L103

Estimating Kick Velocity of PNS with $T = 20$ **MeV and** $B = 2\times 10^{17}$**G**

<u>Poloidal</u> Magnetic Field 2-3% Asymmetry in Absorption

§ 4-1 Neutrino Transportation

Neutrino Phase Space Distribution Function

$$f(\boldsymbol{p},\boldsymbol{r}) \approx f_0(\boldsymbol{p},\boldsymbol{r}) + \Delta f(\boldsymbol{p},\boldsymbol{r}), \quad f_0(\boldsymbol{p},\boldsymbol{r}) = 1/\left\{1+\exp\left[\left(\left|\boldsymbol{p}\right| - \varepsilon_\nu\right)/T\right]\right\}$$

Equib. Part

Non-Equib. Part

Neutrino Propagation ⇒ Boltzmann Eq.

$$c\frac{\partial}{\partial \boldsymbol{r}} f_0(\boldsymbol{p},\boldsymbol{r}) \approx c\frac{\partial}{\partial \boldsymbol{r}} f_0(\boldsymbol{p},\boldsymbol{r}) + c\frac{\partial}{\partial \boldsymbol{r}}\Delta f(\boldsymbol{p},\boldsymbol{r}) = I_{coll} \approx -c\frac{\sigma_A}{V}\Delta f(\boldsymbol{p},\boldsymbol{r})$$

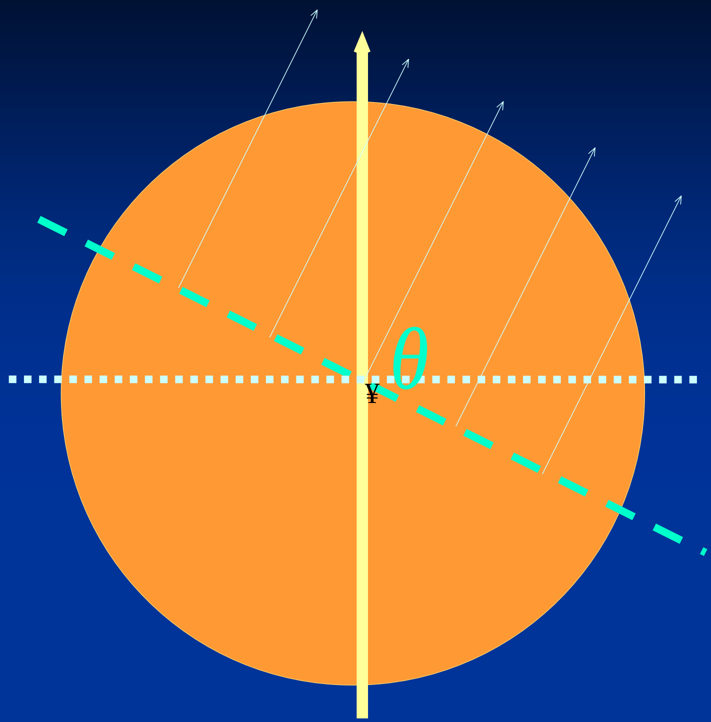
Neutrinos Propagate on **Strait Line**

only absorption

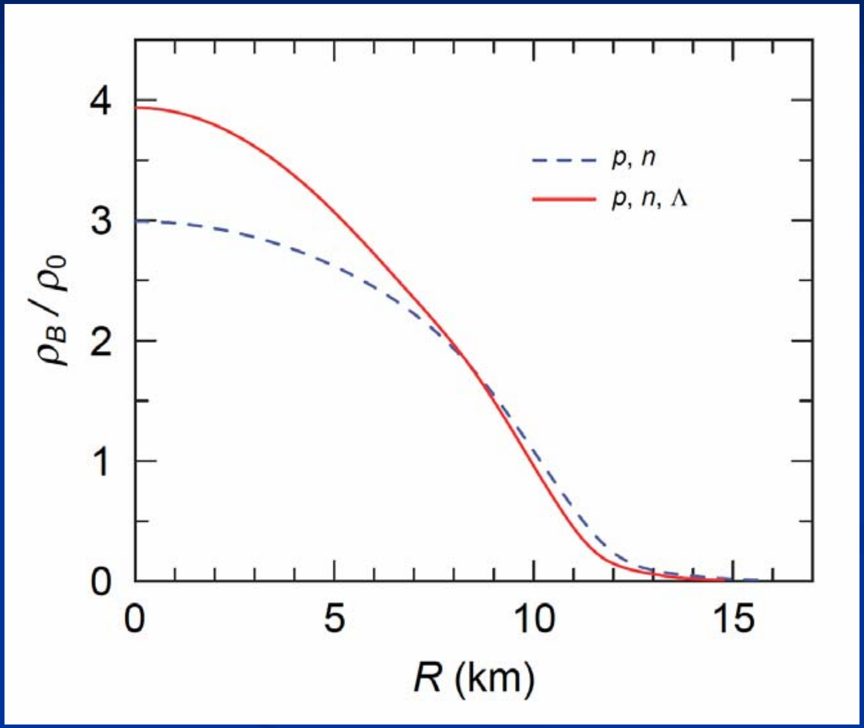
Solution ⇒

$$\Delta f(\boldsymbol{p},\boldsymbol{r}_T,z) = \int_0^z dx\left[-\frac{\partial}{\partial x} f_0(p,r_T,x)\right]\exp\left[-\int_x^z dy\,\frac{\sigma_A(y)}{V}\right],$$

$$z = \boldsymbol{r}\cdot\hat{p}, \qquad \frac{\partial}{\partial z} f_0(p,r_T,z) = \frac{d\varepsilon_\nu}{dz}\frac{\partial}{\partial \varepsilon_\nu} f_0(p,r_T,z)$$



Baryon density in PNS



1) **Neutrinos propagate on the straight lines**

2) **Neutrino are produced and absorbed at all positions on the lines**

Mean-Free path : σ_{ab}/V

$M = 1.68M_{solar}$, $Y_L = 0.4$, $T = 20$ MeV

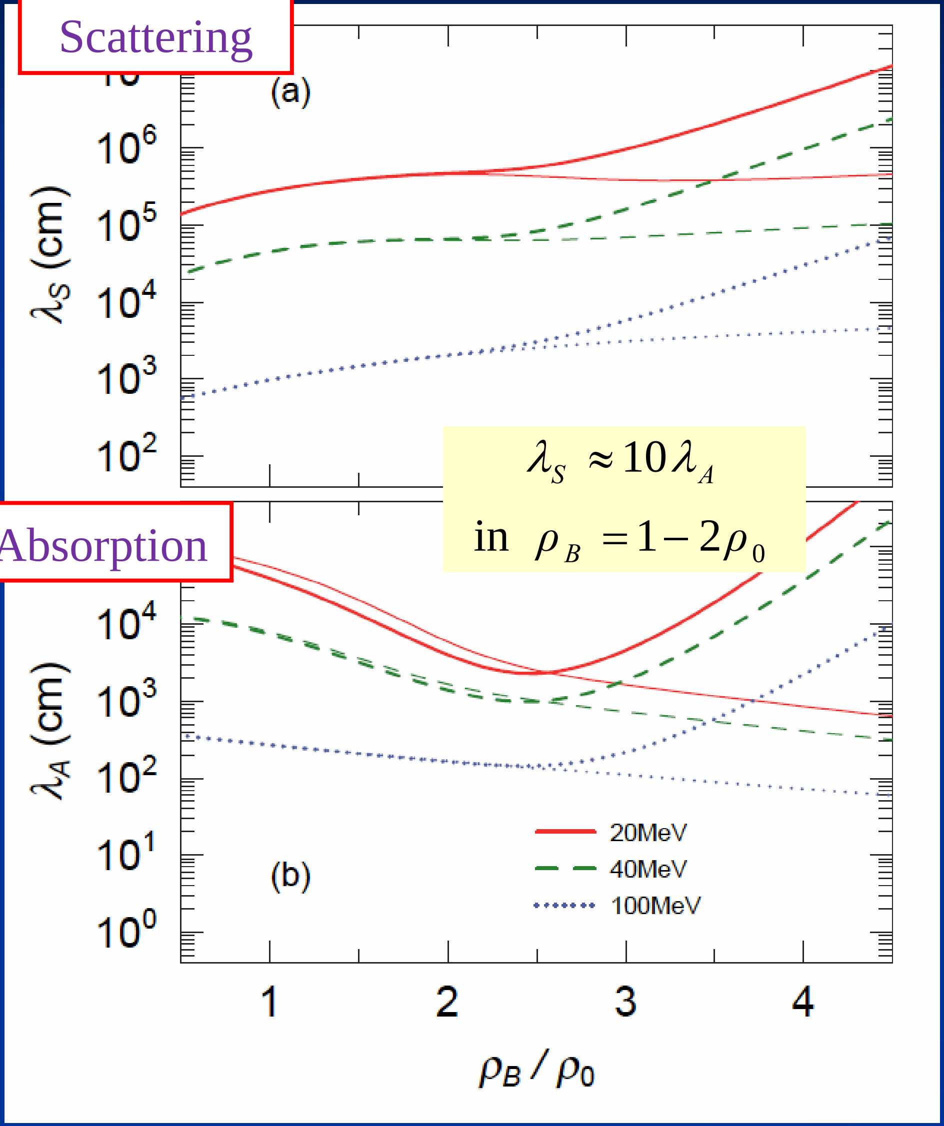
Calculating Neutrino Propagation above $\rho_B = \rho_0$ ρ_0

