Speed of Sound beyond the conformal bound in Dense Two-Color QCD - Lattice Simulation Results -

Etsuko Itou (YITP, Kyoto U./ iTHEMS, RIKEN)

Based on K.Iida and EI, PTEP 2022 (2022) 11, 111B01

+ work in progress

第18回Hadron Spectroscopy Cafe, 東京工業大学　大岡山キャンパス, 2023/09/21

Conformal bound (Holography bound)

conjecture (A.Cherman et al., 2009)

maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

for a broad class of 4-dim. theories

A bound on the speed of sound from holography

Aleksey Cherman[*] and Thomas D. Cohen[†]
Center for Fundamental Physics, Department of Physics,
University of Maryland, College Park, MD 20742-4111

Abhinav Nellore[‡]
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

$c_s^2/c^2 > 1/3$ is found by Lattice Simulation in Dense Two-Color QCD

Etsuko Itou (RIKEN/ Keio U. / Osaka U.)

Based on K.Iida and EI, PTEP 2022 (2022) 11, 111B01

Our projects (2color QCD)

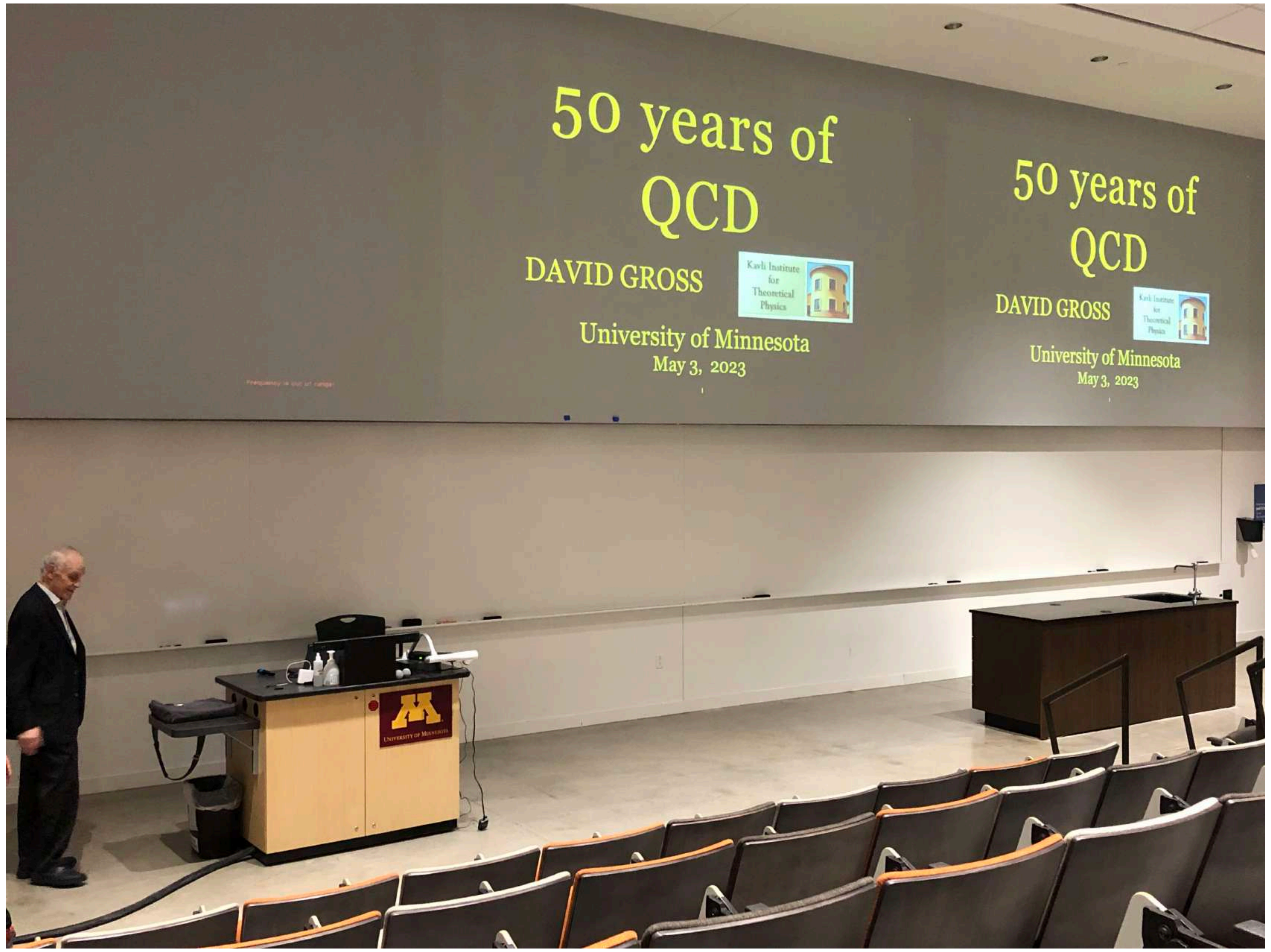
- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
  Phase diagram by Lattice simulation
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253
  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
  Scale setting of Lattice simulation
- K.Ishiguro, K.Iida, EI, arXiv: 2111.13067 (PoS, Lattice 2021)
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01
  Velocity of sound by Lattice simulation
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023)
  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472 (PoS, Lattice 2022)
  Mass spectrum by Lattice simulation
- K.Murakami, K.Iida, EI, arXiv:2309.08143
  Hadron scattering (HAL QCD method) by Lattice simulation

Plans

- Introduction
- 2color QCD phase diagram
- Equation of State
- Hadron spectrum and Hadron interaction

Why 2-color QCD at finite density?

Introduction



May, 2023 @ U. of Minnesota

Finite density QCD

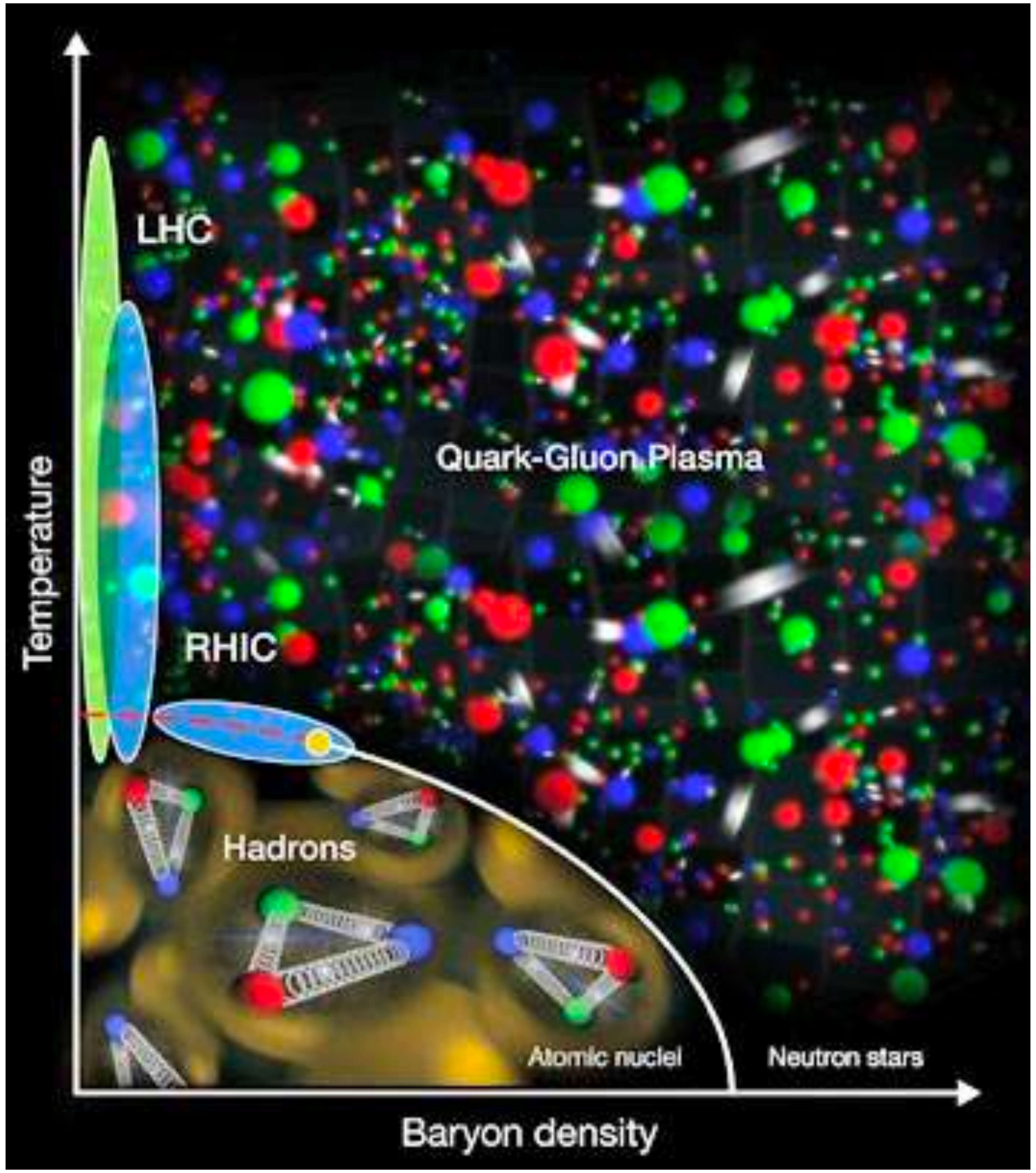
$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- At $\mu = 0$, QCD phenomena has been well understood for this 50 years
- We know almost nothing about QCD at finite density

Introduction

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

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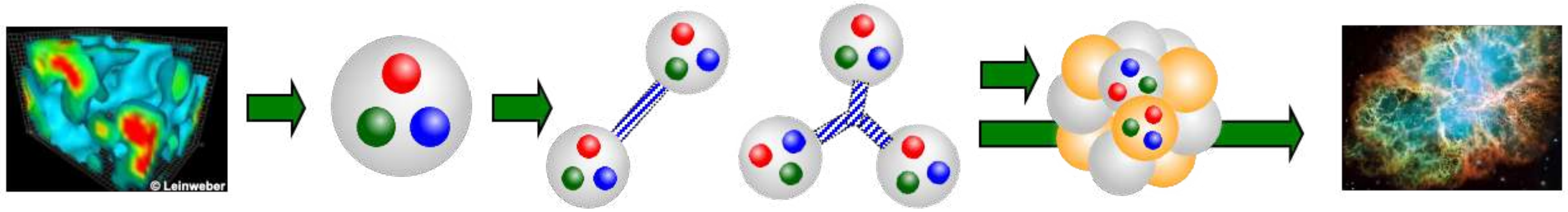
素粒子から宇宙まで第一原理計算を繋いだ理解



有限温度
ゼロ温度
QCD

ハドロン質量

ハドロン間
相互作用

核子多体系

宇宙の時間発展
現在の物質の性質

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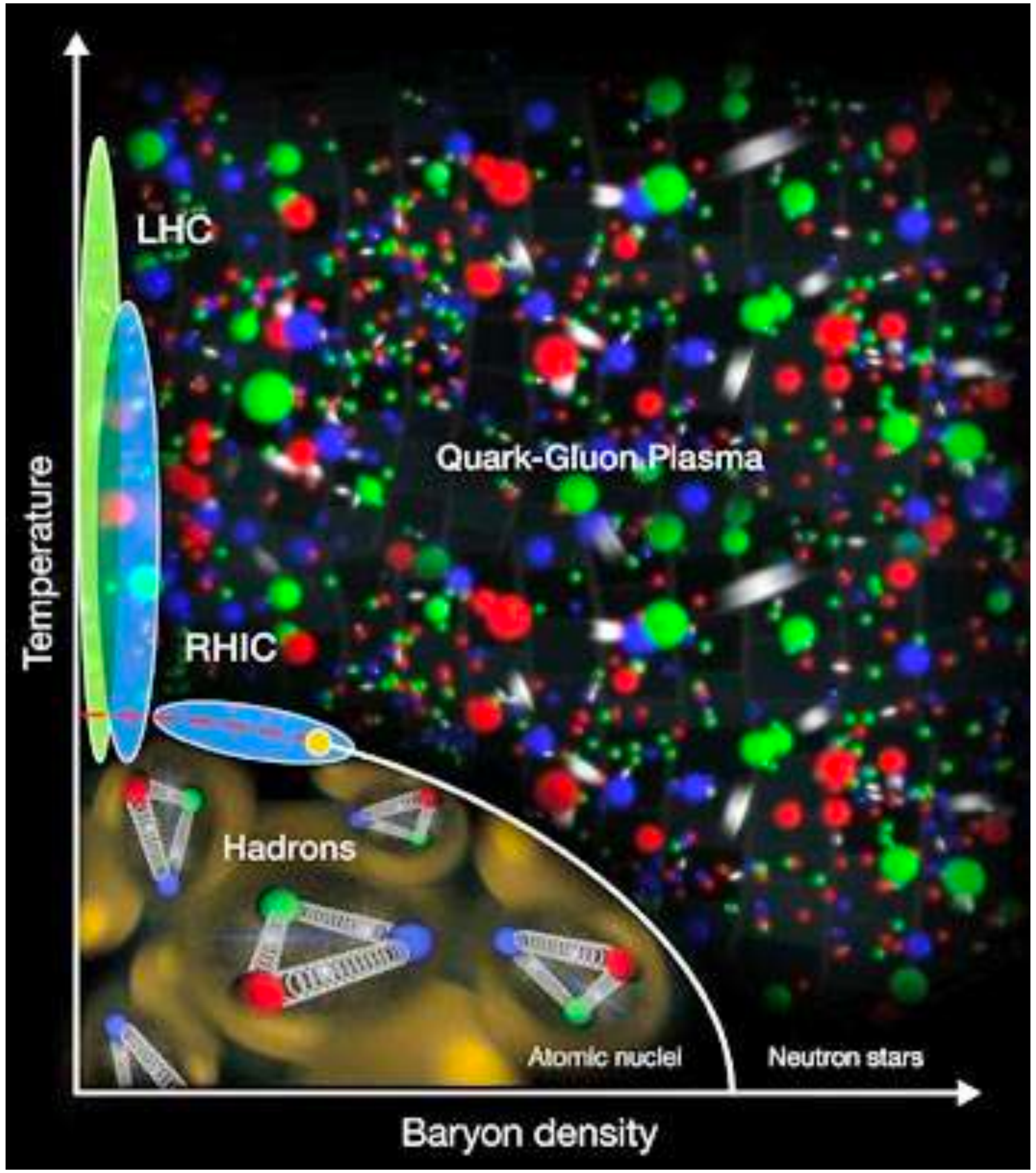

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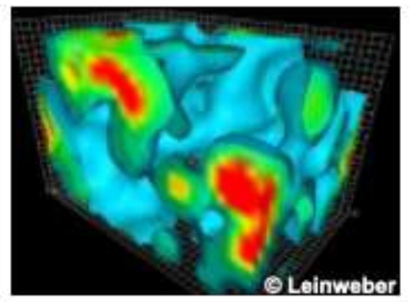

有限温度
ゼロ温度
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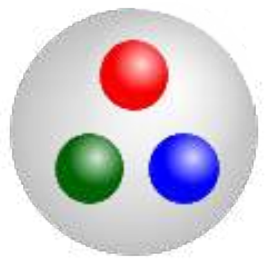
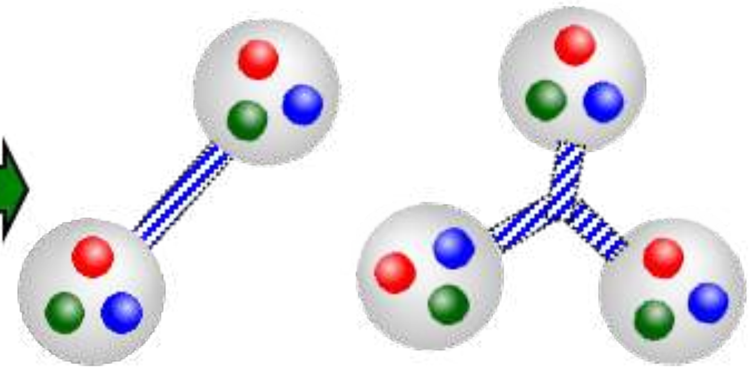
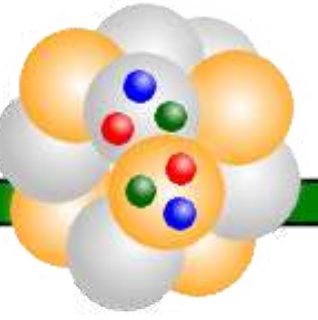


ハドロン質量



ハドロン間
相互作用



核子多体系

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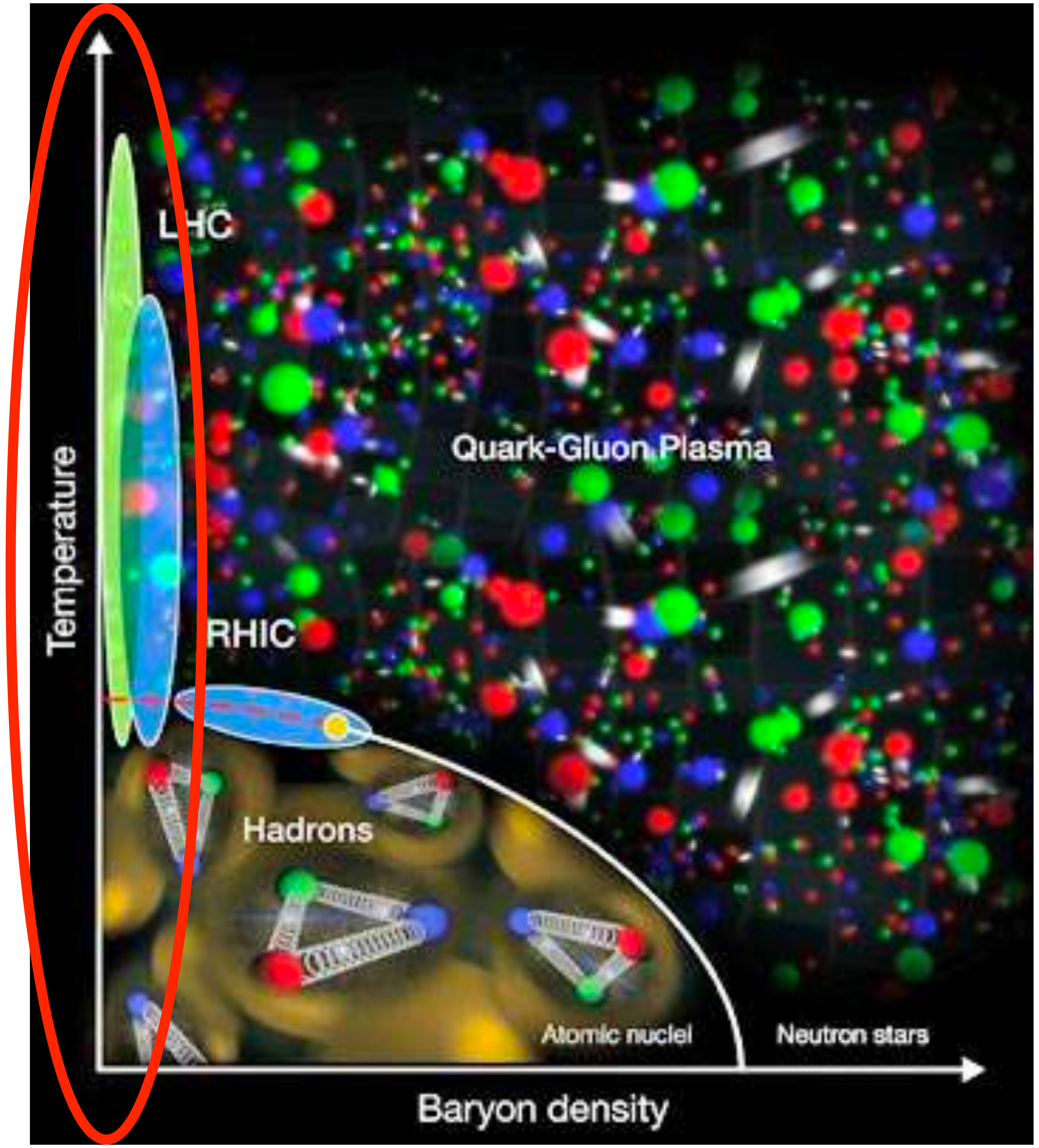

$\propto \mu$ ©BNL/RHIC

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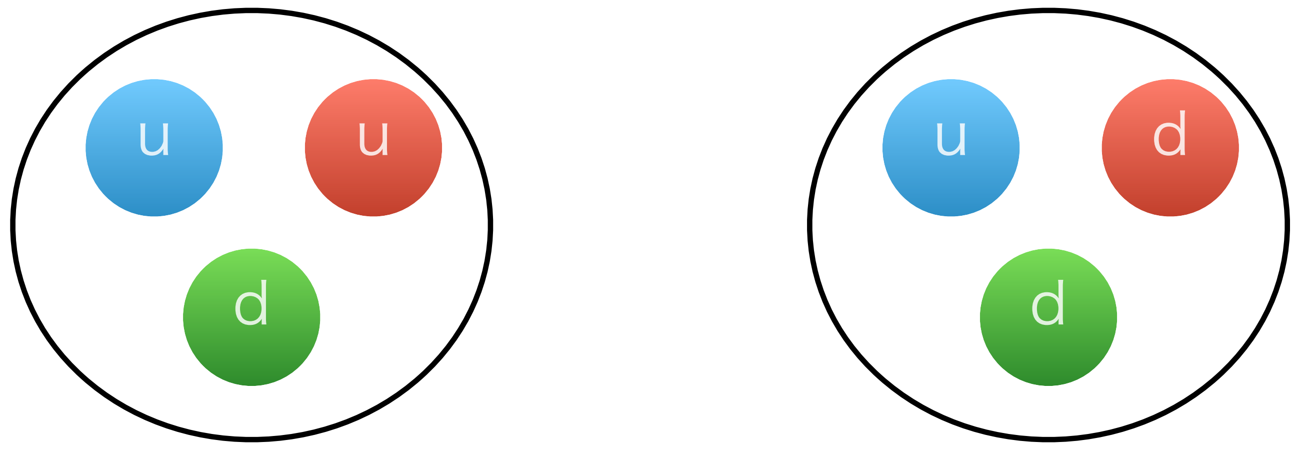
- Lattice gauge theory is only known nonperturbative and gauge invariant regularization method
- Finite-T QCD at $\mu = 0$ axis:

  studied by lattice MC and collider experiments

シミュレーションで明らかになったQCDの性質

- クォークの閉じ込め(1980年, M.Creutz)
- 真空の様子
  (作用密度(~エネルギー)の変化)
- クォークの質量とハドロンの質量
  (カイラル対称性の自発的破れ)
  u quark mass ~ 2-3MeV
  d quark mass ~ 5MeV



陽子~938MeV　中性子~940MeV

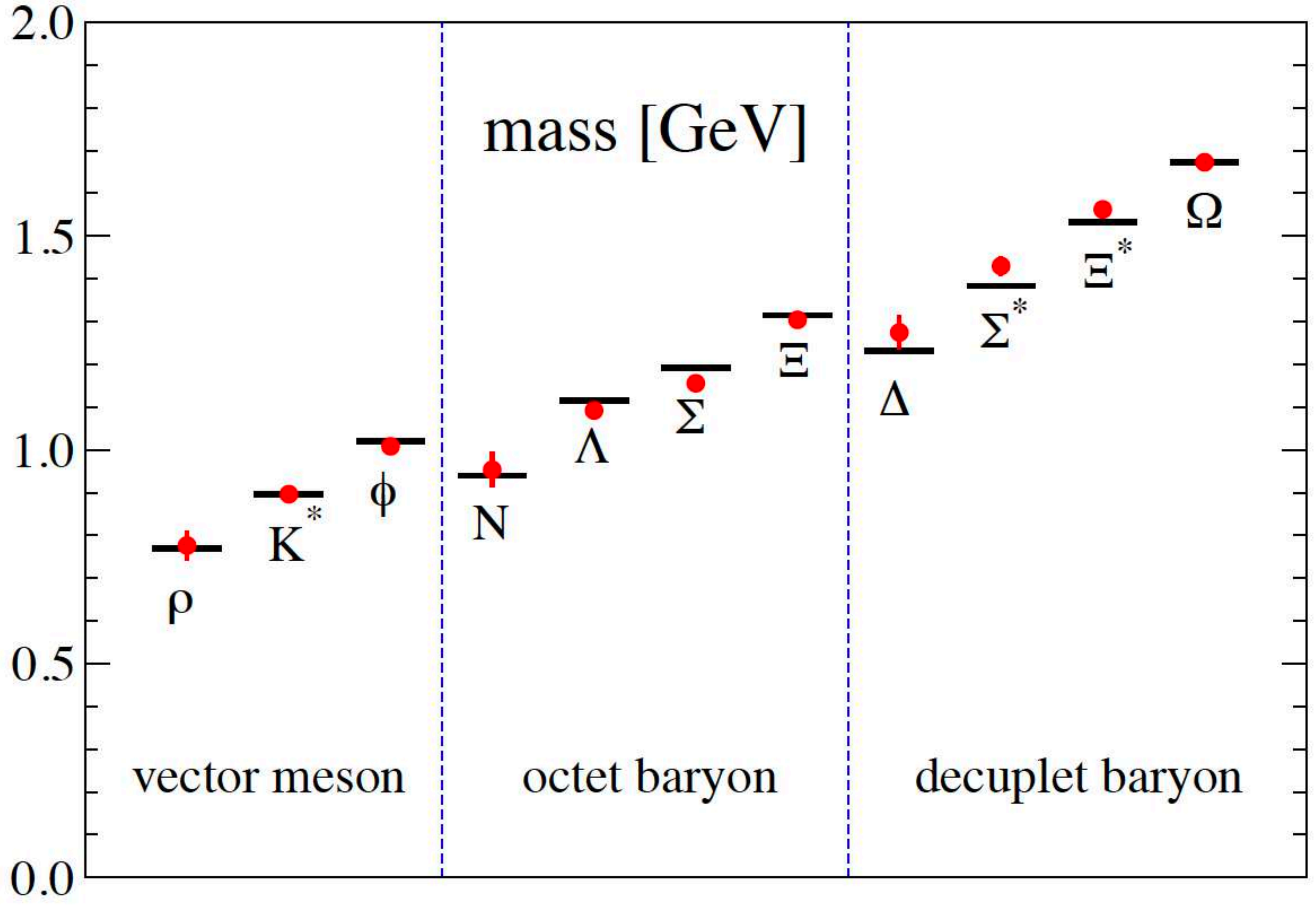


FIG. 20 The extrapolated $N_f = 2 + 1$ light hadron spectrum results from the PACS-CS collaboration. Experimental data are from (Amsler *et al.*, 2008). The plot is reproduced from (Aoki *et al.*, 2009a) with friendly permission of the PACS-CS collaboration.

Z.Fodor and C.Hoelbling
arXiv:1203.4789

QCD作用には３つのインプットしかないが, その３つでたくさん種類のハドロンの質量が実験結果と合う

シミュレーションで明らかになったQCDの性質

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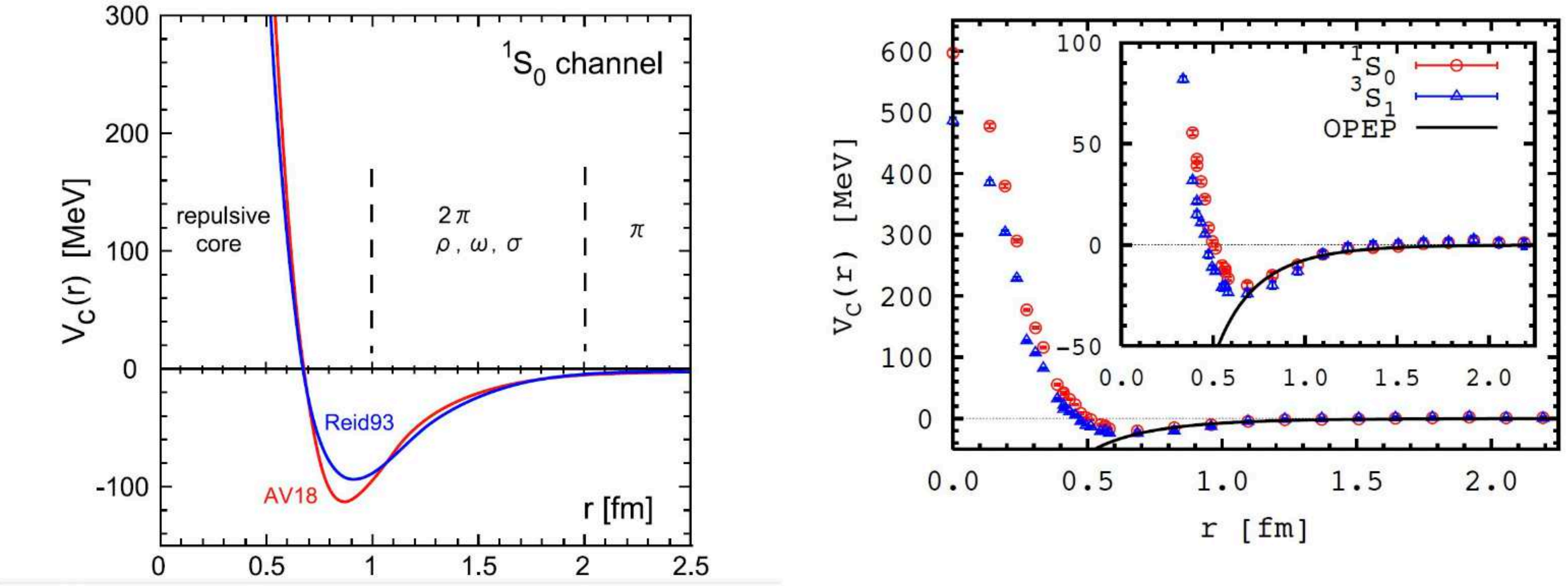
(カイラル対称性の自発的破れ)

- 核力ポテンシャル
核子同士が近いとパウリの排他率で斥力
その後,近距離(原子核の大きさ ~1fm)にだけ働く引力

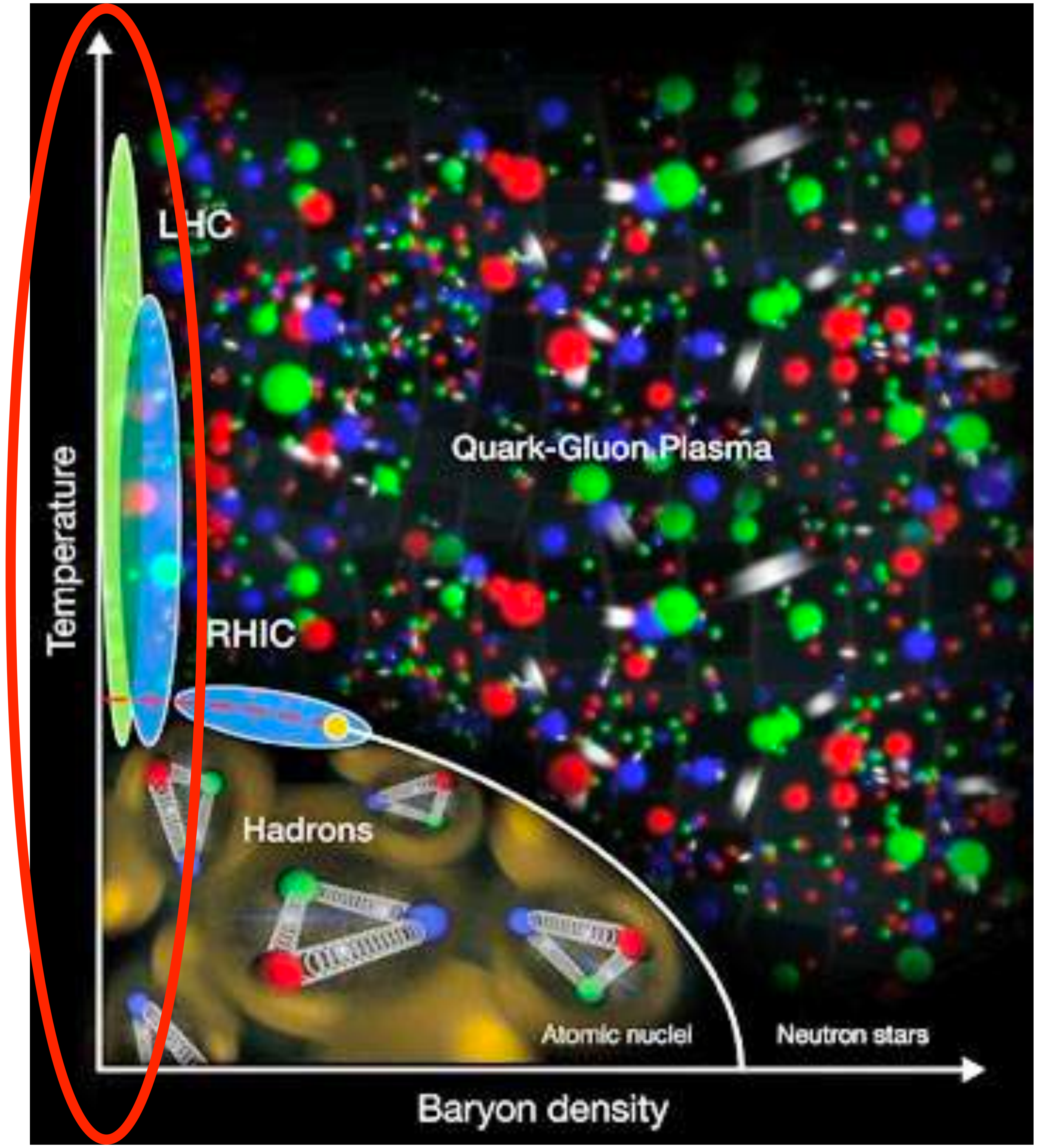


(左)現象論的核力ポテンシャルの例，(右)格子QCDを用いて計算した核力ポテンシャル

基礎物理学研究所HPから

Sound velocity: finite-T transition

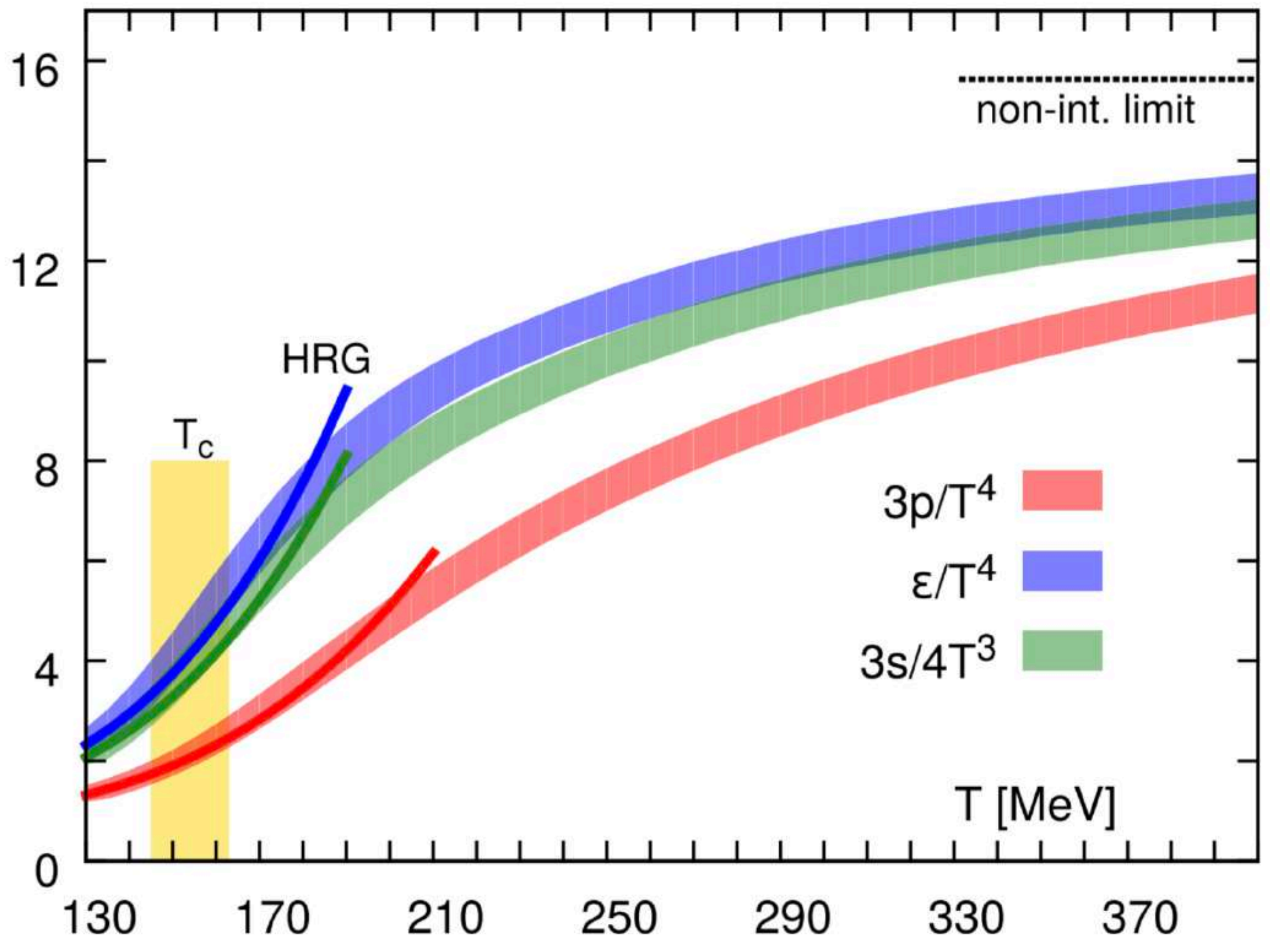
EoS and sound velocity at zero-μ



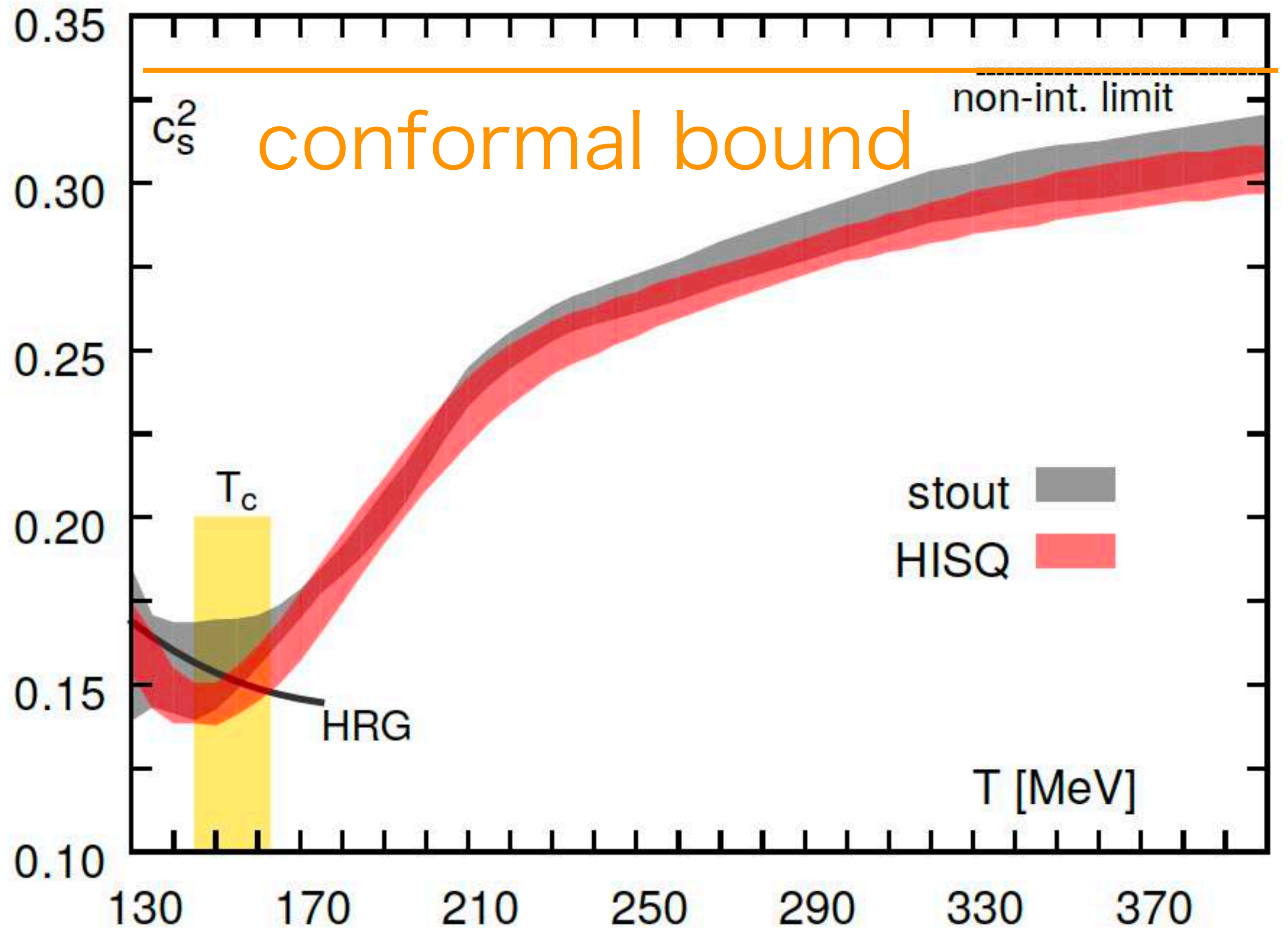
©BNL/RHIC

Finite Temperature transition (Nf=2+1 QCD)

EoS (p and ε)



Sound velocity $c_s^2 = \partial p / \partial \epsilon$

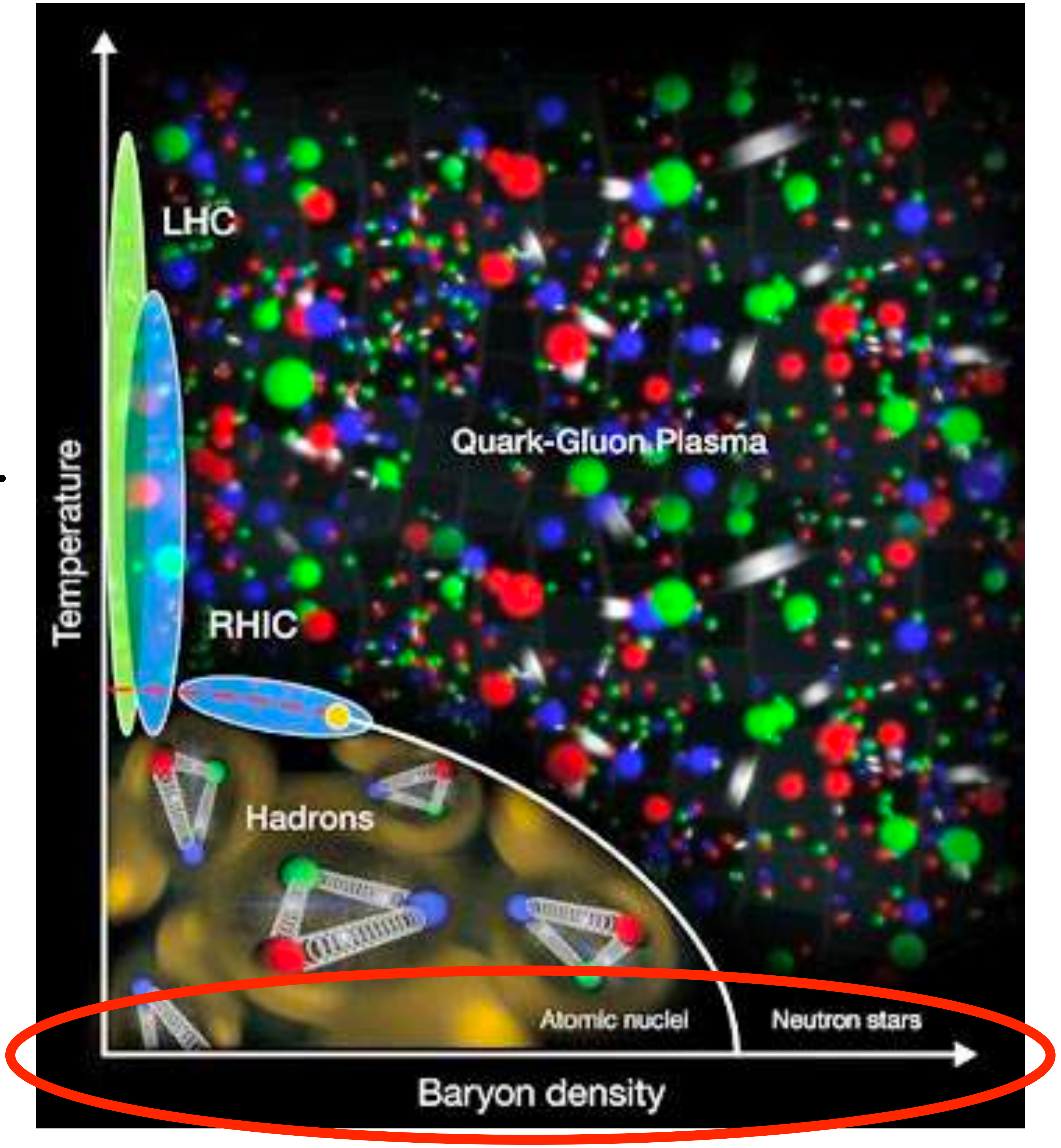


HotQCD (2014)

Introduction

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

Finite density QCD

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- In $\mu \neq 0$ regime, MC simulation suffers from the sign problem
  (理論を変えるか,アルゴリズムを変えるか)

  永田桂太郎：「有限密度格子QCDと符号問題の現状と課題」
  素粒子論研究Vol.31(2020) No.1
  Prog.Part.Nucl.Phys. 127 (2022) 103991・e-Print: 2108.12423 [hep-lat]

- Experiments:
  Neutron star observations are (will be) ongoing
  Gravitational wave, LISA, NICER,...

