Chiral quark-soliton model, a pion mean-field approach for the structure of baryons

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Contents

- Starting point: Nonperturbative QCD

The QCD partition function

$$\mathcal{Z}_{\mathrm{QCD}} = \int D\psi D\psi^\dagger DA_\mu \exp\{-S[\psi, \psi^\dagger, A]\}$$

$$S = \int d^4x \left[i\psi^\dagger(i\not{D} + i\hat{m})\psi - \frac{1}{4}G^2 \right]$$: QCD Action

The quark is relatively easy to handle but the gluon matters.

$$\mathcal{Z}_{\mathrm{QCD}} = \int \boxed{DA_\mu} \mathrm{Det}[i\not{D} + i\hat{m}] \exp\{-S[A]\}$$

Assertion: Instantons are the most dominant one.

Gain: Chiral symmetry and its spontaneous breakdown is realized.

Price to pay: No explicit effects of the quark confinement

- Instanton Vacuum (Instanton Liquid Model)

Refs. D.I. Diakonov & V. Yu Petrov, NPB 245 (1984) 259, **NPB 272 (1986) 457**

Review: Diakonov, hep-ph/0212026

$\mathcal{Z}_{\rm eff} = \overline{{\rm Det}[i\not{D} + i\hat{m}]}$: Instanton ensemble average

$\bar{\rho}/\bar{R} \simeq \frac{1}{3} \longrightarrow$ Packing fraction: $\pi^2 \frac{\bar{\rho}^4}{\bar{R}^4} \simeq \frac{1}{8}$

Dilute instanton liquid

$\longrightarrow$ Integrations over zero modes can be performed independently (no overlapping between instantons).

$N_c^2 - 1 - (N_c - 2)^2$

$\int DA_\mu[\cdots] \to \int dz_\mu D\rho DR[\cdots]$ Saddle-point approx. 4+1+(4Nc-5) zero modes

- Gluon condensate (Nonperturbative nature of the QCD vacuum)

$$\frac{1}{32\pi^2}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle \simeq (200\,\mathrm{MeV})^4 > 0 \longleftarrow$$ QCD sum rules

$$\frac{1}{32\pi^2}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle \simeq \frac{N}{V} := \frac{1}{\overline{R}^4} \longrightarrow \overline{R} \simeq \frac{1}{200\,\mathrm{MeV}} = 1\,\mathrm{fm}$$

⟶ Gluon Energy of the QCD vacuum

Instanton-antiinstanton ensemble stabilizes at a certain density related to the QCD Lambda:

$$\overline{\rho} \simeq 0.48/\Lambda_{\overline{MS}} \simeq 0.35\,\mathrm{fm}$$ ⟶ **Natural scale of the model**

$$\overline{R} = \left(\frac{N}{V}\right)^{-1/4} \simeq 1.35/\Lambda_{\overline{MS}} \simeq 0.95\,\mathrm{fm}$$

with $\Lambda_{\overline{MS}} = 280\,\mathrm{MeV}$ used from the DIS data.

Spontaneous breakdown of chiral symmetry (SBχS)

- Chiral condensate as an order parameter

$$\langle\overline{\psi}\psi\rangle = -\frac{1}{V}\mathrm{sign}(m)\pi\boxed{\overline{\nu}(0)}$$

:Banks-Casher relation NPB169 (1980) 103

Zero eigenvalue spectrum (zero mode) of the Dirac operator in QCD

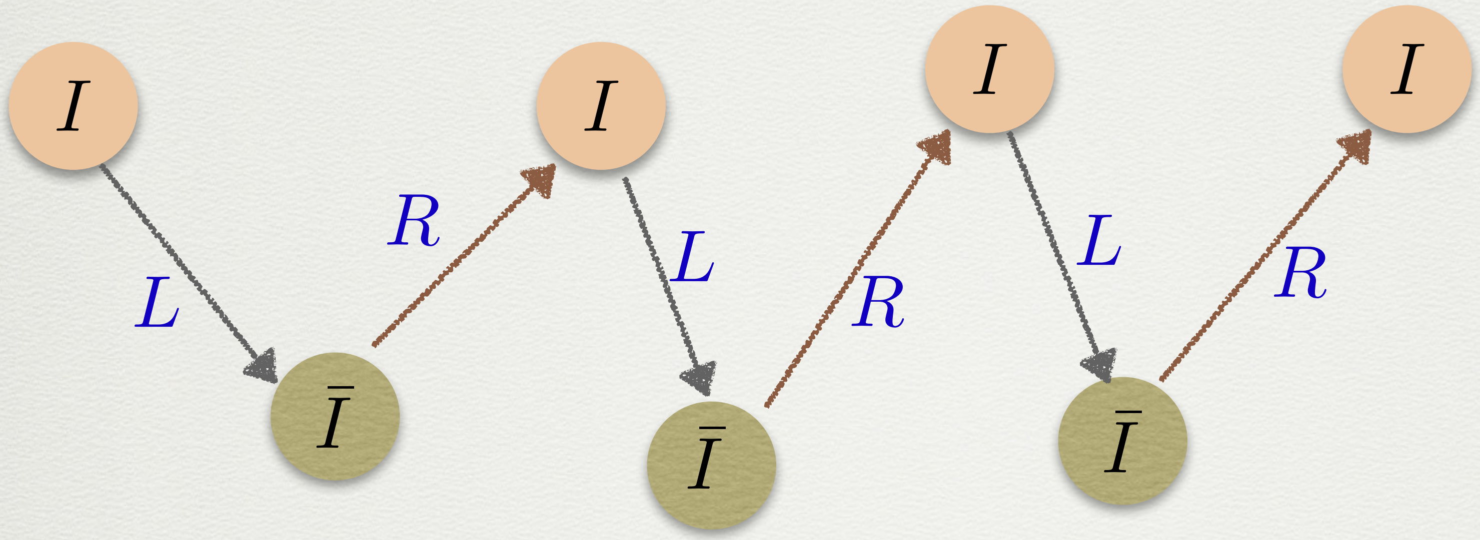
$$\langle\overline{\psi}\psi\rangle = -\frac{\pi}{V_4}\overline{\nu}(0) \sim -\frac{1}{\overline{R}^2\overline{\rho}}$$

- Consequences of SBxS
  - Final values of the chiral condensate
  - The Dynamical quark mass:
  - pseudo-NG bosons
  - Pion decay constant & GOR relation (explicit breaking of XS)

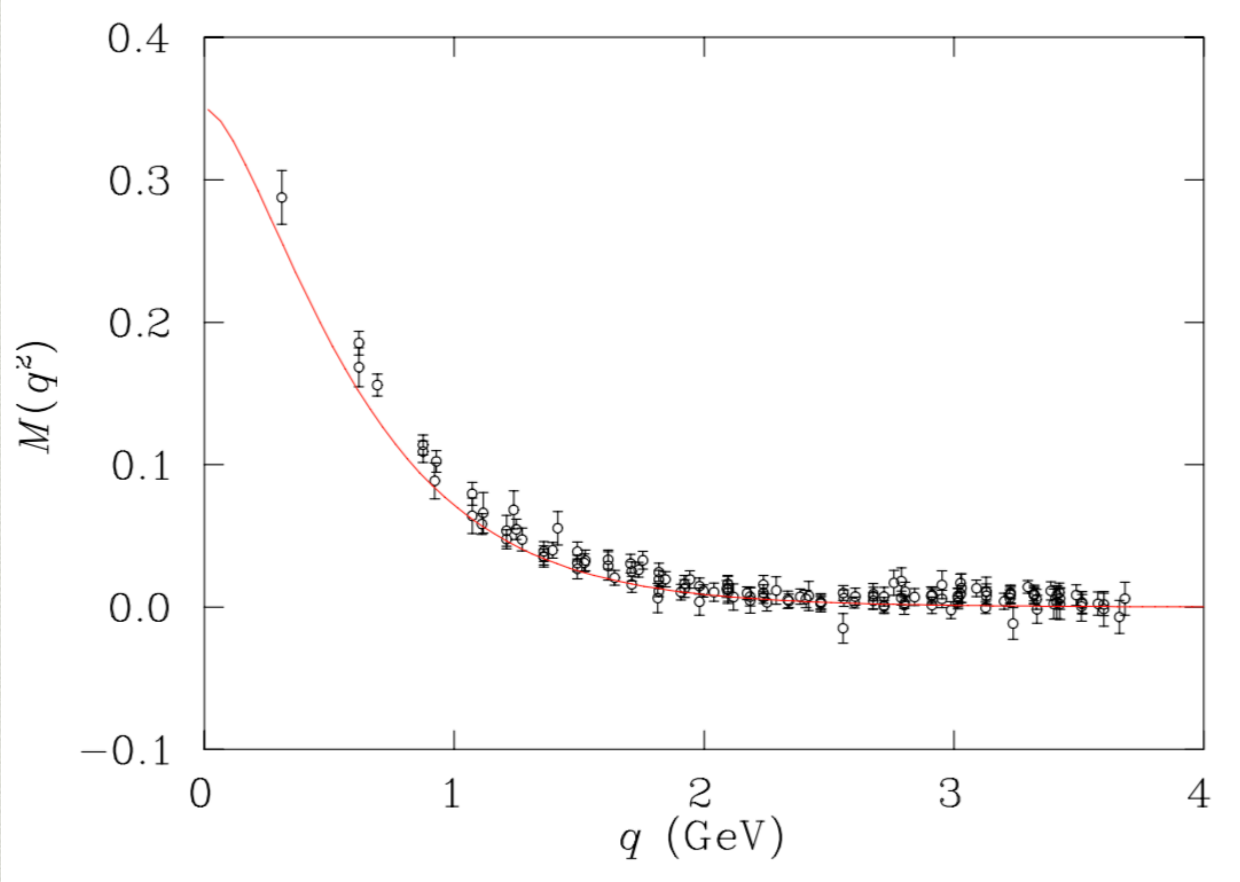
Instanton vacuum provides a beautiful mechanism of SBχS

-D.I. Diakonov-

- Mechanism of how the quark acquires the dynamical mass



- The helicity of quarks flips in the course of hopping from instantons to antiinstantons and



$$-\langle\overline{\psi}\psi\rangle_M = i\langle\psi^\dagger\psi\rangle_E = 4N_c\int d^4p\frac{M(p)}{p^2+M^2(p)}$$

$$= \text{const.}\sqrt{\frac{NN_c}{\pi^2 V_4\bar{\rho}^2}} = -(253\,\text{MeV})^3$$

$M(p)$: Dynamical quark mass from the Fourier transform of the quark zero mode in the instanton background

Merit of the instanton vacuum

- Given $\Lambda_{\overline{MS}} = 280\,\mathrm{MeV}$,

  We obtain $\overline{\rho} \approx 0.35\,\mathrm{fm}$ and $\overline{R} \approx 1\,\mathrm{fm}$.

$$\longrightarrow \quad M(0) \approx 350\,\mathrm{MeV}$$

$$\langle \overline{\psi}\psi \rangle_M = -(253\,\mathrm{MeV})^3$$

$$f_\pi^2 \approx 4N_c \int d^4p \frac{M^2(p)}{(p^2 + M^2(p))^2} \approx (94\,\mathrm{MeV})^2$$

- 2Nf quark-quark interactions: Chiral dynamics of the quark and pseudo-NG bosons.

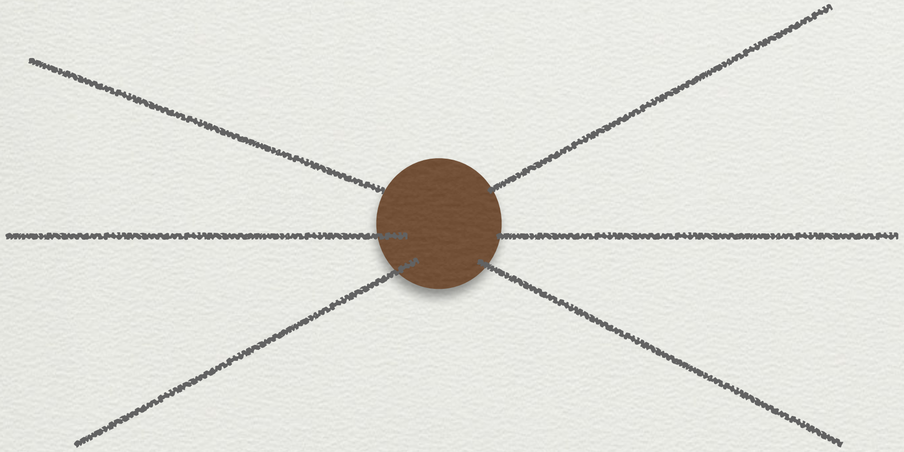
- All low-energy theorems are satisfied.
- U(1) Axial anomaly is also explained.

- Low-energy QCD partition function

$$\mathcal{Z} = \int D\psi D\psi^\dagger \exp\left(\int d^4x \sum_{f=1}^{N_f} \bar{\psi}_f i\partial\!\!\!/ \psi^f\right)\left(\frac{Y^+_{N_f}}{VM_1^{N_f}}\right)^{N_+}\left(\frac{Y^-_{N_f}}{VM_1^{N_f}}\right)^{N_-}$$

$$Y^+_{N_f} = \int d\rho\,\nu(\rho)\int dU \prod_{f=1}^{N_f}\left\{\int\frac{d^4k_f}{(2\pi)^4}\,2\pi\rho F(k_f\rho)\int\frac{d^4l_f}{(2\pi)^4}\,2\pi\rho F(l_f\rho)\right.$$

$$\left.\cdot(2\pi)^4\delta(k_1+...+k_{N_f}-l_1-...-l_{N_f})\cdot U^{\alpha_f}_{i'_f}U^{\dagger j'_f}_{\beta_f}\,\epsilon^{i_f i'_f}\epsilon_{j_f j'_f}\left[i\psi^\dagger_{Lf\alpha_f i_f}(k_f)\psi_L^{f\beta_f j_f}(l_f)\right]\right\}$$



2Nf quark interations

- Bosonization: Effective chiral action - our starting point

$$\begin{aligned}\mathcal{Z}^{\mathrm{eff}}_{\mathrm{QCD}} &= \int D\psi D\psi^\dagger D\pi^a \exp\left[\int d^4x\psi^\dagger(i\partial\!\!\!/ + i\sqrt{M(i\partial)}U^{\gamma_5}[\pi^a]\sqrt{M(i\partial)} + i\hat{m})\psi\right] \\ &= \int D\pi^a \exp(-S_{\mathrm{eff}}[\pi^a])\end{aligned}$$

- Nonlocal Effective chiral action (Nonlocal chiral quark model)

$$S_{\mathrm{eff}}[\pi^a] = -N_c\mathrm{Trlog}\left(i\partial\!\!\!/ + i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)} + i\hat{m}\right)$$

$U^{\gamma_5} = \exp\left(i\frac{\pi^a\lambda^a}{f_\pi}\right)$: Chiral field (pseudo-NG boson field)

We used this action to compute all the properties of the pions and kaons.

- Local version: Chiral quark model (Regularization is inevitable.)

$S_{\mathrm{eff}}[\pi^a] = -N_c\mathrm{Trlog}\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$: starting point

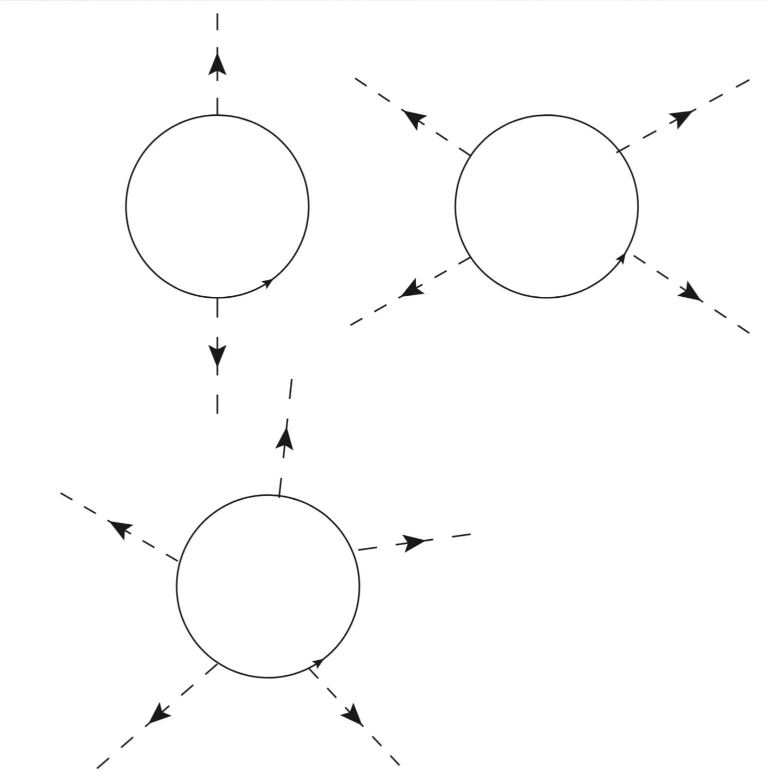
- In even lower energies

$$S_{\text{eff}}[\pi^a] = -N_c \text{Trlog}\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

Small pion momentum ⟶ Derivative expansion

$$\text{Re}S_{\text{eff}} = \int d^4x \left[\mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \cdots\right],$$

$$\delta\text{Im}S_{\text{eff}} = \frac{iN_c}{48\pi^2}\int d^4x \varepsilon_{\alpha\beta\gamma\delta}\text{Tr}\left(\partial_\alpha U^\dagger \partial_\beta U \partial_\gamma U^\dagger \partial_\delta U U^\dagger \delta U\right)$$



$$\mathcal{L}^{(2)} = \frac{f_\pi^2}{4}\langle \partial_\mu U^\dagger \partial_\mu U\rangle$$

$$f_\pi^2 = 4N_c \int \frac{d^4k}{(2\pi)^4} \frac{M^2(k) - \frac{1}{2}M(k)M'(k)k + \frac{1}{4}M'^2(k)k^2}{(k^2 + M^2(k))^2}$$

$$\mathcal{L}^{(4)} = L_1\langle \partial_\mu U^\dagger \partial_\mu U\rangle^2 + L_2\langle \partial_\mu U^\dagger \partial_\nu U\rangle^2 + L_3\langle \partial_\mu U^\dagger \partial_\mu U\rangle \partial_\nu U^\dagger \partial_\nu U\rangle$$

H. Choi and HChK,PRD **69** (2004) 054004.

- Witten's seminal idea: Baryon in the large Nc

NPB, 149(1979)285

\- Problem in low-energy QCD: Large value of the strong coupling constant

→ The number of color as an implicit expansion parameter

✱A baryon can be viewed as a state of Nc quarks bound by mesonic **mean fields**.

Its mass is proportional to *Nc*, while its width is of order *O(1)*.

→ Mesons are weakly interacting

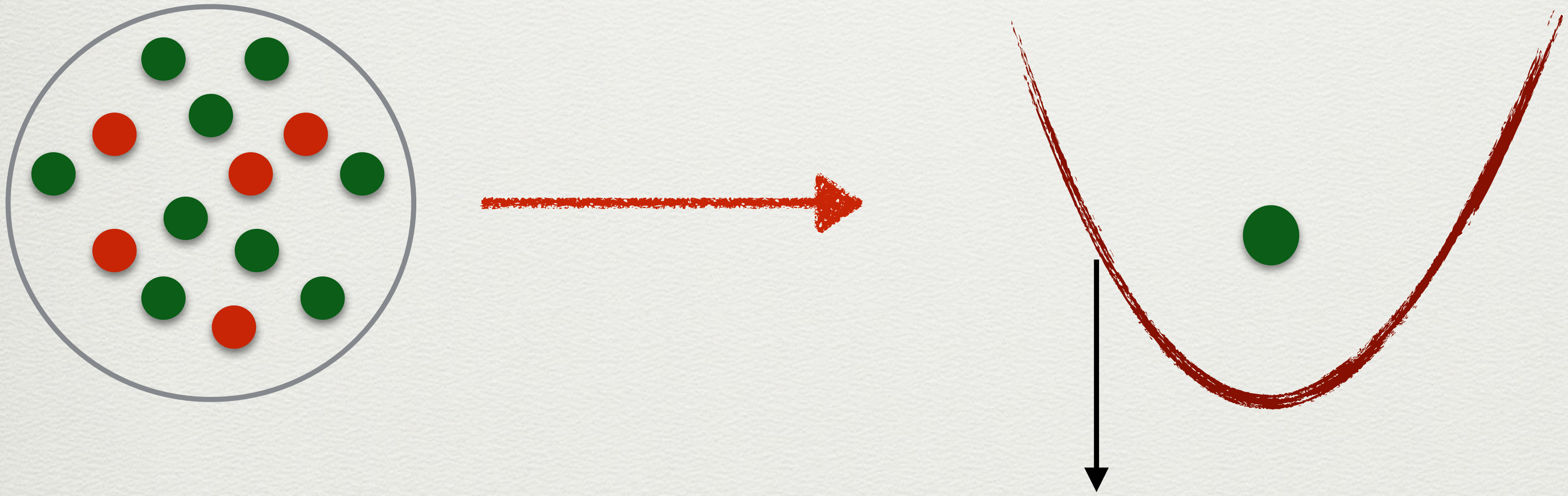
(Quantum fluctuations are suppressed by 1/Nc: *O(1/Nc)*).

Mean fields in Quantum field theory

Given action $S[\phi]$,

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

→ This classical solution is regarded as a mean field.



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

> “For the baryons, things are not so good. Witten’s theory is an analytical triumph but a phenomenological disaster.”
>
> \- S. Coleman-

S. Coleman, 1/N, in Aspects of Symmetry (1985)

A.V. Manohar, Large N QCD, hep-ph/9802419

- In fact, Witten discussed the baryons including excited ones in detail in his seminal paper.

We will show here a possible realization of Witten’s idea, and the pion mean-field approach indeed describes the structure of baryons very well.

Pion mean-field approach (Chiral Quark-Soliton Model)

- ✱Baryons as a state of Nc quarks bound by mesonic mean fields.

