Theoretical study of Deeply Bound Pionic Atoms

(A cúrtain-ràiser)

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N. Ikeno (Tottori Univ., now in TEXAS, USA)
(Many Ikeno-san’s slides are used.)

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Nara Women's University

鳥取大学
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TEXAS A&M UNIVERSITY

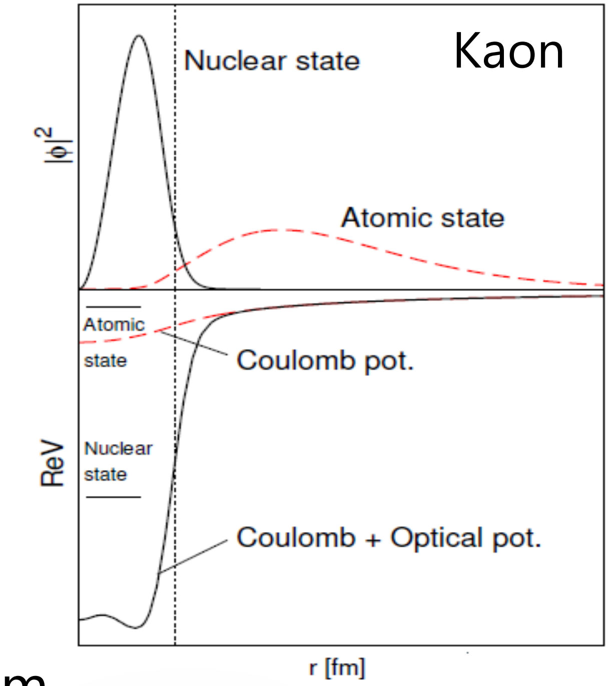
Time/Date : 13:00-14:30 Satoru Hirenzaki, 15:00-16:30 Kenta Itahashi, 6 February (Mon.) 2023

Place: Tokyo Institute of Technology Ookayama Campus, Lecture Theatre

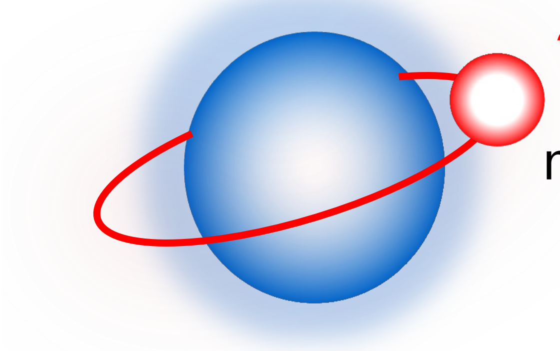
Meson-nucleus bound system

REAL meson exists inside and/or very close to the nuclear surface

- **Mesonic Atoms:** Strong+Coulomb interaction
  Pionic atom, Kaonic atom
- **Mesonic Nuclei:** Strong interaction
  K, η, η′, … meson-nucleus



Pionic atom

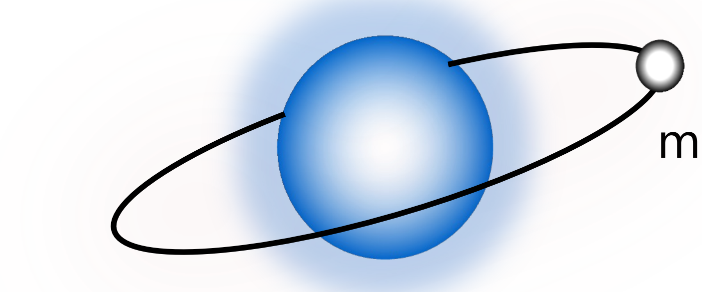


π^- meson

$m_\pi \sim 140$ MeV

Coulomb + Strong Interaction

(cf.) Normal atom



electron

$m_e \sim 0.5$ MeV

Coulomb Interaction

Typical energy scale: B.E ∼ keV- MeV B.E. ∼ eV

Interest and Motivation

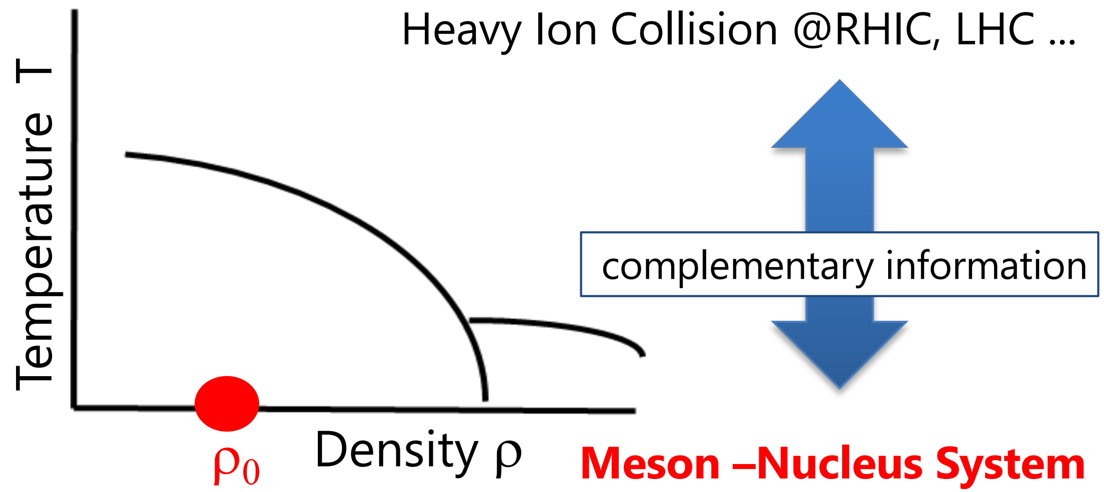
1. Exotic Many Body Physics:
Interaction, Structure, Formation

Like unstable nuclei, hypernuclei … etc.
⇒ Extension of the research area of nuclear physics

2. Meson properties at finite density:
Aspect of QCD symmetries

<u>Chiral symmetry</u>
Spontaneous, Explicit breaking@Vacuum

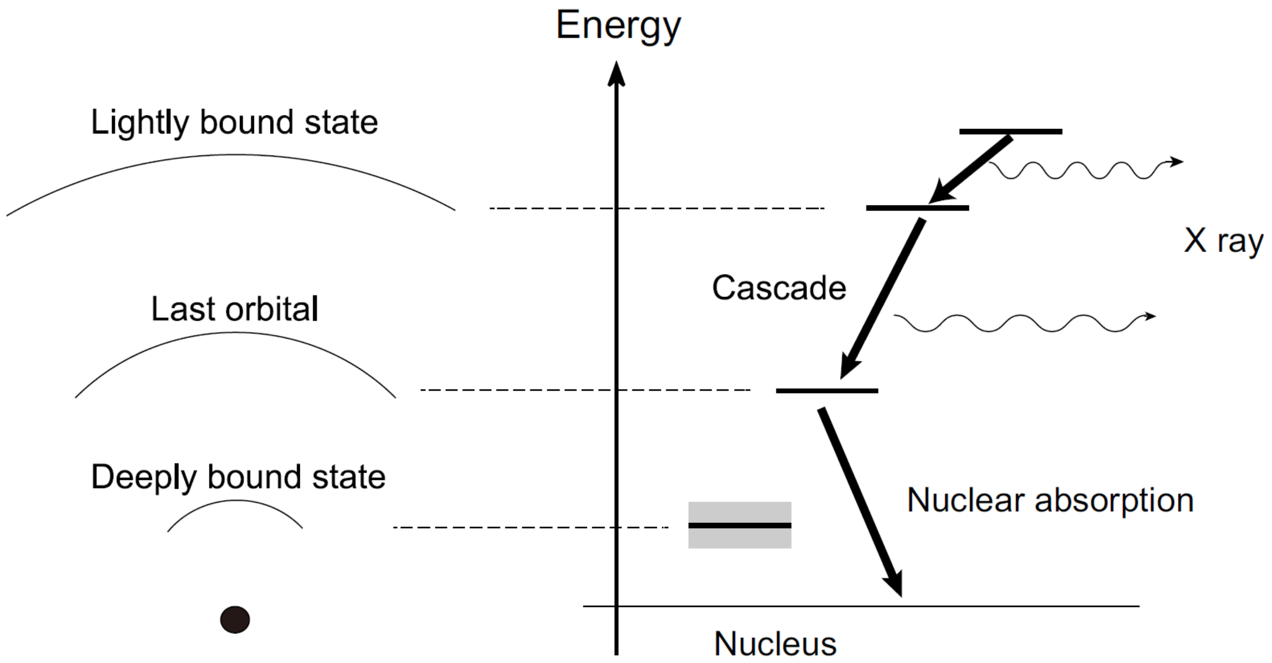
Partial restoration @Nuclear density



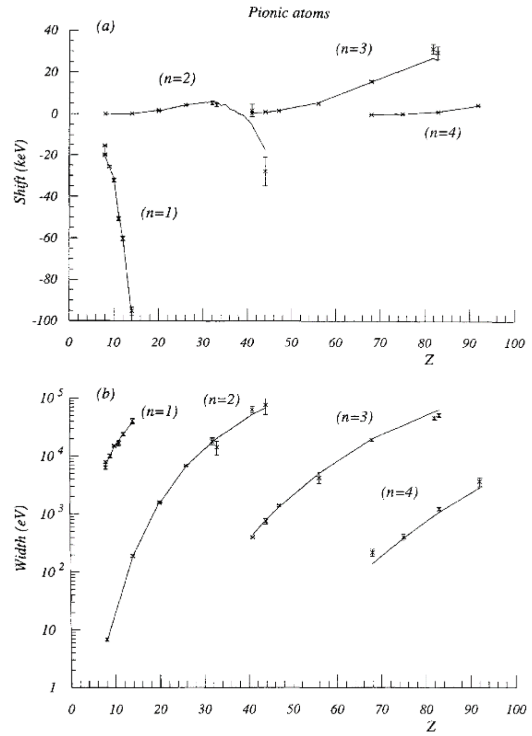
How do the hadron properties change in the nucleus from the vacuum?

How to observe the exotic atoms (traditional)

X-ray spectroscopy (1950s~)



C. Batty, E. Friedman, and A. Gal, Phys. Rep. 287(97)385



The deeply bound states such as 1s and 2p states in heavy nuclei could not be observed because of the absorption

➔ **``Deeply" Bound Pionic Atom**

Difficulity in X-ray spectroscopy

Pionic X-ray data

C.T.A.M. De Laat, et al., Nucl.Phys.A523:453-487,1991.

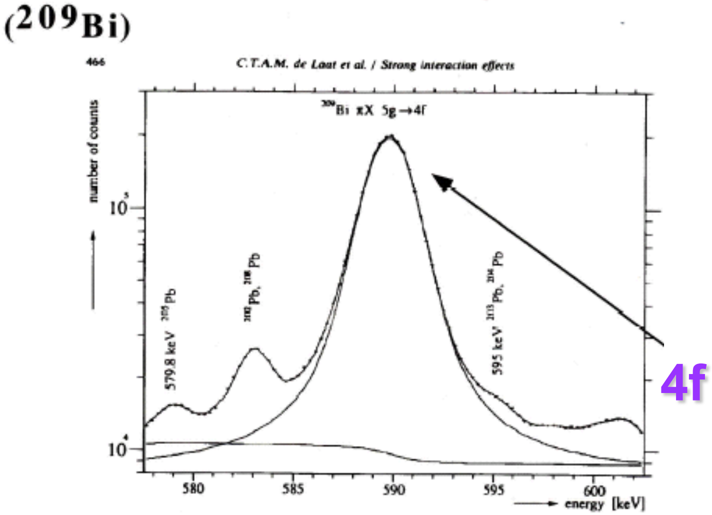


Fig. 5. Part of the pionic X-ray spectrum shown in fig. 4, showing the energy region of the pionic 5g→4f hyperfine complex from pions stopped in a ^{209}Bi target. The fit to the experimental data points is represented by the solid line. The background including the step function is also displayed by a solid line in the figure.

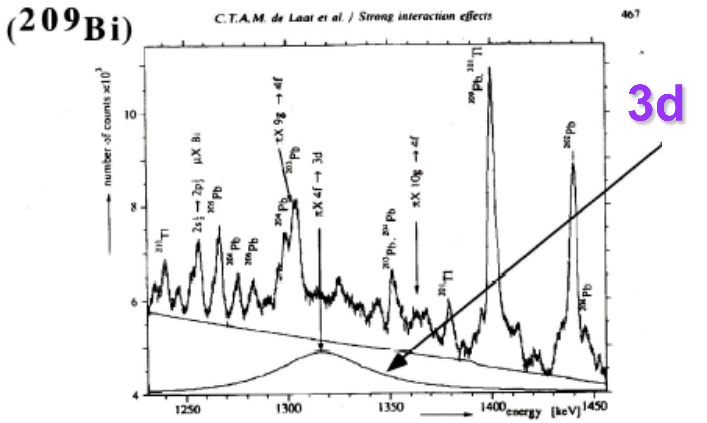


Fig. 6. This figure displays the 4f→3d hyperfine complex of the prompt pionic ^{209}Bi X-ray spectrum. The solid line represents the fit to the experimental data points. Also shown are the background (see text) and the resulting 4f→3d line. The various γ-rays also included in the fitting procedure have been identified as transitions mainly in Pb isotopes in the mass region A = 200–208 as a result of pion and muon capture.