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expected QCD phase diagram

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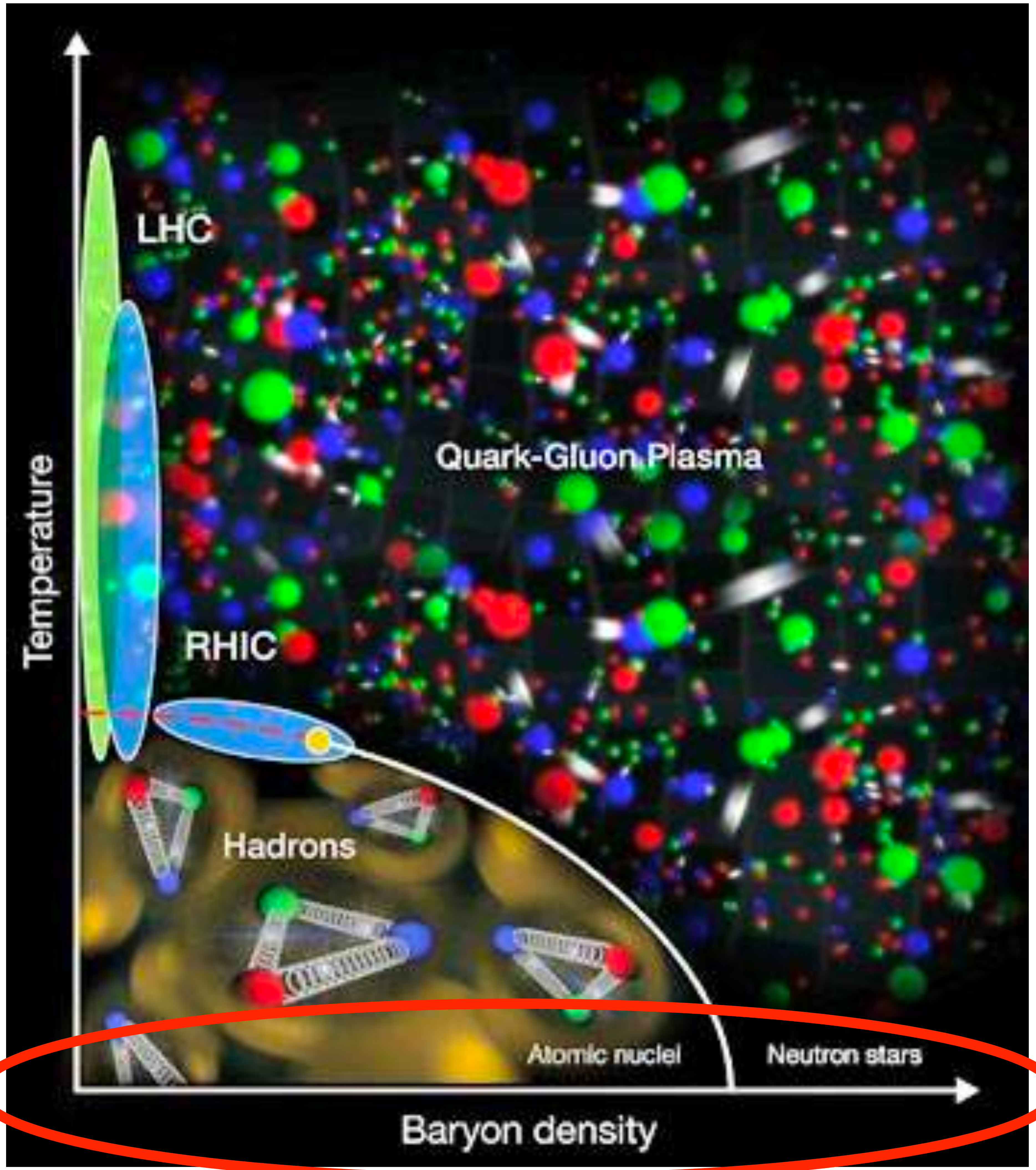

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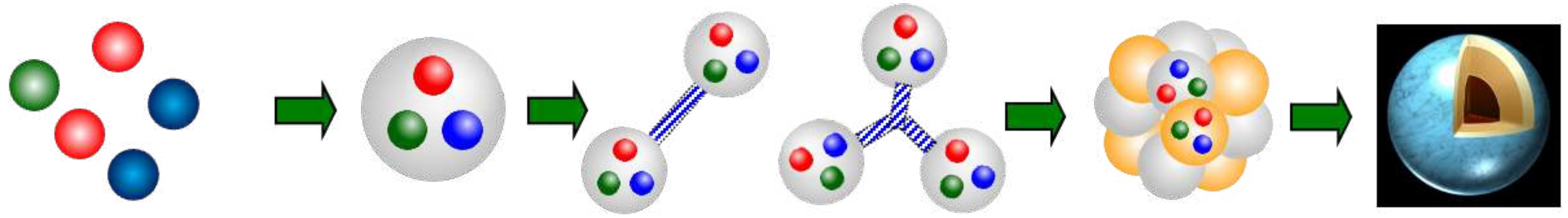

有限密度 QCD → 超流動相のハドロンの性質 → 超流動相のハドロン間の性質 → 多体系 → 中性子星

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expected QCD phase diagram

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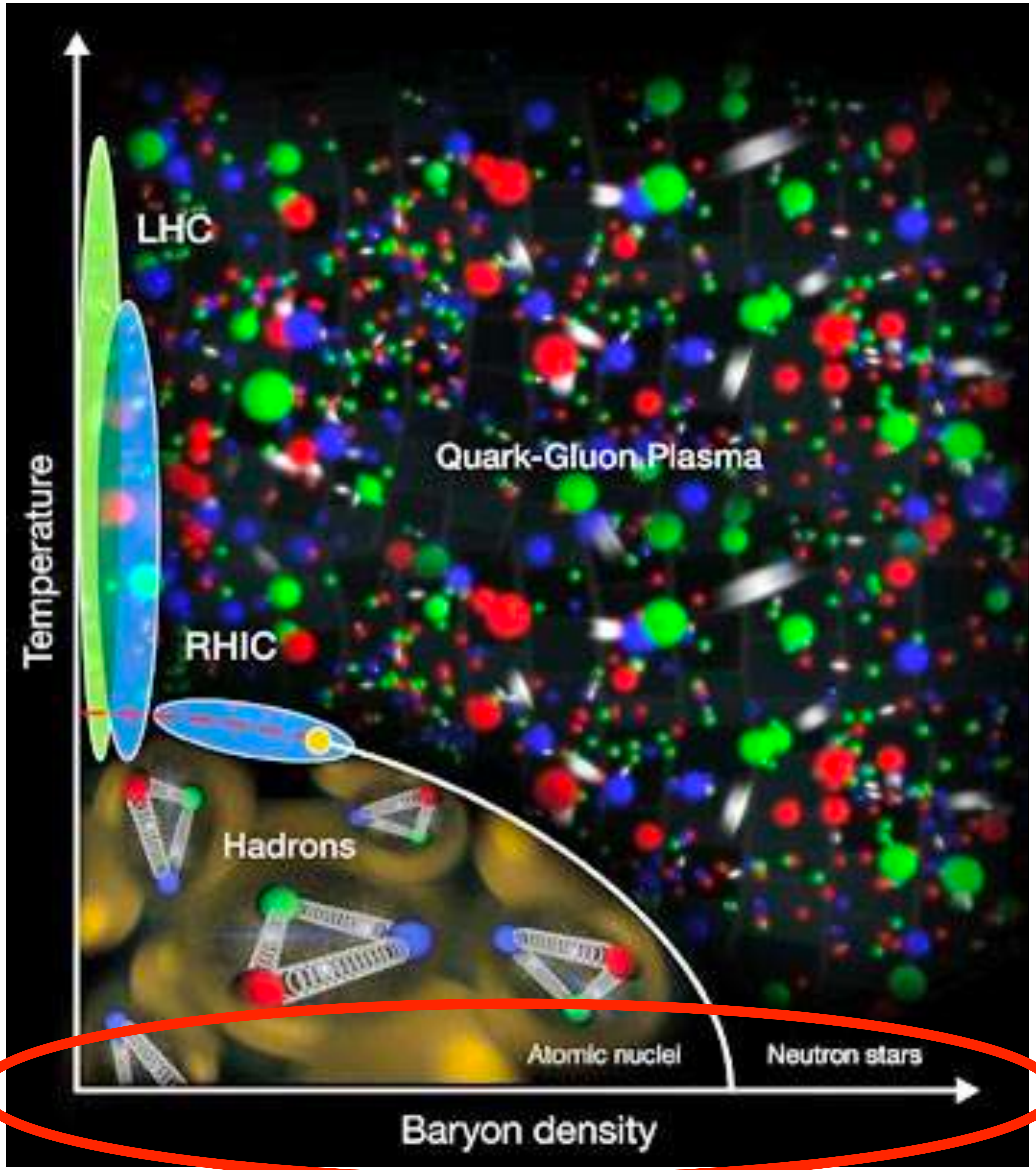

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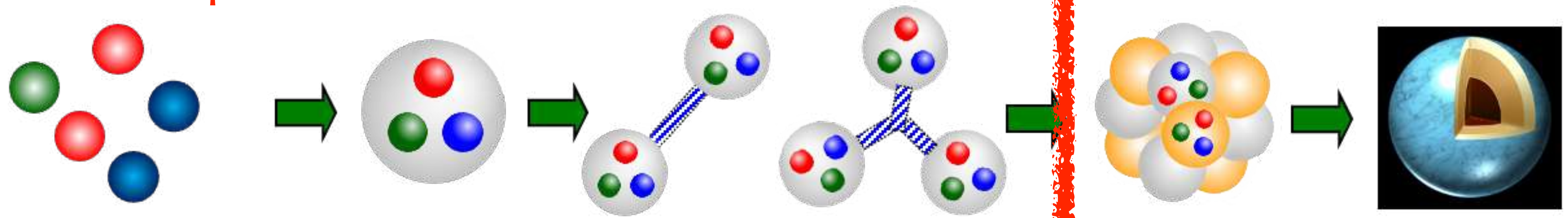
Cannot perform Lattice Monte Carlo



Introduction

expected QCD phase diagram

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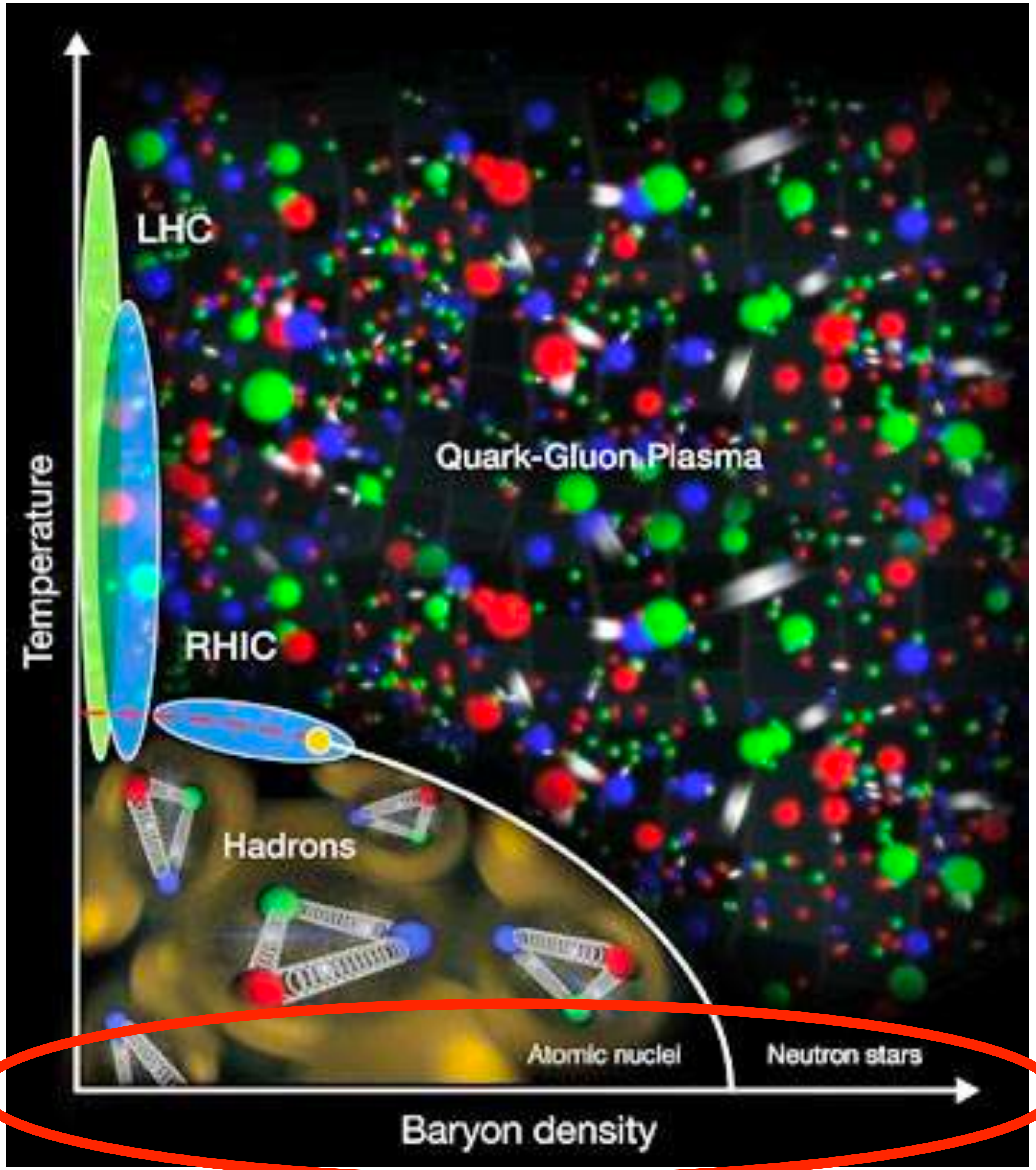

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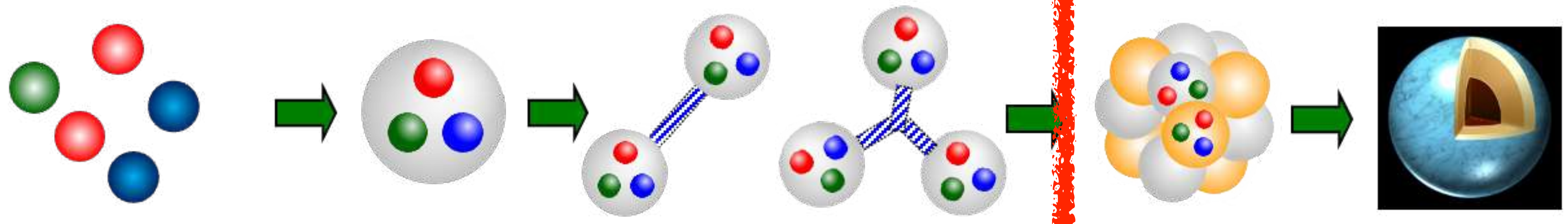
- In $\mu \neq 0$ regime, MC simulation suffers from the sign problem
(理論を変えるか,アルゴリズムを変えるか)

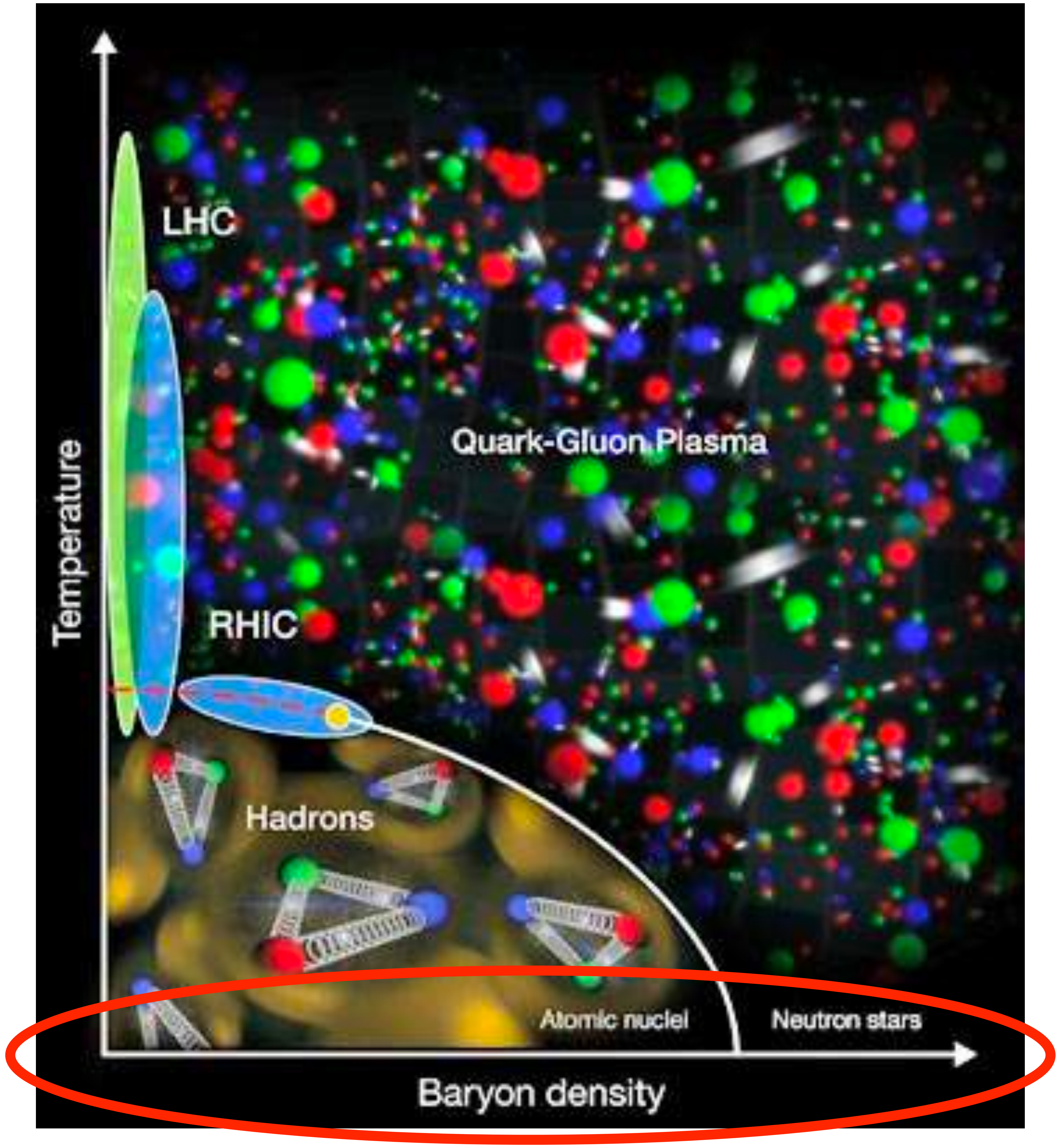
2color dense QCD is doable!!



有限密度QCD → 超流動相のハドロンの性質 → 超流動相のハドロン間の性質 → 多体系 → 中性子星

Sound velocity: finite density regime

EoS and sound velocity at low-T and high-μ



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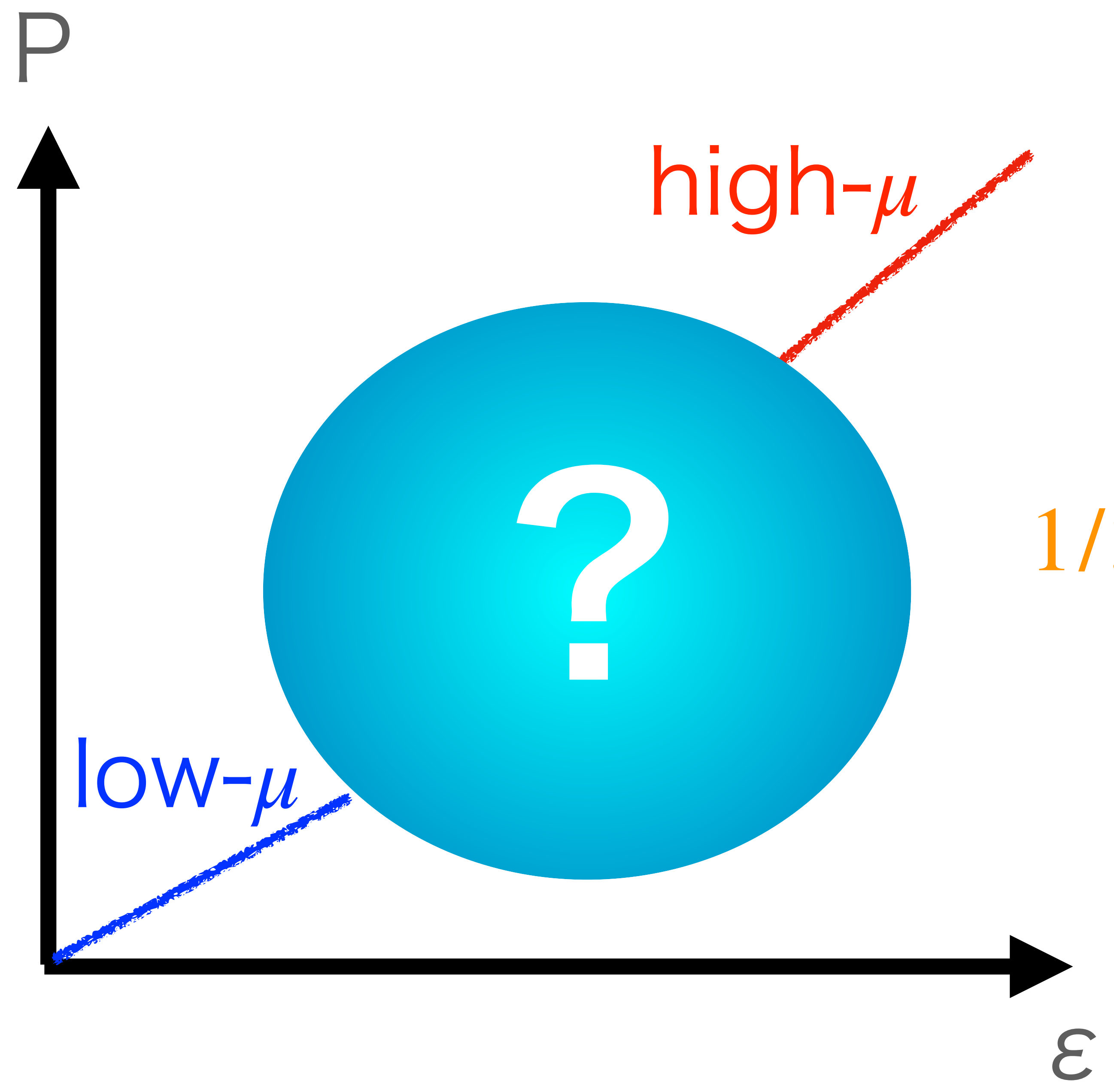
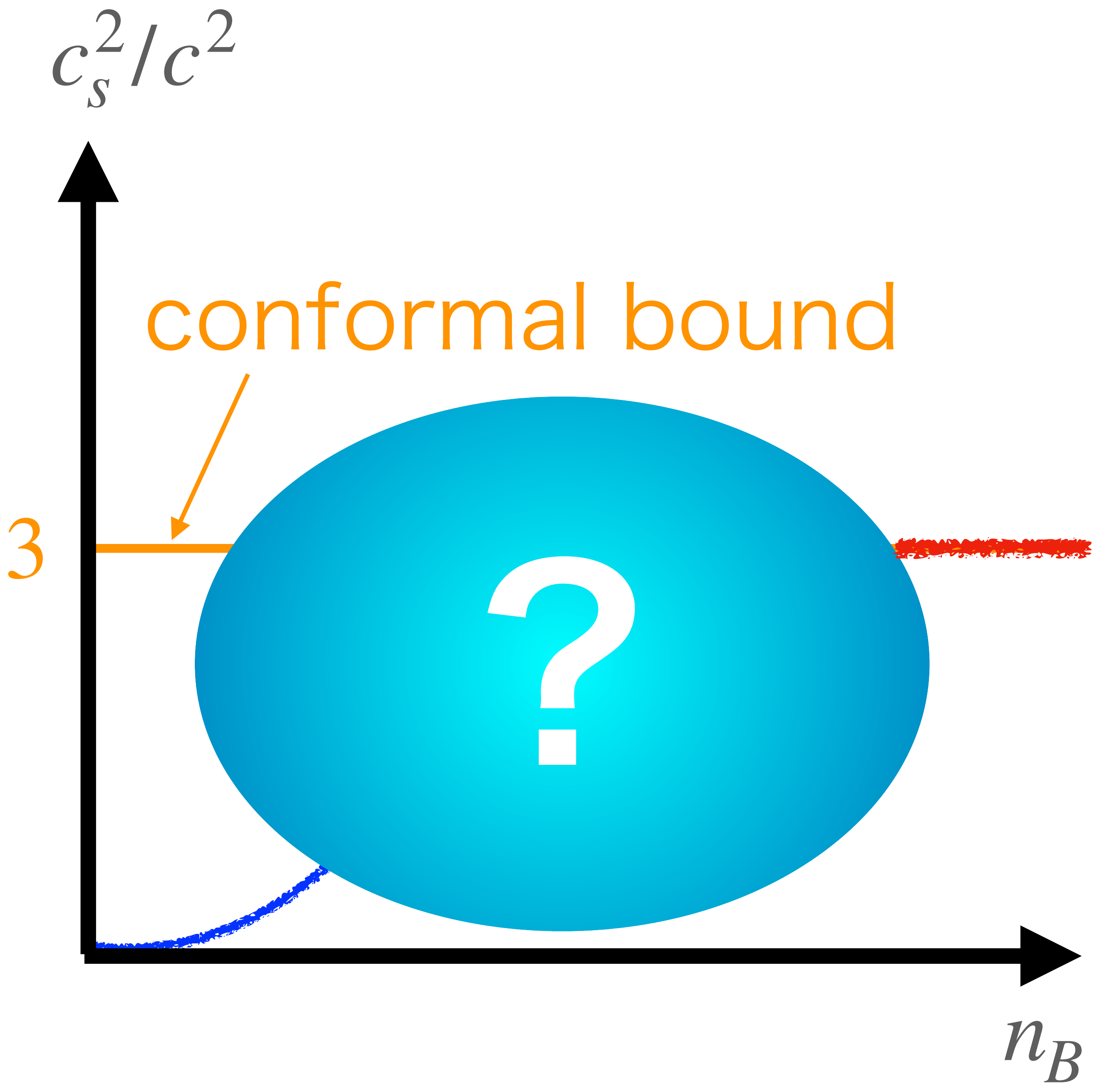
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$



low -μ ($n_B \lesssim 2n_0$): Hadronic matter

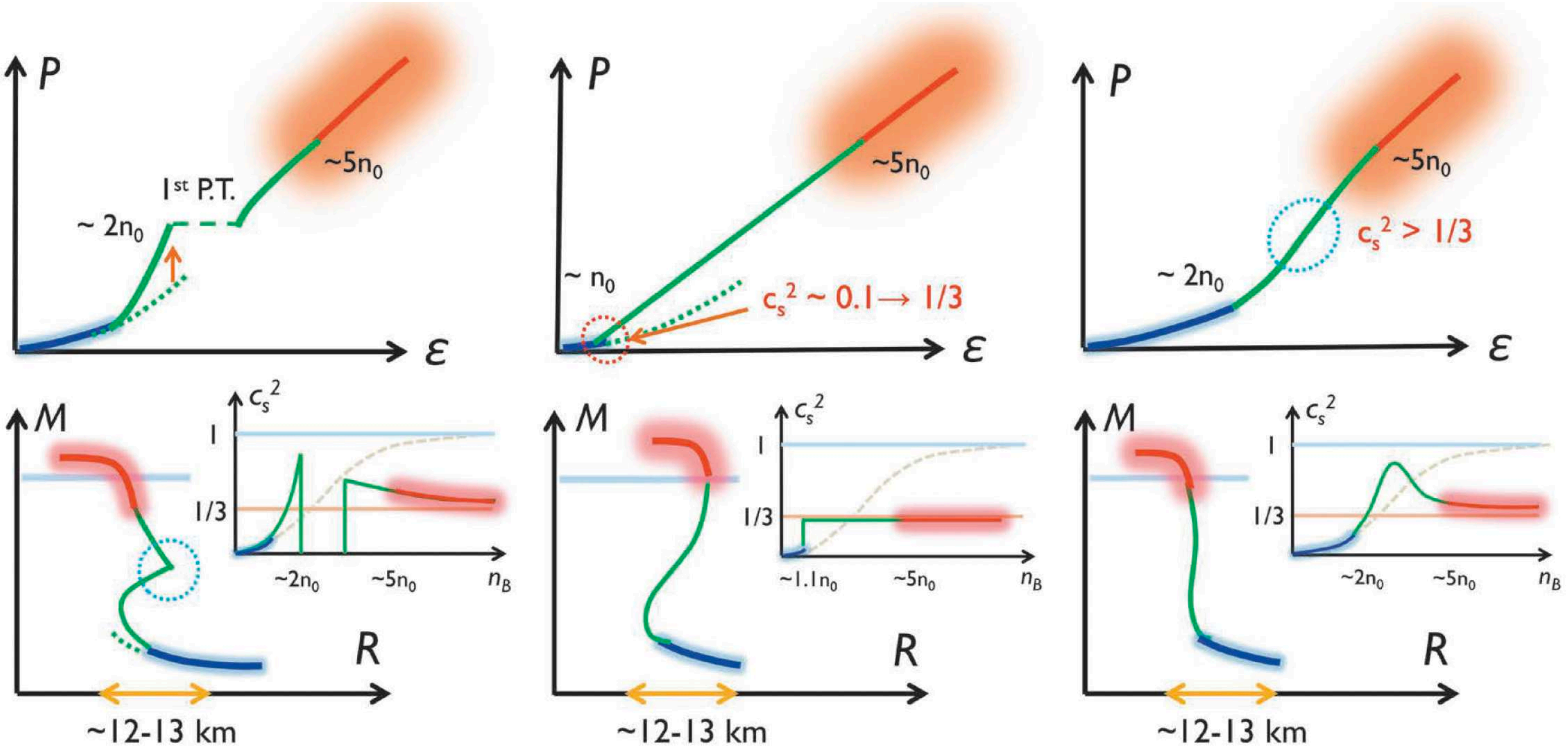
high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

EoS, c_s and neutron star

Mass and radius of neutron star

T. Kojo, arXiv:2011.10940



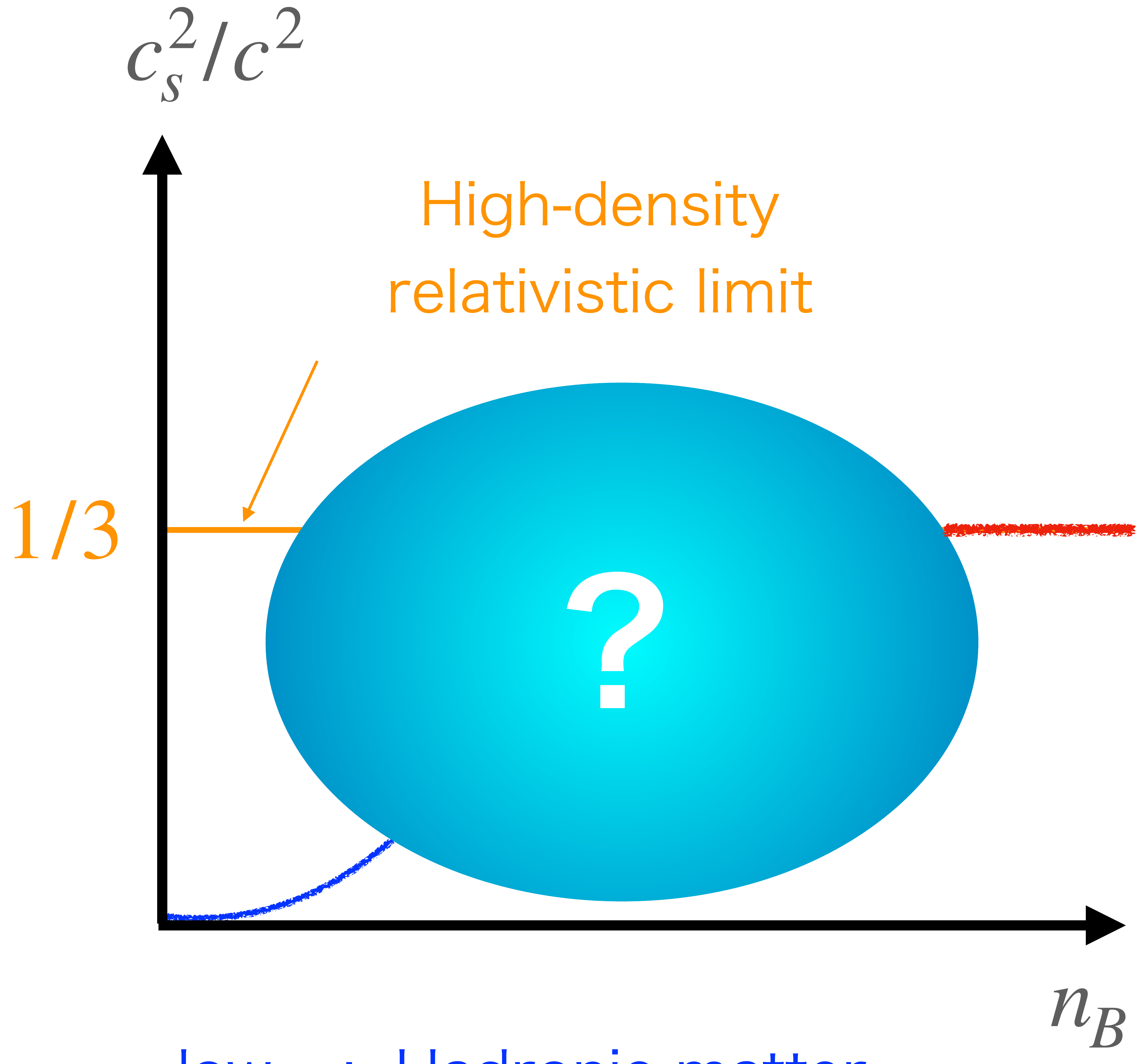
Sound velocity

$$c_s^2 = \partial p / \partial \epsilon$$

Mass-Radius of neutron star <=> EoS in dense QCD

Prediction by phenomenology and effective models

Sound velocity has a peak?



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

Masuda,Hatsuda,Takatsuka (2013)
Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model c_s^2 peaks at $n_B = 1 - 5n_0$

McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (work for any # of color)

Kojo (2021), Kojo and Suenaga (2022)

→ Lattice study on 2color dense QCD the sign problem is absent!!

Two problems at low-T high-μ QCD

- Sign problem (at $\mu \neq 0$ $S_E[U]$ takes complex value)

➡ Reduce the color dof, 2color QCD
quarks becomes pseudo-real reps.
The sign problem is absent from 2color QCD with even Nf

- Onset problem in low-T, high-μ (e.g. $\mu_q > m_\pi/2, \ m_N/3$),

It comes from the phase transition to superfluid phase(SSB of baryon sym.)

Kogut et al. NPB642 (2002)18

➡ Add an explicit breaking term of the sym., then take $j \to 0$ limit

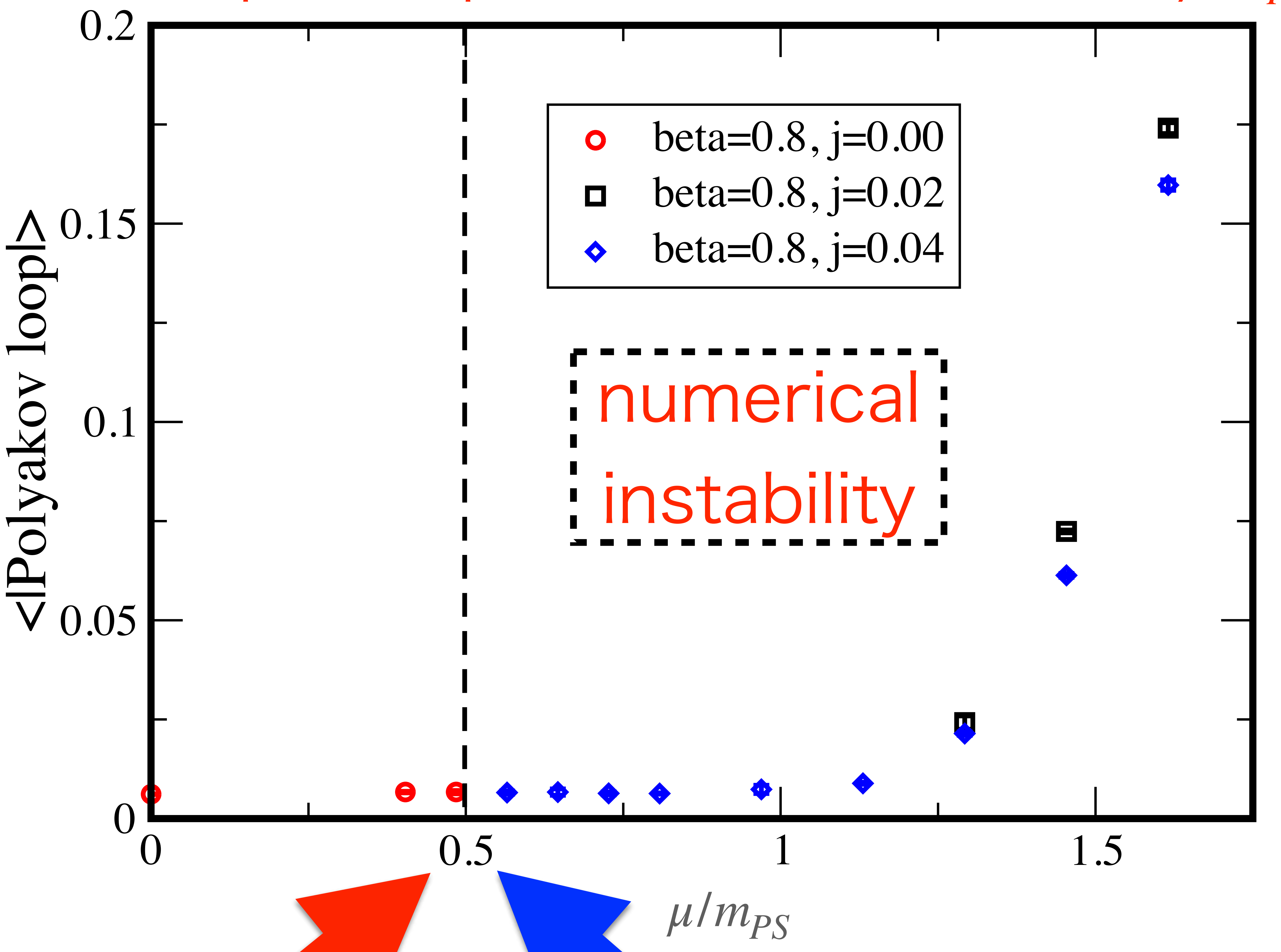
$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

HMC simulations for whole T-μ regime are doable!