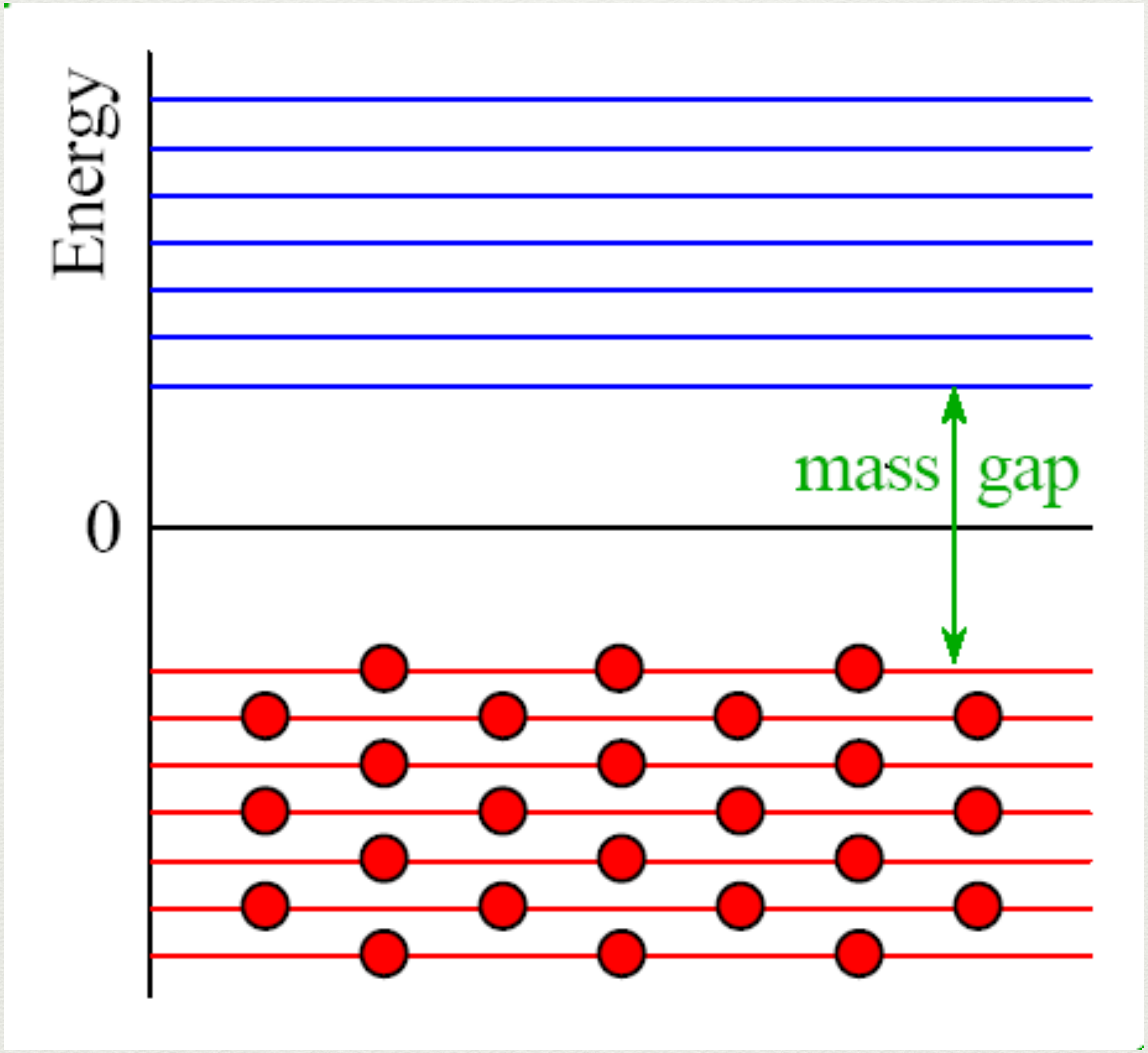
$$S_{\rm eff}[\pi^a] = -N_c{\rm Trlog}\,(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m})$$

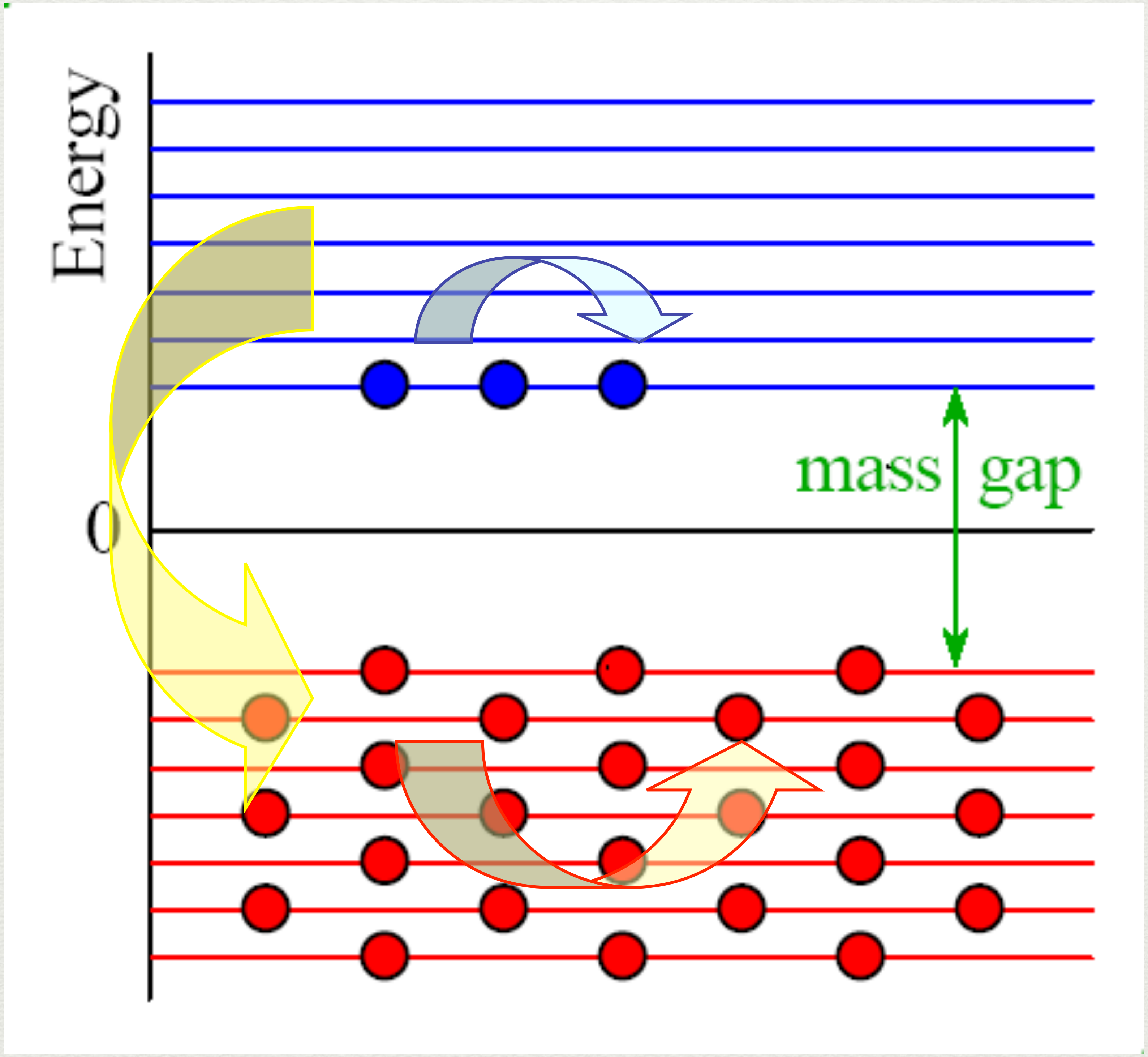
✱ Key point: Hedgehog Ansatz

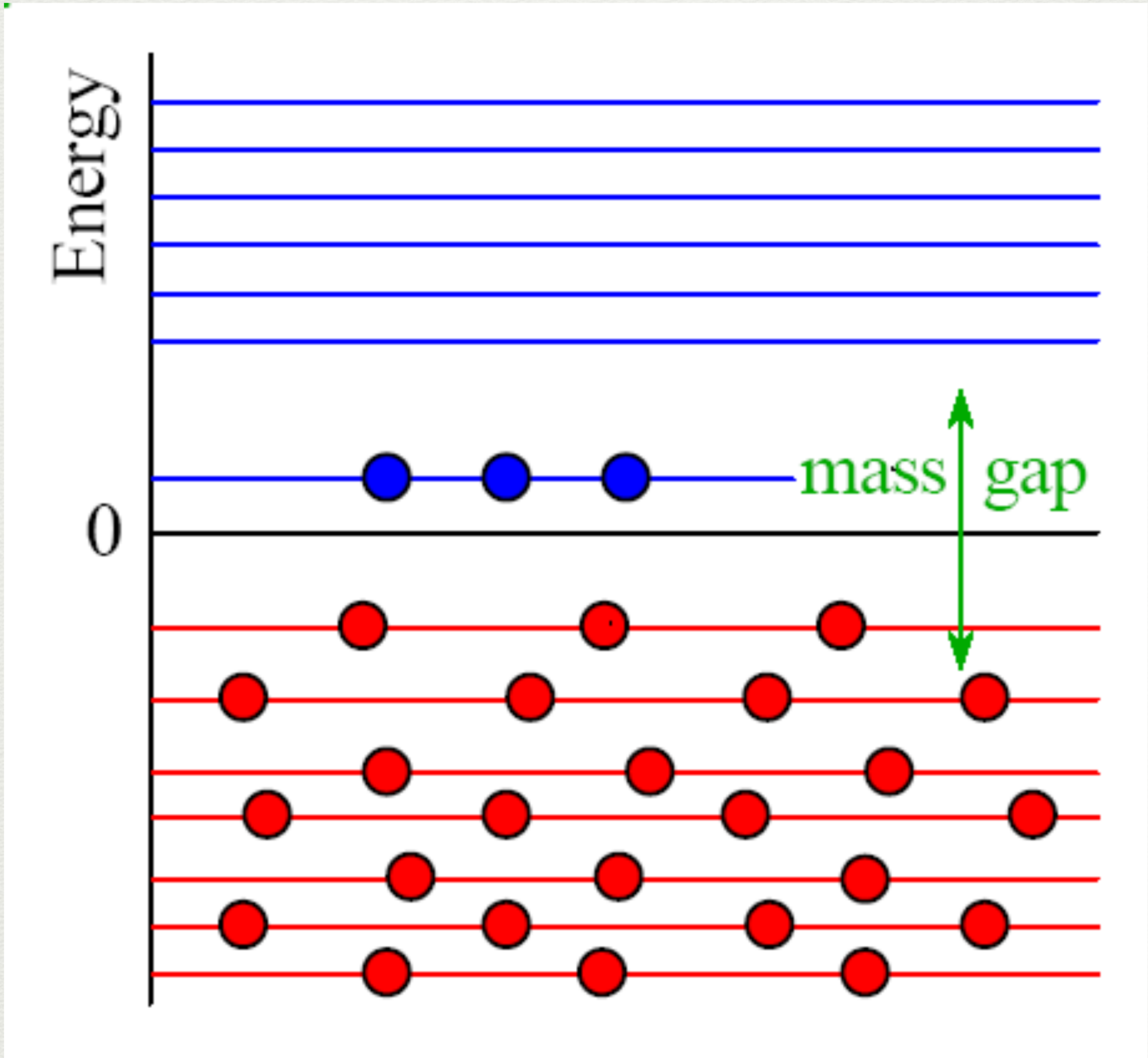
$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\, n^a = x^a/r, & a = 1,\, 2,\, 3 \\ 0, & a = 4,\, 5,\, 6,\, 7,\, 8. \end{cases}$$

It breaks spontaneously $\mathrm{SU(3)_{flavor}} \otimes \mathrm{O(3)_{space}} \to \mathrm{SU(2)_{isospin+space}}$

Witten's trivial embedding $U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$


Energy
0
mass gap


Energy
0
mass gap

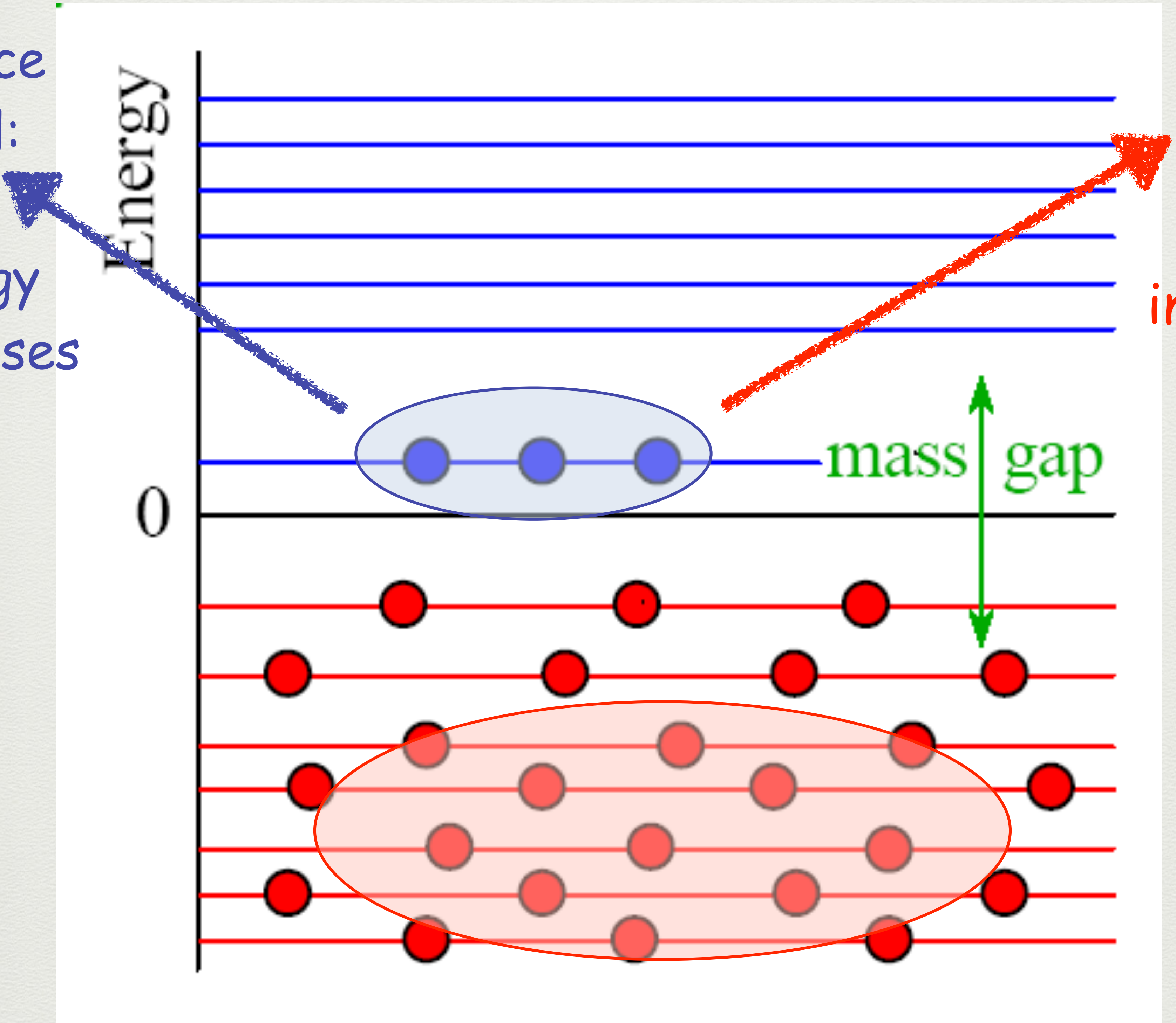

Energy
0
mass gap

valence level:

energy decreases

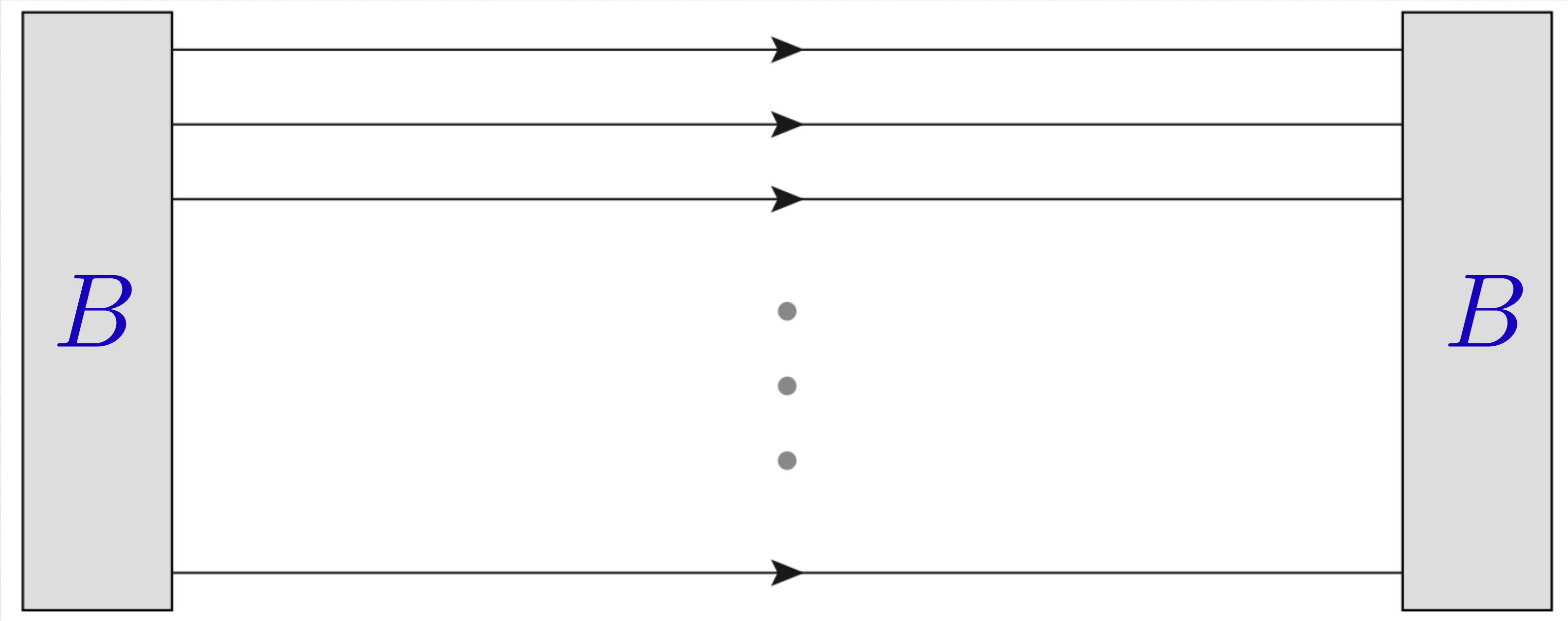
sea levels:

energy increases



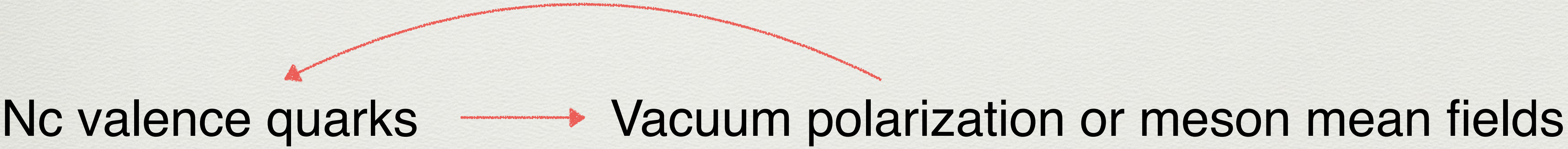
system is stabilized

Baryon as Nc valence quarks bound by pion mean fields



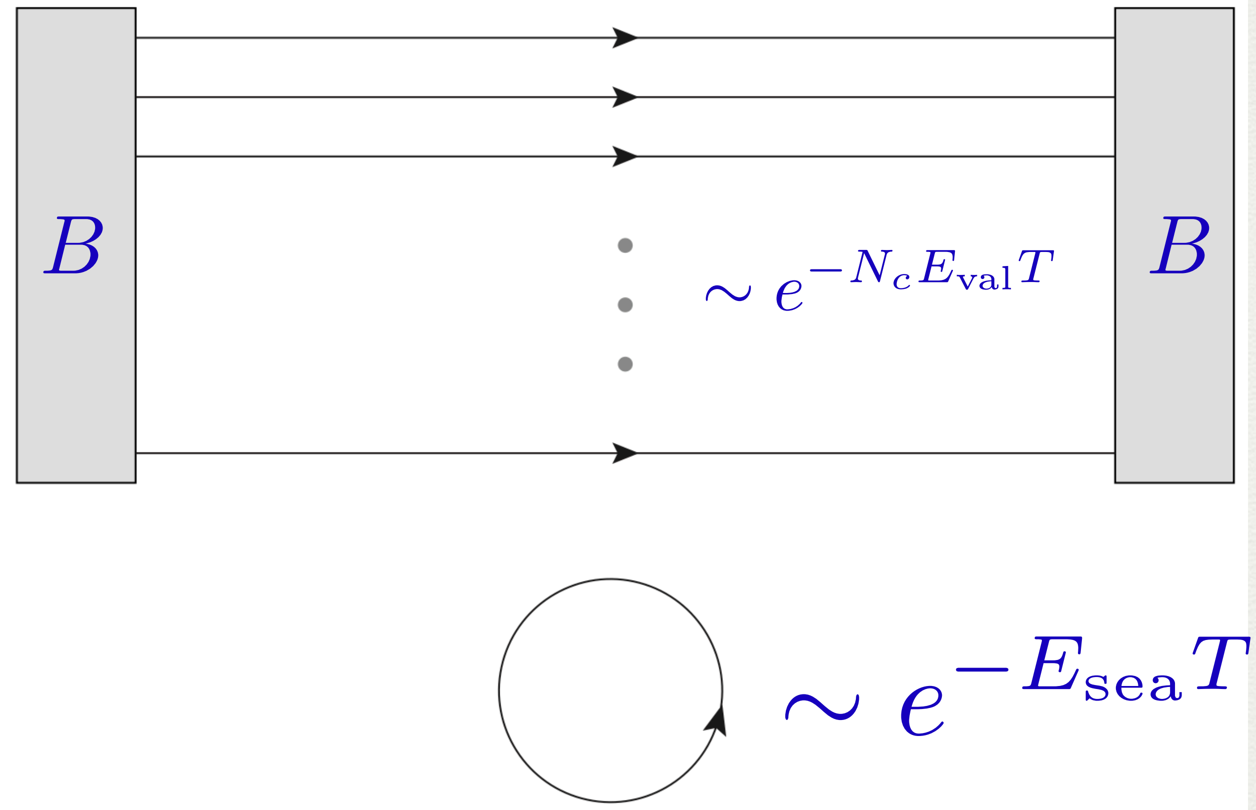
$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\rm val} T}$$

Presence of Nc quarks will polarize the vacuum or create mean fields.



Nc valence quarks → Vacuum polarization or meson mean fields

Baryon as Nc valence quarks bound by pion mean fields



$$E_{\mathrm{cl}} = N_c E_{\mathrm{val}} + E_{\mathrm{sea}}$$

Classical Nucleon mass is described by the Nc valence quark energy and sea-quark energy.

$$\begin{tikzcd}
\frac{\delta E_{cl}}{\delta U} = 0 \arrow[r] & M_{\mathrm{cl}} \arrow[r] & P(r) \arrow[l, bend right]
\end{tikzcd}$$

P(r): Soliton profile function
or Soliton field

Baryon as Nc valence quarks bound by pion mean fields

$$\Pi_N = \langle 0|J_N(0,T/2)J_N^\dagger(0,-T/2)|0\rangle$$

$$\lim_{T\to\infty}\Pi_N(T) \simeq e^{-M_N T}$$

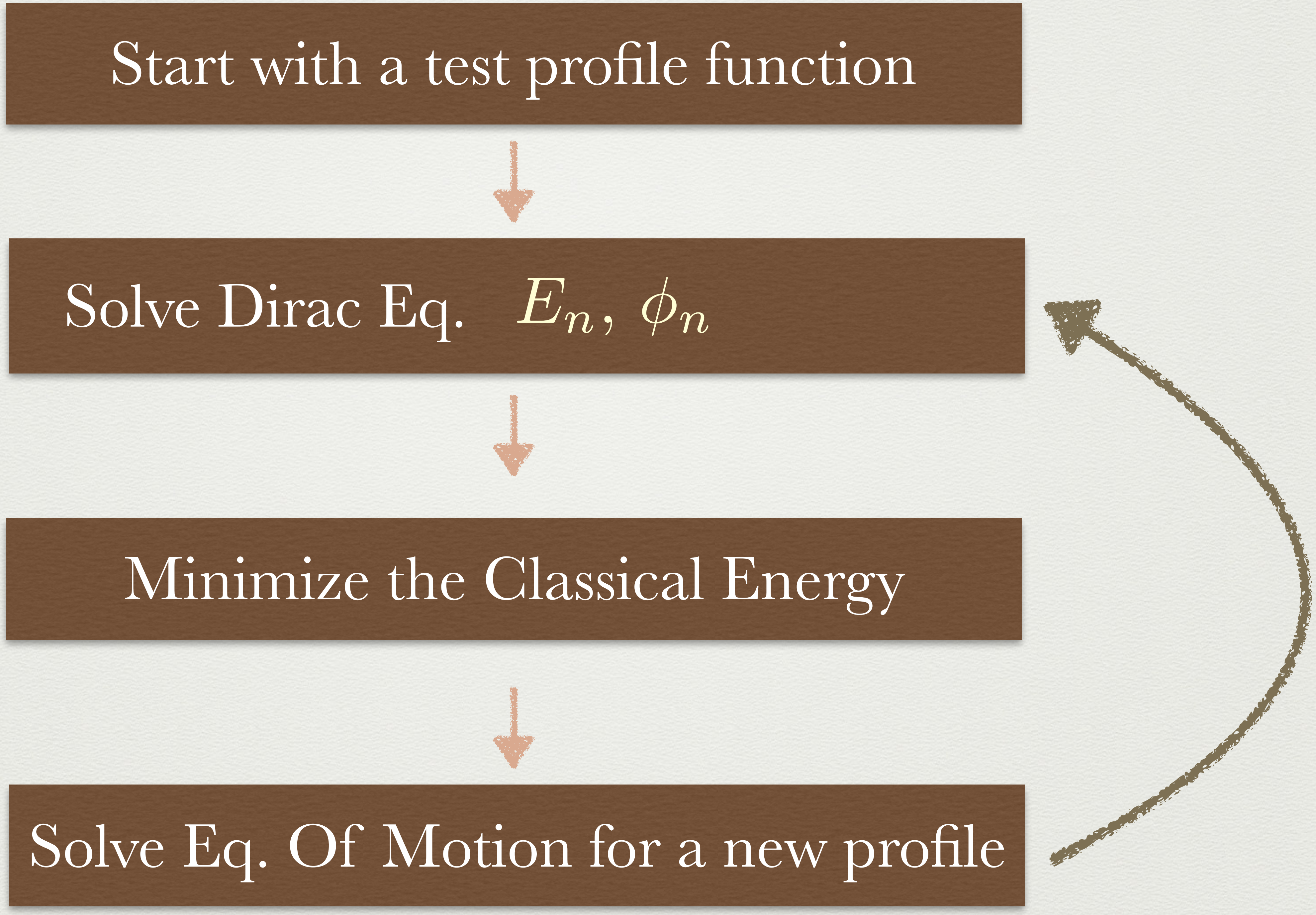
$$\Pi_N(\vec{x},t) \quad = \quad \Gamma_N^{\{f\}}\Gamma_N^{\{g\}*}\frac{1}{Z}\int dU\prod_{i=1}^{N_c}\left\langle 0,T/2\left|\frac{1}{D(U)}\right|0,-T/2\right\rangle_{f,g} \quad e^{-S_{\mathrm{eff}}}$$

$$\longrightarrow \quad \exp[-E_{\mathrm{cl}}T] = \exp\left[-(N_c E_{\mathrm{val}} + E_{\mathrm{sea}})T\right]$$

$$\frac{\delta}{\delta U}(N_c E_{\mathrm{val}} + E_{\mathrm{sea}}) = 0 \quad \rightarrow \quad M_{\mathrm{cl}} = N_c E_{\mathrm{val}}(U_c) + E_{\mathrm{sea}}(U_c)$$

→ Classical equation of motion

Mean-field approximation (Hartree approximation)



Start with a test profile function

Solve Dirac Eq. E_n, ϕ_n

Minimize the Classical Energy

Solve Eq. Of Motion for a new profile

Stop if the classical soliton energy is converged enough.