Structure of the pionic atoms

- Klein-Gordon equation:

$$[-\nabla^2+\mu^2+2\mu V_{\text{opt}}(r)]\phi(\mathbf{r}) = [E-V_{\text{coul}}(r)]^2\phi(\mathbf{r})$$

- Pion-Nucleus Optical Potential ：

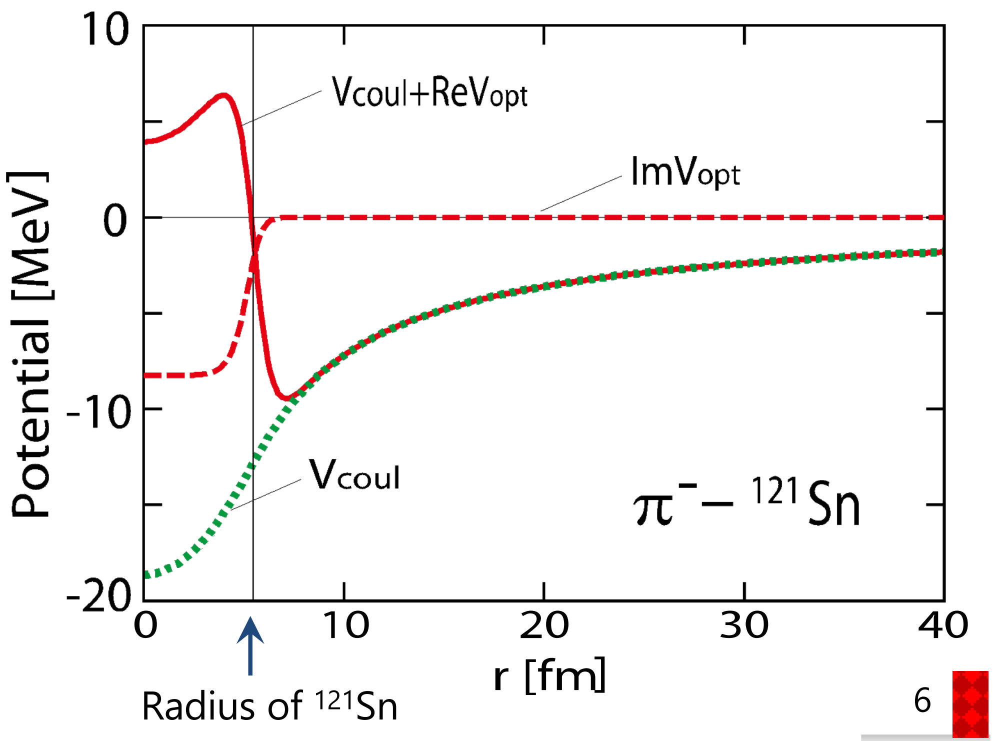
$$2\mu V_{\text{opt}}(r) = \underbrace{-4\pi[b(r)+\varepsilon_2 B_0\rho^2(r)]}_{\text{s-wave term}} + \underbrace{4\pi\nabla\cdot[c(r)+\varepsilon_2^{-1}C_0\rho^2(r)]L(r)\nabla}_{\text{p-wave term}}$$

$$b(r) = \varepsilon_1\{b_0\rho(r)+b_1\ [\rho_n(r)-\rho_p(r)]\}$$

$$c(r) = \varepsilon_1^{-1}\{c_0\rho(r)+c_1[\rho_n(r)-\rho_p(r)]\}$$

$$L(r) = \{1+\tfrac{4}{3}\pi\lambda[c(r)+\varepsilon_2^{-1}C_0\rho^2(r)]\}^{-1}$$

M. Ericson, T. E. O Ericson, Ann. Phys.36(66)496
R. Seki, K. Masutani, PRC27(83)2799

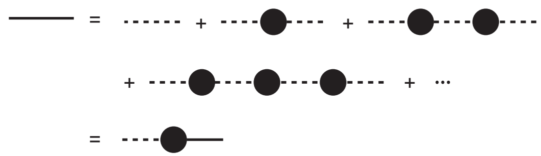


- ✓ Strong interaction s-wave terms are repulsive
- ✓ Pocket structure near the nuclear surface

Optical pot. by Dyson eq.

$$iD^0(q^\mu) = \frac{i}{q^2 - m_\pi^2 + i\varepsilon}$$

$$\begin{aligned} iD(q^\mu) &= iD^0 + iD^0(-i\Pi)iD^0 + iD^0(-i\Pi)iD^0(-i\Pi)iD^0 + \ldots \\ &= i\,(D^0 + D^0\Pi D^0 + D^0\Pi D^0\Pi D^0 + \ldots) \\ &= iD^0(q^\mu) + iD^0(q^\mu)\Pi(q^\mu)D(q^\mu), \end{aligned}$$



$$iD(q^\mu) = \frac{iD^0(q^\mu)}{1 - D^0(q^\mu)\Pi(q^\mu)} = \frac{i}{(D^0(q^\mu))^{-1} - \Pi(q^\mu)} = \frac{i}{q^2 - m_\pi^2 - \Pi(q^\mu)}$$

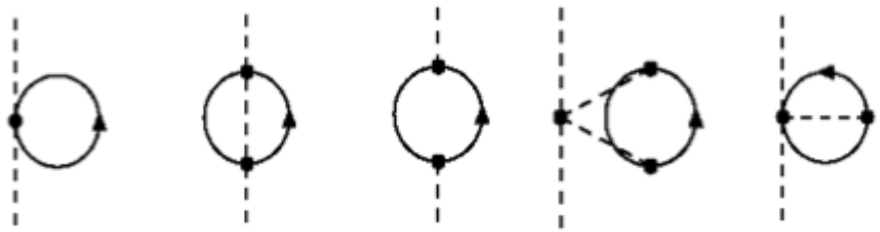
$$H = D^{-1} = q^2 - m_\pi^2 - \Pi(q^\mu)$$

$$\Pi(q^\mu) = \Sigma\,\Pi^{\text{irreducible}}$$ ➔ Optical Potential

Ex.) Kolomeitsev, Kaiser, & W. Weise, Phys.Rev.Lett. 90 ('03)

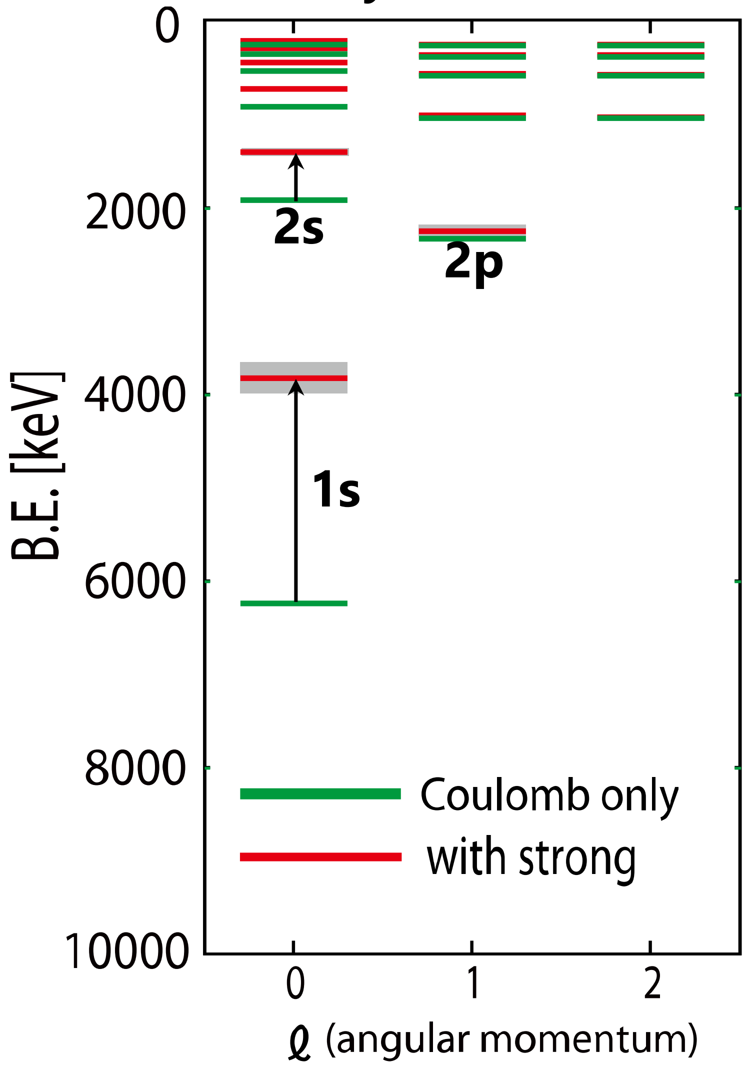
$$\begin{aligned} \Pi_{\text{tot}}(\omega; \rho_p, \rho_n) = \Pi(\omega) &+ \Delta\Pi_{\text{S}}(\omega; \rho_p, \rho_n) \\ &+ \Pi_{\text{P}}(\omega; \rho_p, \rho_n), \end{aligned}$$

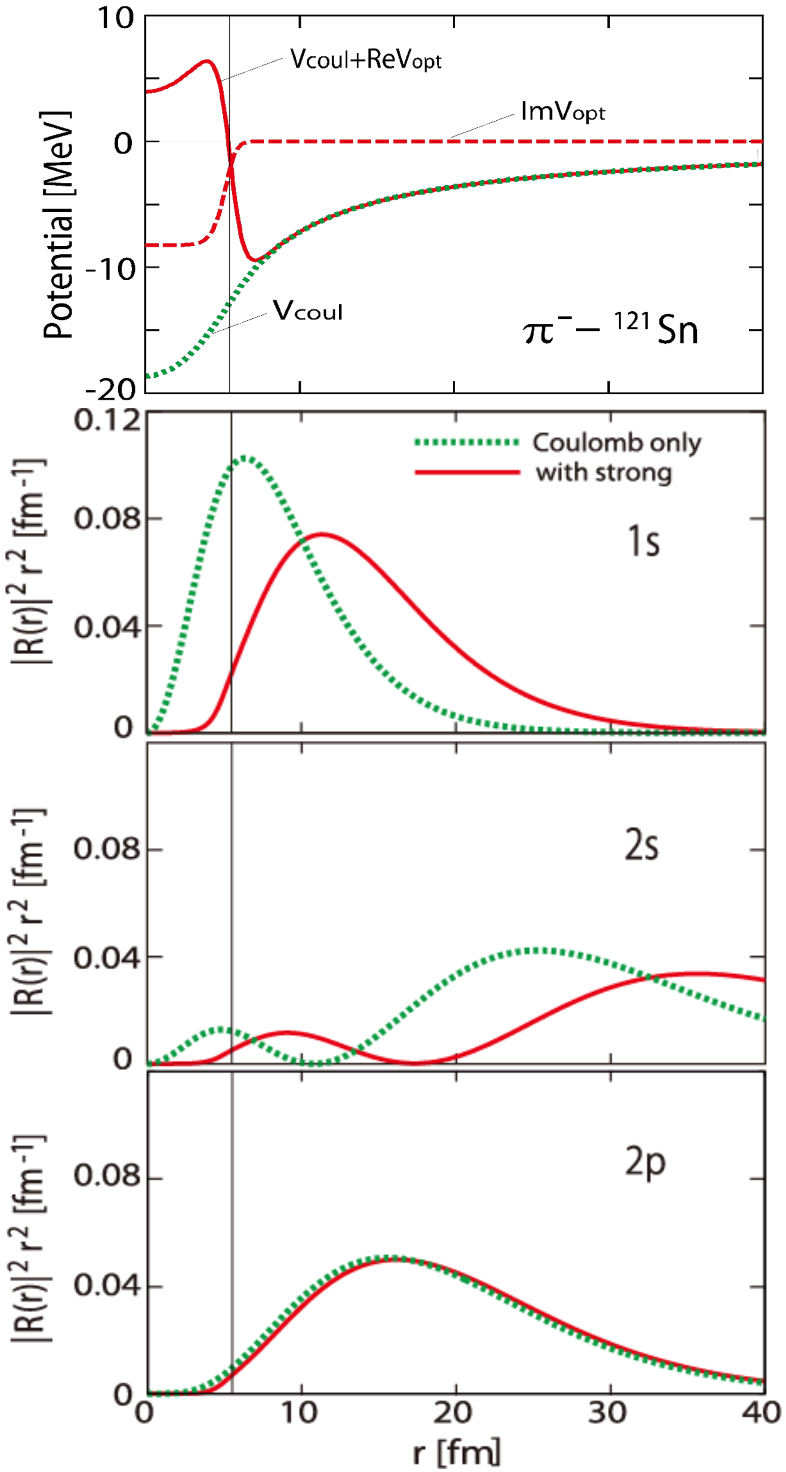
(ChPT up to NNLO) + (Pheno. 2-body abs.)
+(Pheno. Pwave)

$\Pi(\omega) =$ 

Structure of the pionic atoms

- π-^{121}Sn system





Deeply bound pionic states **1s, 2s, 2p** :