Mean-Free Paths

$$\lambda_{S,A} = V / \sigma^0_{S,A}$$

Scattering

Absorption



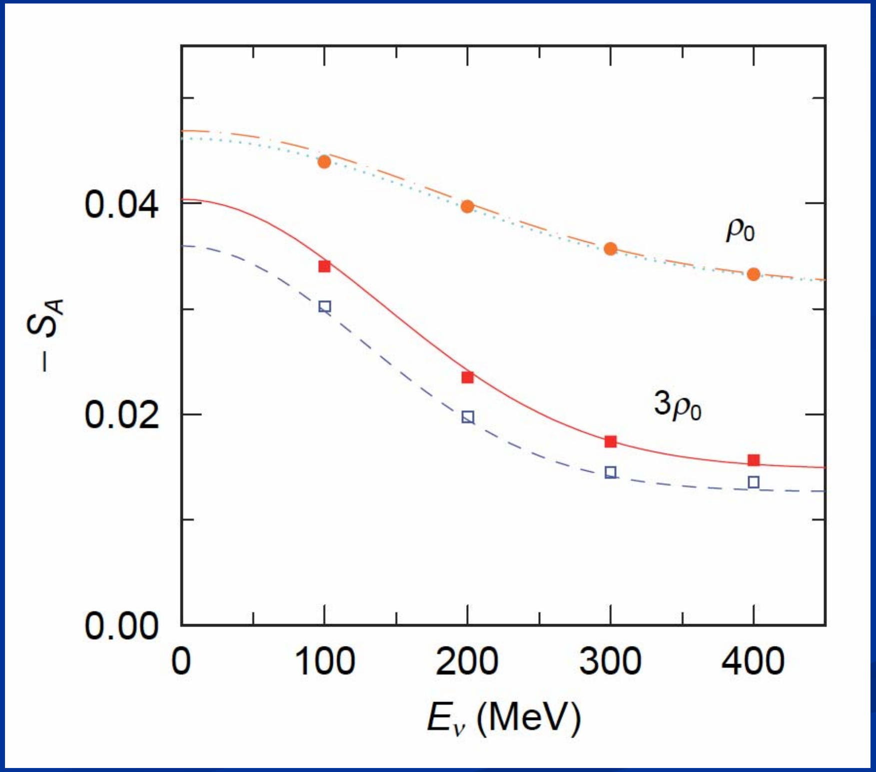
$$\lambda_S \approx 10\lambda_A$$

$$\text{in } \rho_B = 1 - 2\rho_0$$

Magnetic Parts

$$\sigma_A = \sigma^0_A \left(1 + S_A \cos\theta_{i,f}\right)$$

S_A: fitting function



Angular Dep. of Emitted Neutrinos in Uniform Poloidal Mag. Field

$$M_{NS} = 1.68 M_{solar}\ [\mathrm{g}],$$
$$E_T = 3\times 10^{53}\ [\mathrm{erg}]$$

$$\frac{\langle p_z \rangle}{E_T} = \frac{P_1}{3P_0} = 2.0\times 10^{-2} \qquad T = 20\ \mathrm{MeV}$$

$$< p(\boldsymbol{n}) > \approx P_0 + P_1 \cos\theta$$

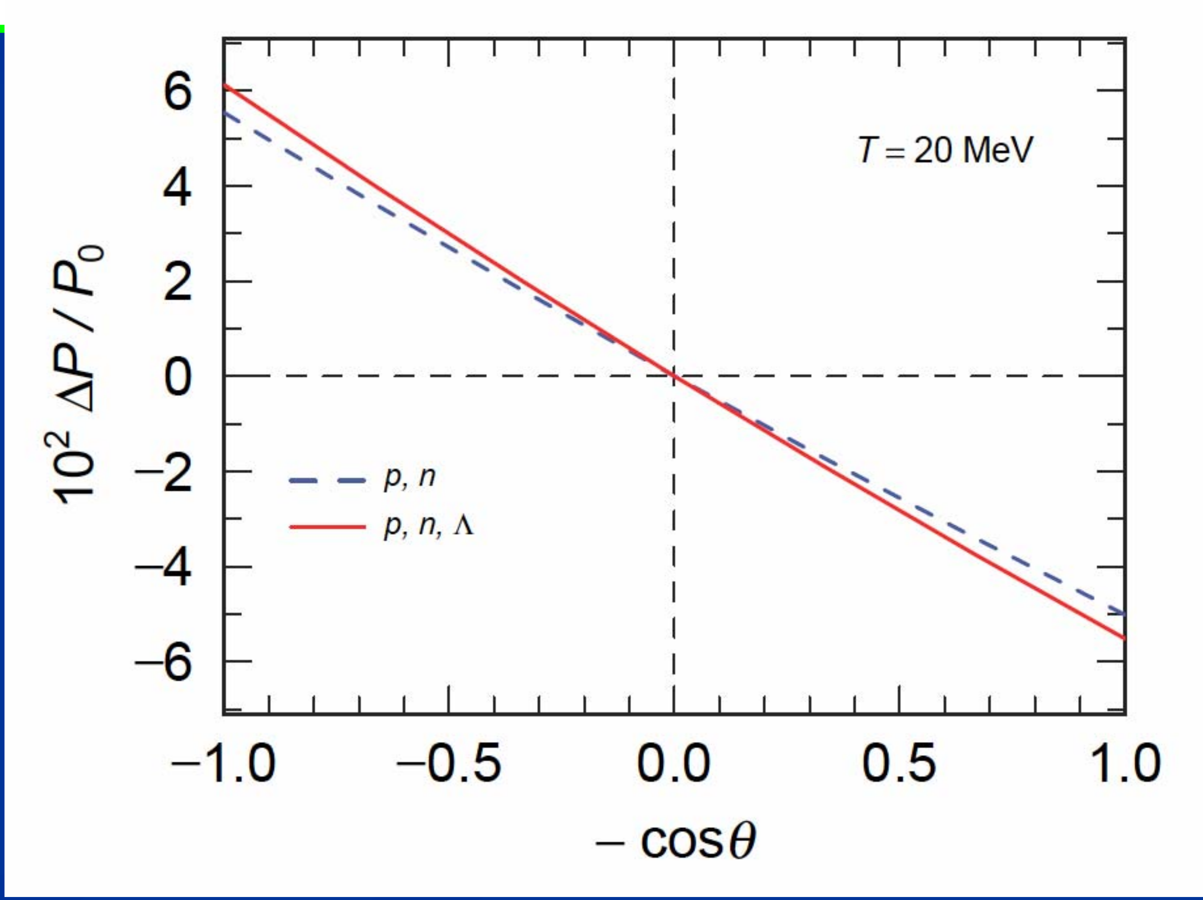
$$B = 2\times 10^{17}\ [\mathrm{G}]$$

$$v_{kick} = \frac{\langle p_z \rangle}{M} \approx 600\,[\mathrm{km/s}]$$

$T = 20$ MeV

Observable Average

… 400km/s,



§ 5 *Summary*

- EOS of Neutron-Star-Matter with p, n, Λ in RMF Approach
- Exactly Solving Dirac Eq. with Magnetic Field in Perturbative Way
- Cross-Sections of Neutrino Scattering and Absorption under Strong Magnetic Field, calculated in Perturbative Way
- Neutrinos are More Scattered and Less Absorbed in Direction Parallel to Magnetic Field

$\Rightarrow$ More Neutrinos are Emitted in Arctic Area

Scattering 1.7 %

Absorption 2.2 % at $\rho_B = 3\rho_0$ and $T = 20$ MeV

$\Rightarrow$ Convection, Pulsar-Kick

Estimating Pulsar Kick in Proto-Neutron-Star with Strong Magnetic Field

$B = 2 \times 10^{17}$G $\Rightarrow$ Perturbative Calculations

$\boxed{V_{kick} = 300 \text{ km/s (p,n), 290 km/s (p,n,}\Lambda\text{) at } T = 20 \text{ MeV}}$

Antarctic Direction

400 km/s (Average of Observed Values)

Future Plans

Fixed Temp. $\Rightarrow$ Constant Entropy

Exact Solution of Dirac Eq. Non-Perturbative Cal.

Low Density Contribution is large $E_\nu < 50$ MeV

$\rightarrow$ Landau Level at least for Electron

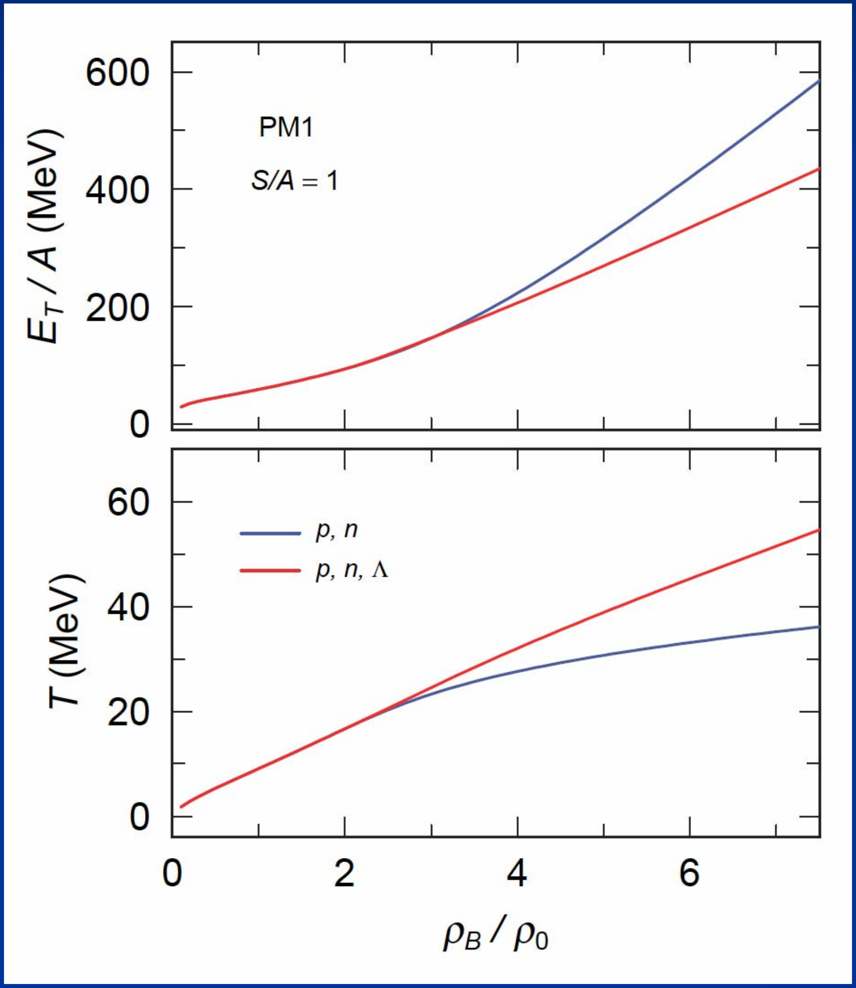
$E_\nu \approx \varepsilon_\nu$ Lower T & Smaller B

