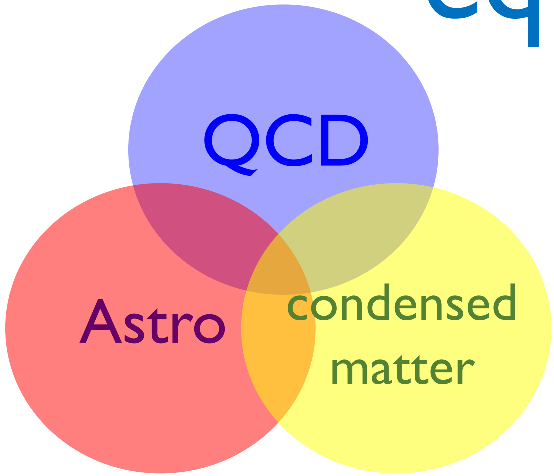
Quark substructure of hadrons and equations of state in dense QCD



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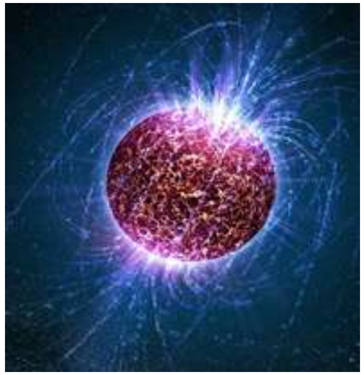


Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

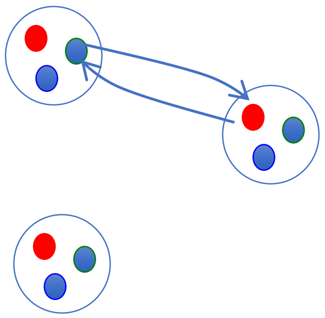
Fujimoto-TK-McLerran, "IdylliQ matter model" arXiv: 2306.04304 [nucl-th]

State of matter: overview

(n_0 = 0.16 fm^{-3})

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

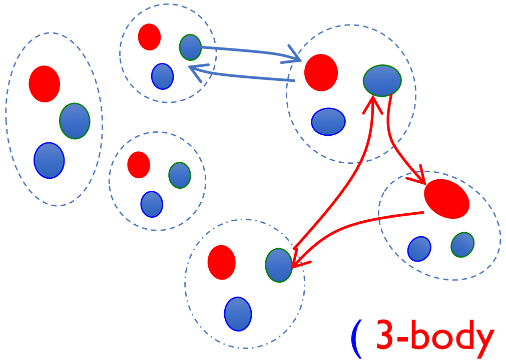


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

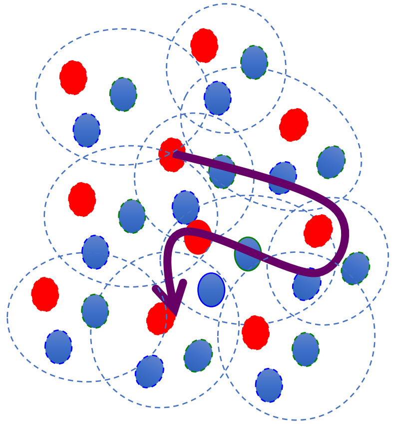


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ 1.4 $M_\odot$ ~ 2 $M_\odot$