→ Final profile function

Collective (Zero-mode) quantization

$$U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$$

Witten's embedding

Collective (Zero-mode) quantization

$$U(\boldsymbol{x},t) \;=\; R(t)U_c(\boldsymbol{x}-\boldsymbol{Z}(t))R^{\dagger}(t)$$

$$\int DU[\cdots] \;\;\rightarrow\;\; \int DADZ[\cdots]$$

$$\mathcal{L} = -M_{sol} + \frac{I_1}{2}\sum_{i=1}^{3}{\Omega_i}^2 + \frac{I_2}{2}\sum_{i=4}^{7}{\Omega_i}^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Collective rotational Hamiltonian

$$H^{\rm rot}_{(p,q)} = M_{\rm sol} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{a=4}^{7}\hat{J}_a^2$$

$$\mathcal{E}^{\rm rot}_{(p,\,q)} = \boxed{M_{\rm sol}} + \frac{J(J+1)}{2I_1} + \frac{C_2(p,q) - J(J+1) - 3/4\,\boxed{Y'^2}}{2I_2}$$

classical nucleon mass

$$Y' = \frac{N_c}{3}$$

Right hypercharge: Constraint on the quantization of the chiral soliton This constraint selects a tower of the allowed rotational excitations of the SU(3) hedgehog.

Success of the XQSM in the light baryon sector

- Connection to QCD via the instanton vacuum (natural scale) $\rho \approx 600\,\mathrm{MeV}$
- The mass splittings of the lowest-lying hyperons
- **All different types of baryon form factors**
- Parton distribution amplitudes (u-d asymmetry, transversity)
- Quasi-parton distribution amplitudes
- GPDs

Two different directions for further development

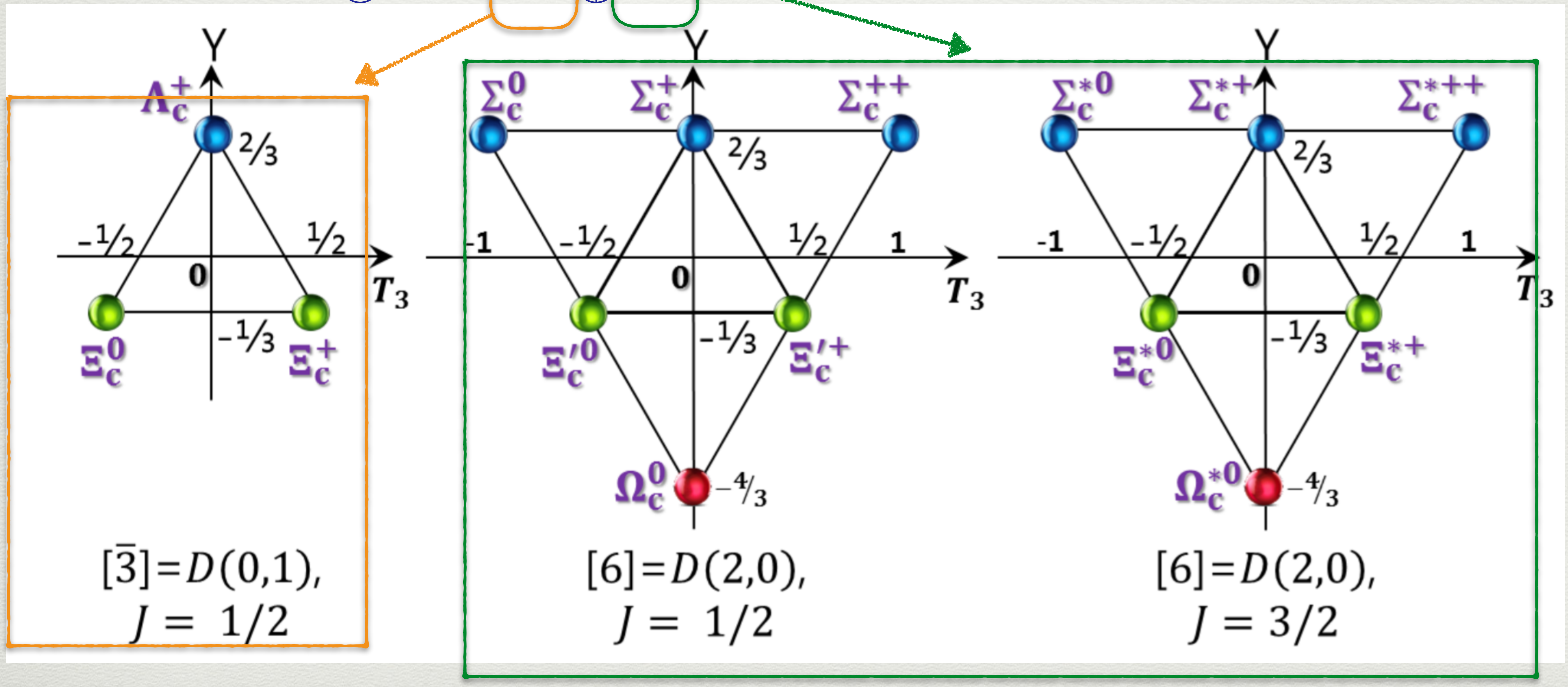
Heavy baryon sector

Excited baryon sector

Singly heavy baryons in SU(3)

- In the heavy quark mass limit, a heavy quark spin is conserved, so light-quark spin is also conserved.
- In this limit, a heavy quark can be considered as **a color static source**.
- Dynamics is governed by light quarks.

$$3 \otimes 3 = \boxed{\bar{3}} \oplus \boxed{6}$$



$[\bar{3}] = D(0,1)$, $J = 1/2$

$[6] = D(2,0)$, $J = 1/2$

$[6] = D(2,0)$, $J = 3/2$

Heavy baryons in the XQSM

- Valence quarks are bound by the meson mean fields.
- Light quarks govern a heavy-light quark system.
- Heavy quarks can be simply viewed as **static color sources.**

$$\boldsymbol{K} = \boldsymbol{J} + \boldsymbol{T} = 0, \quad T_8 = \frac{N_c - 1}{2\sqrt{3}}$$ Ground-state heavy baryons

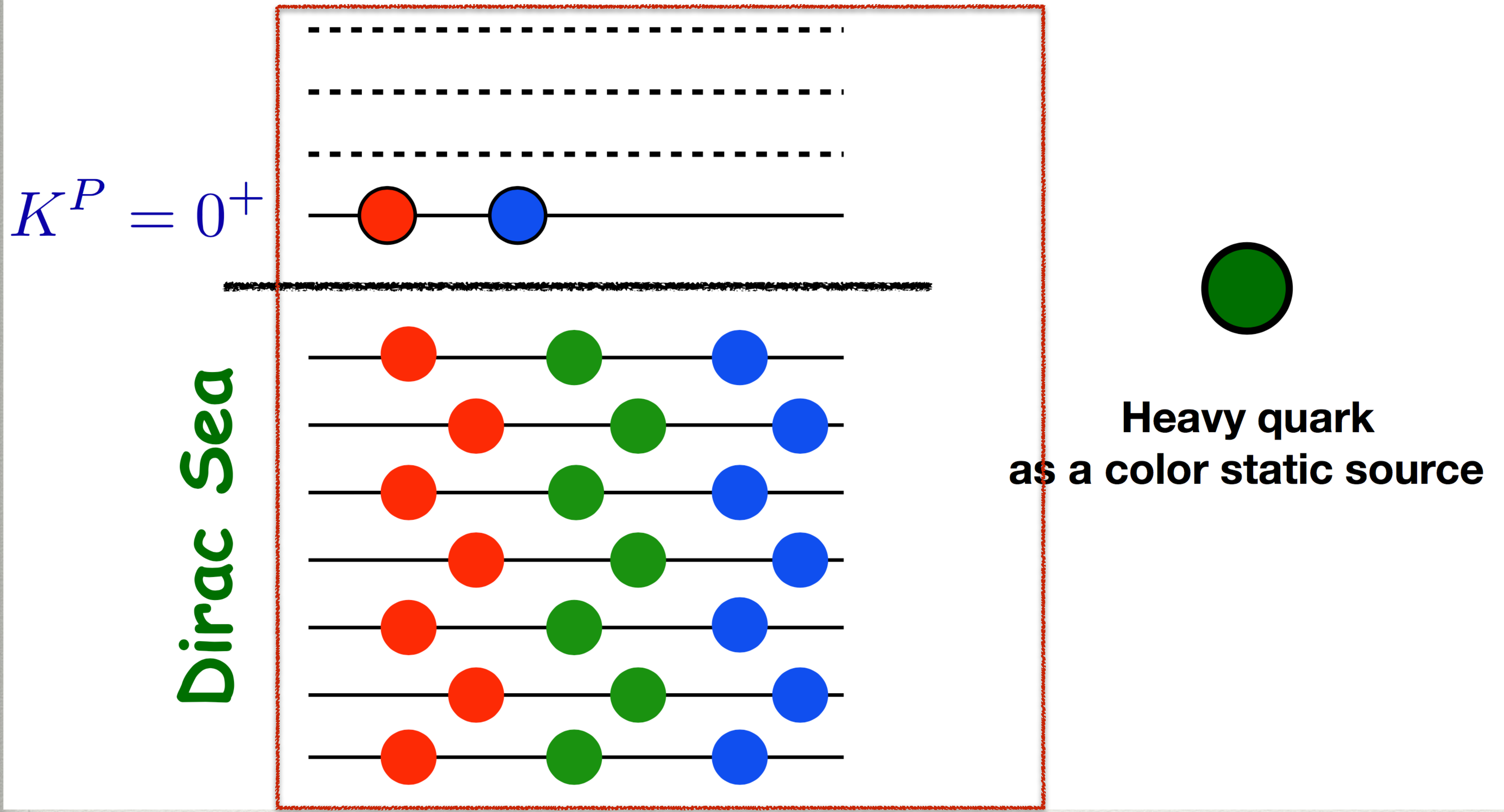
Right hypercharge $Y' = \dfrac{N_c - 1}{3}$

A heavy quark: Static color source to make a heavy baryon color singlet.

D. Diakonov, arXiv:1003.2157 [hep-ph].

Gh.S. Yang, HChK, M. Polyakov, M. Praszalowicz, PRD **94**, 071501(R) (2016)

Heavy baryons in the XQSM



Nc-1 light quarks govern a singly heavy baryon.

Heavy baryons in the XQSM

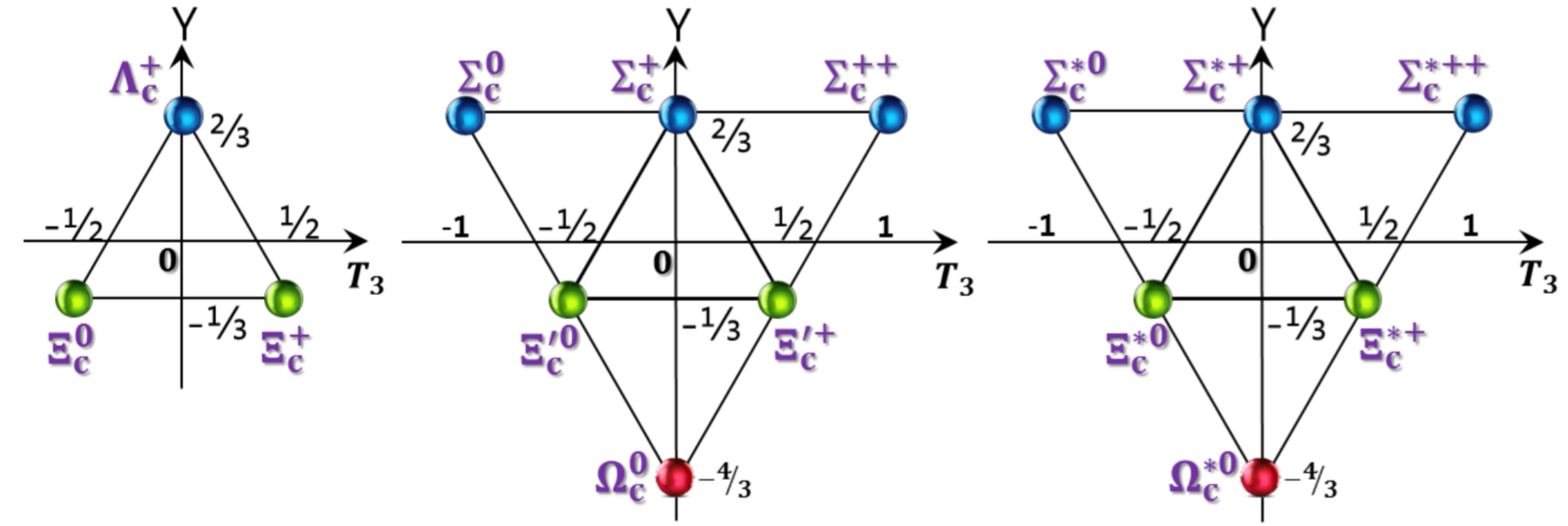
Nc-1 quarks represent heavy-baryon spectra.

$$Y' = \frac{N_c - 1}{3}$$

Grand spin: $K = 0 \to T = J$

- The lowest rotationally excited states $\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6}$
- T=0 for a anti-triplet: J=0 for it. Combining a charm quark with spin 1/2, we have one anti-triplet.
- T=1 for a sextet: J=1. We have two sextets with a charm quark. (1/2, 3/2).

$$Y' = 2/3$$



$[\overline{3}] = D(0,1)$, $J = 1/2$

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$[6] = D(2,0)$, $J = 3/2$

SU(3) symmetry breaking

- The collective Hamiltonian for SU(3) symmetry breaking

$$H_{\mathrm{br}} = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D_{8i}^{(8)} J_i$$

In the light-quark sector, we have fixed already these dynamical parameters as

$$\alpha = -\frac{2m_s}{3}\sigma - \beta Y' = -(255.03 \pm 5.82)\,\mathrm{MeV}$$

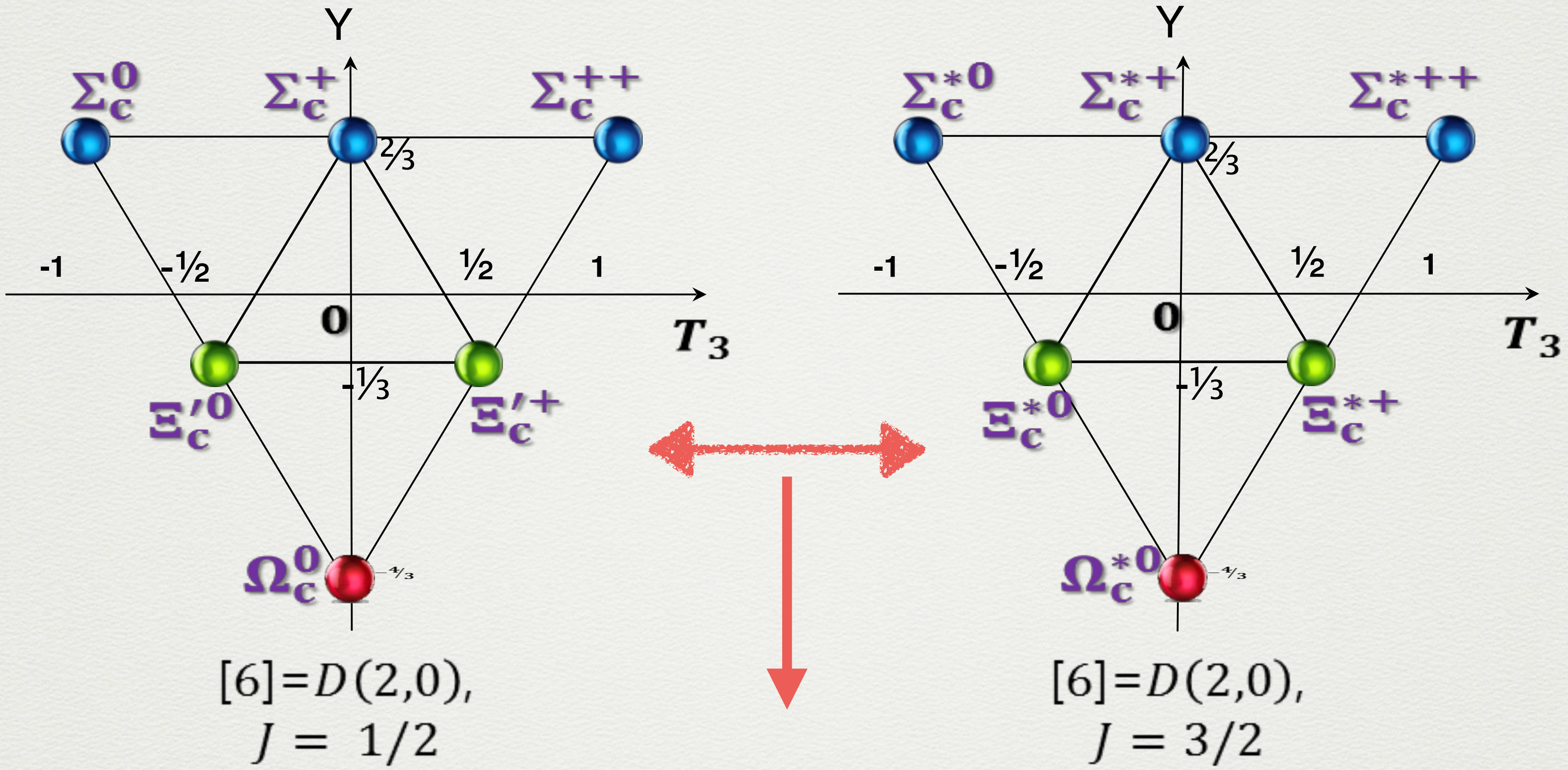
$$\beta = -\frac{m_s K_2}{I_2} = -(140.04 \pm 3.20)\,\mathrm{MeV}$$

$$\gamma = \frac{2m_s K_1}{I_1} + 2\beta = -(101.08 \pm 2.33)\,\mathrm{MeV}$$

$$\alpha \to \bar{\alpha} = \frac{N_c - 1}{N_c}\alpha$$

G. S. Yang, HChK, PTP, 128, 397 (2012).

Hyperfine mass splittings (only new parameter)



$[6]=D(2,0)$,
$J = 1/2$

$[6]=D(2,0)$,
$J = 3/2$

Hyperfine splitting between different spin states

$$H_{LQ} = \frac{2}{3}\frac{\kappa}{m_Q M_{\rm sol}}\mathbf{S}_{\rm L}\cdot\mathbf{S}_Q = \frac{2}{3}\boxed{\frac{\varkappa}{m_Q}}\mathbf{S}_{\rm L}\cdot\mathbf{S}_Q$$

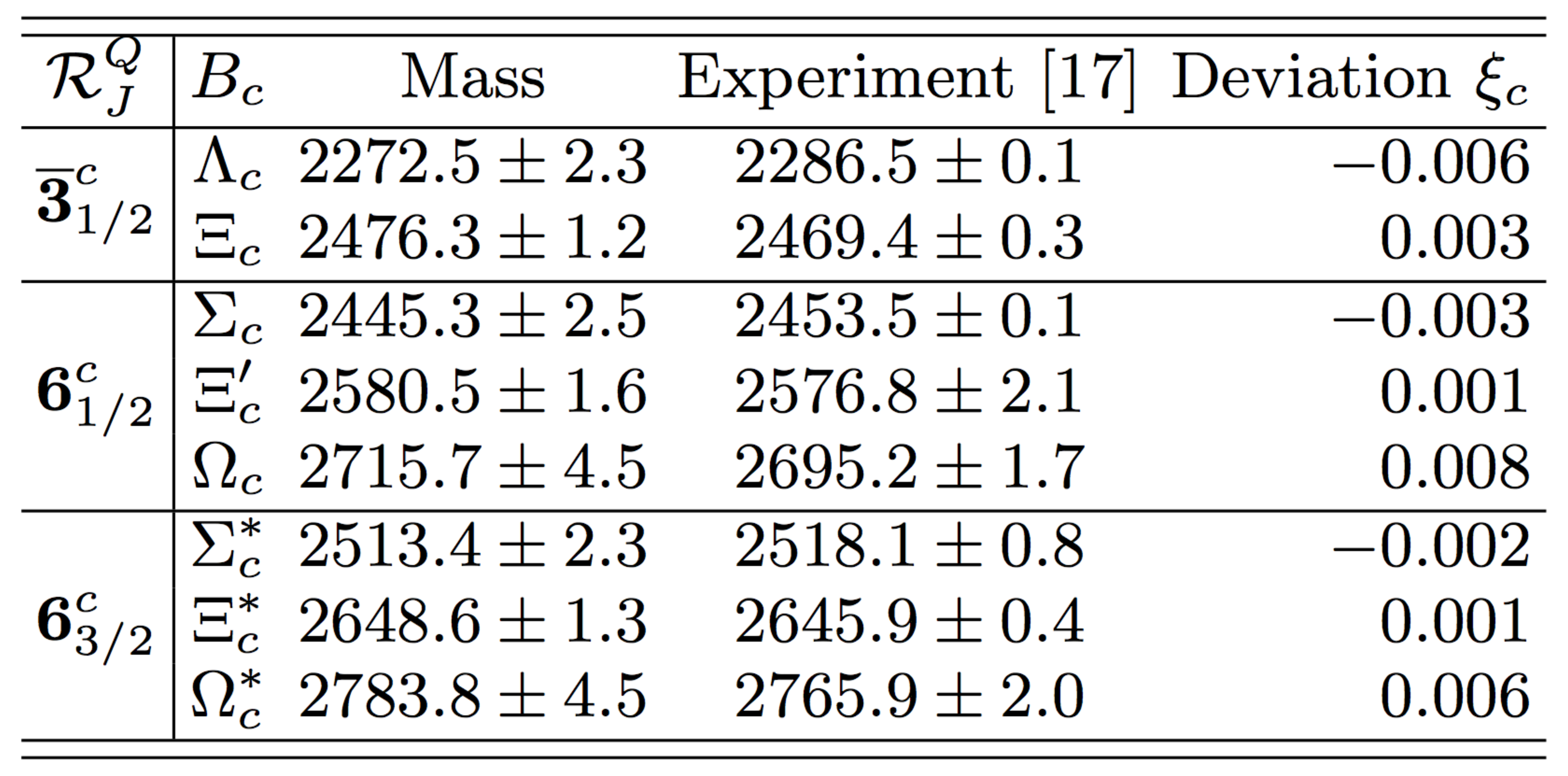
The ratio can be determined by the center values of the sextet masses

$$\frac{\varkappa}{m_c} = (68.1 \pm 1.1)\,\mathrm{MeV}$$
$$\frac{\varkappa}{m_b} = (20.3 \pm 1.0)\,\mathrm{MeV}$$

Remind you that all the parameters are the same as in the light baryon sector except for the hyperfine interaction.