(j->0 extrapolation is taken in all plots today)

HMC calculation w or w/o diquark source term

According to chiral perturbation theory,
the hadronic-superfluid phase transition occurs at $\mu/m_{PS} \sim 0.5$



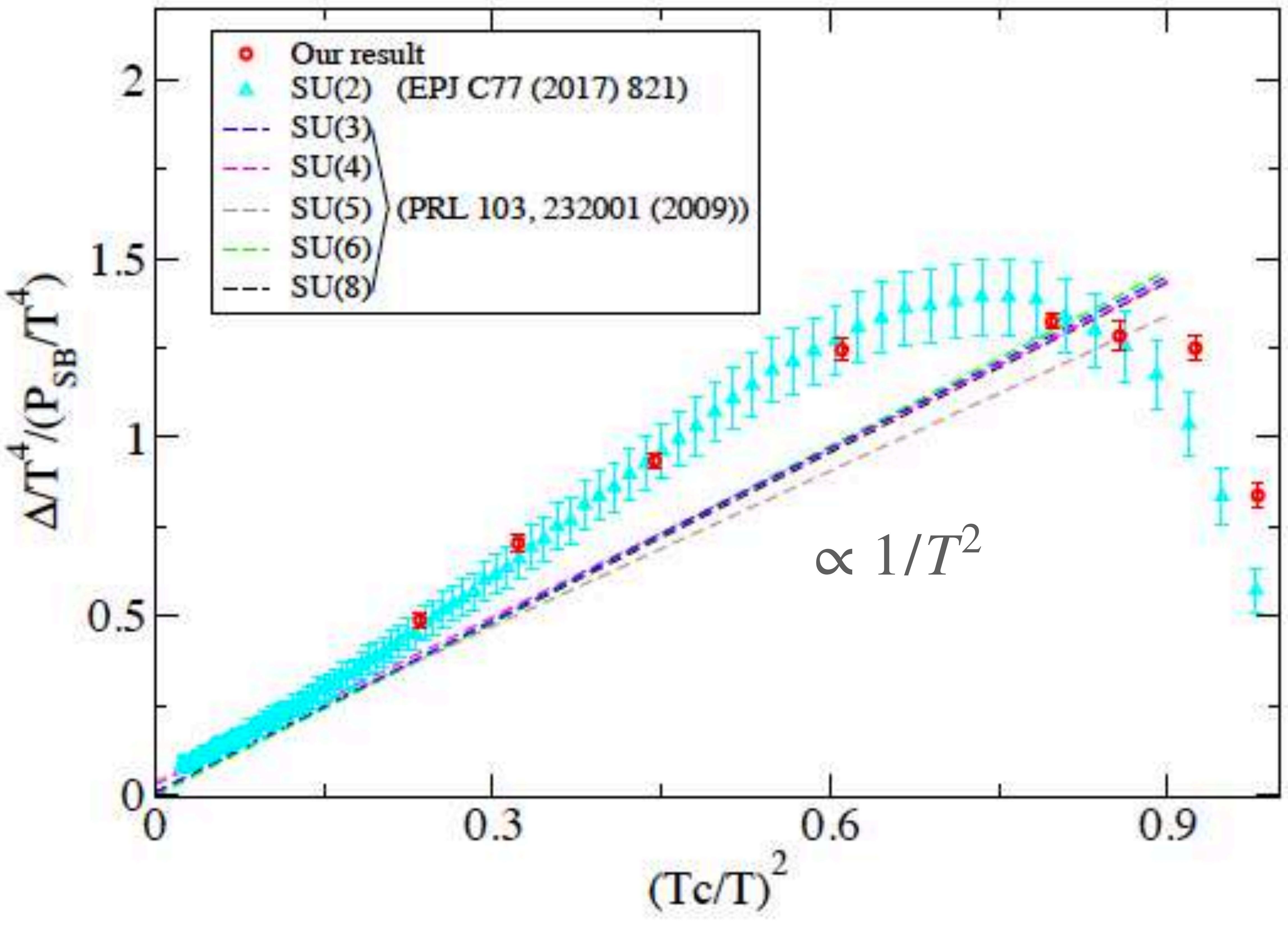
HMC without j is doable
(minimum MC step ~ 1/800)

HMC without j cannot run even with
a tiny MC step(~1/1000)

2color QCD ≈ 3color QCD at $\mu = 0$

EoS shows very similar at least quenched QCD case

Trace anomaly ($\Delta = (\epsilon - 3p)$) of pure SU(Nc) gauge theories with several Nc



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3color QCD: a=1 - 8

2color QCD: a=1 - 3

T. Hirakida, EI, H. Kouno
PTEP 2019 (2019) 033B01

2color QCD phase diagram

(1) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067

(2) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0

(3) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

(4) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Flavor symmetry and its breaking

Nf=2 Nc=3

standard symmetry

$SU(2)_L \times SU(2)_R \times$ ~~$U(1)_A$~~ $\times U(1)_B$

$m = 0, \mu = 0$

chiral condensation

$SU(2)_V \times U(1)_B$

enhanced flavor symmetry in Nc=Nf=2

$SU(4)$ $\quad m = 0, \mu = 0$

$m > 0$

$Sp(4) \simeq SO(5)$

5 pseudo-Goldston bosons

$3\pi, \langle qq \rangle, \langle \bar{q}\bar{q} \rangle$

$\mu \gg 0$

$Sp(1)_L \times Sp(1)_R \times U(1)_B$

partially restored chiral sym.

$\mu > 0$

$m > 0$

$SU(2)_V \times U(1)_B$

meson-baryon sym.

diquark condensation

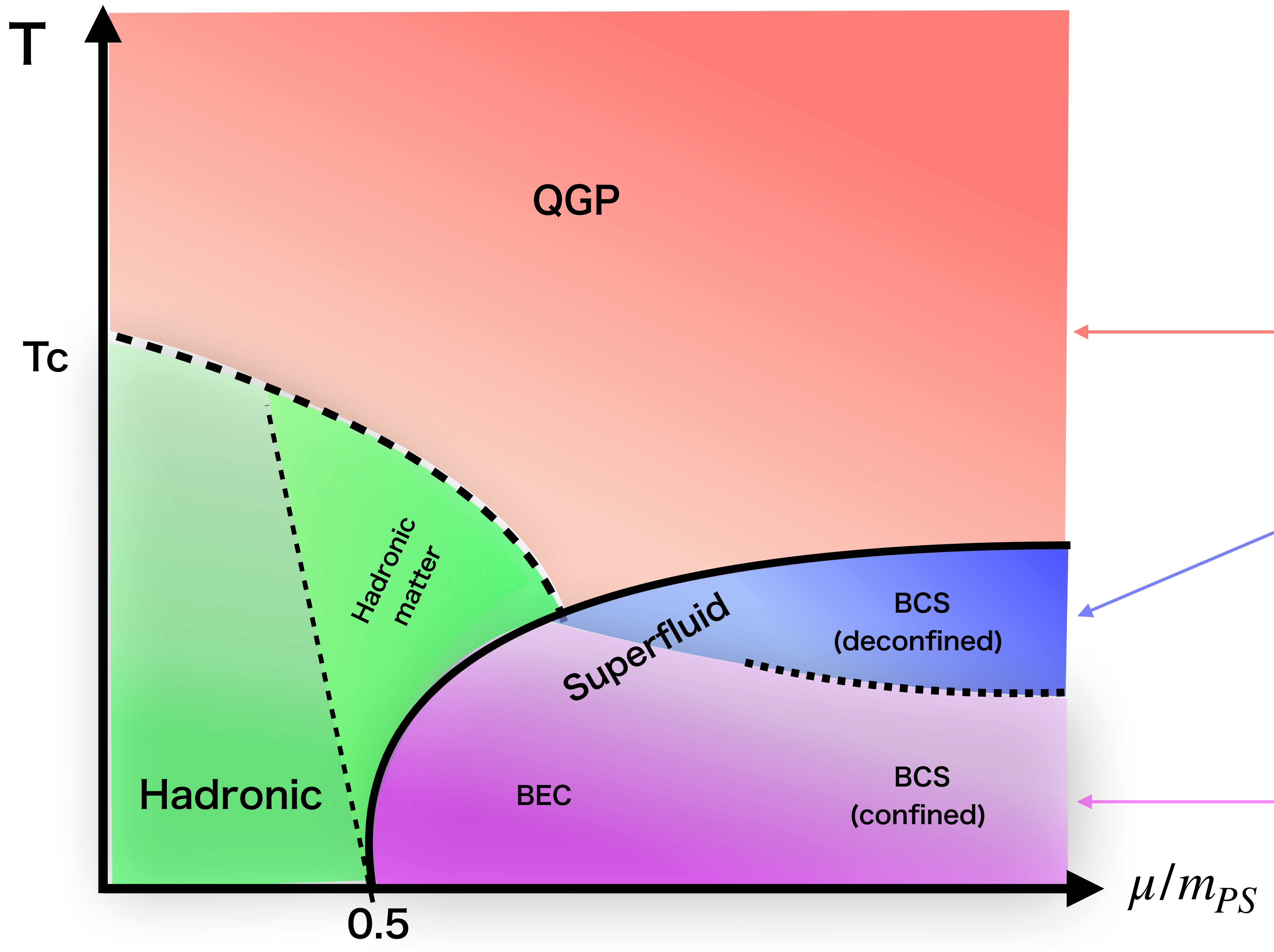
$Sp(1)_V \simeq SU(2)_V$

U(1)B

$\psi \to e^{i\alpha}\psi \quad \bar{\psi} \to \bar{\psi}e^{-i\alpha}$

diquark condensate plays an alternative role of chiral condensate to study phase in finite μ regime.

Current status on 2color QCD phase diagram



At least Four independent group studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

- T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
- T=130 MeV (**deconfined? QGP phase?** , 2019)

- T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
- T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

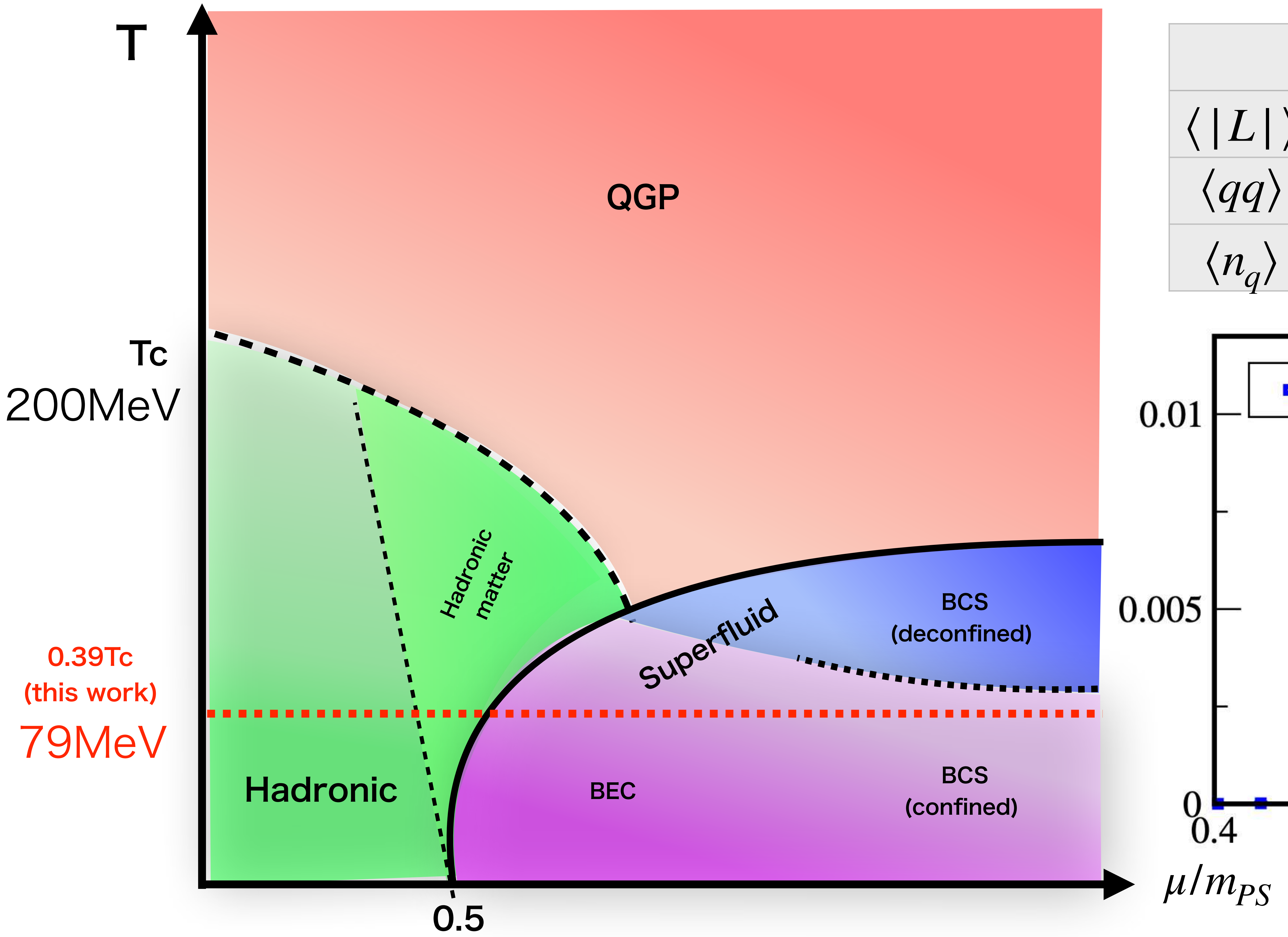
- T=87 MeV (**confined** in 2019)
- T=79 MeV (**confined** even in high mu)
- T=55 MeV (**confined** in high mu, 2016)
- T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
- T=45 MeV (**confined** in 2019)

- Even $T \approx 100\mathrm{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq T_{SF}$ (superfluid/QGP) : constraint from 't Hooft anomaly matching

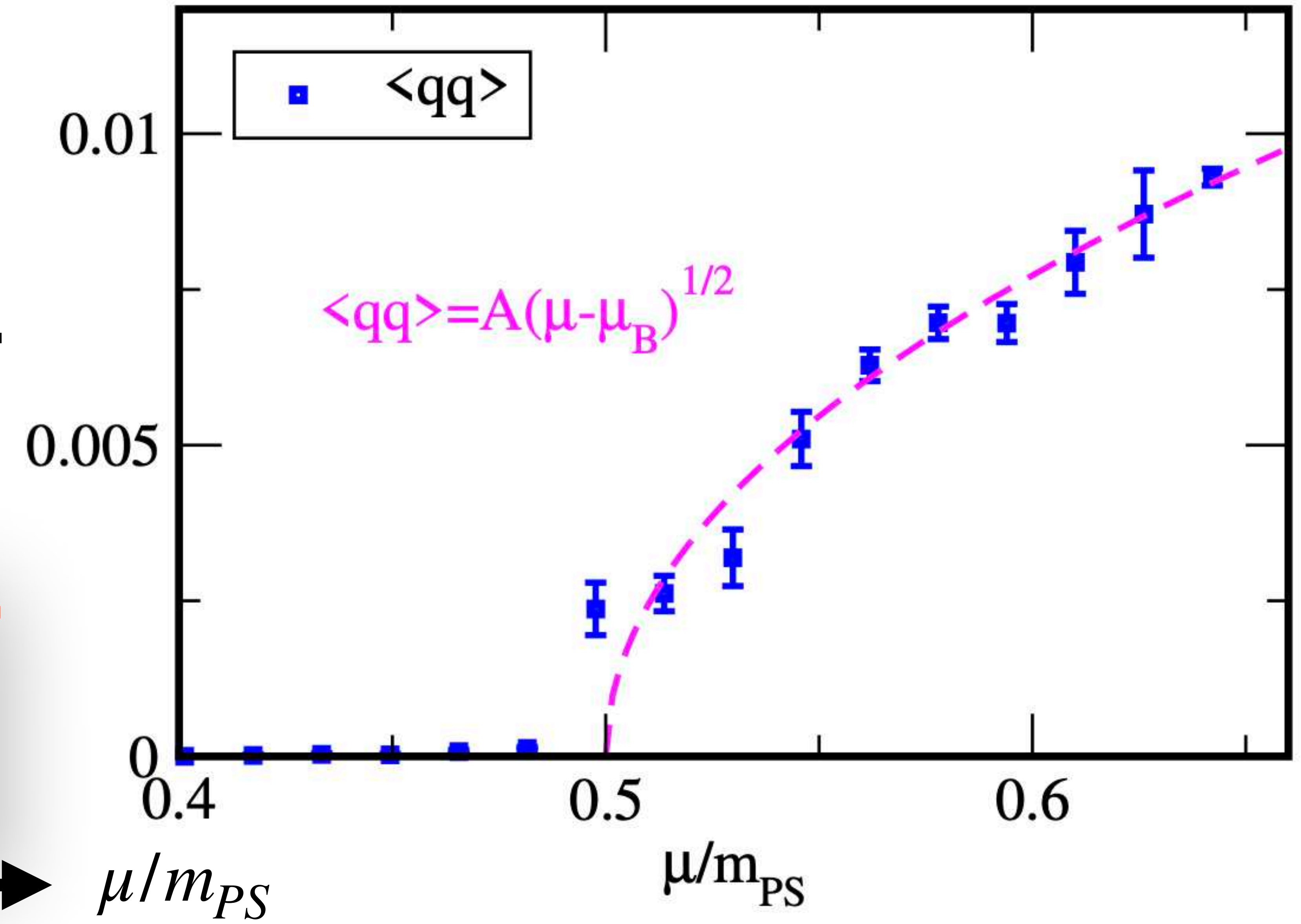
T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Phase diagram of 2color QCD

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



	Hadronic	Hadronic-matter	QGP	Superfluid BEC	BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{tree} \approx 1$

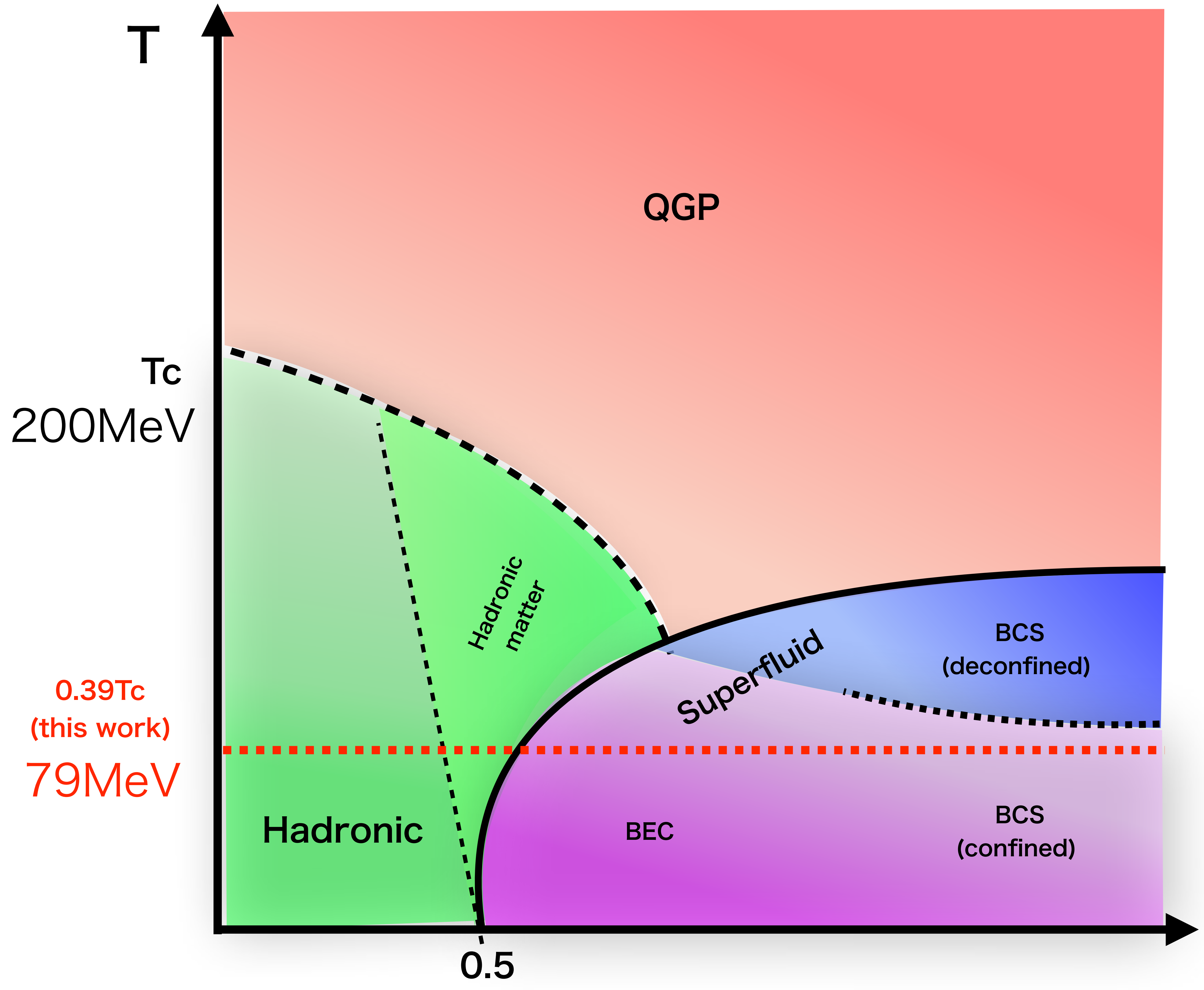


Scaling law of order param. is consistent with ChPT.

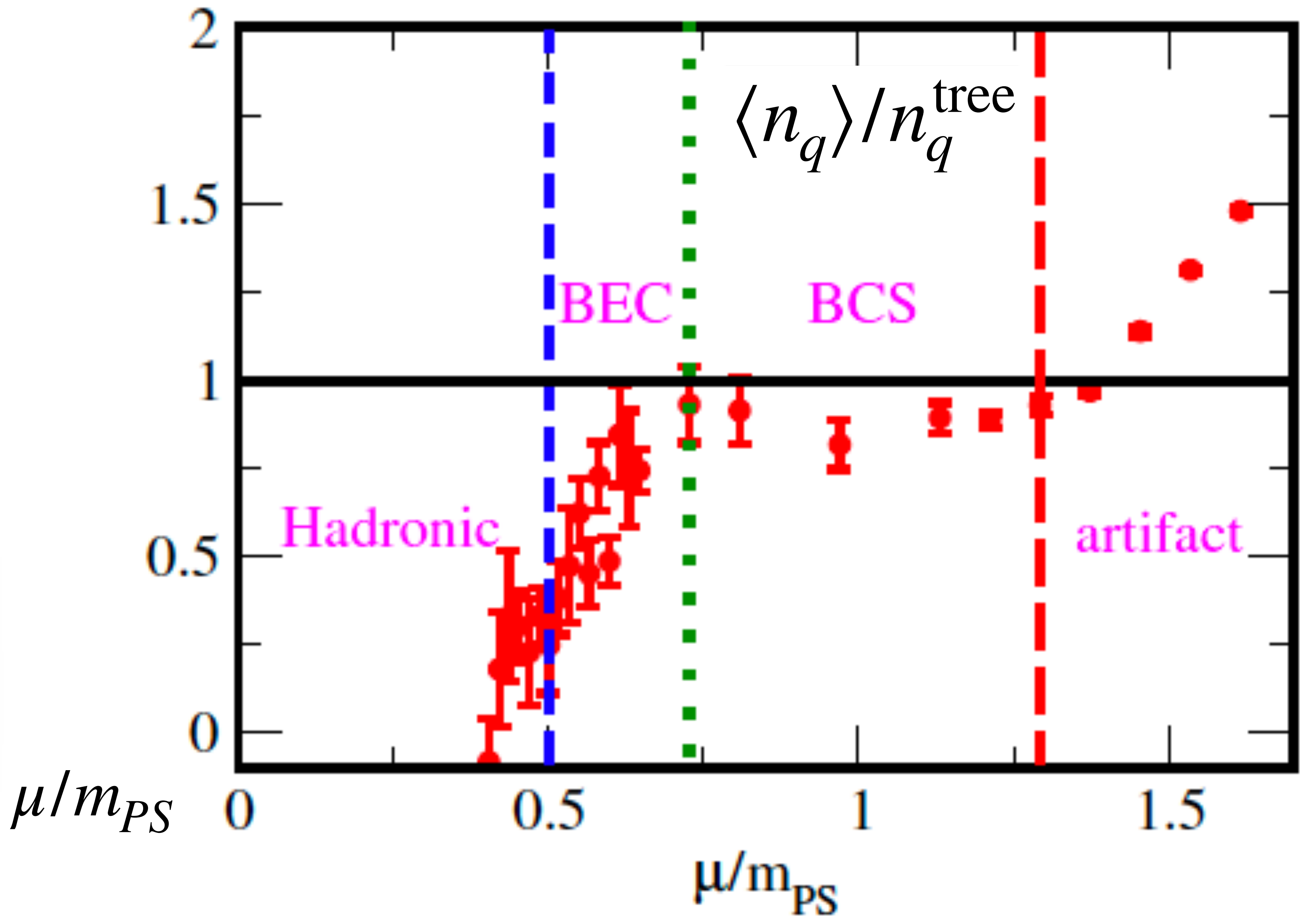
Kogut et al., NPB 582 (2000) 477

Phase diagram of 2color QCD

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



	Hadronic	Hadronic-matter	QGP	Superfluid BEC	Superfluid BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\mathrm{tree}} \approx 1$



In high-μ, $\langle n_q \rangle \approx n_q^{\mathrm{tree}}$

number density of free particle

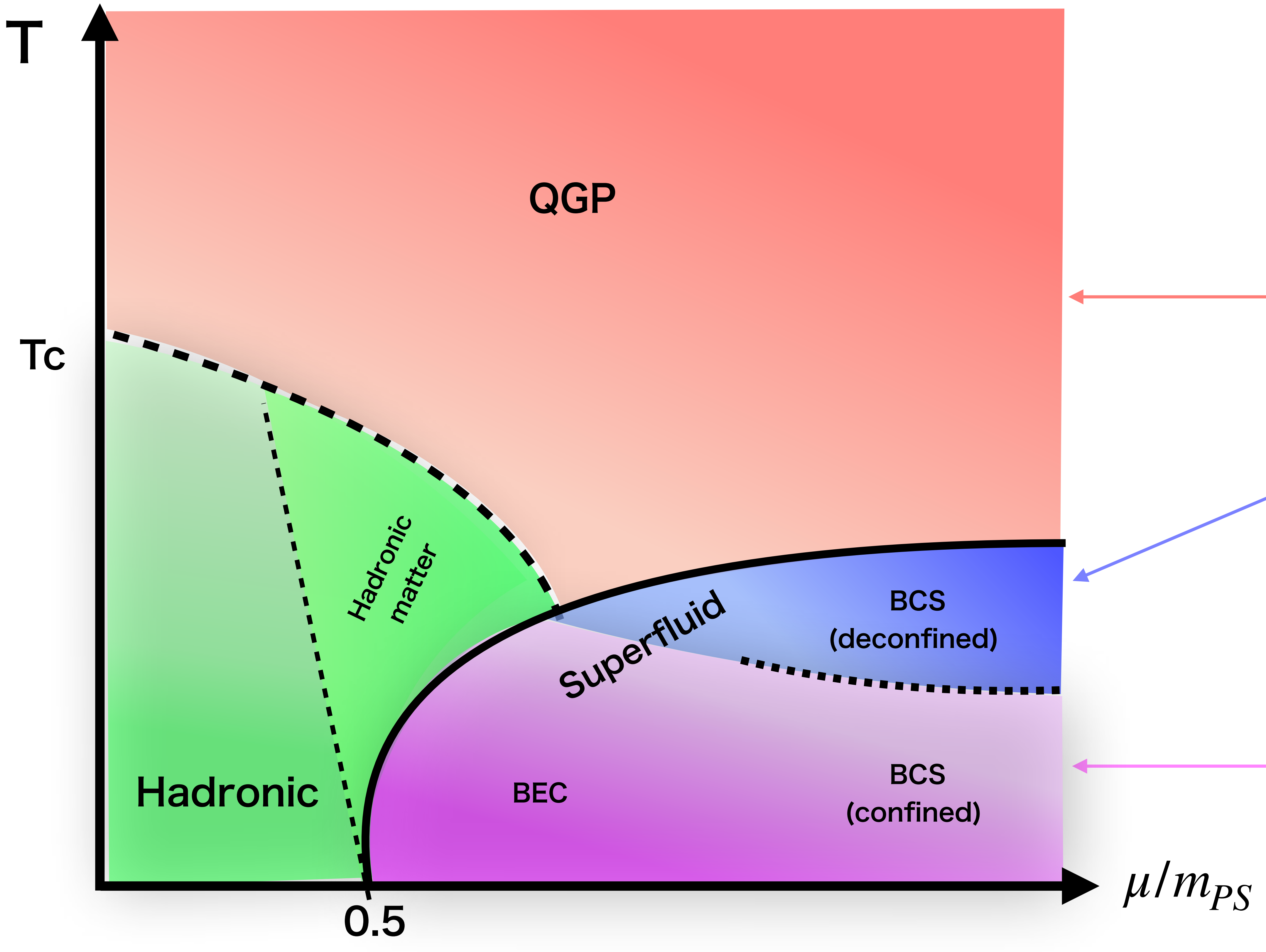
BEC-BCS crossover

Other quantities in 2color QCD phase

Confinement? deconfinement? at large μ

In 3color QCD, at ultra high-density, it is believed that quark d.o.f. is fundamental. (Color-Flavor-Locking phase)

The lattice results of 2color QCD suggest Hadronic superfluid phase even at high-density



T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

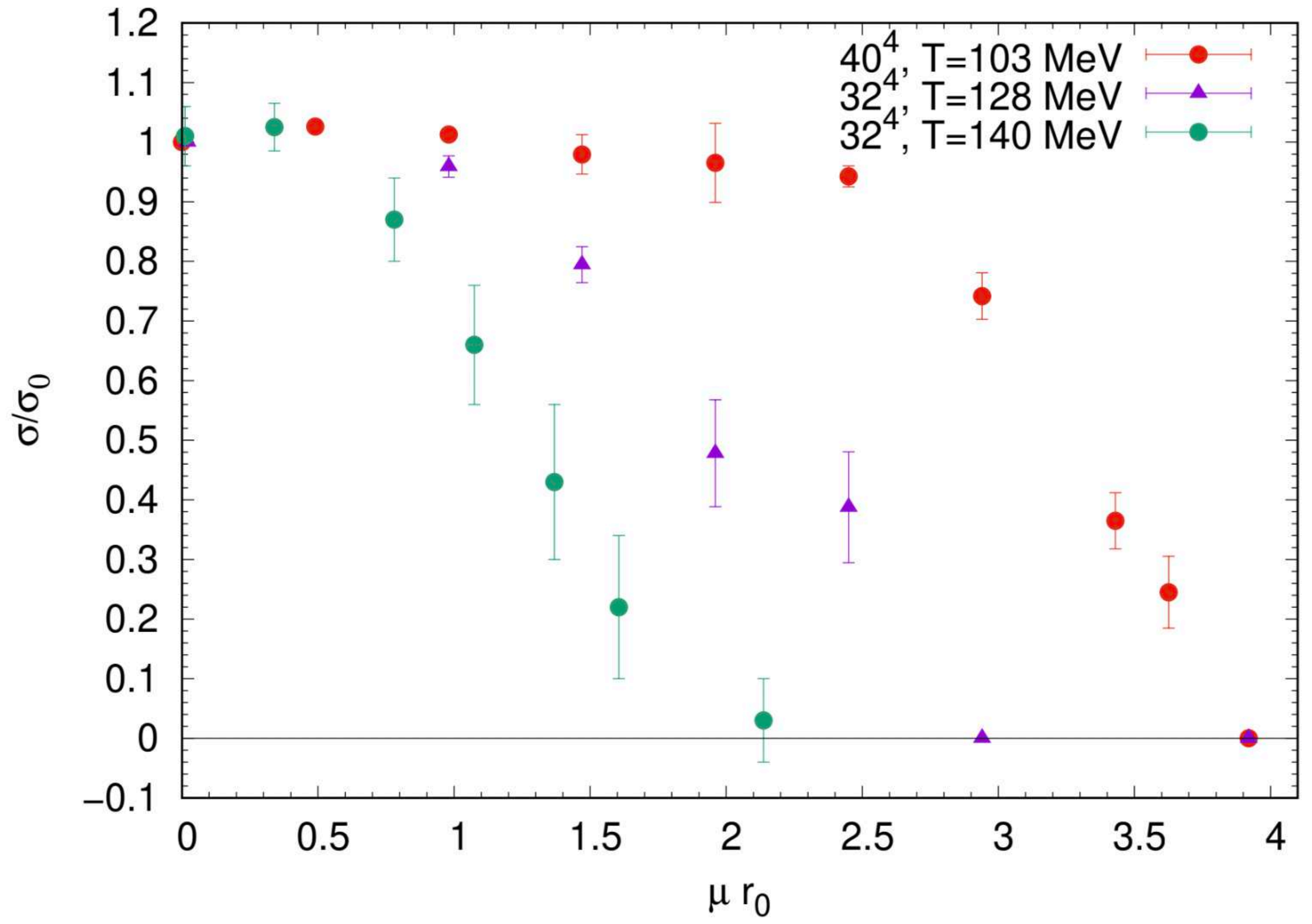
T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

Other quantities in 2color QCD phase

Confinement? deconfinement? at large μ

A. Begun, V. G. Bornyakov, V. A. Goy, A. Nakamura, R. N. Rogalyov, arXiv:2203.04909



string tension becomes nonzero
in low-T (and large vol.) even in high-μ

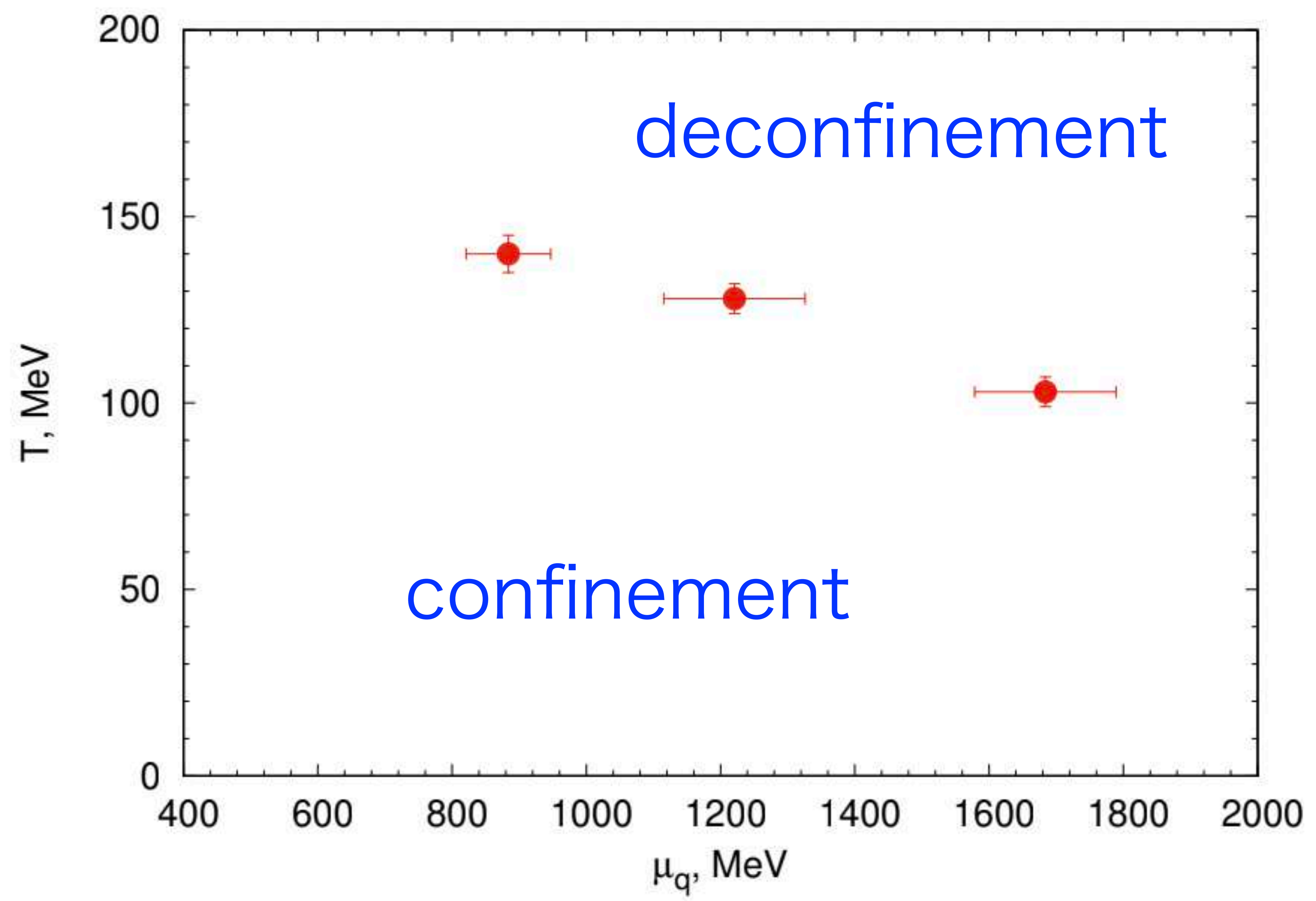


FIG. 4: The confinement-deconfinement transition in (μ_q, T) plane.

At $\mu_q \gtrsim 1.8$ GeV, $T_d \gtrsim 100$MeV

Other quantities in 2color QCD phase

Topological susceptibility

Nonperturbative topological configuration (instantons)

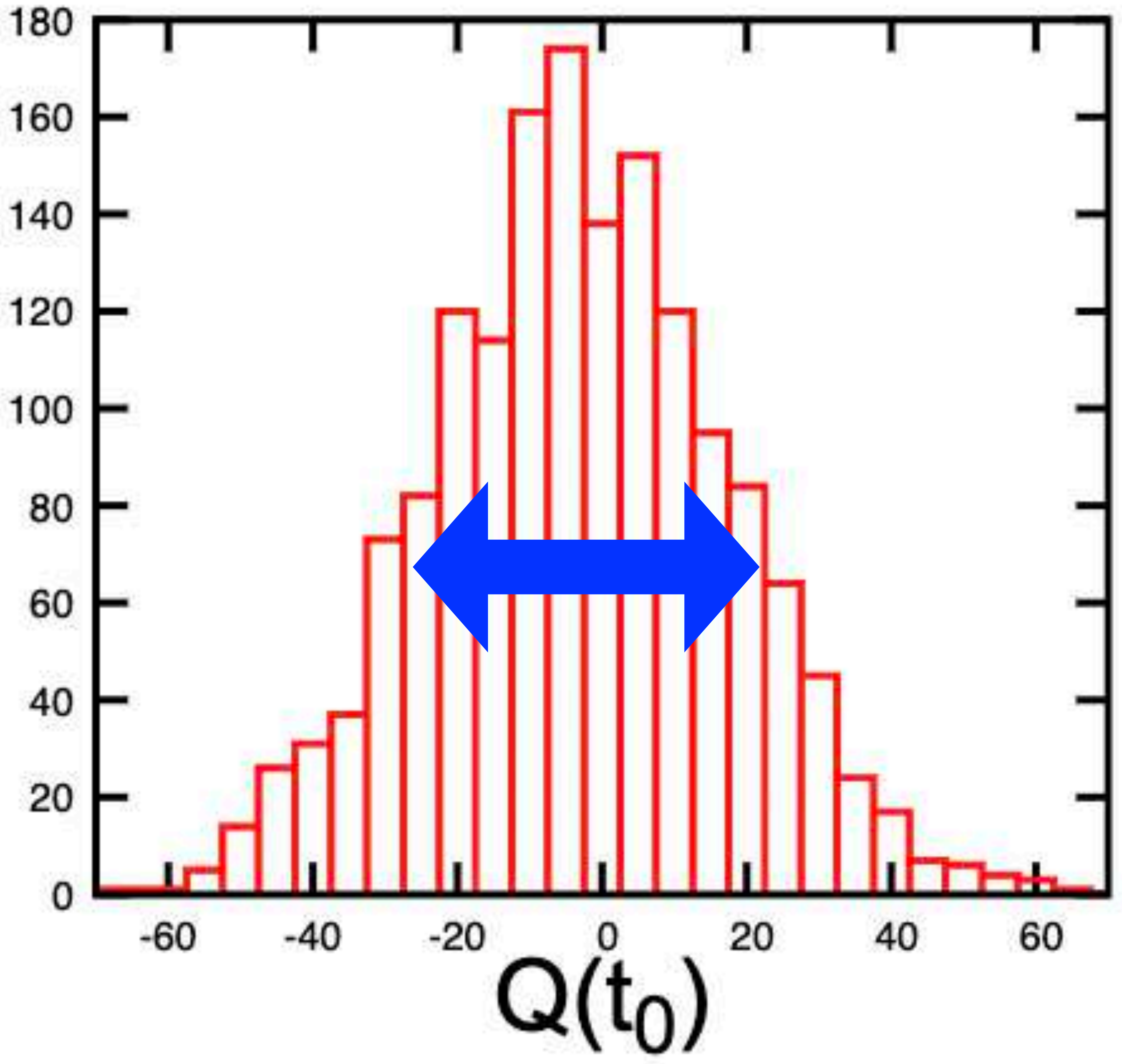
Instanton charge:

$$Q = \int dx F_{\mu\nu} F_{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$$

Topological susceptibility (Distribution of instanton charge)

$$\chi_Q = \frac{1}{V}(\langle Q^2 \rangle - \langle Q \rangle^2)$$

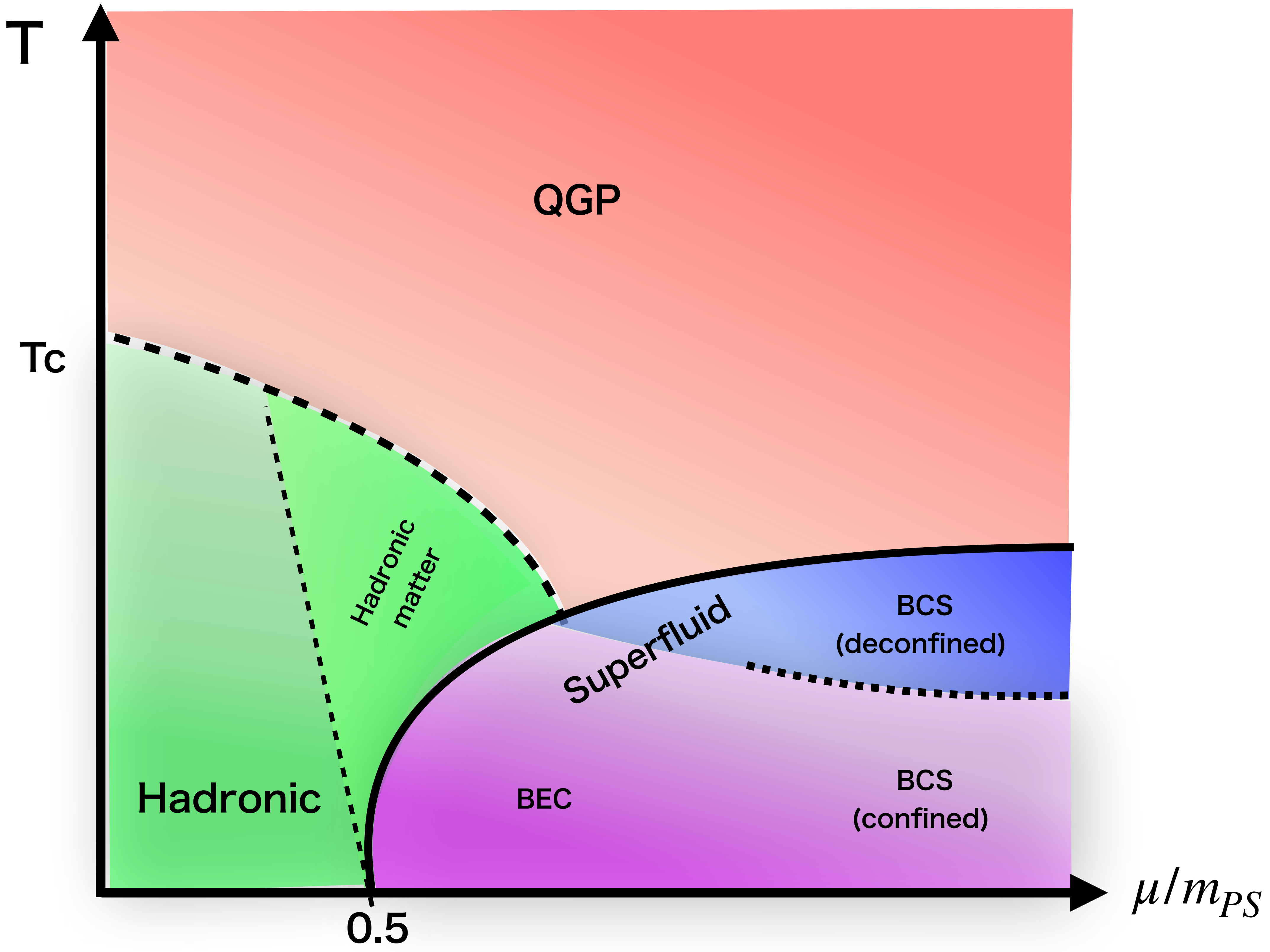
Typical distribution of Q at $T = 0, \quad \mu = 0$



Other quantities in 2color QCD phase

Topological susceptibility

Hands. (1104.0522)



Nf=4, T=0 (12^3x24)