n_B

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

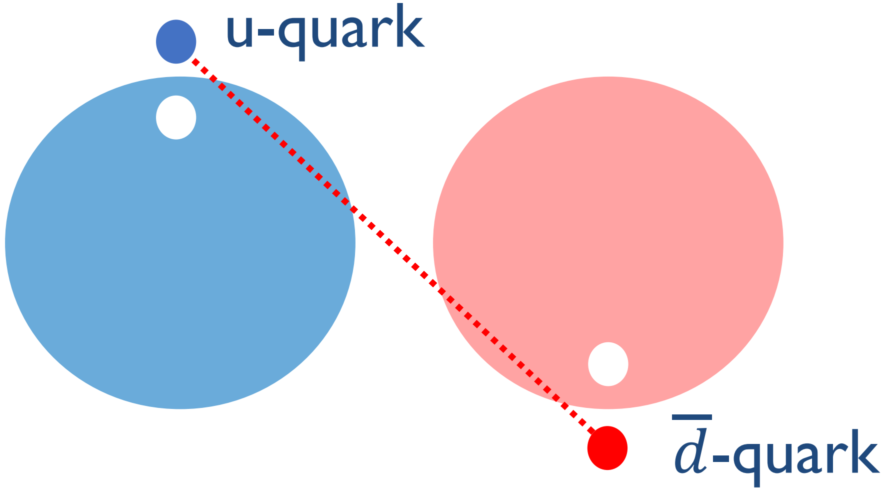
Hints: isospin QCD

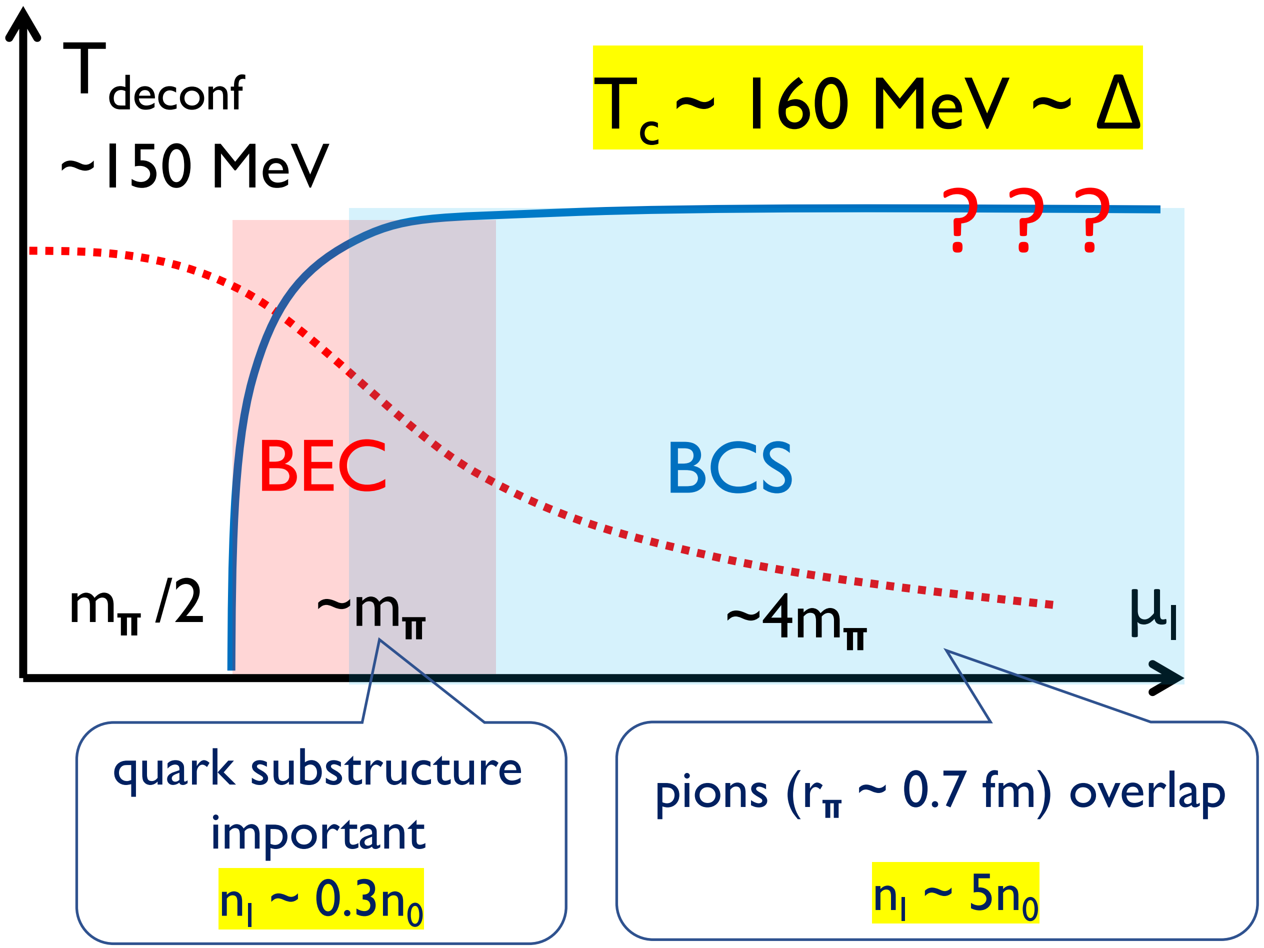
isospin chemical pot. $\mu_I = \mu_u - \mu_d$ $n_0 = 0.16\ \text{fm}^{-3}$

[Son-Stephanov, ...]

[for **2-color QCD**, see Itou-san's talk]

・ onset of finite density → begin with pions (instead of nucleons)





[e.g., Chiba-TK '23]

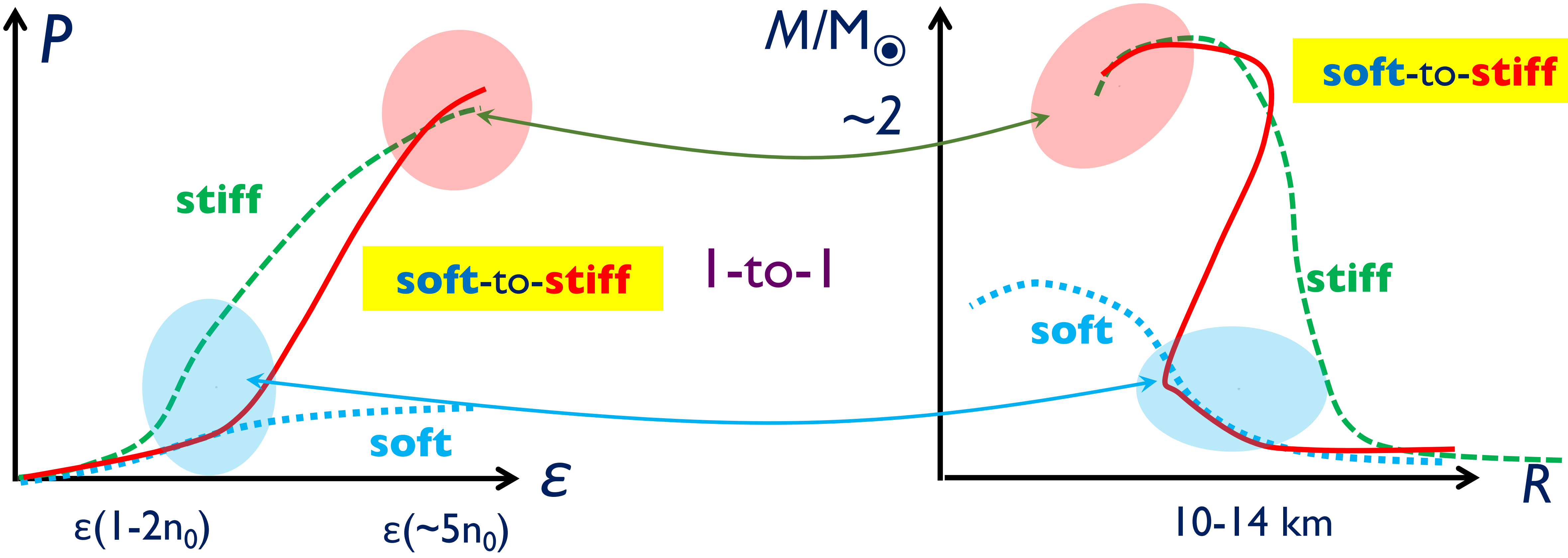
pairing → pion cond. π^+ (p-a) & π^- (h-ah)

→ quark mass gap Δ

~ $T_c/0.57$ ~ 280 MeV   BCS estimate (naive)

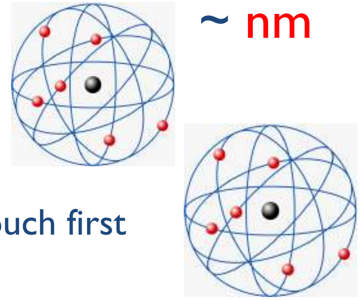
EoS & Neutron Star M-R relation

Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ ······ QCD (+EW) EoS



gravitational compression & matter content

atom
(nucleus + electrons)



