“Application of Hyperspherical Three-Body Variables to Lattice QCD Data: Is the Three-Quark Confining Potential Y-string or Δ-string?”

“Kantou Hadron Cafe"
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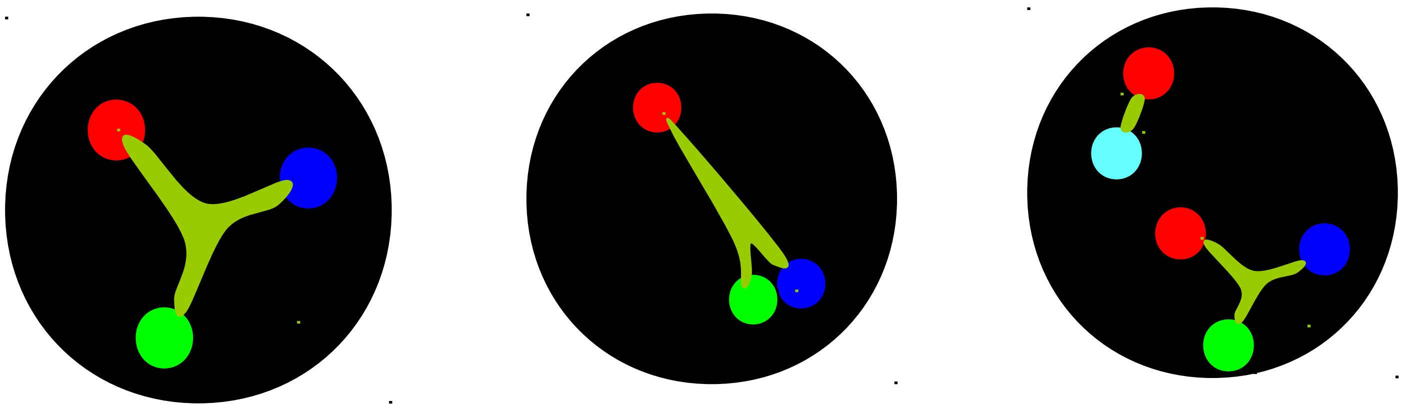
Outline:

- Open question: Δ- or Y-string in baryons?
- Review of 3-static-quark potentials in lattice QCD
- Intro to 3-body hyperspherical vbls
- Lattice data in terms of h.s. variables
- Error bars(?)
- (Possible) Intrepretation of results
- Dynamical O(2) symmetry of Y-string
- Summary and Outlook

1. The question: Delta or Y-string confinement?

Strings as the source of confinement?



S-matrix "dual resonance models" (Veneziano, 1969) led to first notions of "hadronic strings"

Advent of QCD (1973) led some (Mandelstam,'tHooft) to talk about possible mechanisms for string formation in QCD.

QCD flux-tubes in baryons I

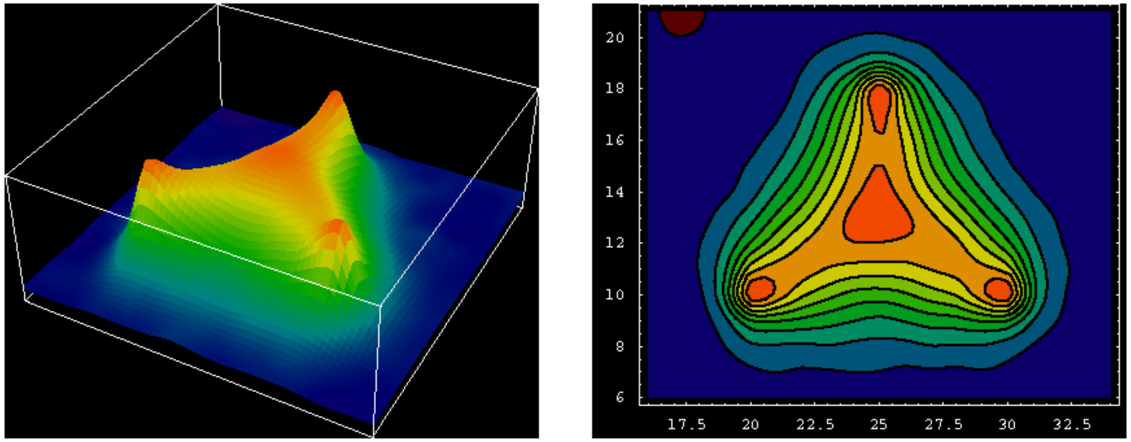


Figure 2. The flux-tube profile in the spatially-fixed 3Q system, in the MA projected QCD.[6] The distance between the junction and each quark is about 0.5 fm.

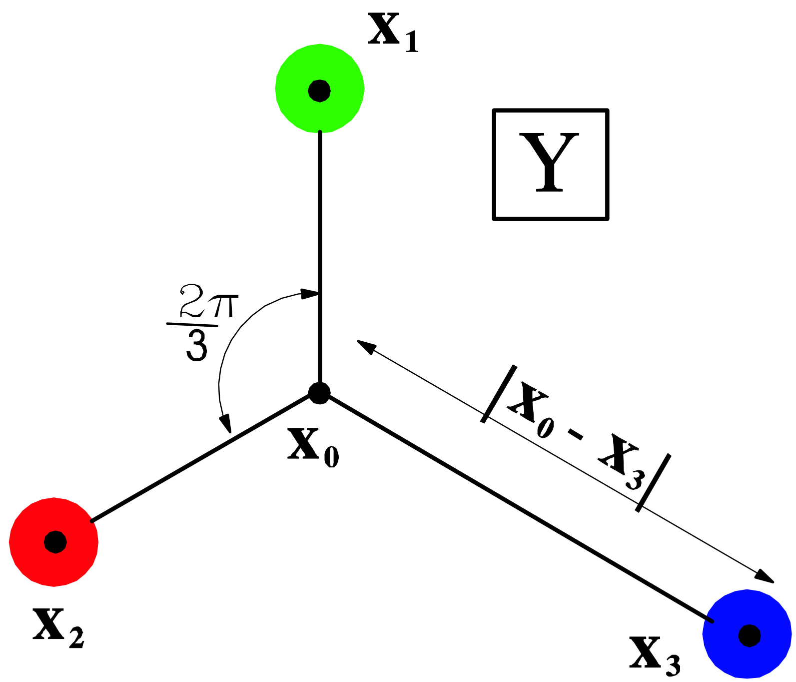
- Color flux-tube profiles in lattice QCD, Takahashi, Ichie and Suganuma, ("Wako 2003", p. 470-474), see also Bornyakov et al. PRD70,054506 (2004)
- Looks like Y-string – can we check this quantitatively?

The Y-string

- Defined as the shortest sum of string lengths; this means that the strings pointing to the three quarks form 120 degree angles at the juncture (Fermat-Torricelli-Steiner point)
- Support claimed from lattice QCD Takahashi,Matsufuru, Nemoto and Suganuma, PRL86,18('01); PRD65, 11409 ('02).

All these fit analyses support the *Y Ansatz*

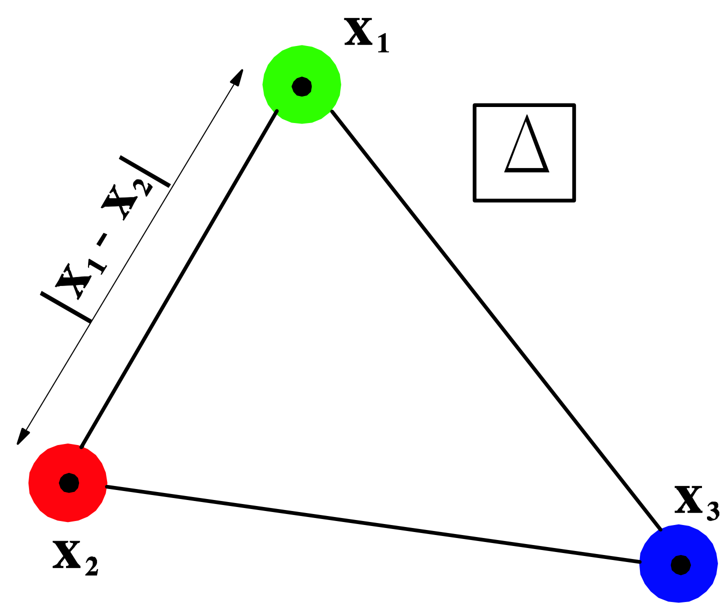
- also Sakumichi and Suganuma PRD92, 094513 (2015)



The Δ-string

- Sum of two-body potentials
- Also support claimed by Lattice QCD:

  (Alexandrou, deForcrand Tsapalis, PRD65, 054503 ('02) also claim Delta.



The Y-string potential

- The minimal Y-string length (potential) contains two square-roots: complicated!
- How to discriminate between Δ and Y on the lattice?

$$L_{\min}=\left[\frac{1}{2}(a^2+b^2+c^2)+\frac{\sqrt{3}}{2}\times\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}\right]^{1/2},$$

$$\boxed{L_{\min}=a+b+c-\max(a,b,c).}$$

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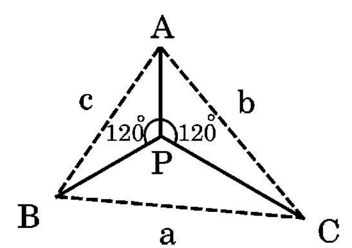


FIG. 1. The flux-tube configuration of the 3Q system with the minimal value of the total flux-tube length. There appears a physical junction linking the three flux tubes at the Fermat point *P*.

Distinction between Y- and Δ-strings

How can one distinguish between the Y and Delta string potentials?

Answer, in principle, based on symmetry, offered by V.D., T. Sato and M. Šuvakov, PRD 80,054501(2009)

In spring of 2017, Yoshiaki and Miho Koma, PRD95, 094513 reported a new calculation that is neither pure Delta nor pure Y string!

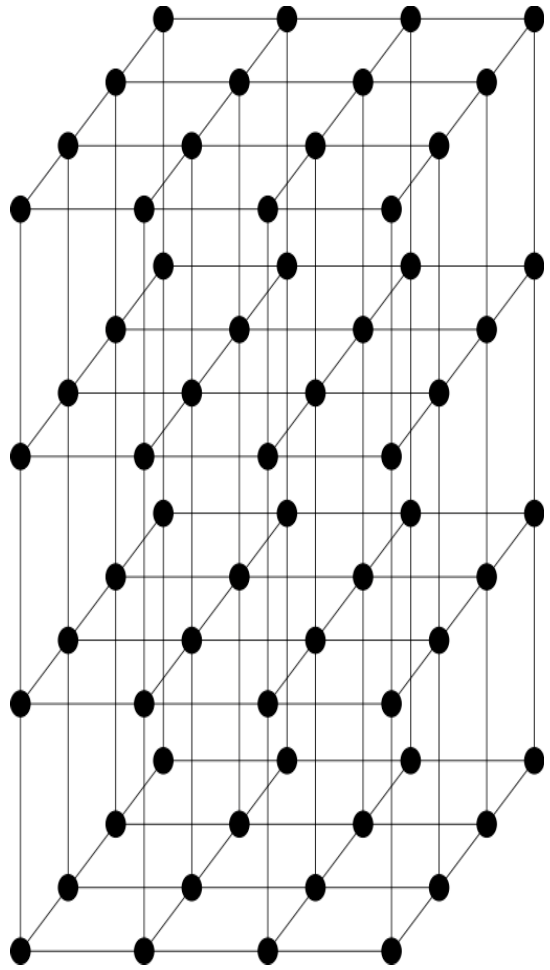
2. Brief review of lattice QCD static 3-quark potentials

Three Lattice QCD calculations

1) Takahashi,Matsufuru, Nemoto and Suganuma, PRL86, 18 ('01); PRD65, 11409 (2002) used smaller lattices: 12^3x24, at β =5.7 and 16^3x32, at β=5.8, 6.0

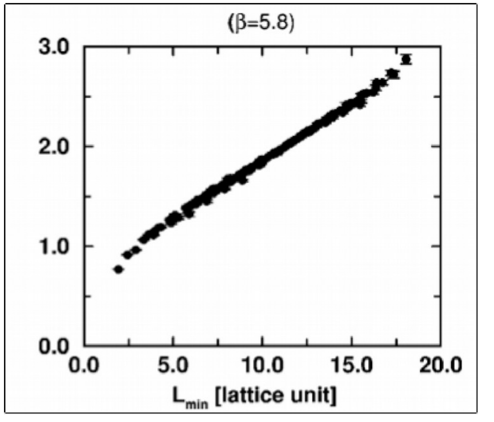
2) Sakumichi & Suganuma PRD92, 094513 (2015), used a larger lattice: 16^3x32 at β = 5.8, and 20^3x32 at β = 6.0