Strong interaction effects are large

s-wave interaction ➔ s state
p-wave interaction ➔ higher partial wave states

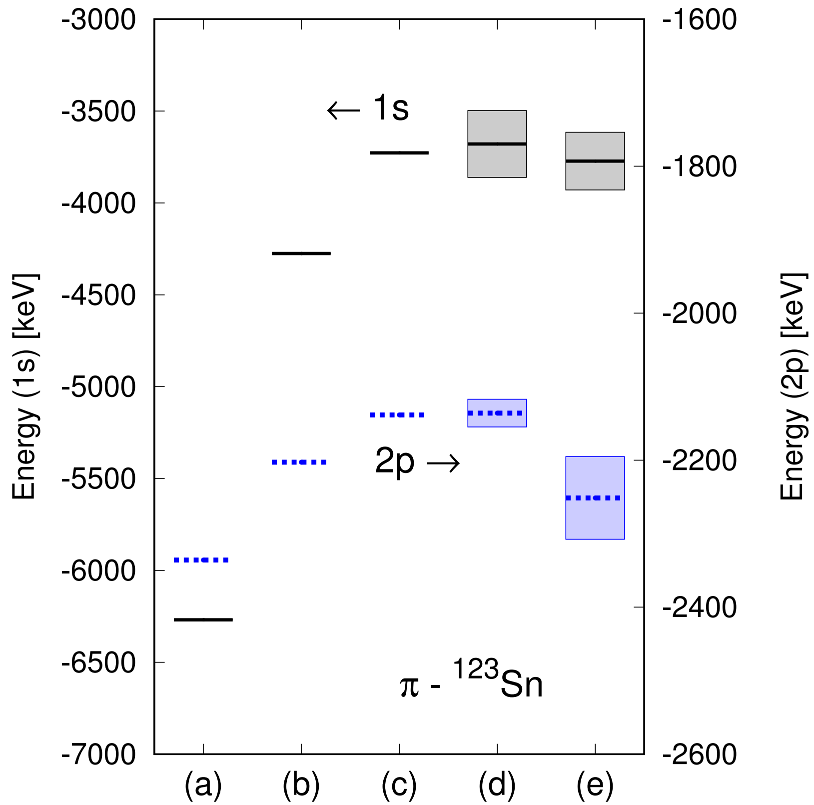
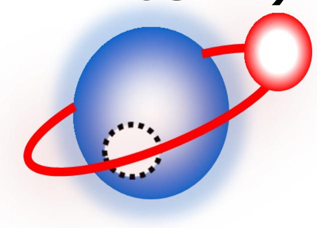


Fig. 8 The calculated energy levels with the several combinations of the potential terms are shown in the solid lines for $1s$ state and dotted lines for the $2p$ state in ^{123}Sn. The level widths are indicated by the hatched area. The potential terms included in the calculation for the energy levels are (a) the electromagnetic interaction V_{em}, (b) V_{em} and the isoscalar s-wave interaction (b_0 term), (c) V_{em}, b_0 term, and the isovector s-wave interaction (b_1 term), (d) V_{em} and the whole part of the local potential (b's and B_0 terms), and (e) the full potential (V_{em} and V_{opt}).

How to observe meson in nucleus (modern)

- Missing mass: 1+2→3+X (d + A → 3He+πAtom)



$$M_{\rm miss}^2 = (E_1 + E_2 - E_3)^2 - (\vec{p_1} + \vec{p_2} - \vec{p_3})^2$$

(X is NOT observed)

- Invariant mass: X→1+2 ($\phi \rightarrow e^- + e^+$)

$$p_\phi^\mu = p_-^\mu + p_+^\mu \qquad m_{\rm inv} = \sqrt{(p_- + p_+)^2}$$

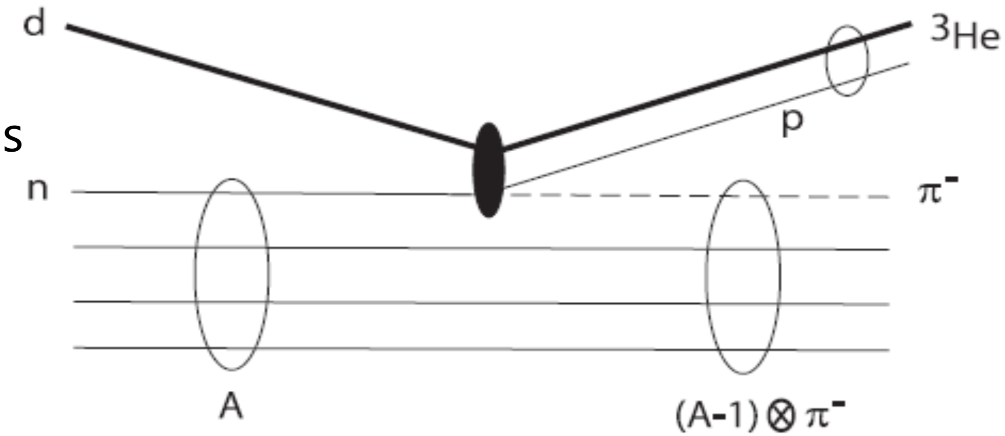
(Outside decay background, FSI)

Effective number approach

- Factorization of elementary process cross section
- Use of known experimental data (e.g., nuclear response)
- Good approach to the mesonic atom systems with relatively narrow widths

➢ (d,^{3}He) reaction H. Toki *et al.*, NPA530(91)679; S. Hirenzaki *et al.*, PRC44(91)2472

d + Nucleus $\longrightarrow$ ^{3}He + π atom 1 neutron pick up reaction



Heavy target case:
Formation cross section of pionic atoms

$$d\sigma = \sum_f \frac{V^2}{v_{\rm rel}} \frac{1}{VT} |S_{fi}|^2 \frac{V}{(2\pi)^3} d\mathbf{p}_{\rm He}$$

S matrix:

$$S_{fi} = \int dt d\mathbf{r} \sqrt{\frac{M_{\rm He}}{E_{\rm He}}} \frac{1}{\sqrt{V}} e^{iE_{\rm He}t} \chi^*_{\rm He}(\mathbf{r}) \sqrt{\frac{1}{2E_\pi}} e^{iE_\pi t} \phi^*_\pi(\mathbf{r}) iT(p_d, p_n, p_{\rm He}, p_\pi) \sqrt{\frac{M_d}{E_d}} \frac{1}{\sqrt{V}} e^{-iE_d t} \chi_d(\mathbf{r}) \sqrt{\frac{M_n}{E_n}} e^{-iE_n t} \psi_n(\mathbf{r})$$

wave function for each particle

will be replaced by the elementary cross section later

Effective number approach

$$\left(\frac{d^2\sigma}{dE_{\rm He}d\Omega_{\rm He}}\right)_A^{\rm lab} = \left(\frac{d\sigma}{d\Omega_{\rm He}}\right)_{\rm ele}^{\rm lab} \sum_{ph} K \left(\frac{\Gamma}{2\pi}\frac{1}{\Delta E^2 + \Gamma^2/4} N_{\rm eff} + \frac{2p_\pi E_\pi}{\pi} N_{\rm eff}\right)$$

$\Delta E = Q + m_\pi - B_\pi + Sn - 6.787\mathrm{MeV}$ **Bound region** **Quasi-elastic region**

- **Elementary cross section** $\left(\frac{d\sigma}{d\Omega_{\rm He}}\right)_{\rm ele}^{\rm lab}$: Experimental data ($d+n\rightarrow{}^3He+\pi^-$)
- **Kinematical correction factor:** Difference of kinematics between $d+n\rightarrow{}^3He+\pi^-$ and $A(d,{}^3He)(A-1)\otimes\pi^-$

$$K = \left[\frac{|\vec{p}^{A}_{\rm He}|}{|\vec{p}_{\rm He}|}\frac{E_n E_\pi}{E_n^A E_\pi^A}\left(1 + \frac{E_{\rm He}}{E_\pi}\frac{|\vec{p}_{\rm He}| - |\vec{p}_d|\cos\theta_{d\rm He}}{|\vec{p}_{\rm He}|}\right)\right]^{\rm lab}$$

- **Effective Number:**

$$N_{\rm eff} = \sum_{JMm}\left|\int d\vec{r} e^{i\vec{q}\cdot\vec{r}} D(\vec{r})\xi^\dagger_{\frac{1}{2}m}[\phi^*_{\ell_\pi}(\vec{r})\otimes\psi_{j_n}(\vec{r})]_{JM}\right|^2$$

D(r) : Distortion factor
q : Momentum transfer
ψ_{j_n} : Neutron wave function

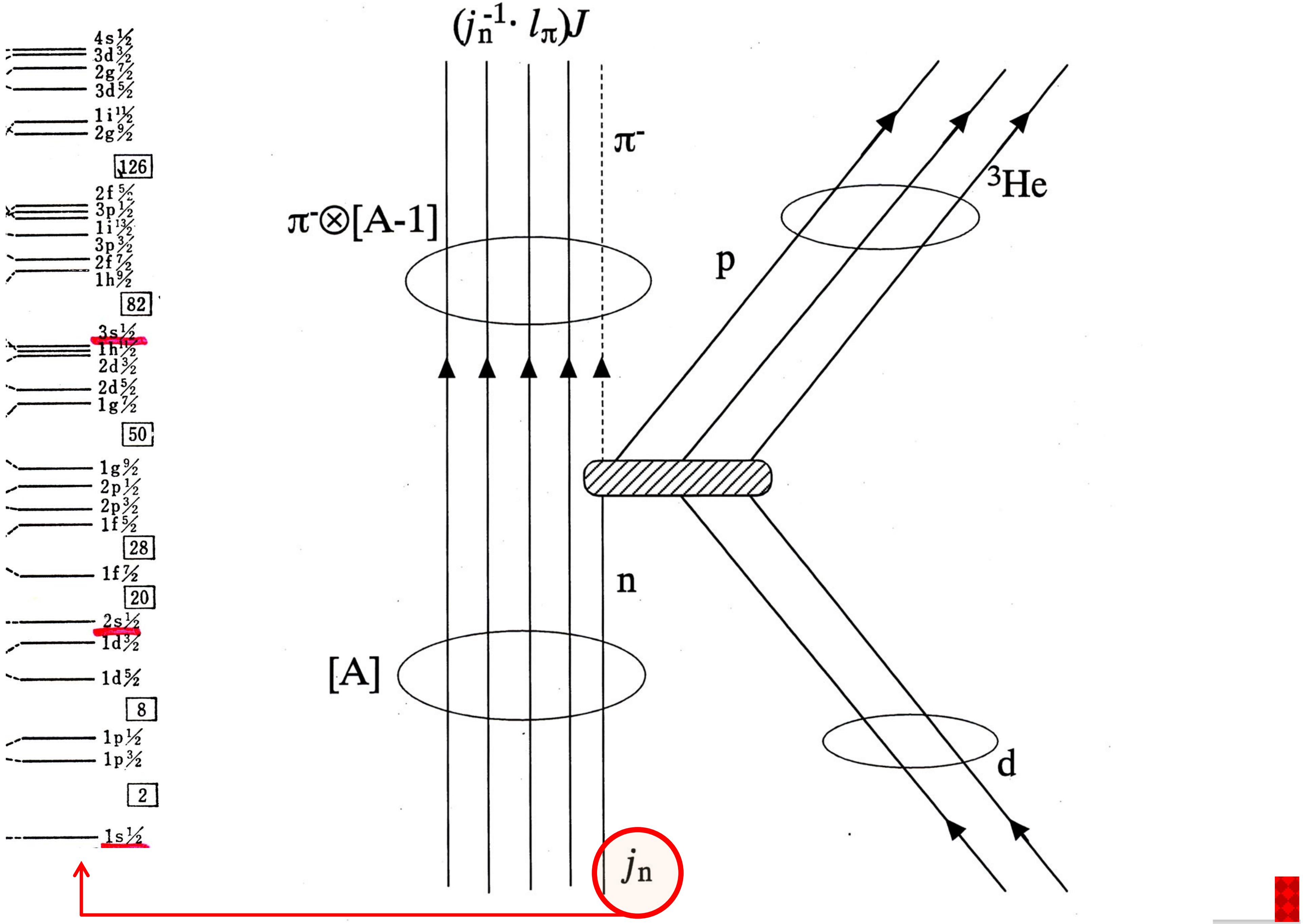
∗ Information on the nuclear response in one neutron pick-up reaction

$$N_{\rm eff} \rightarrow N_{\rm eff}(\ell_\pi \otimes j_n^{-1}) \times F_O(j_n) \times \begin{cases} F_R((j_n^{-1})_1), \\ F_R((j_n^{-1})_2), \\ \cdots \\ F_R((j_n^{-1})_N), \end{cases}$$