Gh.S. Yang, HChK, M. Polyakov, M. Praszalowicz, PRD **94**, 071501(R) (2016)

Results for the charmed baryon masses

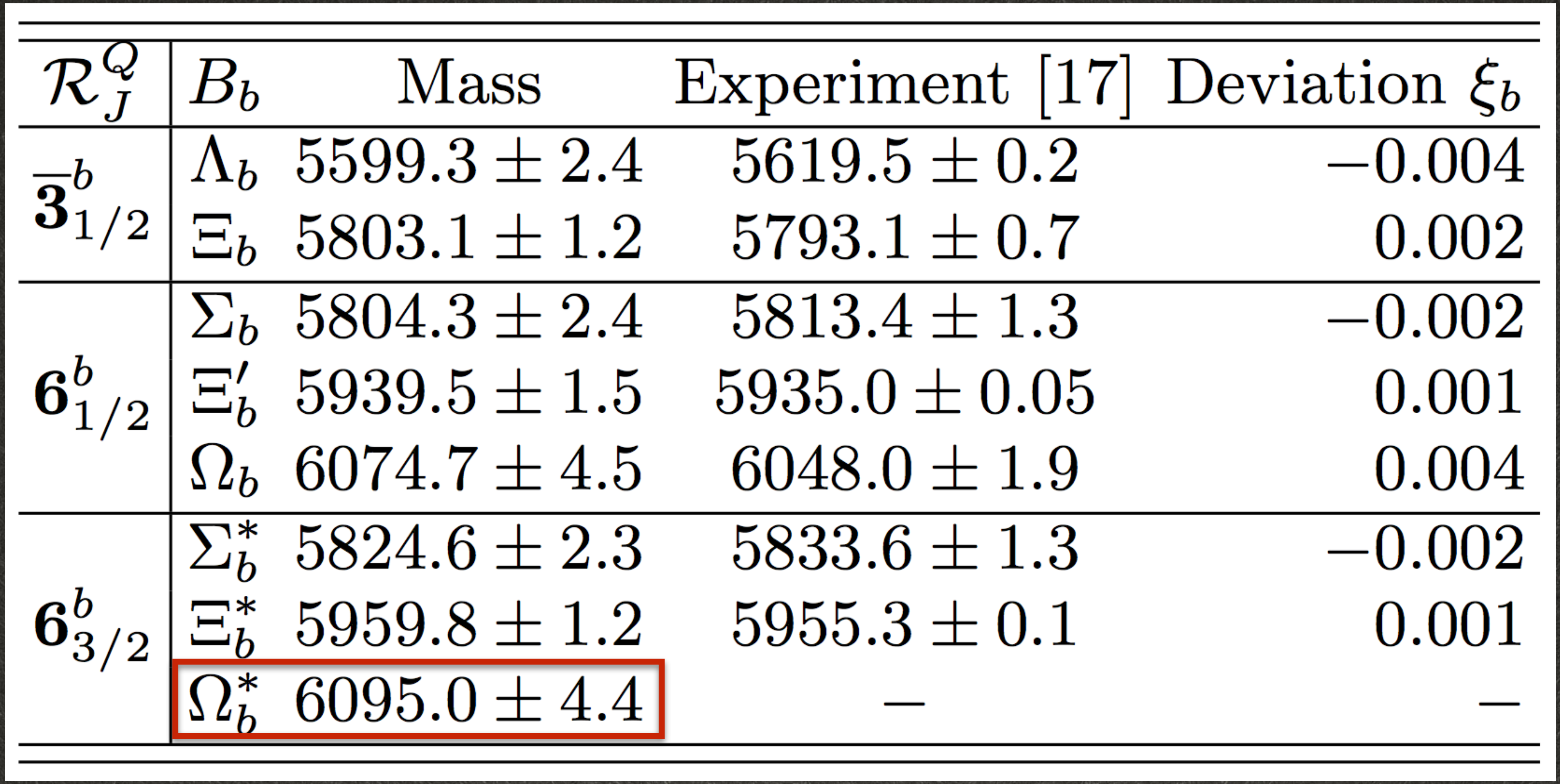


$\mathcal{R}_J^Q$	B_c	Mass	Experiment [17]	Deviation ξ_c
$\overline{\mathbf{3}}^c_{1/2}$	Λ_c	2272.5 ± 2.3	2286.5 ± 0.1	-0.006
	Ξ_c	2476.3 ± 1.2	2469.4 ± 0.3	0.003
$\mathbf{6}^c_{1/2}$	Σ_c	2445.3 ± 2.5	2453.5 ± 0.1	-0.003
	Ξ'_c	2580.5 ± 1.6	2576.8 ± 2.1	0.001
	Ω_c	2715.7 ± 4.5	2695.2 ± 1.7	0.008
$\mathbf{6}^c_{3/2}$	Σ^*_c	2513.4 ± 2.3	2518.1 ± 0.8	-0.002
	Ξ^*_c	2648.6 ± 1.3	2645.9 ± 0.4	0.001
	Ω^*_c	2783.8 ± 4.5	2765.9 ± 2.0	0.006

$$\xi_c = (M^{B_b}_{\mathrm{th}} - M^{B_b}_{\mathrm{exp}})/M^{B_b}_{\mathrm{exp}}$$

Gh.S. Yang, HChK, M. Polyakov, M. Praszalowicz, PRD **94**, 071501(R) (2016)

Results for the bottom baryon masses



$\mathcal{R}_J^Q$	B_b	Mass	Experiment [17]	Deviation ξ_b
$\overline{\mathbf{3}}_{1/2}^b$	Λ_b	5599.3 ± 2.4	5619.5 ± 0.2	-0.004
	Ξ_b	5803.1 ± 1.2	5793.1 ± 0.7	0.002
$\mathbf{6}_{1/2}^b$	Σ_b	5804.3 ± 2.4	5813.4 ± 1.3	-0.002
	Ξ_b'	5939.5 ± 1.5	5935.0 ± 0.05	0.001
	Ω_b	6074.7 ± 4.5	6048.0 ± 1.9	0.004
$\mathbf{6}_{3/2}^b$	Σ_b^*	5824.6 ± 2.3	5833.6 ± 1.3	-0.002
	Ξ_b^*	5959.8 ± 1.2	5955.3 ± 0.1	0.001
	Ω_b^*	6095.0 ± 4.4	–	–

Prediction from the present work

The results are in remarkable agreement with the experimental data.

$$\xi_c = (M_{\rm th}^{B_c} - M_{\rm exp}^{B_c})/M_{\rm exp}^{B_c}$$

Gh.S. Yang, HChK, M. Polyakov, M. Praszalowicz, PRD **94**, 071501(R) (2016)

Magnetic moments of heavy baryons

- Collective operators for the magnetic moments

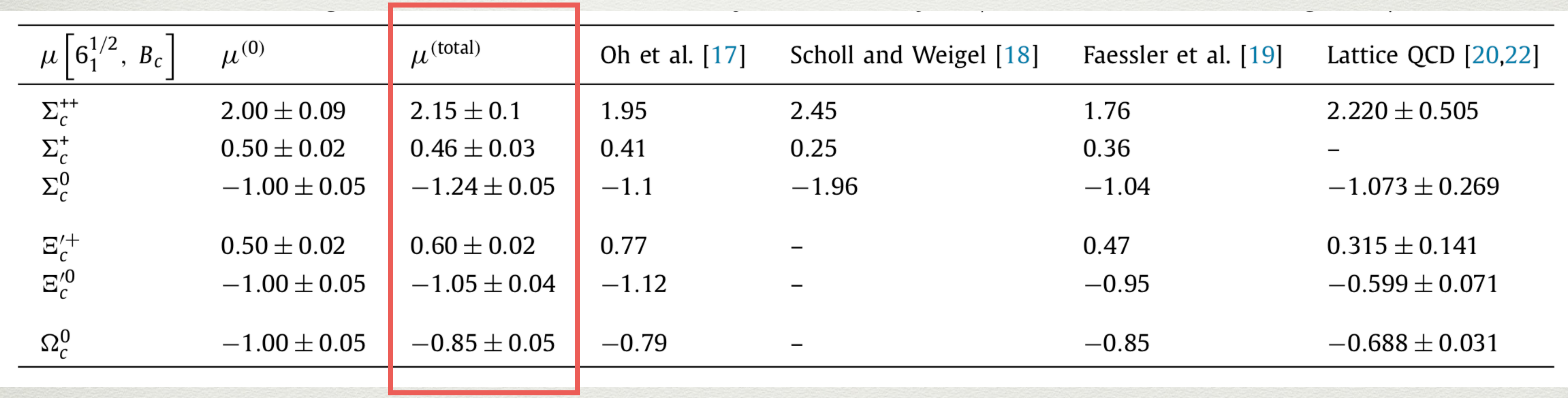
$$\hat{\mu}^{(0)} = w_1 D^{(8)}_{\mathcal{Q}3} + w_2 d_{pq3} D^{(8)}_{\mathcal{Q}p} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{\mathcal{Q}8} \hat{J}_3,$$

$$\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{\mathcal{Q}p} D^{(8)}_{8q} + w_5 \left(D^{(8)}_{\mathcal{Q}3} D^{(8)}_{88} + D^{(8)}_{\mathcal{Q}8} D^{(8)}_{83} \right) + w_6 \left(D^{(8)}_{\mathcal{Q}3} D^{(8)}_{88} - D^{(8)}_{\mathcal{Q}8} D^{(8)}_{83} \right)$$

  - The parameter wi’s are determined by the experimental data on the magnetic moments of the baryon octet.

No additional free parameter!

- Results of the magnetic moments of the baryon sextet with spin 1/2

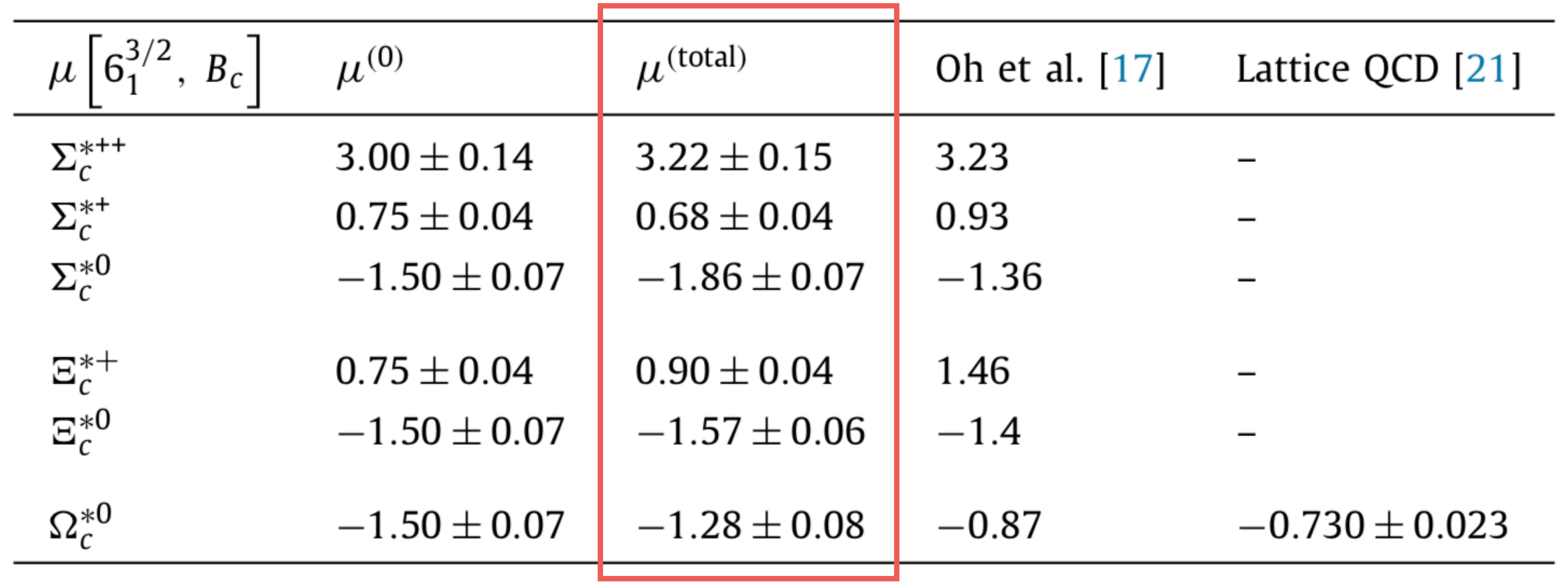


$\mu\left[6_1^{1/2},\ B_c\right]$	$\mu^{(0)}$	$\mu^{(\text{total})}$	Oh et al. [17]	Scholl and Weigel [18]	Faessler et al. [19]	Lattice QCD [20,22]
Σ_c^{++}	2.00 ± 0.09	2.15 ± 0.1	1.95	2.45	1.76	2.220 ± 0.505
Σ_c^{+}	0.50 ± 0.02	0.46 ± 0.03	0.41	0.25	0.36	–
Σ_c^{0}	-1.00 ± 0.05	-1.24 ± 0.05	−1.1	−1.96	−1.04	-1.073 ± 0.269
$\Xi_c^{\prime +}$	0.50 ± 0.02	0.60 ± 0.02	0.77	–	0.47	0.315 ± 0.141
$\Xi_c^{\prime 0}$	-1.00 ± 0.05	-1.05 ± 0.04	−1.12	–	−0.95	-0.599 ± 0.071
Ω_c^{0}	-1.00 ± 0.05	-0.85 ± 0.05	−0.79	–	−0.85	-0.688 ± 0.031

Yang & HChK, PLB 781, 601 (2018).

Magnetic moments of heavy baryons

- Results of the magnetic moments of the baryon sextet with spin 3/2



$\mu\left[6_1^{3/2},\ B_c\right]$	$\mu^{(0)}$	$\mu^{(\text{total})}$	Oh et al. [17]	Lattice QCD [21]
Σ_c^{*++}	3.00 ± 0.14	3.22 ± 0.15	3.23	–
Σ_c^{*+}	0.75 ± 0.04	0.68 ± 0.04	0.93	–
Σ_c^{*0}	-1.50 ± 0.07	-1.86 ± 0.07	-1.36	–
Ξ_c^{*+}	0.75 ± 0.04	0.90 ± 0.04	1.46	–
Ξ_c^{*0}	-1.50 ± 0.07	-1.57 ± 0.06	-1.4	–
Ω_c^{*0}	-1.50 ± 0.07	-1.28 ± 0.08	-0.87	-0.730 ± 0.023

No additional free parameter!

Yang & HChK, PLB 781, 601 (2018).

Strong decay rates

- Collective operator for the strong vertices in SU(3) symmetric case

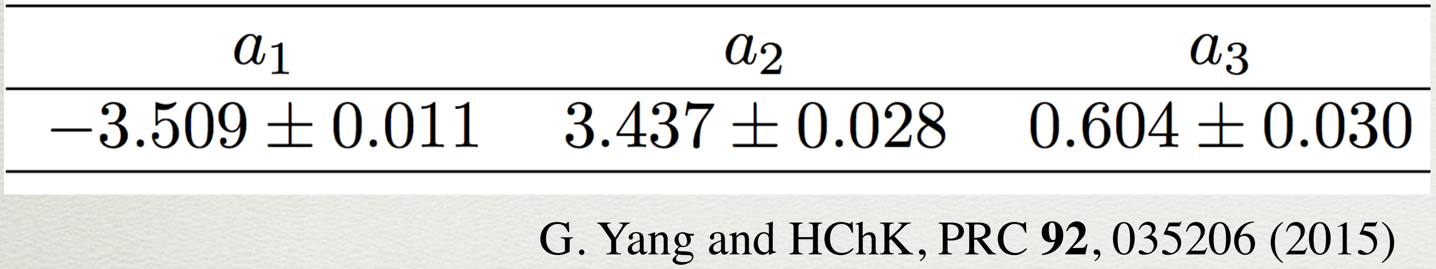
$$\mathcal{O}_\varphi = \frac{3}{M_1 + M_2} \sum_{i=1,2,3} \left[G_0 D^{(8)}_{\varphi\, i} - G_1\, d_{ibc} D^{(8)}_{\varphi\, b} \hat{S}_c - G_2 \frac{1}{\sqrt{3}} D^{(8)}_{\varphi\, 8} \hat{S}_i \right] p_i$$

- Decay widths

$$\Gamma_{B_1 \to B_2 + \varphi} = \frac{1}{2\pi} \overline{\langle B_2 \,|\mathcal{O}_\varphi|\, B_1 \rangle}^2 \frac{M_2}{M_1} p$$

$$G_0 = -\frac{M + M'}{6 f_\varphi} a_1$$

$$G_{1,2} = \frac{M + M'}{6 f_\varphi} a_{2,3}$$



a_1	a_2	a_3
-3.509 ± 0.011	3.437 ± 0.028	0.604 ± 0.030

G. Yang and HChK, PRC **92**, 035206 (2015)

No additional free parameter!

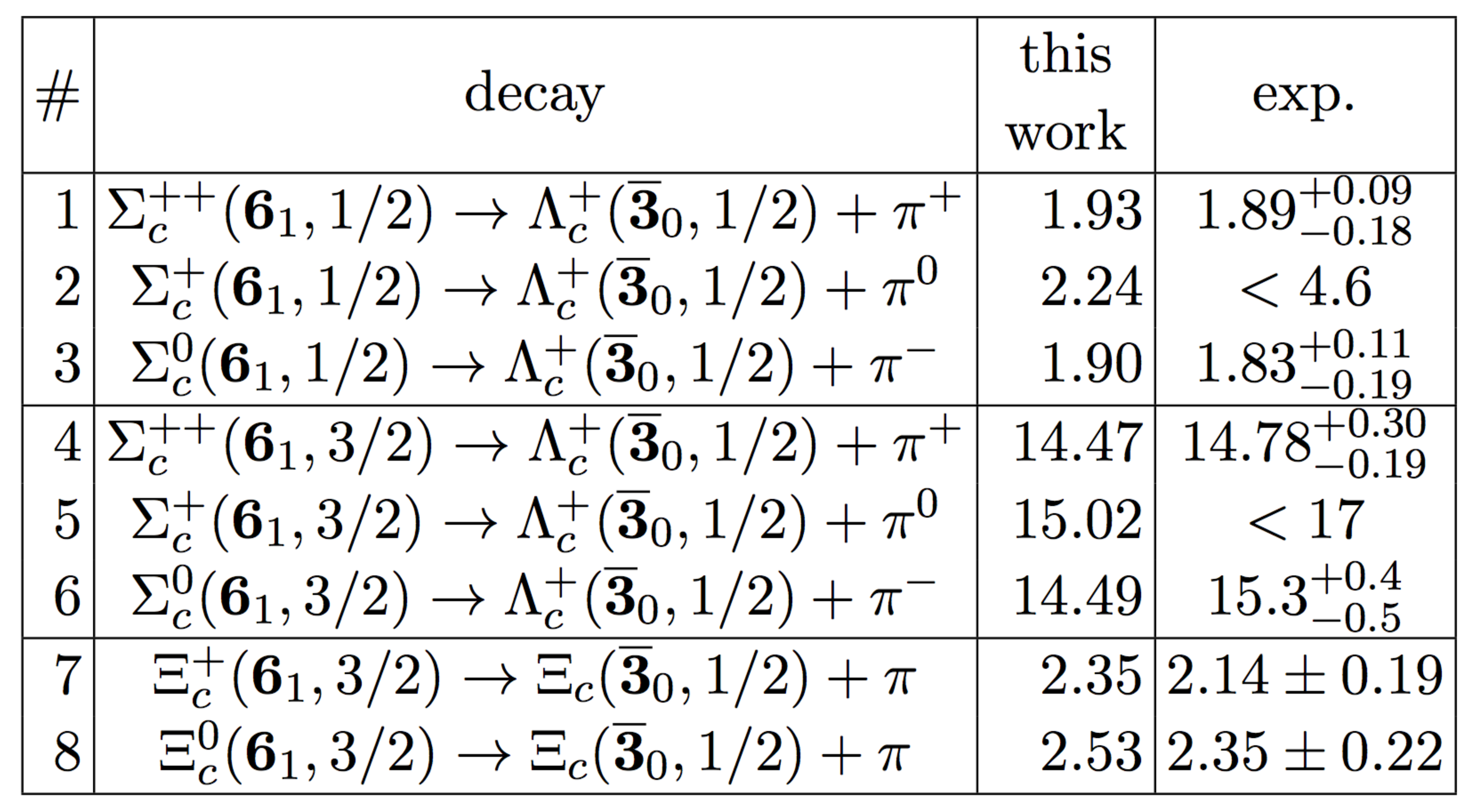
$$f_\pi = 93\,\mathrm{MeV}, \quad f_K = 1.2 f_\pi$$

- These parameters a_i have been determined by the hyperon semileptonic decays.