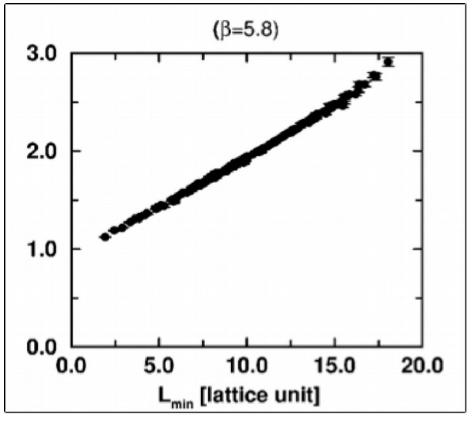
3) Koma & Koma PRD95, 094513 (2017), used the largest lattice: 24^4 at β = 5.85, 6.0, 6.3

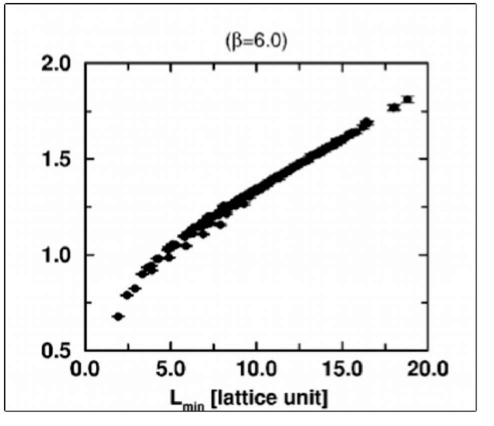


1. Takahashi et al. (2002)

- Takahashi et al.'s results: with and with/o Coulomb
- L_min = length of Y string
- All quark configurations (geometries) included







2. Sakumichi & Suganuma (2015)

Naoyuki Sakumichi, Hideo Suganuma

Phys. Rev. D 92, 034511 (2015)

did calculation with

$\beta = 5.8$ on 16^3x32;

$\beta = 6.0$ on 20^3x32

with 1000–2000 gauge configurations

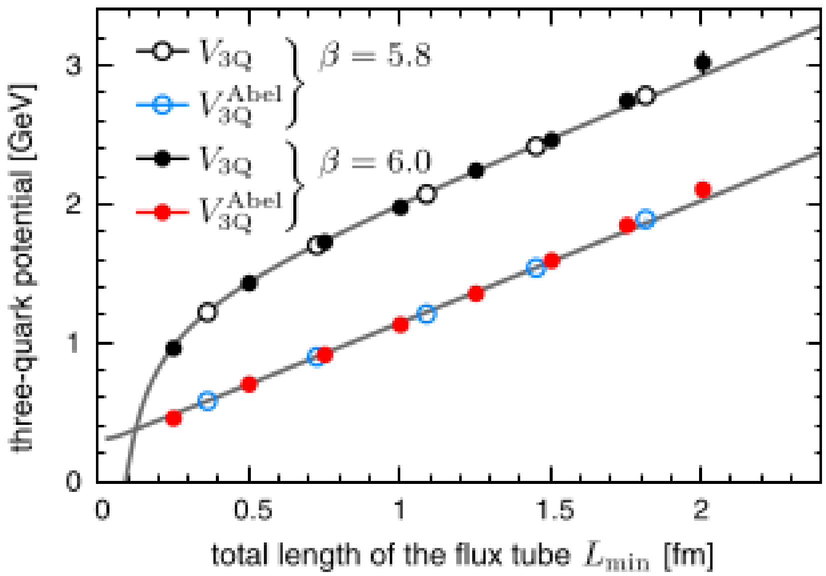
Sakumichi & Suganuma results

β = 5.8 on 16^3x32;

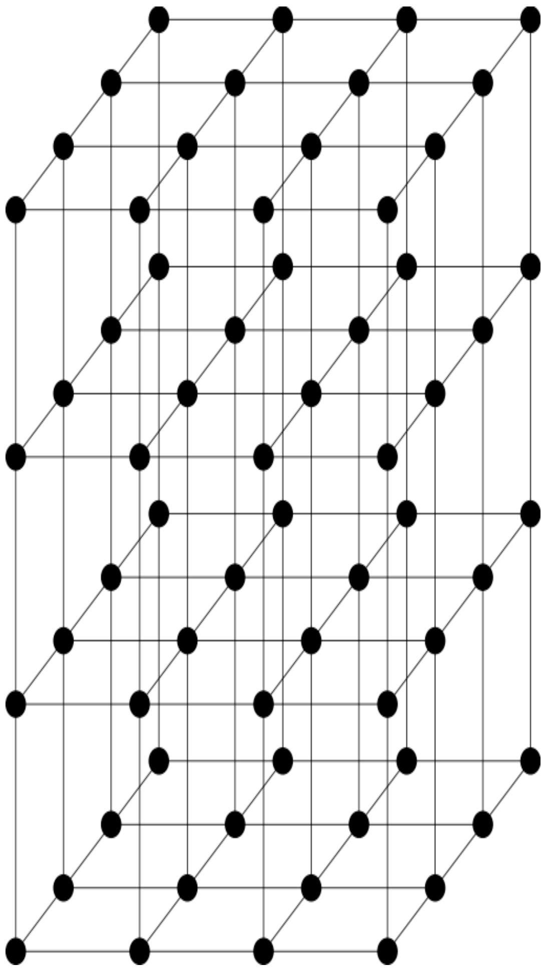
2000 gauge configurations

β = 6.0 on 20^3x32

1000 gauge configurations



3. Koma & Koma (2017)



24 x 24 x 24

Komas use a "multilevel algorithm" → all results from one gauge configuration!

Koma & Koma 3-source (quark) configurations include:

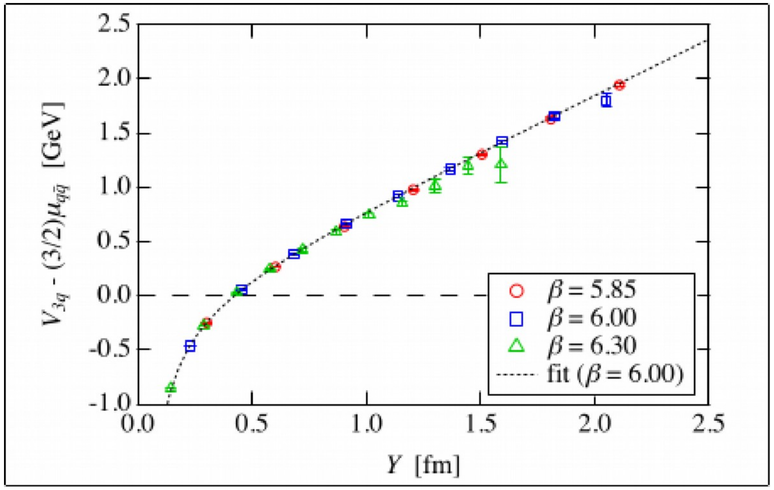
Isosceles triangles

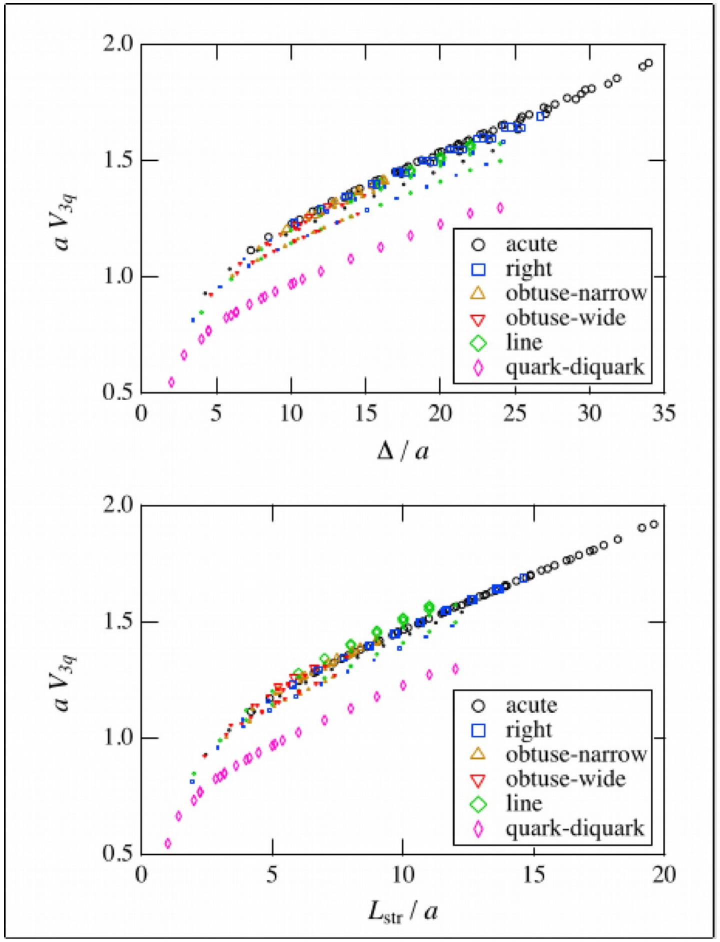
Right-angled triangles

Many collinear and close to collinear geometries

Komas' results

- Komas' final results:
- L_str is the length of the Y string
- Different quark configurations separated – still little to discriminate between Delta and Y-string





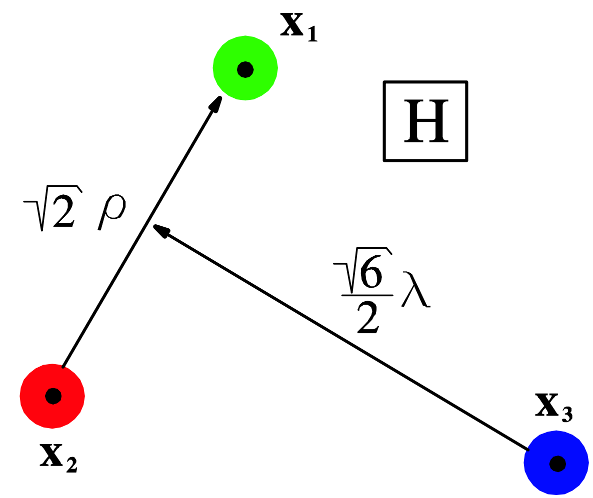
3. Quick introduction to 3-body hyper-spherical coordinates and dynamical O(2) symmetry of Y-string

Required Properties of 3-body potentials:

- **Invariances/symmetries:**
- **Translations: must be independent of CM variable; can depend only on two relative vectors!**
- **Rotations: must be only a function of 3 scalar products of two relative vectors!**
- **Permutation symmetry of 3 particles (non-trivial implications)**
- **Smoothness requirement: the gradient of the potential (force) must be continuous everywhere, except, perhaps, at 3 isolated singular points**

Jacobi and hyper-spherical variables

- Jacobi relative coordinate vectors $(\vec{\rho}, \vec{\lambda})$
- They form the basis of two-dimensional representation of permutation group S_3



$$\vec{\rho} = \frac{1}{\sqrt{2}}\left(\vec{r}_1 - \vec{r}_2\right)$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}\left(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3\right)$$

Hyper-spherical variables

- The hyper-radius R sets the overall scale ("size") of the triangle.
- Two hyper-angles determine the shape of the triangle, or a point on the shape sphere.

$$R^2 = \vec{\rho}^{\,2} + \vec{\lambda}^{2}$$

$$2\chi = \tan^{-1}\left(\frac{2\,\vec{\rho}\cdot\vec{\lambda}}{\vec{\rho}^{\,2} - \vec{\lambda}^{2}}\right)$$