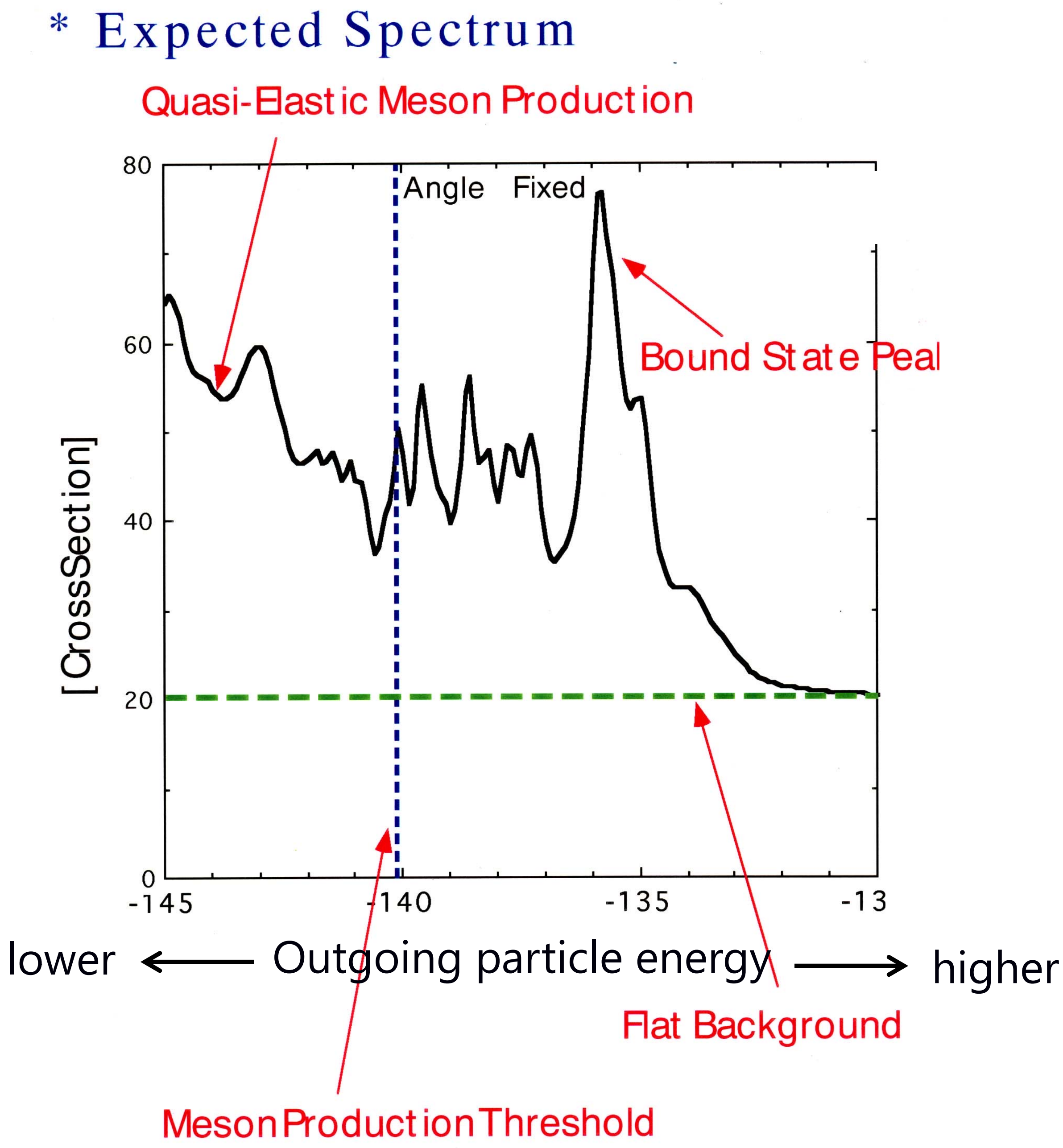
F_O : Neutron occupation probabilities in the target

F_R : Relative strength factor of the N-th excited states in the daughter nucleus

Pick-up Reaction : d + Nucleus ⟶ ^{3}He + π atom



Expected (d,^{3}He) spectra

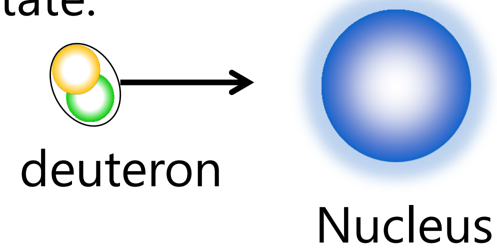


Observed (d,^{3}He) spectra

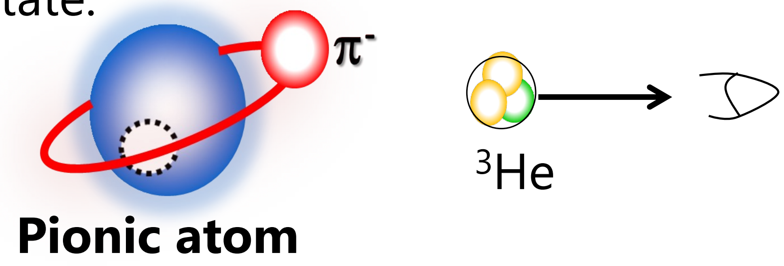
(d,^{3}He) reaction

H. Toki, *et al.*, NPA530(91)679; S. Hirenzaki *et al.*, PRC44(91)2472

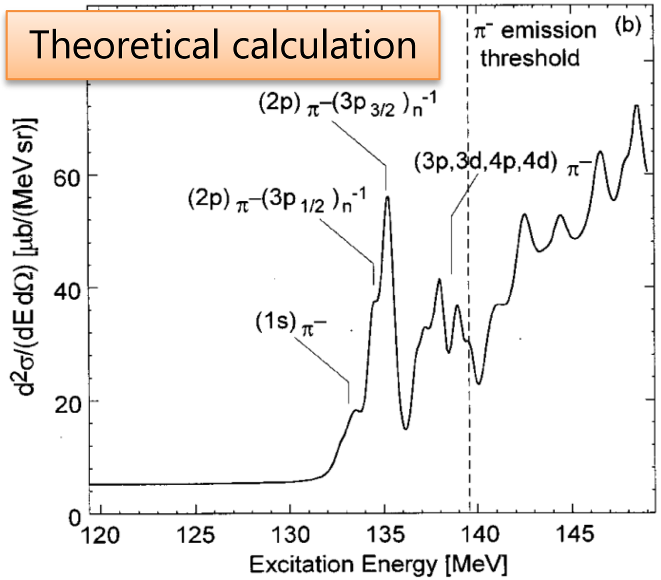
Initial state:



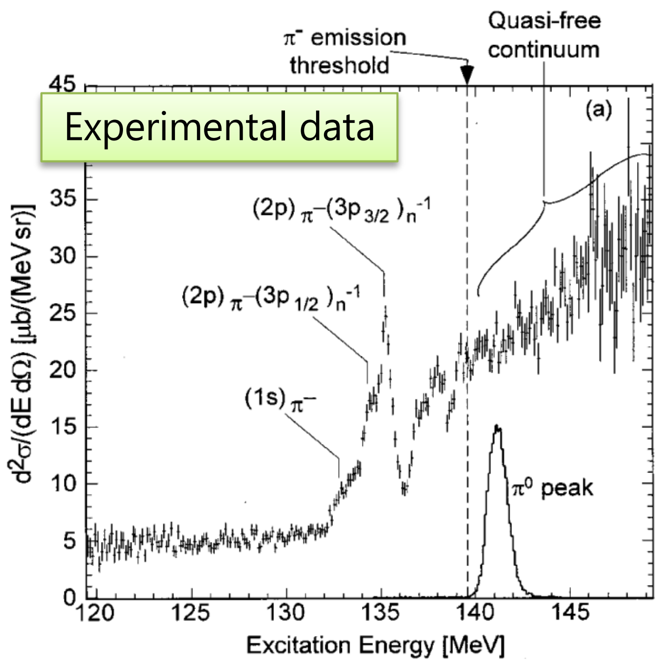
Final state:



^{208}Pb target



S. Hirenzaki *et al.*, PRC44(91)2472



Experiment@GSI

K. Itahashi *et al.*, PRC62(00)025202

Observation of the deeply bound pionic state for the first time

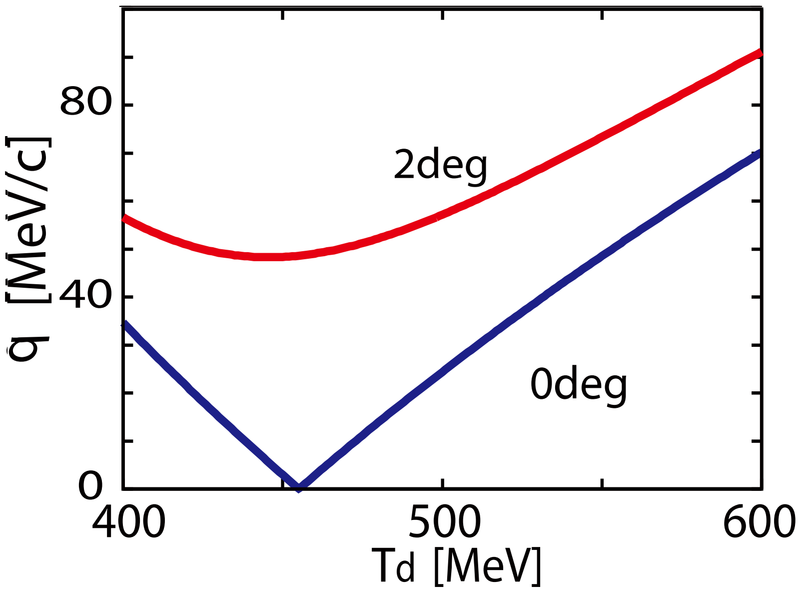
Role of the momentum transfer q in the reaction

- Large q ⇒ Cross section becomes small (generally)
- **Matching condition** of the angular momentum transfer L and the momentum transfer q

$$L = [\ell_\pi \otimes \ell_n^{-1}] \simeq qR$$ = q × (Nuclear Radius)

=> the matching condition plays an important role in determining the largely populated subcomponents

➢ (d,^{3}He) reaction **q=|p$_d$-p$_{He}$|**



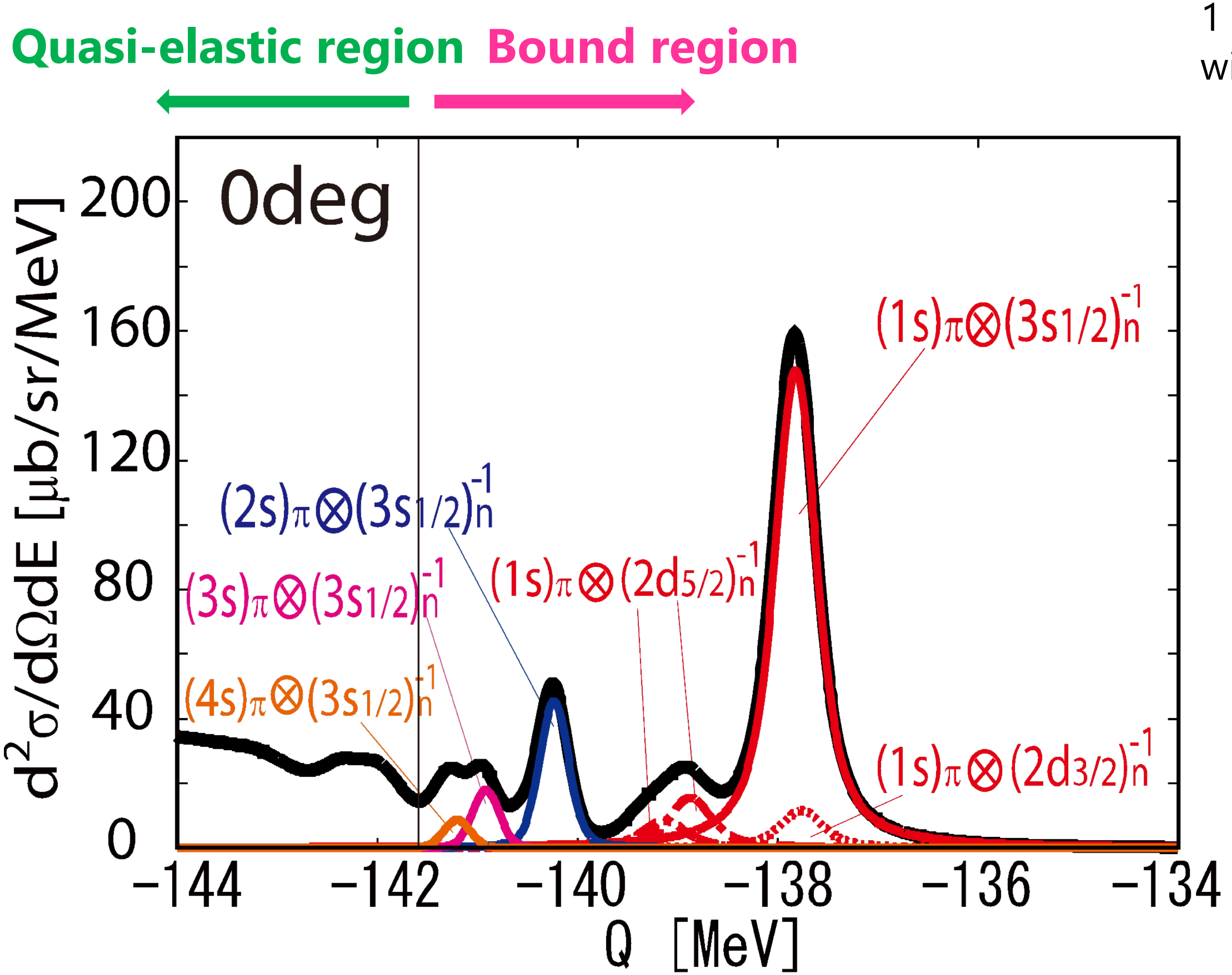
- **Forward angle:** It can be recoilless (q~0)
⇒ Enhanced formation with L~0 state
(s-state contributions relatively large)