~ nm

e- touch first

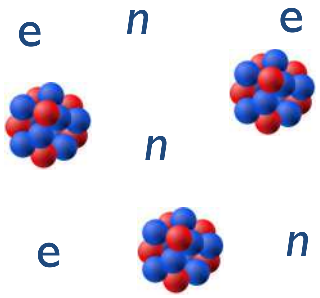
compressions

e-dripped

$e + p \rightarrow n + \nu_e$

(see below)

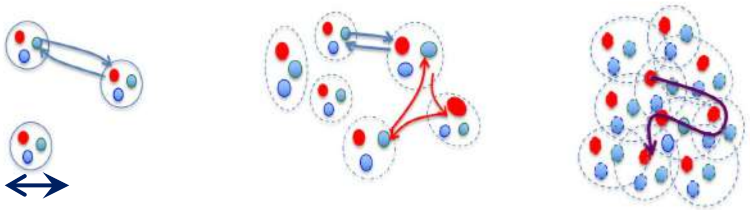
nuclei + dripped n & e



~ 5 fm

nuclear liquid to **quark matter**

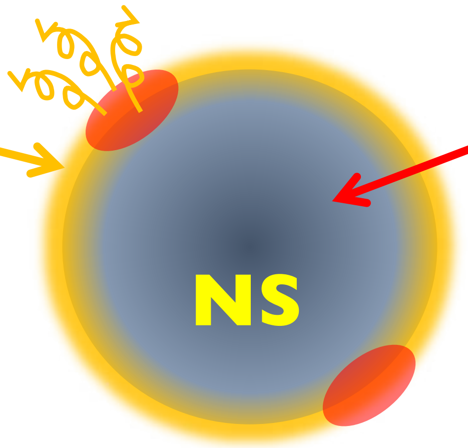
n or p



~ 1-2 fm

$n_B \sim 10^{-9}\, n_0$ ~ $10^{-3}\, n_0$ ~ **1-2**n_0 ~ **2-5**n_0 ~ **5-10** n_0

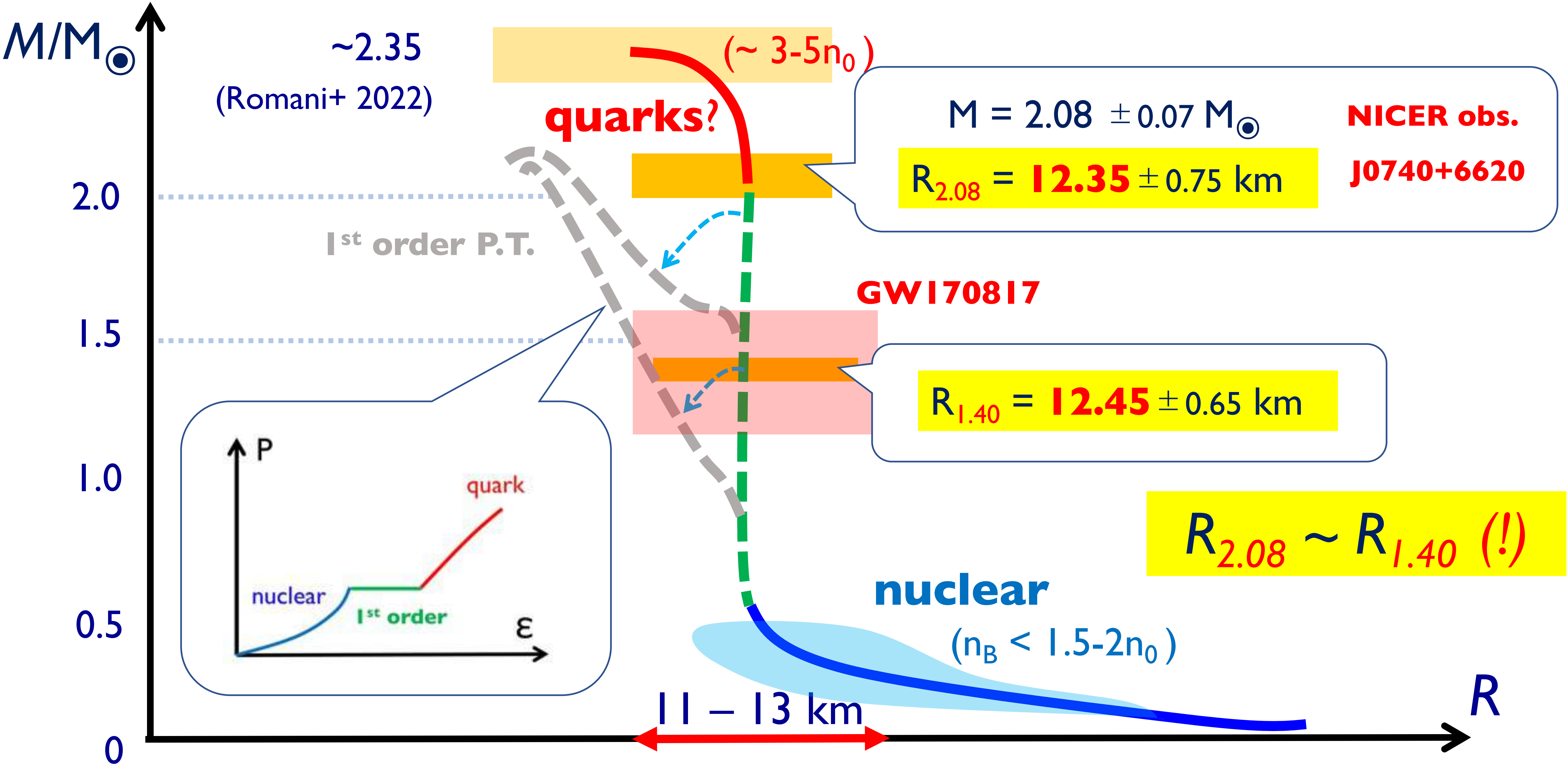
crust ⇕ surface phenomena



core ⇕ *M-R* relation

Observations: (NICER, GW170817, nuclear)