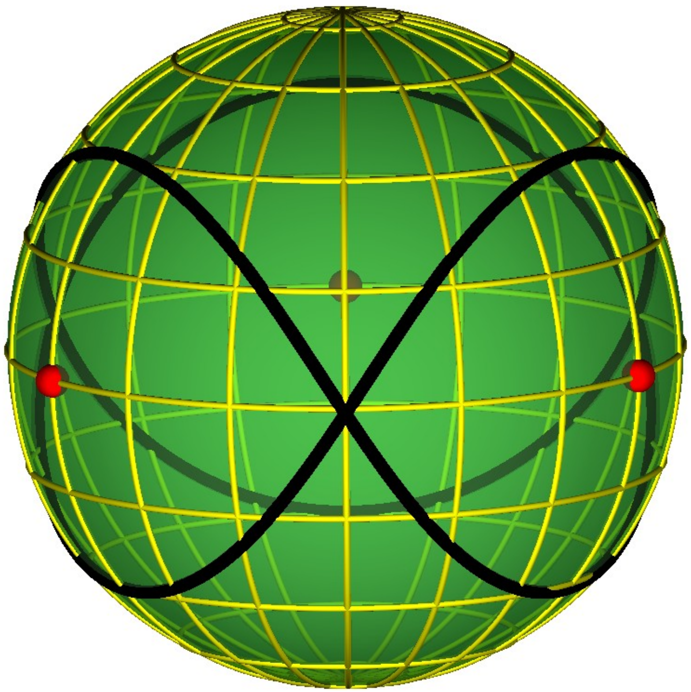
$$\theta = \cos^{-1}\left(\frac{\vec{\rho}\cdot\vec{\lambda}}{\rho\lambda}\right)$$

The shape-space sphere

$$X=\left(\frac{2\vec{\rho}\cdot\vec{\lambda}}{\vec{\rho}^{\,2}+\vec{\lambda}^{2}}\right)$$

$$Y=\left(\frac{\vec{\rho}^{\,2}-\vec{\lambda}^{2}}{\vec{\rho}^{\,2}+\vec{\lambda}^{2}}\right)$$

$$Z=\left(\frac{2\left(\vec{\rho}\times\vec{\lambda}\right)_z}{\vec{\rho}^{\,2}+\vec{\lambda}^{2}}\right)$$

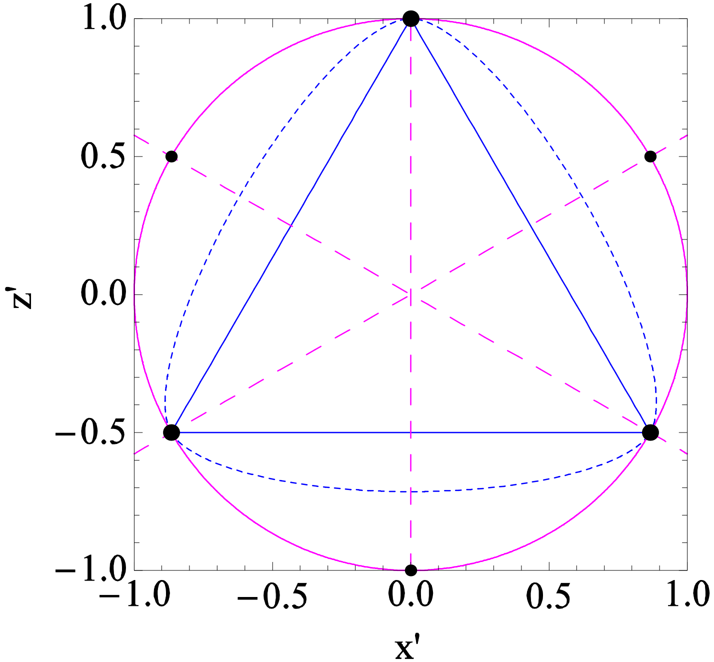


- Define a unit sphere with (X,Y,Z) coordinates
- What we showed before was a projection form infinity above the North Pole.
- Red points correspond to equi-distant collinear configurations (“Euler” points)
- The solid black line is the boundary between integrations regions (plus its “reflection” for triangles of opposite orientation).

Permutation-symmetric hyper-angles

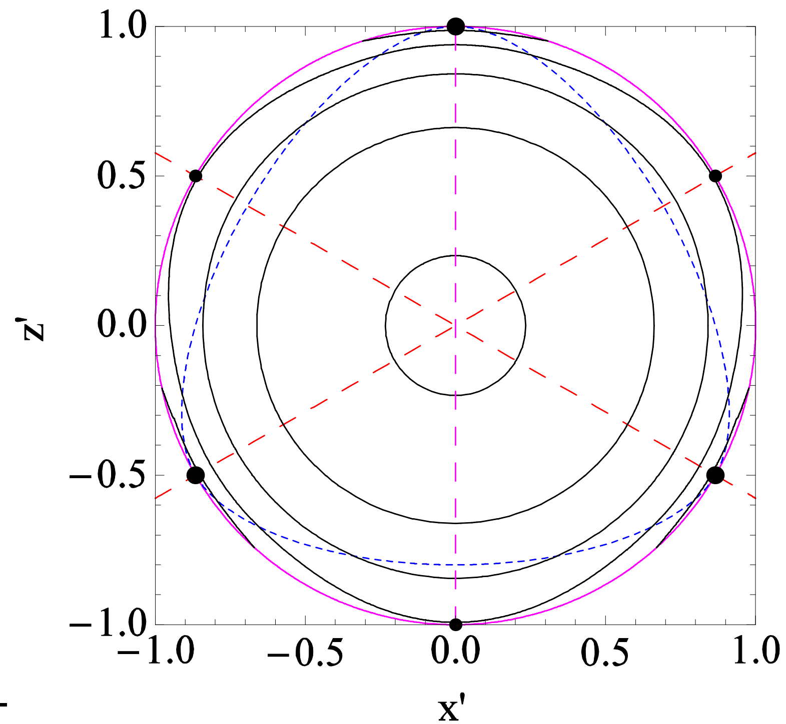
- Define the new (permutation-symmetric) hyper-angles by view from "infinity above the North Pole".

- The discrete symmetry group consisting of three reflections (about the vertical and two slanted (magenta dashes) axes) and two rotations through $^{2\pi}\!/_{3}$ correspond to 5 elements of the permutation group S_3 of three quarks.



The Y-string in terms of new hyper-angles

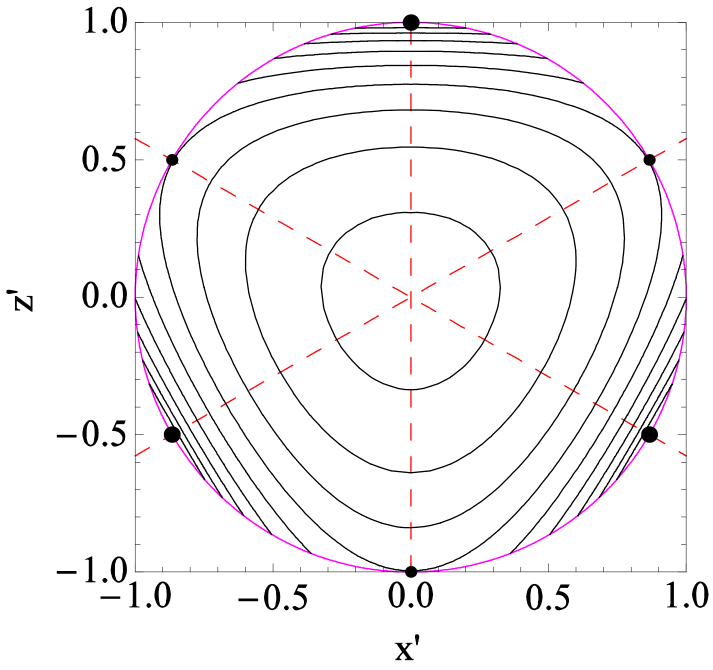
- The contour plot of the Y-string potential consists of concentric circles (solid black)
- The Y-string potential is axially symmetric under rotations: not a function of the (new) hyperangle ***Φ***



$$V_Y(R,\alpha,\varphi)=\sigma R\sqrt{\frac{3}{2}\left(1+|\cos\alpha|\right)}$$

The Delta-string potential & hyper-angles

- This O(2) is not a symmetry of sums of two-body potentials, like the Δ-string.
- The Δ-string potential has only three (discrete) reflection symmetries
- Other sums of two-body potentials - e.g. Coulomb - have the same symmetry as Delta and similar contours
- This difference between the Δ and Y-strings must be detectable on the lattice!



Dynamical O(2) symmetry of the Y-string

- Consequently there is a new constant of motion G that is associated with "hyper-rotations" in the $\left(\vec{\rho},\vec{\lambda}\right)$ plane
- This G is the "hyper-angular momentum" conjugate to the new hyper-angle *Φ*.
- Permutation group S3 is a discrete subgroup of this O(2) -> G must be a good quantum number of h.s. harmonics
- How and where does this O(2) symmetry fit in?

$$G = \boldsymbol{\lambda} \cdot \mathbf{p}_\rho - \boldsymbol{\rho} \cdot \mathbf{p}_\lambda,$$

$$\delta \boldsymbol{\rho} = \varepsilon \boldsymbol{\lambda}$$

$$\delta \boldsymbol{\lambda} = -\varepsilon \boldsymbol{\rho}.$$

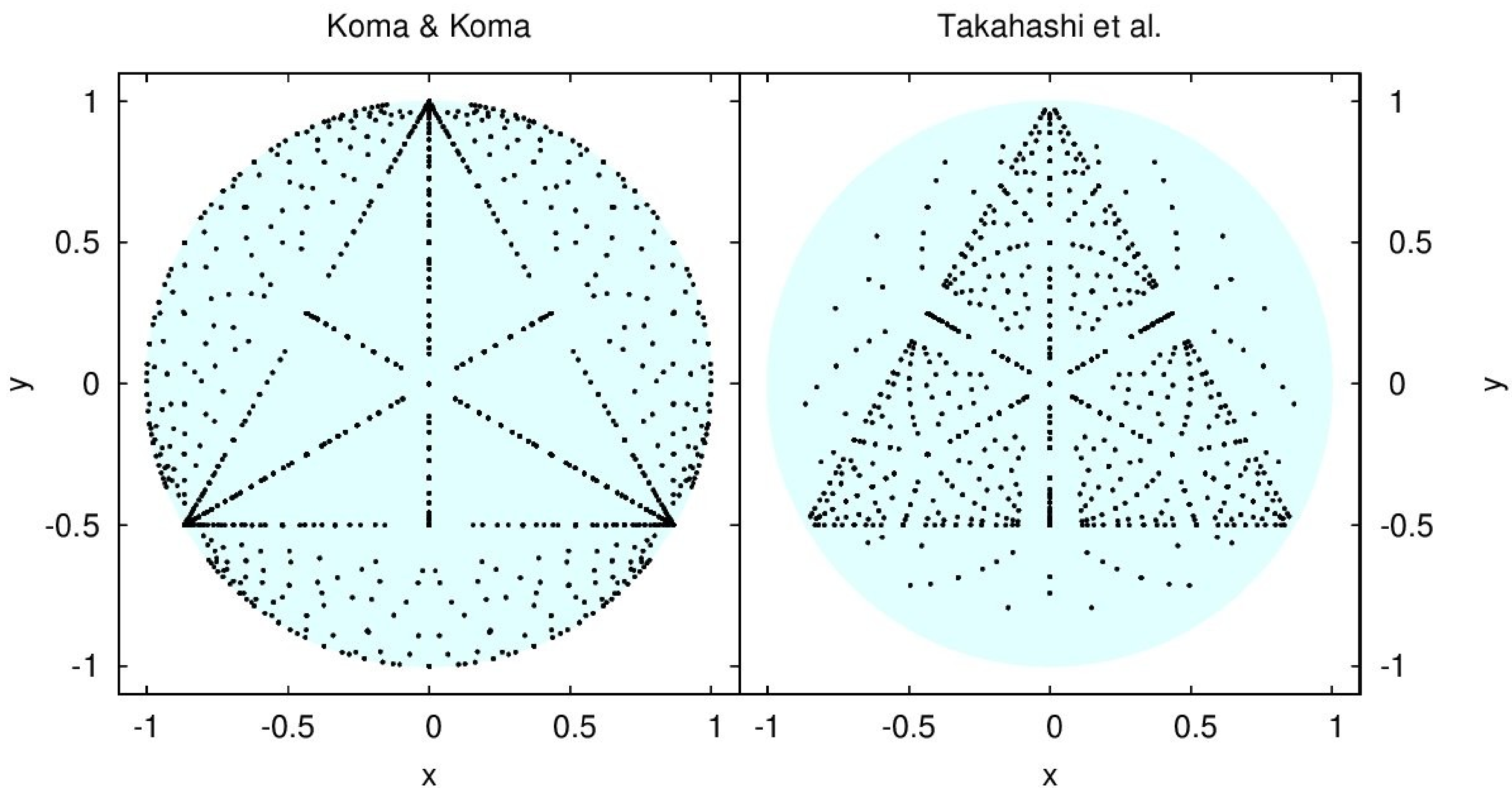
$$G = \frac{L}{2}\cos\alpha - \frac{m}{4}\left(R\sin\alpha\right)^2 \dot{\varphi}$$

$$\varphi = \tan^{-1}\left(\frac{2\vec{\rho}\cdot\vec{\lambda}}{\vec{\rho}^2 - \vec{\lambda}^2}\right)$$