- **Finite angles:** Larger q
⇒ Enhanced formation with large L state

Sn=0 MeV, B.E.=0 MeV

We can observe selectively the different pionic states by adjusting q

^{122}Sn(d,^{3}He) spectrum $[(n\ell)_\pi \otimes (n\ell_j)_n^{-1}]$



1 neutron pickup reaction without π production

Exp. Data: ^{122}Sn(d,t)^{121}Sn
E. J. Schneid et al.,
Phys. Rev. 156 (1967) 1316

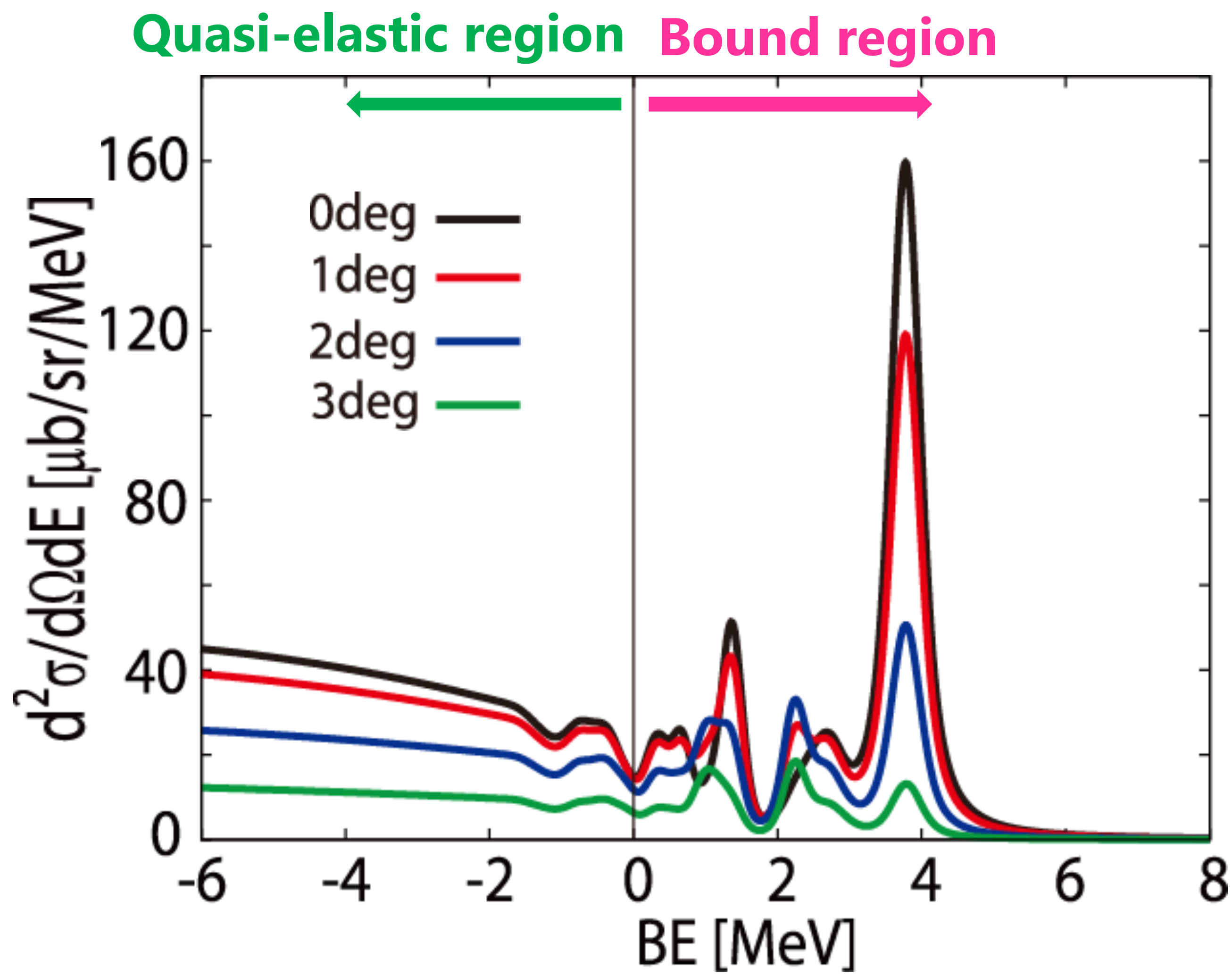
Neutron hole orbit j_h	Ex [MeV]
3s1/2	0.06
2d3/2	0.00
2d5/2	1.11
	1.37
1g7/2	0.90
1h11/2	0.05

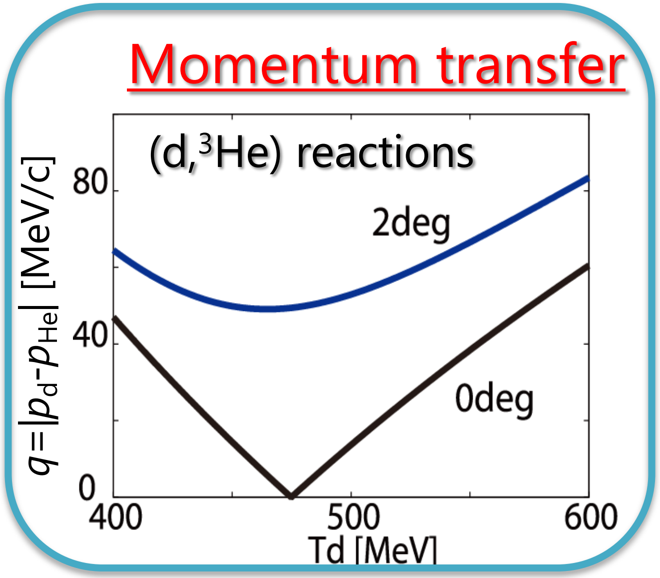
New Exp. data:
S.V. Szwec et al.,
PRC104 (2021) 054308

Energy resolution
ΔE=300keV

- We can see the large peak structure of pionic 1s state
- Combination of the pionic 1s state and neutron-hole $3s_{1/2}$ state

^{122}Sn(d,^{3}He) spectra at Finite angles





Energy resolution
ΔE=300keV

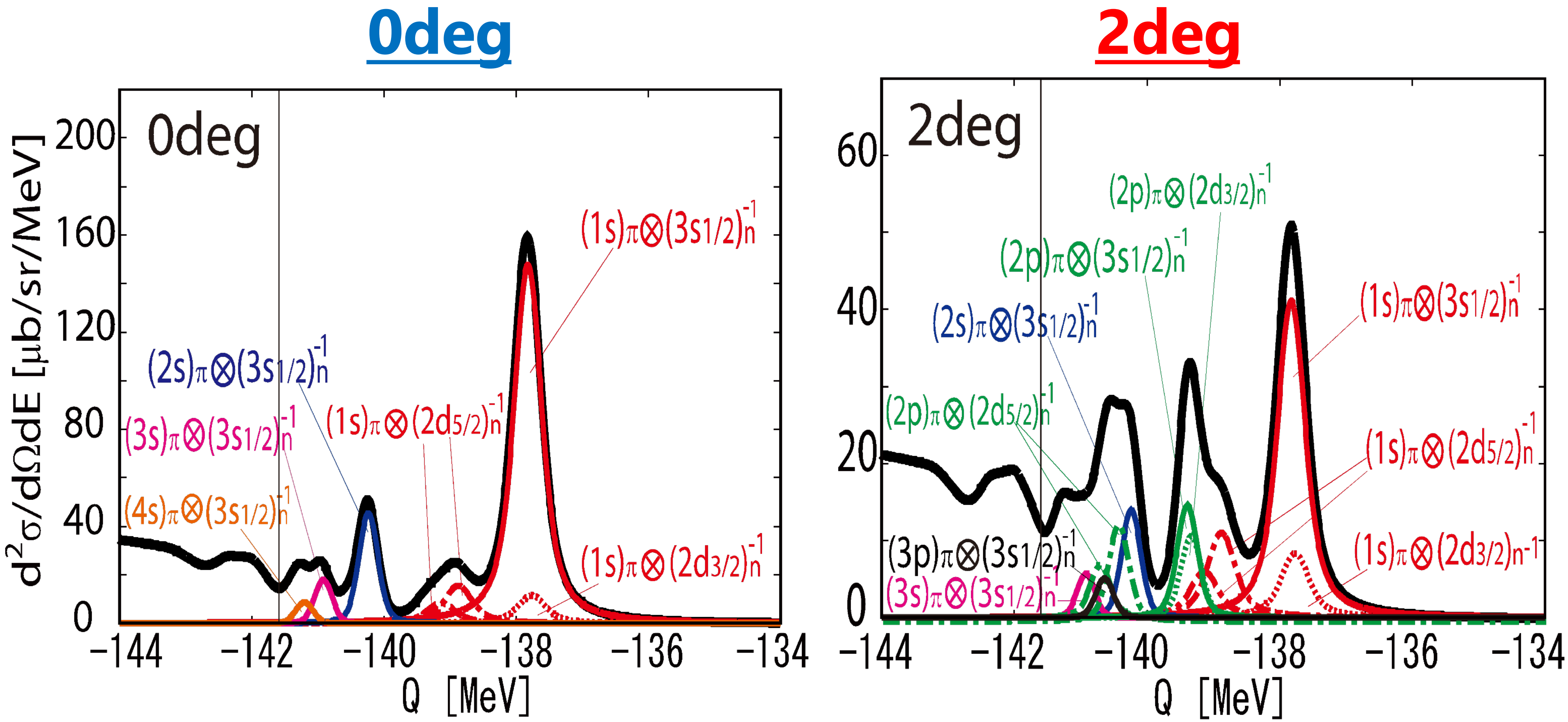
Neutron wave function:
H. Koura *et al.*,
NPA671(2000)96

Spectra have a strong angular dependence.

$^{122}Sn(d,^{3}He)$ spectra at Finite angles

Dominant Subcomponent $[(n\ell)_\pi \otimes (n\ell_j)_n^{-1}]$

Energy resolution
ΔE=300keV

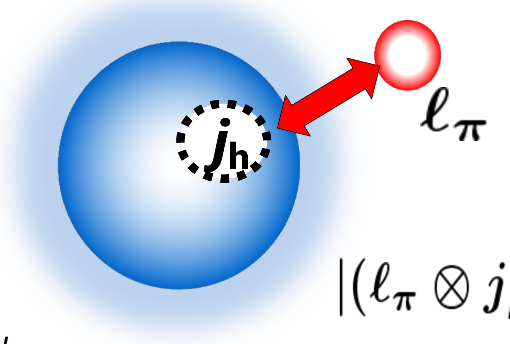


We can obtain information on the deeply bound pionic **2*p*** state in addition to **1*s*** and **2*s*** states.

Extension to the study of the odd-neutron nuclear target

<u>Even-Even Nucleus: J^p=0^+</u>

Pionic atoms: pion particle - neutron hole $[\pi \otimes n^{-1}]$



$|(\ell_\pi \otimes j_h^{-1})_J\rangle$

<u>``Residual interaction effect''</u>

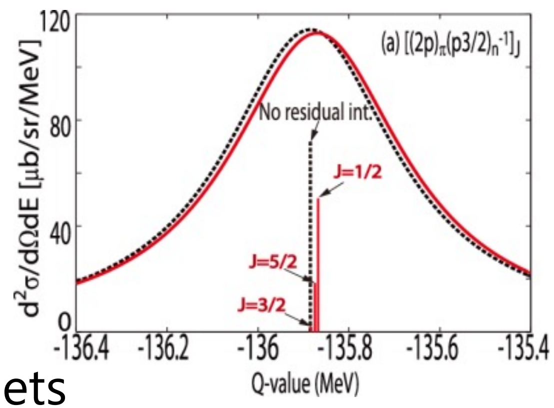
S. Hirenzaki *et al.* PRC60(99)058202; N. Nose-Togawa *et al.* PRC71(05)061601(R)

- Energy shift
- Level splitting between different J state

Shift of Peak position in the spectra

Additional difficulty to determine B.E. and pion property in the nucleus

^{116}Sn complex energy shift			
j_h^{-1}	1s [keV]	2p [keV]	
$3s_{1/2}^{-1}$	-15.4-4.2i	J=1/2	-4.0-1.1i
		J=3/2	-4.0-1.1i
$2d_{3/2}^{-1}$	-15.9-4.8i	J=1/2	-9.1-3.1i
		J=3/2	0.3+0.3i
		J=5/2	-5.2-1.8i
Exp. Error ± 24 [keV] @GSI			



- Interests of Odd target

Sn:	^{115}Sn	^{116}Sn	^{117}Sn	^{118}Sn	^{119}Sn	^{120}Sn	^{121}Sn	^{122}Sn	^{123}Sn	^{124}Sn
	1/2$^+$	0$^+$	1/2$^+$	0$^+$	1/2$^+$	0$^+$	3/2$^+$	0$^+$	11/2$^-$	0$^+$

Pionic state free from residual interaction effect $[\pi^- \otimes 0^+]$

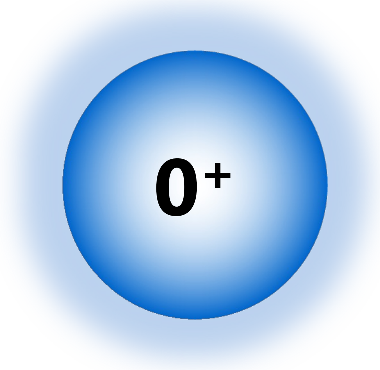
$\Rightarrow$ Expect to extract more accurate information than even targets from data.