Spin-Spin interaction

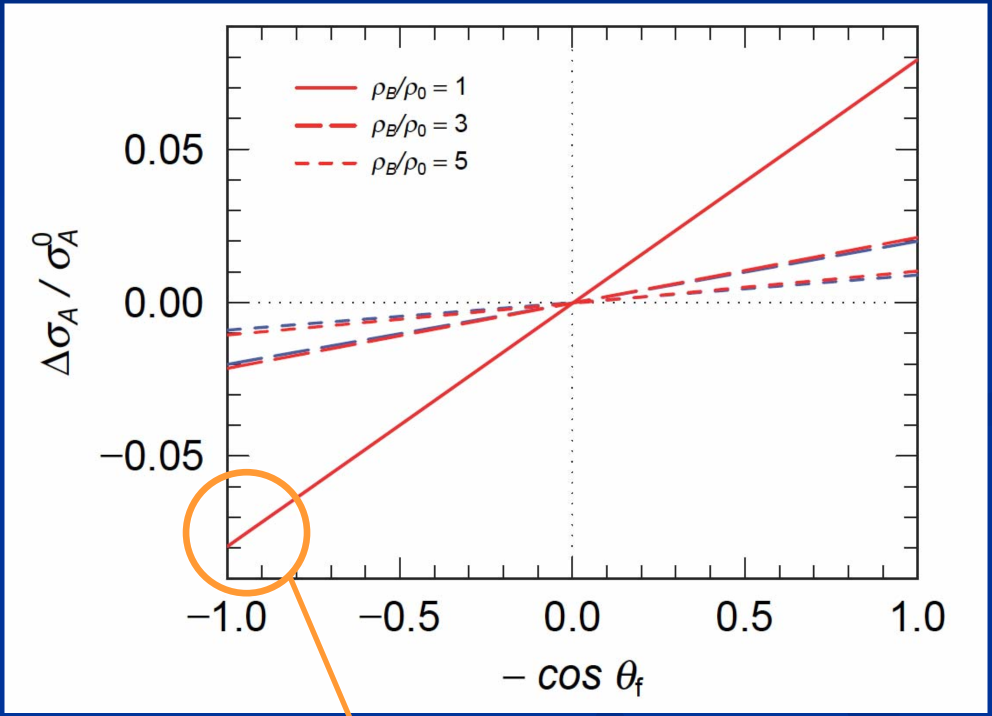
Poloidal Mag. Field $\Rightarrow$ Toroidal Mag. Field

Results in Iso-Entropical Model ($S/A = 1$)

EOS with $S/A = 1$



Absorption Cross-Section



two times larger that at T=20 MeV

Angular Deceleration in Toroidal Magnetic Field

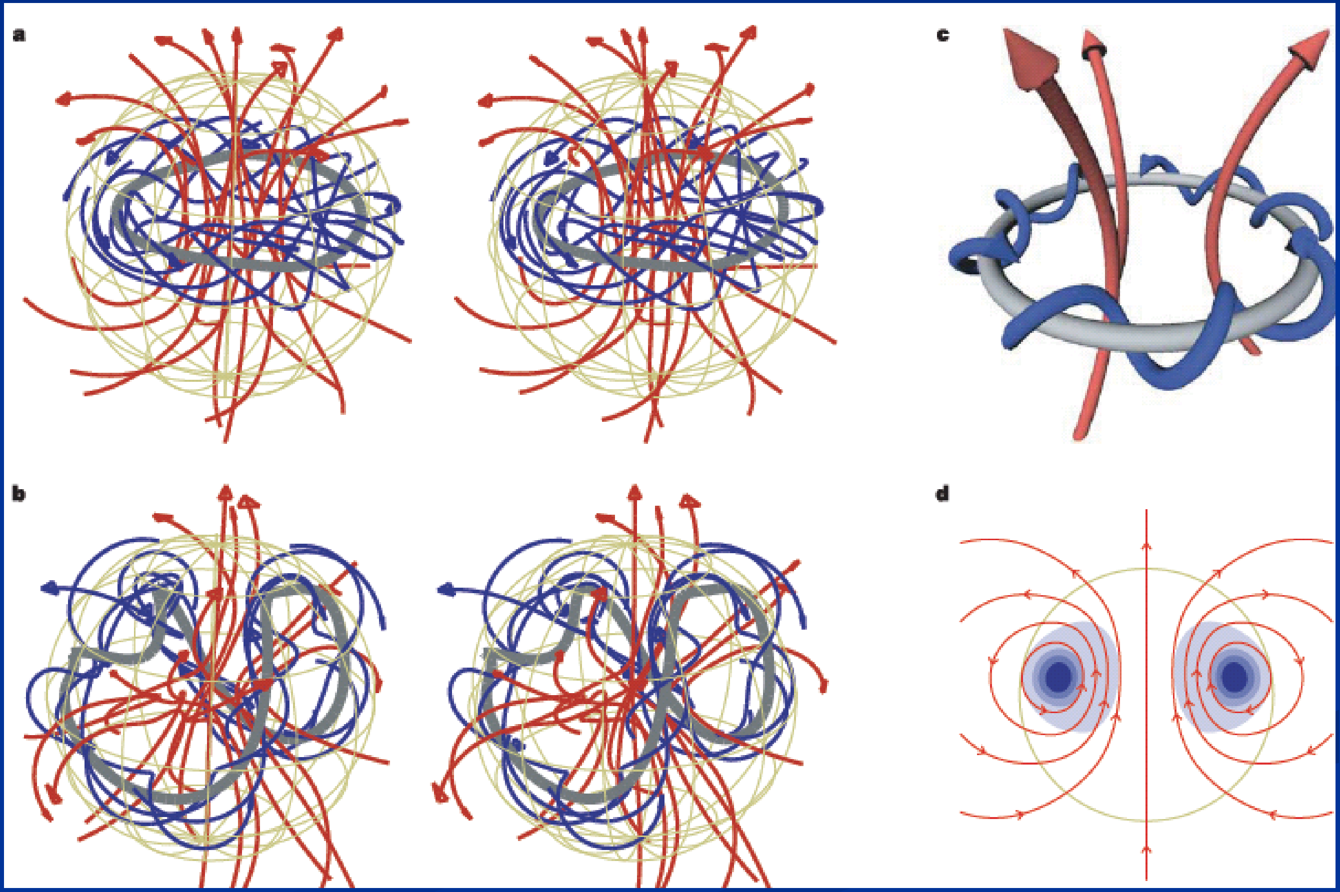
Stability of Magnetic Field in Compact Objects

(*Braithwaite & Spruit 2004*)

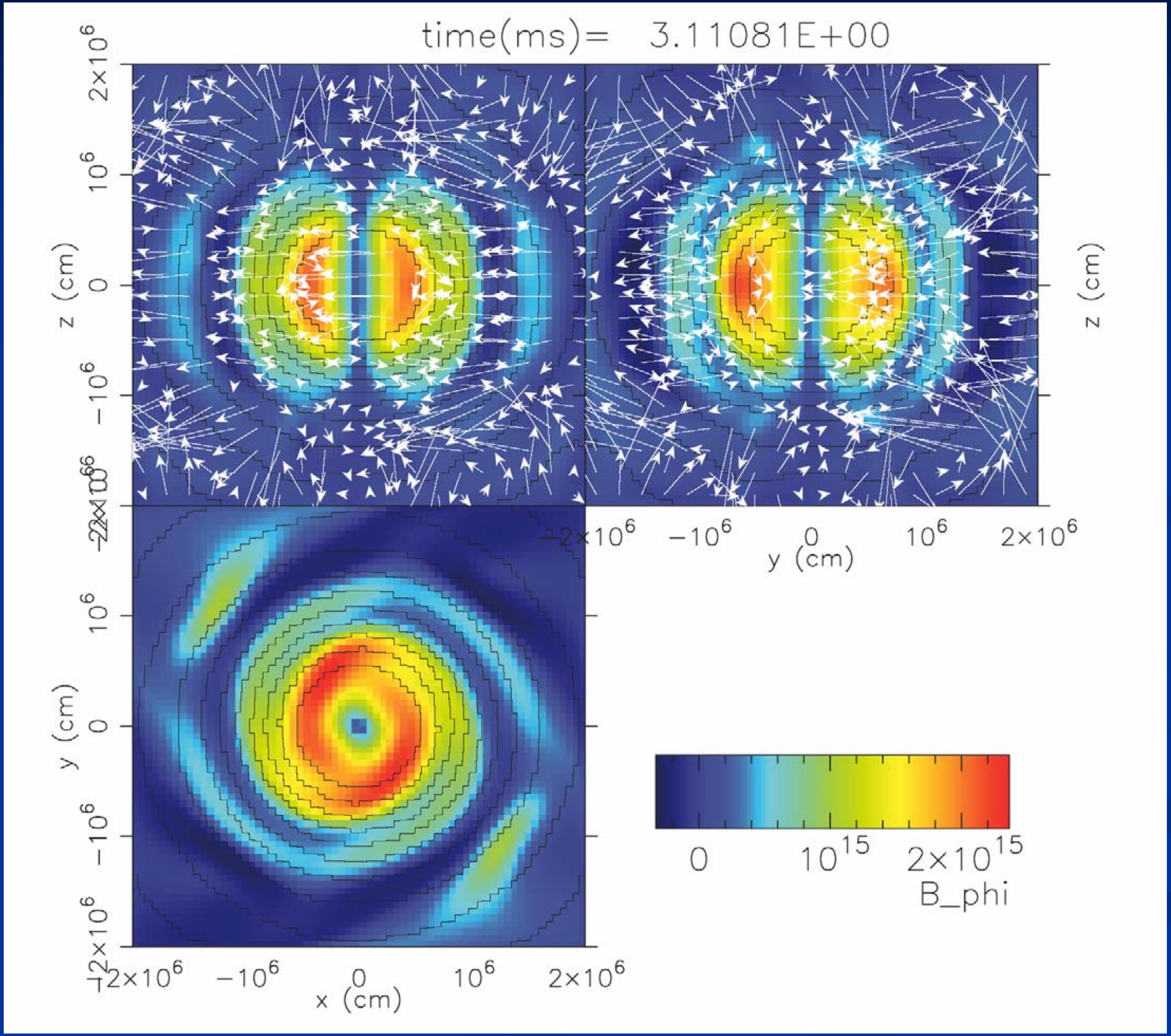
Toroidal Magnetic Field is stable !!