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Strong decays of heavy baryons

- Decay widths of the **charm** baryon sextet



#	decay	this work	exp.
1	$\Sigma_c^{++}(\mathbf{6}_1, 1/2) \to \Lambda_c^+(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	1.93	$1.89^{+0.09}_{-0.18}$
2	$\Sigma_c^{+}(\mathbf{6}_1, 1/2) \to \Lambda_c^+(\overline{\mathbf{3}}_0, 1/2) + \pi^0$	2.24	< 4.6
3	$\Sigma_c^{0}(\mathbf{6}_1, 1/2) \to \Lambda_c^+(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	1.90	$1.83^{+0.11}_{-0.19}$
4	$\Sigma_c^{++}(\mathbf{6}_1, 3/2) \to \Lambda_c^+(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	14.47	$14.78^{+0.30}_{-0.19}$
5	$\Sigma_c^{+}(\mathbf{6}_1, 3/2) \to \Lambda_c^+(\overline{\mathbf{3}}_0, 1/2) + \pi^0$	15.02	< 17
6	$\Sigma_c^{0}(\mathbf{6}_1, 3/2) \to \Lambda_c^+(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	14.49	$15.3^{+0.4}_{-0.5}$
7	$\Xi_c^{+}(\mathbf{6}_1, 3/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + \pi$	2.35	2.14 ± 0.19
8	$\Xi_c^{0}(\mathbf{6}_1, 3/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + \pi$	2.53	2.35 ± 0.22

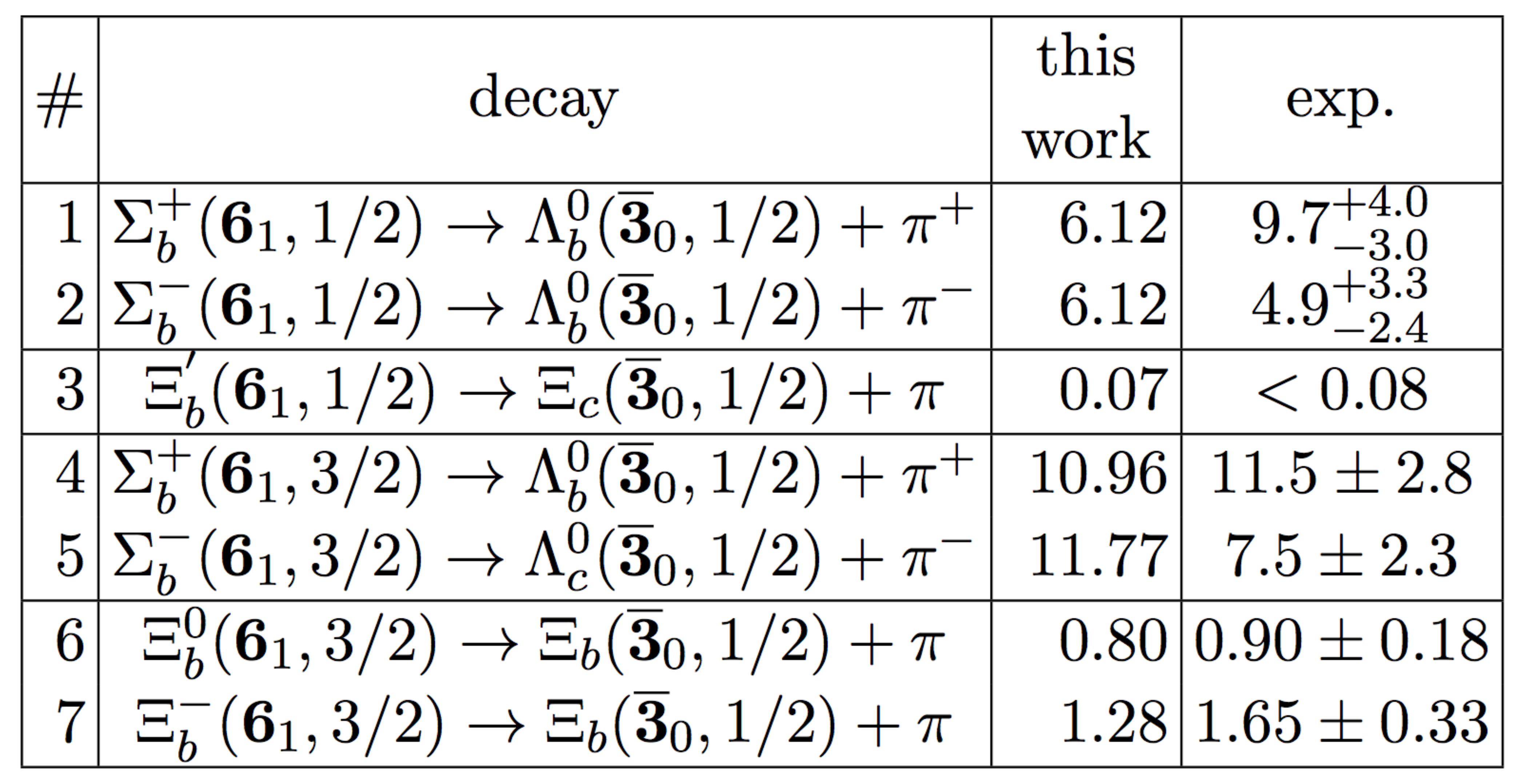
Experimental data are taken from the PDG Book.

No additional free parameter!

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Strong decays of heavy baryons

- Decay widths of the **bottom** baryon sextet



#	decay	this work	exp.
1	$\Sigma_b^+(\mathbf{6}_1, 1/2) \to \Lambda_b^0(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	6.12	$9.7^{+4.0}_{-3.0}$
2	$\Sigma_b^-(\mathbf{6}_1, 1/2) \to \Lambda_b^0(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	6.12	$4.9^{+3.3}_{-2.4}$
3	$\Xi_b'(\mathbf{6}_1, 1/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + \pi$	0.07	< 0.08
4	$\Sigma_b^+(\mathbf{6}_1, 3/2) \to \Lambda_b^0(\overline{\mathbf{3}}_0, 1/2) + \pi^+$	10.96	11.5 ± 2.8
5	$\Sigma_b^-(\mathbf{6}_1, 3/2) \to \Lambda_c^0(\overline{\mathbf{3}}_0, 1/2) + \pi^-$	11.77	7.5 ± 2.3
6	$\Xi_b^0(\mathbf{6}_1, 3/2) \to \Xi_b(\overline{\mathbf{3}}_0, 1/2) + \pi$	0.80	0.90 ± 0.18
7	$\Xi_b^-(\mathbf{6}_1, 3/2) \to \Xi_b(\overline{\mathbf{3}}_0, 1/2) + \pi$	1.28	1.65 ± 0.33

Experimental data are taken from the PDG Book.

No additional free parameter!

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Electromagnetic form factors

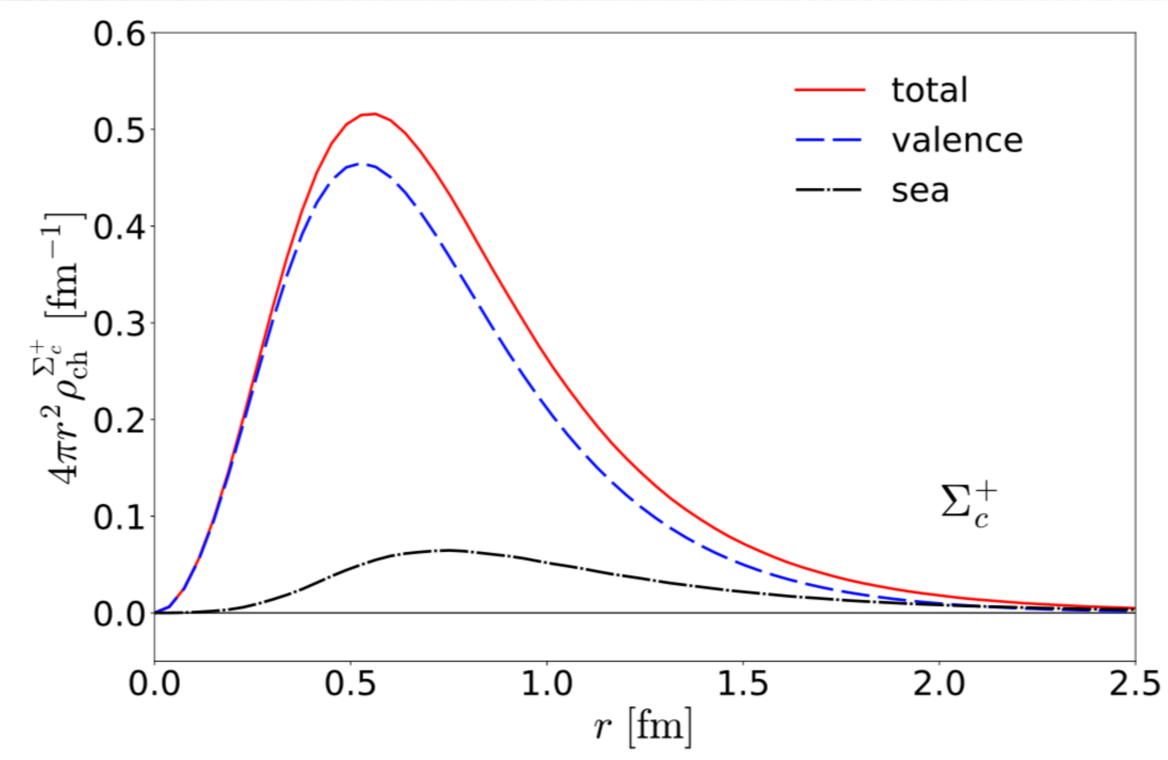
$$J_\mu(x) = \bar{\psi}(x)\gamma_\mu \hat{\mathcal{Q}}\psi(x) + e_Q \bar{\Psi}(x)\gamma_\mu \Psi(x)$$

- Heavy quark: point-like structure

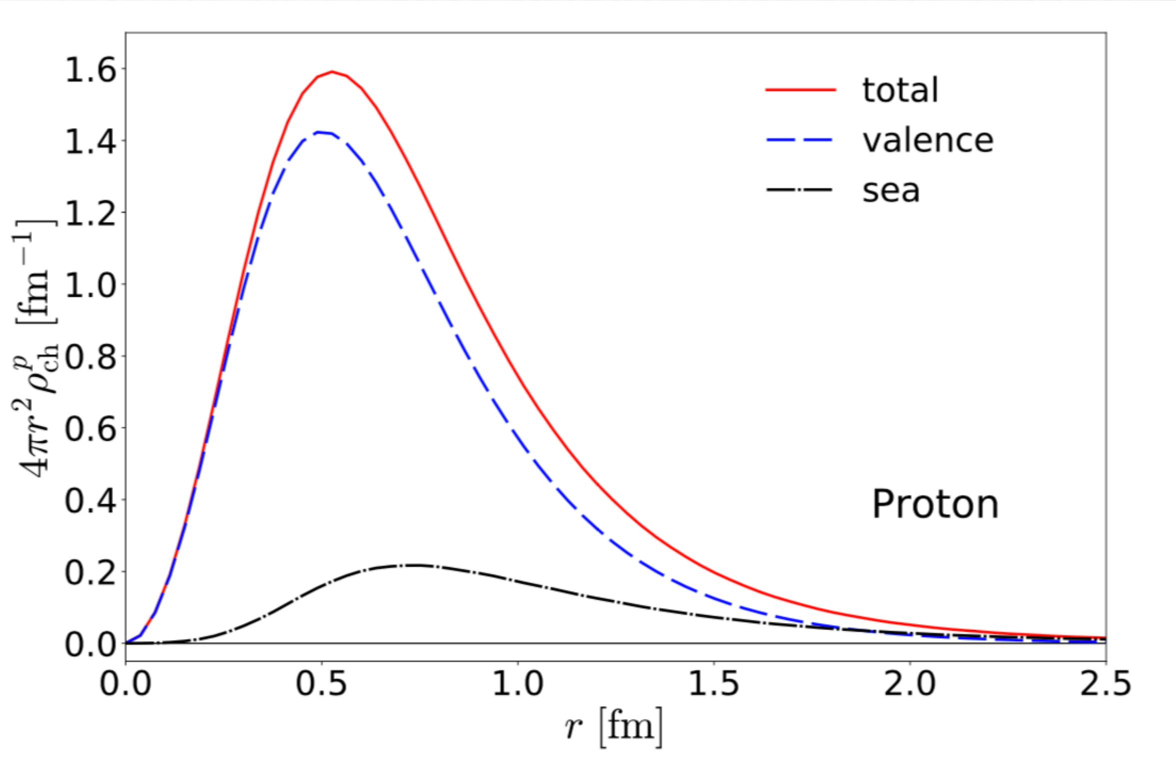
$$(m_Q \to \infty)$$

Electric charge densities

$\Sigma_c^+ \ (udc)$



proton (uud)



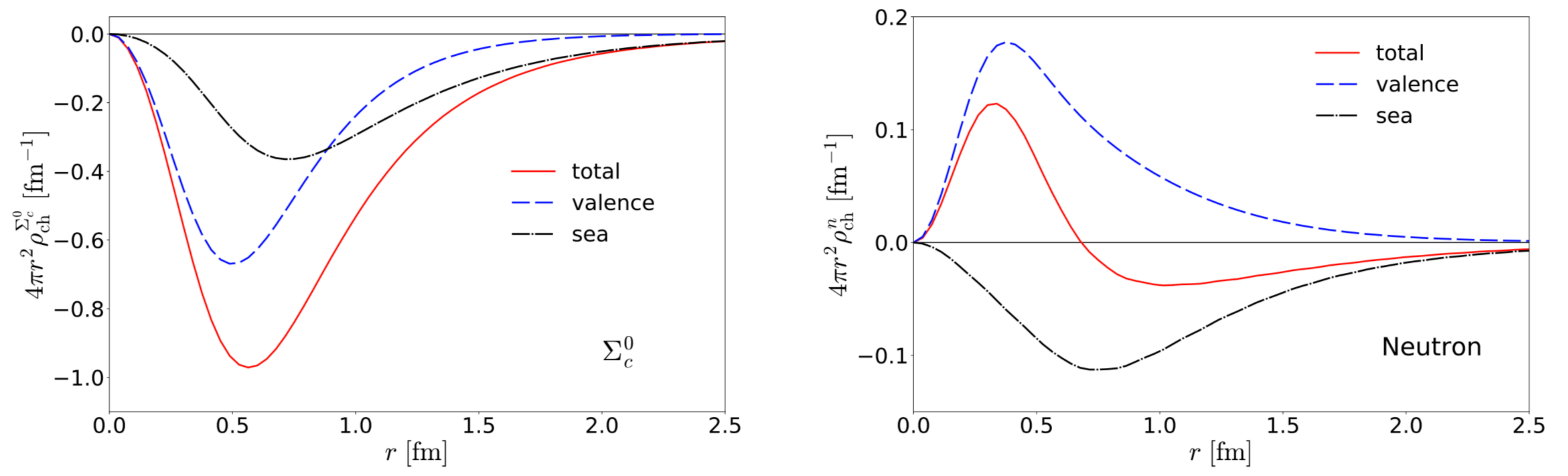
J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Electric charge densities

$\Sigma_c^0\,(ddc)$

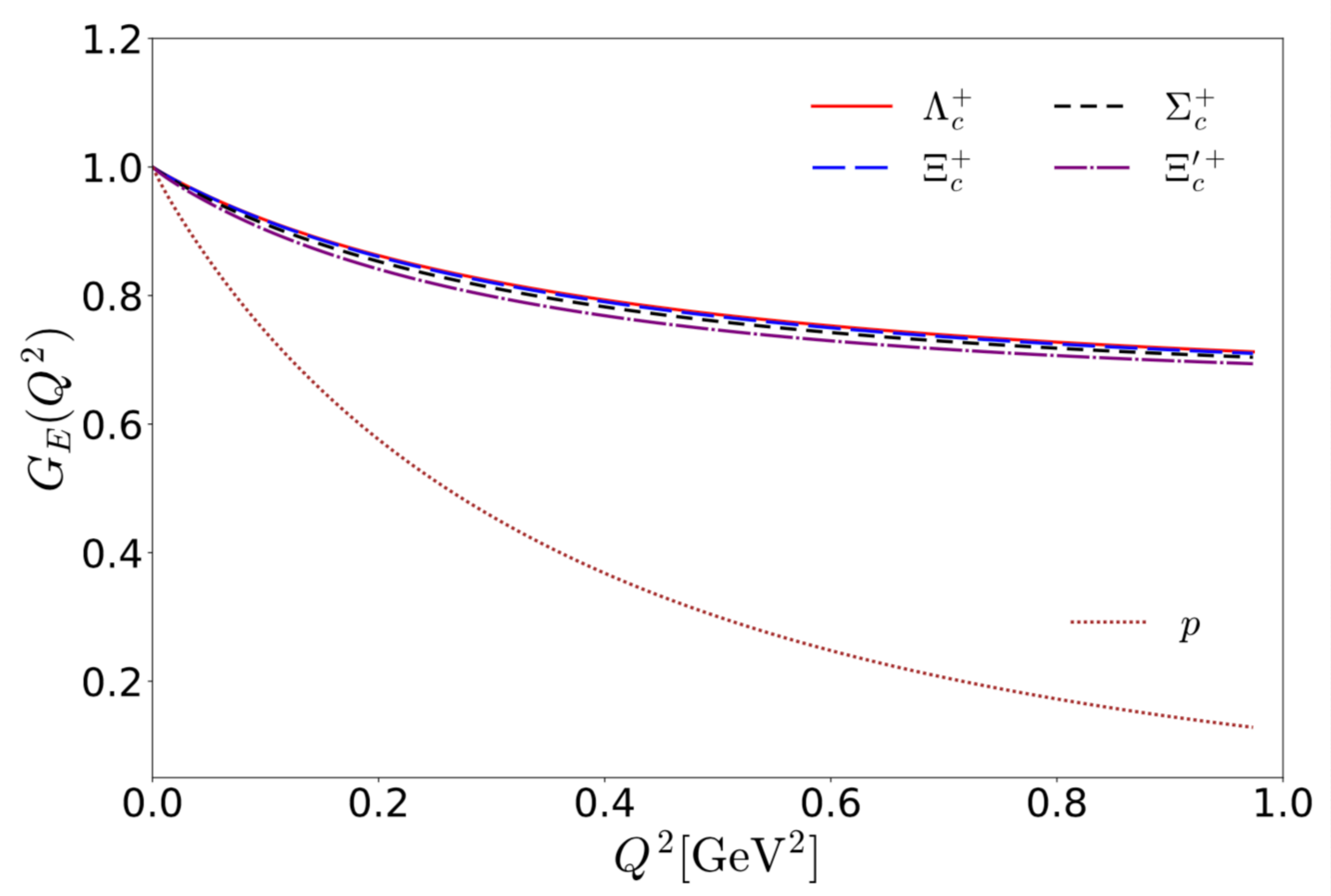
Neutron (udd)



J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Electric form factors



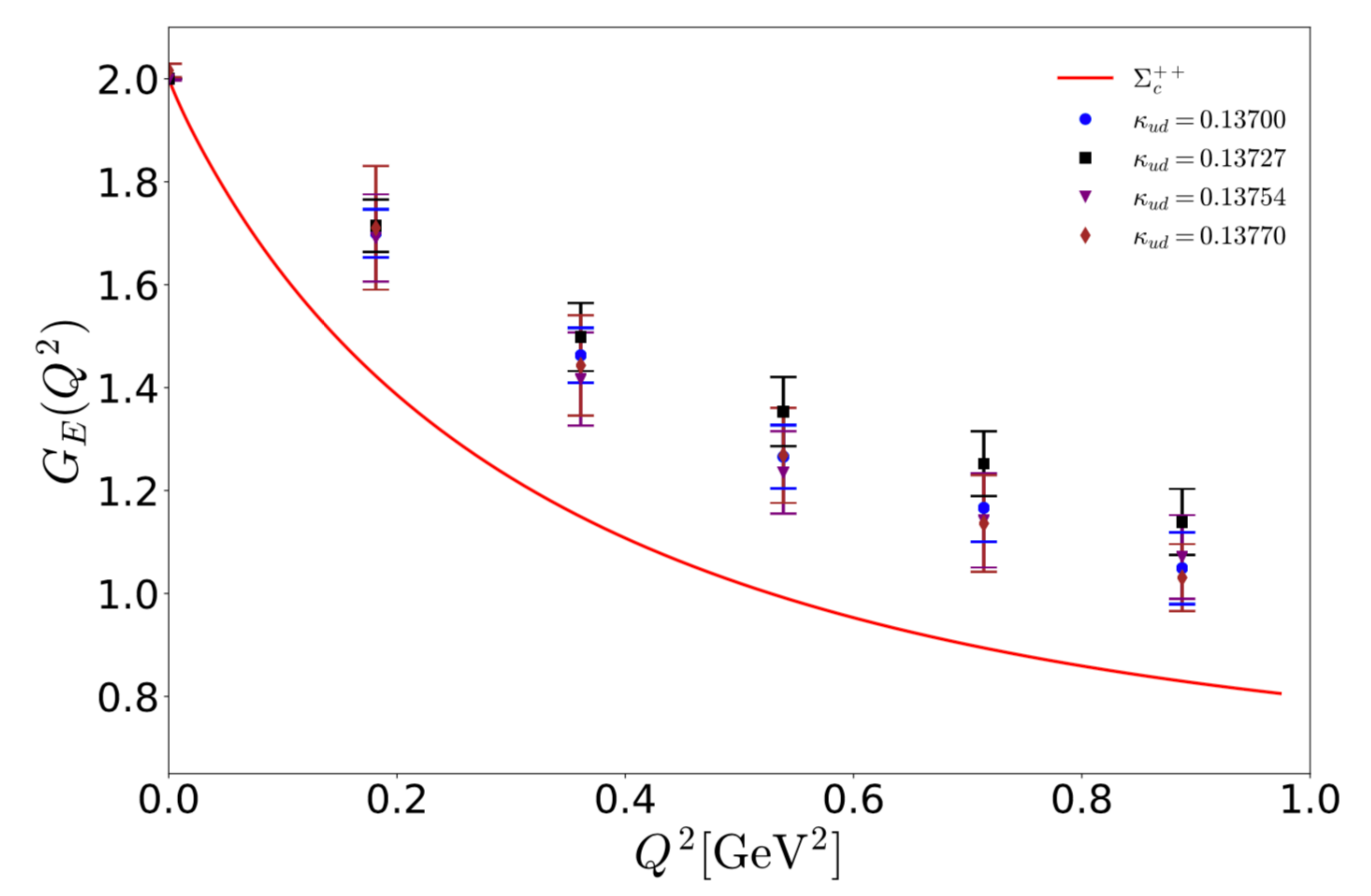
→ **Heavy baryons are electrically more compact than the proton!**

J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Electric form factors

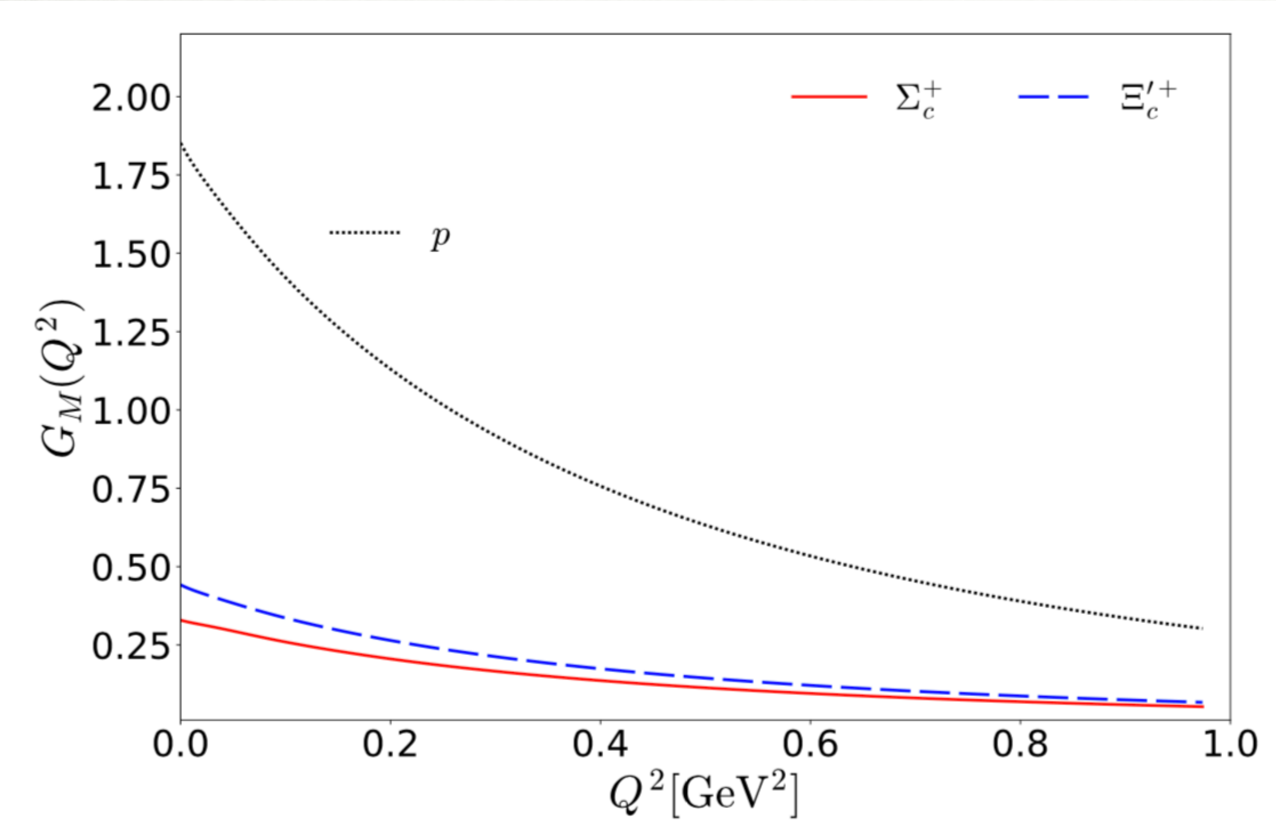
Lattice data: K. U. Can et al., JHEP 05 (2014) 125.