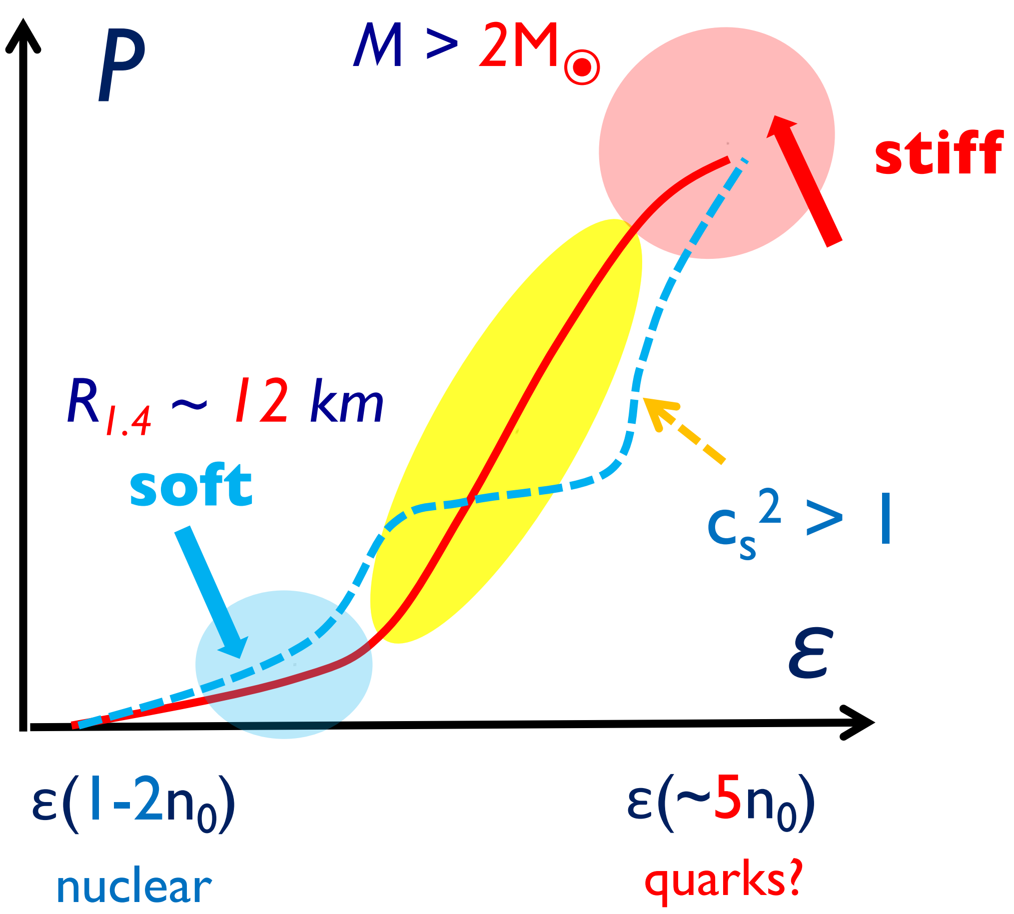
[e.g., Miller+ '21]

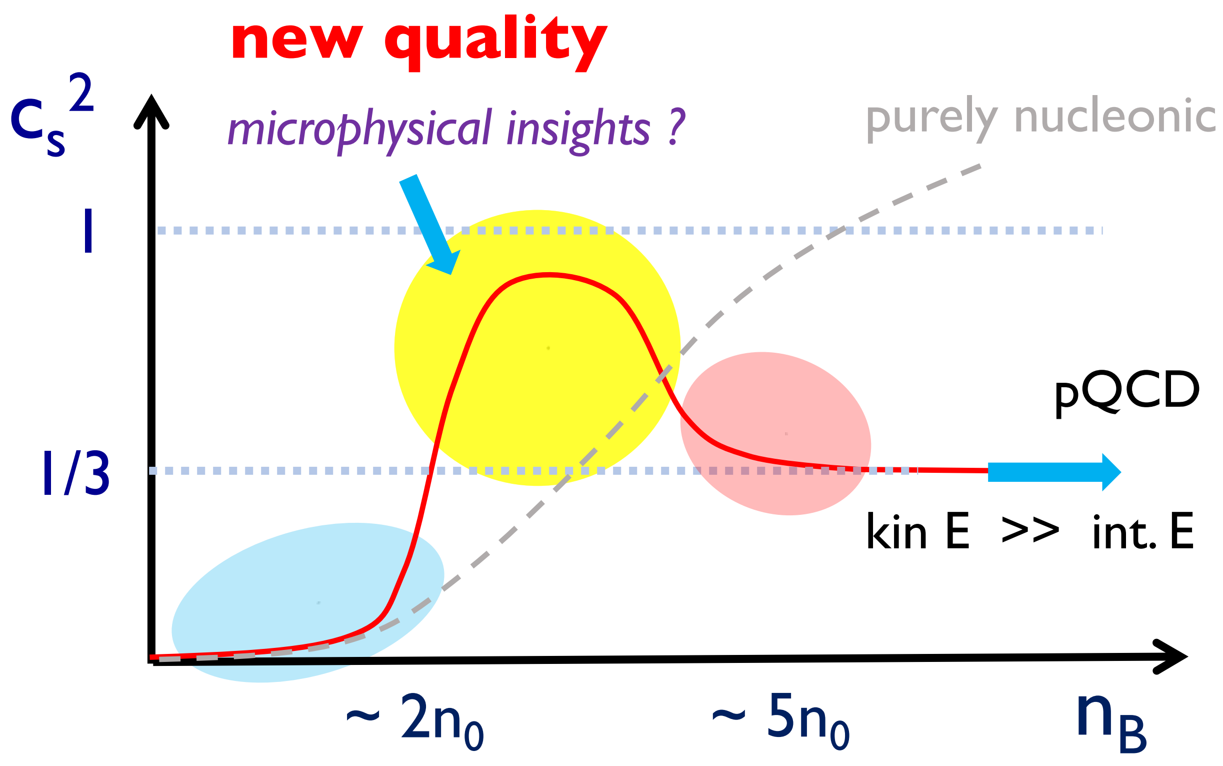


Soft to stiff is challenging:

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)

nuclear & *quark* physics constrain each other





baseline: *quark-hadron continuity* **(QHC)**

Contents

pressure from ε(n_B)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle

e.g.) gas of heavy particles (massive limit)

$$\varepsilon(n_B) = m_N n_B \quad \Rightarrow \quad \varepsilon/n_B = m_N \quad \Rightarrow \quad P = 0$$

gas of relativistic particles (massless limit)

$$\varepsilon(n_B) = a n_B^{4/3} \quad \Rightarrow \quad \varepsilon/n_B = a n_B^{1/3} \quad \Rightarrow \quad P = \frac{\varepsilon}{3}$$

c_s^2 in purely nucleonic models

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha}$$

large (!) small (!)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

$$P = \frac{2}{3} a \frac{n_B^{5/3}}{m_N} + b(\alpha - 1) n_B^{\alpha}$$

small (!)

at LO: p << ε (!)

if interactions dominate (at large n_B):

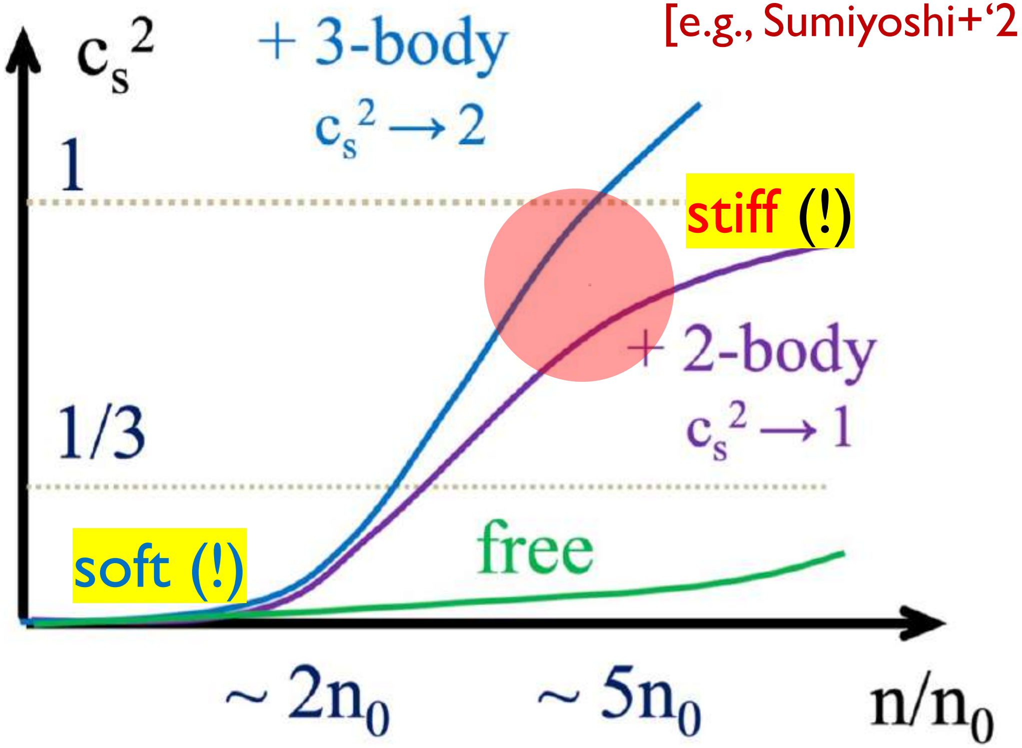
$$P \sim (\alpha - 1)\varepsilon \;\rightarrow\; c_s^2 \sim (\alpha - 1)$$

2-body int. → α = 2 3-body int. → α = 3

(contact type)

causality & convergence ??

[e.g., Sumiyoshi+'21]