Formulation: Effective Number

Acknowledgement
H. Nakada-san

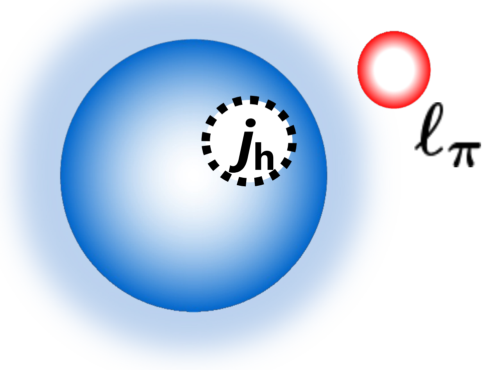
Even target: ^{122}Sn (0^+)

Initial:



$|0^+\rangle$

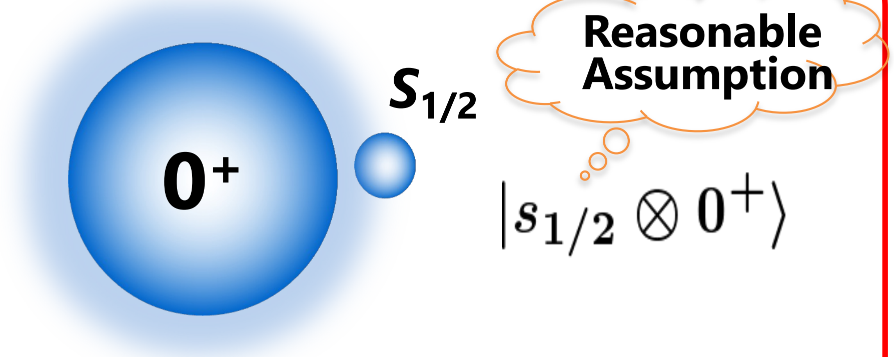
Final:



$|(\ell_\pi \otimes j_h^{-1})_J\rangle$

Odd target: $^{117,\,119}$Sn ($1/2^+$)

Initial:



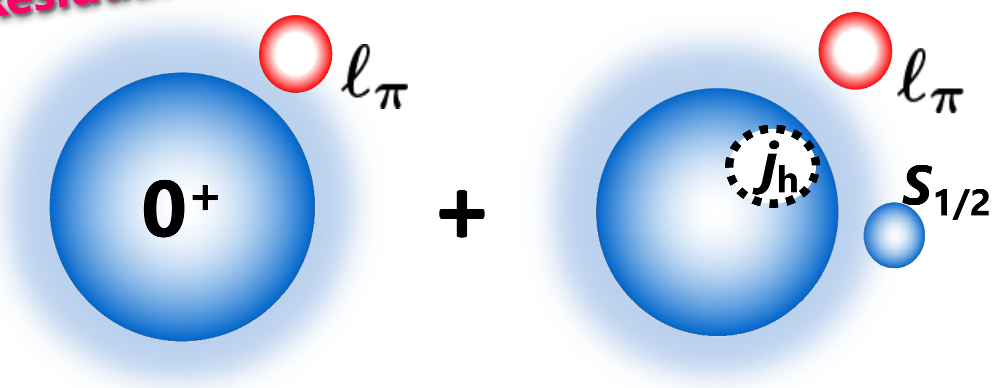
$|s_{1/2} \otimes 0^+\rangle$

Final:

(1) neutron pick-up from $s_{1/2}$ orbit

(2) neutron pick-up j_h orbit from other than $s_{1/2}$

``No Residual Interaction"



$|(\ell_\pi \otimes 0^+)\rangle$ $\quad$ $|(\ell_\pi \otimes [s_{1/2} \otimes j_h^{-1}]_J)_{J_f}\rangle$

- Realistic neutron configurations for the target and the daughter nucleus: Exp. Data

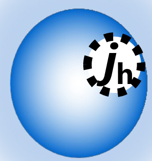
Even target: ^{122}Sn (0^+)

Excited level of ^{121}Sn

Exp. Data: $^{122}Sn(d,t)^{121}Sn$

E. J. Schneid et al., Phys. Rev. 156 (1967) 1316

Neutron hole orbit j_h	Ex [MeV]
3s1/2	0.06
2d3/2	0.00
2d5/2	1.11
	1.37
1g7/2	0.90
1h11/2	0.05



New Exp. data: S.V. Szwec et al., PRC104 (2021) 054308

- ✓ Many excited levels
- ✓ Large excitation energies (Ex)

➡ **Pionic atom formation spectra: Expected to be complicated and broad spectra**

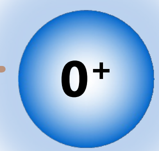
Odd target: ^{117}Sn ($1/2^+$)

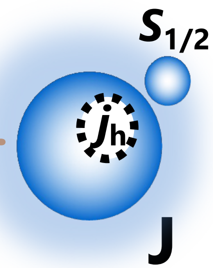
Excited level of ^{116}Sn

Exp. Data: $^{117}Sn(d,t)^{116}Sn$,

J. M. Schippers et al., NPA510(1990)70

J^p	Neutron hole orbit j_h	Ex [MeV]
0+	3s1/2	0.00
		1.76
		2.03
		2.55
1+	2d3/2	2.59
		2.96
2+	2d3/2 and 2d5/2	1.29
		2.23
		3.23
		3.37
		3.47
		3.59
		3.77
		3.95
3+	2d5/2 and 1g7/2	3.00
		3.42
		3.71
	1g7/2	3.18
4+	1g7/2	2.39
		2.53
		2.80
		3.05
		3.10
5-	1h11/2	2.37
6-	1h11/2	2.77

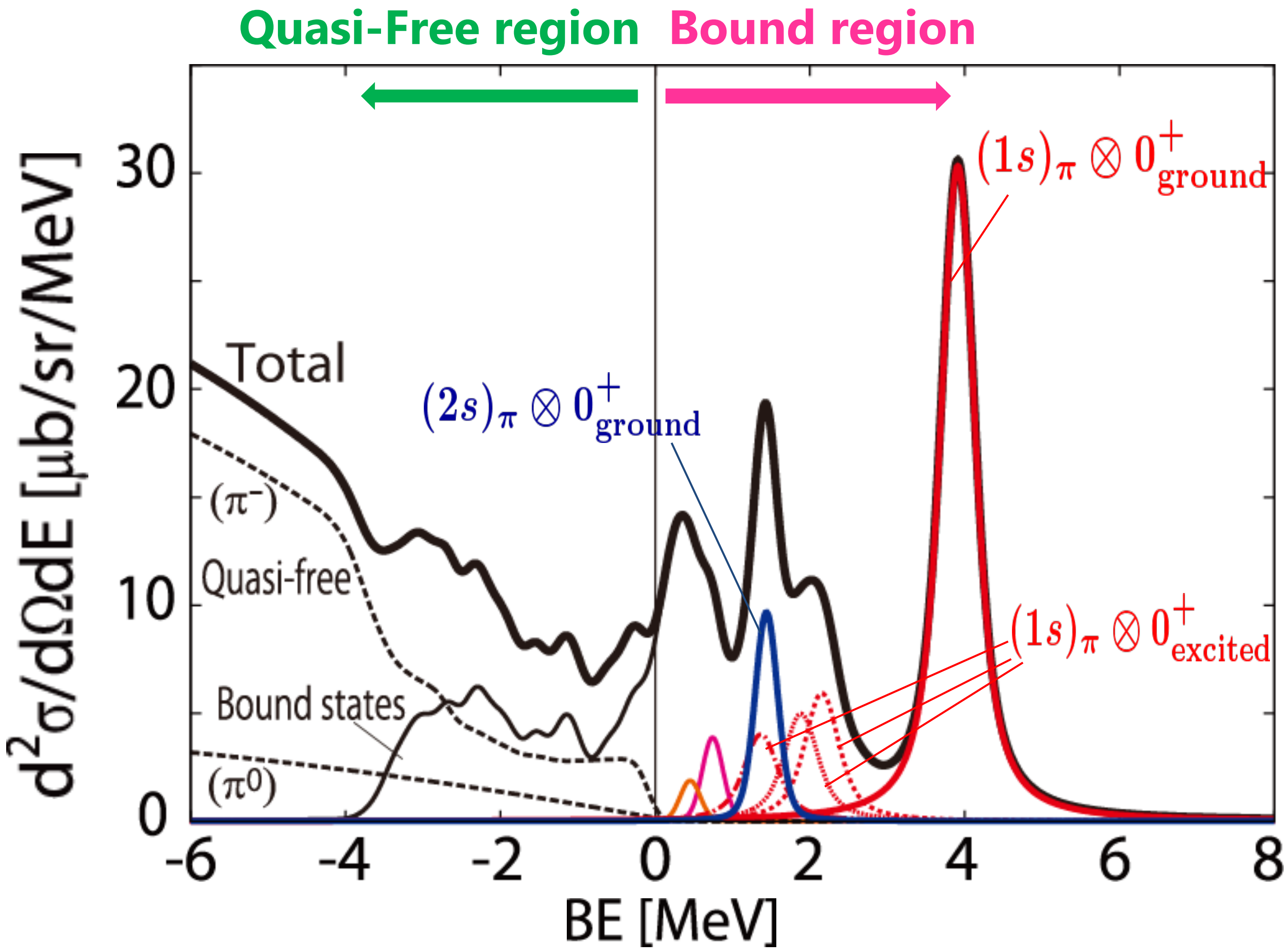




Odd target

- ^{117}Sn(d,^{3}He) spectra at 0 degrees

Neutron wave function:
H. Koura *et al.*, NPA671(00)96



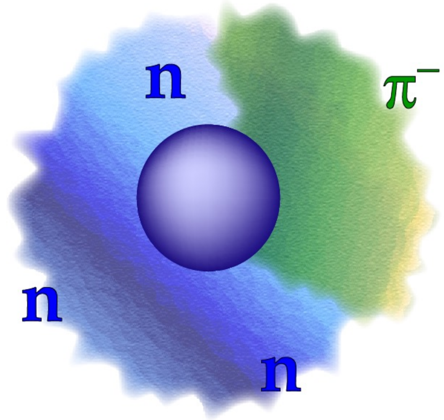
Energy resolution
ΔE=300keV

Dominant Subcomponent:

$$[(n\ell)_\pi \otimes J^P]$$

- We can see clear peak structure of $[(1s)_\pi \otimes {}^{116}\text{Sn}(0^+)]$
  - No residual interaction effect

Interest and Motivation

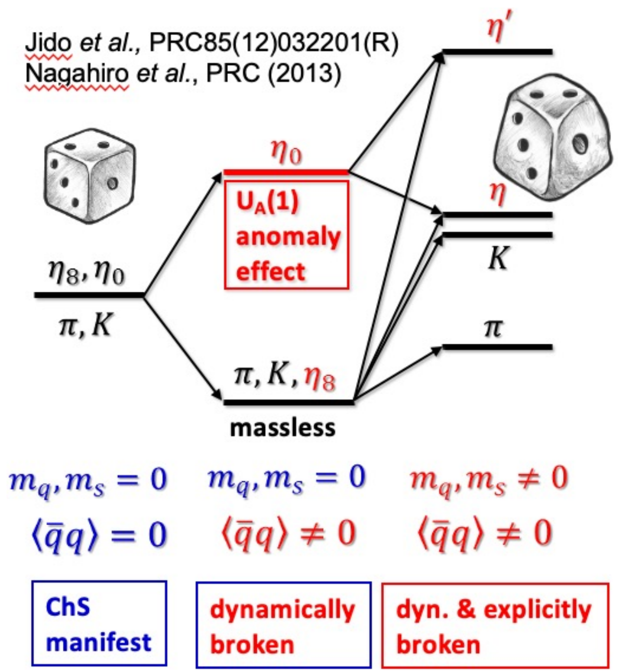


(1) New exotic Hadron many body systems

(2) Baryon resonances at finite density

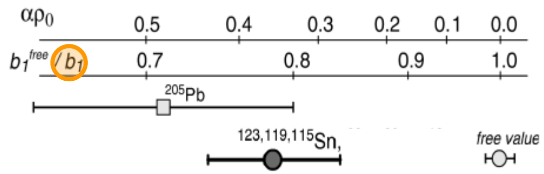
$$N^*(1535) \quad \Lambda(1405)$$

(3) Aspects of the Strong Int.Symmetry



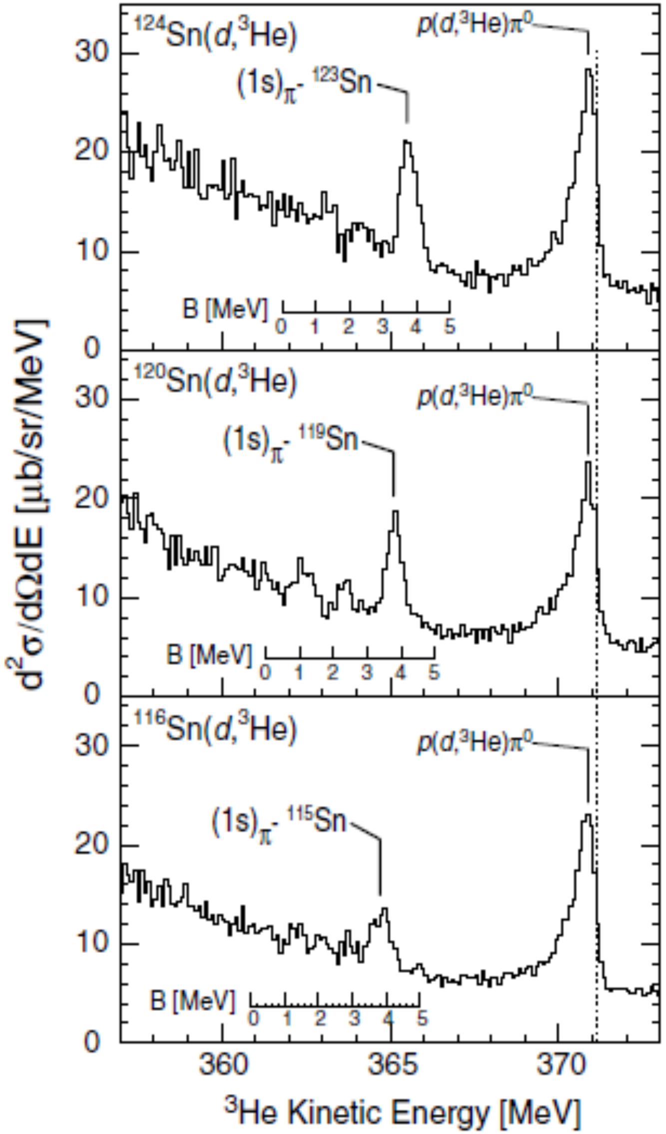
In-medium pion and Chiral sym.