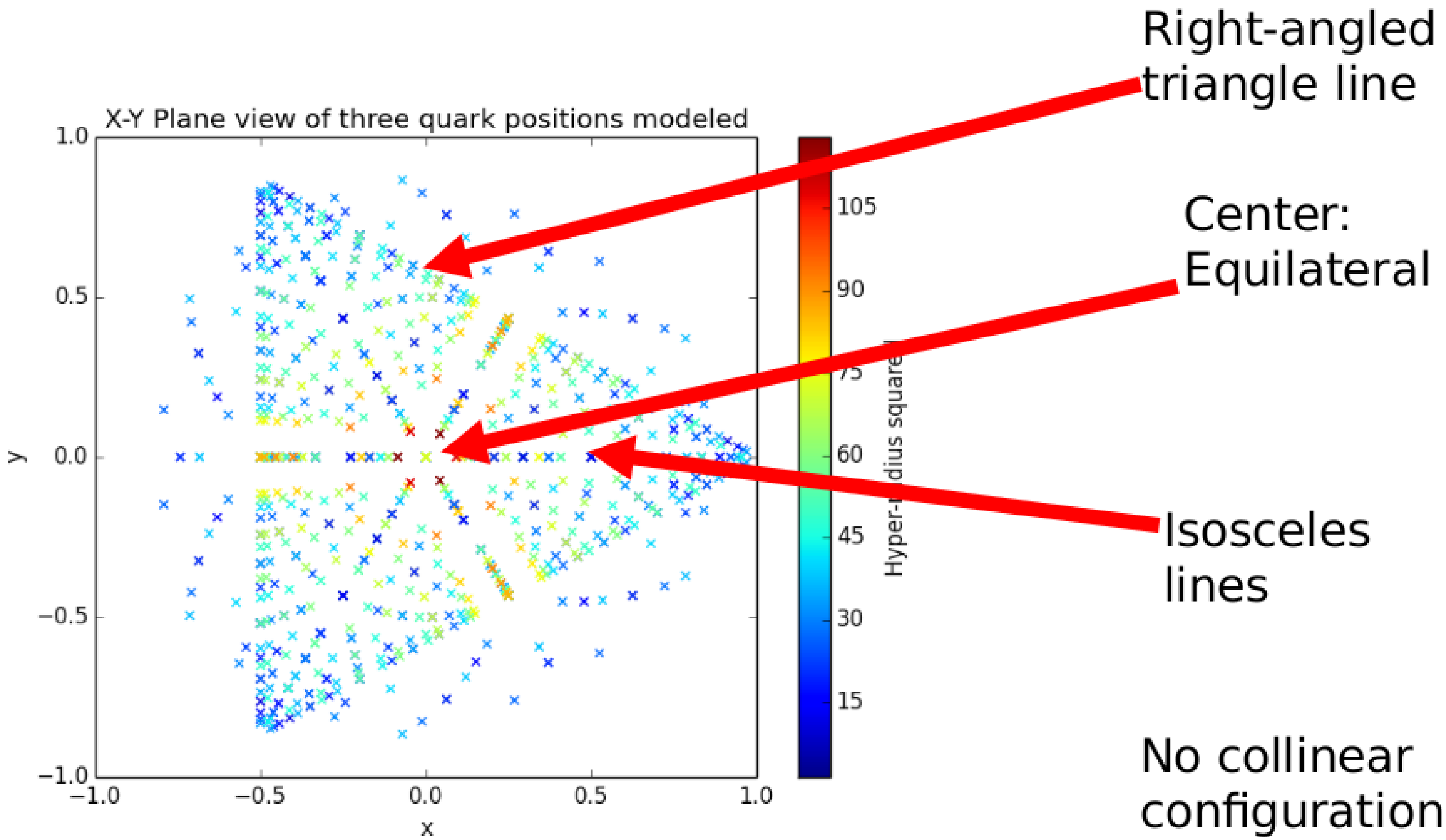
$$\alpha = \cos^{-1}\left(\frac{2\left(\vec{\rho}\times\vec{\lambda}\right)_z}{\vec{\rho}^2 + \vec{\lambda}^2}\right)$$

4. Lattice QCD 3-body potentials in terms of hyper-spherical variables

Koma vs. Takahashi geometries



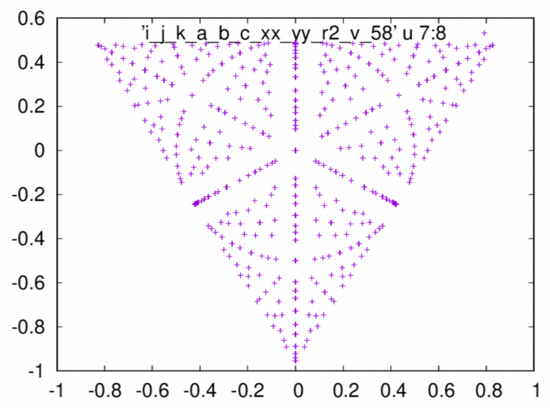
Takahashi data – Hyper-radius as a function of hyper-angles:



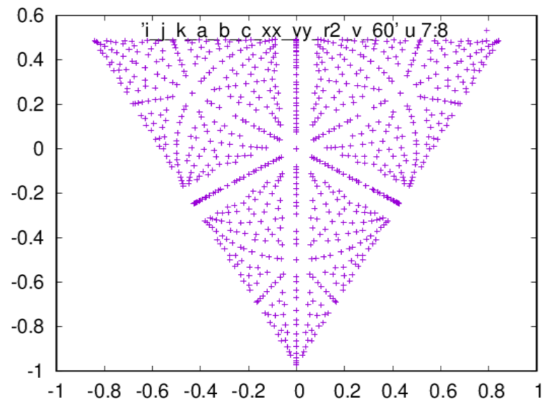
No collinear configurations! (equator of sphere)

Sakumichi & Suganuma data

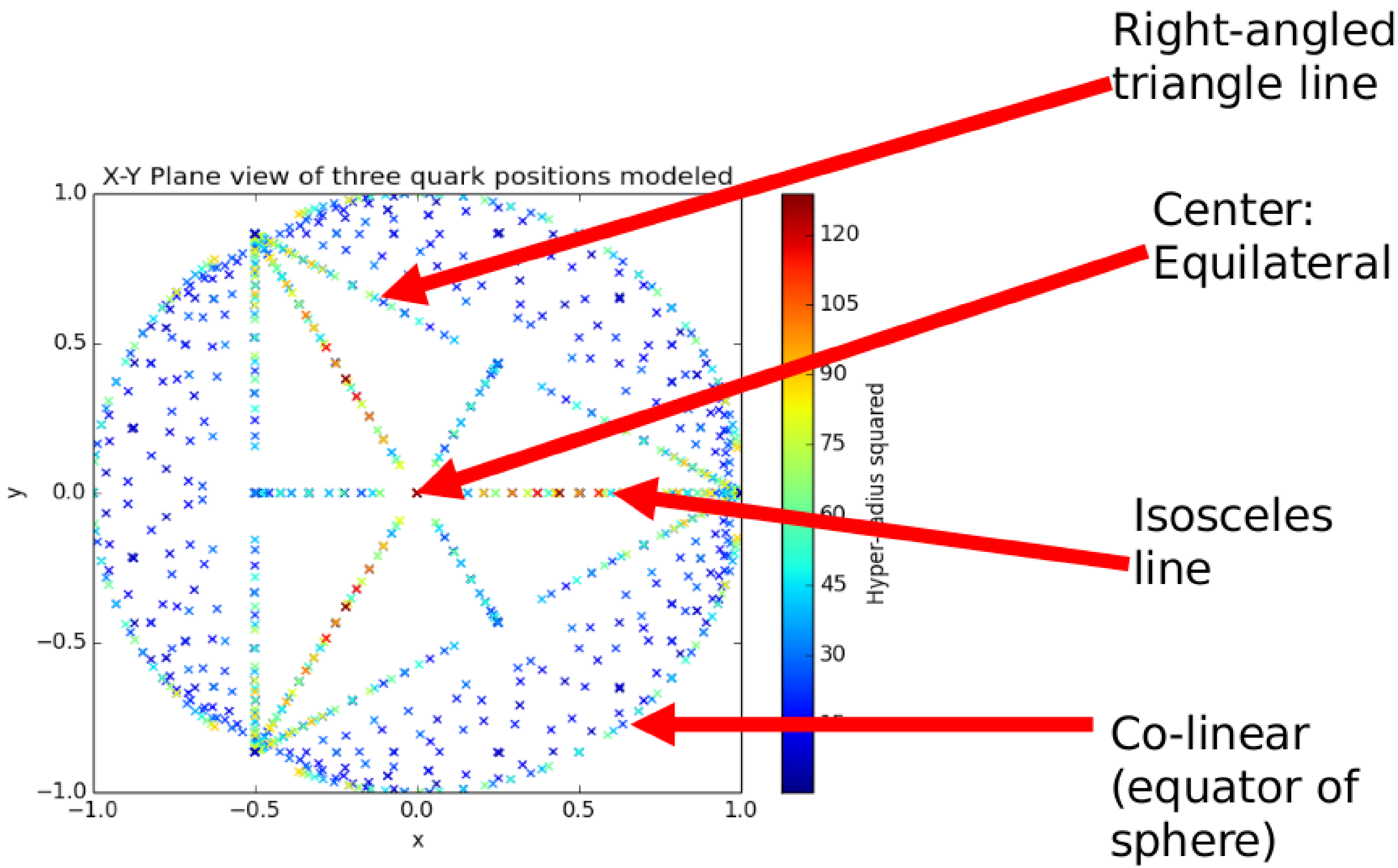
β = 5.8 on 16^3x32; with 2000 gauge configurations



β = 6.0 on 20^3x32 with 1000 gauge configurations



Koma data – Hyper-radius as a function of hyper-angles:

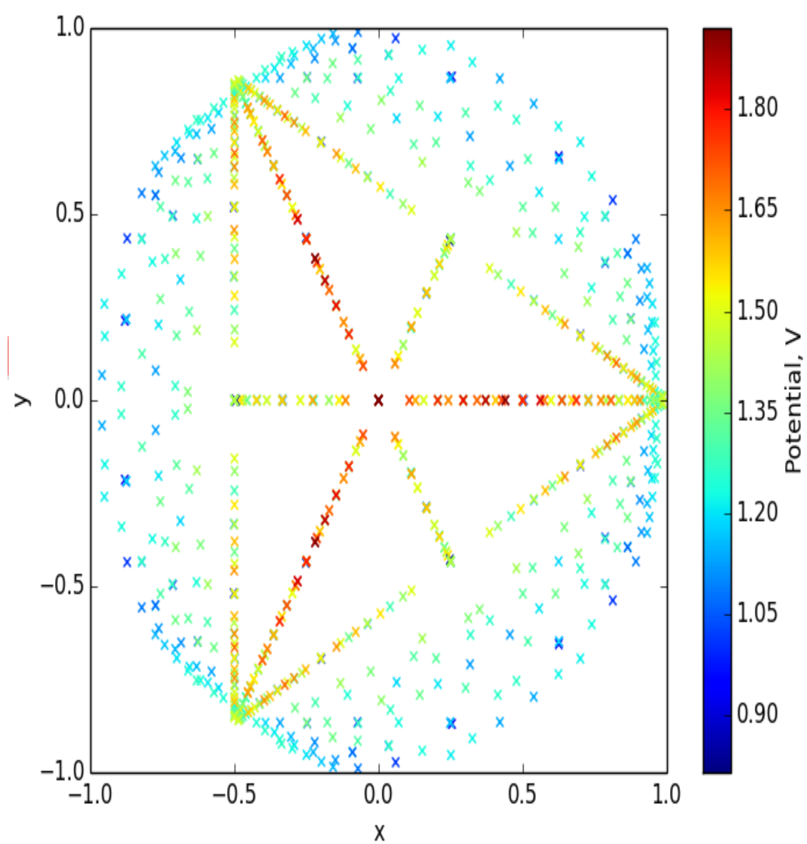


Hyper-spherical Coordinates

Table of Koma & Koma data:

No.	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$	r_1	r_2	r_3	θ_{max}	V_{eq}
1	(2,0,0)	(1,0,0)	(0,0,0)	1.00	2.00	1.00	180	0.8478
2	(3,0,0)	(1,0,0)	(0,0,0)	1.00	3.00	2.00	180	0.9914
3	(4,0,0)	(1,0,0)	(0,0,0)	1.00	4.00	3.00	180	1.0786
4	(4,0,0)	(2,0,0)	(0,0,0)	2.00	4.00	2.00	180	1.1203
5	(5,0,0)	(1,0,0)	(0,0,0)	1.00	5.00	4.00	180	1.1457
6	(5,0,0)	(2,0,0)	(0,0,0)	2.00	5.00	3.00	180	1.2017
7	(6,0,0)	(1,0,0)	(0,0,0)	1.00	6.00	5.00	180	1.2044
8	(6,0,0)	(2,0,0)	(0,0,0)	2.00	6.00	4.00	180	1.2661
9	(6,0,0)	(3,0,0)	(0,0,0)	3.00	6.00	3.00	180	1.2803
10	(7,0,0)	(1,0,0)	(0,0,0)	1.00	7.00	6.00	180	1.2588
11	(7,0,0)	(2,0,0)	(0,0,0)	2.00	7.00	5.00	180	1.3233
12	(7,0,0)	(3,0,0)	(0,0,0)	3.00	7.00	4.00	180	1.3434
13	(8,0,0)	(1,0,0)	(0,0,0)	1.00	8.00	7.00	180	1.3108
14	(8,0,0)	(2,0,0)	(0,0,0)	2.00	8.00	6.00	180	1.3769
15	(8,0,0)	(3,0,0)	(0,0,0)	3.00	8.00	5.00	180	1.3998
16	(8,0,0)	(4,0,0)	(0,0,0)	4.00	8.00	4.00	180	1.4055
17	(9,0,0)	(1,0,0)	(0,0,0)	1.00	9.00	8.00	180	1.3612
18	(9,0,0)	(2,0,0)	(0,0,0)	2.00	9.00	7.00	180	1.4283
19	(9,0,0)	(3,0,0)	(0,0,0)	3.00	9.00	6.00	180	1.4527
20	(9,0,0)	(4,0,0)	(0,0,0)	4.00	9.00	5.00	180	1.4614
21	(10,0,0)	(1,0,0)	(0,0,0)	1.00	10.00	9.00	180	1.4108
22	(10,0,0)	(2,0,0)	(0,0,0)	2.00	10.00	8.00	180	1.4786
23	(10,0,0)	(3,0,0)	(0,0,0)	3.00	10.00	7.00	180	1.5040
24	(10,0,0)	(4,0,0)	(0,0,0)	4.00	10.00	6.00	180	1.5142
25	(10,0,0)	(5,0,0)	(0,0,0)	5.00	10.00	5.00	180	1.5171
26	(11,0,0)	(1,0,0)	(0,0,0)	1.00	11.00	10.00	180	1.4591
27	(11,0,0)	(2,0,0)	(0,0,0)	2.00	11.00	9.00	180	1.5279
28	(11,0,0)	(3,0,0)	(0,0,0)	3.00	11.00	8.00	180	1.5540
29	(11,0,0)	(4,0,0)	(0,0,0)	4.00	11.00	7.00	180	1.5654
30	(11,0,0)	(5,0,0)	(0,0,0)	5.00	11.00	6.00	180	1.5697
31	(12,0,0)	(1,0,0)	(0,0,0)	1.00	12.00	11.00	180	1.4987
32	(12,0,0)	(2,0,0)	(0,0,0)	2.00	12.00	10.00	180	1.5740

Koma & Koma data – Potential V as a function of hyper-angles:



Analysis of lattice QCD data: the basic Ansatz

The QCD 3-static-quarks potential is expected to contain a Coulomb term, a confining term – both with unknown coupling constants - and an unknown additive constant C, [see H. J. Rothe, “Lattice Gauge Theories: An Introduction”, (3rd edition, 2005), or C. Gattringer, and C. B. Lang, “Quantum Chromodynamics on the Lattice - An Introductory Presentation” (2010)]

$$\boxed{V = \frac{-A}{R} + BR + C}$$

Each term’s degree of homogeneity is known (-1,0,+1), so one can factor out each hyper-radial (R) dependence.

This means that there are 2 (possibly 3, only 1 of which – the Coulomb -- is known) shape-dependent functions A,B,C with 3 corresponding unknown multiplicative constants.

If one has sufficiently many data points at one and the same triangle shape, but different sizes (R’s), one can determine the multiplicative constants A,B,C.

There are only two such configurations in both data sets: 1) equilateral, 2) right-angled isosceles.

Analysis of lattice QCD data: the basic Ansatz

$$\boxed{V = \frac{-A}{R} + BR + C}$$

Lattice data potential
QCD Coulomb interaction – QCD coupling constant alpha_C unknown
Confining potential – string tension sigma and shape dependence unknown
Additive constant C unknown

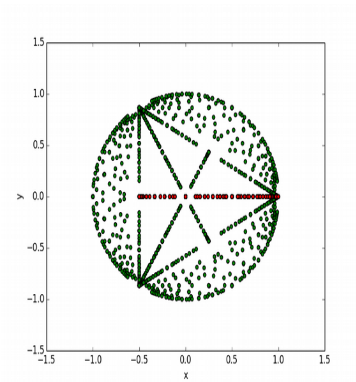
Aim: to remove QCD Coulomb interaction and the hyper-radial dependence of confining potential - thus isolate the hyper-angular dependence of confining potential

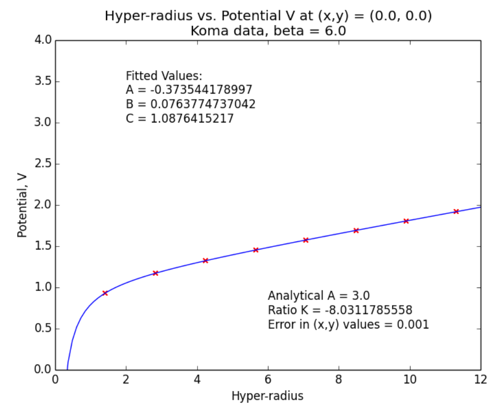
We have multiple data points with different R at these two configurations:

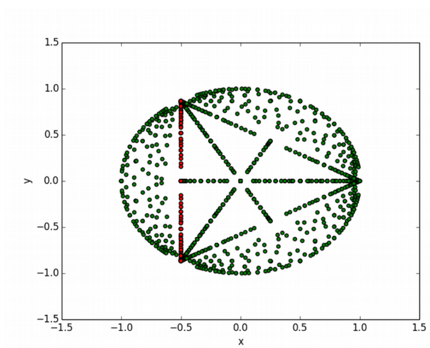
1) equilateral,

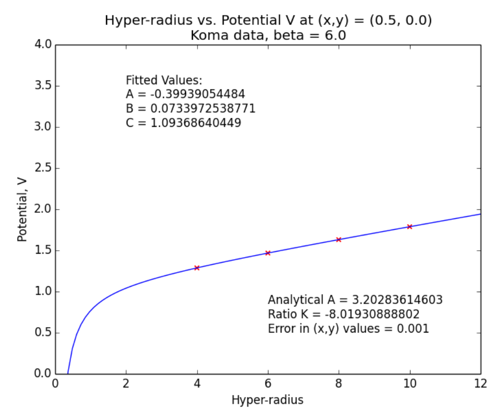
2) right-angled isosceles.

Fitting Komas' data at two points on the shape sphere









Analysis of lattice data

There are only two such configurations in both data sets: 1) equilateral, 2) right-angled isosceles.

The potential takes the form: $V = \frac{-A}{R} + BR + C$

Do a least-square fit for constants A, B, C at each point.

(x,y)	A_{fitted}	B_{fitted}	C_{fitted}	K	δK
(0, 0)	-0.37354418	0.0763775	1.0876415	-8.0311786	-0.07%
(0, -0.5)	-0.3993905	0.07339725	1.0936864	-8.019309	+0.07%

The percentage difference in K is much larger for the Takahashi data sets: for $\beta = 5.8$ and $\beta = 6.0$ data sets they are 22.5 % and 18.1 %, respectively.

Hyper-angular analysis of lattice data

Having done a least-square fit for constants A, B, C, at two different geometries, define K, the ratio of fitted (Coulomb) A and theoretical A – so as to remove the Coulomb potential from the lattice total.

$$K = \frac{A_{analytical}}{A_{fitted}} = \frac{1}{\alpha_S}$$

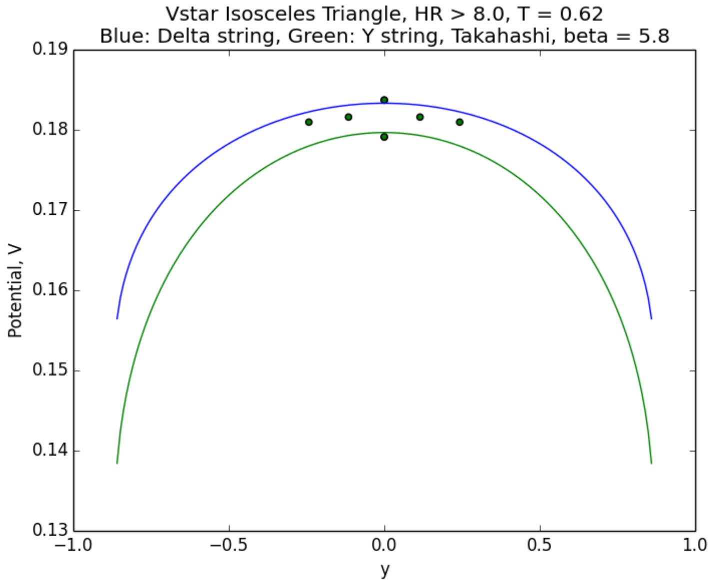
$$\frac{A_K(\alpha,\phi)}{R} = \frac{1}{K}\left(\frac{1}{a(\alpha,\phi,R)} + \frac{1}{b(\alpha,\phi,R)} + \frac{1}{c(\alpha,\phi,R)}\right)$$

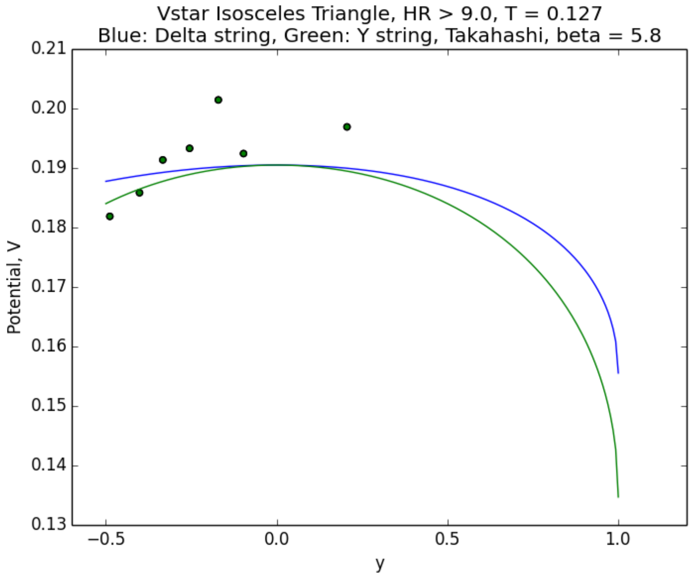
If K is constant, then one may define the hyper-angular part of the confining potential:

$$V^* = \frac{1}{R}\left(V + \frac{A}{R} - C\right) = B$$

Which is independent of hyper-radius.