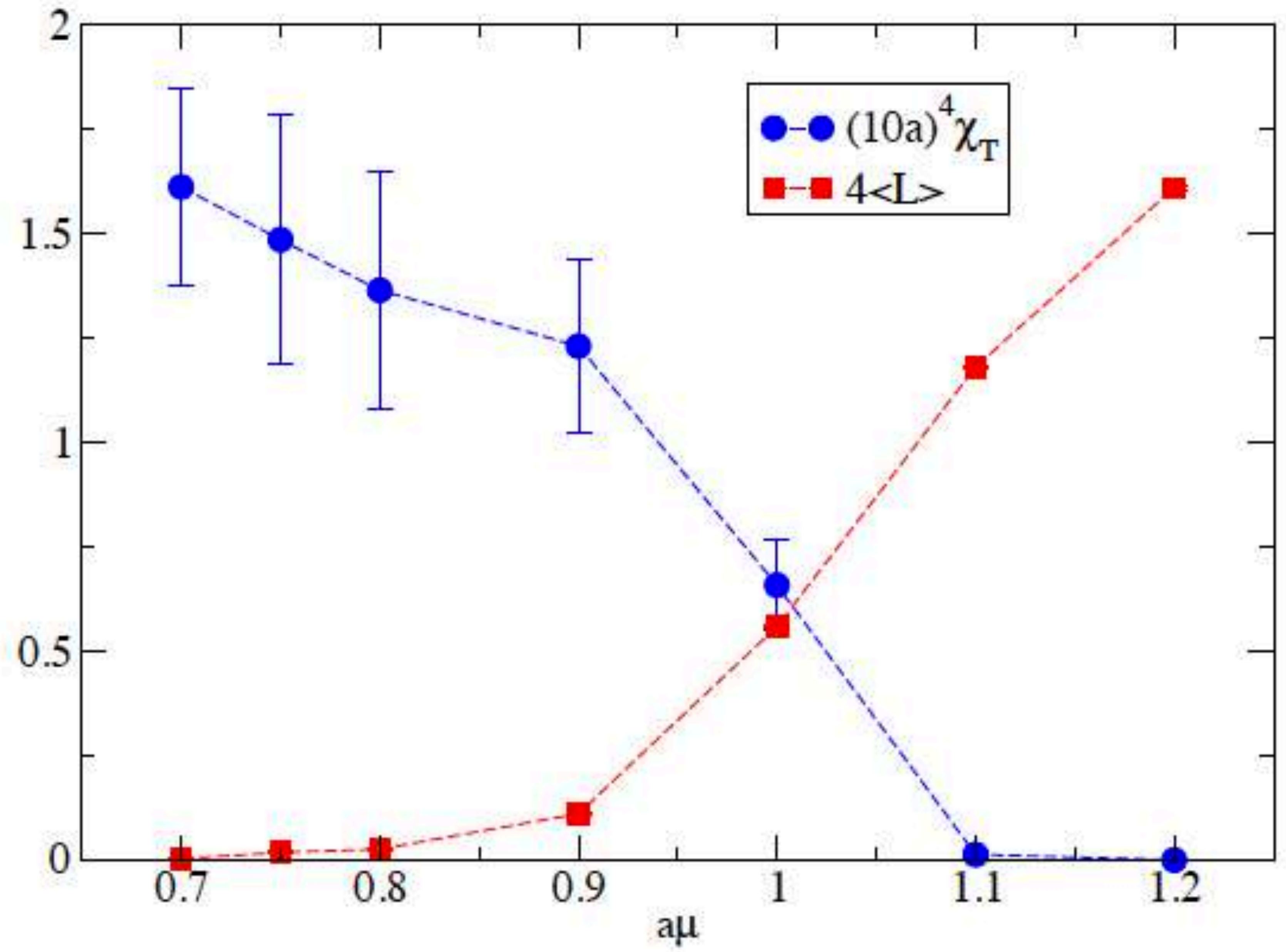


Figure 2: *The suppression of χ_T coinciding with the rise in $\langle L\rangle$ for $N_f = 4$. Note $\langle L\rangle$ has been rescaled for clarity.*

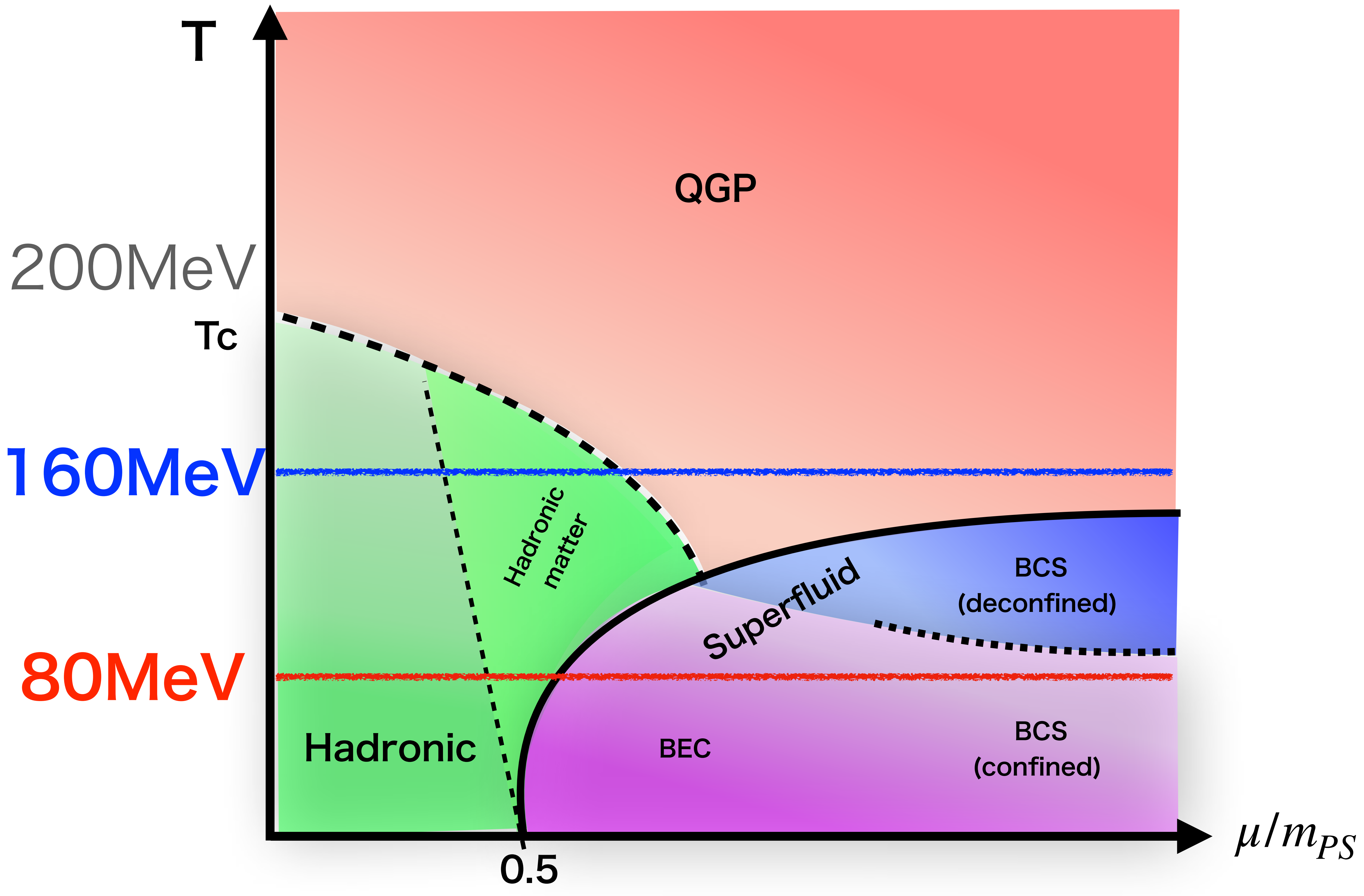
Polyakov loop increasing

||

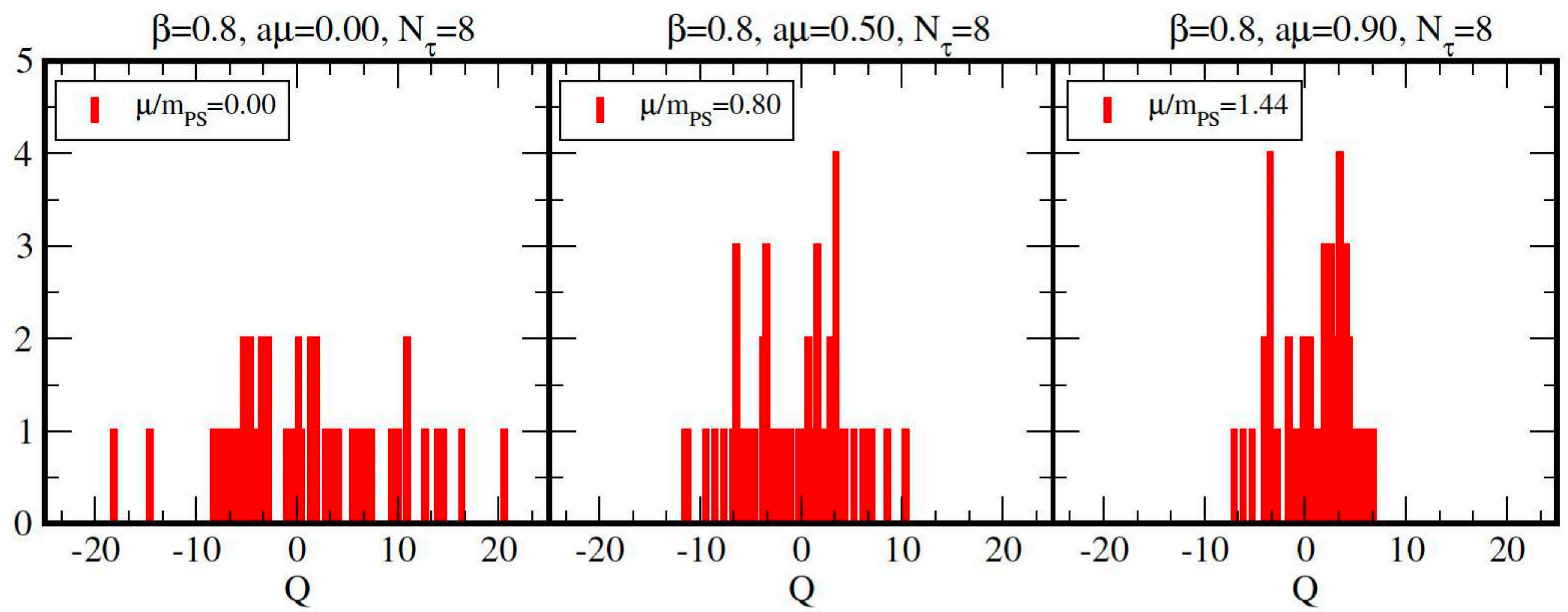
Topological suscep. decreasing

Other quantities in 2color QCD phase

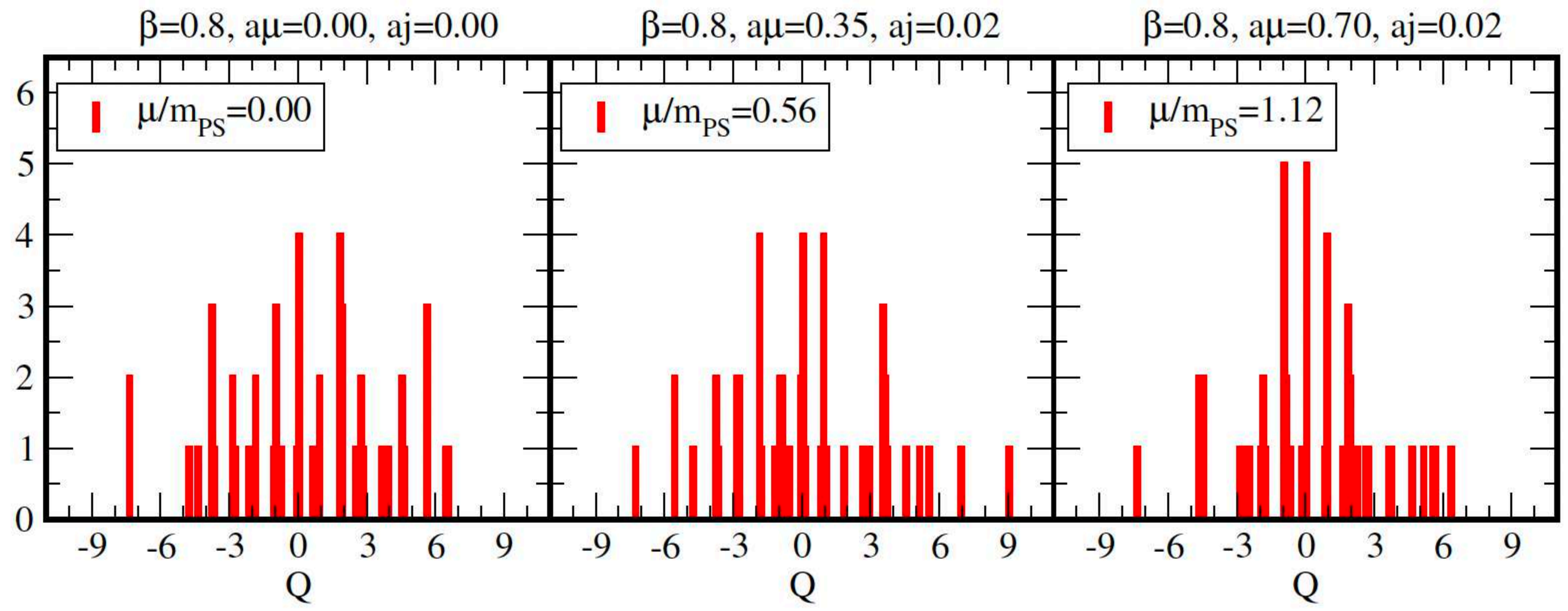
Topological susceptibility



T=160MeV, χ_Q decreases as μ increases

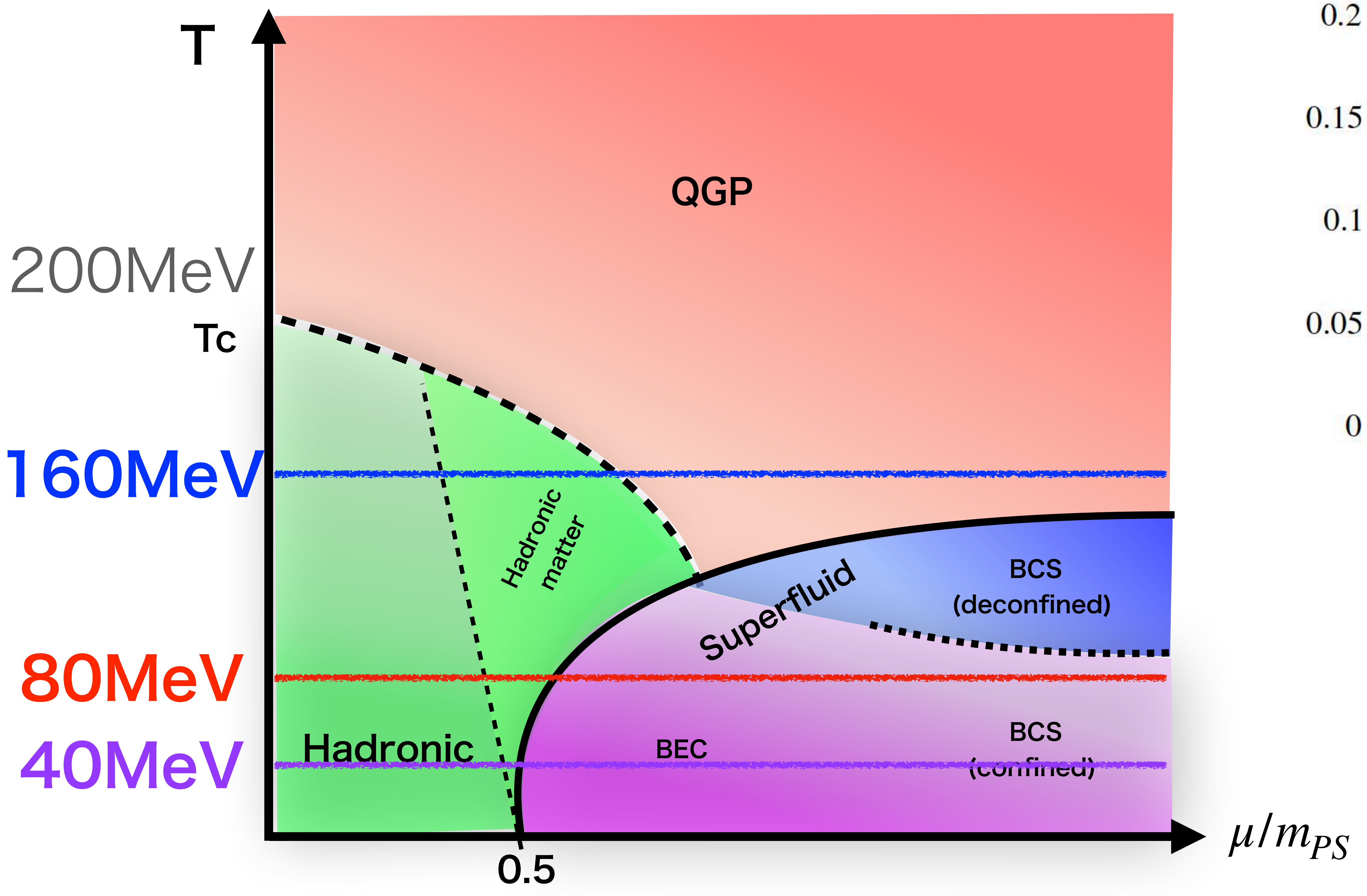


T=80MeV, χ_Q is independent of μ

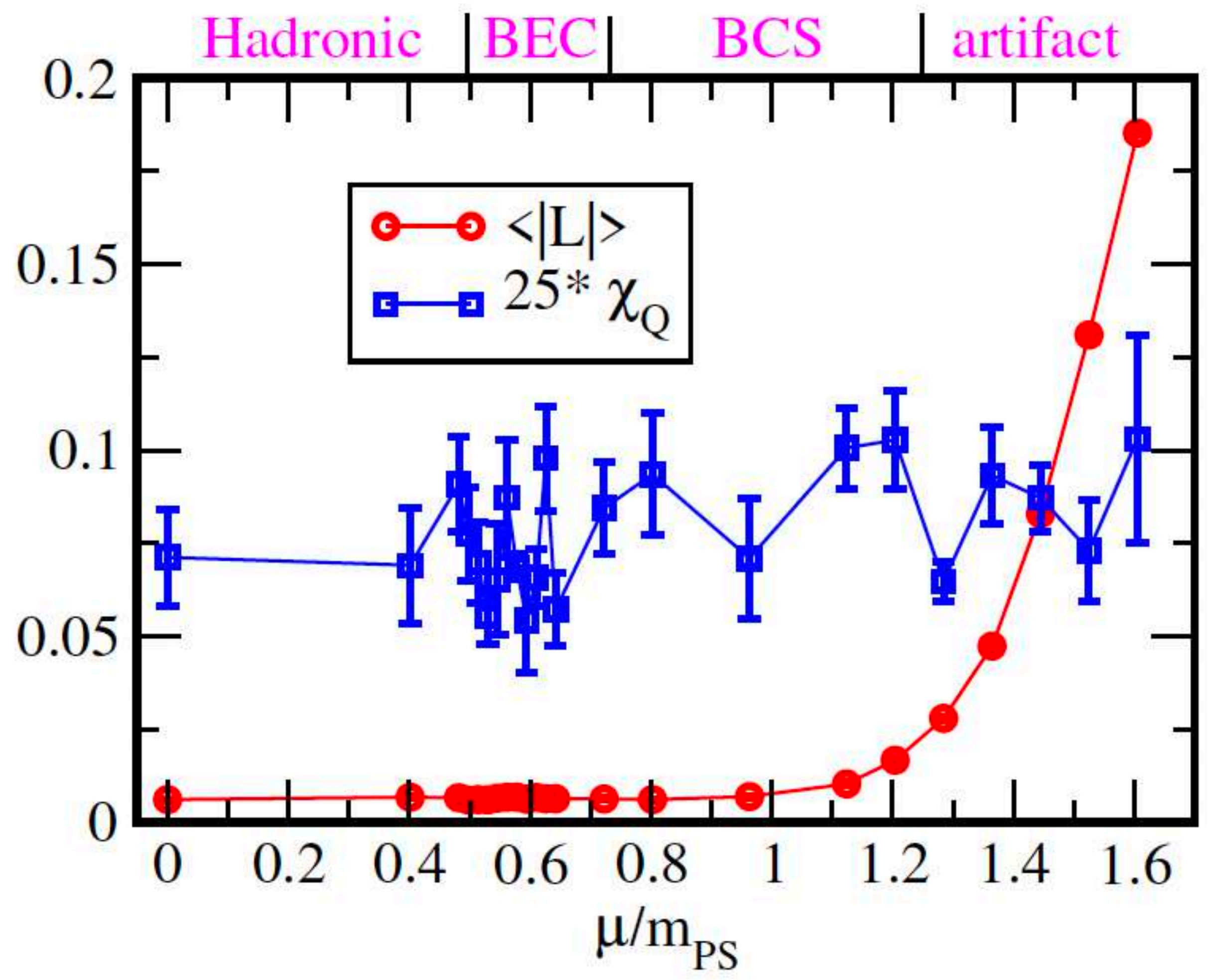


Other quantities in 2color QCD phase

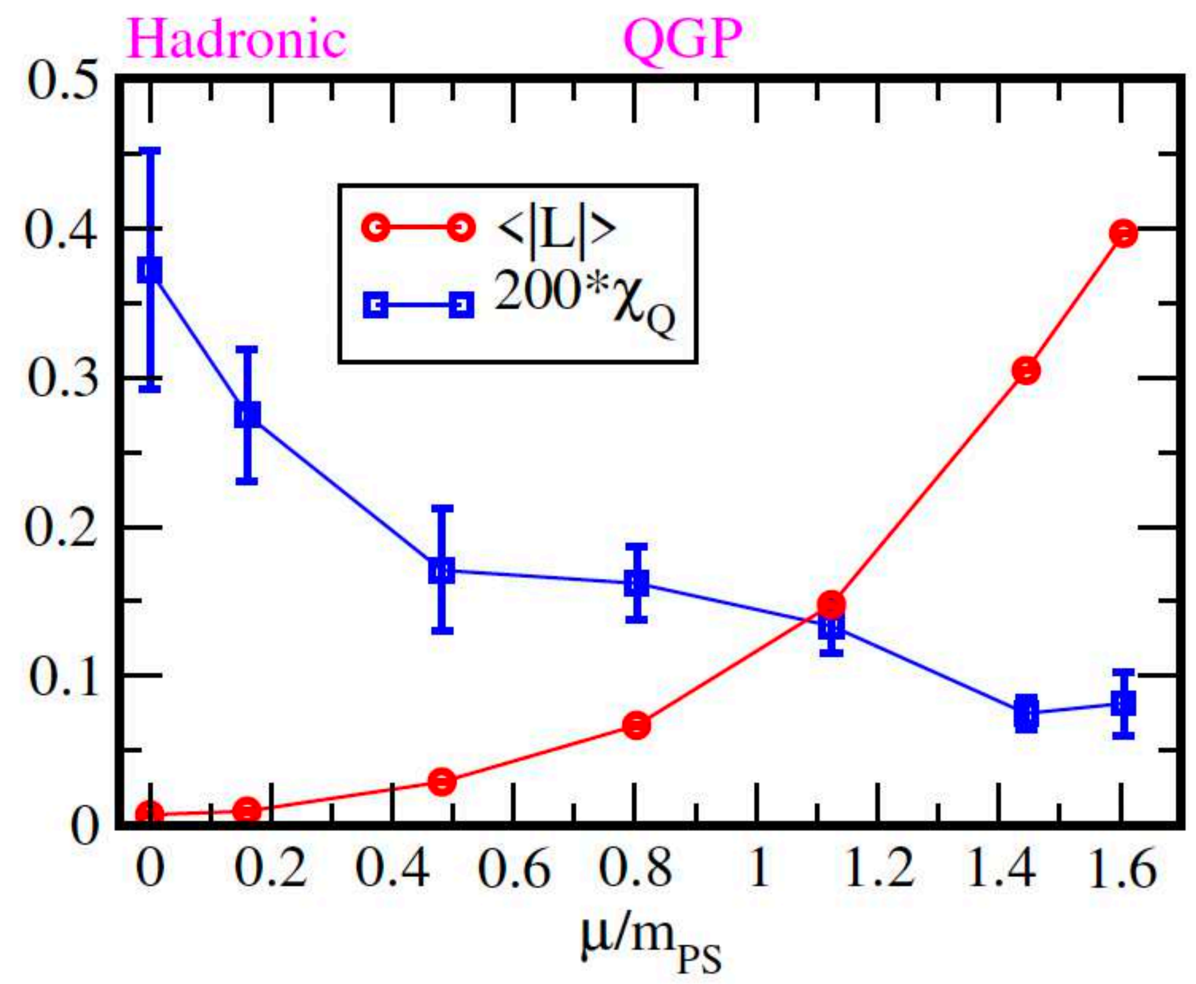
Topological susceptibility



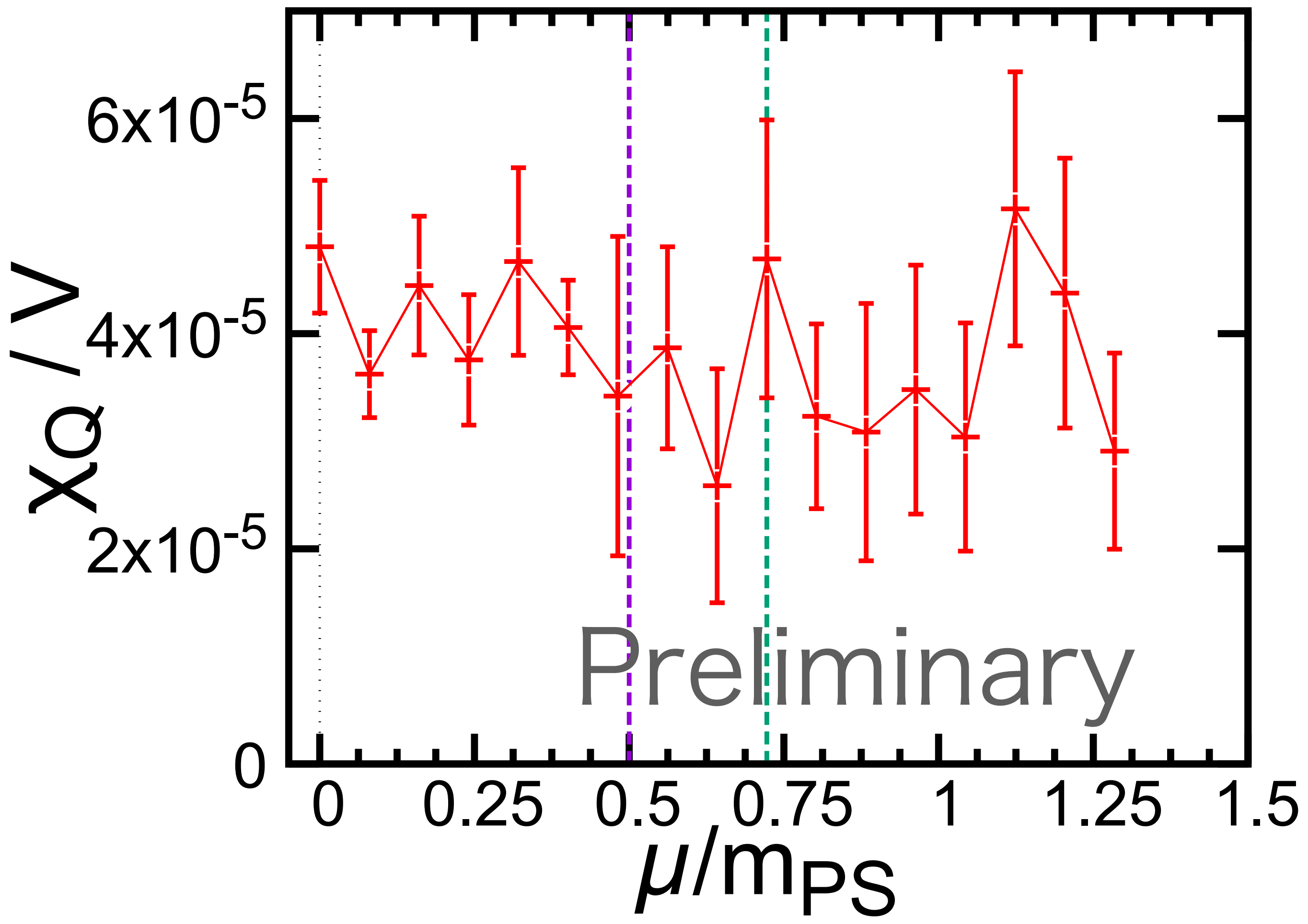
T=80MeV



T=160MeV

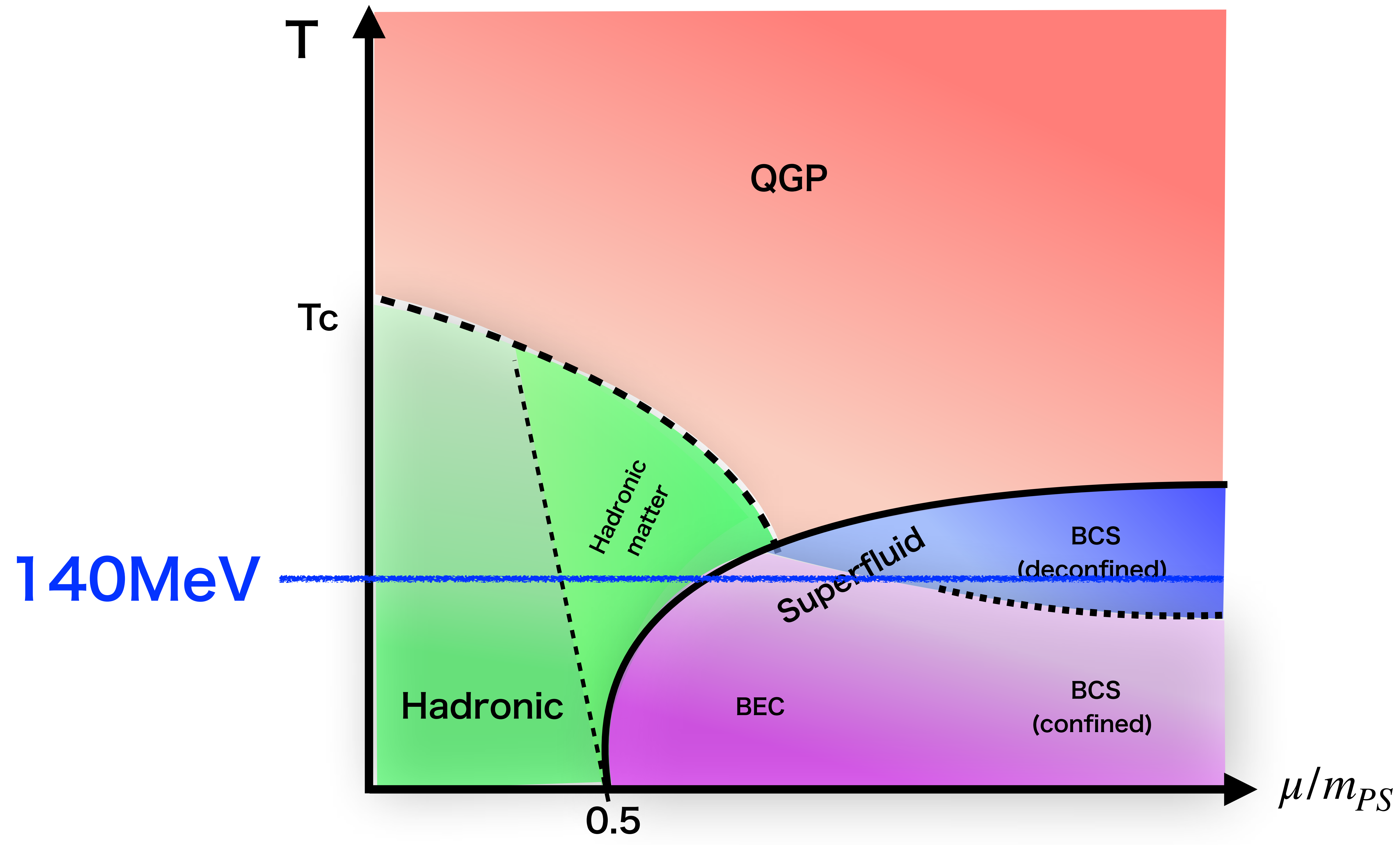


T=40MeV (new!)



Other quantities in 2color QCD phase

Topological susceptibility



Staggered fermion (small mass)

Astrakhantsev at al., PRD 2022

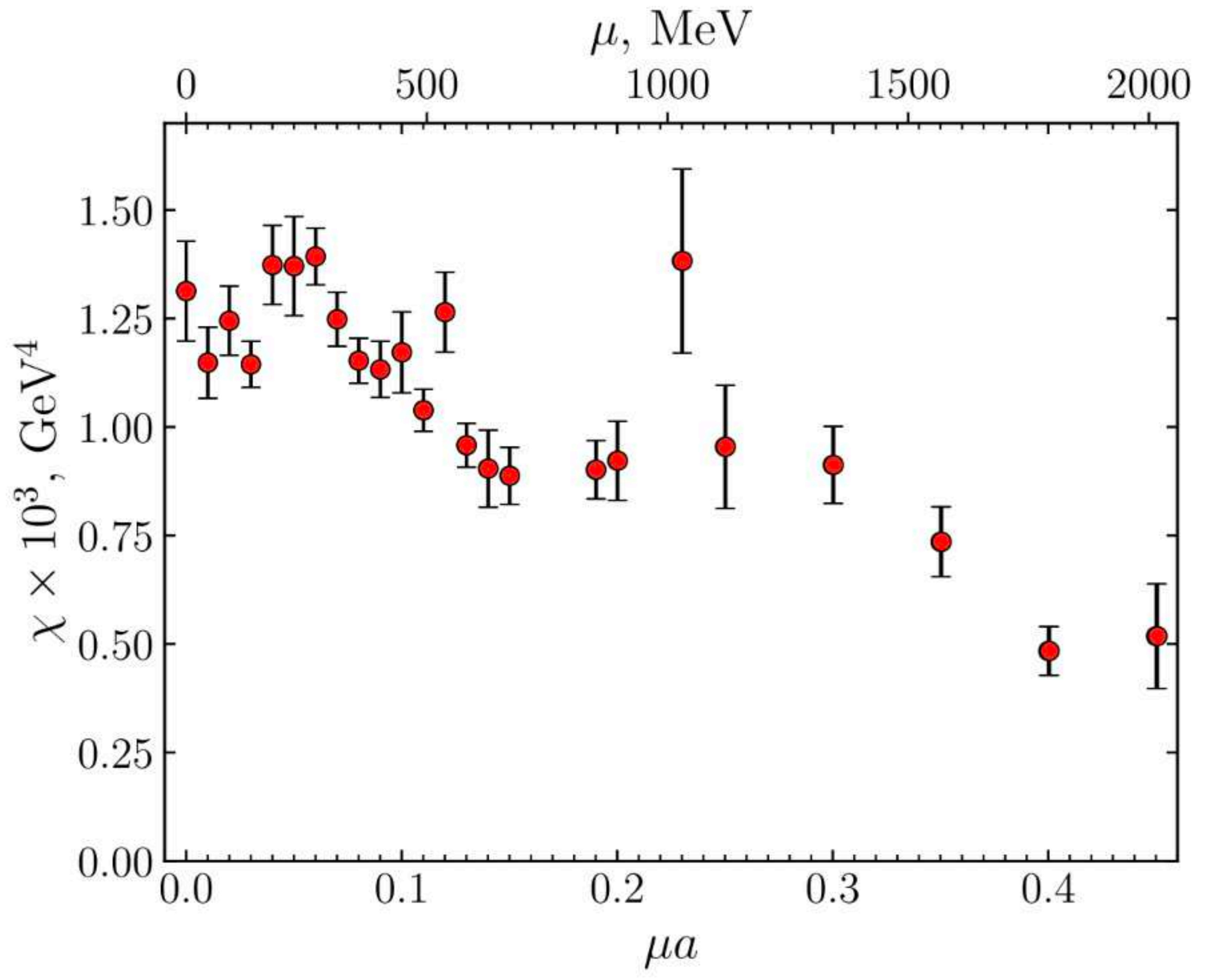


FIG. 8. Topological susceptibility in energy units, scaled by 10^3 for the better visual presentation, as a function of the chemical potential.

Cf.) Kawaguchi and Suenaga, arXiv:2305.18682
linear sigma model (~2color QCD with Nf=2)
χ_Q decreases in superfluid phase

small a simulation reach large mu?
high-T (T~140MeV) effect?

Summary of QCD phase diagram

- 2color QCD phase diagram has been determined by independent works!

- Even $T \approx 100\mathrm{MeV}$, superfluid phase emerges

- Difference between 3color and 2color confinement/deconfinement at ultra high-density?

- Confinement/deconfinement might be related with topology distribution

Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge)

  lattice size = 16^4

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle \right)$

  No renormalization for μ

  $\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu} - \langle \cdot \rangle_{\mu=0}$

  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge)

  lattice size = 16^4

  T=79MeV, j->0 extrapolation is taken

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  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Technical steps

(1) Measure $\langle \cdot \rangle$ on the generated configuration

(2) **Nonperturbatively** calculate beta fn. at $\mu = 0$

(3) Numerical integration of n_q

Equation of state

- Fixed scale approach ($\mu \neq 0$ version)

  beta=0.80 (Iwasaki gauge)

  lattice size = 16^4

  T=79MeV, j->0 extrapolation is taken

EoS in dense 2color QCD
Hands et al. (2006)
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  Zero at $j \to 0$

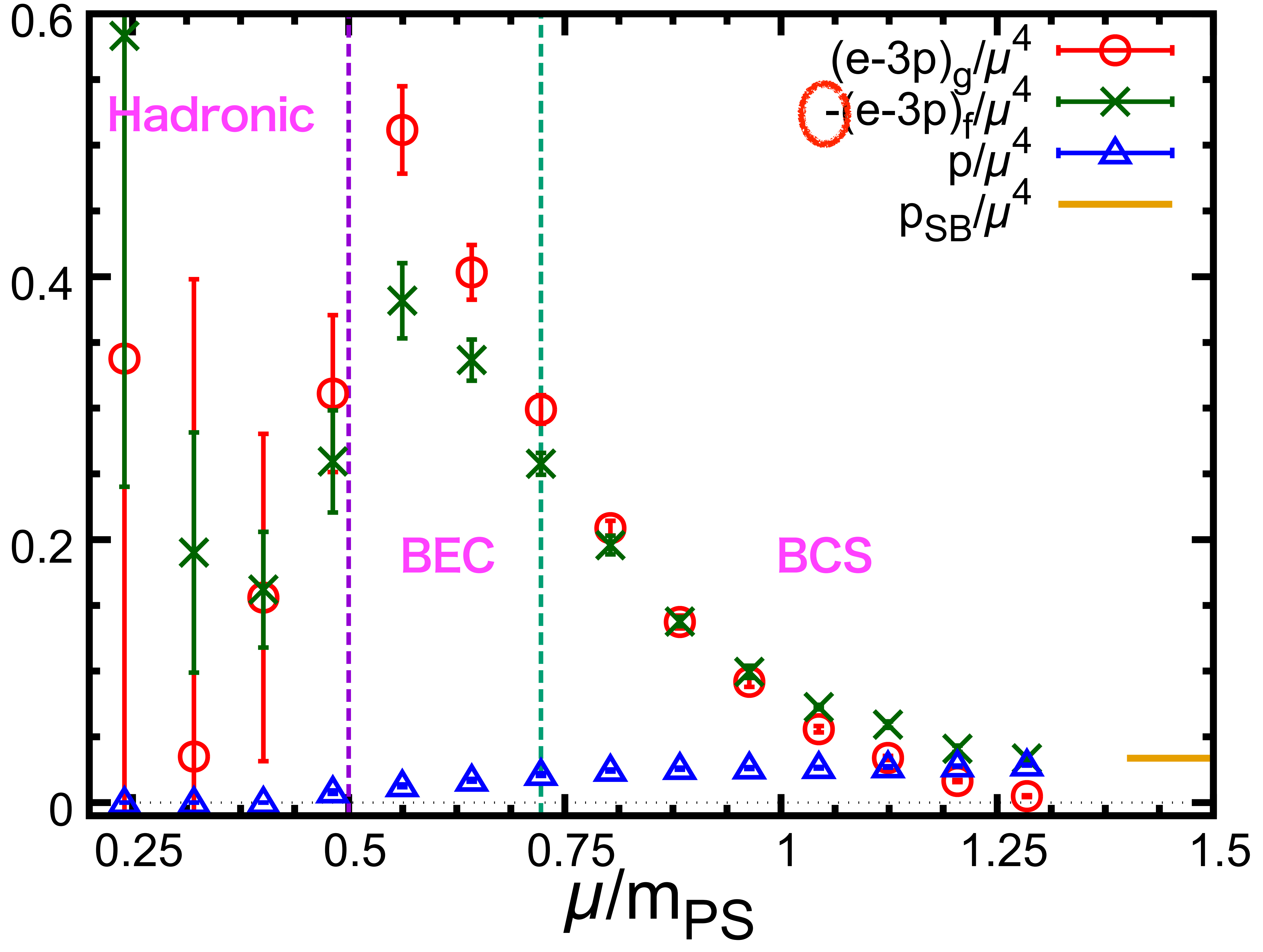
- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Nonperturbative beta-fn.