alternative **baseline**: quark EOS

e.g.) free massless quarks

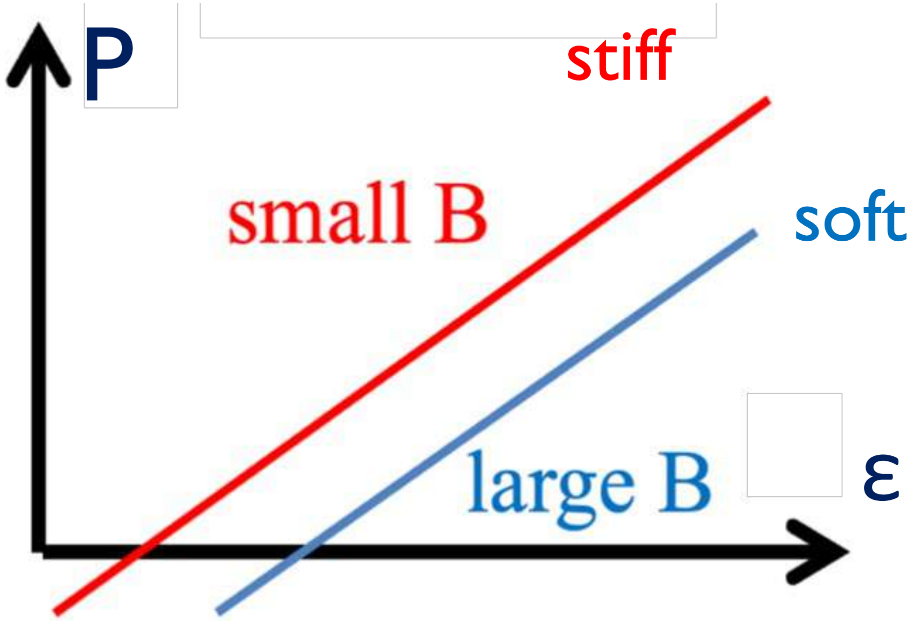
$$P = \frac{\varepsilon}{3} - B'$$

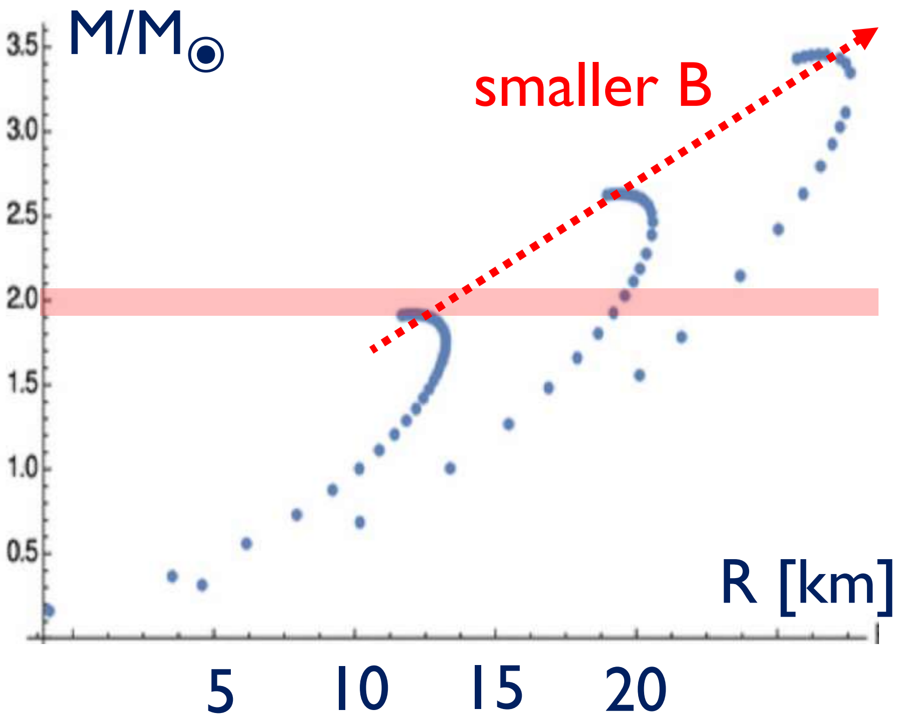
normalization

$c_s^2 = 1/3$

quark kin. E ~ N_c^2 x nucl. kin. E

~ $N_c \times p_F^2/M_q$ ~ $P_F^2/N_c M_q$



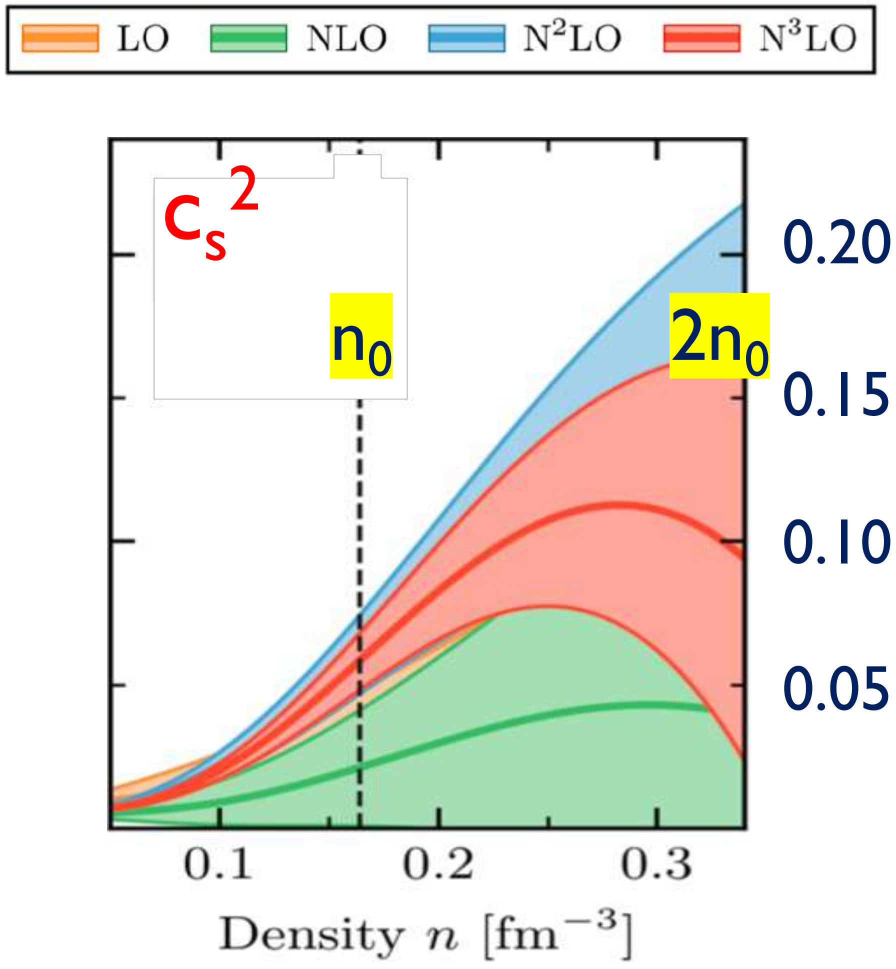


relativistic pressure → stiff EOS

can be a good starting point!?

c_s^2 = 1/3 = 0.33... (at 1-3n_0) is large

[e.g., ChEFT, Drischler+ '21]

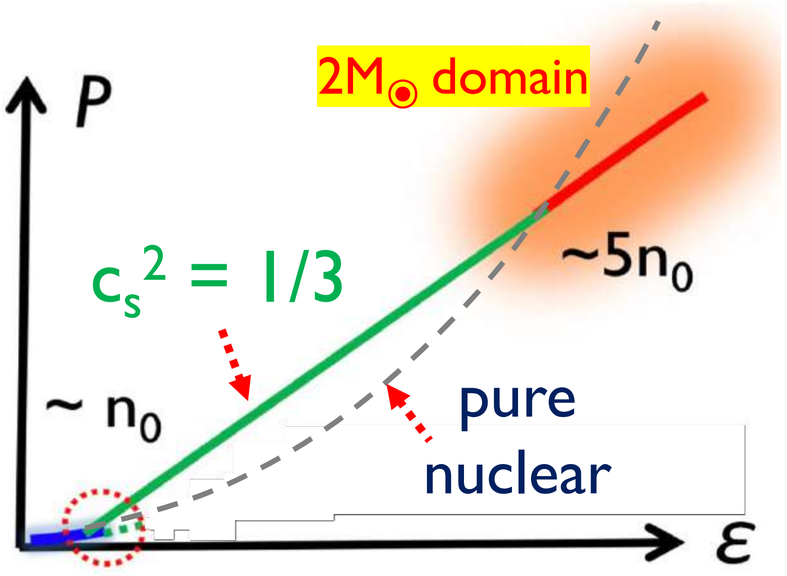


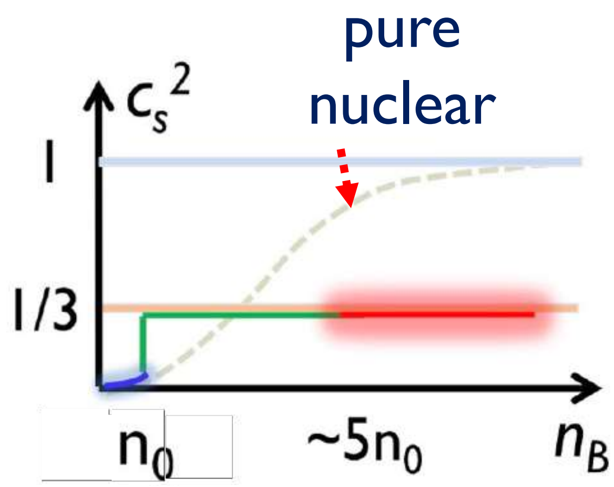
ChEFT (to N³LO)

small..