In-medium property of pion



K. Suzuki *et al.*, PRL92 (04) 072302

Exp@GSI



- Pion-Nucleus optical potential

$$2\mu V^s_{\rm opt} = -4\pi[\varepsilon_1\{b_0\rho(r) + b_1\delta\rho(r)\} + \varepsilon_2 B_0\rho^2(r)]$$

✓ b1 determination, Comparison with value in vacuum

✓ Relation between $b_1 \Leftrightarrow f_\pi \Leftrightarrow \langle\bar{q}q\rangle$

- Tomozawa – Weinberg (TW) relation

$$T^{(-)}_{\pi A} = -4\pi\varepsilon_1 b_1 = \frac{\omega}{2f^{*2}_\pi}$$

- Gell-Mann – Oakes – Renner (GOR) relation

$$m^2_\pi f^{*2}_\pi = -2m_q\langle\bar{q}q\rangle_\rho$$

$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0} \simeq \frac{f^{*2}_\pi}{f_\pi{}^2} \simeq \frac{b_1^{\rm free}}{b_1^*(\rho)} = 0.78 \ @\ \rho \simeq 0.6\rho_0$$

$$\sim 0.67\ @\ \rho = \rho_0$$

Some memos for pi atom

Basic Story (Prediction, Observation, Feedback)

- Observe meson in nucleus (B.E., Width, , , ,)
- Deduce in-medium meson properties (b1, , ,)
- Relate them to fundamental parameters
  (Condensate, , ,)

Some points

＊States with well-defined quantum numbers
(something like “selection rule”)

＊Exclusive information (s-wave isovector int., , ,)

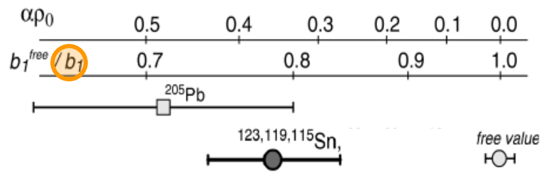
＊Reliable connection between Theoretical formula
and Exp. Result

＊Model independent theoretical treatment
(… for feedback/fitting)

In reality, we need some phenomenological pieces.

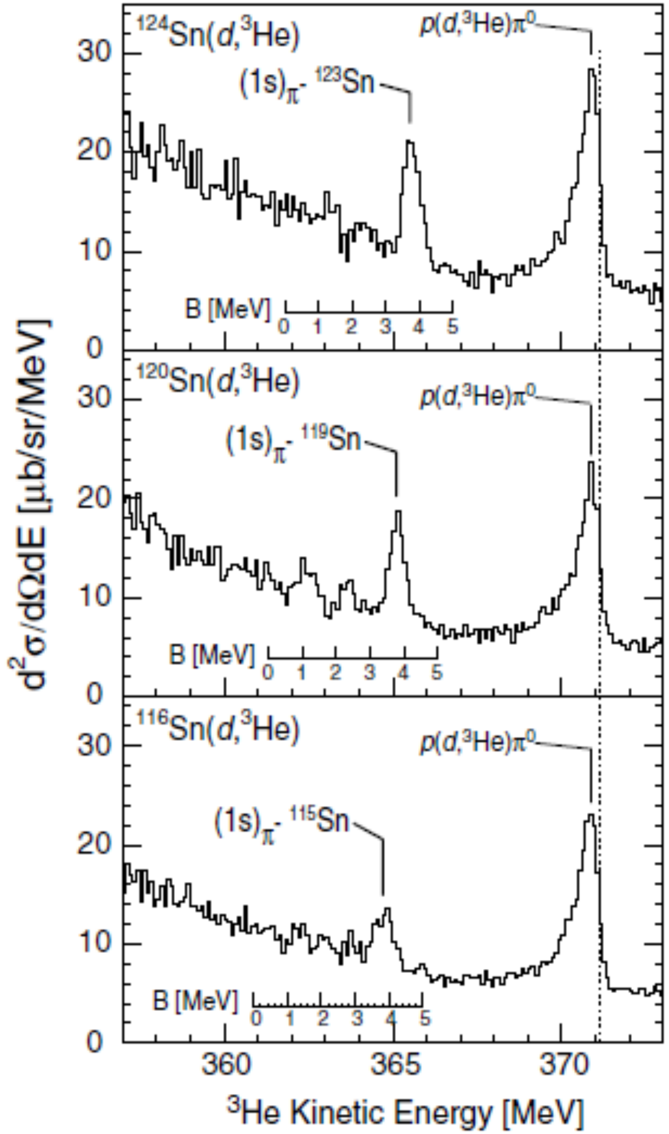
In−medium Chiral sym.

In-medium property of pion



K. Suzuki *et al.*, PRL92 (04) 072302

Exp@GSI



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$$\sim 0.67 \ @\ \rho = \rho_0$$

- Data corresponds to info. at Effective Density

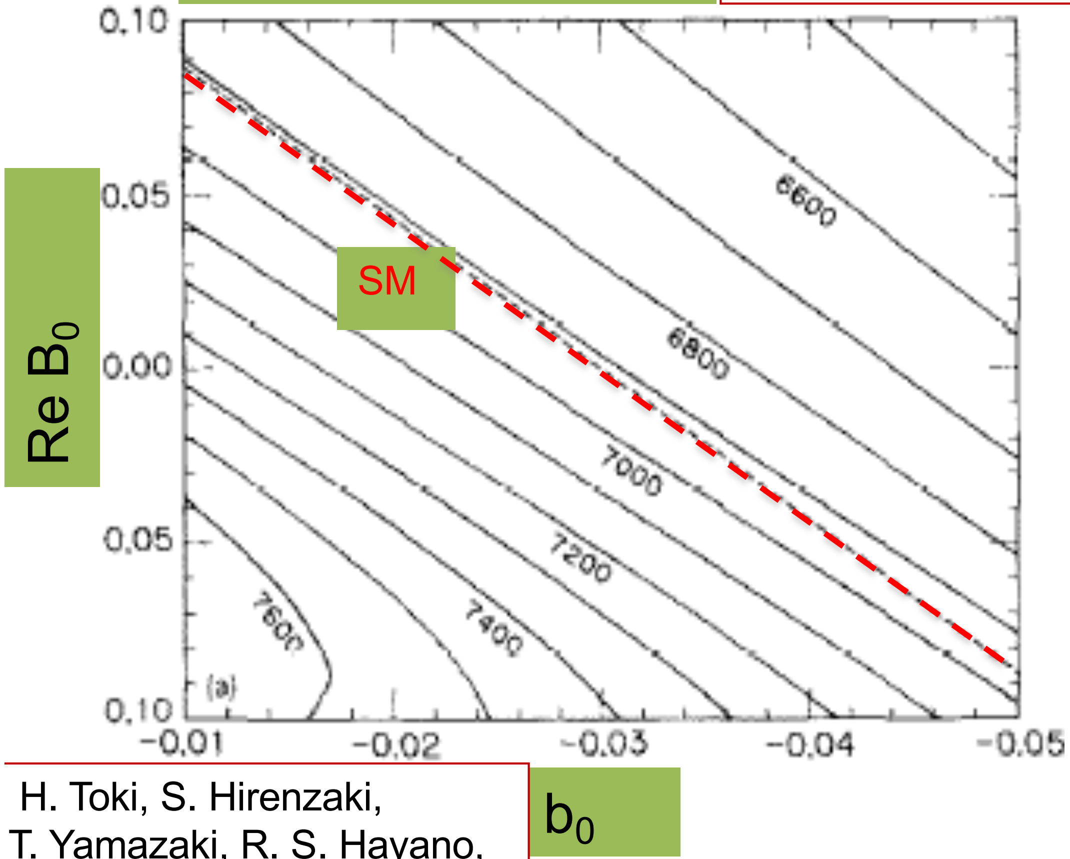
Parameter correlation and Effective density

R. Seki, K. Masutani,
Phys. Rev. C27(1983)2799

$$b_0 + \alpha_s B_0 = \beta_s = (0.003 + 0.01i)\mathrm{m}_\pi^{-1}$$
$$\alpha_s \simeq 0.23\mathrm{m}_\pi^3$$

All potentials which satisfies the SM relation between potential parameters reproduce the experimental data.

Contour plot of B.E. ^{208}Pb 1s state



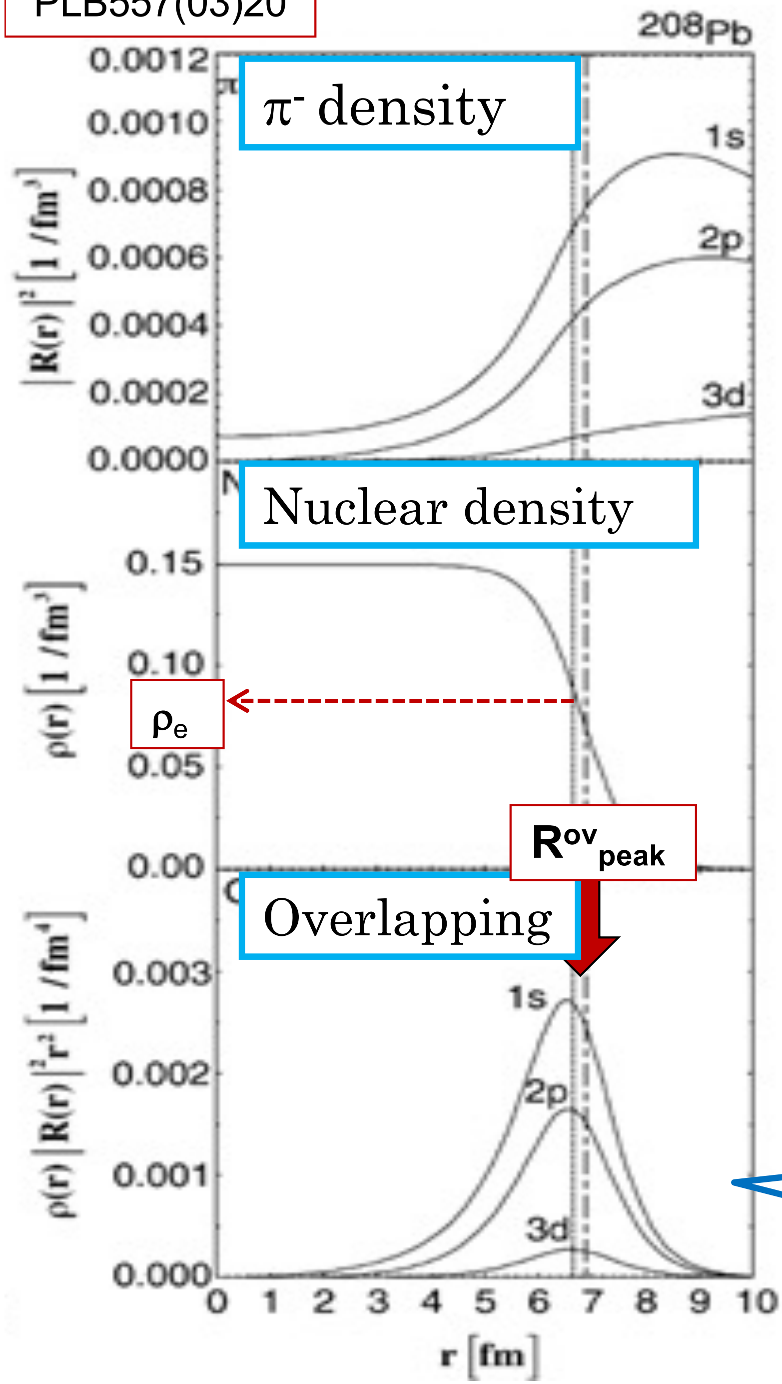
H. Toki, S. Hirenzaki,
T. Yamazaki, R. S. Hayano,
Nucl. Phys. 501(1989)653

Binding energies are almost unchanged along the line of SM relation.

=> Pionic atom properties are determined by potential strength at $\rho_e \sim 1/2\rho_0$

T. Yamazaki,
S. Hirenzaki
PLB557(03)20

Parameter correlation and Effective density



◆Peak positions of the overlapping density are almost same for all states.