Mag. Field Parallel to Baryonic Flow

Assym. of ν-Emit. must decelerate PNS Spin



No poloidal Magnetic Field at the beginning



Single Toroidal

by T. Kuroda

(NAO)

Distribution of Magnetic Field inside Neutron Star

Toroidal Distribution is Stable inside NS

Spin –Direction is Parallel to Magnetic Field

ν Asymmetric Emission must affect PNS Spin

More Significantly than Magnetic Dipole γ-Radiation

The Details are presented in Poster by Jun Hidaka

Electron

Landau Level	$e_e(n,k_z;s) = \sqrt{k_z^2+m_e^2+eB(2n+1+s)},$

When B → 0 , the wave function → plane wave

$$e_e \approx \sqrt{\boldsymbol{k}^2+m_e^2} - \frac{eBs}{2\sqrt{\boldsymbol{k}^2+m_e^2}} \approx |\boldsymbol{k}| + \frac{m_e}{|\boldsymbol{k}|}\mu_e Bs$$

$$a_e(\boldsymbol{k}) \approx \frac{1}{m_e}\left(k_z, \ \frac{k_z\boldsymbol{k}_T}{|\boldsymbol{k}|}, \ \frac{k_z^2}{|\boldsymbol{k}|}\right), \quad \mu_e = -e/2m_e$$

Toroidal Magnetic Field

$$\boldsymbol{B}(r_T, z) = B_0 G_T(r_T) G_L(z) \hat{e}_\phi$$

$$G_T(r_T) = \frac{16 \exp[-(r_T - R_0)/\Delta r]}{\{1 + \exp[-(r_T - R_0)/\Delta r]\}^2}$$

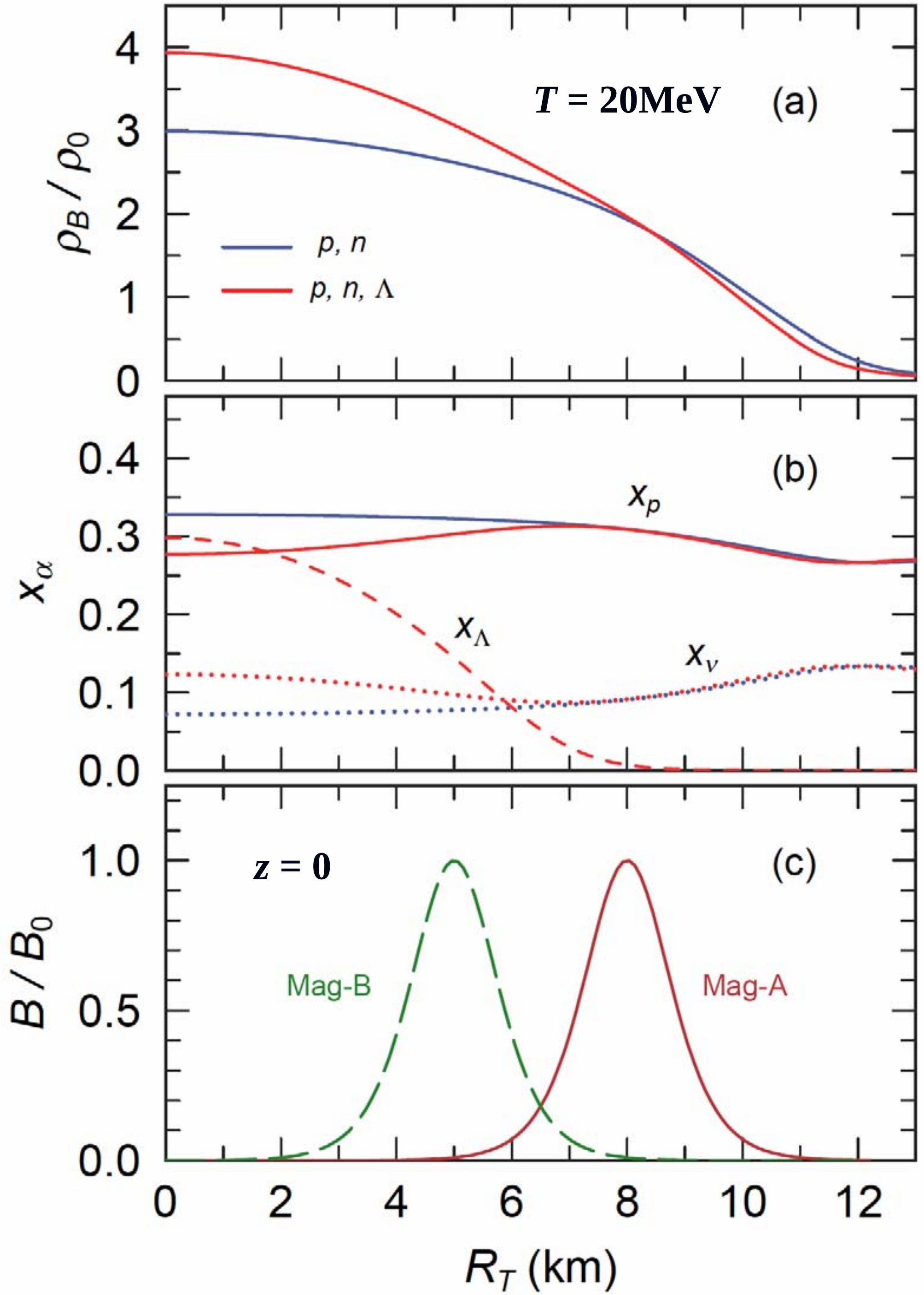
$$G_L(z) = \frac{\exp[-z/\Delta r]}{\{1 + \exp[-z/\Delta r]\}^2}$$

$$\hat{e}_\phi = (-\sin\varphi, \cos\varphi, 0)$$

$\Delta r = 0.5$ (km)

R_0 = 8 (km) Mag.-A

R_0 = 5 (km) Mag.-B



Angular Deceleration

$$\frac{dL_z}{dt} = c\int_{S_N} d\mathbf{r}\int d\mathbf{n}\int dp_L \Delta f(\mathbf{r}, p_L, \mathbf{n})(\mathbf{r}\times\mathbf{p})_z$$

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{1}{I_{NS}}\frac{dL_Z}{dt} = \frac{1}{I_{NS}}\left(\frac{dE_T}{dt}\right)_\nu \frac{cdL_Z/dt}{dE_T/dt}$$

Neutrino Luminosity

$(dE_T/dt)_\nu \sim 3\times 10^{52}$ **erg/s**

$M_{NS} = 1.68 M_{solar}$

Period $P = 10$s

Magnetic Dipole Rad. $B = 3.2\times 10^{39}\sqrt{P\dot{P}}$ [G]

Mag Distr.	Bary.	$-\frac{cdL_Z/dt}{dE_T/dt}$ (cm)	$-\dot{\omega}$ (rad/s)	$\dot{P}/P$		
				ρ_0	$0.1\rho_0$	(Mg.Dpl-rad.)
Mag-A	p,n	66.9	4.94×10^{-2}	7.82×10^{-2}	1.03×10^{-2}	5.32×10^{-12}
	p,n,Λ	109	7.07×10^{-2}	11.26×10^{-2}	1.11×10^{-2}	6.77×10^{-11}
Mag-B	p,n	9.64	7.09×10^{-3}	1.13×10^{-3}	4.50×10^{-5}	5.32×10^{-12}
	p,n,Λ	7.81	5.07×10^{-3}	8.07×10^{-4}	2.29×10^{-5}	6.77×10^{-11}

In Early Stage (**∼ 10 s**) ν Asymmetric Emission must affect PNS Spin

More Significantly than Magnetic Dipole γ-Radiation

Pulsar Kick in Poroidal Mag. Field $B = 2\times 10^{17}$G $\Rightarrow$ Perturbative Cal.

v_{kick} = 580 km/s (p,n) , 610 km/s (p,n, Λ) at T = 20 MeV
= 230 km/s (p,n) , 270 km/s (p,n, Λ) at T = 30 MeV

Antarctic Dir.

400 km/s (Average of Observed Values)

Spin Deceleration in Toroidal Magnetic Field

Reestimating when $B = 10^{15}$G in the surface with $\rho = \rho_0$

$\Longrightarrow$

Mag Distr.	$\dot{P}/P\,[\mathrm{s}^{-1}]$ p,n	(P = 10 s) p,n,Λ
Mag-A	4.8×10^{-3}	5.2×10^{-3}
Mag-B	6.1×10^{-3}	7.2×10^{-3}
Dpl. Rad.	9.8×10^{-11}	

Asymmetric ν-Emission plays an important role in PNS Spin in Early Time

Future Plans

- ν-Scattering
- Fixed Temp. $\Rightarrow$ Constant Entropy
- Exact Solution of Dirac Eq. in Non-Perturbative Cal.
  $\rightarrow$ Landau Level at least for Electron

§ 2-1 Neutron-Star Matter in RMF Approach

RMF Lagrangian, *N*, *Λ*, *σ*, *ω*, *ρ*

$$
\begin{aligned}
\mathcal{L}_{RMF} \;=\;& \bar{\psi}_N(i\partial\!\!\!/ - M_N)\psi_N + \bar{\psi}_\Lambda(i\partial\!\!\!/ - M_\Lambda)\psi_\Lambda + g_\sigma\bar{\psi}_N\psi_N\sigma + g_\sigma^\Lambda\bar{\psi}_\Lambda\psi_\Lambda\sigma \\
& + g_\omega\bar{\psi}_N\gamma_\mu\psi_N\omega^\mu + g_\rho\bar{\psi}_N\gamma_\mu\vec{\tau}\psi_N\vec{\rho}^\mu + g_\omega^\Lambda\bar{\psi}_\Lambda\gamma_\mu\psi_\Lambda\omega^\mu \\
& - \widetilde{U}[\sigma] + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu
\end{aligned}
$$

$\widetilde{U}[\sigma]$: the self-energy potential of the scalar mean-field.