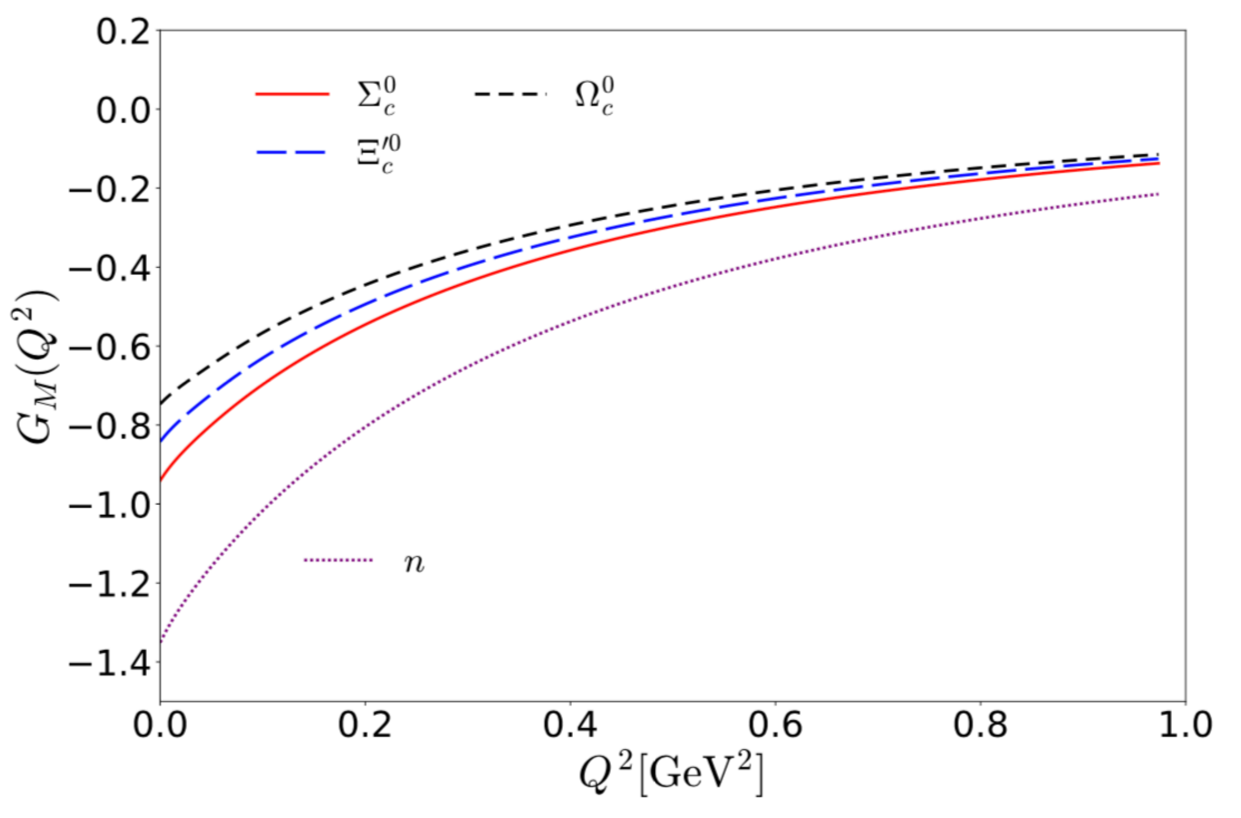


J.Y. Kim and HChK, PRD D97, 114009 (2018).

Electromagnetic form factors

Magnetic form factors

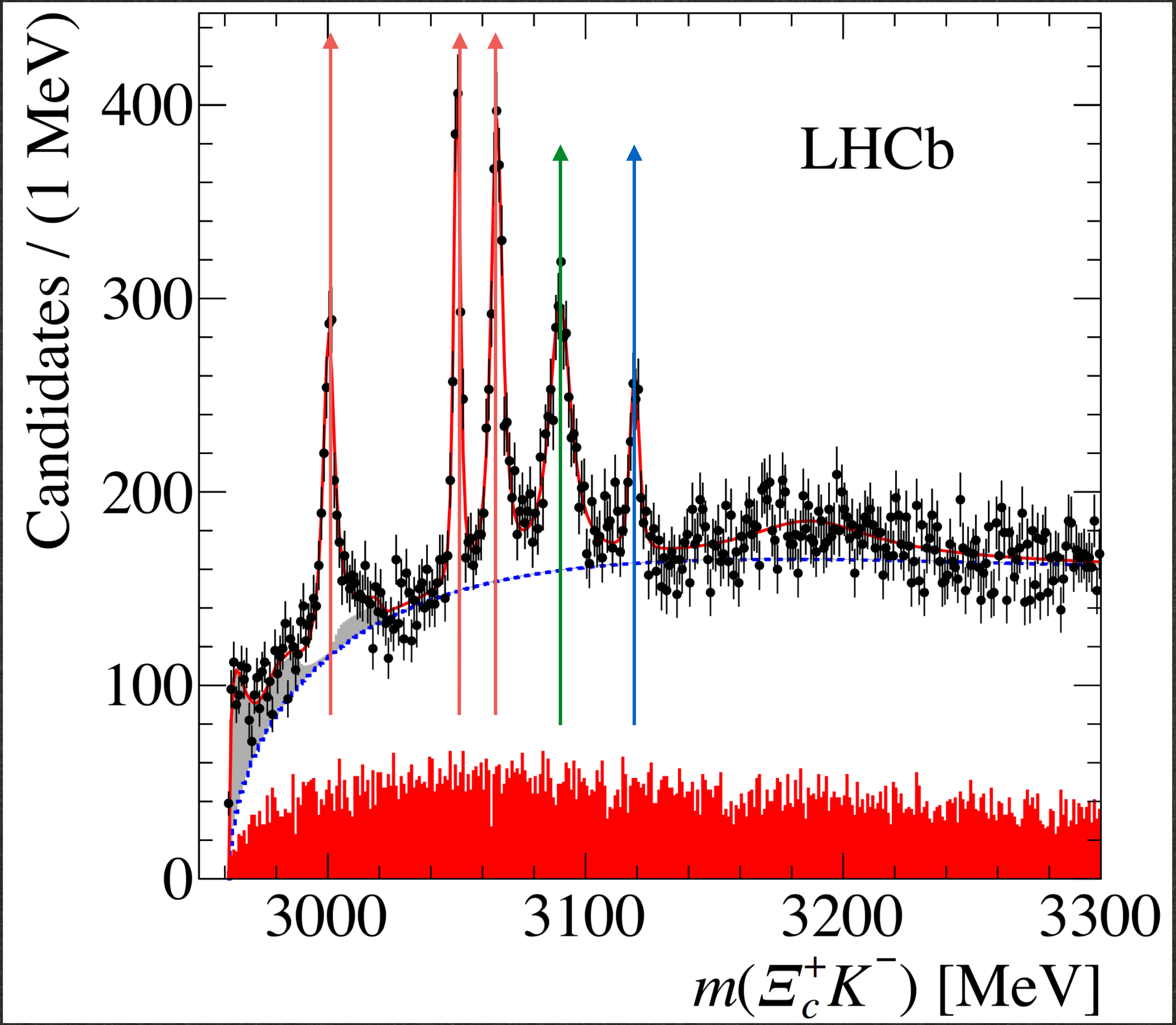




The singly heavy baryons are less magnetized than the proton and the neutron.

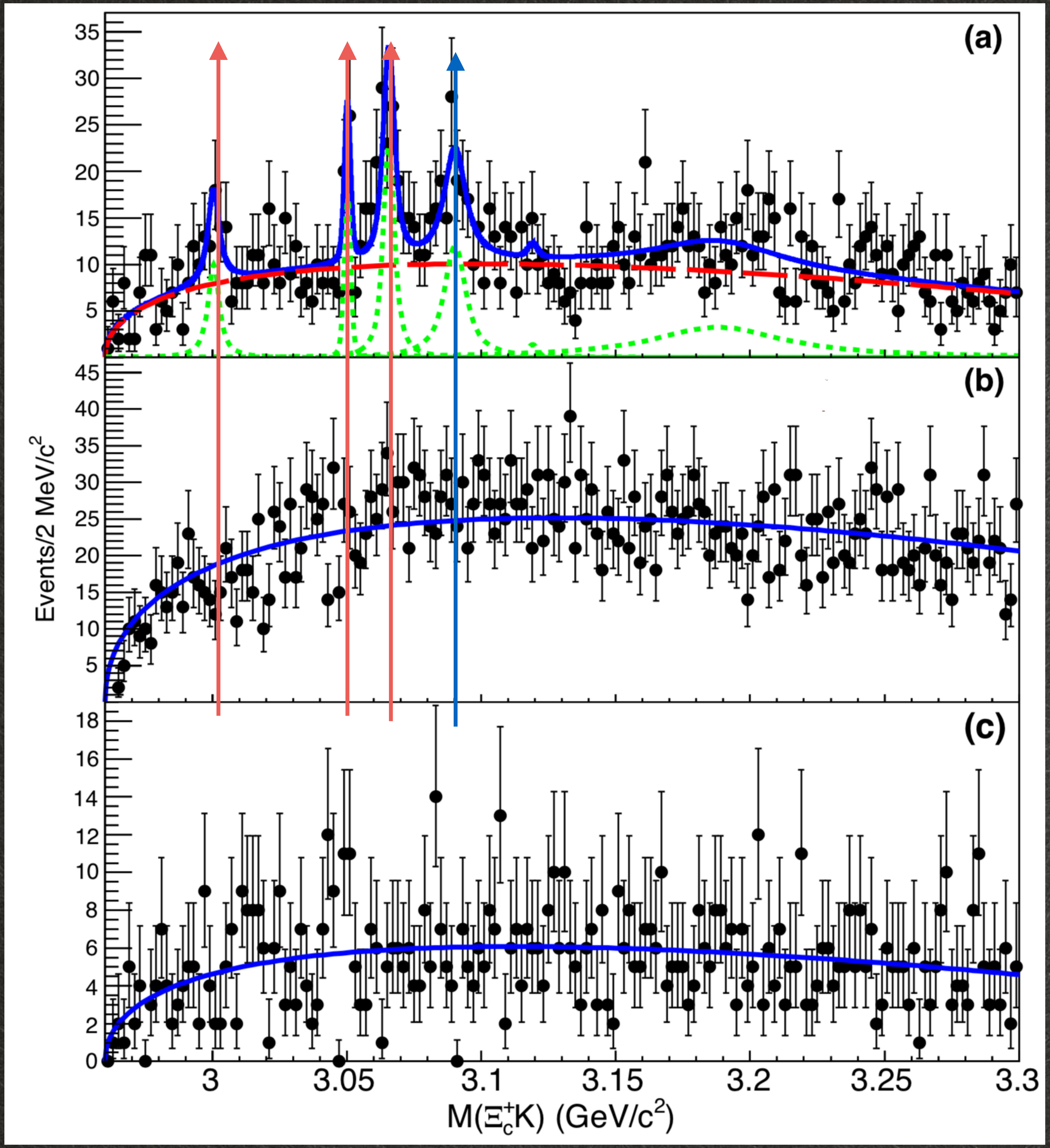
J.Y. Kim and HChK, PRD D97, 114009 (2018).

LHCb Findings: New five Omega_cs



LHCb Collaboration, PRL 118 (2017) 182001

Belle Confirmation: Four Omega_cs

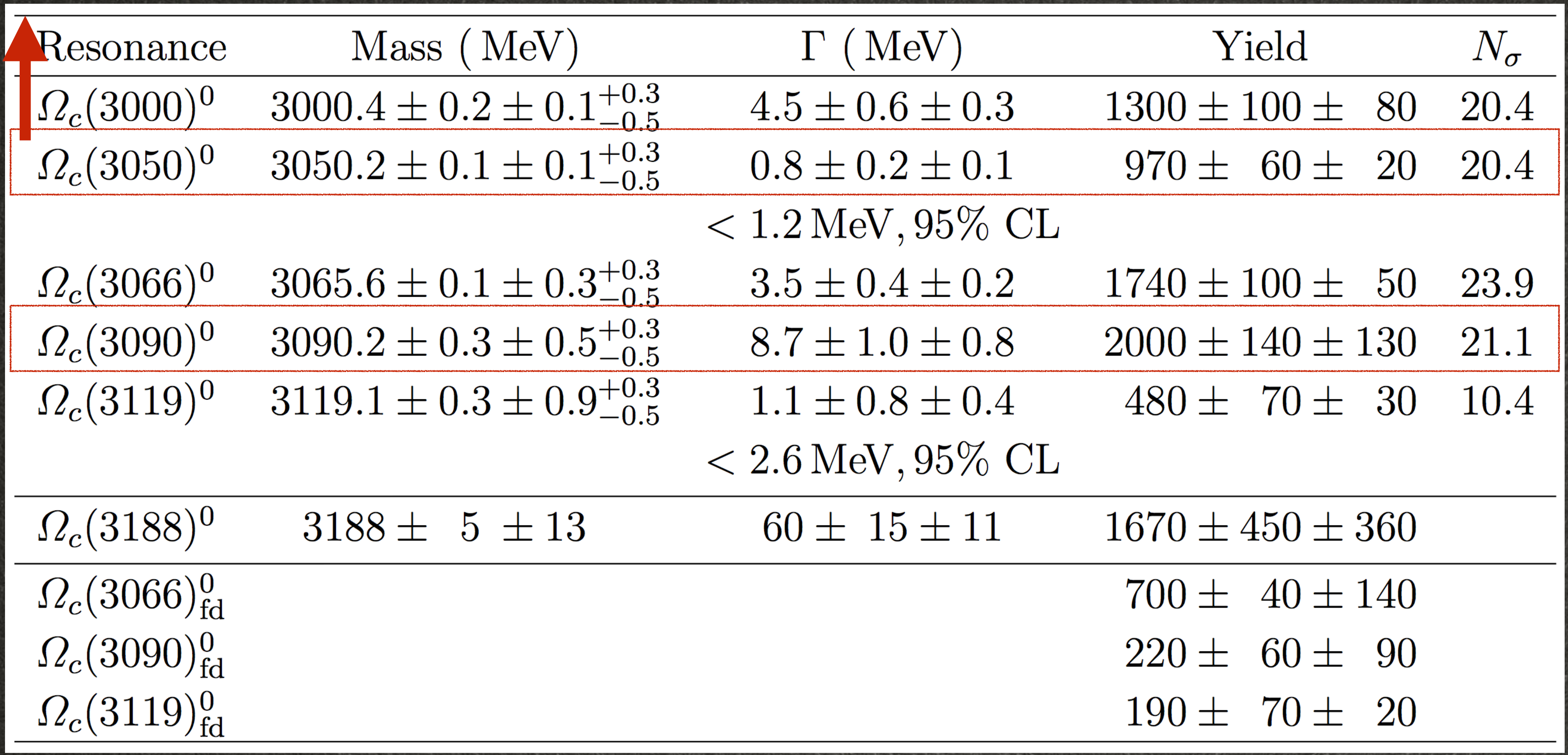


Four Ω_c^*'s were confirmed by Belle Coll.

Belle Collaboration, PRD 97 (2018) 051102

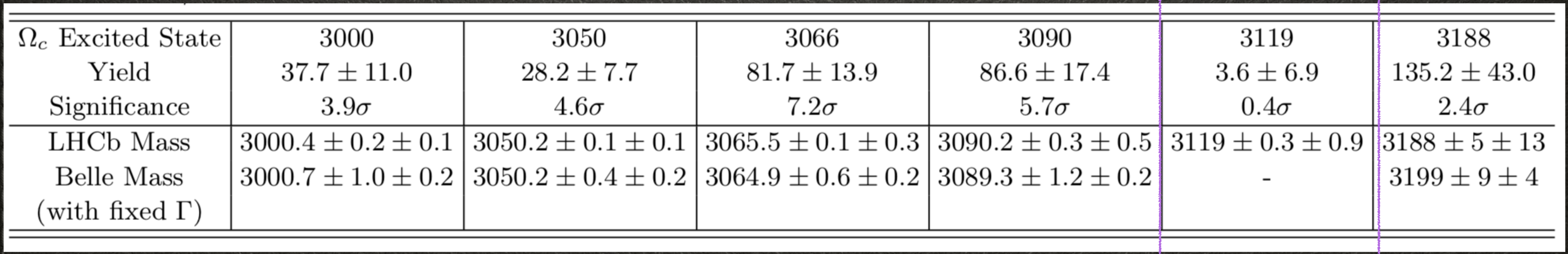
Five Omega_cs

The Widths are rather small, even if we consider the fact that heavy baryons have smaller widths than light ones.



Resonance	Mass (MeV)	Γ (MeV)	Yield	N_σ
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$	$1300 \pm 100 \pm 80$	20.4
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$	$970 \pm 60 \pm 20$	20.4
		$< 1.2\,\mathrm{MeV}, 95\%$ CL		
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$	$1740 \pm 100 \pm 50$	23.9
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$	$2000 \pm 140 \pm 130$	21.1
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$	$480 \pm 70 \pm 30$	10.4
		$< 2.6\,\mathrm{MeV}, 95\%$ CL		
$\Omega_c(3188)^0$	$3188 \pm 5 \pm 13$	$60 \pm 15 \pm 11$	$1670 \pm 450 \pm 360$	
$\Omega_c(3066)^0_{\mathrm{fd}}$			$700 \pm 40 \pm 140$	
$\Omega_c(3090)^0_{\mathrm{fd}}$			$220 \pm 60 \pm 90$	
$\Omega_c(3119)^0_{\mathrm{fd}}$			$190 \pm 70 \pm 20$	

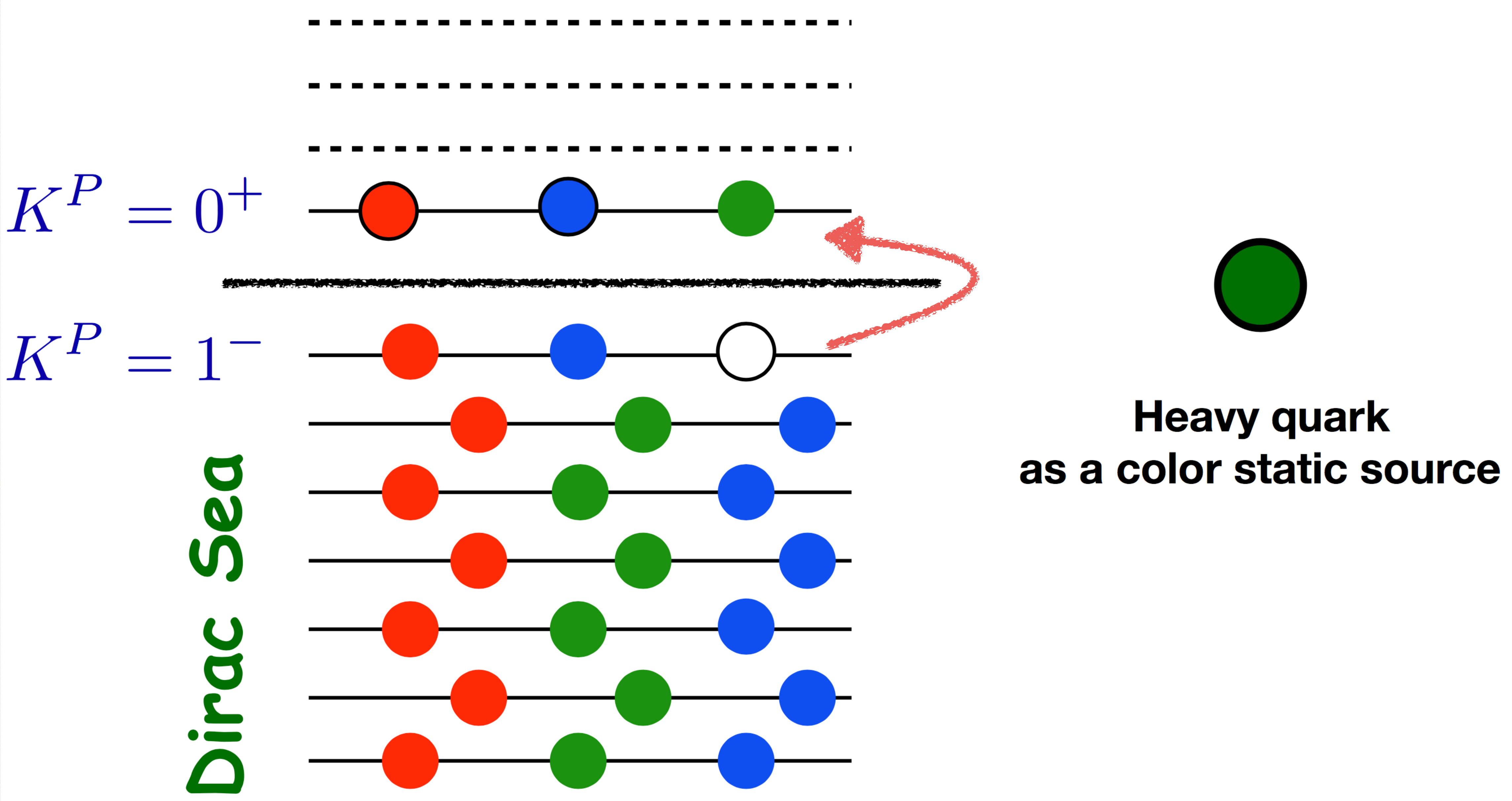
LHCb Collaboration, 2017



Ω_c Excited State	3000	3050	3066	3090	3119	3188
Yield	37.7 ± 11.0	28.2 ± 7.7	81.7 ± 13.9	86.6 ± 17.4	3.6 ± 6.9	135.2 ± 43.0
Significance	3.9σ	4.6σ	7.2σ	5.7σ	0.4σ	2.4σ
LHCb Mass	$3000.4 \pm 0.2 \pm 0.1$	$3050.2 \pm 0.1 \pm 0.1$	$3065.5 \pm 0.1 \pm 0.3$	$3090.2 \pm 0.3 \pm 0.5$	$3119 \pm 0.3 \pm 0.9$	$3188 \pm 5 \pm 13$
Belle Mass (with fixed Γ)	$3000.7 \pm 1.0 \pm 0.2$	$3050.2 \pm 0.4 \pm 0.2$	$3064.9 \pm 0.6 \pm 0.2$	$3089.3 \pm 1.2 \pm 0.2$	-	$3199 \pm 9 \pm 4$

Belle Collaboration, PRD 97 (2018) 051102

Excited anti-triplets and sextets



Heavy quark as a color static source

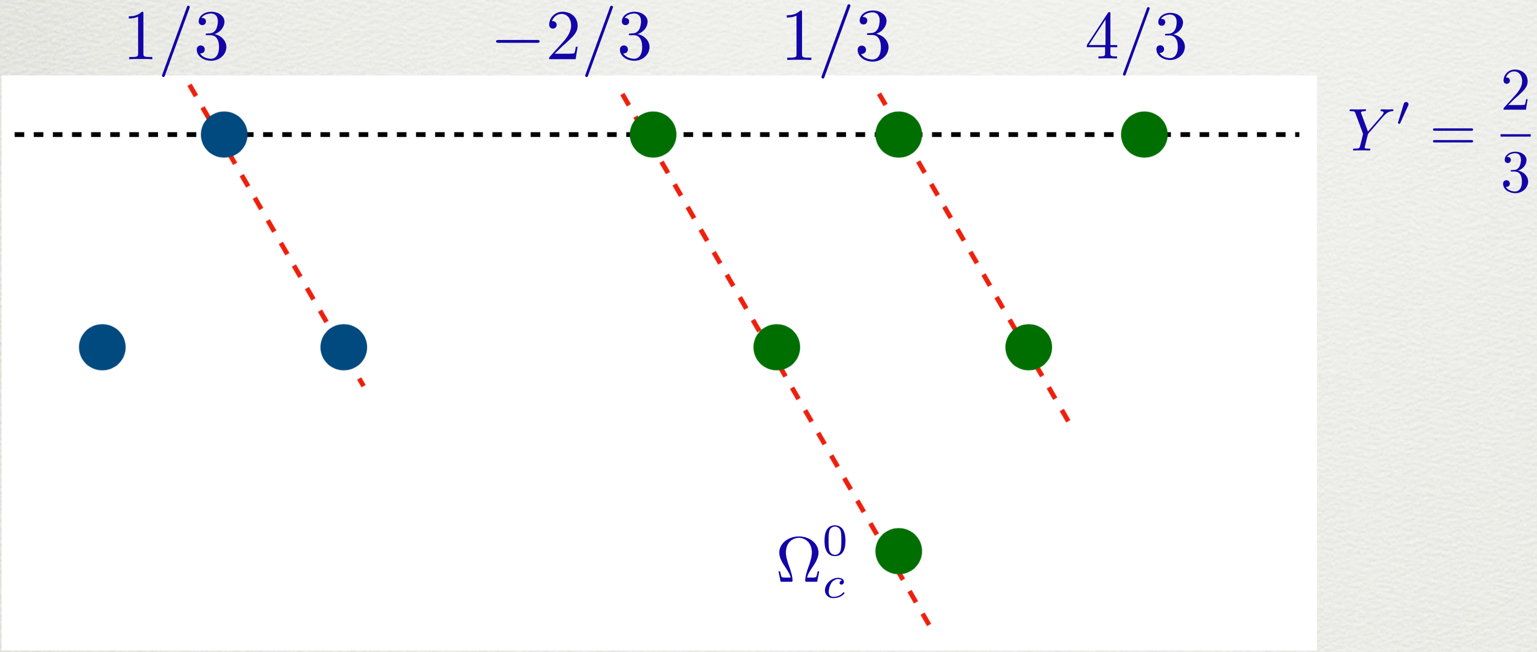
$K = J + T \neq 0$ $J = |T - K|, \cdots |T + K|$