◆The effective nuclear density ρe is almost same, ρe ~ 1/2ρ₀ for all states.

=> consistent with the expectation from the contour plot

$$S(r) = \underbrace{\rho(r)}_{N}\underbrace{|R_{nl}(r)|^2}_{\pi} r^2$$

: Overlapping density

Model independent analysis (here low density expressions)

In-medium pion and partial restoration of chiral symmetry

D. Jido[a,*], T. Hatsuda[b], T. Kunihiro[a,c]

Physics Letters B 670 (2008) 109–113

$$\frac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle} \simeq \left(\frac{b_1}{b_1^*}\right)^{1/2}\left(1-\gamma\frac{\rho}{\rho_0}\right)$$

, where

$$Z_\pi^{*1/2} \equiv \left(\frac{G_\pi^*}{G_\pi}\right)^{1/2} = 1-\gamma\frac{\rho}{\rho_0}$$

- ∗ Model independent (low density expression)
- ∗ Z_π : wave function renormalization
- ∗ Equivalent to GOR
- ∗ m_π* not necessary (but scattering length)

In-medium GOR

$$(F_\pi^t)^2 m_\pi^{*2} = -2m_q\langle\bar{q}q\rangle^*, \quad \rightarrow \quad \left(\frac{F_\pi^t}{F_\pi}\right)^2\left(\frac{m_\pi^*}{m_\pi}\right)^2 = \frac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle}$$

➔ Adopt these theoretical relations at the effective density

Another “prescription”

$$\sigma_{\pi N} = \frac{\bar{m}_q}{2m_N} \sum_{u,d} \langle N|\bar{q}q|N\rangle$$

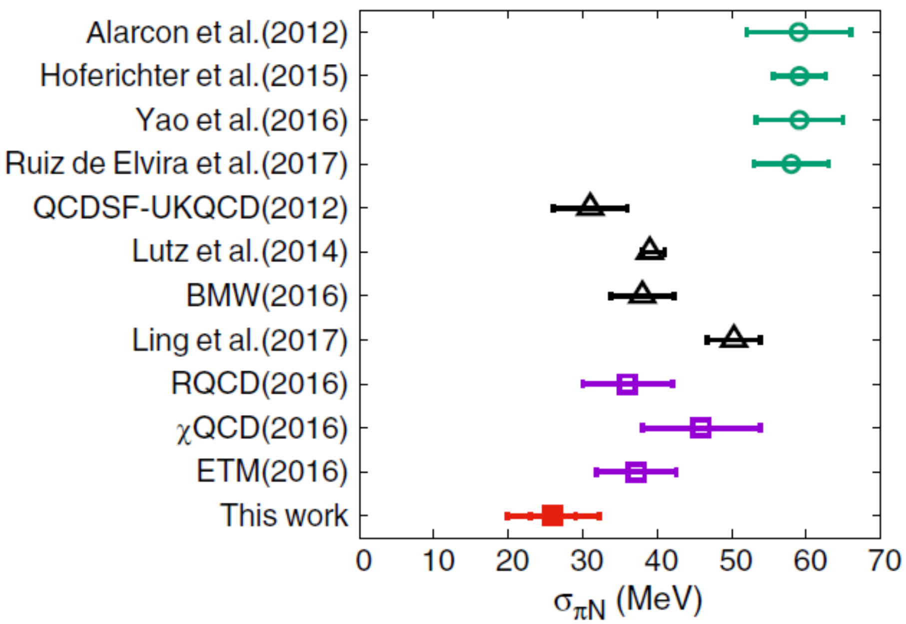
Pion-nucleon sigma term $\sigma_{\pi N}$ ``distribution''

$$\bar{m}_q = \frac{m_u + m_d}{2}$$

Nucleon charges with dynamical overlap fermions

PHYSICAL REVIEW D **98**, 054516 (2018)

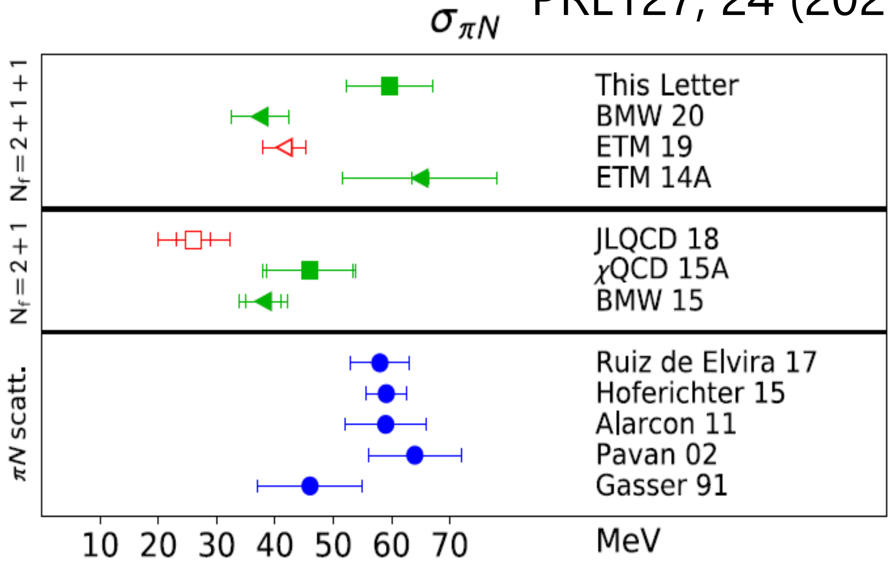
N. Yamanaka* et al., (JLQCD Collaboration)



The nucleon sigma term from lattice QCD

R. Gupta, S. Park, M. Hoferichter, E. Mereghetti, B. Yoon and T. Bhattacharya, [arXiv:2105.12095 [hep-lat]].

PRL127, 24 (2021)



The value of $\sigma_{\pi N}$ has not been determined accurately enough: **$\sigma_{\pi N}$ = 25~60 MeV**

=> It seems to be very interesting to determine the $\sigma_{\pi N}$ value by the deeply bound pionic atoms.

$\sigma_{\pi N}$ term in the optical potential

➢ Pion-Nucleus optical potential

$$2\mu V_{\text{opt}}(r) = -4\pi[b(r) + \varepsilon_2 B_0 \rho^2(r)] + 4\pi\nabla \cdot [c(r) + \varepsilon_2^{-1} C_0 \rho^2(r)] L(r) \nabla$$

$$b(r) = \varepsilon_1 \{b_0 \rho(r) + b_1 [\rho_n(r) - \rho_p(r)]\}$$

$$c(r) = \varepsilon_1^{-1} \{c_0 \rho(r) + c_1 [\rho_n(r) - \rho_p(r)]\}$$

$$L(r) = \{1 + \tfrac{4}{3}\pi\lambda[c(r) + \varepsilon_2^{-1} C_0 \rho^2(r)]\}^{-1}$$

$$b_1(\rho) = b_1^{\text{free}} \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)^{-1},$$

$$b_0(\rho) = b_0^{\text{free}} - \varepsilon_1 \frac{3}{2\pi}(b_0^{\text{free}2} + 2b_1^2(\rho)) \left(\frac{3\pi^2}{2}\rho\right)^{1/3}$$

- The $\sigma_{\pi N}$ value determined by the existing pionic atom data was reported:

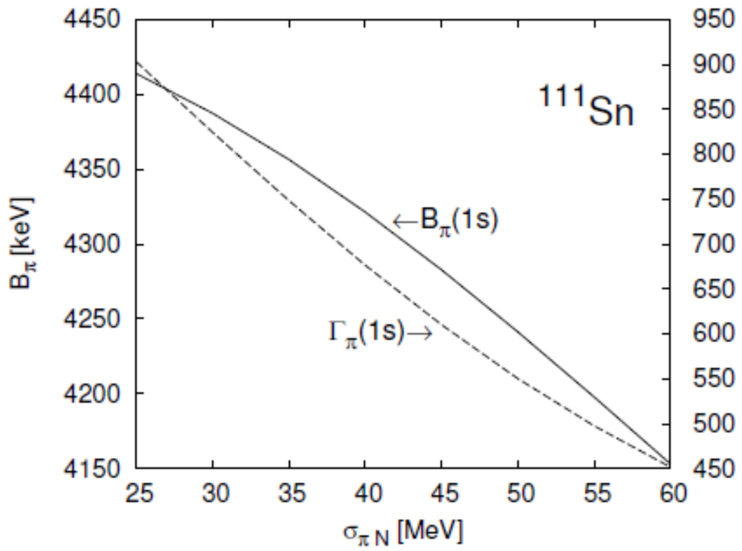
  χ^2 fitting for (all) atomic data (BE, Width)

➡ $\sigma_{\pi N}^{\text{FG}} = 57 \pm 7$ MeV,

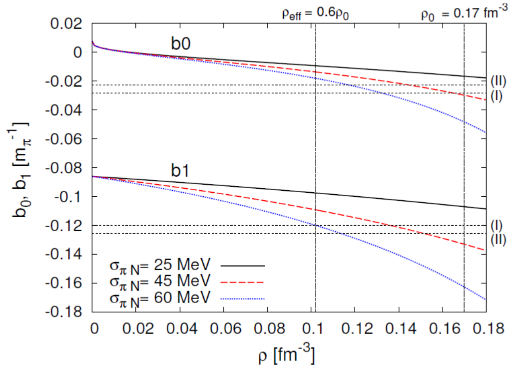
E. Friedman and A. Gal, Phys. Lett. B **792**, 340 (2019).
E. Friedman and A. Gal, Acta Phys. Polon. B **51**, 45-54 (2020).

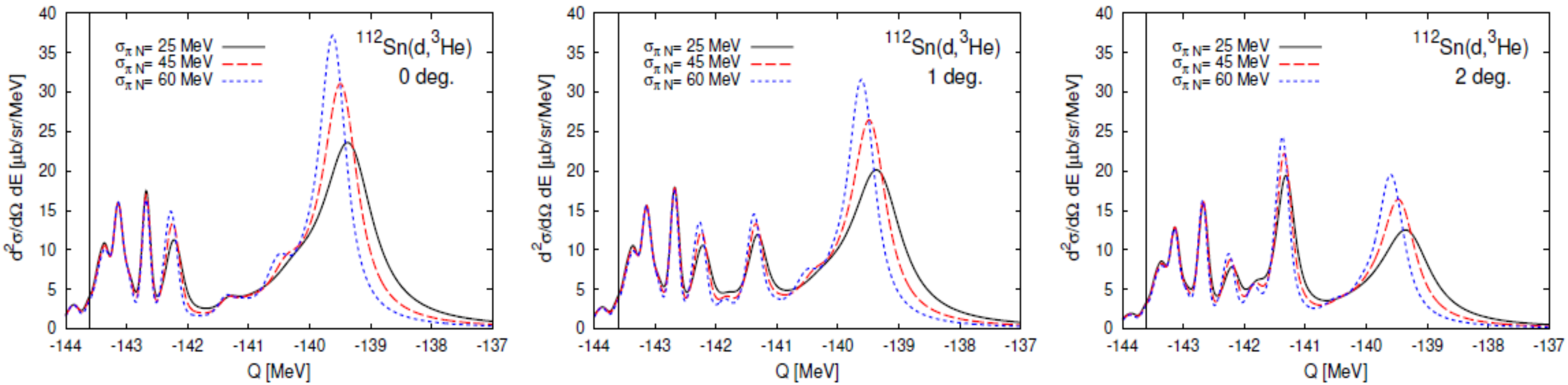
We especially focus on the observables of the high-precision deeply bound pionic states

$\sigma_{\pi N}$ term dependence of the pionic atom observables



N. Ikeno, T. Nishi, K. Itahashi, N. Nose-Togawa, A. Tani, S. Hirenzaki, arXiv:2204.09211 [nucl-th]





We can see clearly the strong sensitivities of the observables to $\sigma_{\pi N}$
=> It would be interesting to determine $\sigma_{\pi N}$ values from experimental data

Future Outlook

- Beyond the linear density (model independent)
- Another prescription ?, <u>GOR with b_0 for mass ?</u>

- Pionic atom in unstable nuclei by inverse kinematics

  chiral symmetry restoration in asymmetric nuclear matter,

  structure of unstable nuclei by pion

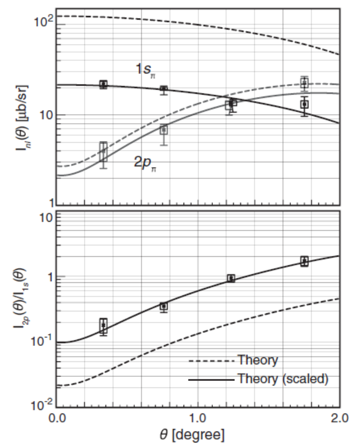
  Exp. of the d(^{136}Xe,^{3}He) at RIKEN in a few years ? K. Itahishi et al.

  Old works: Y. Umemoto et al. NPA679(2001)549, S. Hirenzaki et al. PLB194 (1987)20,

- Improvement of the theoretical calculations:

  To reproduce quantitatively the data by

  T. Nishi, K. Itahashi et al., PRL120, 152505 (2018)



- Extension to other meson systems
- Combined analysis with transport models such as JAM (for heavier meson sys.)

  (Y. Higashi, Master's thesis (Nara Women's 2015))

etc....