Dirac Eq.

$$[p\!\!\!/ - M_N^* - U_0(N)\gamma_0 - U_{IV}\gamma_0\tau_3]\, u_N(p) = 0$$

$$[p\!\!\!/ - M_\Lambda^* - U_0(\Lambda)\gamma_0]\, u_\Lambda(p) = 0$$

Scalar Field ⇒ Effective Mass

$$M_N^* = M_N - g_\sigma\sigma \qquad M_\Lambda^* = M_\Lambda - g_\sigma^\Lambda\sigma$$

$$\frac{\partial}{\partial\sigma}\widetilde{U}[\sigma] = g_\sigma\rho_s(N) + g_\sigma^\Lambda\rho_s(\Lambda)$$

$$U_0(N) = \frac{g_\omega}{m_\omega^2}(g_\omega\rho(N) + g_\omega^\Lambda\rho(\Lambda)) \qquad U_{IV} = \frac{g_\rho^2}{m_\rho^2}(\rho(p) - \rho(n))$$

$$U_0(\Lambda) = \frac{g_\omega^\Lambda}{m_\omega^2}(g_\omega\rho(N) + g_\omega^\Lambda\rho(\Lambda))$$

Spin-indep. part

$$\frac{d^2\sigma_0}{dk_f d\Omega_f} = \frac{G_F^2}{32\pi^5}\frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|}[1-f_{l'}(|\boldsymbol{k}_f|)]\int \frac{d^3p}{E_iE_f} W_0 \times \delta(|\boldsymbol{k}_i|-|\boldsymbol{k}_f|+\varepsilon_i(\boldsymbol{p})-\varepsilon_f(\boldsymbol{p}'))n_B(\varepsilon_i(\boldsymbol{p}))[1-n_{B'}(\varepsilon_f(\boldsymbol{p}'))]$$

$$\begin{aligned} W_0 &= c_V^2\left[(k_f\cdot p)(k_i\cdot p')+(k_f\cdot p')(k_i\cdot p)-M_fM_i(k_f\cdot k)\right] \\ &+ c_A^2\left[(k_f\cdot p)(k_i\cdot p')+(k_f\cdot p')(k_i\cdot p)+M_fM_i(k_i\cdot k_f)\right]-2c_Vc_A\left[(k_f\cdot p')(k_i\cdot p)-(k_f\cdot p)(k_i\cdot p')\right] \\ & q = k_i - k_f = p' - p \end{aligned}$$

Spin-dep. Part

$$\frac{d^2\Delta\sigma}{dk_f d\Omega_f} = \frac{G_F^2}{32\pi^5}B\frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}|}[1-f_{l'}(\boldsymbol{k}_f)](S_1+S_2)$$

$$\begin{aligned} S_1 &= \frac{1}{Q}\int dE_i \int d\phi_p \left\{n'_B(\varepsilon_i)[1-n_{B'}(\varepsilon_i+\omega)]W_i + n'_{B'}(\varepsilon_f)n_B(\varepsilon_i)(W_i-2W_f)\right\}, \\ S_2 &= -\frac{1}{Q^2}\int dE_i \int d\phi_p (E_i+\omega)n_B(\varepsilon_i)[1-n_{B'}(\varepsilon_i+\omega)]\frac{1}{p}\frac{\partial}{\partial t}(W_i-W_f). \end{aligned}$$

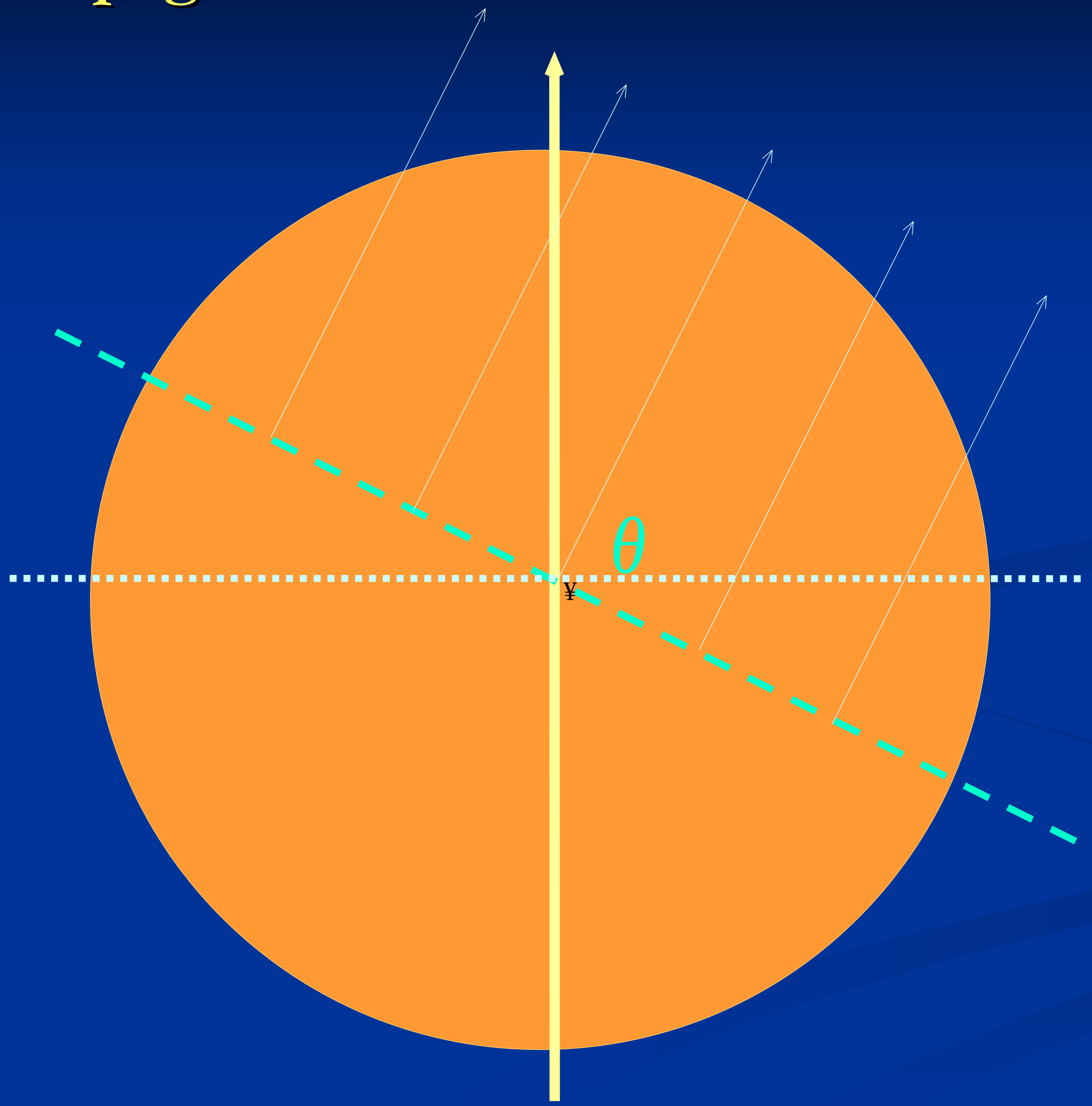
$$t = \frac{\boldsymbol{q}\cdot\boldsymbol{p}}{|\boldsymbol{q}||\boldsymbol{p}|}$$

$$\begin{aligned} W_i/\mu_i &= c_V^2\left\{[k_f\cdot(M_fp-M_ip')](k_i\cdot b_i)-[k_i\cdot(M_fp-M_ip')](k_f\cdot b_i)\right\} \\ & +c_A^2\left\{[-k_f\cdot(M_fp+M_ip')](k_i\cdot b_i)+[k_i\cdot(M_fp+M_ip')\}(k_f\cdot b_i)\right] \\ & -2c_Vc_AM_i\left\{(k_f\cdot p')(k_i\cdot b_i)+(k_f\cdot b_i)(k_i\cdot p')\right\}, \\ W_f/\mu_f &= c_V^2\left\{[k_f\cdot(M_fp-M_ip')](k_i\cdot b_f)-[k_i\cdot(M_fp-M_ip')](k_f\cdot b_f)\right\} \\ & +c_A^2\left\{[k_f\cdot(M_fp+M_ip')](k_i\cdot b_f)-[k_i\cdot(M_fp+M_ip')](k_f\cdot b_f)\right\} \\ & -2c_Vc_AM_f\left\{[(k_f\cdot b_f)(k_i\cdot p)+(k_f\cdot p)(k_i\cdot b_f)]\right\} \end{aligned}$$

$$b_i = \frac{\sqrt{p_T^2+M_i^2}}{E_i(\boldsymbol{p})}a_i,$$

$$b_f = \frac{\sqrt{p_T'^2+M_f^2}}{E_f(\boldsymbol{p}')}a_f$$

Neutrino Propagation



1) **Neutrinos propagate on the straight lines**

2) **Neutrino are created and absorbed at all positions on the lines**

Mean-Free Path

: σ_{ab}/V

$$\hat{K}(p) = \not{p} - M^* - \mu B \sigma_z$$

$$= \begin{pmatrix} p_0 - M^* - \mu B & 0 & -p_z & -p_x \\ 0 & p_0 - M^* + \mu B & -p_x & p_z \\ p_z & p_x & -p_0 - M^* - \mu B & 0 \\ p_x & -p_z & 0 & -p_0 - M^* + \mu B \end{pmatrix}$$