HChK, M. V. Polyakov, M. Praszalowicz, PRD **96**, 014009 (2017)

Excited anti-triplets and sextets

Grand spin: $K^p = 1^-$

- Quantization of excited baryons yield **two** anti-triplet and FIVE sextets.



$$\boldsymbol{K = J + T = 1}$$

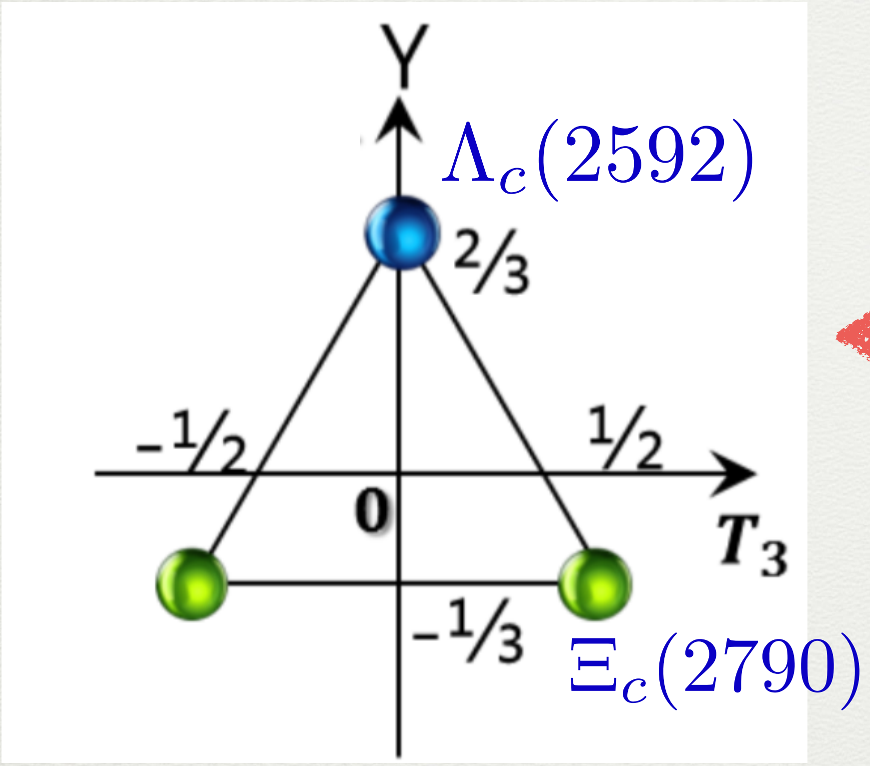
$$J = |T - K|, \cdots |T + K|$$

- **T=0** for an anti-triplet: J=1 for it. Combining a charm quark with spin 1/2, we have two anti-triplets (1/2) and (3/2).
- **T=1** for a sextet: J=0,1,2 for 6. We have five sextets with a charm quark (1/2), (1/2, 3/2), and (3/2, 5/2)!

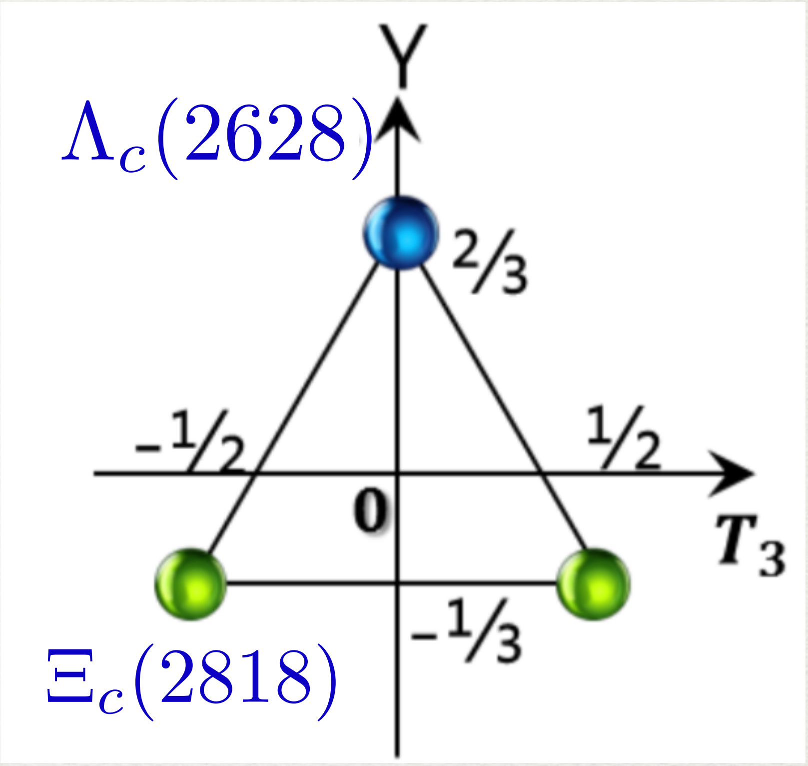
HChK, M. V. Polyakov, M. Praszalowicz, PRD **96**, 014009 (2017)

Hyperfine splittings for excited anti-triplets

- Candidates for excited anti-triplets



$J^p = 1/2^-$

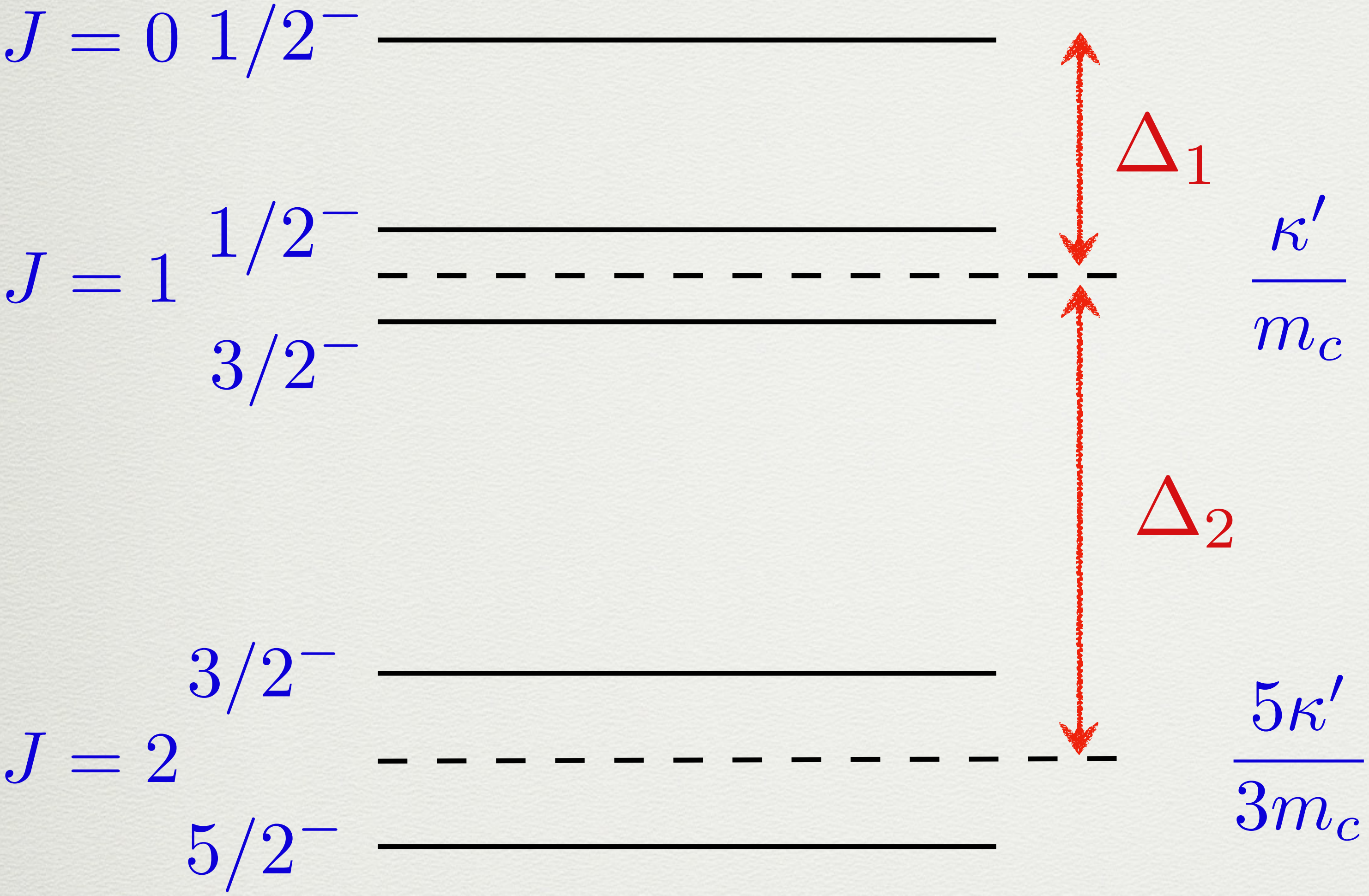


$J^p = 3/2^-$

$$\boxed{\frac{\kappa'}{m_c} \approx 30\,\mathrm{MeV}}$$

HChK, M. V. Polyakov, M. Praszalowicz, PRD **96**, 014009 (2017)

Hyperfine splittings for excited sextets



$$\Delta_1 = \frac{a_1}{I_1} + \frac{3}{20}\delta$$

$$\boxed{\Delta_2 = 2\Delta_1}$$

The robust relation
in the present approach

- The mean-field approach (XQSM) predicts **five excited sextet states.**
- The splitting between J=1 and J=2 is twice as large as that between J=0 and J=1.($\Delta_2 = 2\Delta_1$)

HChK, M. V. Polyakov, M. Praszalowicz, PRD **96**, 014009 (2017)

Scenario I

Assertion: Five Ω_c^*'s belong to excited sextets.

J	S^P	M [MeV]	κ'/m_c [MeV]	Δ_J [MeV]
0	$\frac{1}{2}^-$	3000	–	–
1	$\frac{1}{2}^-$	3050	16	61
	$\frac{3}{2}^-$	3066		
2	$\frac{3}{2}^-$	3090	17	47
	$\frac{5}{2}^-$	3119		

$$\Delta_2 = 2\Delta_1$$

This relation is badly broken.

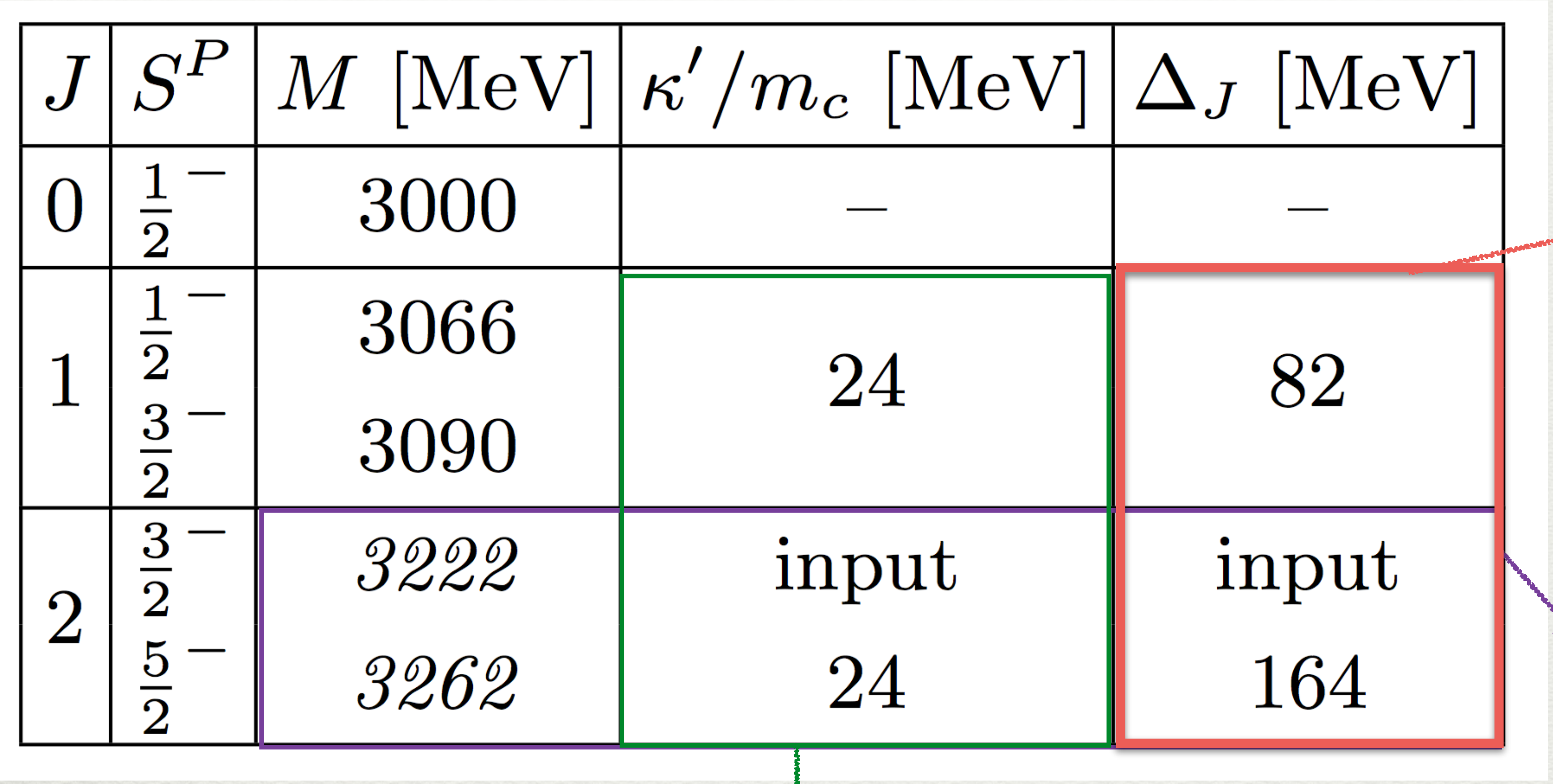
$$\frac{\kappa'}{m_c} = 30\,\mathrm{MeV}$$

The HF splittings are very much **deviated** from what we have determined from the excited anti-triplet.

HChK, M. V. Polyakov, M. Praszalowicz, PRD **96**, 014009 (2017)

Scenario II

Assertion: **Three** Ω_c^* ’s belong to excited sextets, whereas **two** Ω_c^* ’s with smaller widths are the members of the antidecapentaplet.



J	S^P	M [MeV]	κ'/m_c [MeV]	Δ_J [MeV]
0	$\frac{1}{2}^-$	3000	–	–
1	$\frac{1}{2}^-$	3066	24	82
	$\frac{3}{2}^-$	3090		
2	$\frac{3}{2}^-$	*3222*	input	input
	$\frac{5}{2}^-$	*3262*	24	164

$\Delta_2 = 2\Delta_1$

This relation is satisfied.

Bump structure above 3.2 GeV in the data

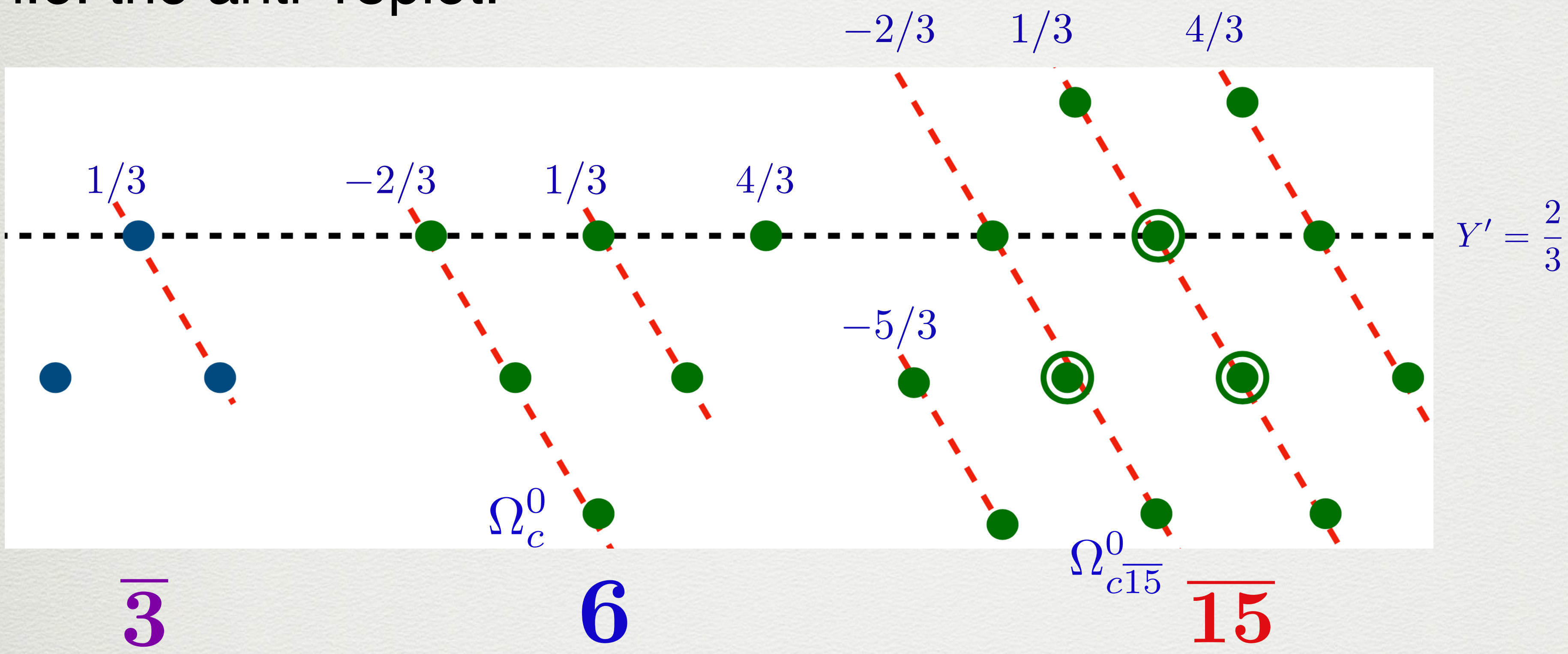
$\kappa'/m_c \approx 30\,\mathrm{MeV}$

What about other two Ω_c^* s?

- We assume that Omega(3050) and Omega(3119) belong to the **third** rotational excitation of the ground states: They will be then **pentaquarks**!

Antidecapentaplet

- In the heavy-quark sector, we have yet the third representation, i.e. the anti-15plet.



For the anti-15plet

$T = 1 \to J = 1$ Combined with a charm quark: $1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2} \in \overline{\mathbf{15}}$

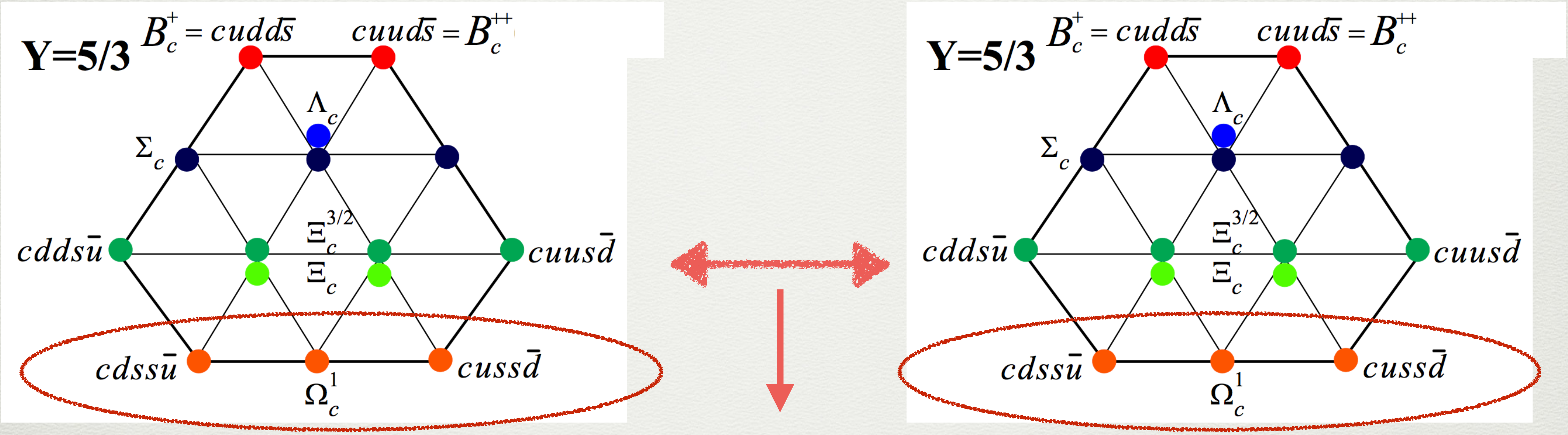
In the limit of infinitely heavy quark mass, 1/2 & 3/2 are degenerate, which will be lifted by a hyperfine interaction.

$$\Omega_c(3050)1/2^+ \quad \Omega_c(3119)3/2^+ : \quad M_{\Omega_c(3/2^+)} - M_{\Omega_c(1/2^+)} \simeq 69\,\mathrm{MeV}\,!$$

$\frac{\kappa}{m_c} = (68.1 \pm 1.1)\,\mathrm{MeV}$ in **excellent agreement** with the ground-state value!

Antidecapentaplet

- Exotic antidecapentaplet naturally arises from the XQSM.