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

Trace anomaly and pressure



- Sum of trace anomaly, $(e-3p)_g + (e-3p)_f$

  zero in Hadronic phase

  positive in BEC phase

  positive -> negative in BCS phase

  Finally, fermions give the larger contribution

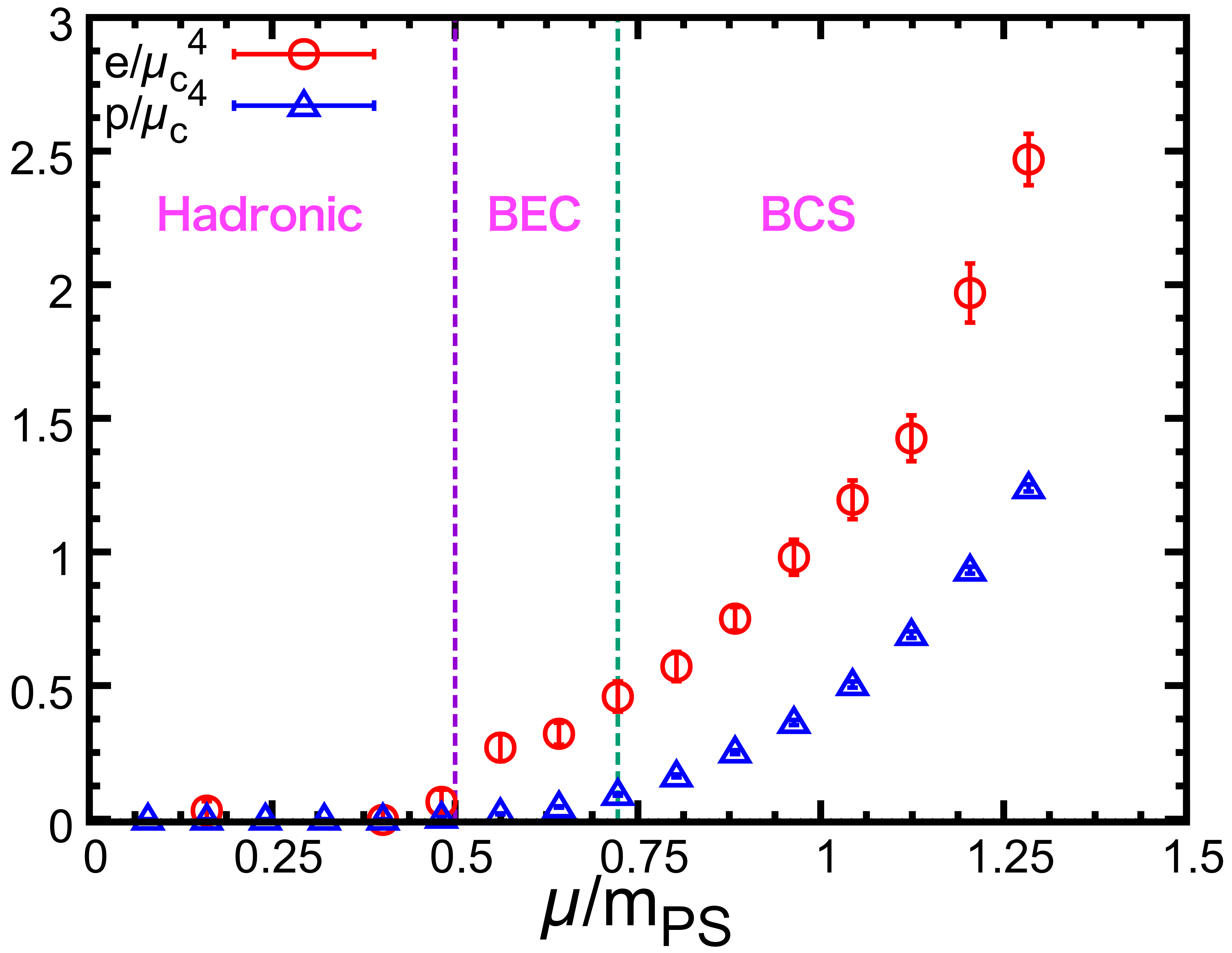
- Pressure increase monotonically

  In high density, it approaches

  $p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$

P and e as a function of μ

(Normalized by $1/\mu_c^4$ to be dim-less)

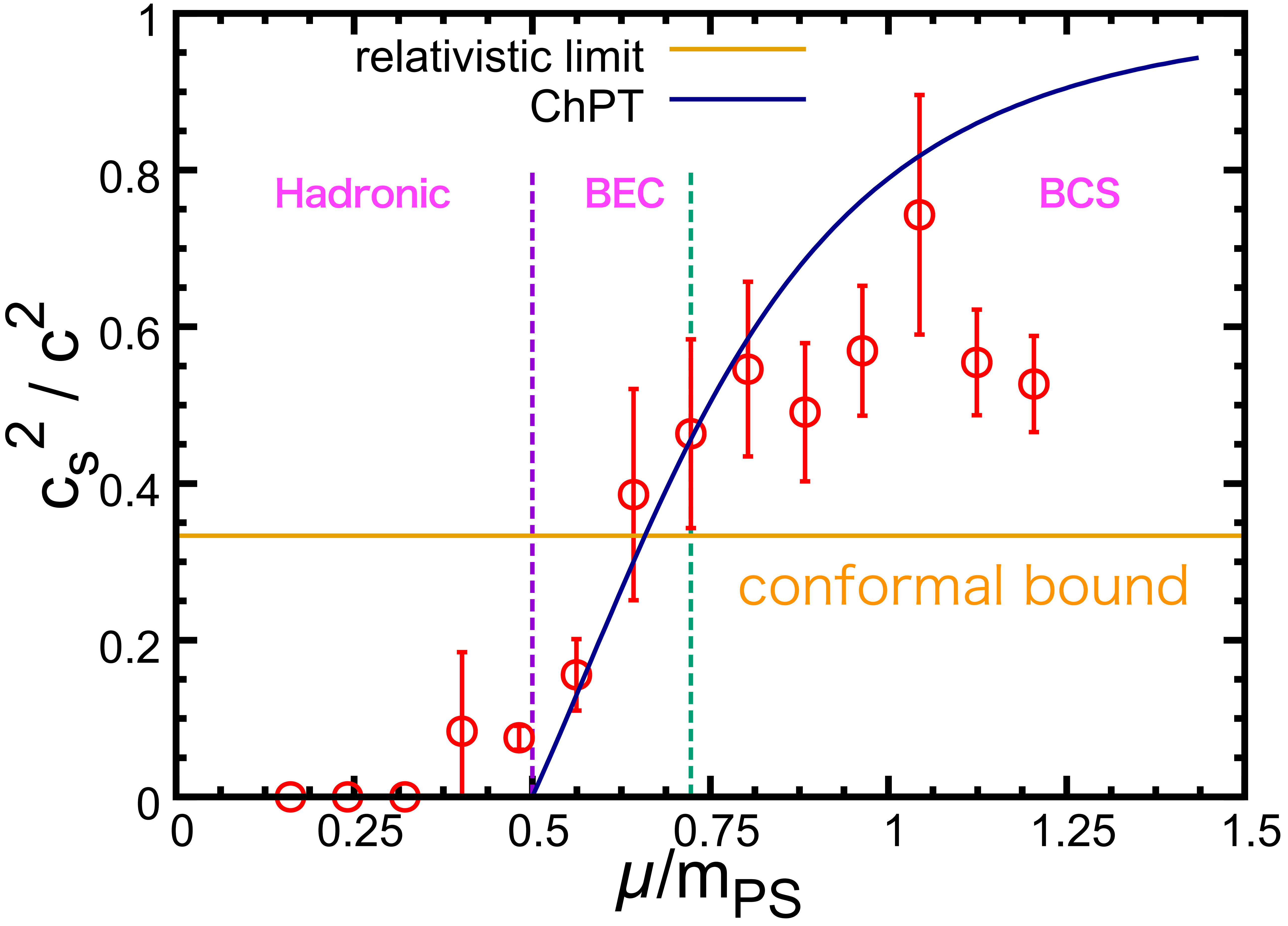


- P is zero in Hadronic phase since $n_q = 0$
- e is also zero in Hadronic phase by the cancelation between $(e-3p)_g$ and $(e-3p)_f$

From these data, the sound velocity is obtained

$$c_s^2/c^2 = \frac{\Delta p}{\Delta e} = \frac{p(\mu+\Delta\mu) - p(\mu-\Delta\mu)}{e(\mu+\Delta\mu) - e(\mu-\Delta\mu)}$$

Sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



Chiral Perturbation Theory (ChPT)

$$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$: no free parameter!!

Son and Stephanov (2001) : 3color QCD with isospin μ

Hands, Kim, Slullerud (2006) : 2color QCD with real μ

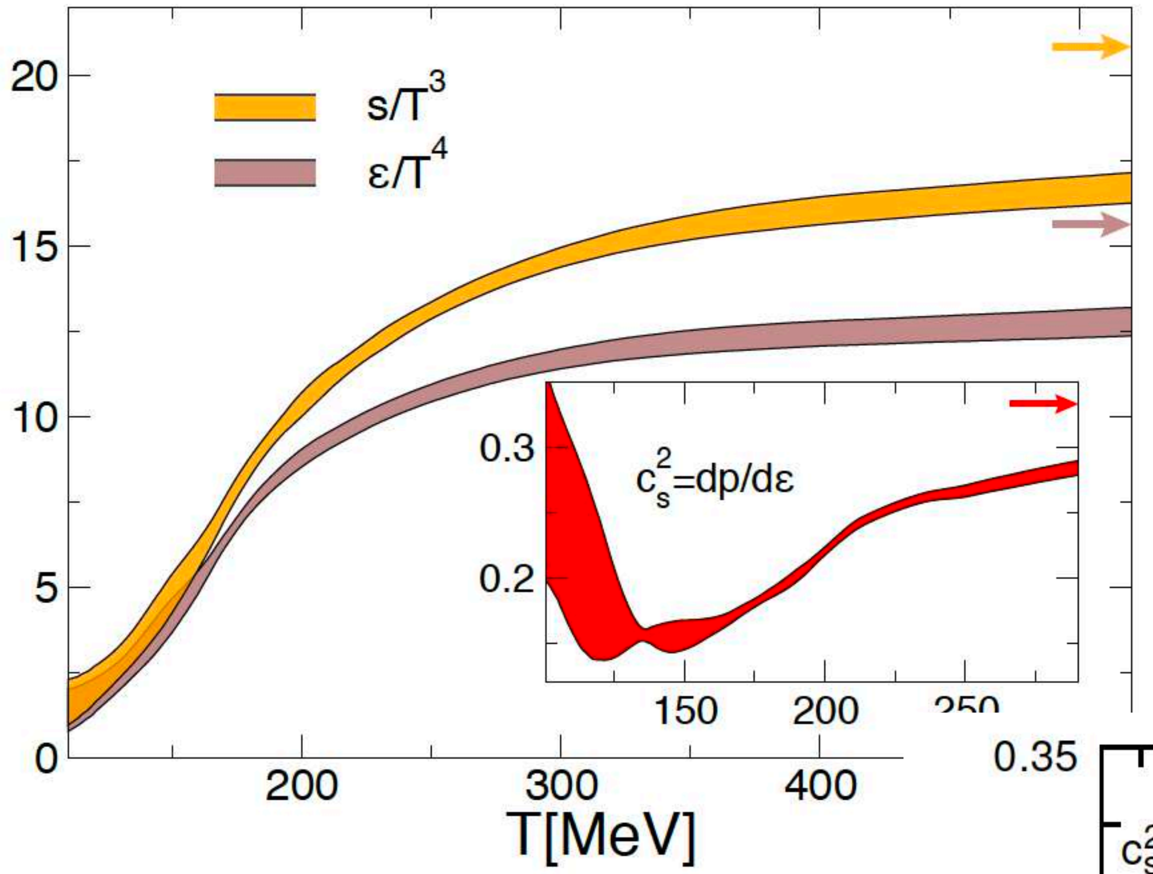
- In BEC phase, our result is consistent with ChPT.
- c_s^2/c^2 exceeds the relativistic limit
- In high-density, it peaks around $\mu \approx m_{PS}$.

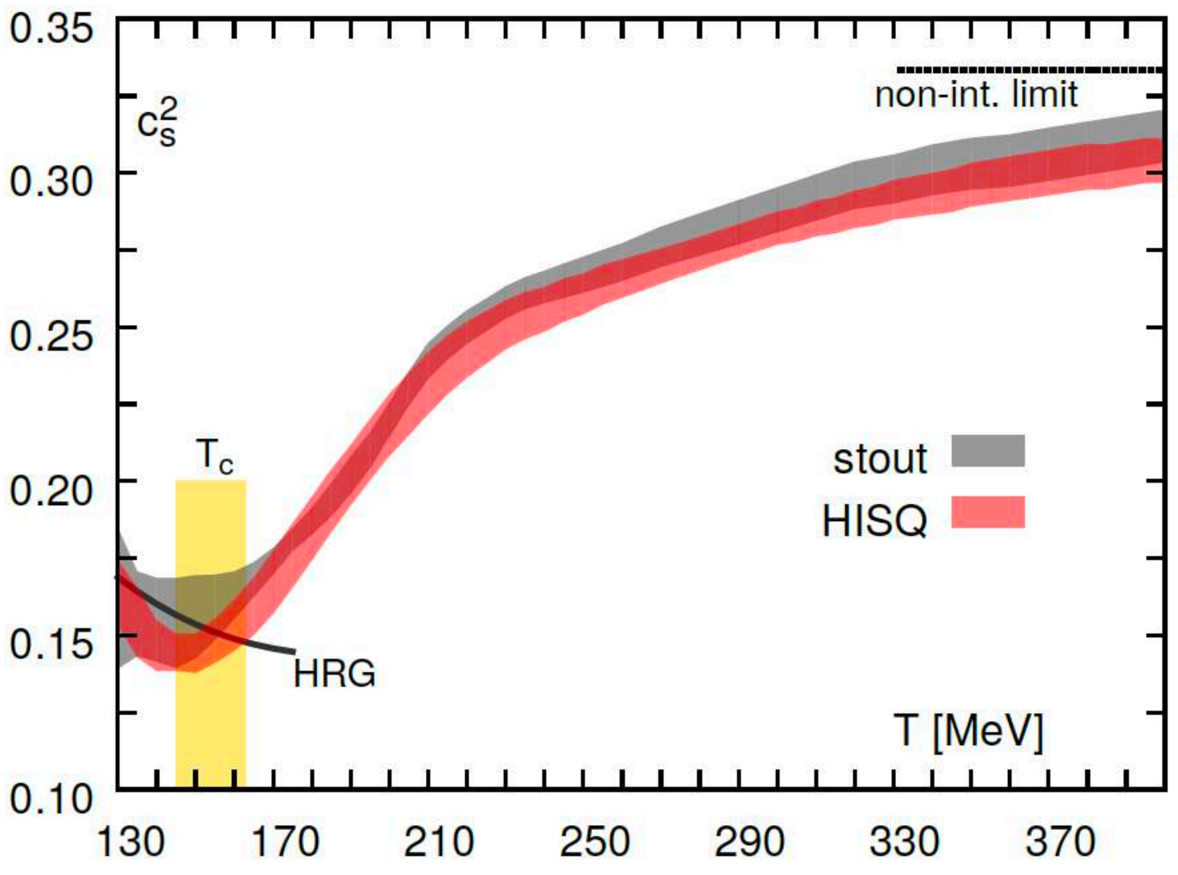
"Stiffen" and then "soften" picture as density increases

Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)

Borsanyi et al. (2013)

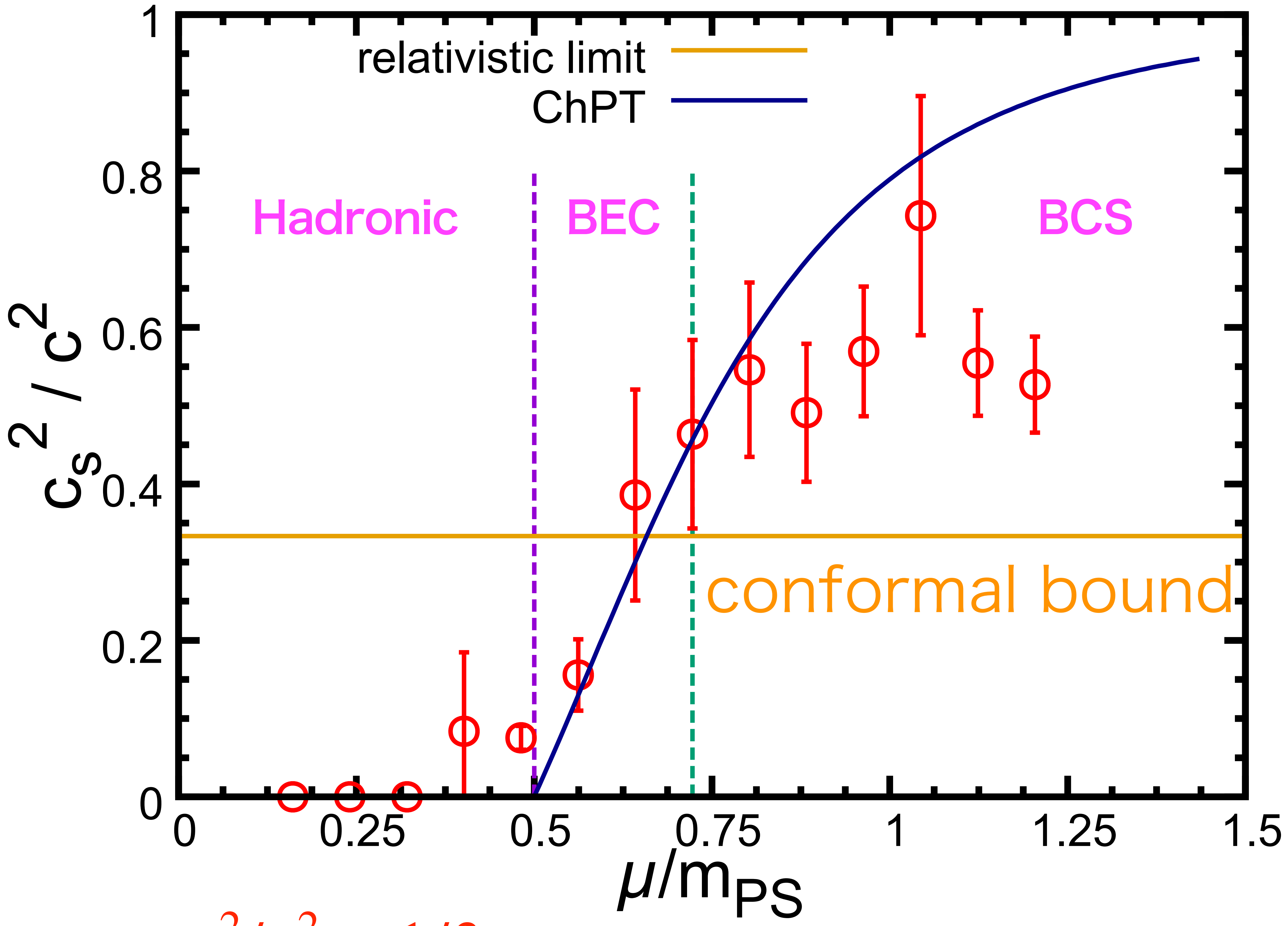




HotQCD (2014)

Finite Density transition (Nf=2 2color QCD)

Iida and El arXiv: 2207.01253



- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

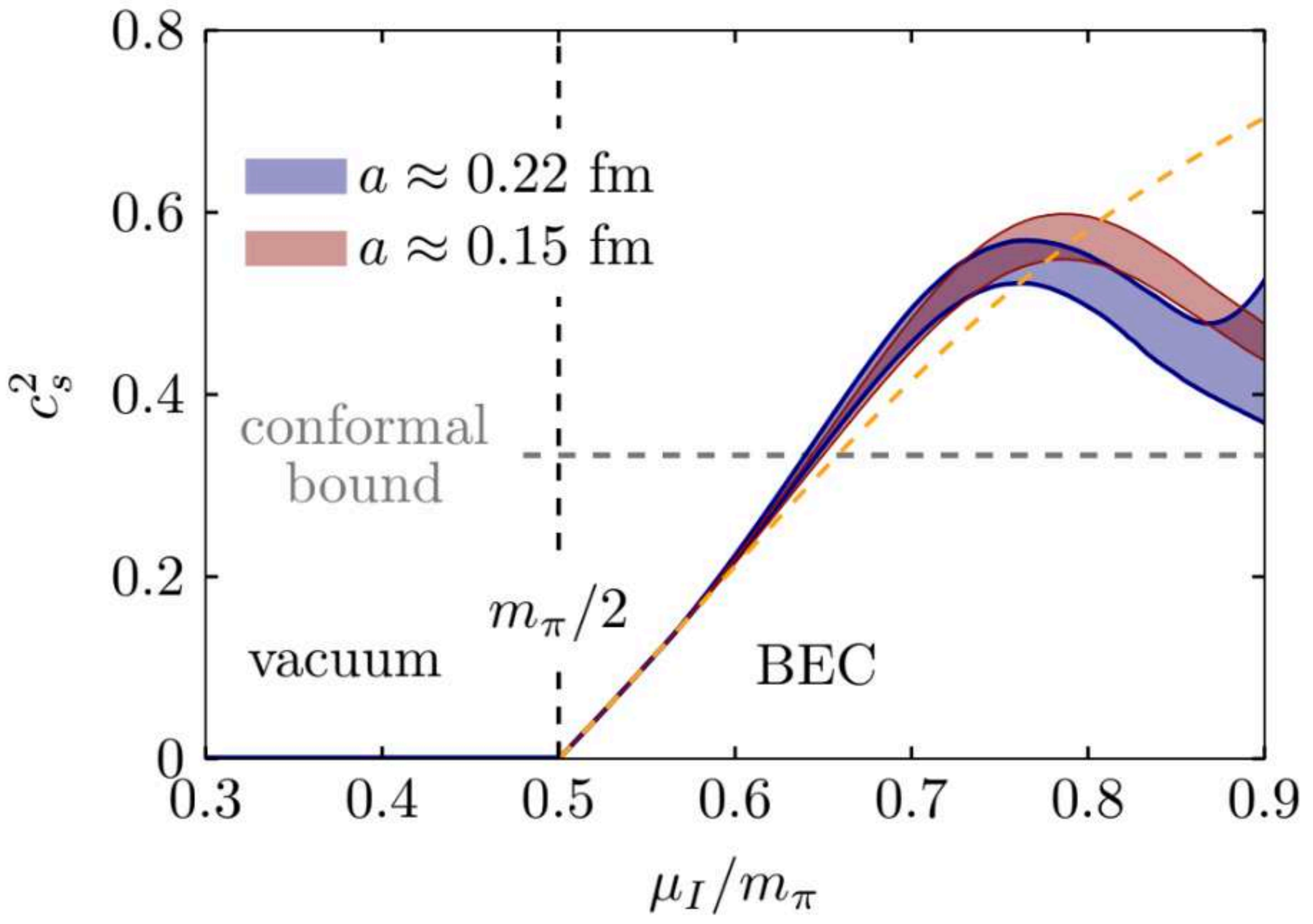
Lattice MC for 3 color QCD with isospin chemical potential

3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

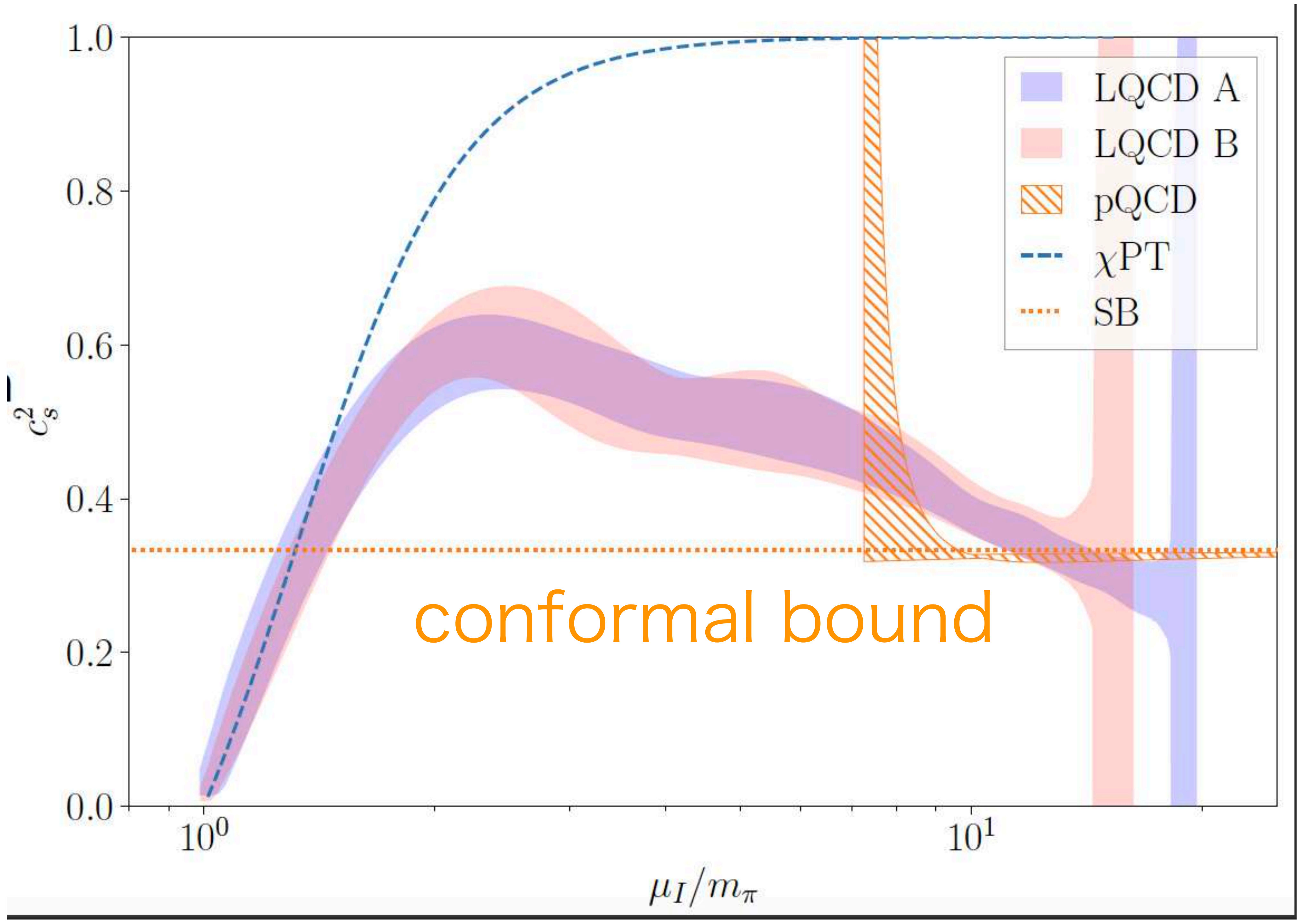
B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

R. Abbott et al. arXiv:2307.15014
(W.Detmold's talk Monday)

Result with spline interpolation



New algorithm for n-point fn. calc.



Counterexamples of conformal bound

PHYSICAL REVIEW D **94**, 106008 (2016)

Breaking the sound barrier in holography

Carlos Hoyos,[1,*] Niko Jokela,[2,†] David Rodríguez Fernández,[1,‡] and Aleksi Vuorinen[2,§]
[1]*Department of Physics, Universidad de Oviedo, Avda. Calvo Sotelo 18, ES-33007 Oviedo, Spain*
[2]*Department of Physics and Helsinki Institute of Physics, P.O. Box 64, FI-00014 University of Helsinki, Finland*
(Received 20 September 2016; published 15 November 2016)

It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

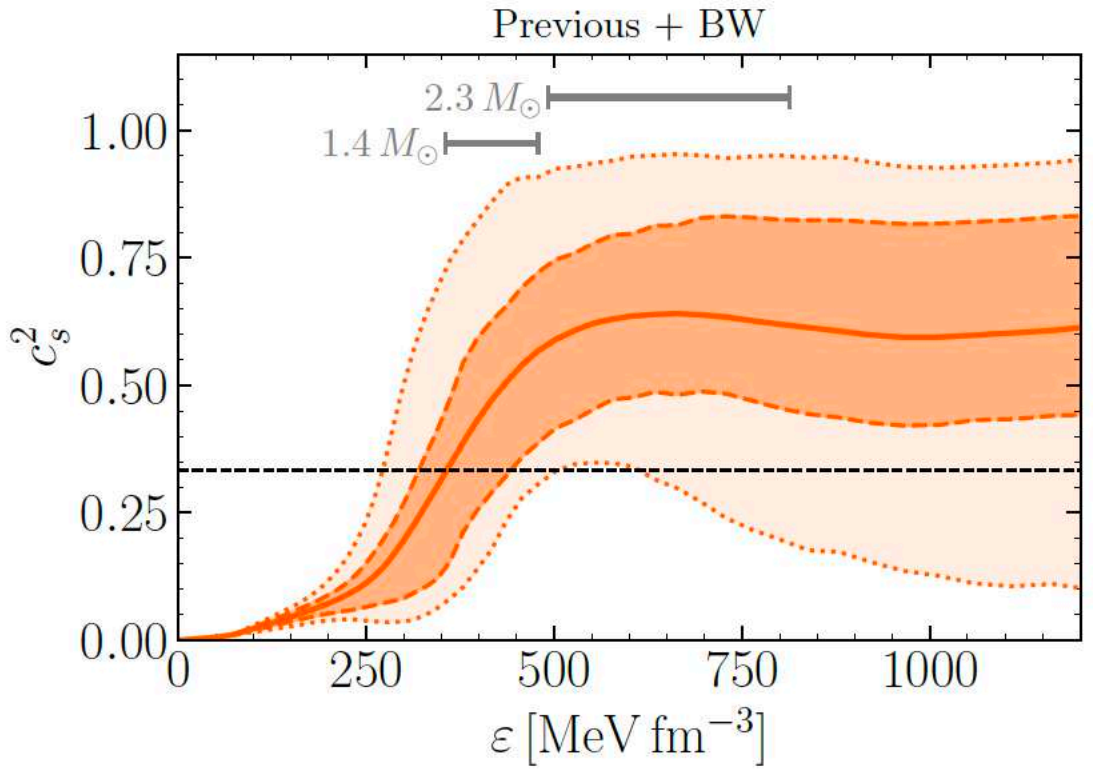
N=4 SYM at finite density

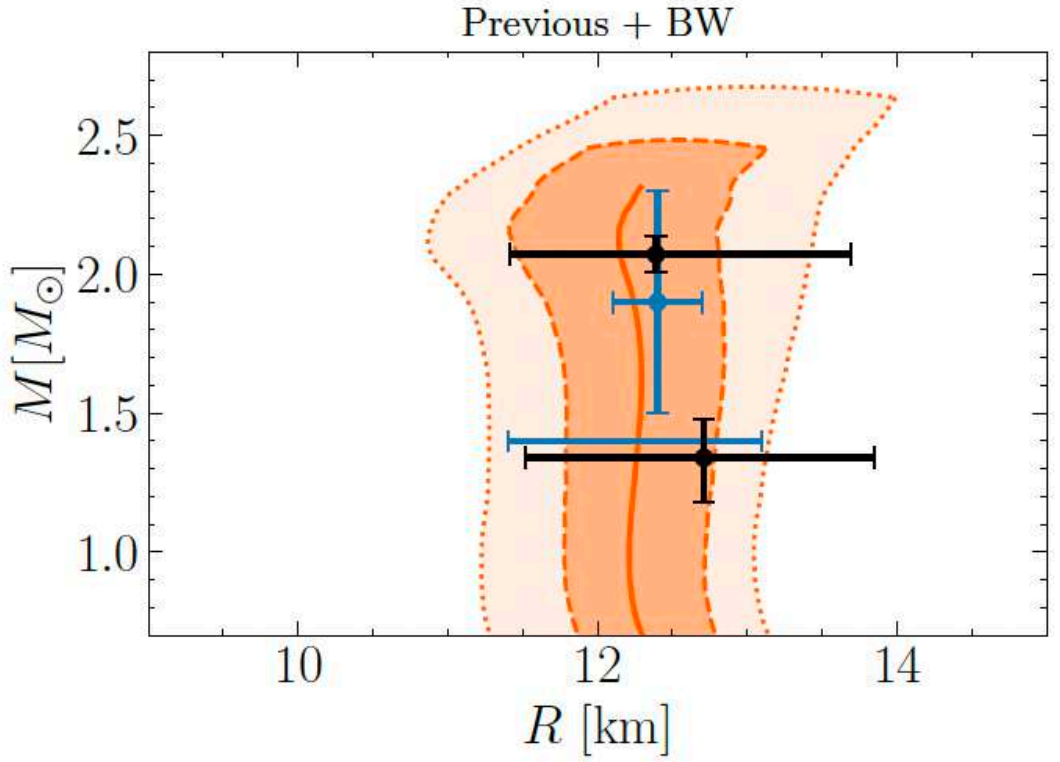
Evidence against a first-order phase transition in neutron star cores: impact of new data

Len Brandes,[*] Wolfram Weise,[†] and Norbert Kaiser[‡]
Technical University of Munich, TUM School of Natural Sciences, Physics Department, 85747 Garching, Germany
(Dated: June 13, 2023)

With the aim of exploring the evidence for or against phase transitions in cold and dense baryonic matter, the inference of the sound speed and equation-of-state for dense matter in neutron stars is extended in view of recent new observational data. The impact of the heavy (2.35 $M_\odot$) black widow pulsar PSR J0952-0607 and of the unusually light supernova remnant HESS J1731-347 is inspected. In addition a detailed re-analysis is performed of the low-density constraint based on chiral effective field theory and of the perturbative QCD constraint at asymptotically high densities, in order to clarify the influence of these constraints on the inference procedure. The trace anomaly measure, $\Delta = 1/3 - P/\varepsilon$, is also computed and discussed. A systematic Bayes factor assessment quantifies the evidence (or non-evidence) of a phase transition within the range of densities realised in the core of neutron stars. One of the consequences of including PSR J0952-0607 in the data base is a further stiffening of the equation-of-state, resulting for a typical 2.1 solar-mass neutron star in a reduced central density of less than five times the equilibrium density of normal nuclear matter. The evidence against the occurrence of a first-order phase transition in neutron star cores is further strengthened.

Bayesian analyses of recent observation data of neutron star

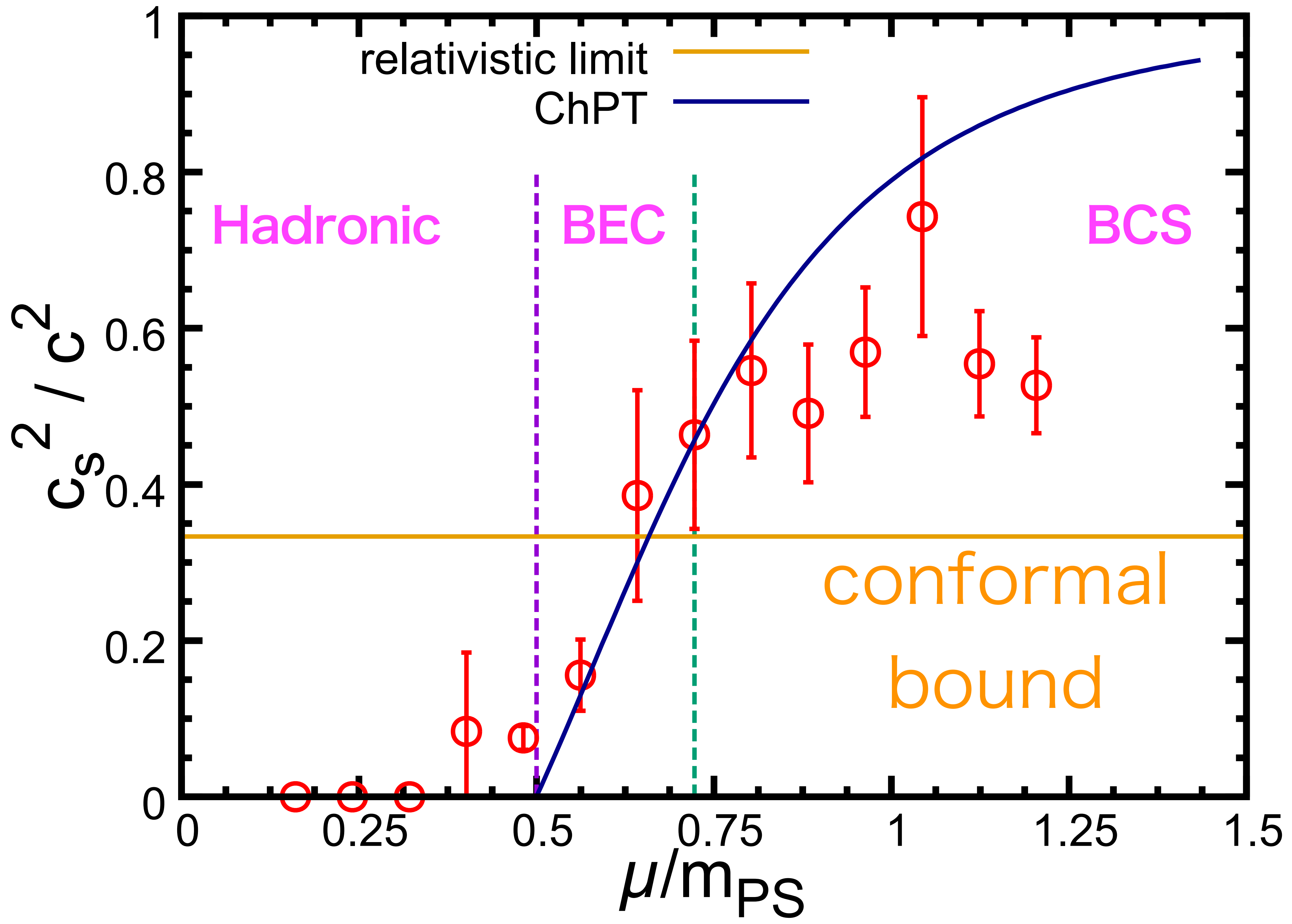




arXiv:2306.06218

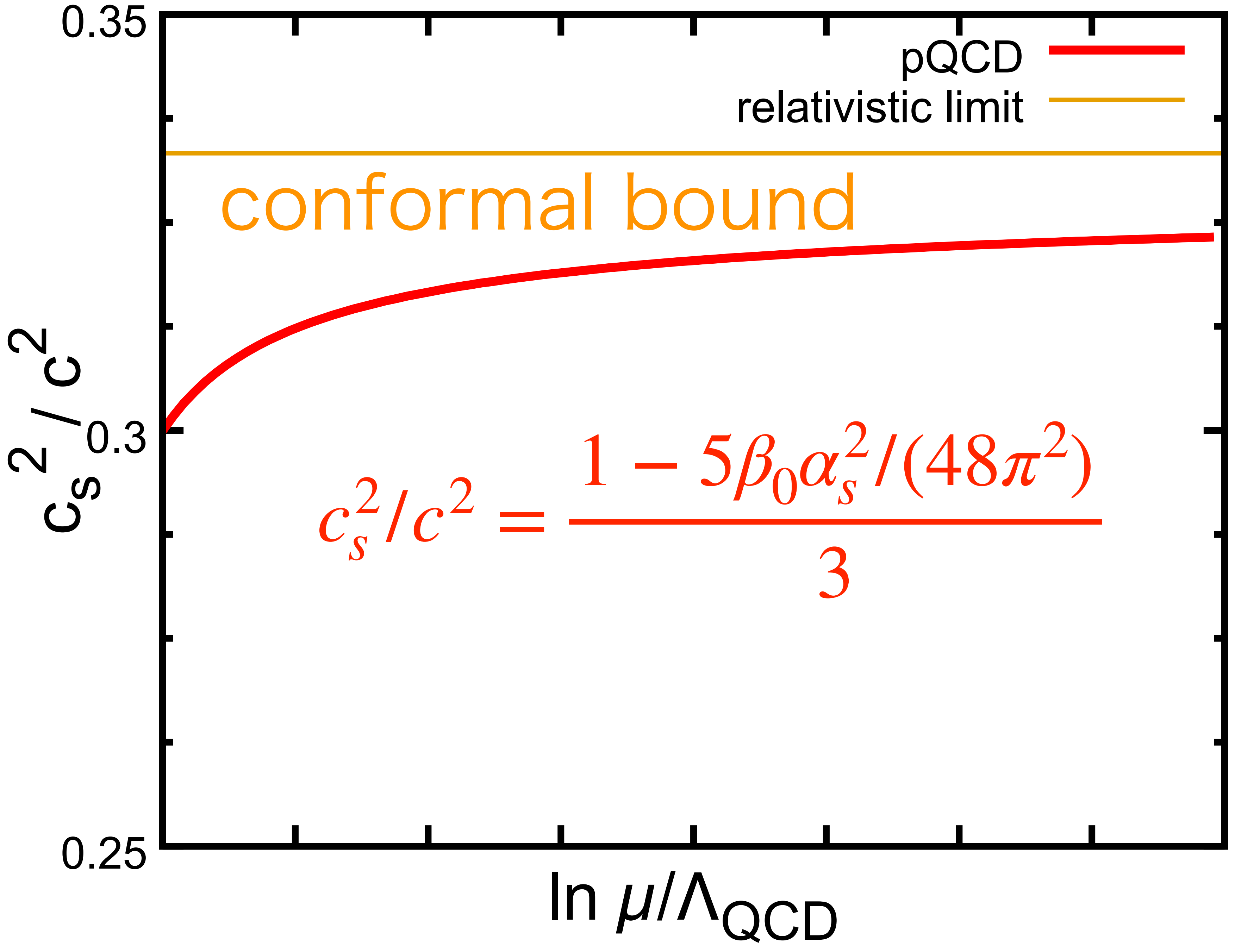
Further high density?

Kojo, Baym, Hatsuda (2021)



pQCD prediction

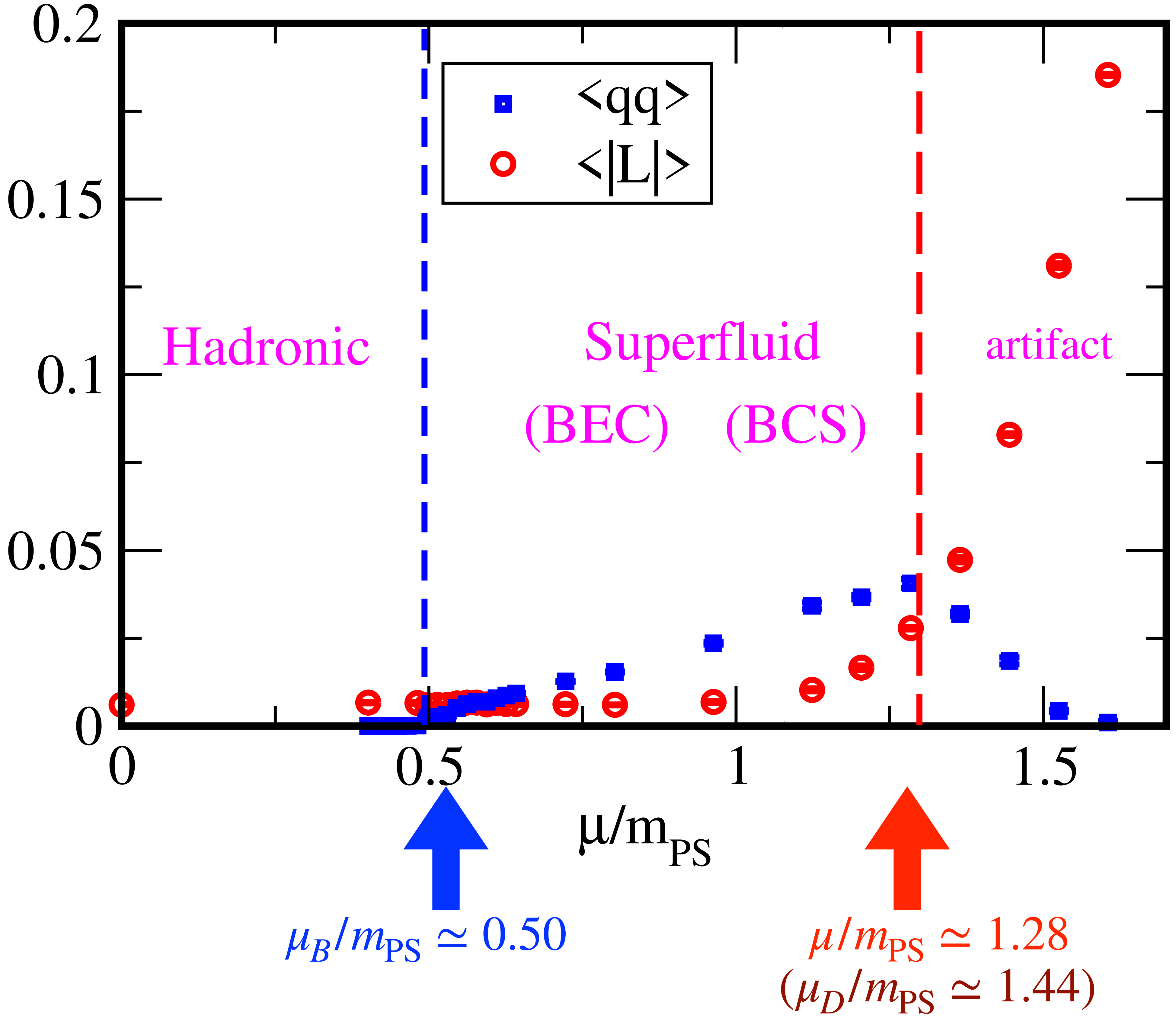
(Ultra high-density regime)



- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$

  (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

Upper limit of $a\mu$



Theoretically, the diquark cond. increases as μ increases.