Baryon Propagator $\hat{K}(p)S(p) = 1$

$$S(p) = \det \hat{K}(S_0 + S_1 U_T + S_2 U_T^2 + S_3 U_T^3)$$

$$\begin{aligned} \det \hat{K} &= p_0^4 - 2p_0^2(\boldsymbol{p}^2 + M^2 + U_T^2) + (\boldsymbol{p}^2 + M^2)^2 + 2U_T^2(p_z^2 - \boldsymbol{p}_T^2 - M^2) + U_T^4 \quad , \\ S_0 &= (p_0^2 - E_p^2)(\not{p} + M) \quad , \\ S_1 &= (p_0^2 + E_p^2)\sigma_z + 2Mp_0\sigma_z\gamma_0 - 2p_z(\boldsymbol{p} \cdot \boldsymbol{\sigma})\gamma_0 \\ &\quad + 2Mp_z\gamma_5\gamma_0 + 2ip_0p_y\gamma^0\gamma^1 - 2ip_0p_x\gamma^0\gamma^2 \quad , \\ S_2 &= -p_0\gamma^0 + p_z\gamma^3 - p_x\gamma^1 - p_y\gamma^2 + M \quad , \\ S_3 &= -\sigma_z \quad . \end{aligned}$$

Baryon Propagator in $B \to 0$ Limit

$$S(p) = \sum_{s=\pm 1} \left\{ \frac{u(\boldsymbol{p}, s)\bar{u}(\boldsymbol{p}, s)}{p_0 - e(\boldsymbol{p}, s) + i\delta} + \frac{v(-\boldsymbol{p}, s)\bar{v}(-\boldsymbol{p}, s)}{p_0 + e(\boldsymbol{p}, s) - i\delta} \right\} = \left[\hat{K}(p)\right]^{-1}$$

$$u(\boldsymbol{p}, s)\bar{u}(\boldsymbol{p}, s) = \left[(p_0 - e(\boldsymbol{p}, s))S(p))\right](p_0 = e(\boldsymbol{p}, s)) \approx \boxed{\frac{1}{4E_p^*}(\not{p} + M^*)(1 + s\gamma_5 \not{a})}$$

$$\boxed{a_z = \frac{E_p^*}{\sqrt{p_T^2 + M^{*2}}} \quad \boldsymbol{a}_T = 0 \quad a_0 = \frac{p_z}{\sqrt{p_T^2 + M^{*2}}}}$$

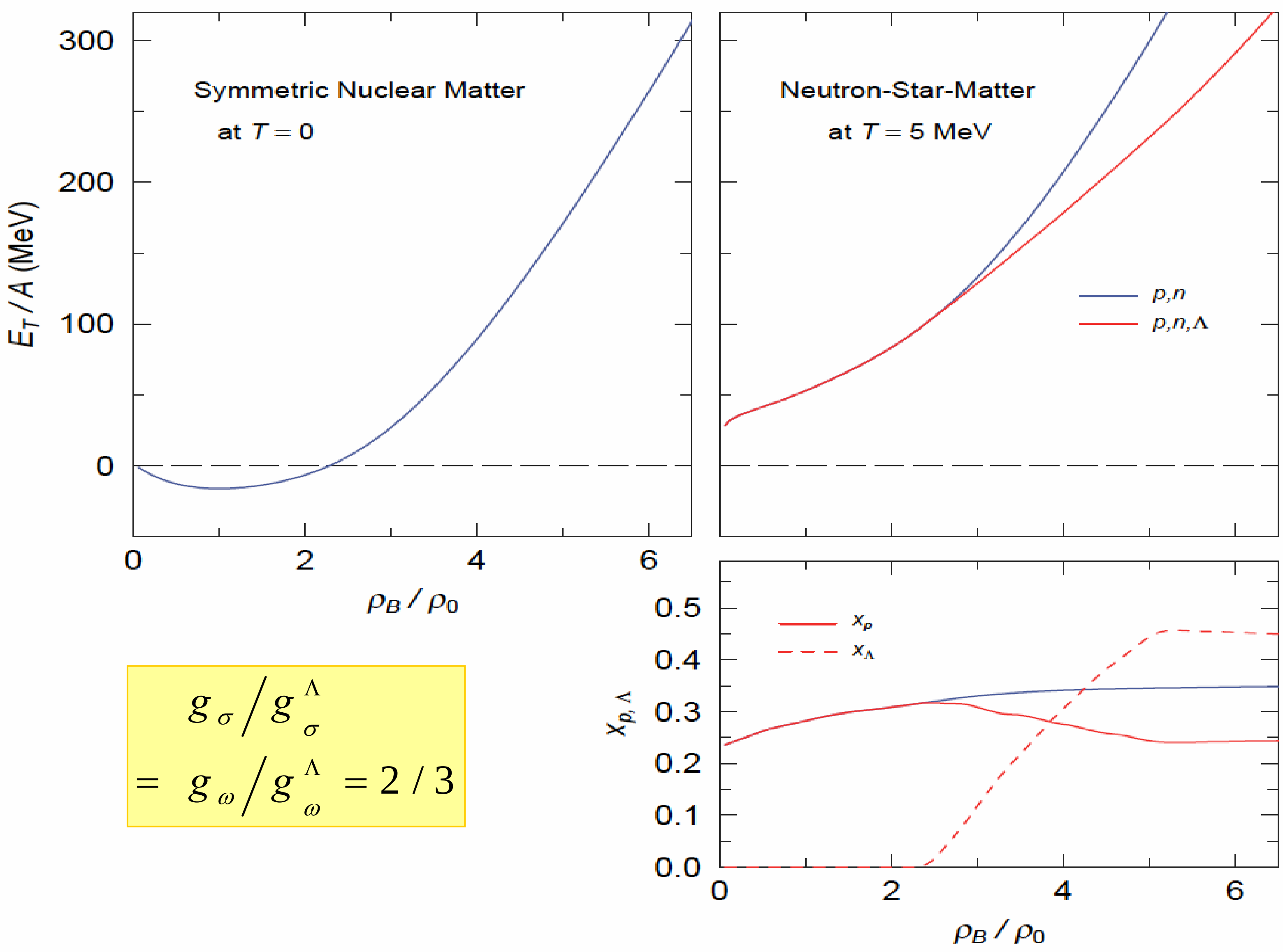
When $\mu B << 1$,

Spin Vector

Dirac Spinor

EOS of PM1-1

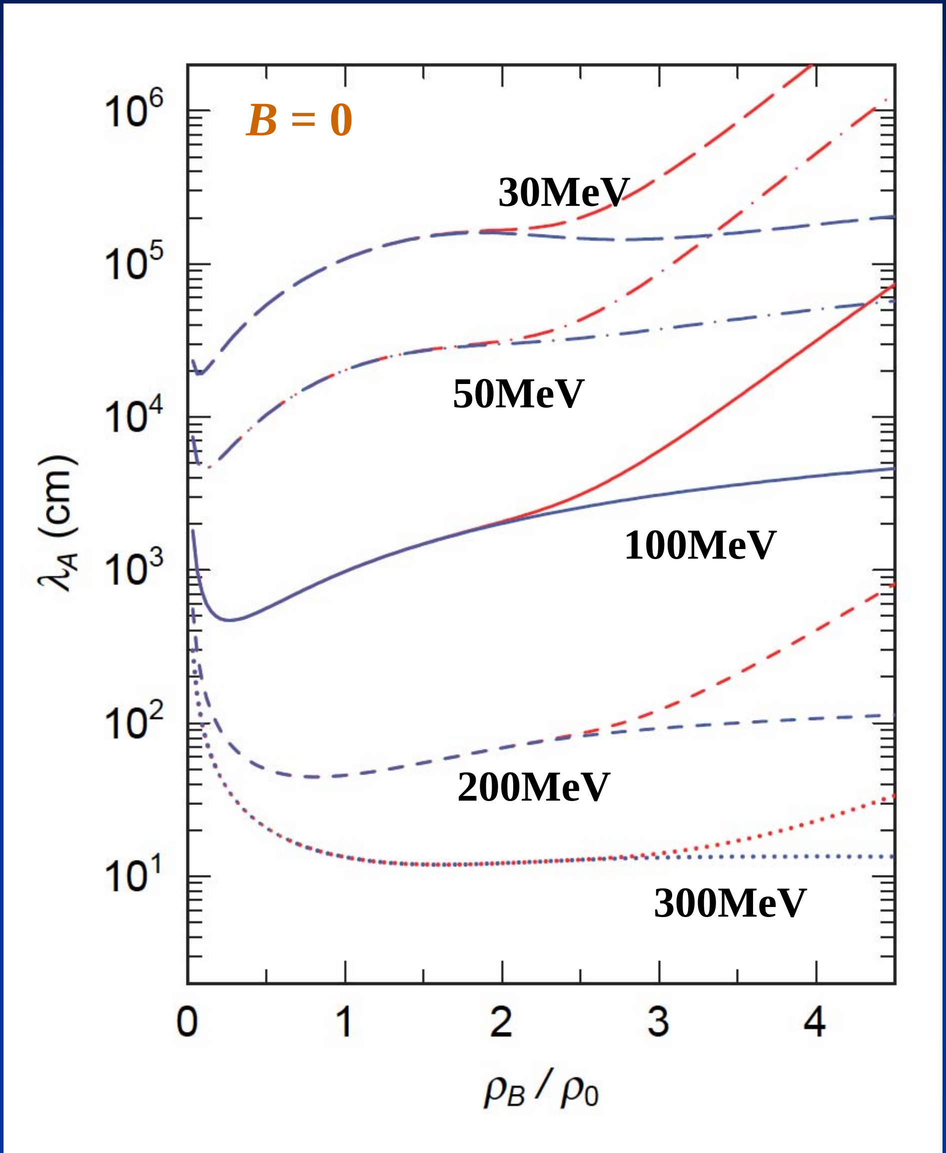
$BE = 16\,\text{MeV},\ M_N^* / M_N = 0.7,\ K = 200\,\text{MeV}$ at $\rho_0 = 0.17\,\text{fm}^{-3}$