$c_s^2(n_0)$ ~ 0.05-0.10 $c_s^2(2n_0)$ ~ 0.1-0.2

If we switch to c_s^2 = 1/3 at **low** density...

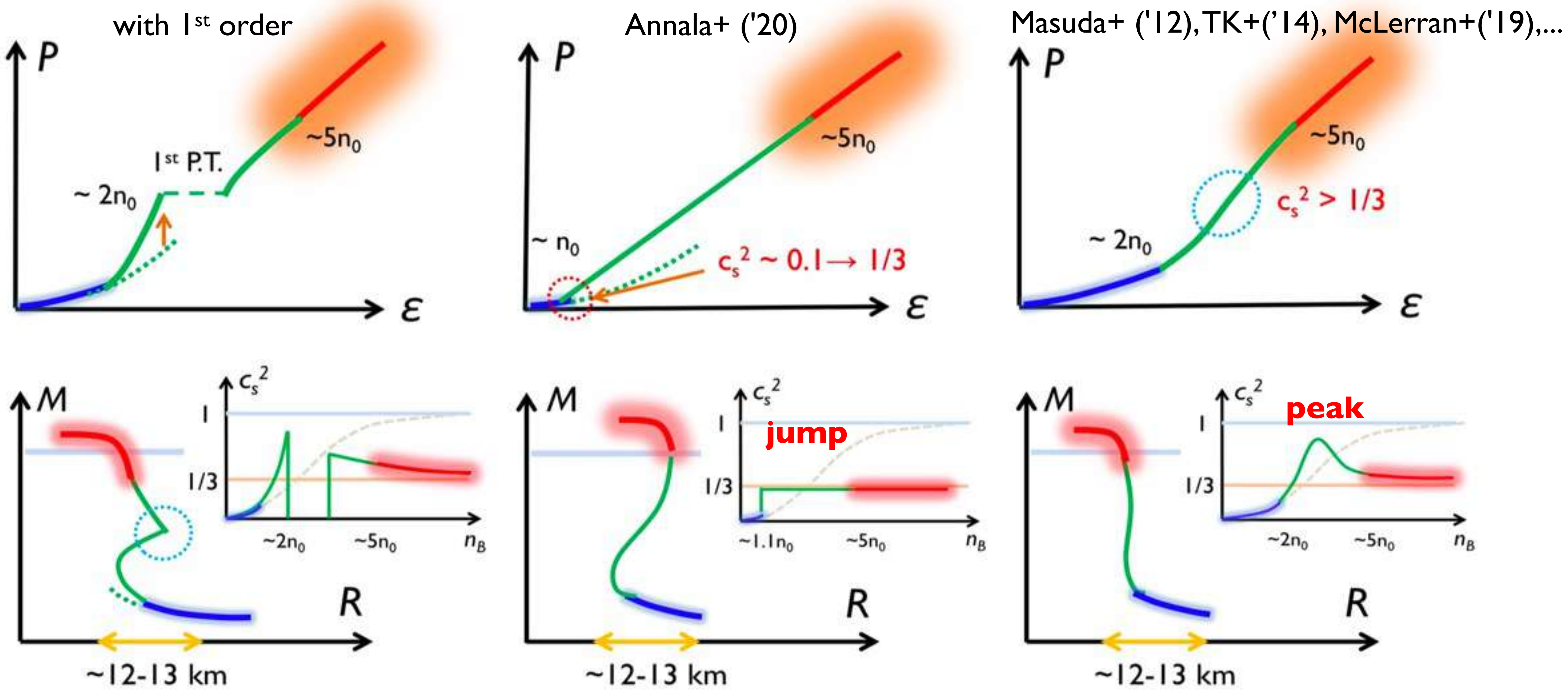




For systematic analyses, see e.g., Annala+ '20

Three possible scenarios

[TK+ ’20: JSPS article, AAPPS bull.]



→ $R_{1.4}$ and $R_{2.1}$?

→ *nuclear physics ?*

→ *my favorite*

Contents

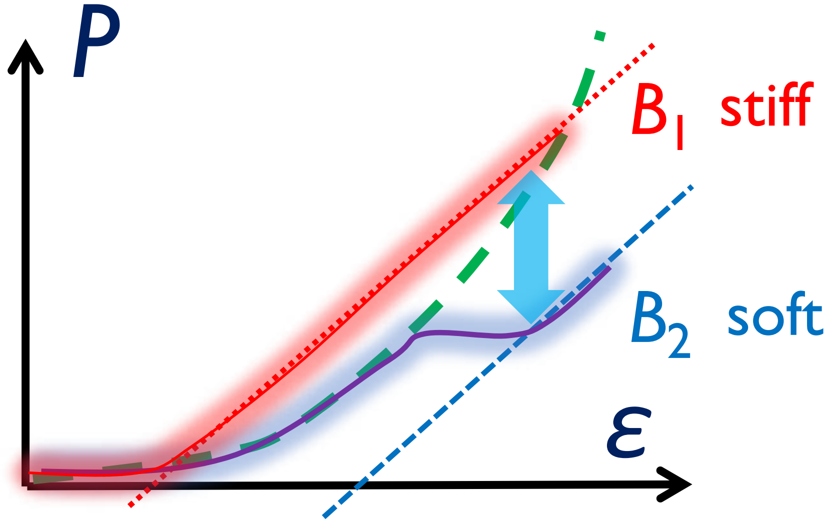
Hadron-to-quark transitions?