But the lattice data turn to decrease at $a\mu \approx 1$.

It must come from a lattice artifact.

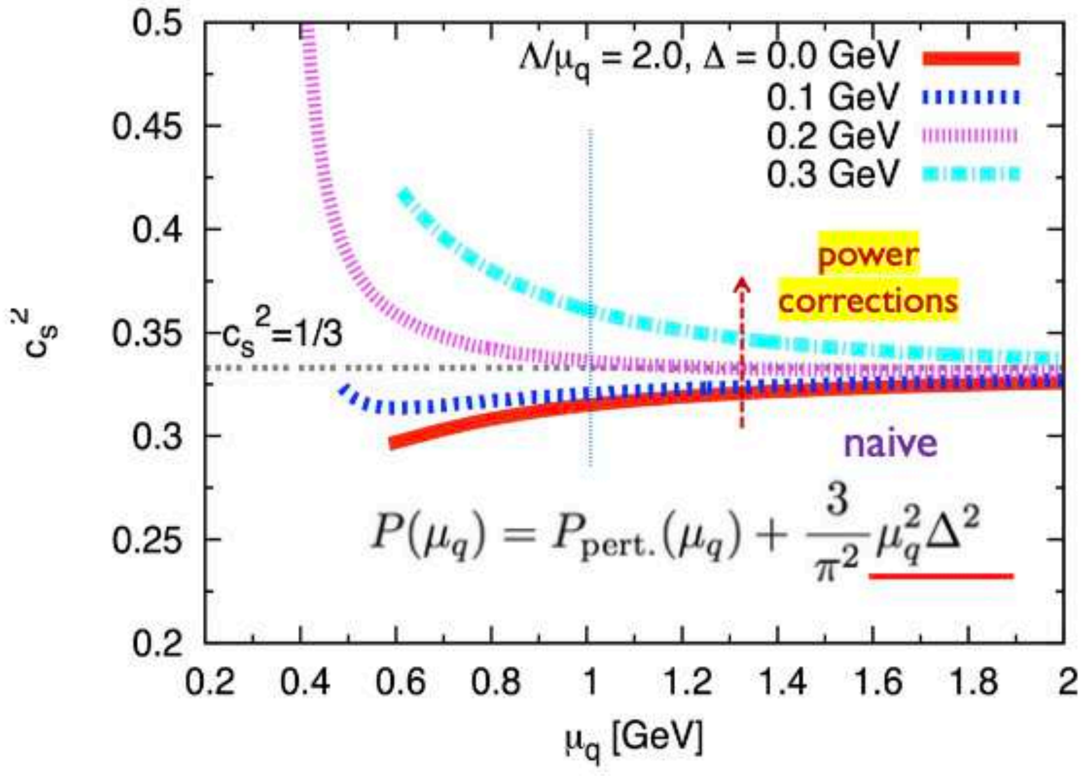
Further high density?

pQCD + power correction due to diquark gap

c_s^2 vs pQCD + power corrections

19/45

Slide by Kojo (2019)



e.g. diquark pairing (CFL) terms

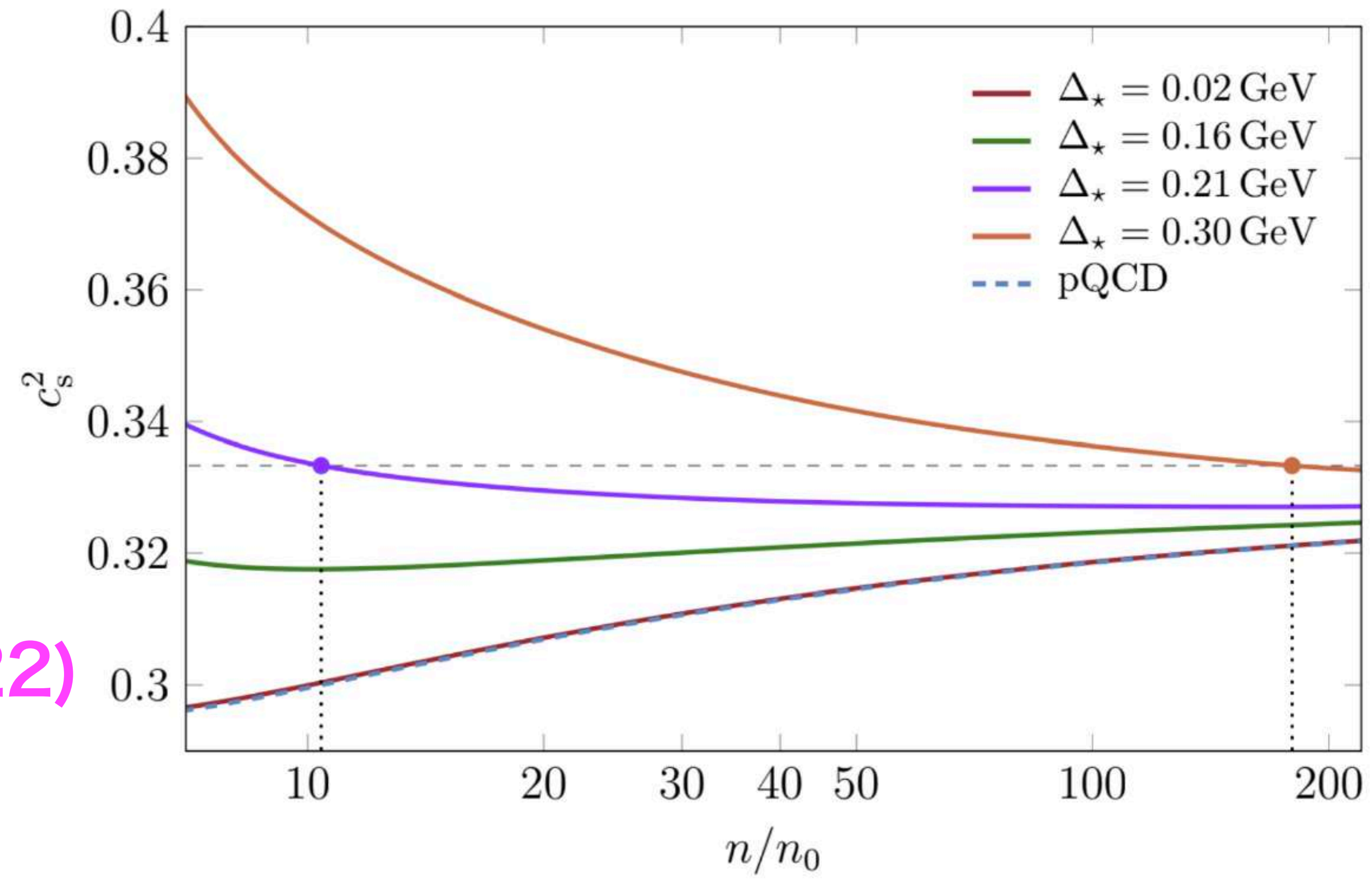
For $\Delta \sim$ **0.2** GeV $\sim \Lambda_{QCD}$

$(\Delta/\mu_q)^2 \sim$ **4 %**

nevertheless,

c_s^2 approach 1/3 from **above**

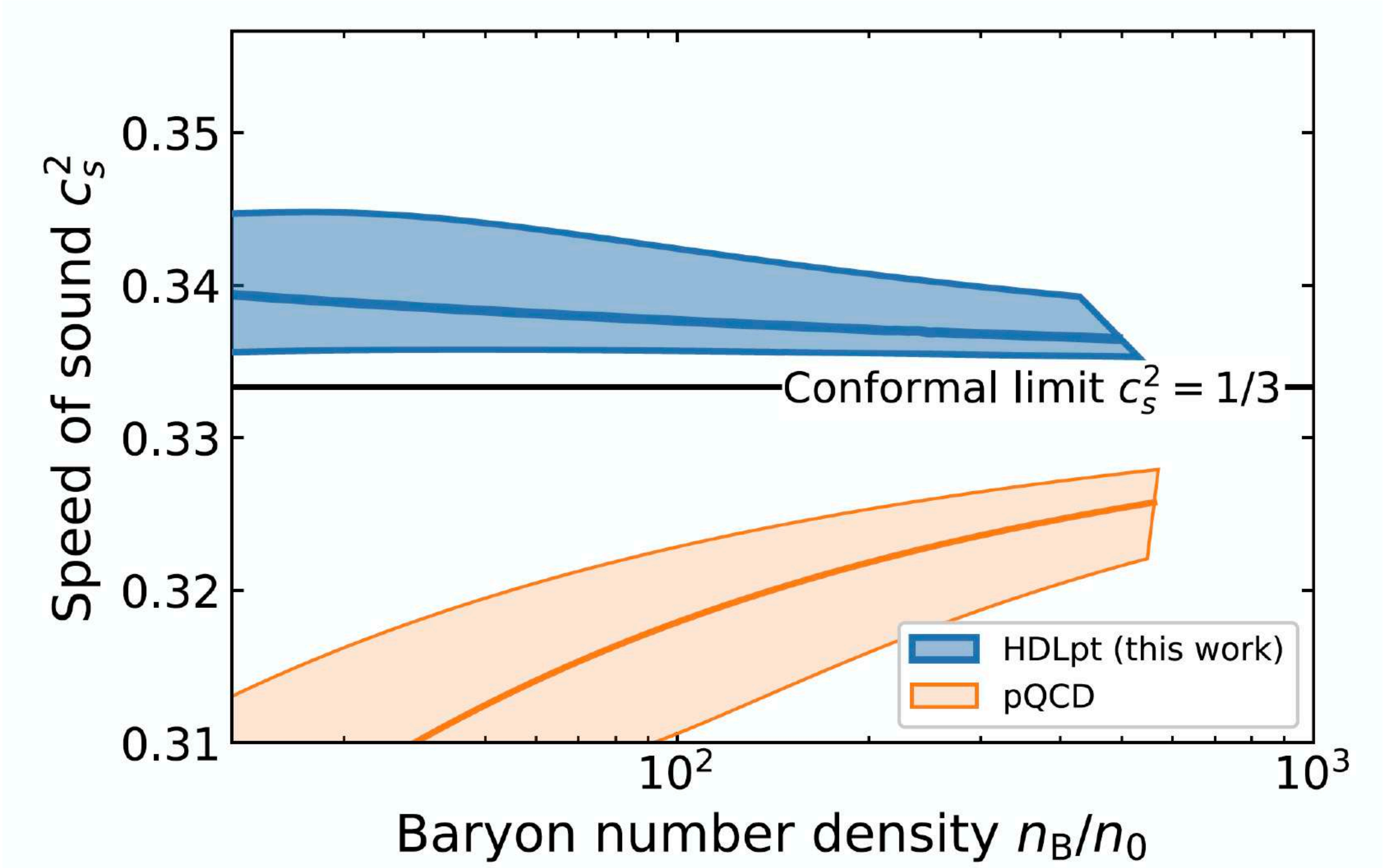
should be more important toward low density



fRG analysis
Braun, Geissel, Schallmo(2022)

Hard thermal loop resummation

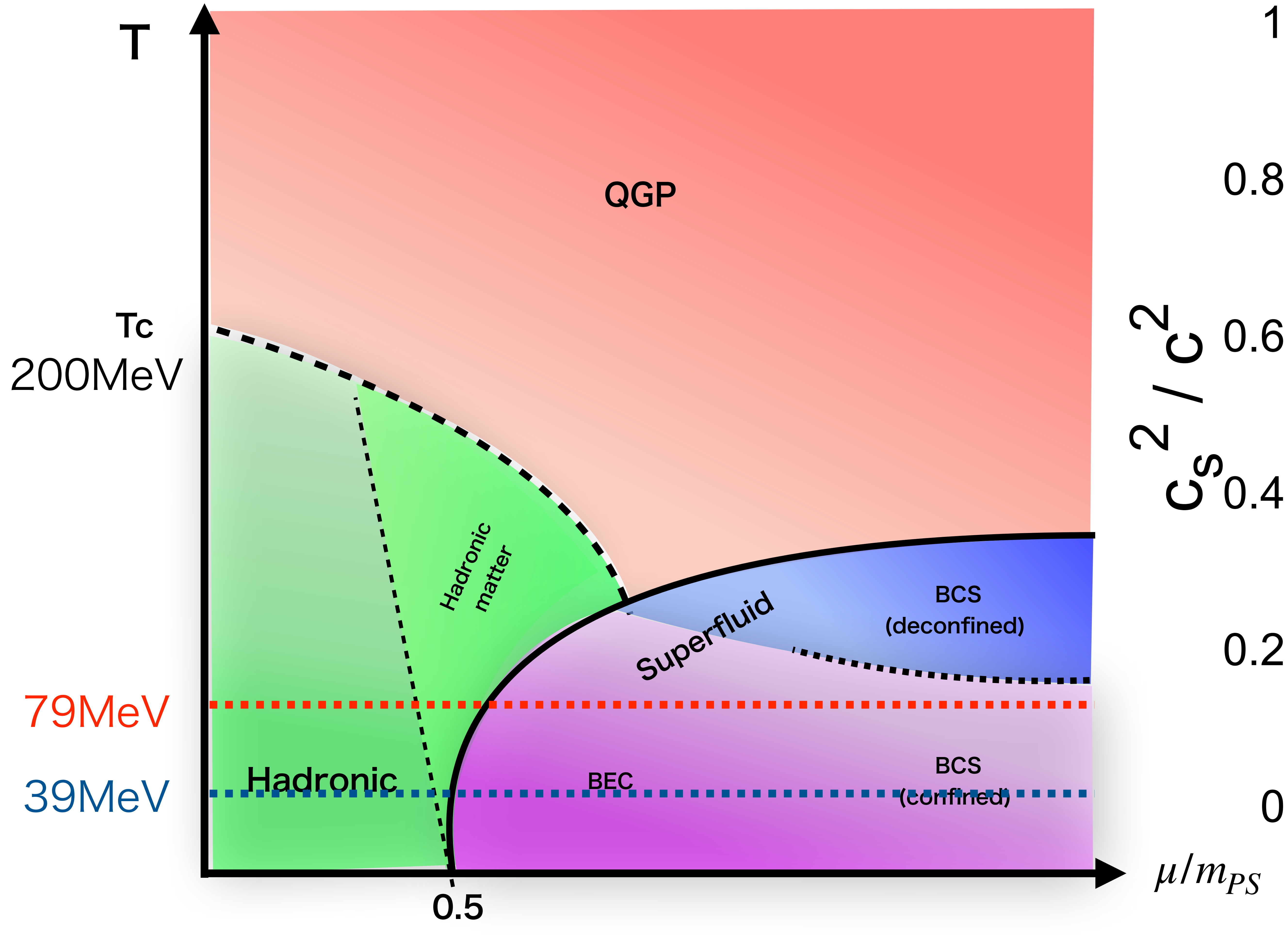
Fujimoto and Fukushima(2021)

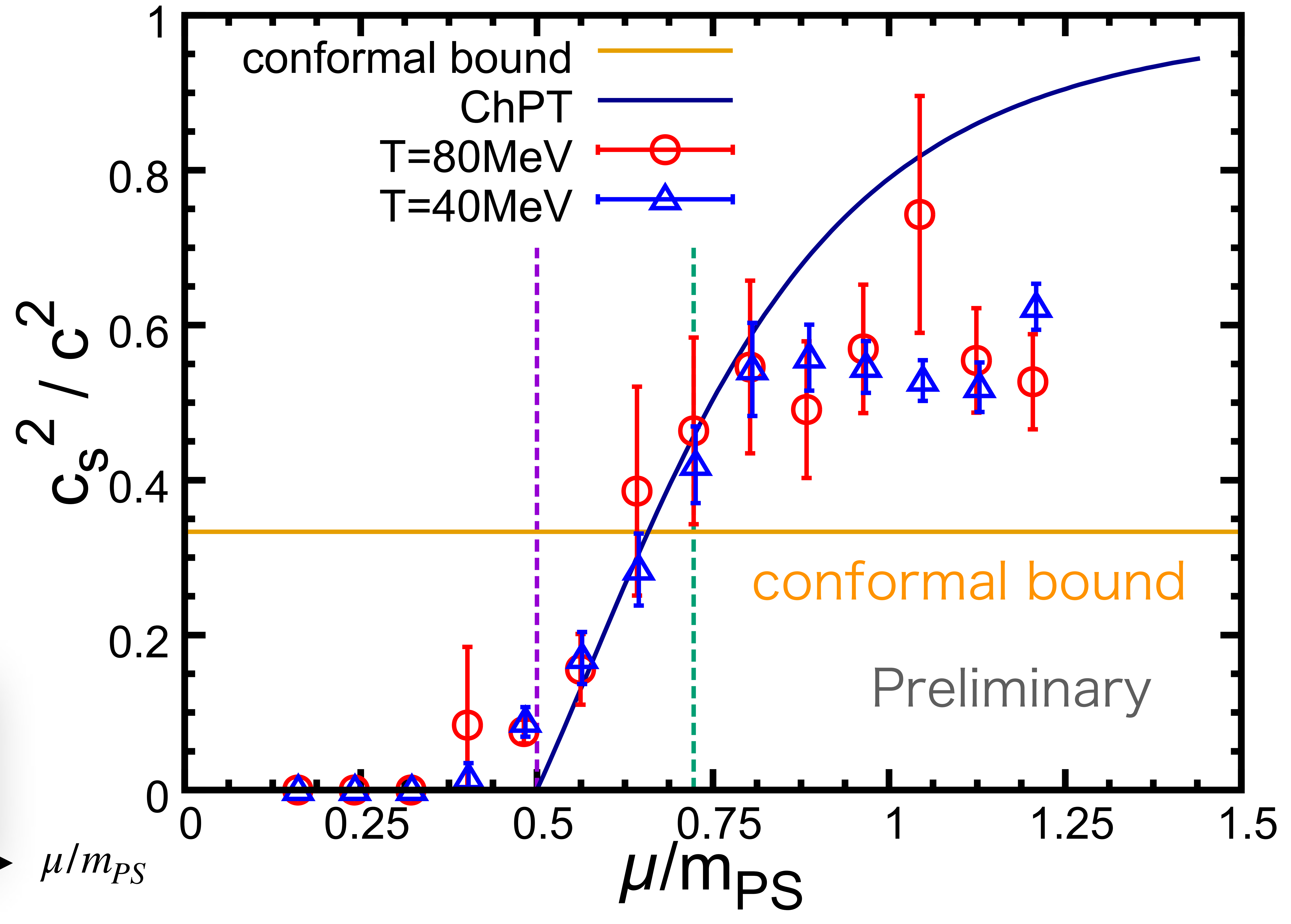


- Open question: How c_s^2/c^2 approaches 1/3; from below or from above?

Further low temperature

T~40MeV data: Phase diagram (μ_c value, BEC-BCS crossover) is not changed





Summary of EoS

- Sound velocity exceeds the relativistic limit in finite-density QCD-like theory
  It seems to have a peak after BEC-BCS crossover
  cf.) cond-mat model study also find it Tajima and Liang (2022)

- How c_s^2/c^2 approaches 1/3; from below or from above?

- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), strong coupling with trace anomaly?(McLerran,Fukushima et al.), others?
  - attractive or repulsive force between hadrons?
    => extended HAL QCD method in finite density
    => mass spectrum in superfluid phase

Hadron spectrum

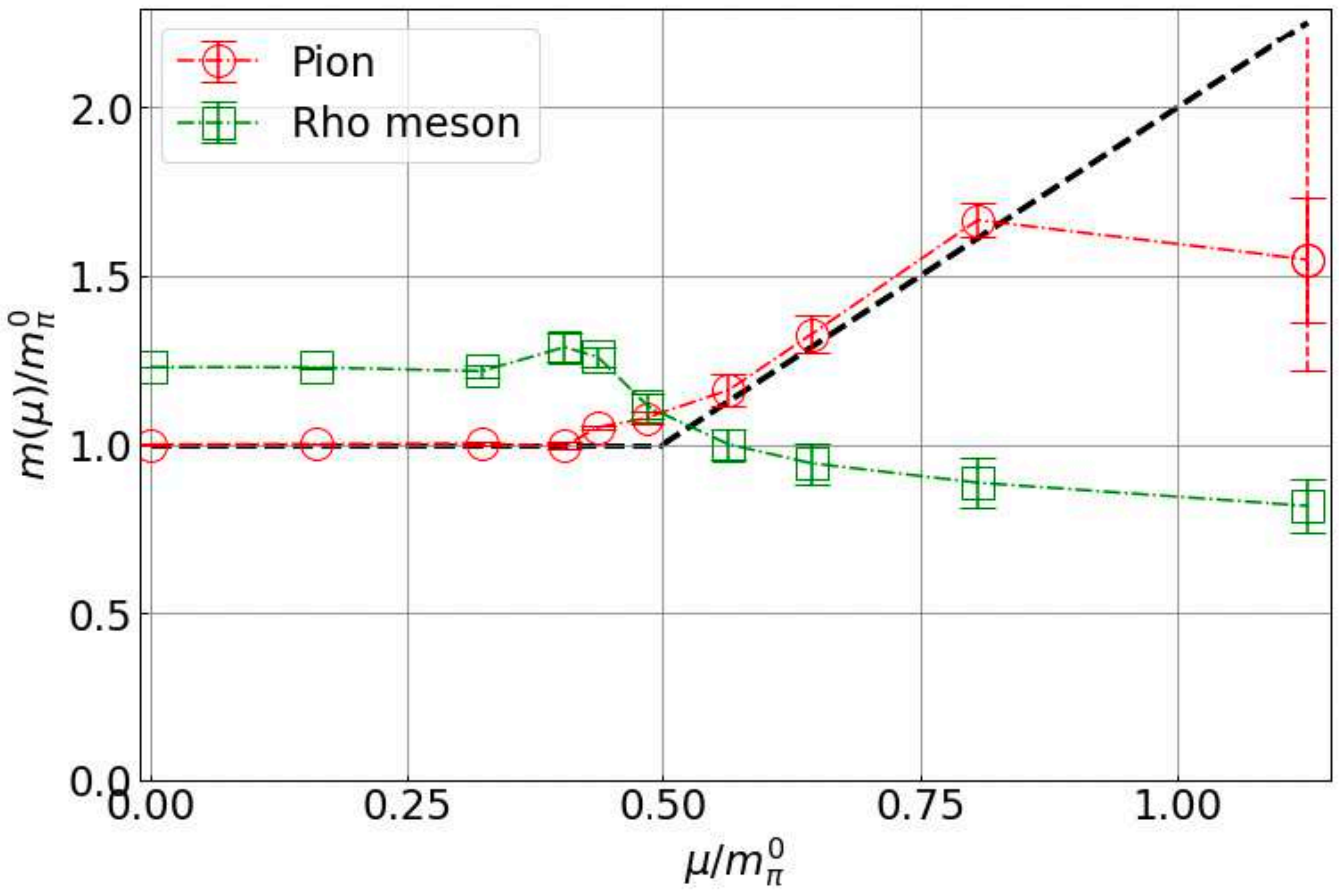
- Pion (pseudo-scalar meson) is lightest hadron at $\mu = 0$

- QCD inequality grantees it
  - no disconnected diagram
  - Gamma_5 Hermitisity

- At $\mu \neq 0$, who is the lightest hadron?

- cf.) Hatsuda-Lee(1992) rho meson mass decreases in nuclear medium

K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472 (PoS, Lattice 2022)



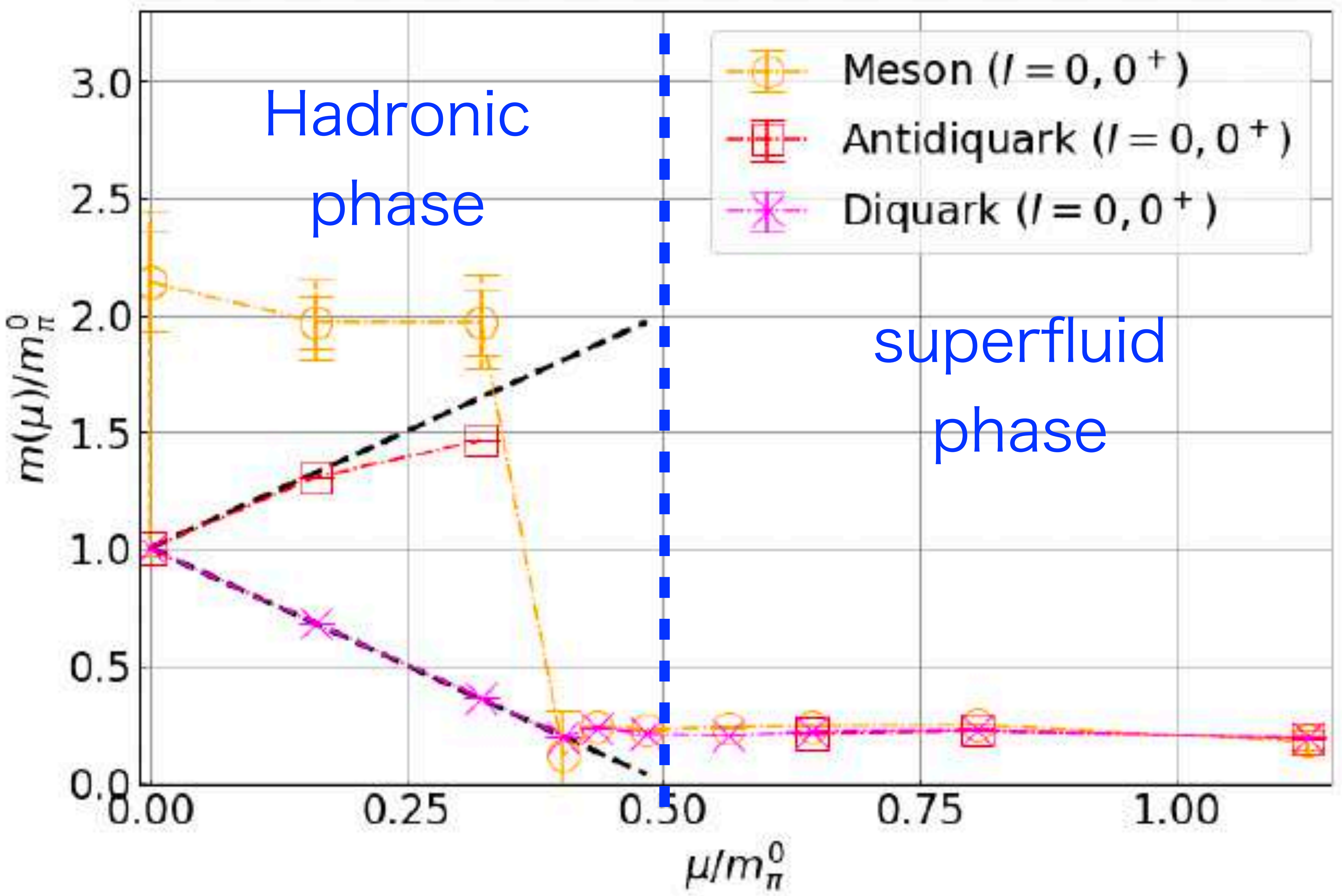
$m_\rho < m_\pi$ in superfluid phase in dense 2color QCD!!

Hadron spectrum

- In 2color QCD,
  QCD inequality
  => isoscalar diquark (NG mod of U(1)B) is the lightest hadron in superfluid phase

Kogut, Stephanov, Toublan
hep-ph/9906346[hep-ph]

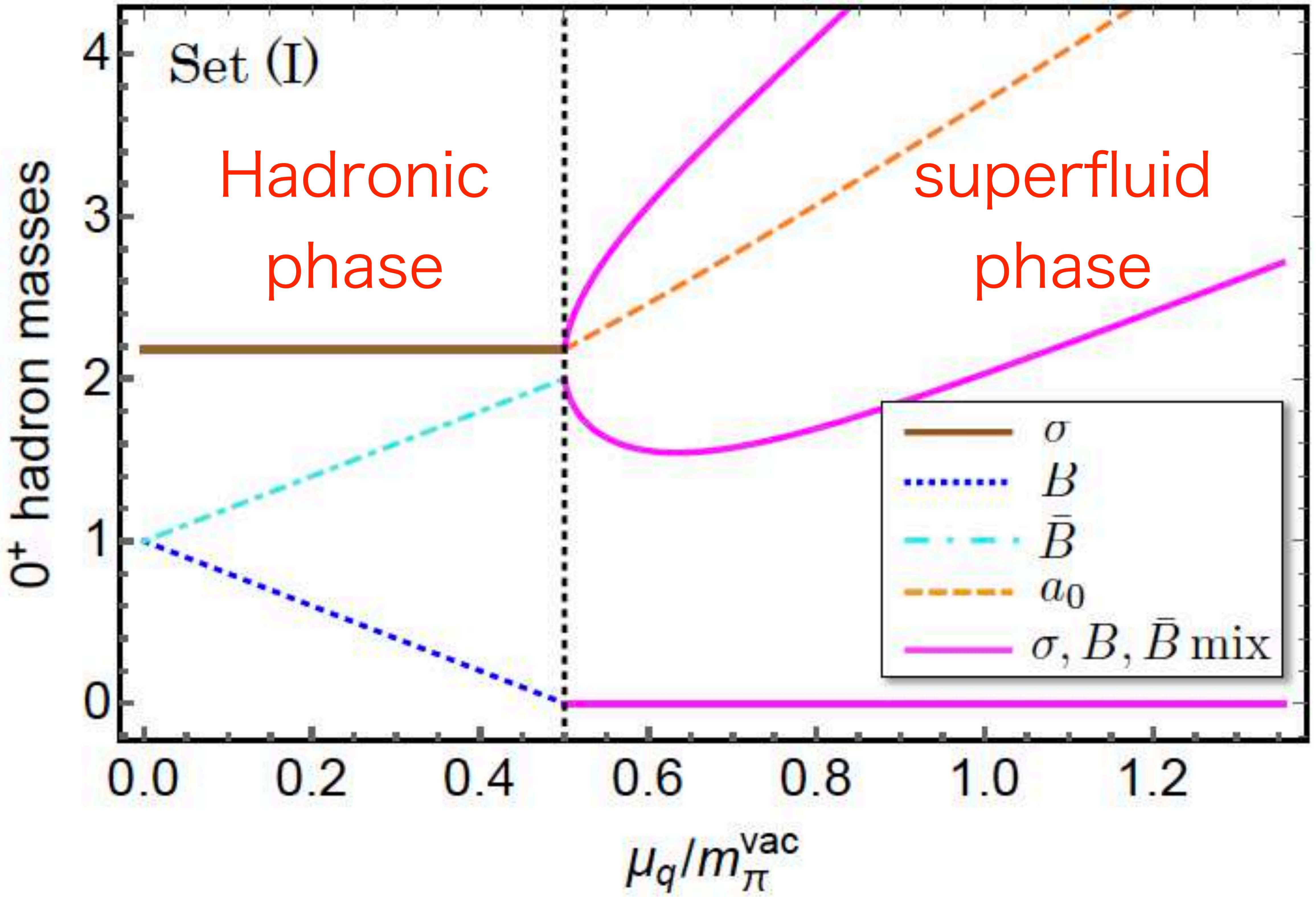
K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472
(PoS, Lattice 2022)



Hadron spectrum

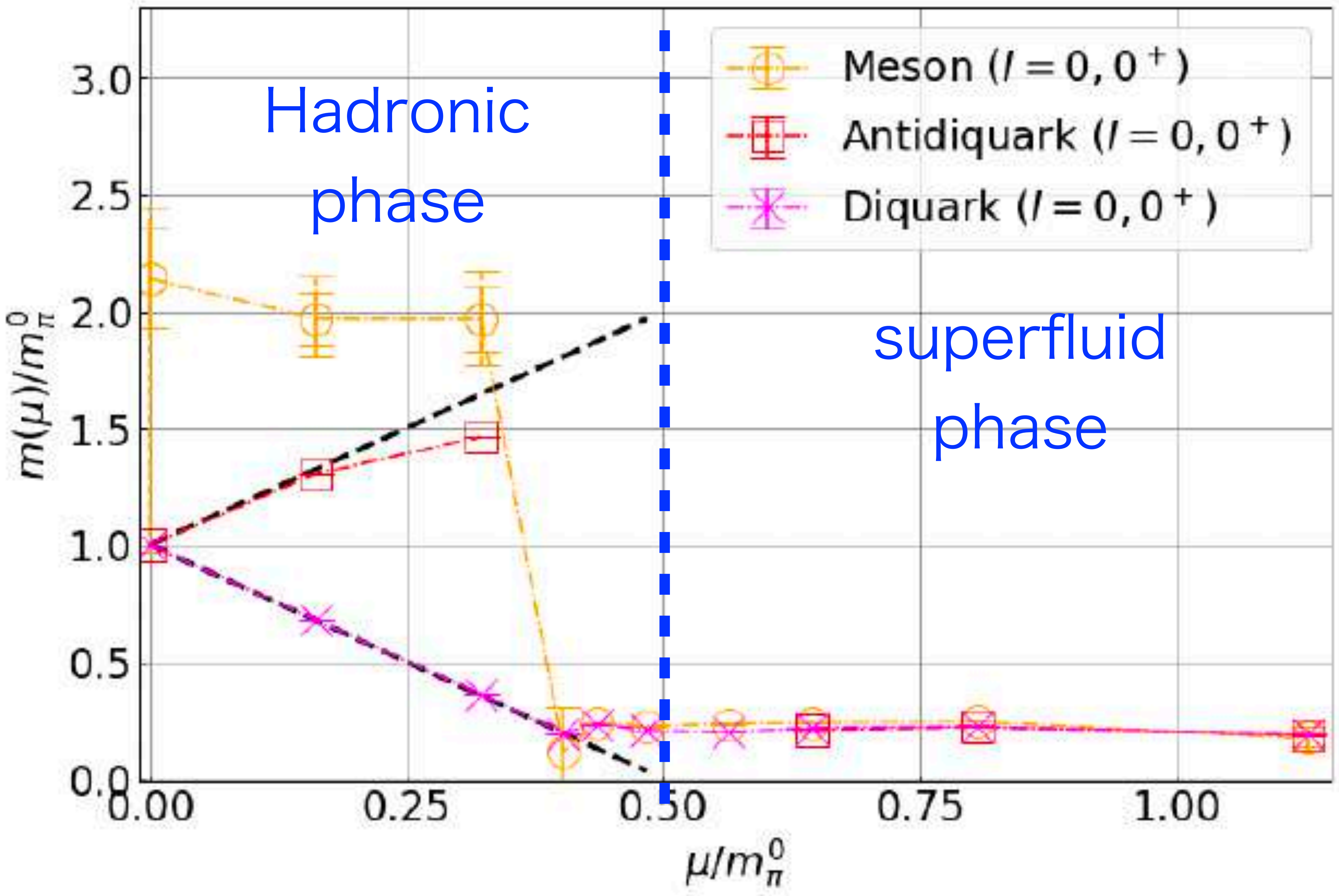
I=0 Scalar channel

Linear sigma model
w/ diquark gap



D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001(2023)

Lattice results



K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472
(PoS, Lattice 2022)

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I=0 Pseudo-Scalar channel

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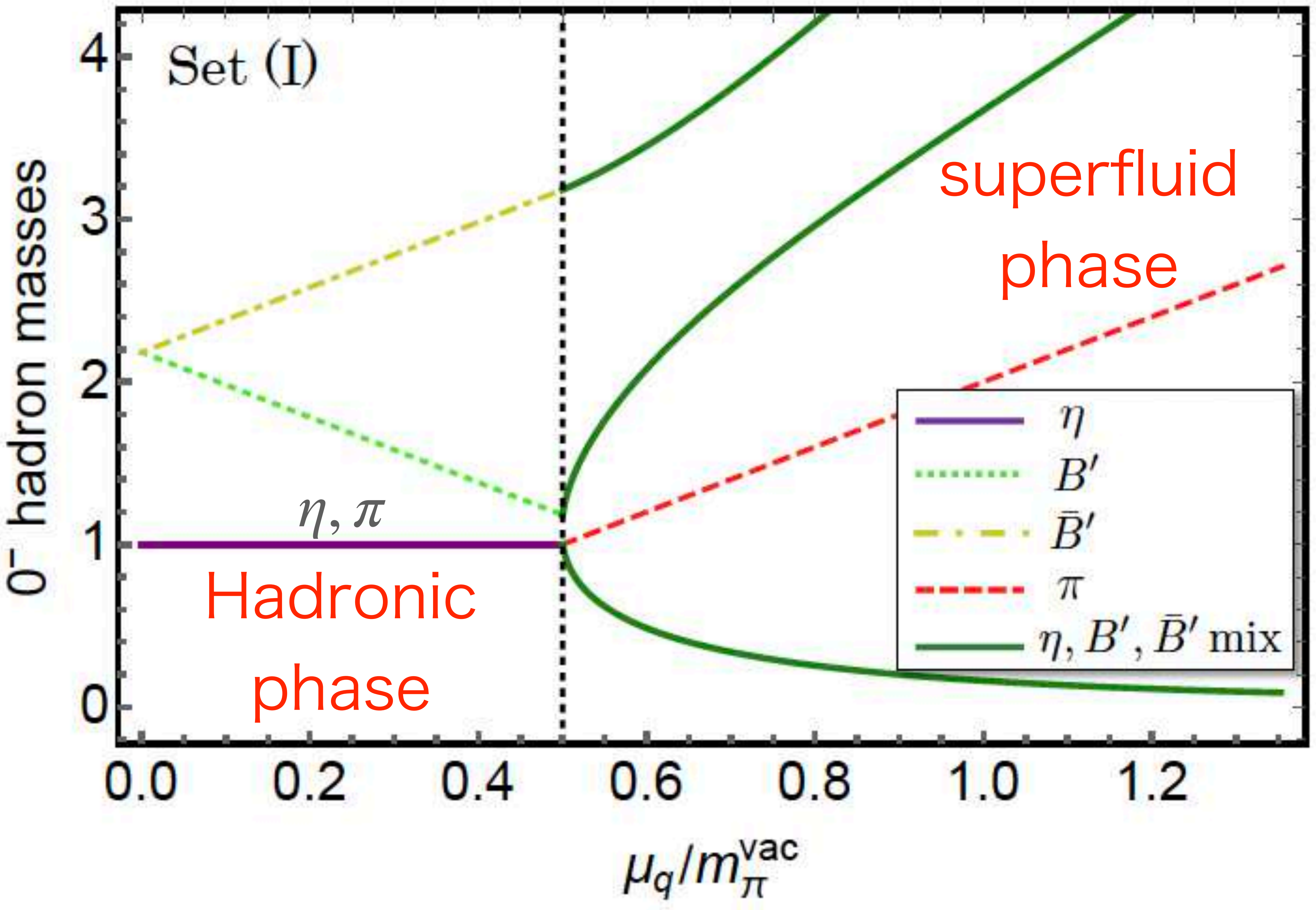
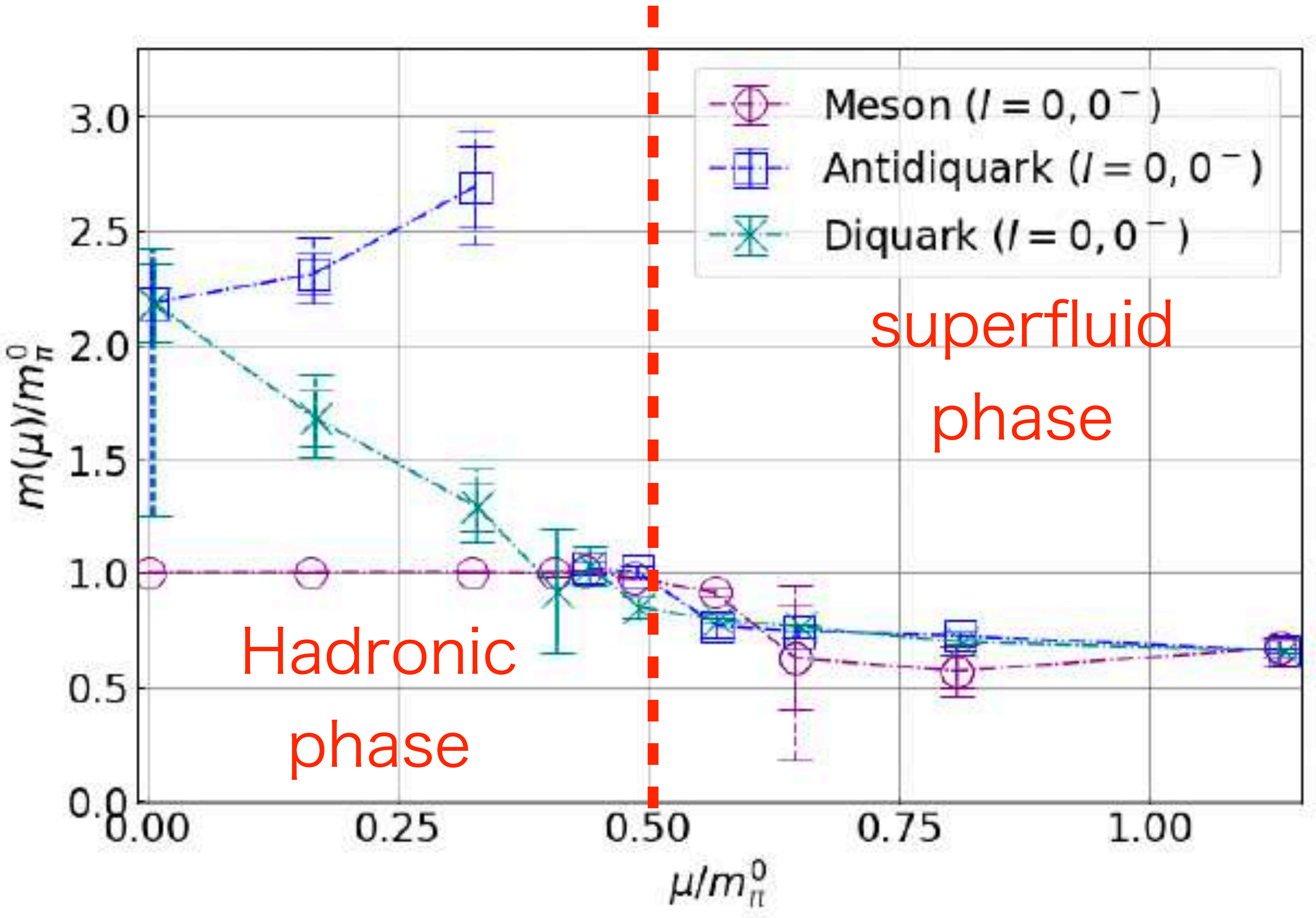


D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001(2023)

Lattice results

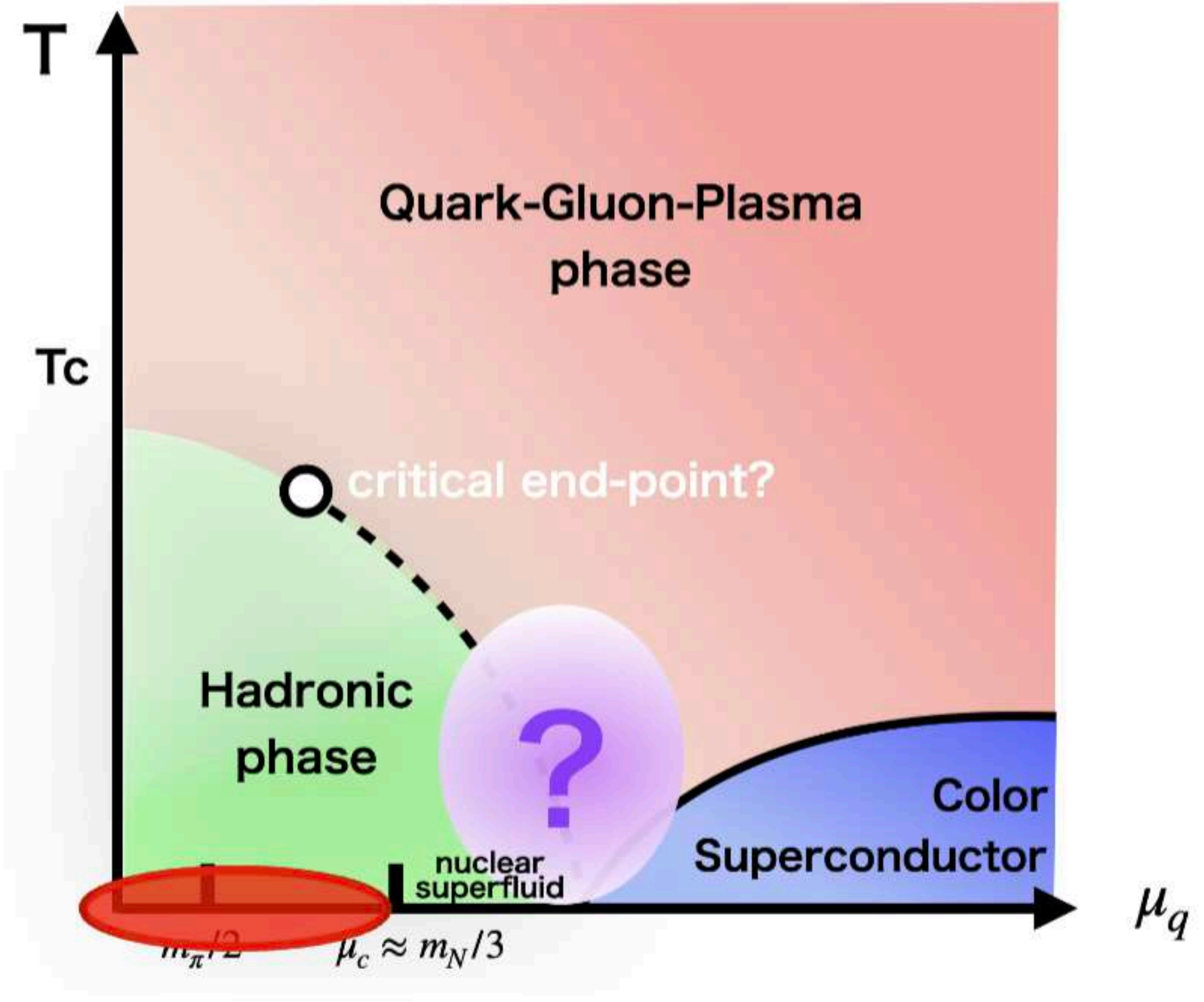


K.Murakami, D.Suenaga, K.Iida, EI, arXiv:2211.13472
(PoS, Lattice 2022)

Hadron interaction (HAL QCD method)

Formulate the HAL QCD method in finite-density

K.Murakami, K.Iida, EI, arXiv:2309.08143



As a first trial:

consider only hadronic phase at $\mu \neq 0$

Physical quantities should be

μ-independent (Silver-Blaze phenom.)