$$g_\sigma / g_\sigma^\Lambda = g_\omega / g_\omega^\Lambda = 2/3$$

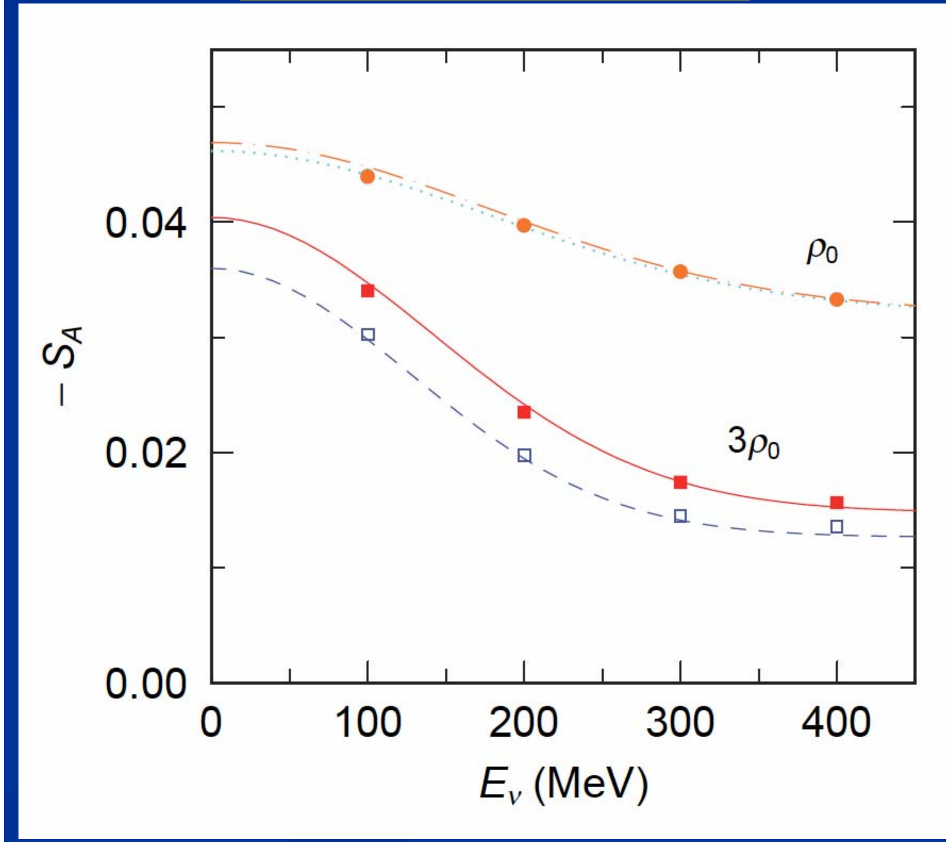
Absorption Mean-Free Path

$$\lambda_A = (\sigma_A / V)^{-1}$$



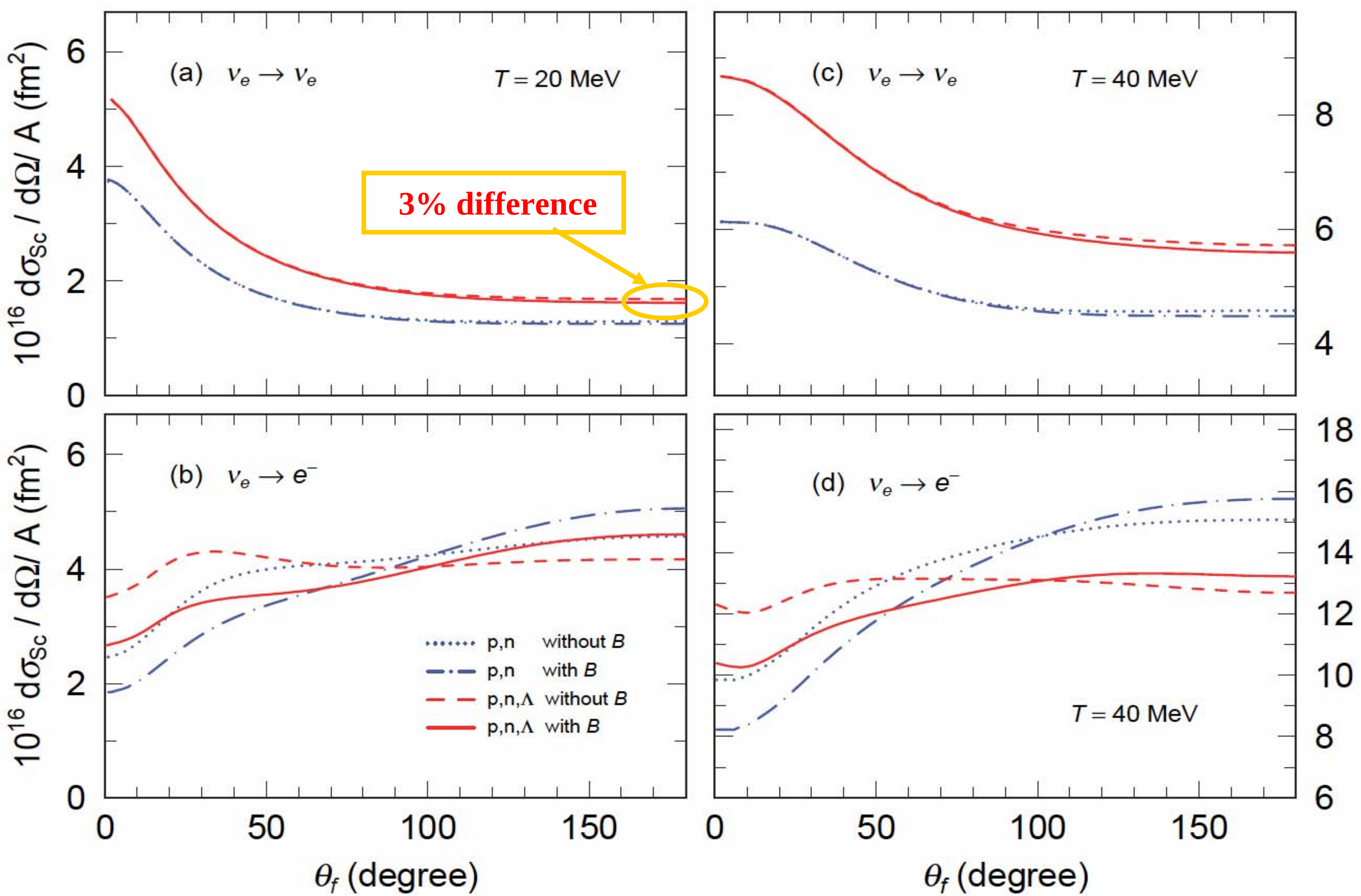
$$\sigma_A = \sigma_A^0 \left(1 + S_A \cos\theta_{i,f}\right)$$