1. Results: Takahashi (beta=5.8)



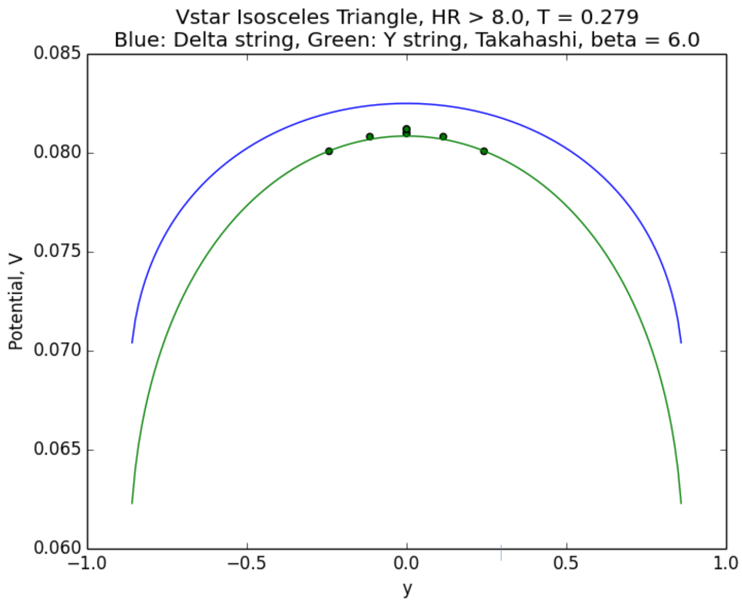


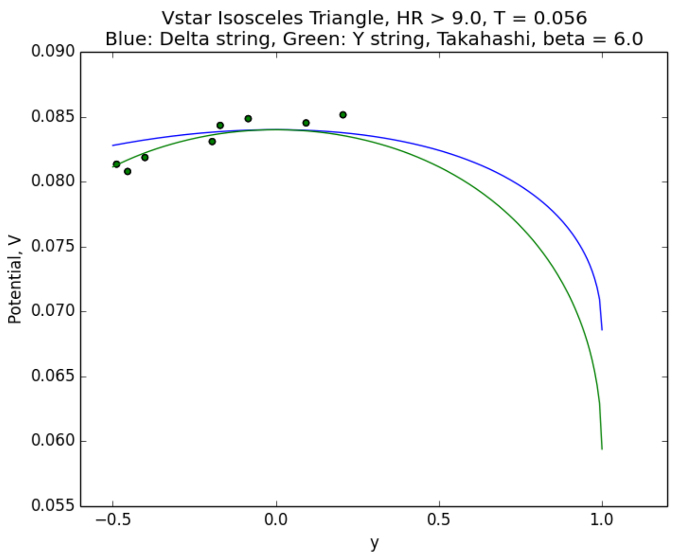
All these fit analyses support the *Y Ansatz*

No sign of agreement with Y-, or Delta string!
(wide scatter of data in the isosceles configuration)

The percentage difference in K is much larger for the Takahashi data sets: for $\beta = 5.8$ and $\beta = 6.0$ data sets they are 22.5 % and 18.1 %, respectively.

Results:Takahashi (beta = 6.0)





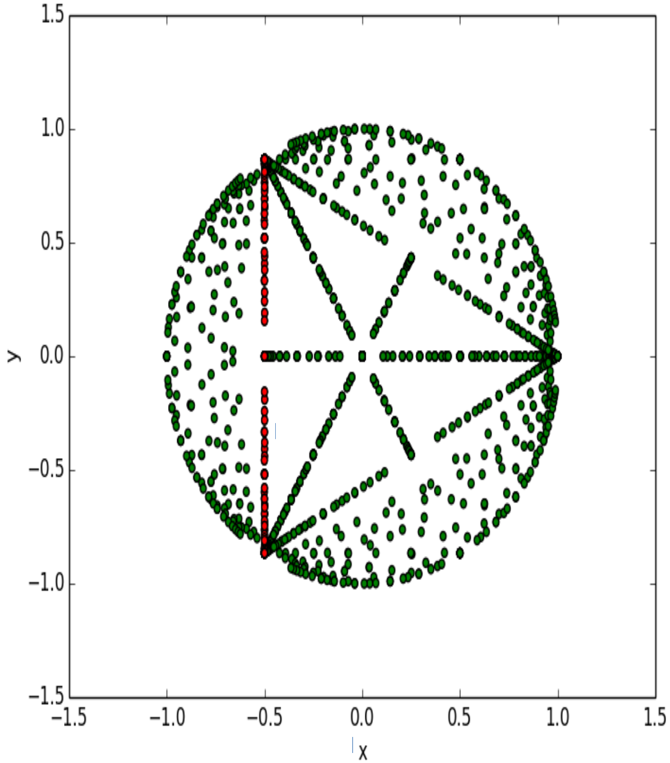
The only example of agreement with the Y -string prediction – insufficient for such a strong claim!

All these fit analyses support the *Y Ansatz*

2. Results:Komas

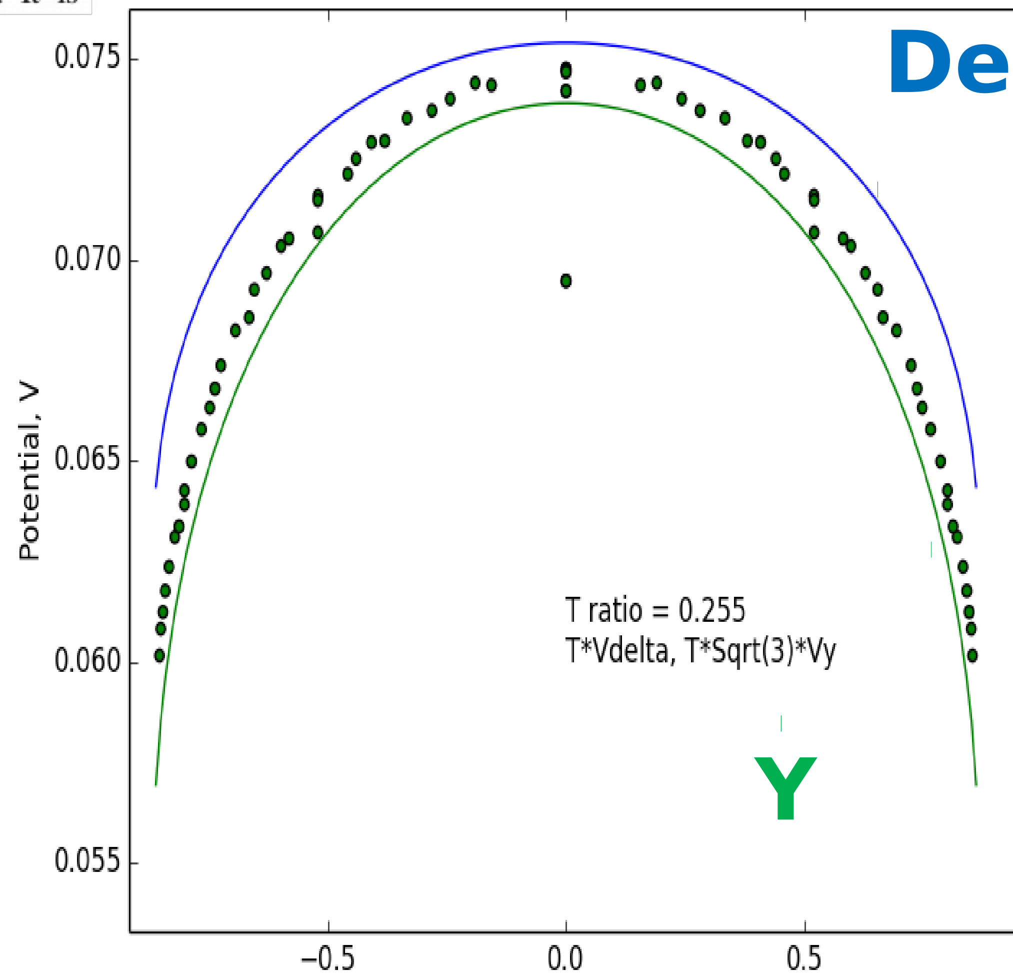
cannot answer here whether or not both functional forms can change continuously depending on the movement of quarks, which should be clarified in future study. It is

V* along highlighted line



Right-angled triangles

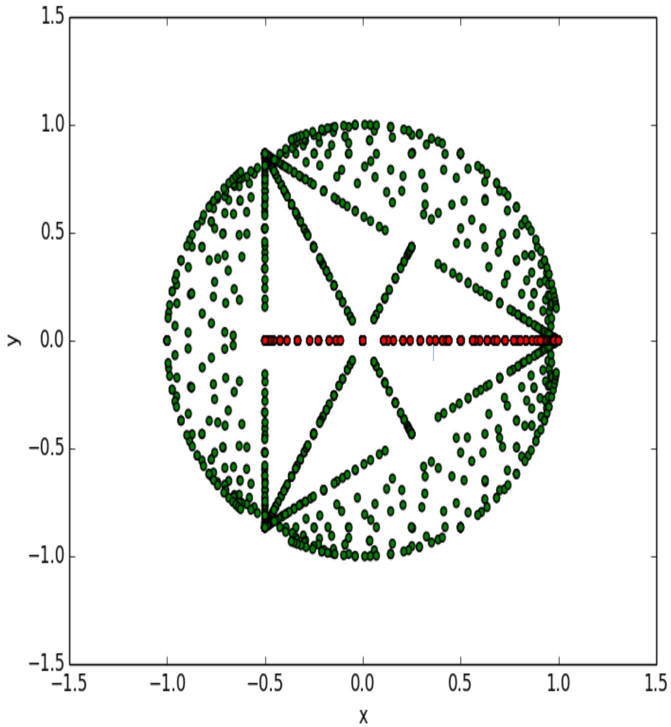
Delta



Some combination of Y & Delta

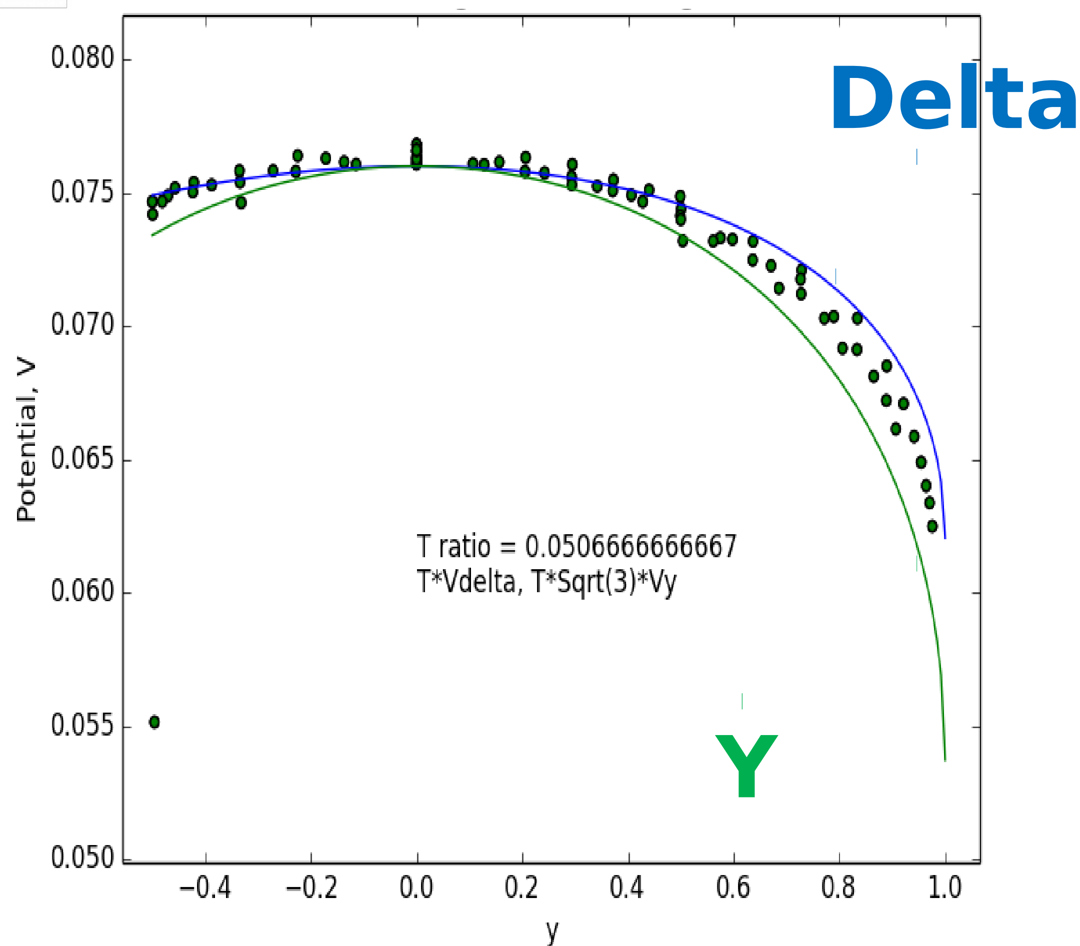
Results: Komas

cannot answer here whether or not both functional forms can change continuously depending on the movement of quarks, which should be clarified in future study. It is



Isosceles triangles

V* along highlighted line



Some combination of Y & Delta

5. Error analysis – zeroth attempt

Error analysis of lattice QCD data

Like any experiment, the lattice QCD calculations suffer from two kinds of uncertainties - errors, for short -

1) statistical;

2) systematic

The statistical ones can be estimated by repeating the simulation N times, whereas the systematic ones must be guessed at (or estimated by changing the set-up of the simulation). Systematic errors can be due to:

1) finite lattice size effects;

2) rotational symmetry breaking;

3) translational symmetry breaking;

4) assumed 3-body Ansatz.