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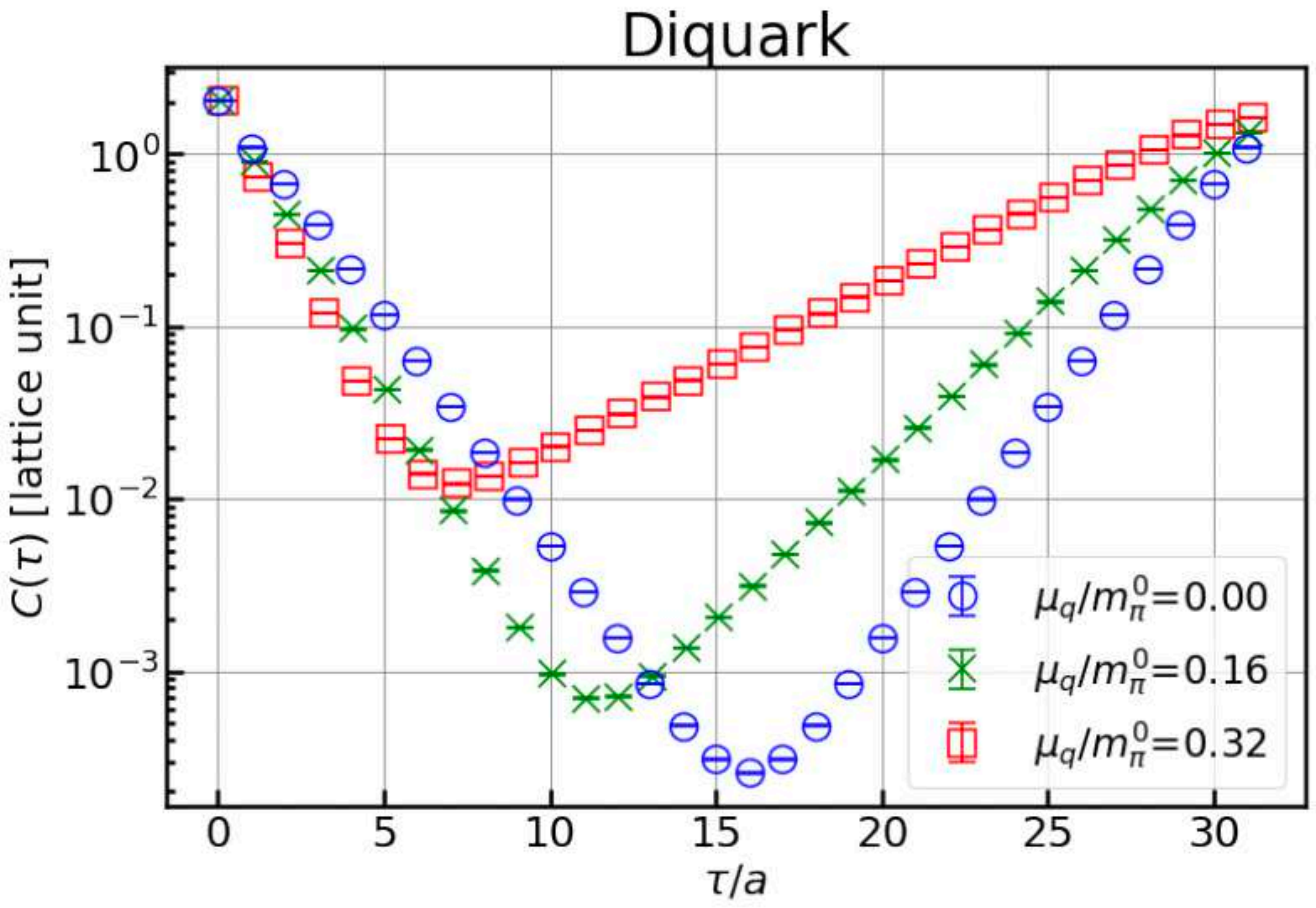

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How we can see μ-independence

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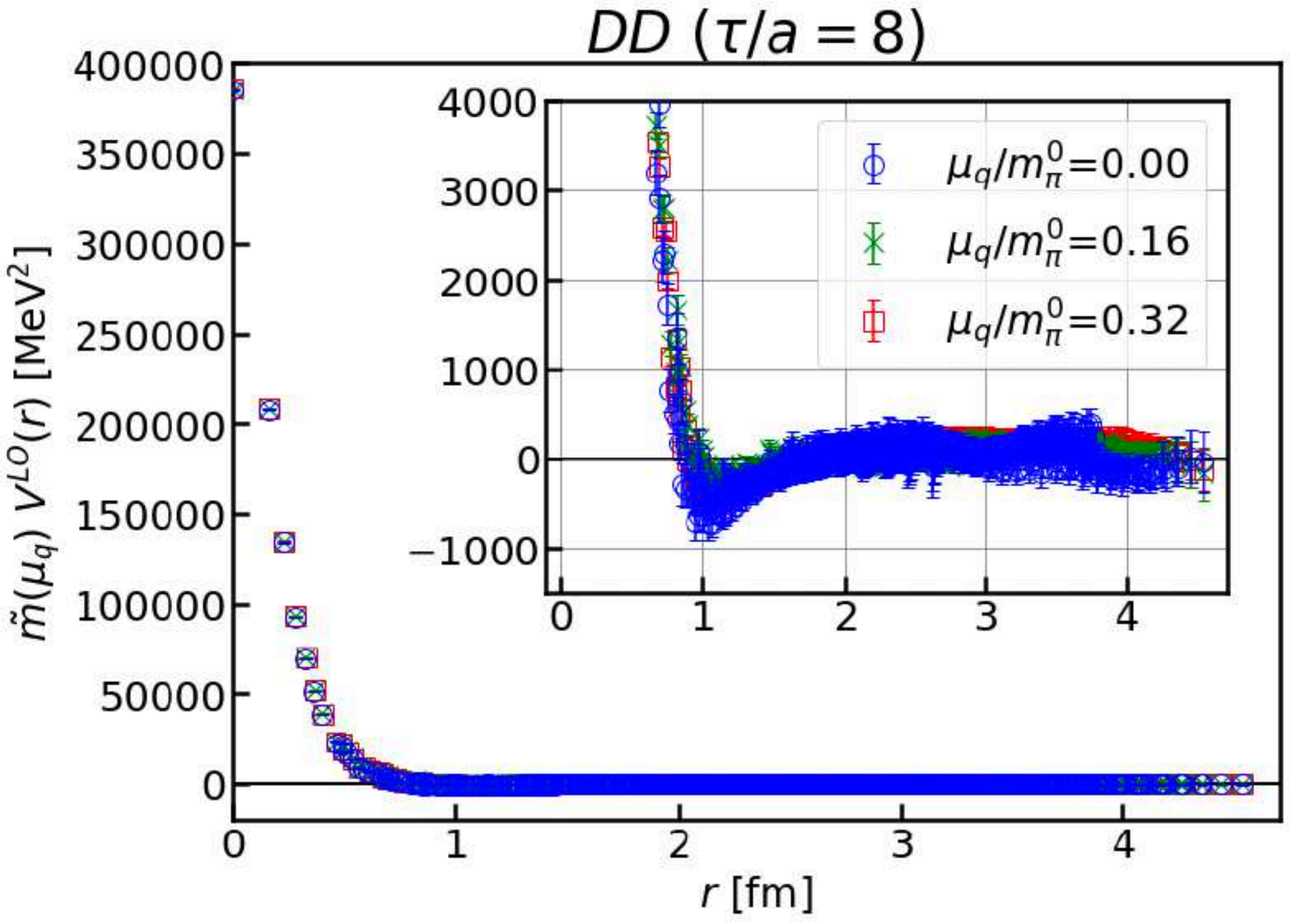
K.Murakami, K.Iida, EI, arXiv:2309.08143



How we can see μ-independence from μ-depend correlation fn?

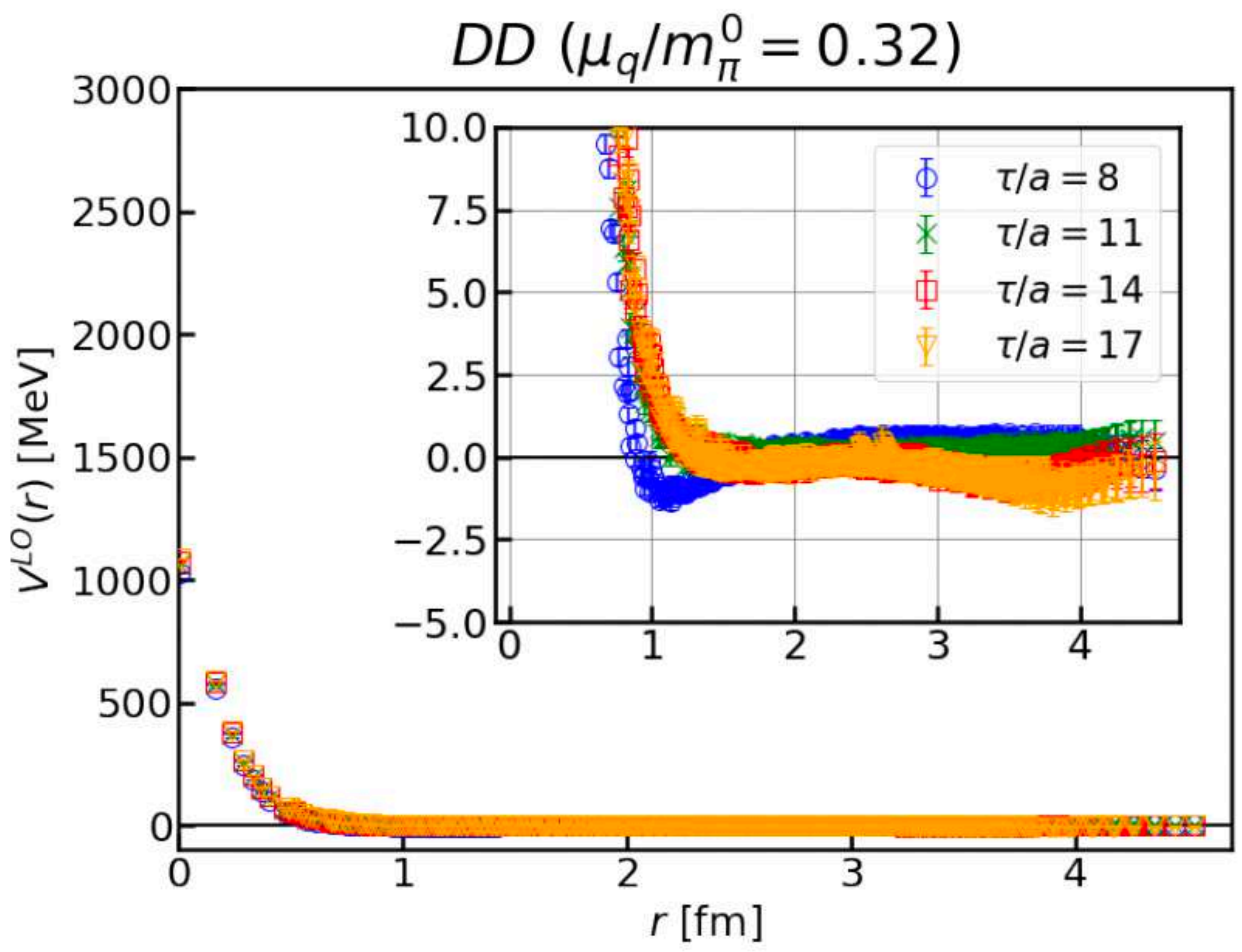
Obtained HAL QCD potential is independent of μ.

Hadron interaction (HAL QCD method)

Potential shape is very similar with 3color QCD

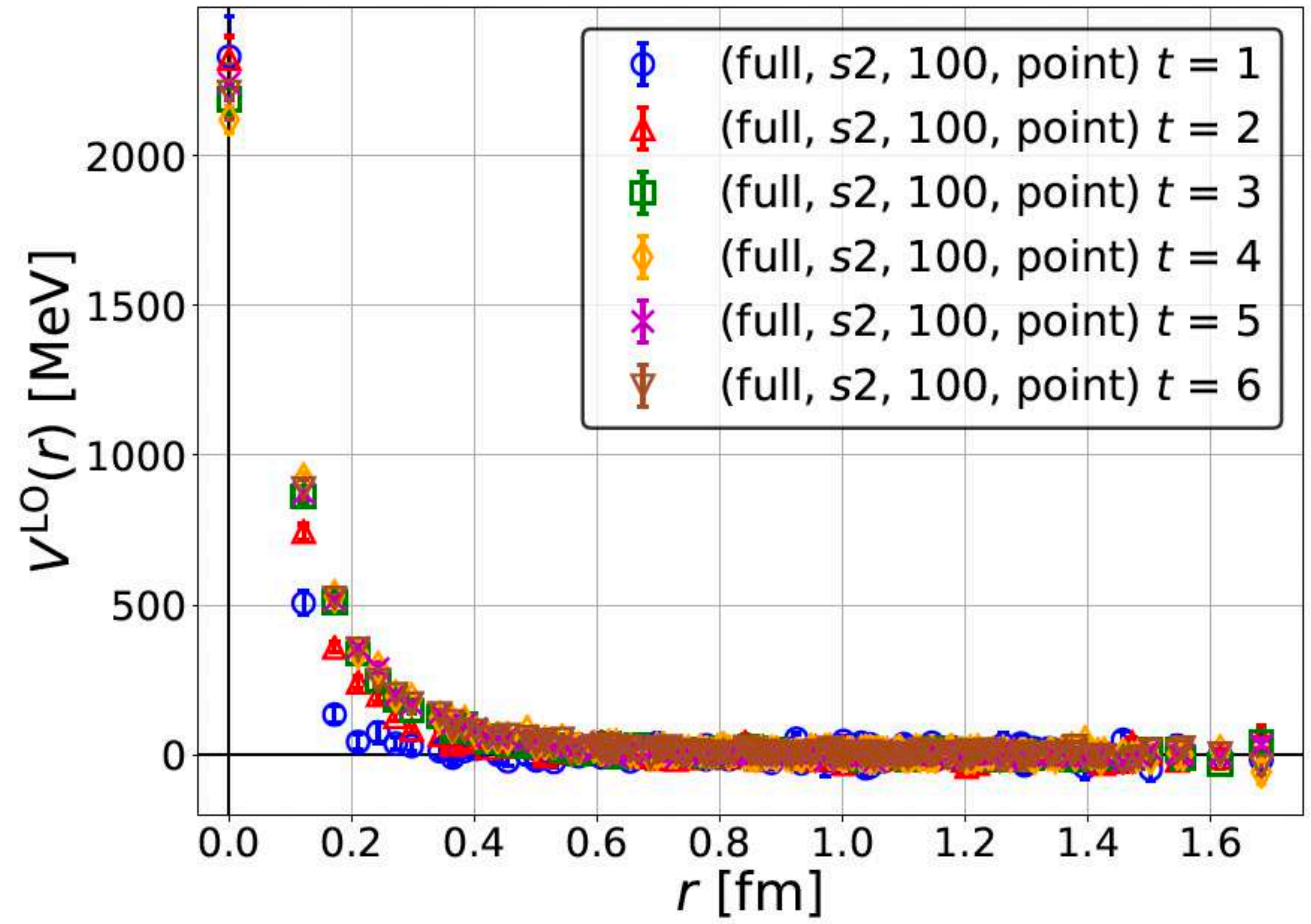
2color QCD, DD channel = $I=2$, $\pi\pi$ channel

K.Murakami, K.Iida, EI, arXiv:2309.08143



3color QCD, $I=2$, $\pi\pi$ channel

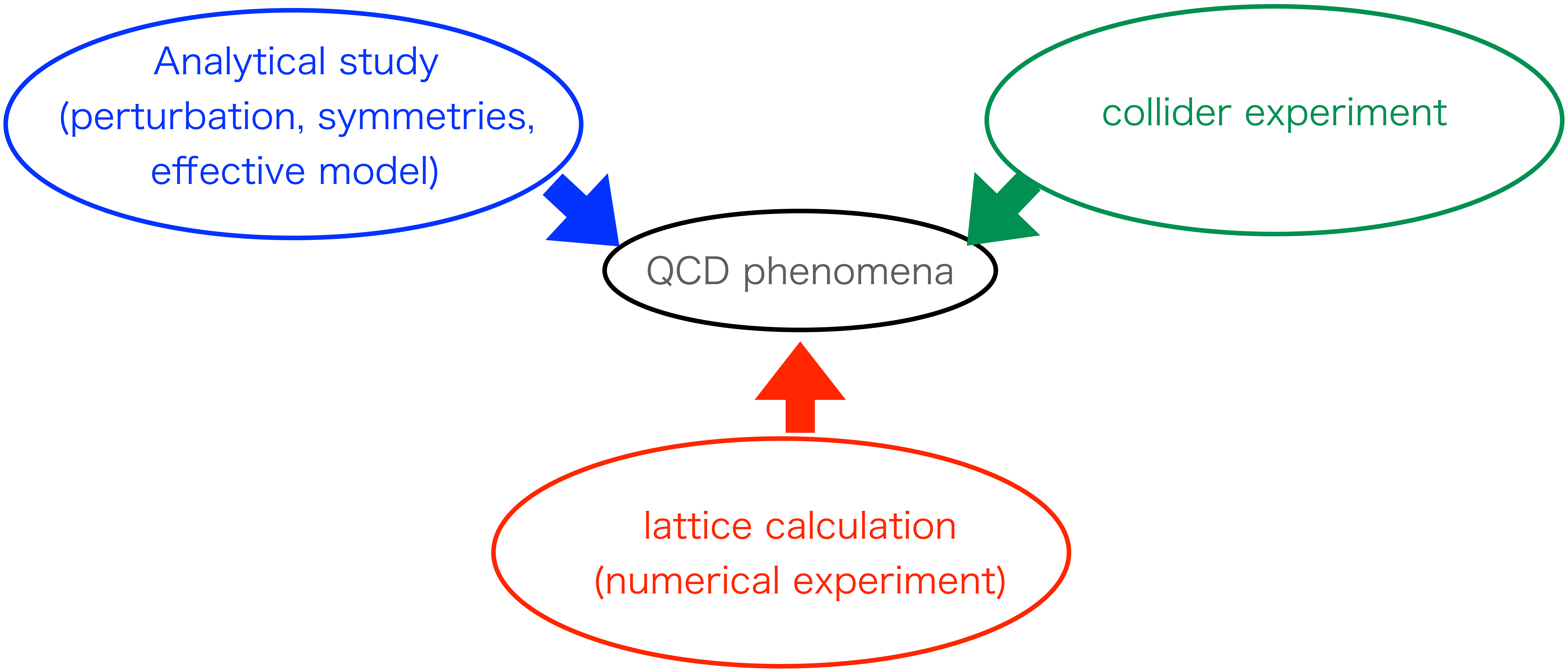
Akahoshi et al., arXiv:1904.09549



In short range, repulsive core comes from fermi statistic of quarks.

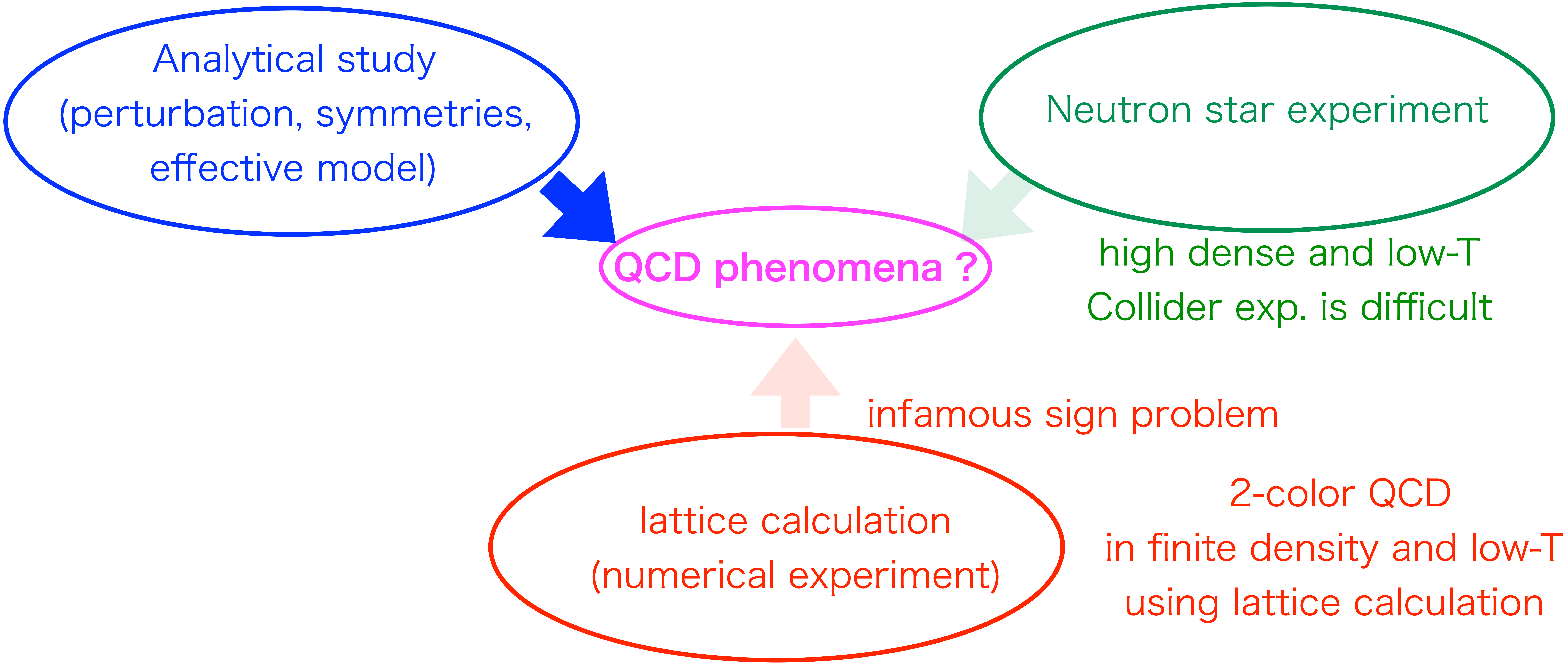
QCD at $\mu = 0$ for $T = 0$ and $T > 0$

- QCD: $\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi$



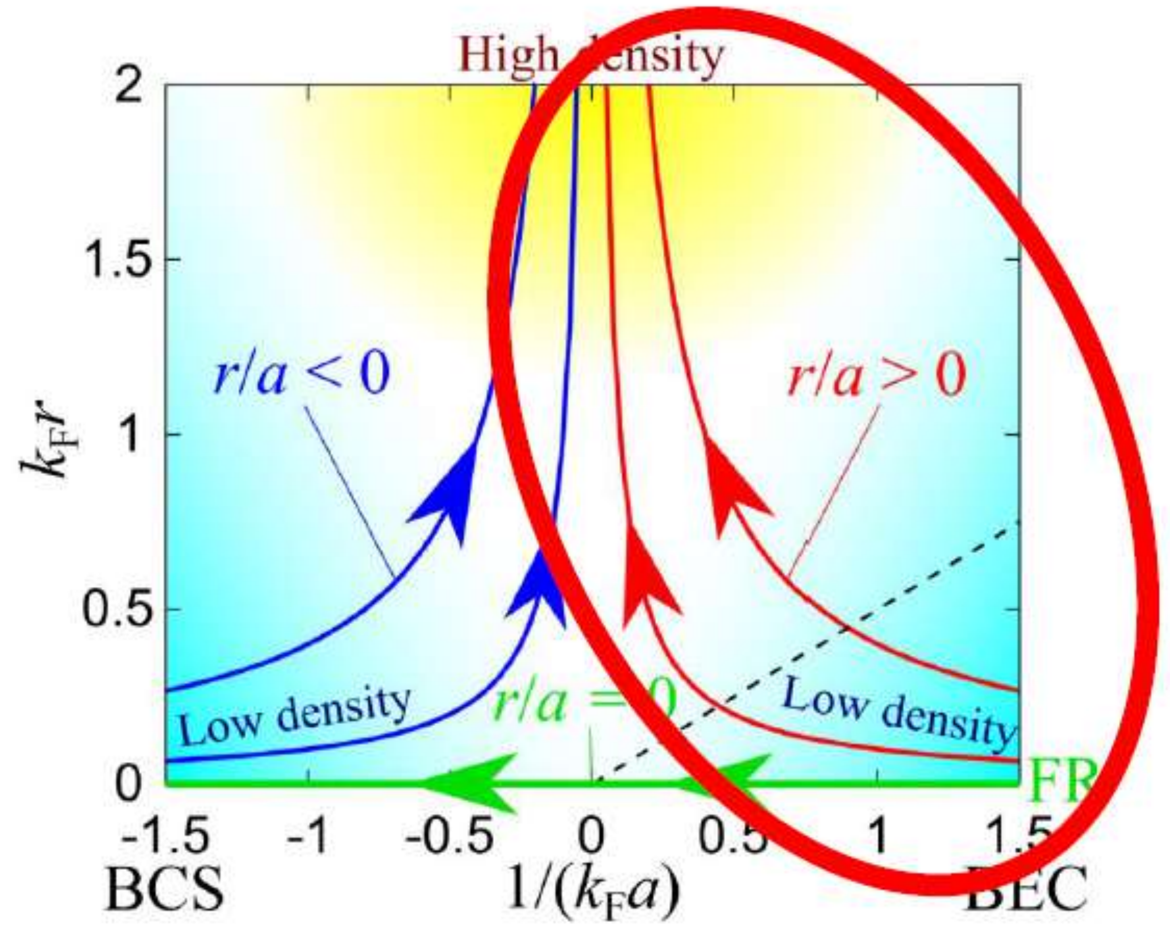
Current status on Dense QCD

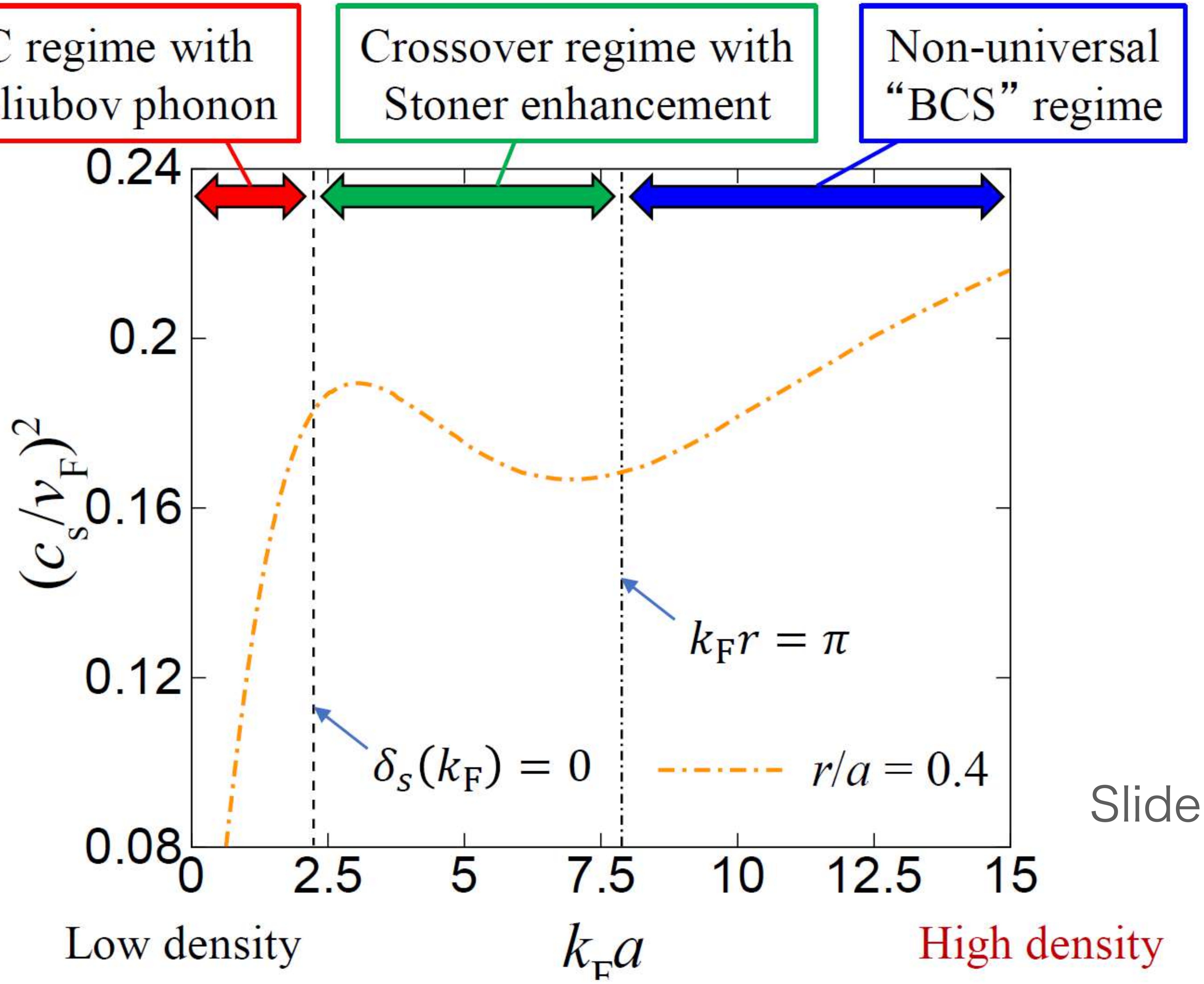
- dense QCD: $\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$



backup

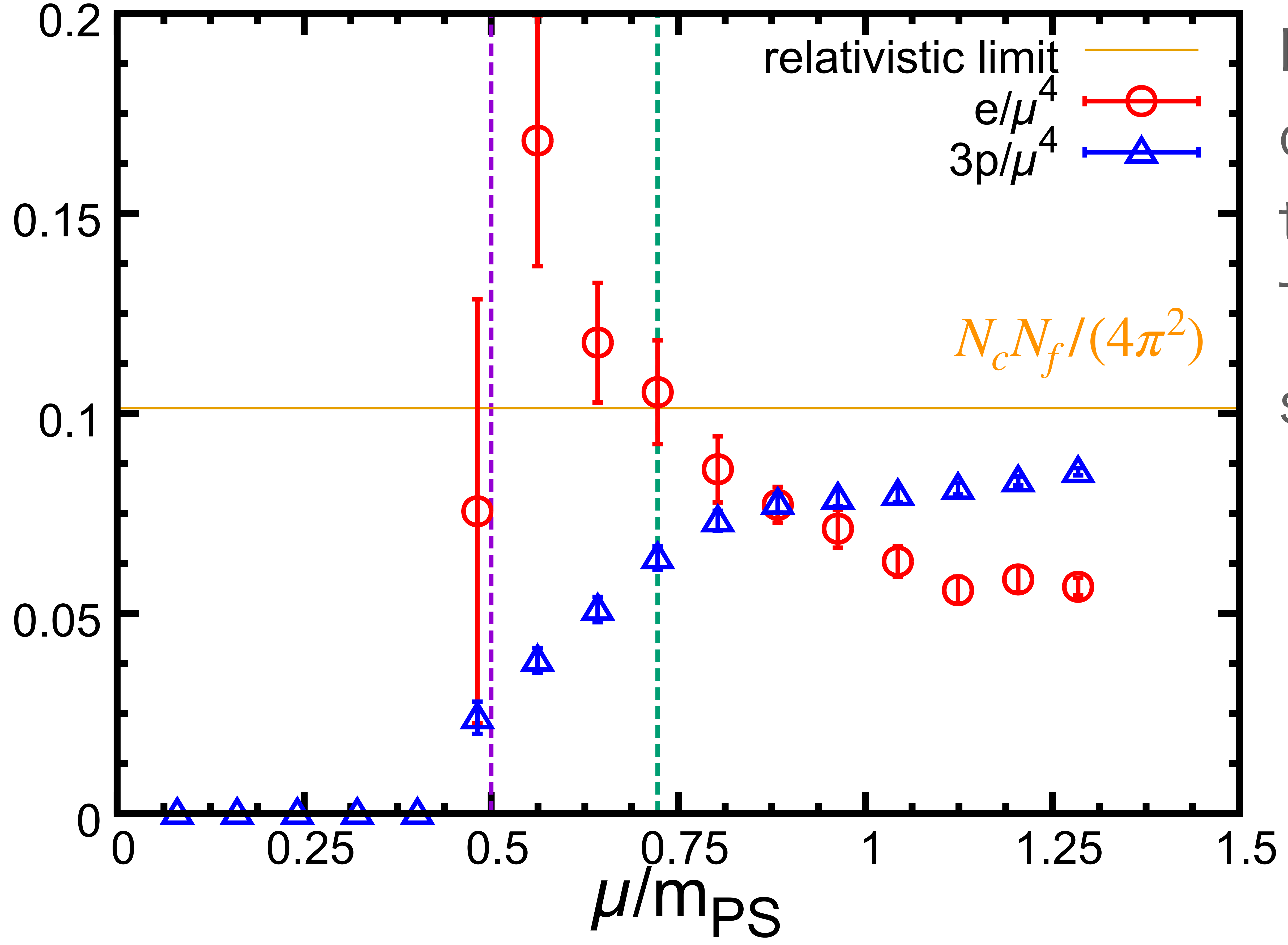
Example of cond.mat. model





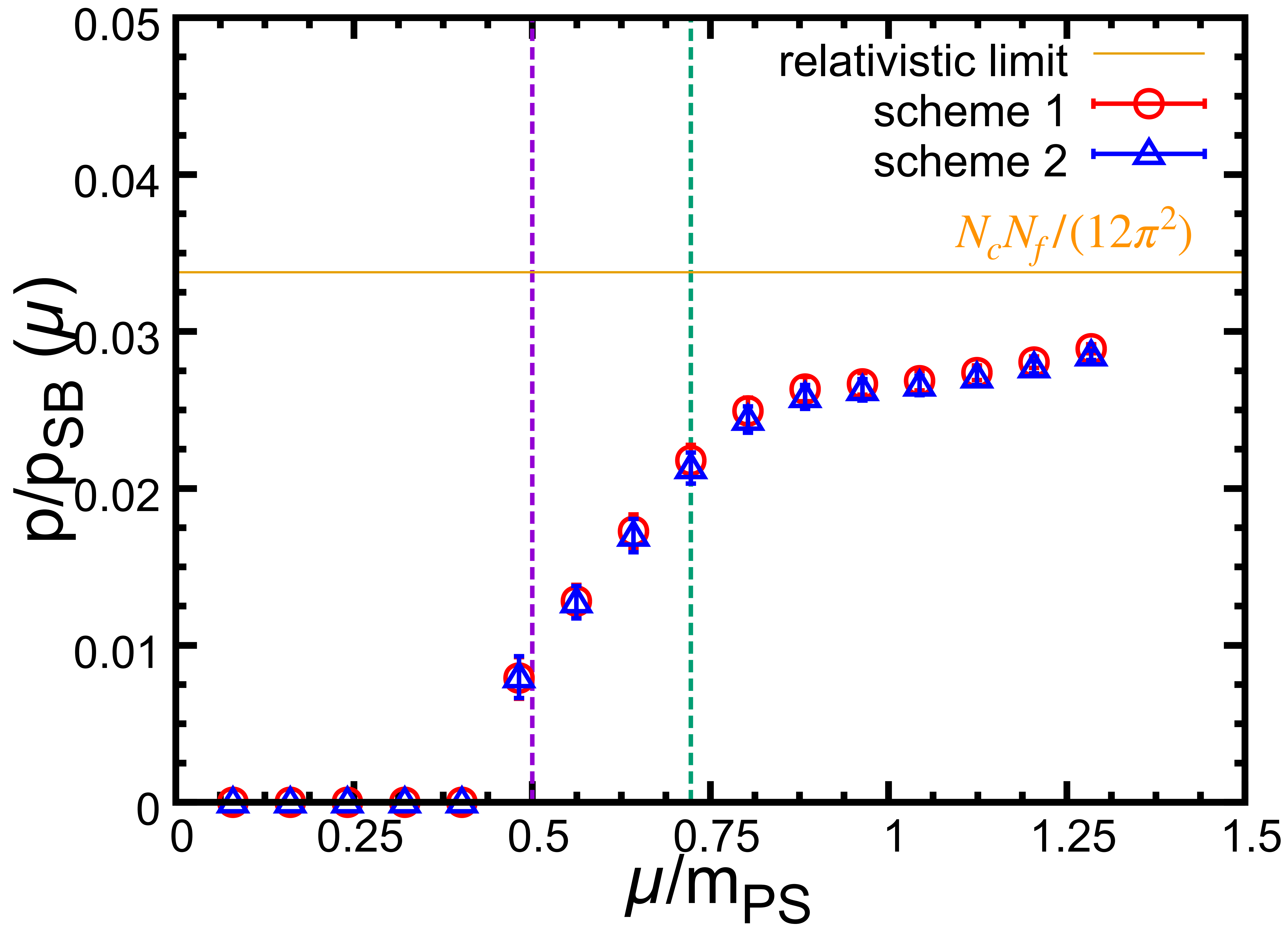
Slide by H.Tajima

scaling of p and e in high density



In massive fermion theory, the trace anomaly does not vanish because the mass term breaks the scale invariance.
The mass term will give a negative contribution, so that we expect $e/\mu^4 < e_{SB}/\mu^4 = N_cN_f/(4\pi^2)$