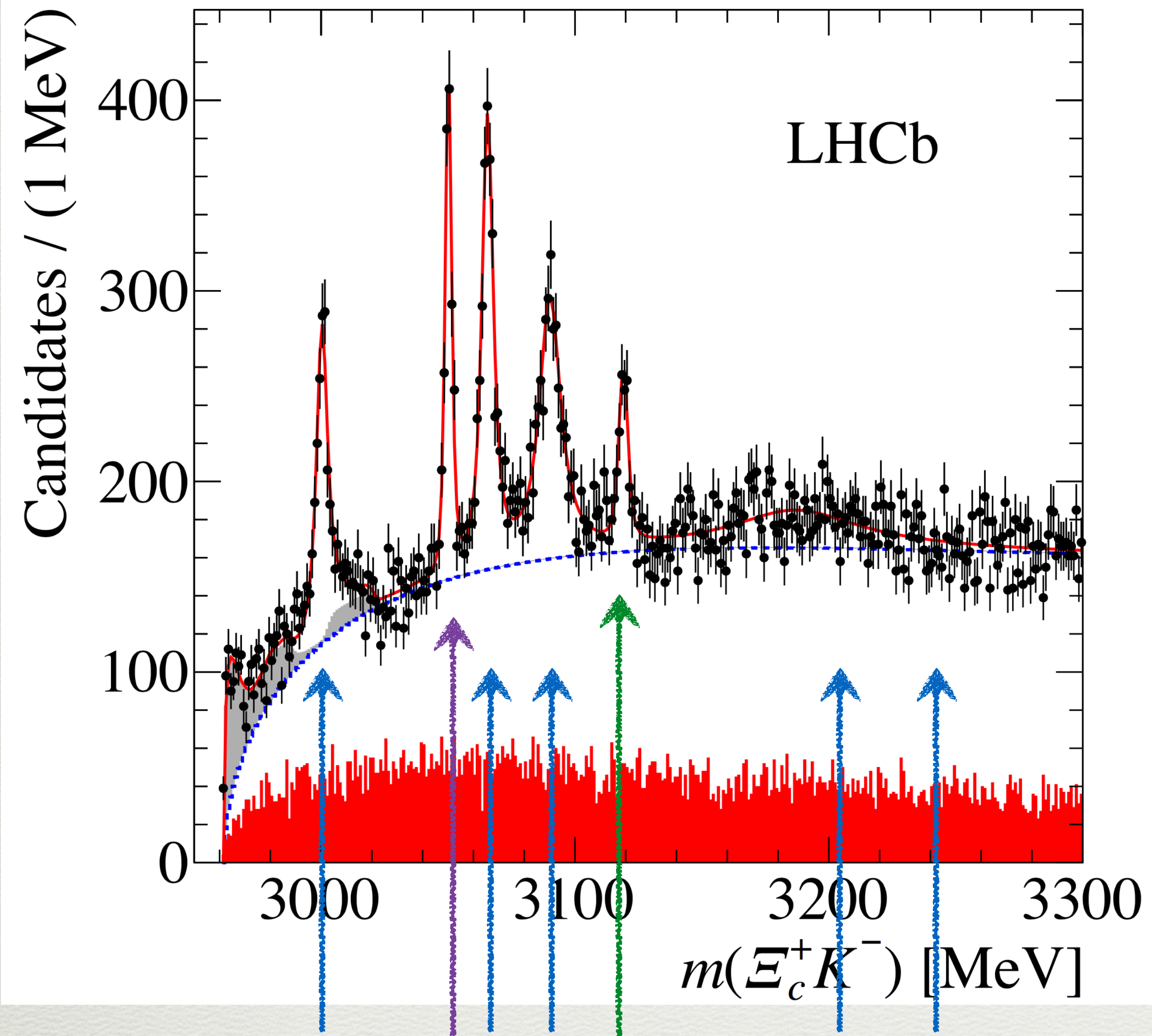
$J = 1/2$ $\qquad$ $\frac{\kappa}{m_c} = (68.1 \pm 1.1)\,\mathrm{MeV}$ $\qquad$ $J = 3/2$

$$\mathcal{M}_{\Omega_c} = (3140 - 3370)\,\mathrm{MeV}$$

- All parameters were fixed in the light baryon sector except for the hyperfine interaction.
- Considering almost all theoretical uncertainties, we get the following:

Interpretation of the LHC data

In the present picture



Resonance	Mass (MeV)	Γ (MeV)
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$ <1.2 MeV, 95% C.L.
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$

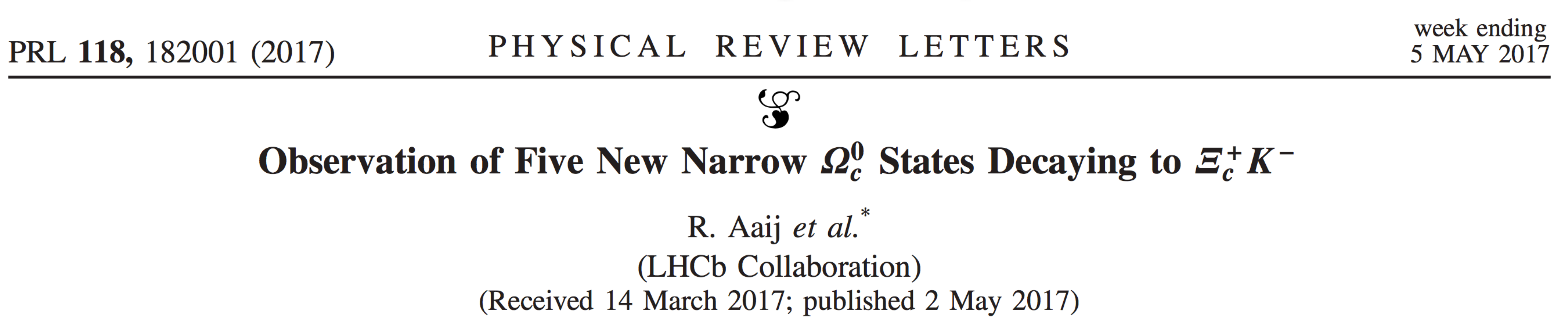
J	S^P	M [MeV]	κ'/m_c [MeV]	Δ_J [MeV]
0	$\frac{1}{2}^-$	3000	–	–
1	$\frac{1}{2}^-$	3066	24	82
	$\frac{3}{2}^-$	3090		
2	$\frac{3}{2}^-$	*3222*	input	input
	$\frac{5}{2}^-$	*3262*	24	164

$\frac{1}{2}^-$ $\frac{1}{2}^-$ $\frac{3}{2}^-$ $\frac{3}{2}^-$ $\frac{5}{2}^-$ $\in \mathbf{6}'$

$\frac{1}{2}^+$

$\frac{3}{2}^+ \in \overline{\mathbf{15}}$ $\Omega_c(3050)$ & $\Omega_c(3119)$

How can one falsify the present idea?

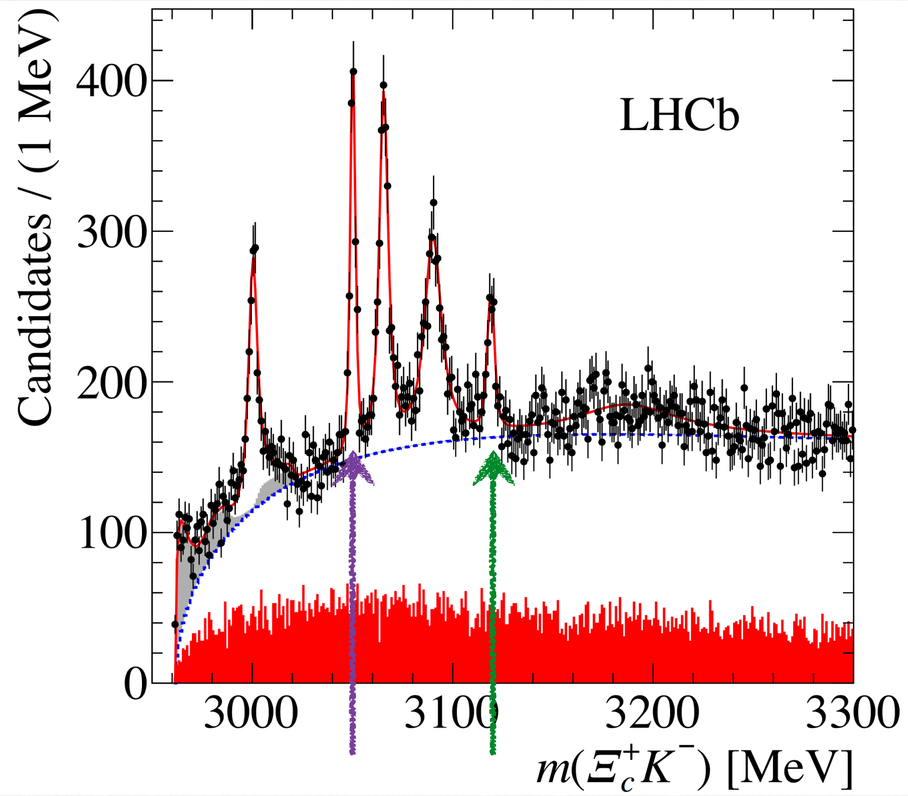


PRL **118,** 182001 (2017) PHYSICAL REVIEW LETTERS week ending 5 MAY 2017

Observation of Five New Narrow Ω_c^0 States Decaying to $\Xi_c^+ K^-$

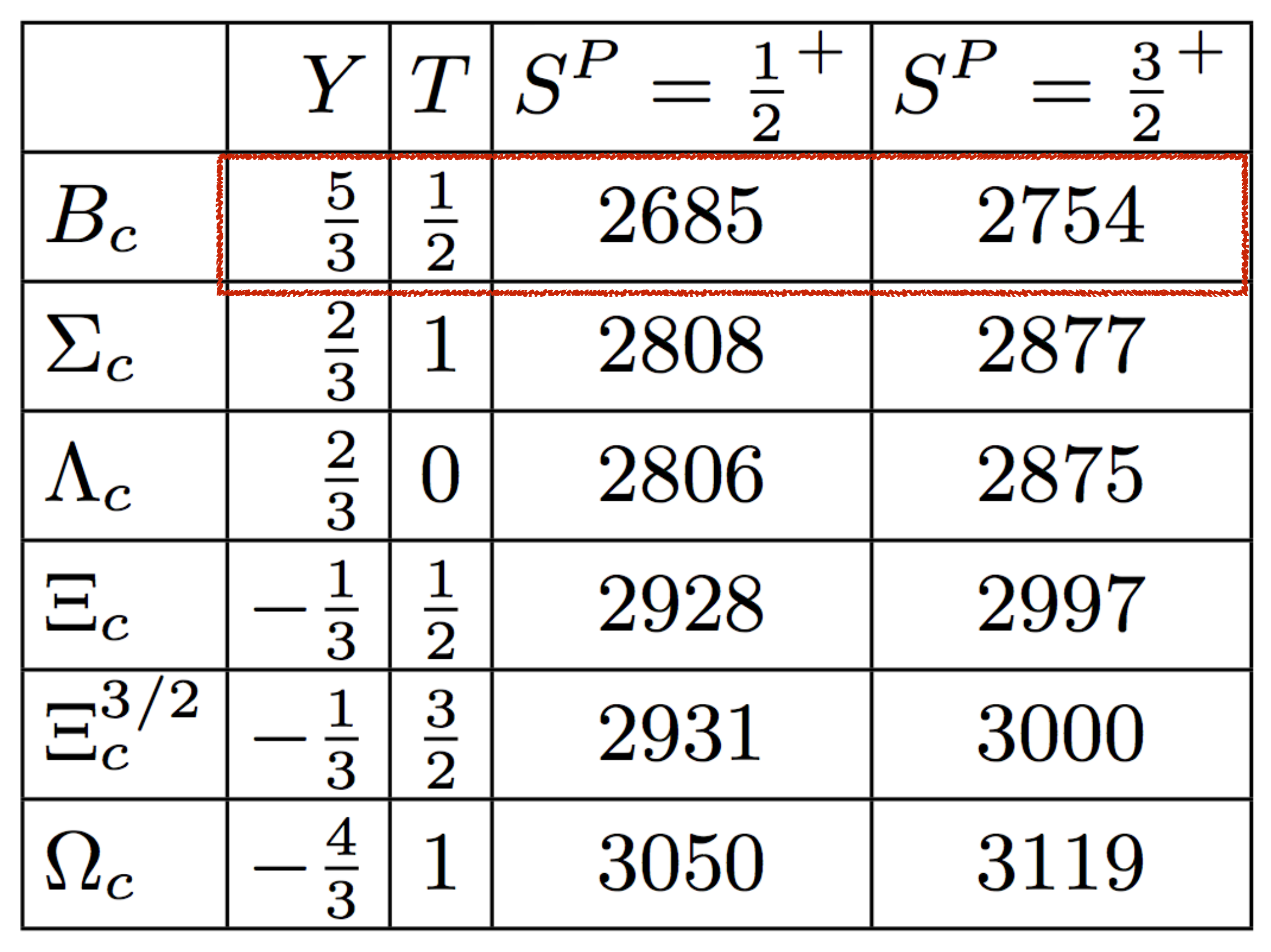
R. Aaij *et al.**
(LHCb Collaboration)
(Received 14 March 2017; published 2 May 2017)

- Anti-15plet contains **three** Omega_c's (Isovector baryons).
- The same peaks with the same strength can be found not only in the $\Xi_c^+ K^-$ channel but also in $\Xi_c^+ K^0$ and $\Xi_c^0 K^-$.

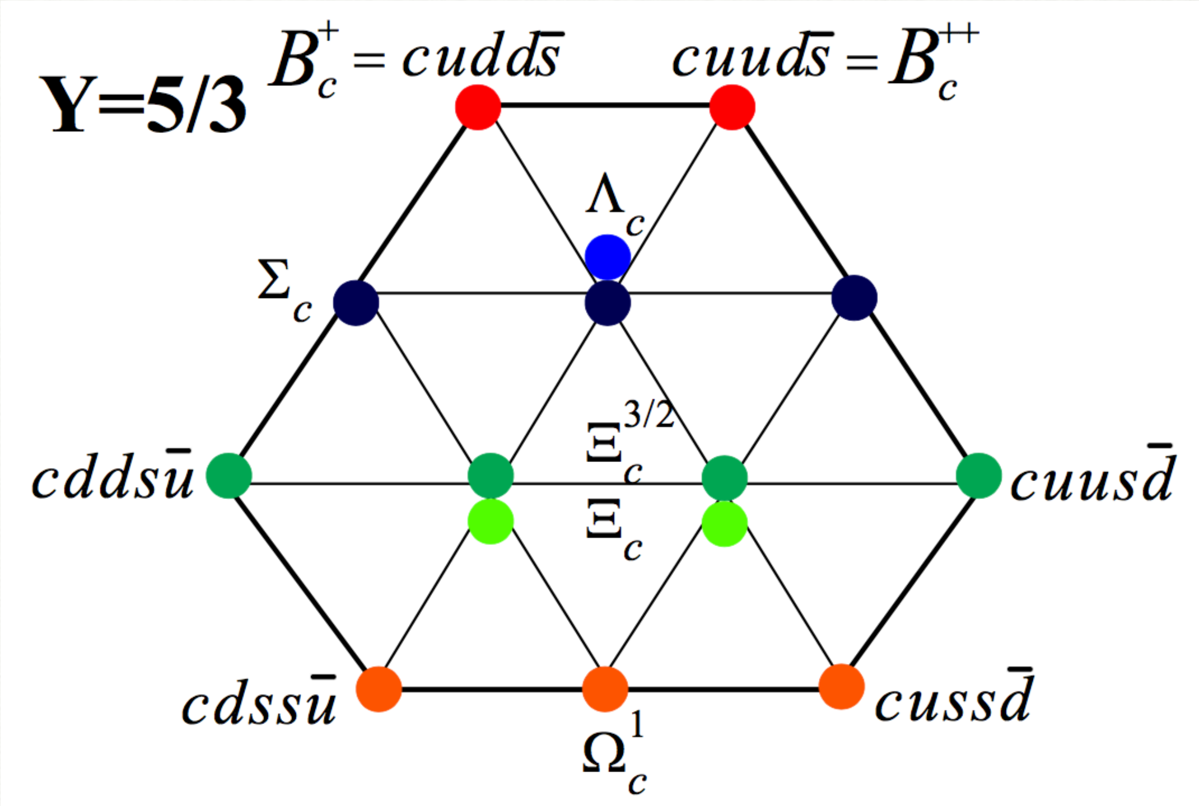


$\Omega_c(3050)$ & $\Omega_c(3119)$

Members of the antidecapentaplet



	Y	T	$S^P = \frac{1}{2}^+$	$S^P = \frac{3}{2}^+$
B_c	$\frac{5}{3}$	$\frac{1}{2}$	2685	2754
Σ_c	$\frac{2}{3}$	1	2808	2877
Λ_c	$\frac{2}{3}$	0	2806	2875
Ξ_c	$-\frac{1}{3}$	$\frac{1}{2}$	2928	2997
$\Xi_c^{3/2}$	$-\frac{1}{3}$	$\frac{3}{2}$	2931	3000
Ω_c	$-\frac{4}{3}$	1	3050	3119



Bc baryons will decay weakly, if they exist.
So, they should be stable.

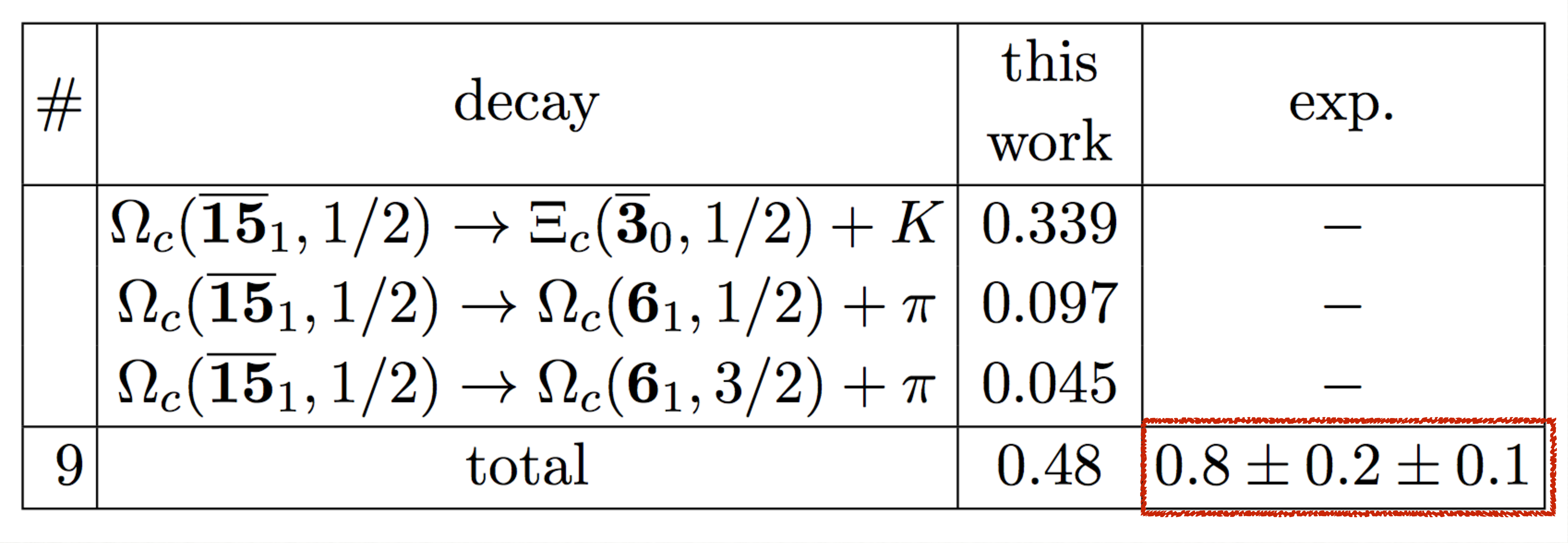
D. Diakonov, arXiv:1003.2157 [hep-ph].

Decays of the Ω_c^*

- Decay widths of the **charm** baryon antidecapentaplet

$$J^P = \frac{1}{2}^+$$

No additional free parameter!



#	decay	this work	exp.
	$\Omega_c(\overline{\mathbf{15}}_1, 1/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + K$	0.339	–
	$\Omega_c(\overline{\mathbf{15}}_1, 1/2) \to \Omega_c(\mathbf{6}_1, 1/2) + \pi$	0.097	–
	$\Omega_c(\overline{\mathbf{15}}_1, 1/2) \to \Omega_c(\mathbf{6}_1, 3/2) + \pi$	0.045	–
9	total	0.48	$0.8 \pm 0.2 \pm 0.1$

Experimental data are taken from the LHCb measurement.

- Note that the widths of Ω_c 's are rather small!

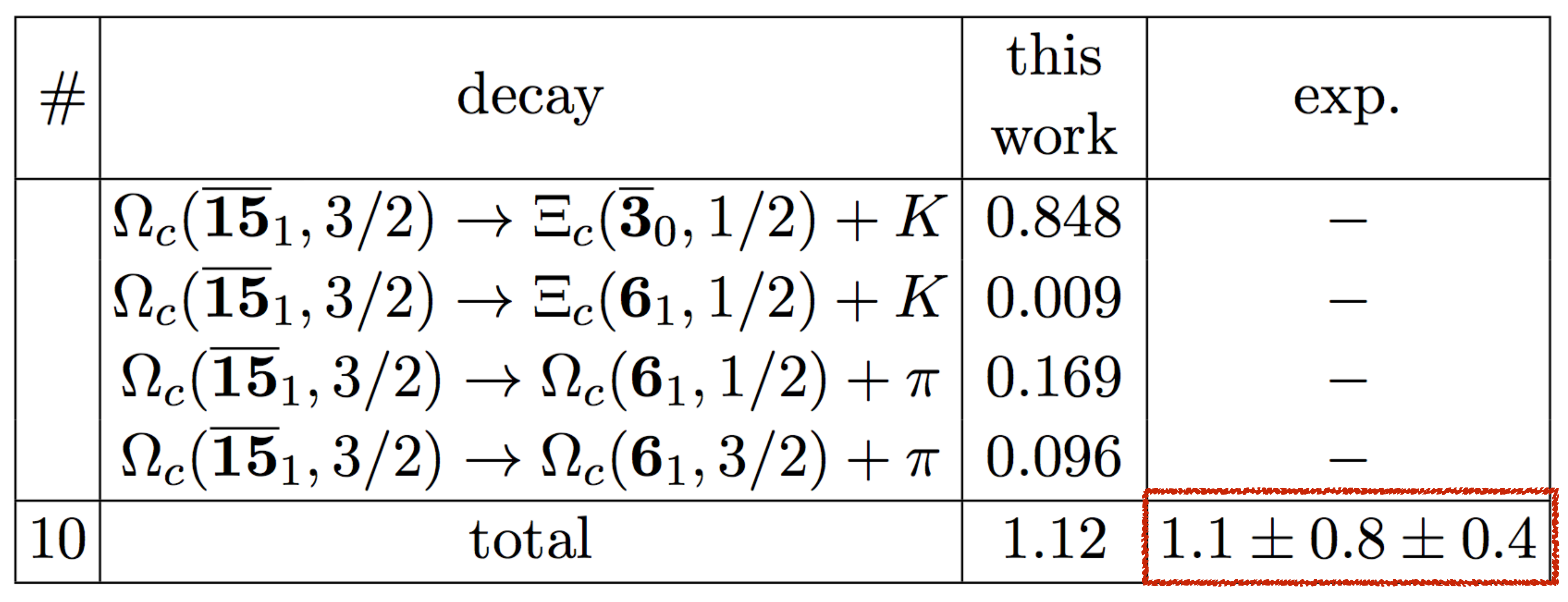
HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

Decays of the Ω_c^*

- Decay widths of the **charm** baryon antidecapentaplet

$$J^P = \frac{3}{2}^+$$

No additional free parameter!



#	decay	this work	exp.
	$\Omega_c(\overline{\mathbf{15}}_1, 3/2) \to \Xi_c(\overline{\mathbf{3}}_0, 1/2) + K$	0.848	–
	$\Omega_c(\overline{\mathbf{15}}_1, 3/2) \to \Xi_c(\mathbf{6}_1, 1/2) + K$	0.009	–
	$\Omega_c(\overline{\mathbf{15}}_1, 3/2) \to \Omega_c(\mathbf{6}_1, 1/2) + \pi$	0.169	–
	$\Omega_c(\overline{\mathbf{15}}_1, 3/2) \to \Omega_c(\mathbf{6}_1, 3/2) + \pi$	0.096	–
10	total	1.12	$1.1 \pm 0.8 \pm 0.4$

Experimental data are taken from the LHCb measurement.

- Note that the widths of Ω_c's are rather small!

HChK, Polyakov, Praszalowicz, Yang, PRD, D96, 094021 (2017).

PERSPECTIVES

Beyond Mean-field approximations

- Inclusion of meson loops (RPA-like) : Excited baryons < 2 GeV
- Modeling the effects of the quark confinement (Broken strings)
- Heavy-quark contributions
- Tcc (Keep in mind that we have the bosonic soliton)
- General mesonic mean fields(vector, axial-vector, tensors)
- Application of the XQSM in magnetic fields and medium

⇒ **Present & Future works of Hadron Theory Group at Inha University**

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

以上で、今日のセミナーを終わります。

どうもありがとうございました。