Estimate of error bars

Koma & Koma estimated statistical error in equilateral geometry as <0.8%

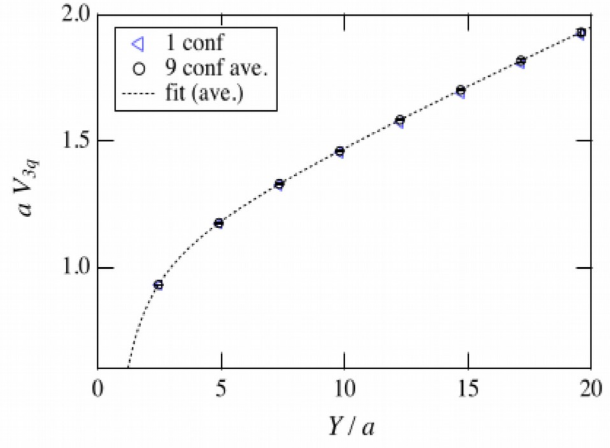


FIG. 6. The three-quark potentials of the equilateral triangle geometries at $\beta = 6.00$ obtained from one gauge configuration and from the average of 9 gauge configurations as a function of Y. The dotted line represents the fit curve to the averaged potential.

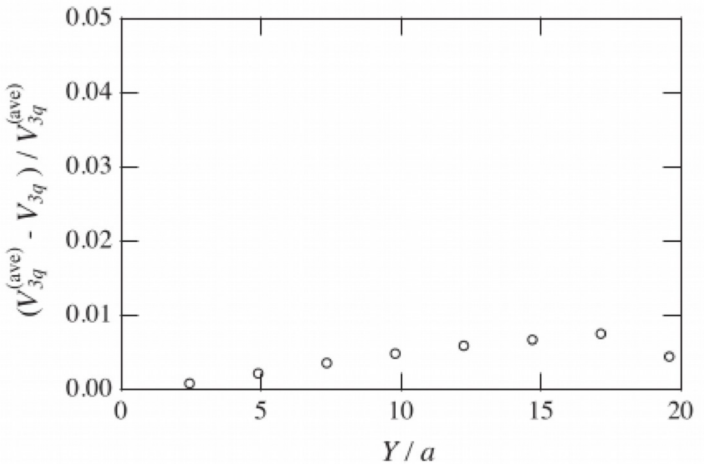


FIG. 7. The relative error between the two potentials in Fig. 6, $(V_{3q}^{(\mathrm{ave})} - V_{3q})/V_{3q}^{(\mathrm{ave})}$.

Estimate of systematic error bars

Koma & Koma noticed significant variation of Coulomb coupling "constant" up to 26%

figure). We find that $A_{3q}^{(\mathrm{R})}$ is significantly smaller than $A_{q\bar{q}}/2 = 0.170$ about 26%, namely,

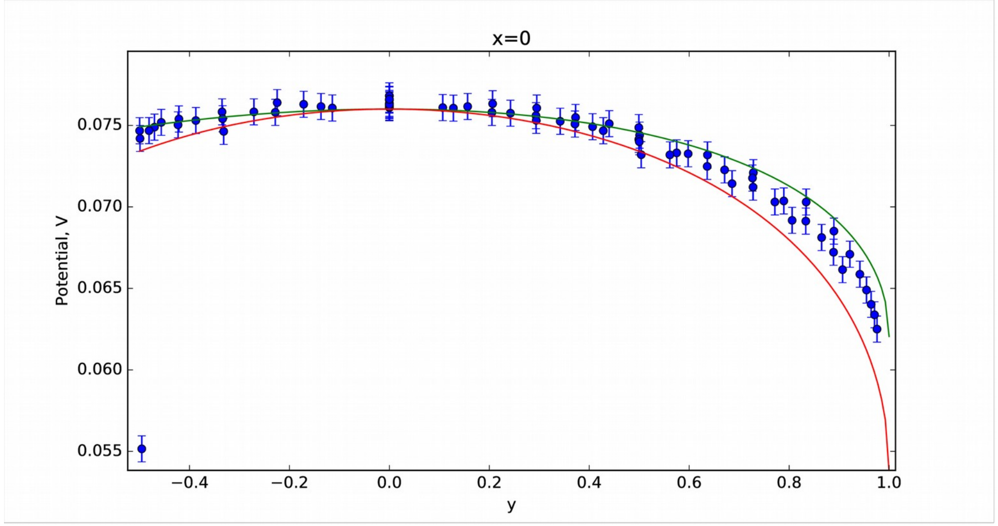
$$A_{3q}^{(\mathrm{R})} = \frac{A_{q\bar{q}}}{2}(1 - 0.259),$$

This variation is on a lattice at a single value of beta! How can this be? Is our Coulomb+constant+ confinement Ansatz valid?

Just for illustration purposes: assign *ad hoc* uniform error bars

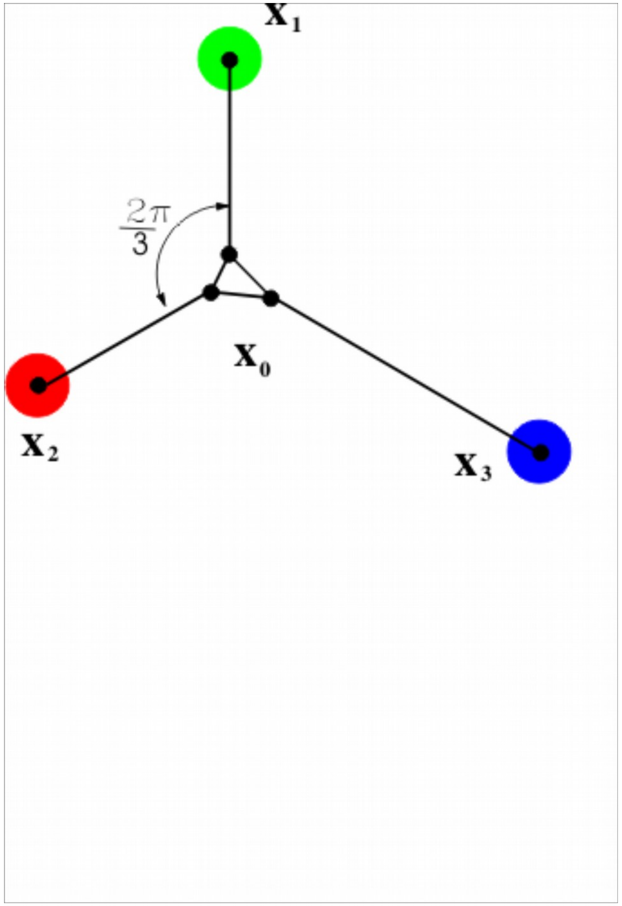
Assign equal error bars to all the points based on the spread of 8 values in the equilateral configuration (y=0)

The distinction between Y and Delta string becomes blurred



6. Possible interpretation

- The Δ and Y-strings have very different topologies - why are their potentials so close?
- (Many) people have suggested a Δ-Y hybrid configuration to allow for a smooth transition from one to another.
- But, it is forbidden! And when allowed, it is equivalent to the Y-string.

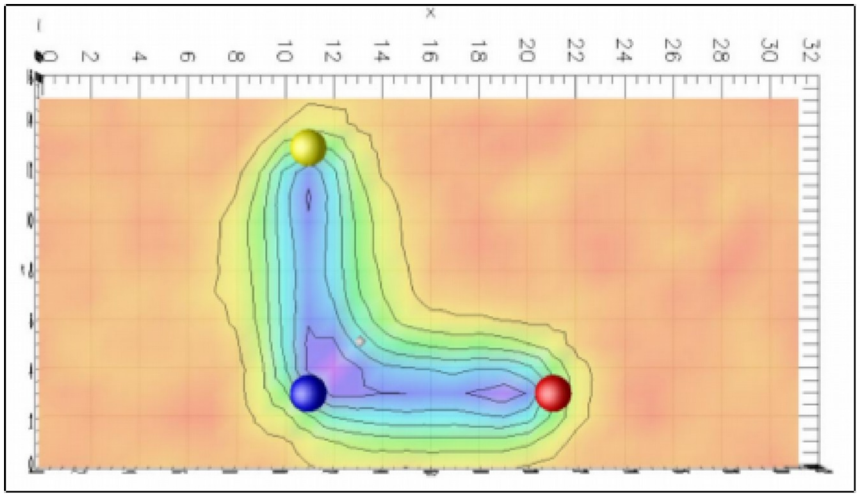


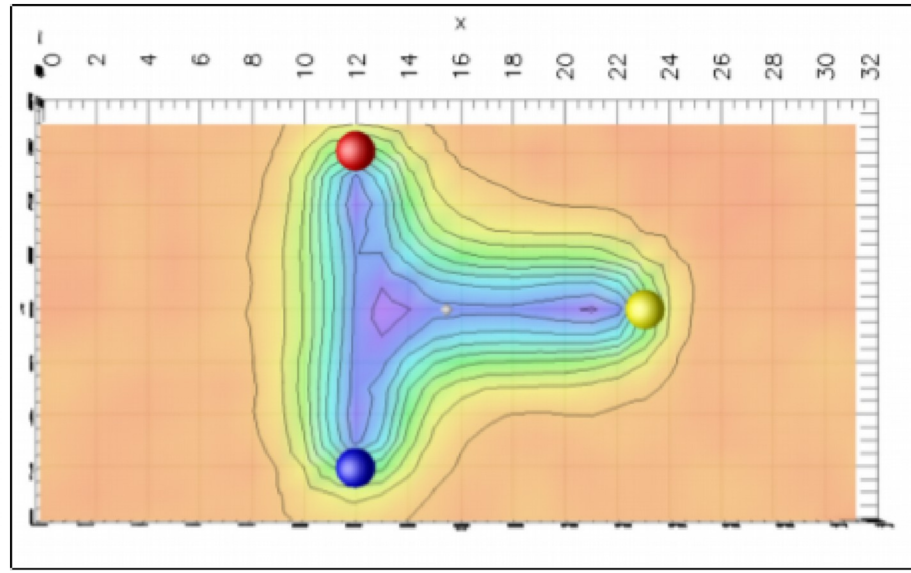
There is a "magical" numerical identity between the Delta and a linear combo of two types of Y-strings: the Fermat-Torricelli-Steiner (Y) and the barycenter (CM) junction ones.

$$V_{\mathrm{Y}} = \frac{1}{2}\left(V_{\mathrm{CM}} + \frac{1}{\sqrt{3}}V_{\Delta}\right) + \mathcal{O}(0.1\%)$$

$$V_{\mathrm{Y}} = \frac{\sigma}{2}\left(\sum_{i=1}^{3}|\mathbf{x_i} - \mathbf{x}_{\mathrm{CM}}| + \frac{1}{\sqrt{3}}\sum_{i<j}^{3}|\mathbf{x}_i - \mathbf{x}_j|\right) + \mathcal{O}(0.1\%)$$

This tells us that the values of “Delta string” potential can be reproduced with Y-string topology: Just shift the junction away from the Fermat-Torricelli point!
This can lead to different junctions (T- and L-) strings!





- Color flux-tube profiles in lattice QCD, Bissey et al. PRD76, 114512 (2007).
- This is an L- or T-string, not a Y-string - what kind of potential does this correspond to?

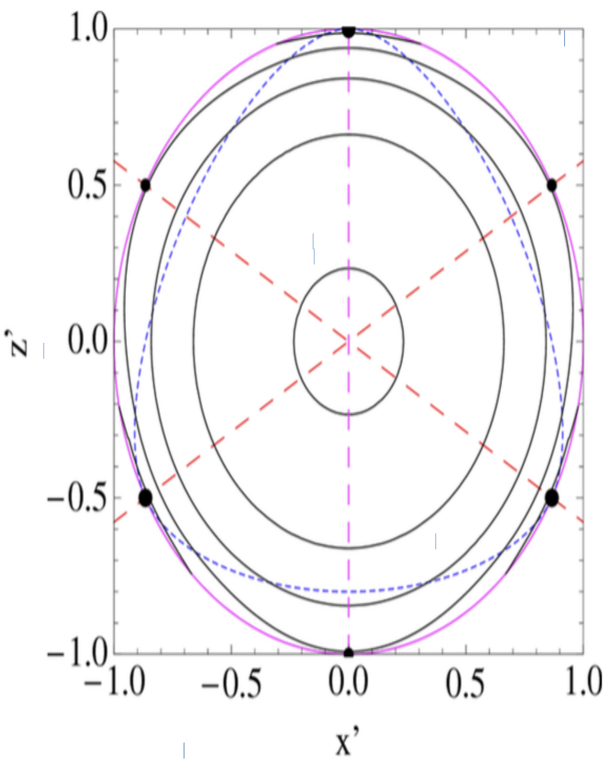
7. Recommendations for future work: How to test the O(2) dynamical symmetry!