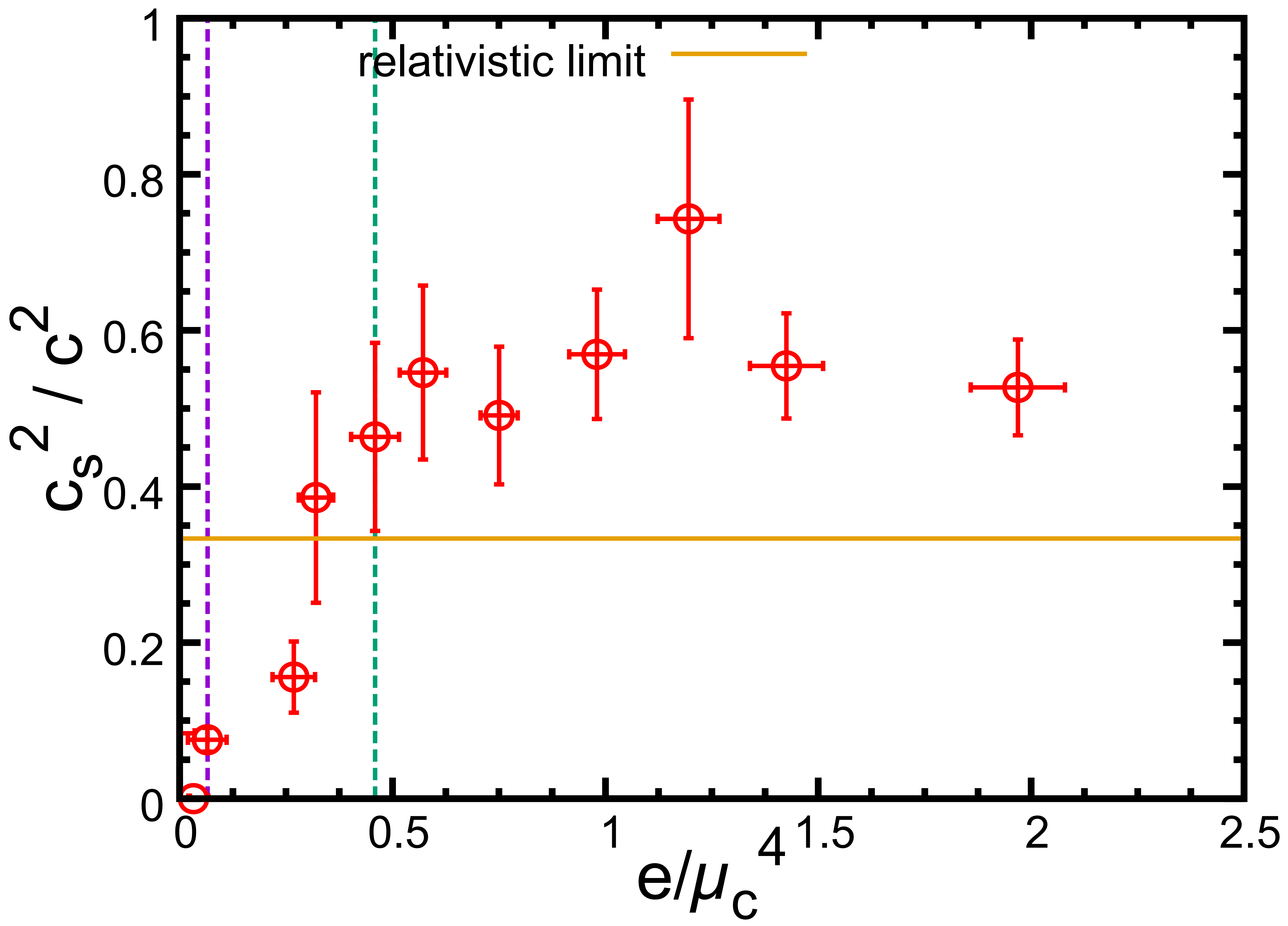
Scheme dependence of pressure

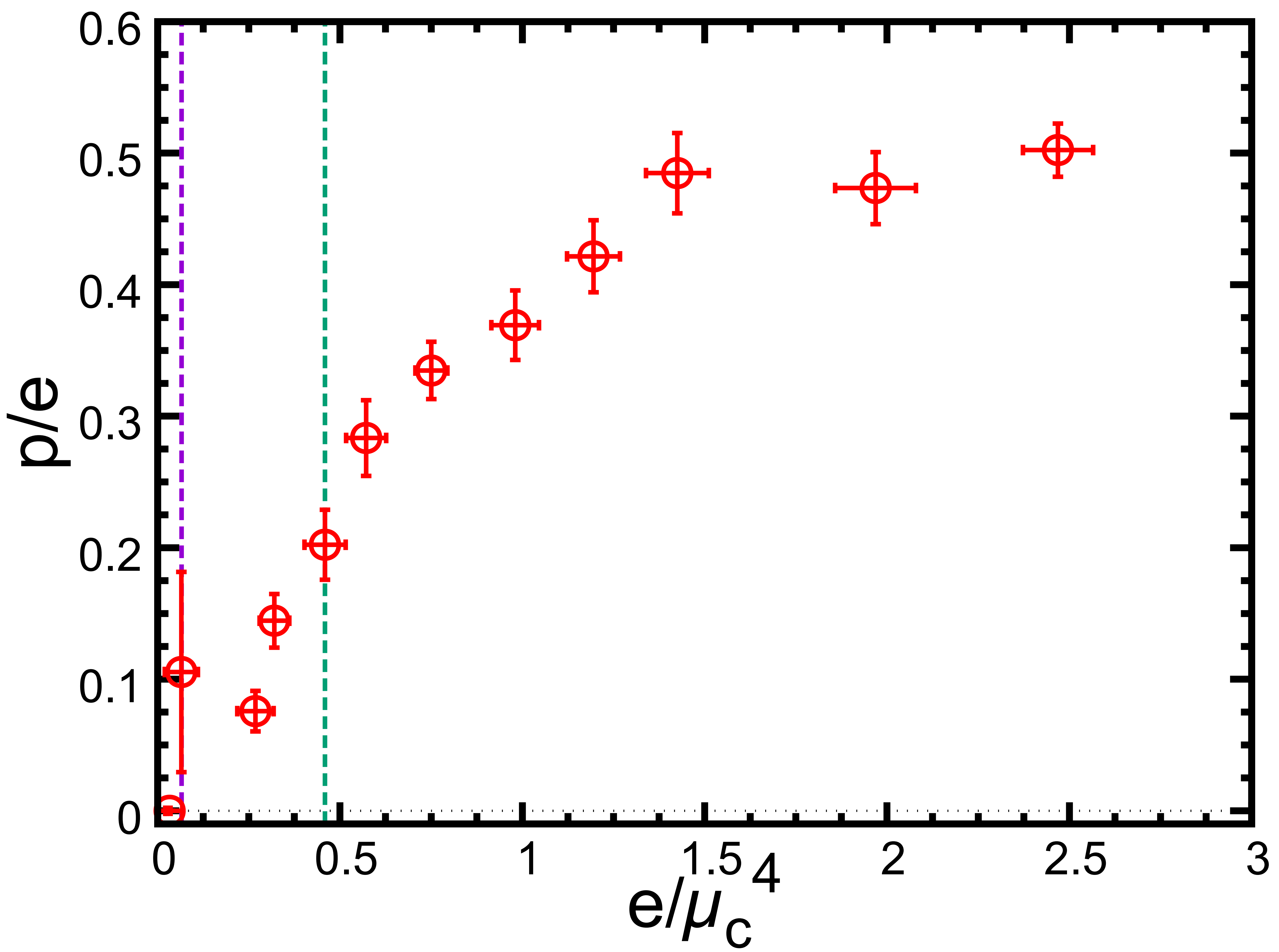


$$\mathrm{I}: \quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\mathrm{lat}}(\mu')d\mu'}; \qquad (28)$$

$$\mathrm{II}: \quad \frac{p}{p_{SB}}(\mu) = \frac{\int_{\mu_o}^{\mu} \frac{n_{SB}^{\mathrm{cont}}}{n_{SB}^{\mathrm{lat}}}(\mu')n_q(\mu')d\mu'}{\int_{\mu_o}^{\mu} n_{SB}^{\mathrm{cont}}(\mu')d\mu'}, \qquad (29)$$

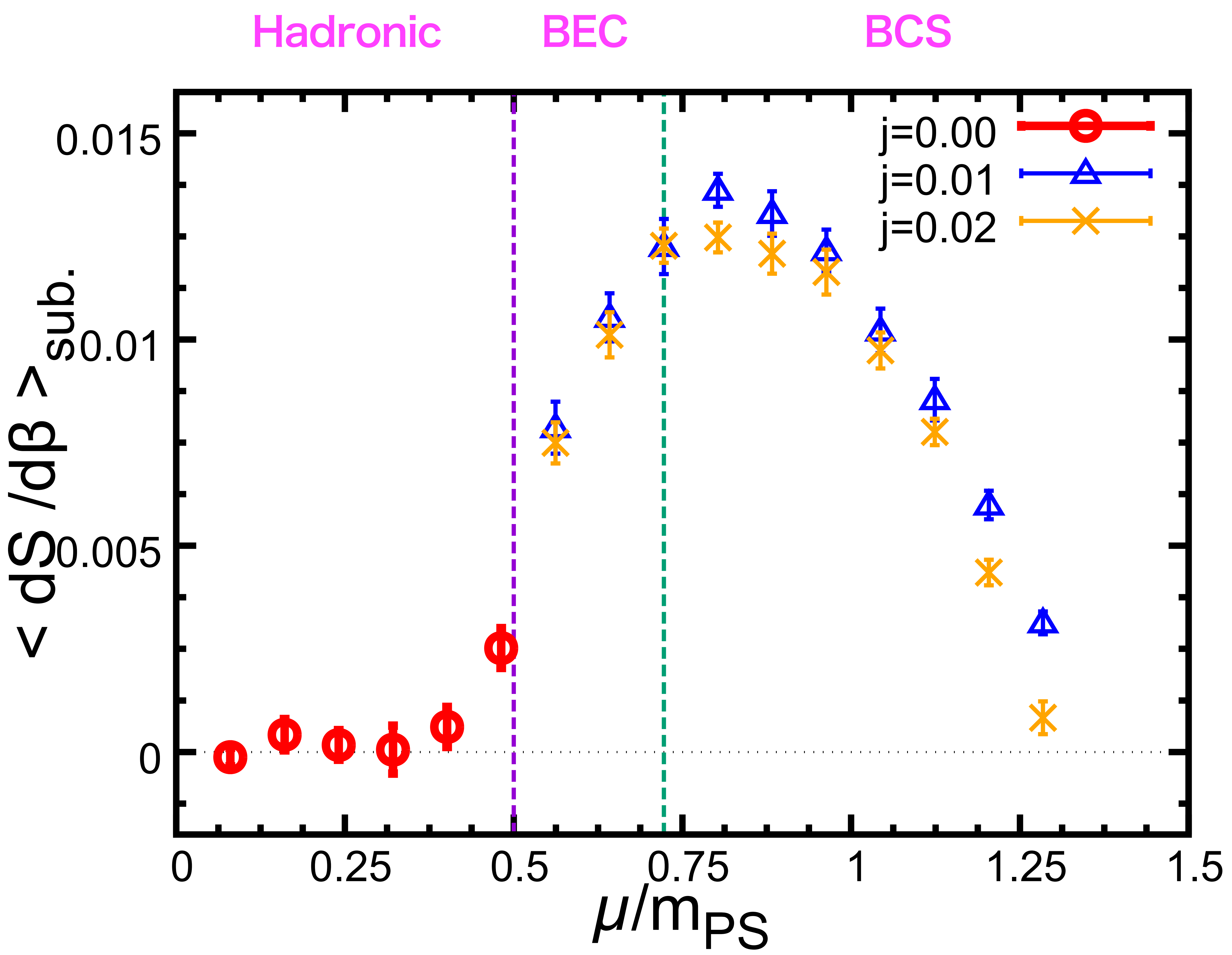
Sound velocity (ratio $\Delta p/\Delta e$) vs energy





μ-dependence of gauge action

value of Iwasaki gauge action knows the phase structure!



Our definition of each phase

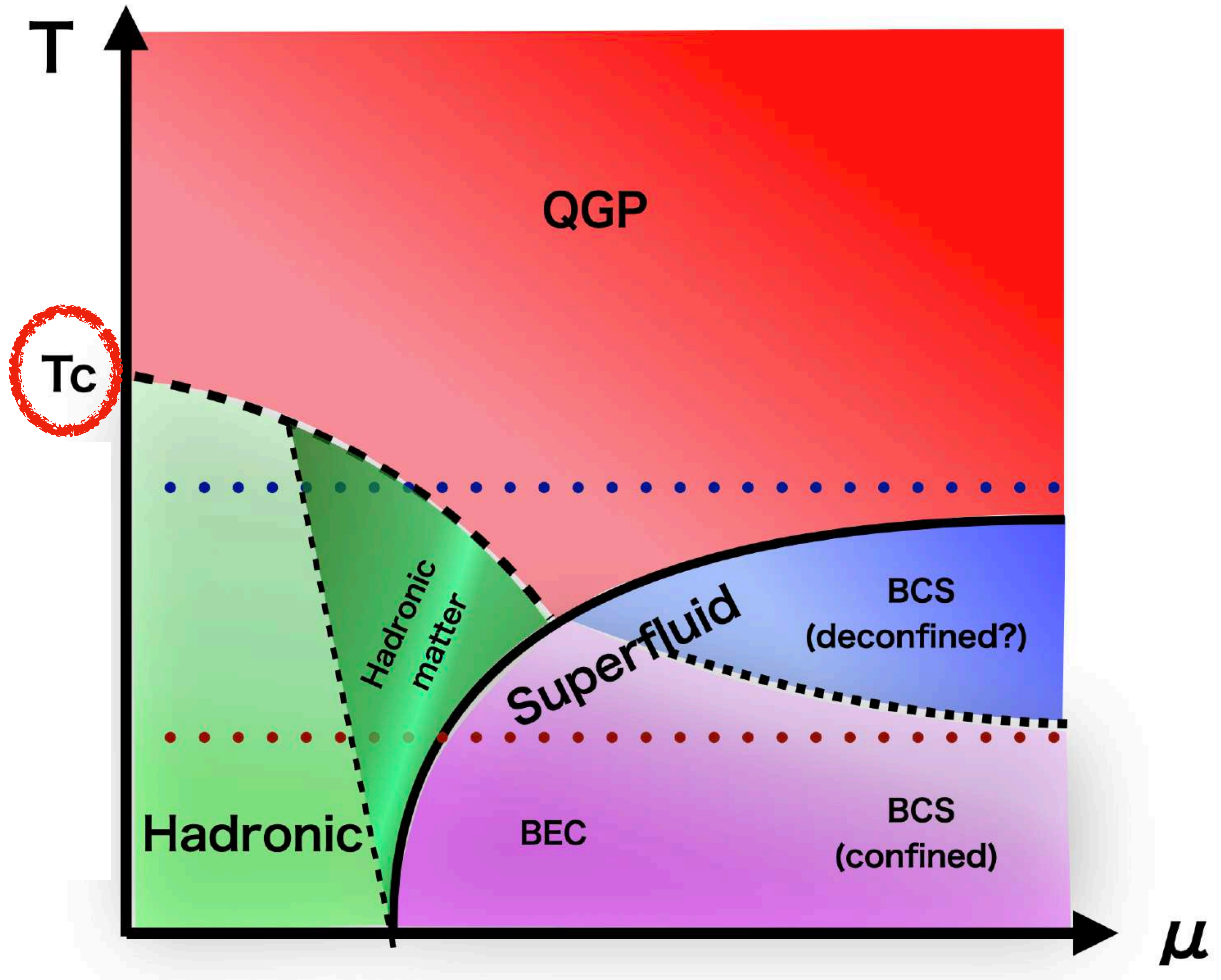
	Hadronic	Hadronic-matter	QGP	Superfluid BEC	Superfluid BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\text{tree}} \approx 1$

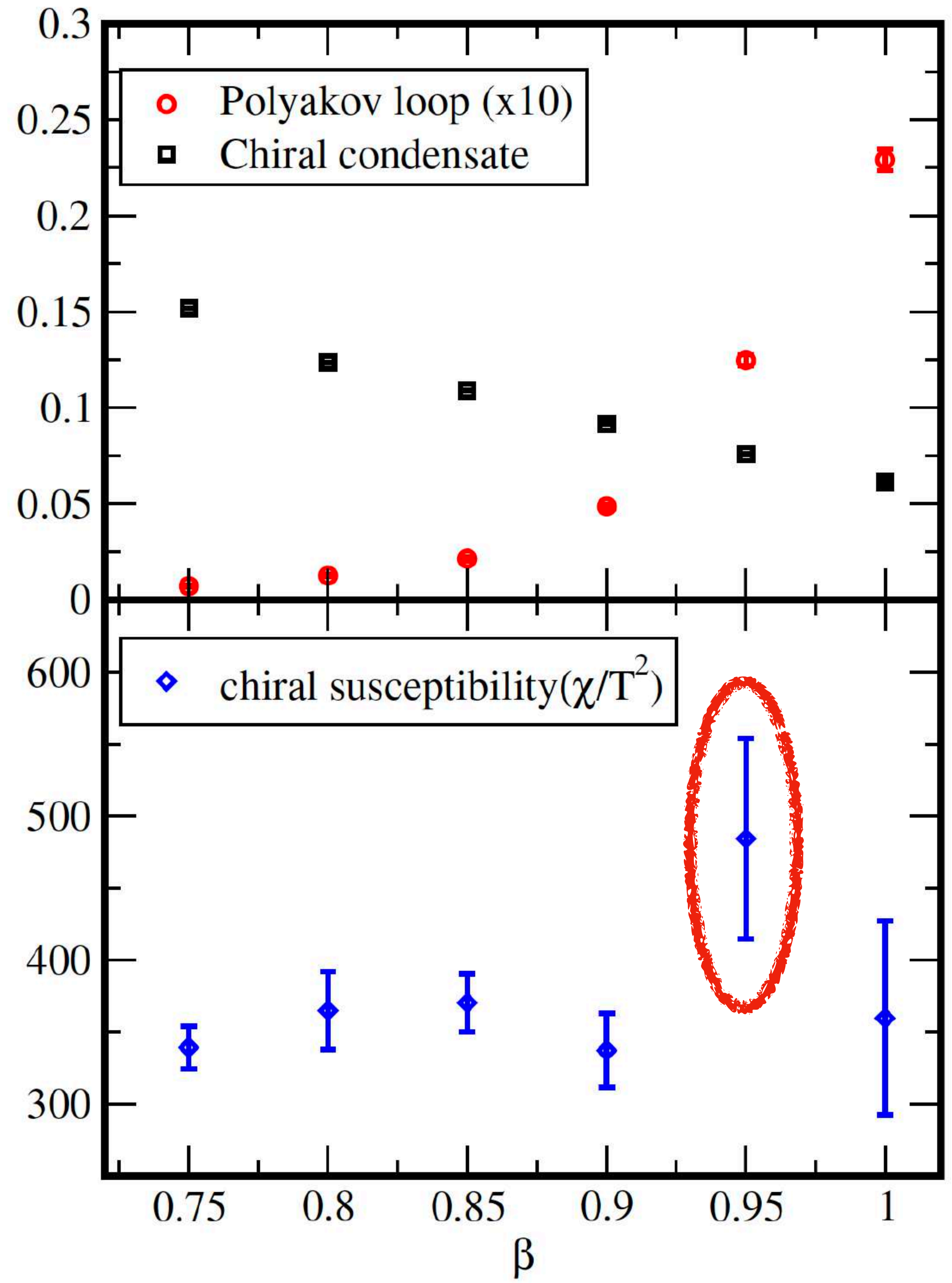
Phase diagram

Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

- Tc at $\mu = 0$ from chiral susceptibility





Scale setting at $\mu = 0$

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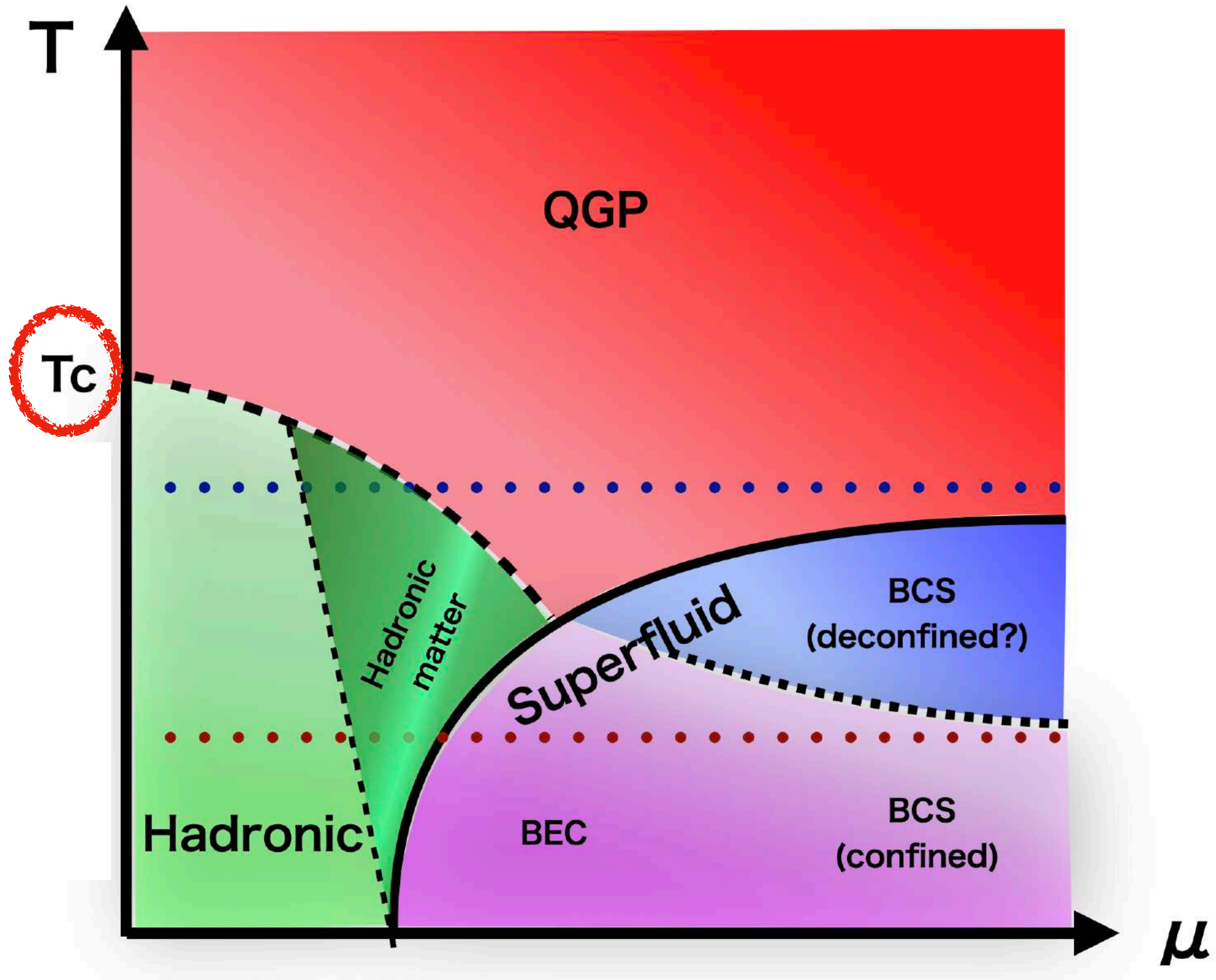


- Tc at $\mu = 0$ from chiral susceptibility

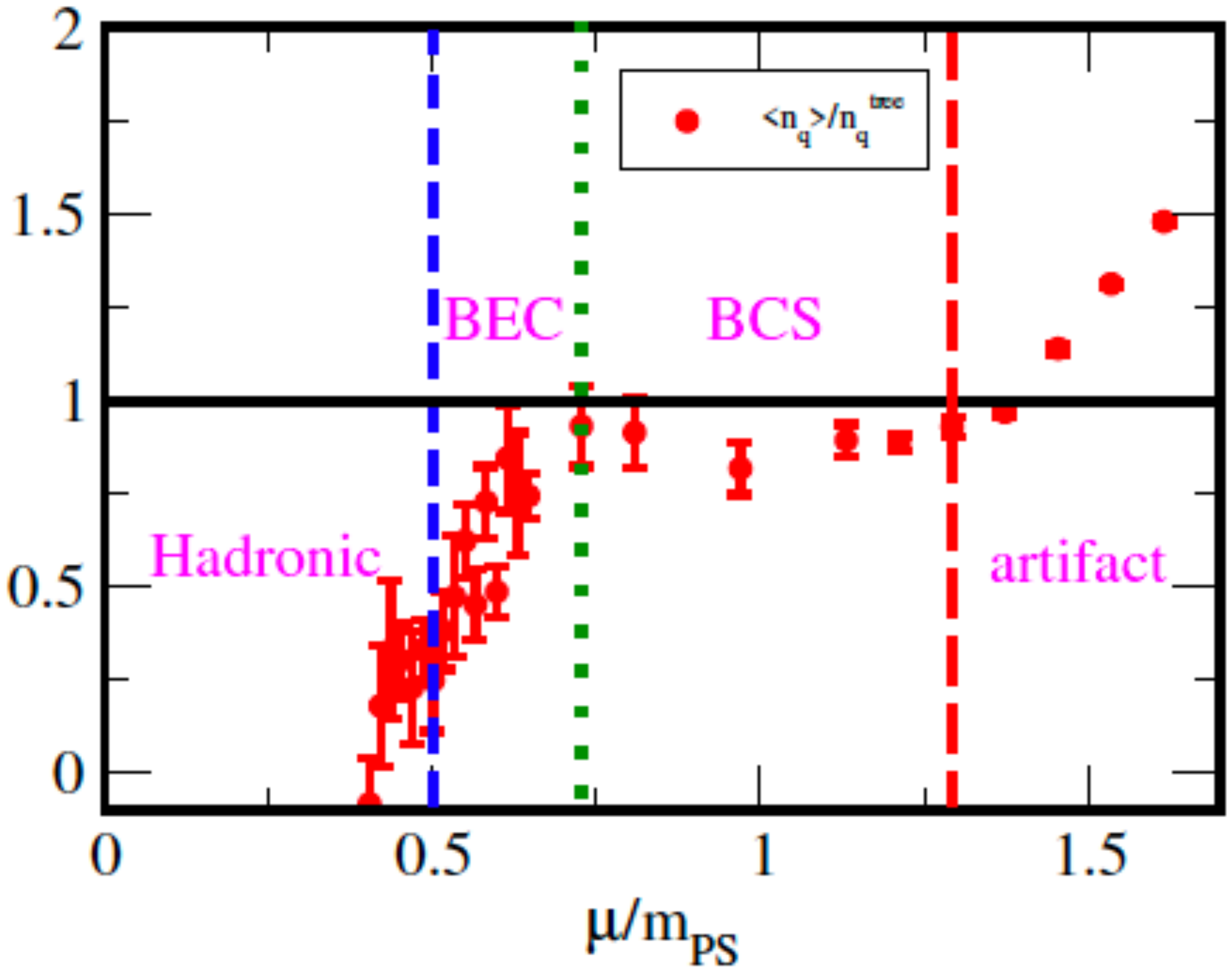
- Assume Tc=200MeV

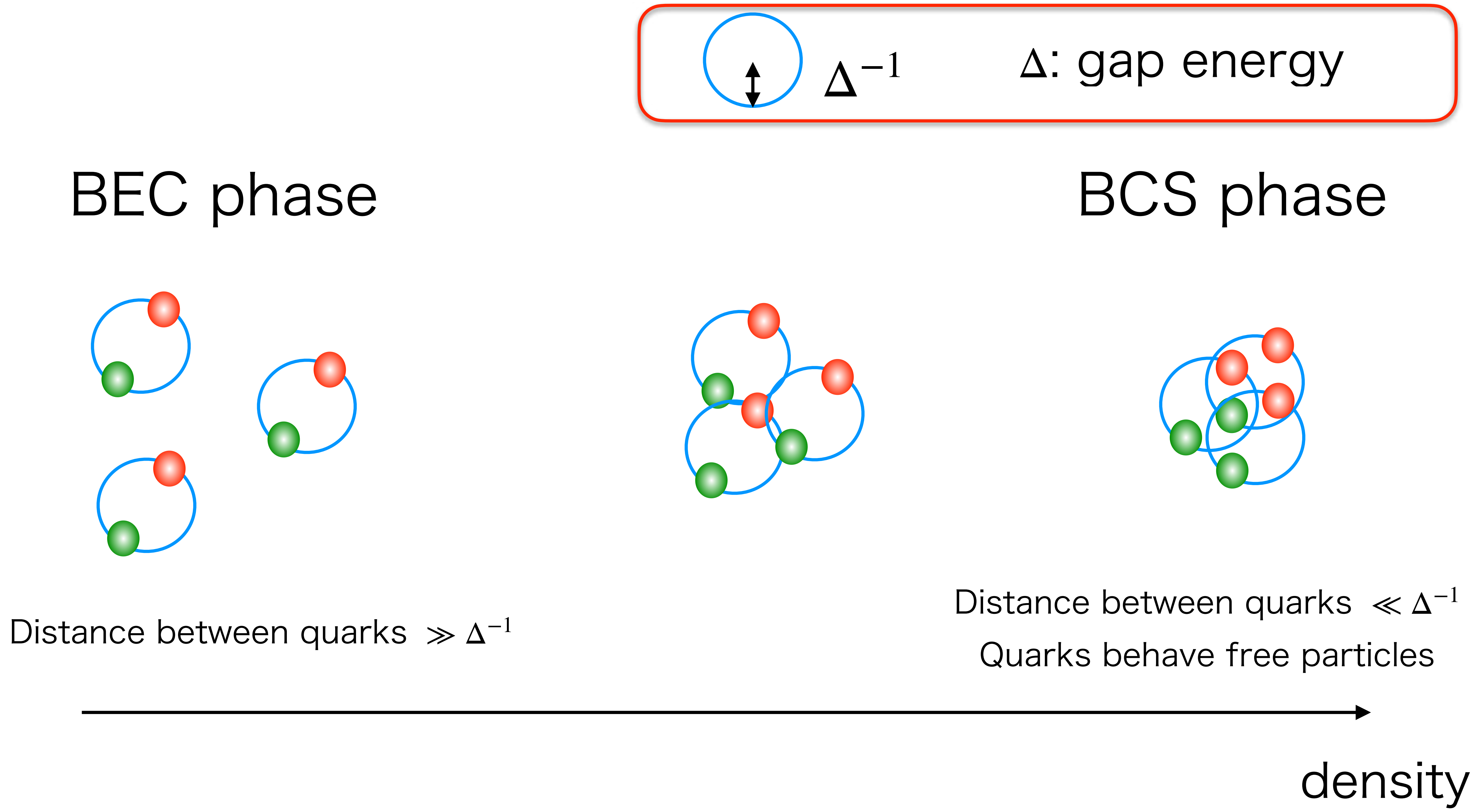
  Tc is realize Nt=10, $\beta = 0.95$ (a=0.1[fm])

- Find relationship between β (lattice bare coupling) and a (lattice spacing)

  In finite density simulation, a=0.1658[fm]

BEC/BCS crossover





Number density of free particle

$$n_q^{\text{tree}}(\mu) = \frac{4N_cN_f}{N_s^3N_\tau}\sum_k \frac{i\sin\tilde{k}_0[\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos\tilde{k}_\nu]^2 + \sum_\nu \sin^2\tilde{k}_\nu}$$

J->0 extrapolation

Diquark condensate has a strong j dependence

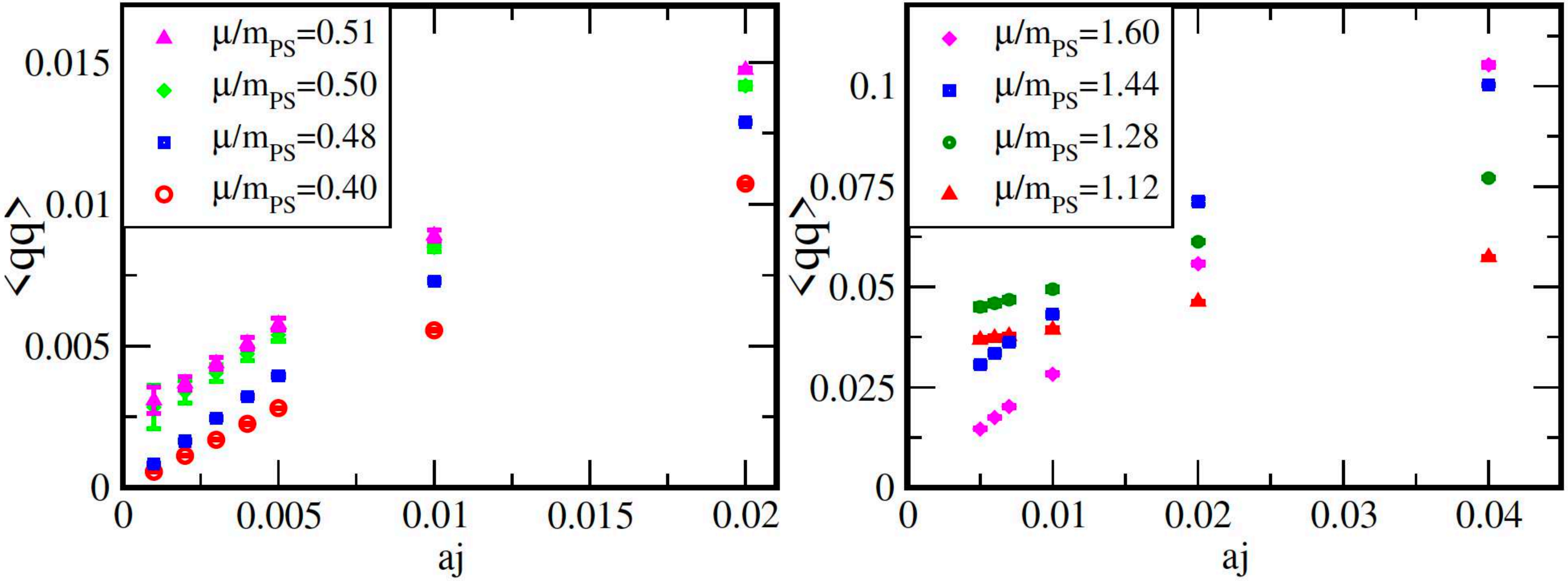
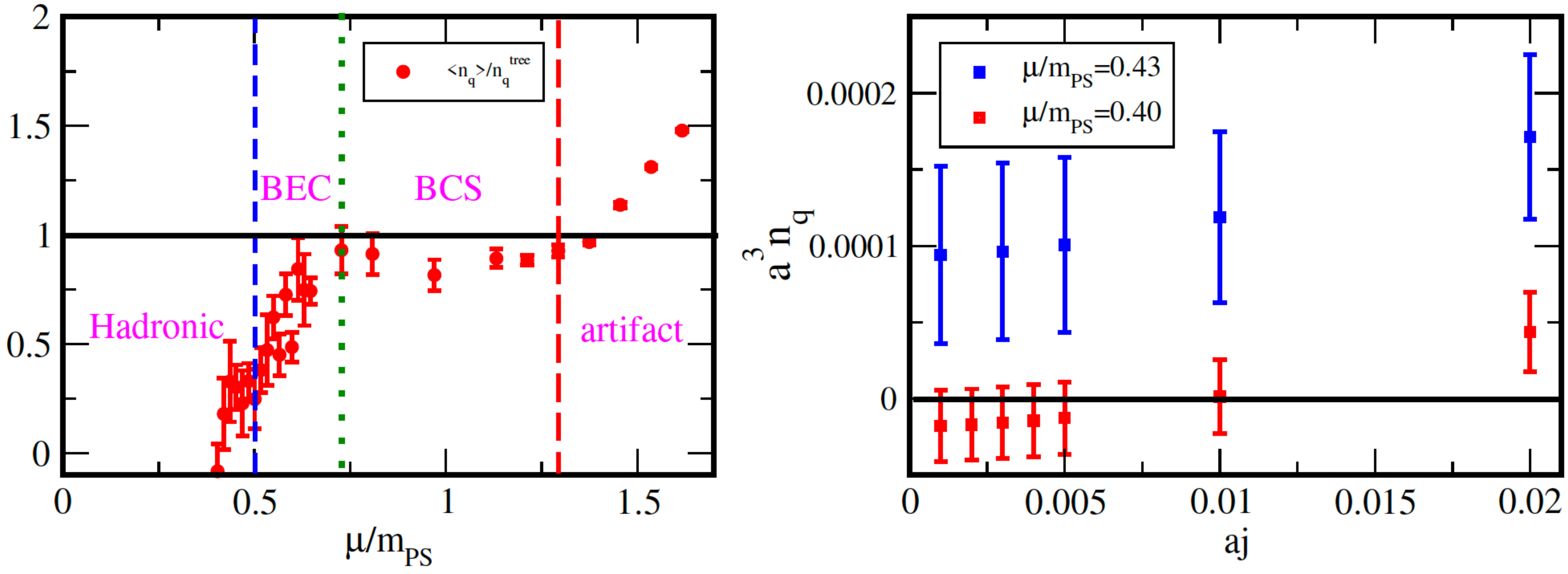


Figure 5. The j-dependence of the diquark condensate for several $\mu/m_{\rm PS}$.

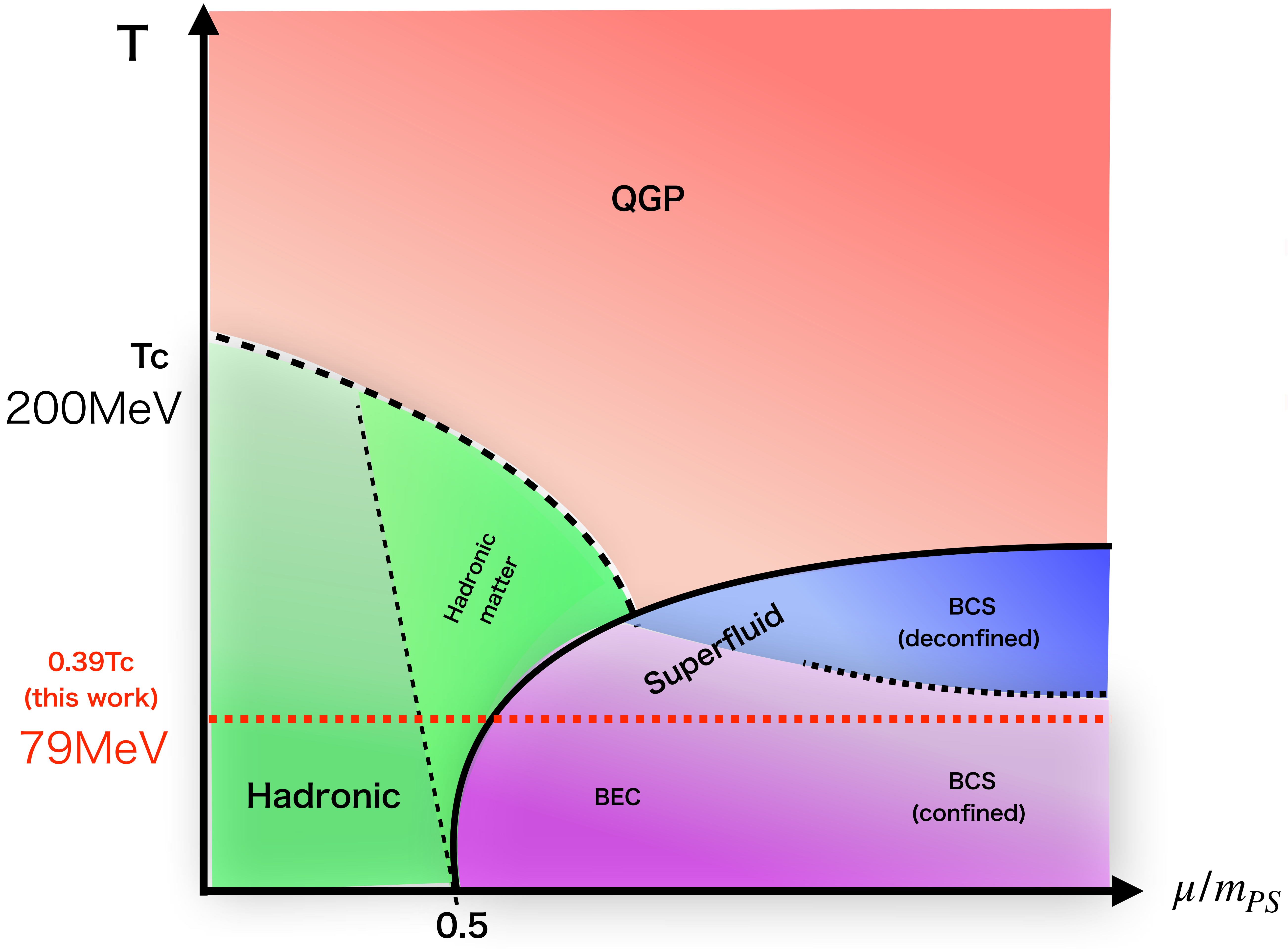
J->0 extrapolation

Chiral condensate and n_q have a mild j-dependence

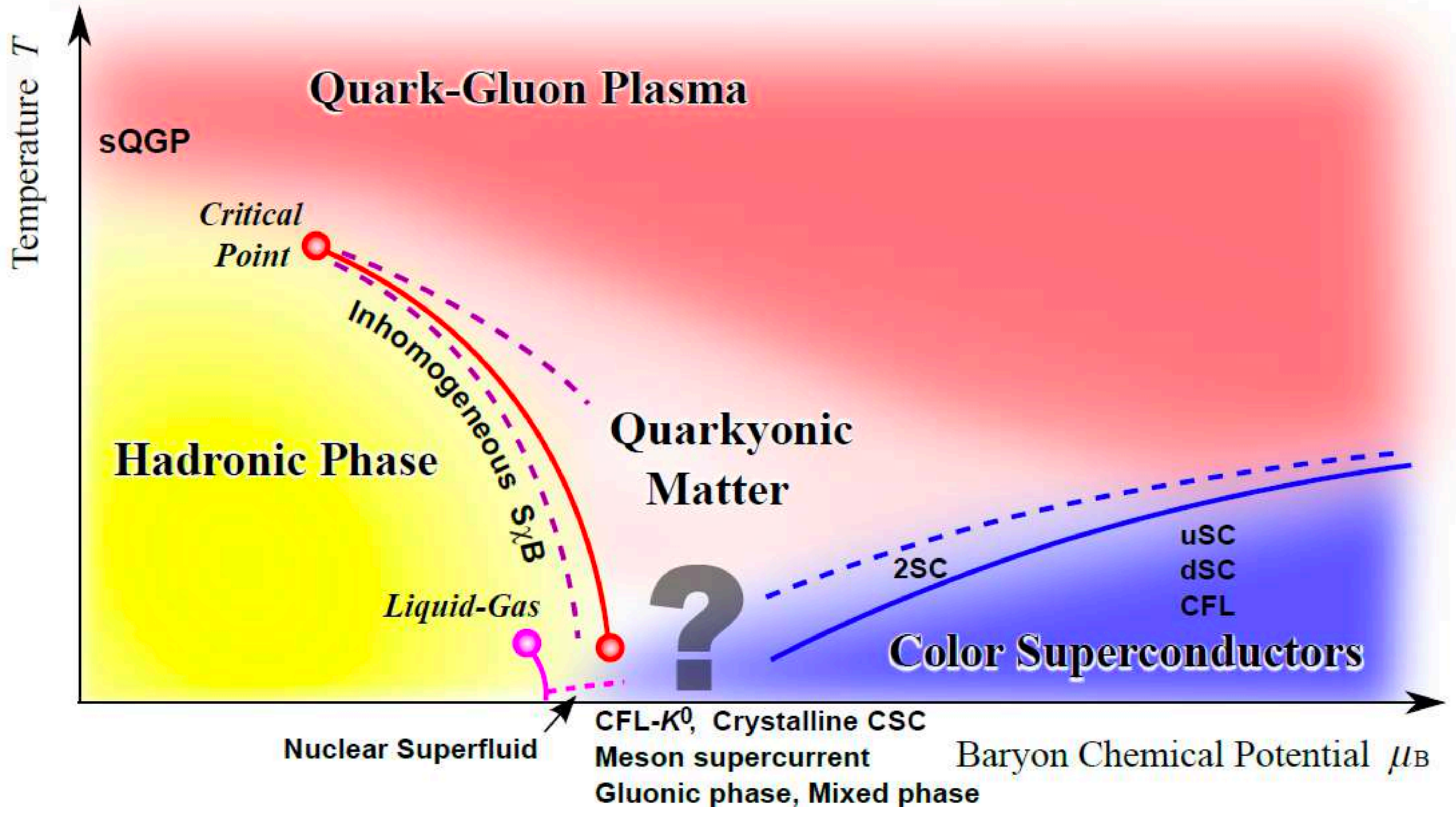


Phase diagram of 2color QCD

Comparison with 3color QCD



Fukushima-Hatsuda (2010)



Implementation QC2D with diquark source term

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

construct a single bilinear form of fermion fields

$$S_F = (\bar{\psi}_1 \ \ \bar{\varphi}) \begin{pmatrix} \Delta(\mu) & J\gamma_5 \\ -J\gamma_5 & \Delta(-\mu) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix} \equiv \bar{\Psi}\mathcal{M}\Psi$$

Here, $\Psi = \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix}$

$$\bar{\varphi} = -\bar{\psi}_2^T C \tau_2, \quad \varphi = C^{-1}\tau_2 \bar{\psi}_2^T$$

$\mathcal{M}$ has non-diagonal components, calculations of det[M] and inverse of M are hard…

$$\mathcal{M}^\dagger \mathcal{M} = \begin{pmatrix} \Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2 & 0 \\ 0 & \Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2 \end{pmatrix}$$

J (=$j\kappa$) term lifts the eigenvalue of Dirac op.

Note that Ψ denotes 2-flavor, $\det \mathcal{M}$ gives Nf=2 action

$\det \mathcal{M}^\dagger \mathcal{M}$ is 4-flavor theory

RHMC algorithm

Introduction

- QCD: $\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi$

- Interesting phenomena have been revealed

  asymptotic freedom

  finite T transition (chiral/confinement)

  small η/s around Tc

  topological objects

  Hadron spectrum

  Equation of State (energy, pressure) as fn. of T