S_A: fitting function

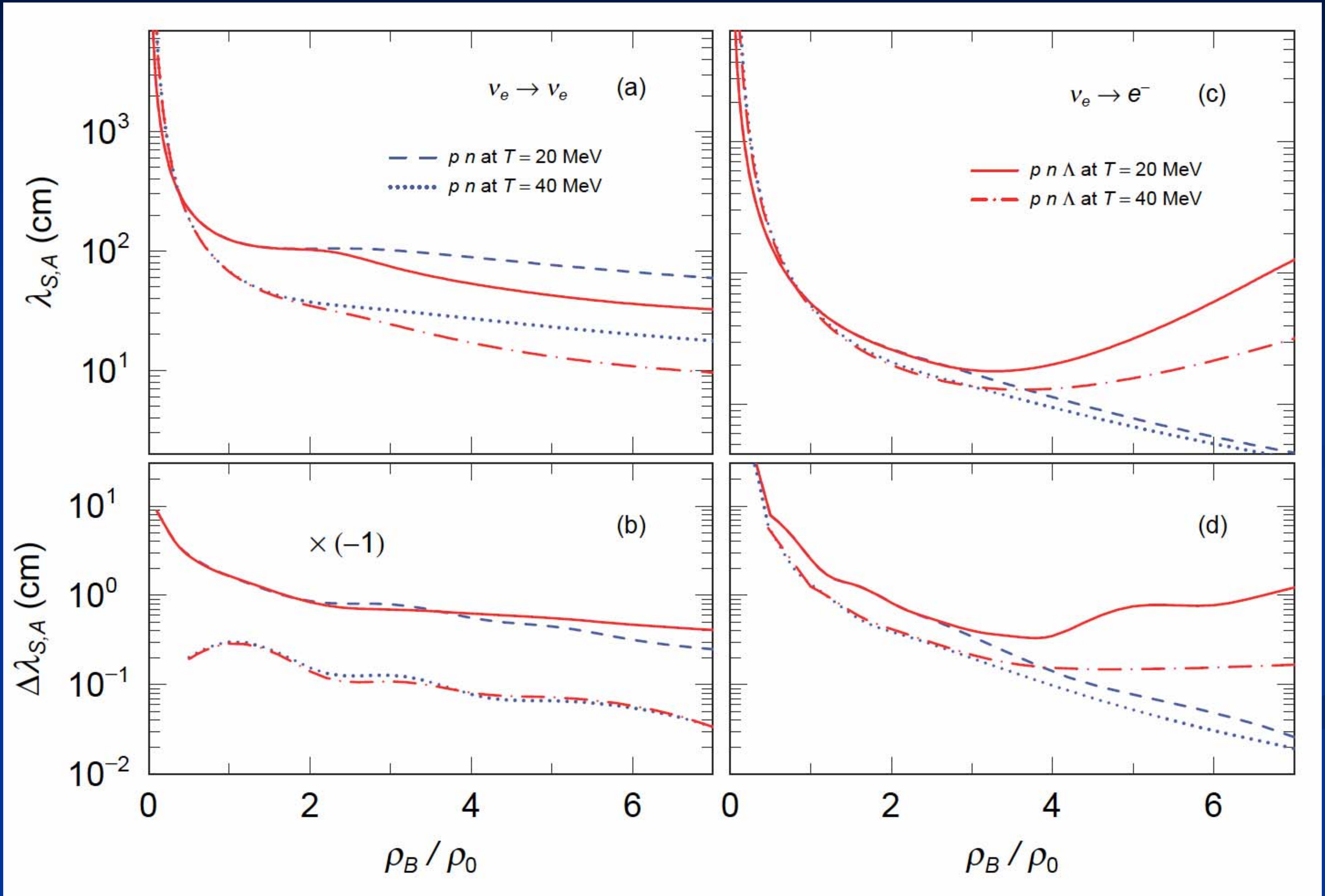


§ 3 Results

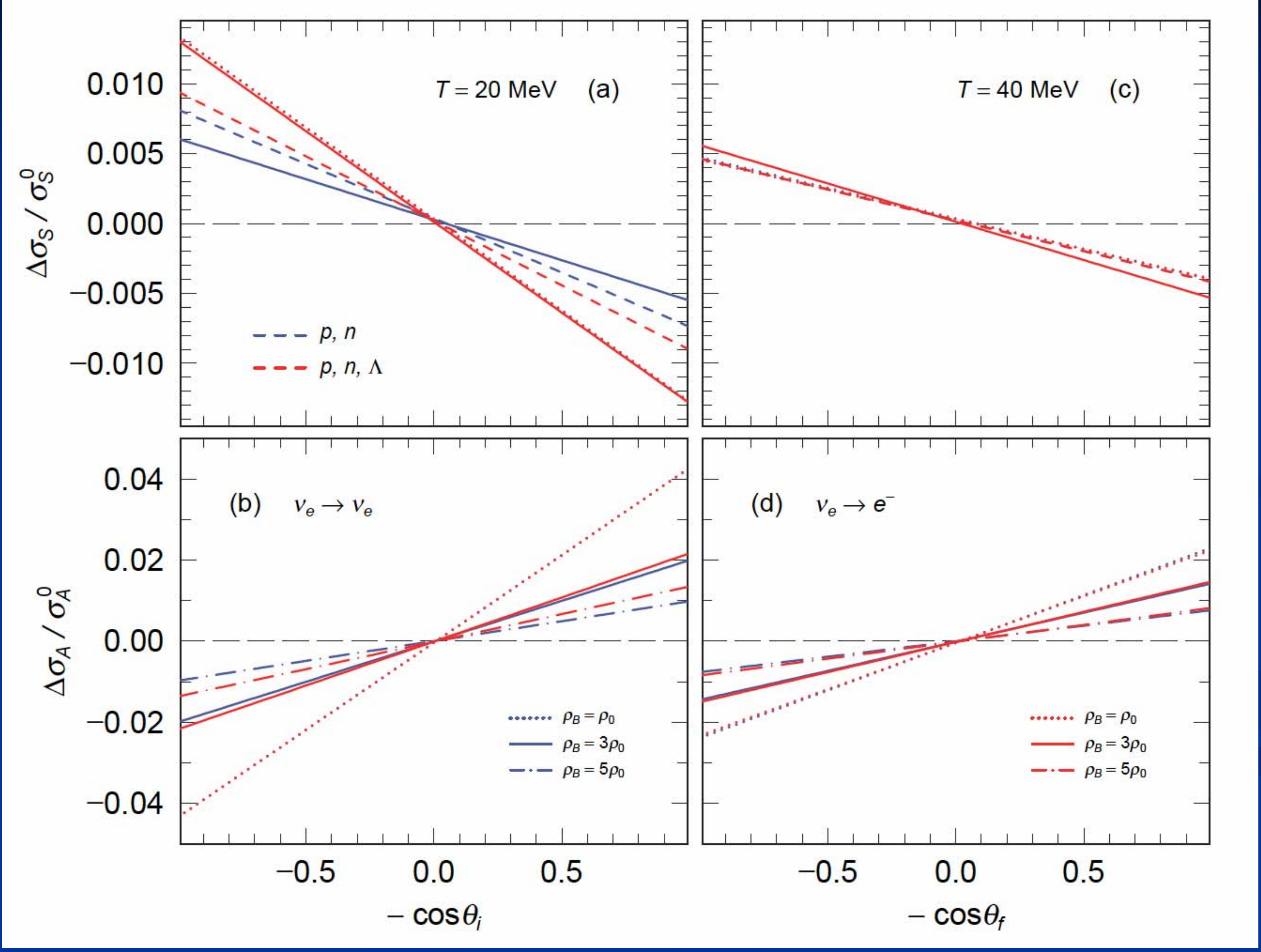
$k_i = \varepsilon_\nu$ (neutrino chem. pot.), $B = 2\times10^{17}$ G and $\theta_i = 0^\circ$



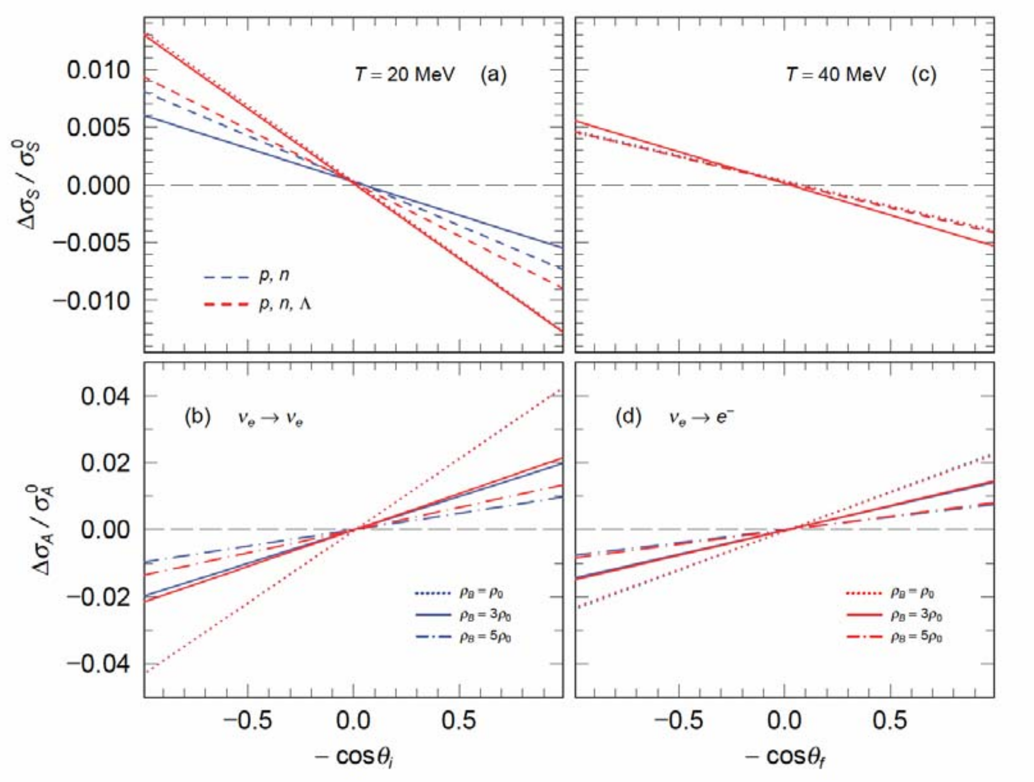
Neutrino Mean-Free-Path at Energy equal to Chem. Potential

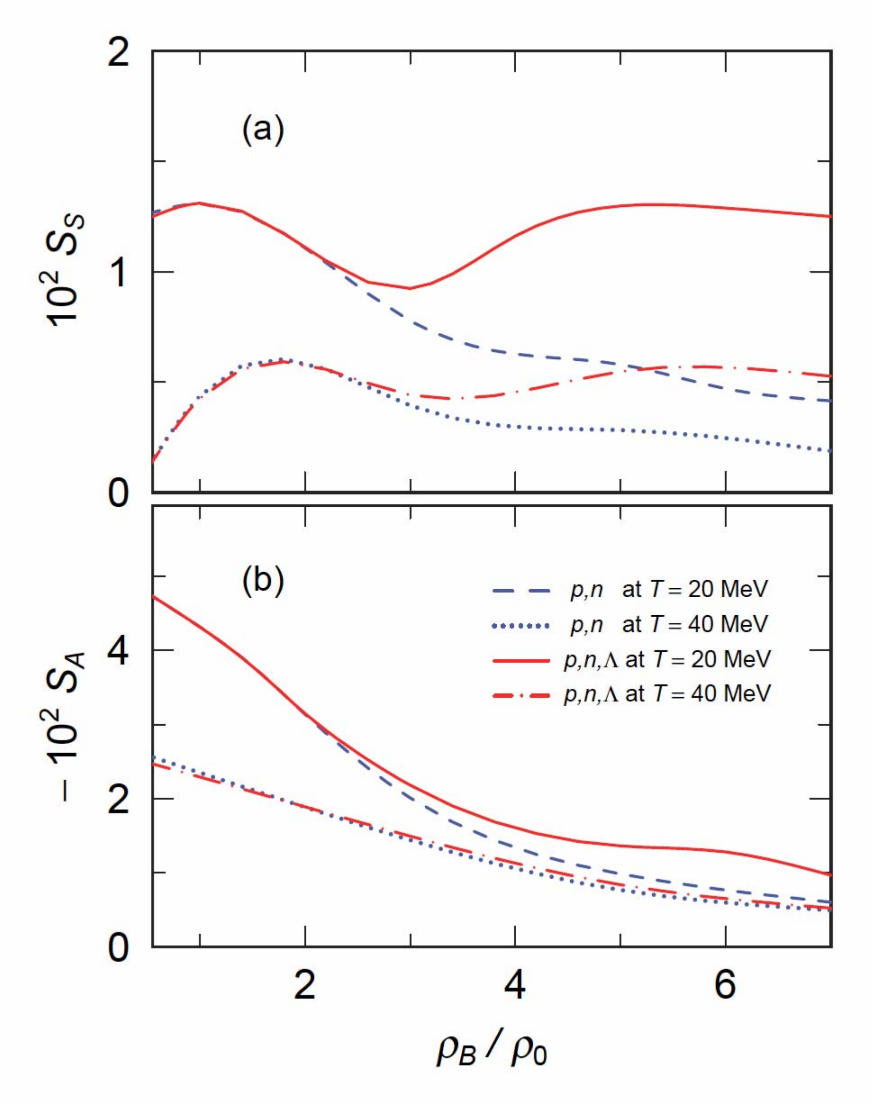


Magnetic Parts of Cross-Sections



Magnetic Parts of Cross-Sections





$$\sigma_{S,A} = \sigma^0_{S,A}\left(1 + S_{S,A}\cos\theta_{i,f}\right)$$