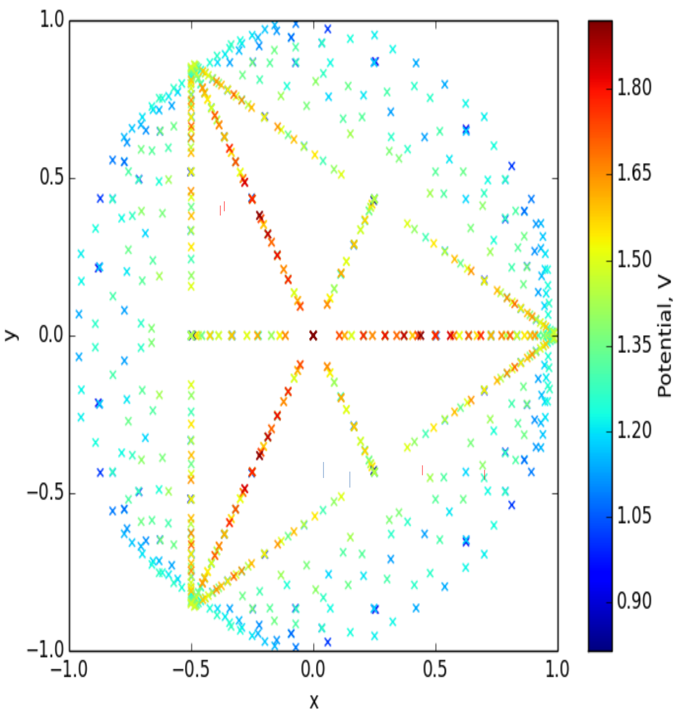
Dynamical O(2) symmetry of Y-string potential

Y potential on plane:

Equipotential circles

Koma & Koma potential on plane:



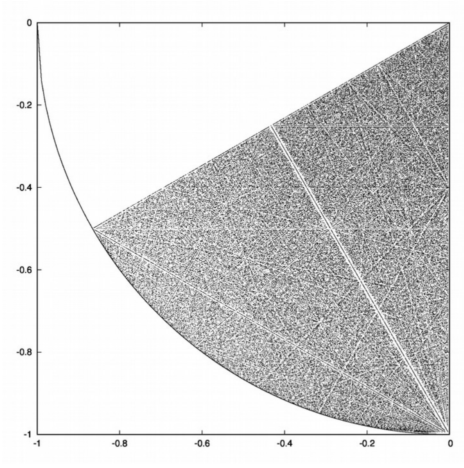


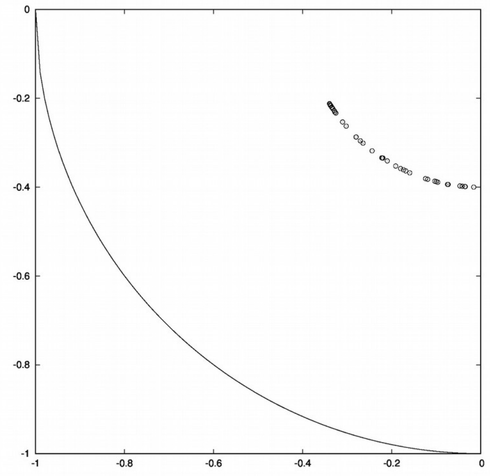
Symmetry slightly broken (“warping” of circles) near collision points

No present data points on circles!

- Calculate 3-quark potential at multiple values of size, and different orientations, but identical shapes of triangles, so as to check:
- Coulomb,
- finite-size,
- rotation and
- translation effects.
- Calculate potential in selected geometries with identical values of the Y-string – test the O(2) dynamical symmetry!

- Generate all possible 3-quark configurations on a lattice of a given size, with hyper-radii larger than some given value
- Select only those configurations that lie on circles in the X-Y plane, that have identical values of Y-string
- Calculate potential in these selected geometries – test the O(2) dynamical symmetry!





8. Summary and Outlook

Summary

- We analyzed lattice QCD results of Takahashi et al. (2002) and of Koma & Koma (2017) in terms of hyper-spherical coordinates.
- Neither choice of shapes and sizes of triangles allows a systematic test of the O(2) dynamical symmetry of the Y-string.
- Neither the Y-string, nor the Delta-string can describe all of the presently extant lattice data.
- We suggest new configurations for lattice QCD calculations.

Outlook

- We suggest new triangle configurations for lattice QCD calculations:

1) to test the size of systematic errors;

2) to test the dynamical O(2) symmetry.

- Must properly estimate the statistical and systematic errors!

Publications

- **V.D., T. Sato and M. Šuvakov, Eur. J. Phys. C 62, 383 (2009)**
- **V.D., T. Sato and M. Šuvakov, Phys. Rev. D 80, 054501 (2009)**
- **M. Šuvakov and V.D., Phys. Rev. E 83, 056603 (2011)**
- **V.D. and I. Salom, Acta Phys. Polon. Supp. 6, 905 (2013)**
- **V.D. and I. Salom, J. Math. Phys. 55, 082105 (2014)**
- **V.D. and I. Salom, Journal of Physics (JPCS) 670: 012044 (2016)**

Publications

- **V.D. and I. Salom, Springer Procs Math & Statistics vol. 191, 431 (2016)**
- **V.D. and I. Salom, Phys. Lett. A 380, 1904 (2016);**
- **I. Salom and V.D., Nucl. Phys. B920, 521 (2017)**
- **V.D. and I. Salom, Phys. Rev. D 97, 094011 (2018)**
- **I. Salom and V.D., "Quantum Theory and Symmetries 2", Springer Proc Math & Statistics vol. 255, 403-410 (2017)**
- **James Leech, M. Šuvakov and V.D., Acta Phys. Polon. Supp. 11, 435 (2018);**

My Collaborators

James Leech; Yuka, Miho and Yoshiaki Koma (2017)

Toru Sato

Milovan Šuvakov

Auxilliary Slides

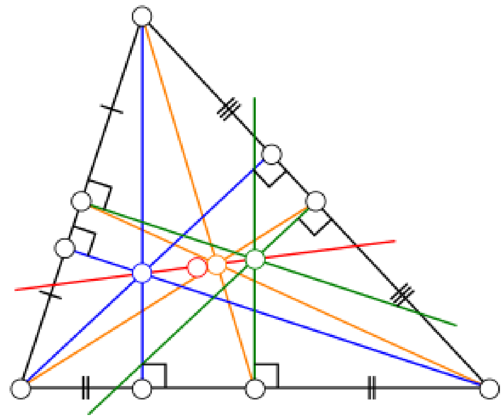
The Y *vs.* Δ-string?

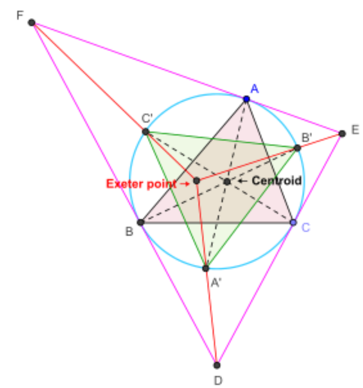
- Bonn group (Metsch, Petry et al.) EJPA 10 ('01) claim equivalence of Δ and Y-strings !?!
- Can one distinguish these two string potentials using (only) the baryon spectra?
- We shall show that, yes, there are clear differences, but only at K=3, 5.
- These differences are related to the dynamical O(2) symmetry of the Y-string.

We would like to note at this stage that we have tested the various radial dependencies (3), (4) and (6) in our Salpeter model. Our investigations, however, clearly showed that the structure of resulting spectra depends only slightly on the various radial dependencies chosen. It turned out that the slope parameter b can always be appropriately rescaled (as, *e.g.*, in eq. (5) with the factor f) to obtain almost the same spectrum for all three choices. We therefore prefer for our model the Δ-shape string potential rising linearly with $r_\Delta(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \sum_{i<j} |\mathbf{x}_i - \mathbf{x}_j|$ which, on the one hand, is favored by the most recent lattice studies anyway and, on the other hand, is also much easier to handle numerically. We found, however, that the structure of the resulting spectra depends much more on the Dirac structure chosen, which we shall consider next.

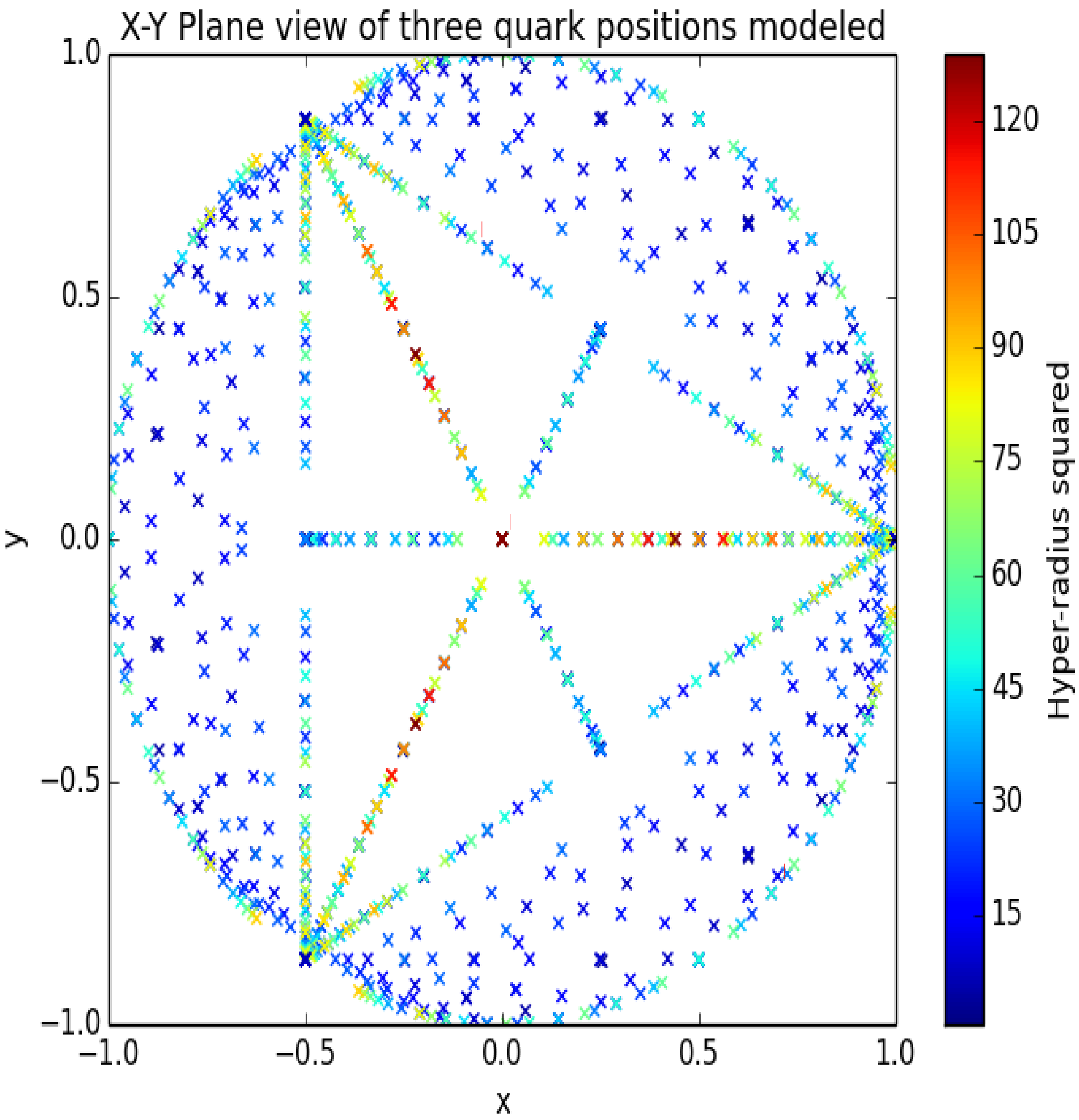
The gross features of the baryon resonances seem to

Euler line exists for any triangle that is not equilateral. It passes through all intrinsic points of the triangle, including the orthocenter, the circumcenter, the centroid, the Exeter point and the center of the nine-point circle of the triangle.





Koma data – Hyper-radius as a function of hyper-angles:



- Right-angled triangle line
  - Equilateral
  - Isosceles line
  - Co-linear (equator of sphere)