Confusing point:

- *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy)



Strategy:

Keep track of quark states from nuclear to quark matter

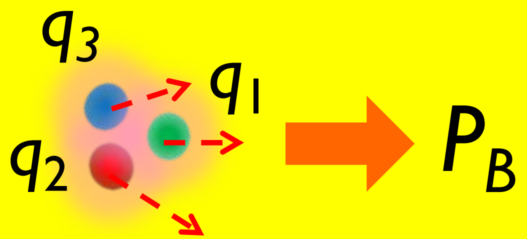
(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

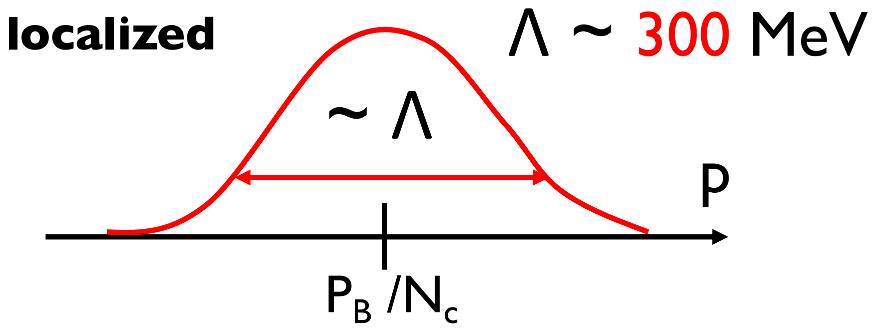
Quarks in a baryon

N_c (=3): number of colors

probability density: e.g.)

$$\varphi(\boldsymbol{q};\boldsymbol{P}_\mathrm{B}) = \mathcal{N} \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{q}-\frac{\boldsymbol{P}_\mathrm{B}}{N_\mathrm{c}}\right)^2}$$





variance: $\left\langle \left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$ *energetic !*

→ large *"mechanical"* pressure

$$\langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B} = \mathcal{N}\int_{\boldsymbol{p}} E_q(\boldsymbol{p})\mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i}\right\rangle_{\boldsymbol{P}_B=0}\left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

x N_c x N_c

$\sim N_c (M_q + E_{kin})$ $\gg$ $\sim P_B^2/(N_c E_q)$

baryon mass baryon kin. energy

A model of quark-hadron-duality

cf) [TK '21, TK-Suenaga '21]

occupation **probability** of **quark** state with p

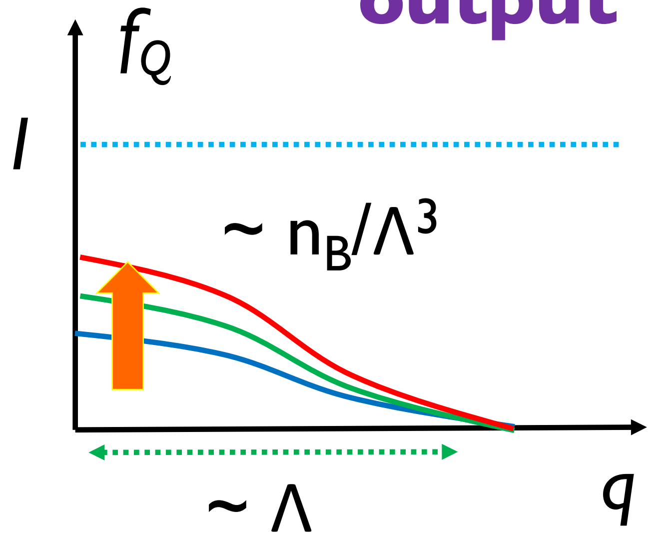
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_Q(\boldsymbol{q}; n_B) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B; n_B)\varphi_Q^B(\boldsymbol{q}; \boldsymbol{P}_B)$$

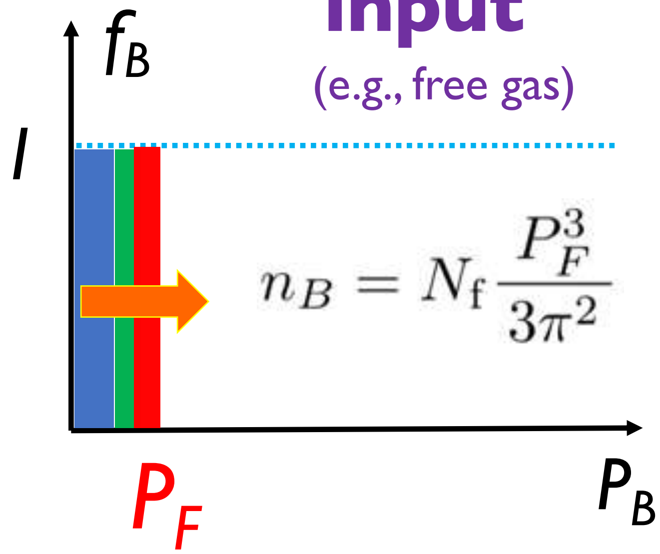
e.g.) in **ideal** baryonic matter

output



input

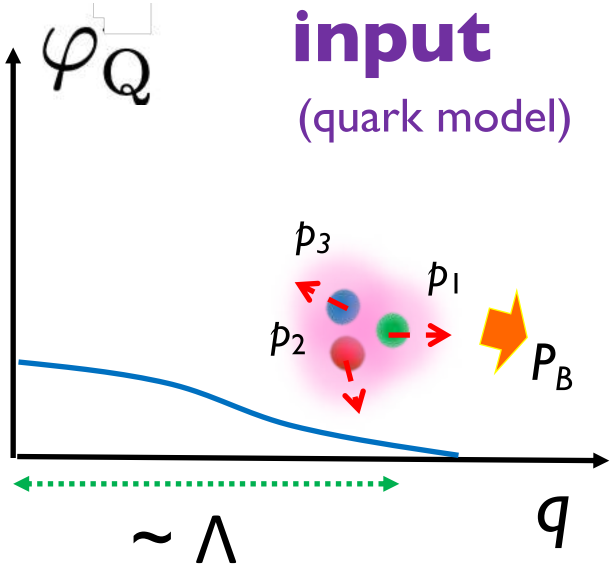
(e.g., free gas)



$\otimes$

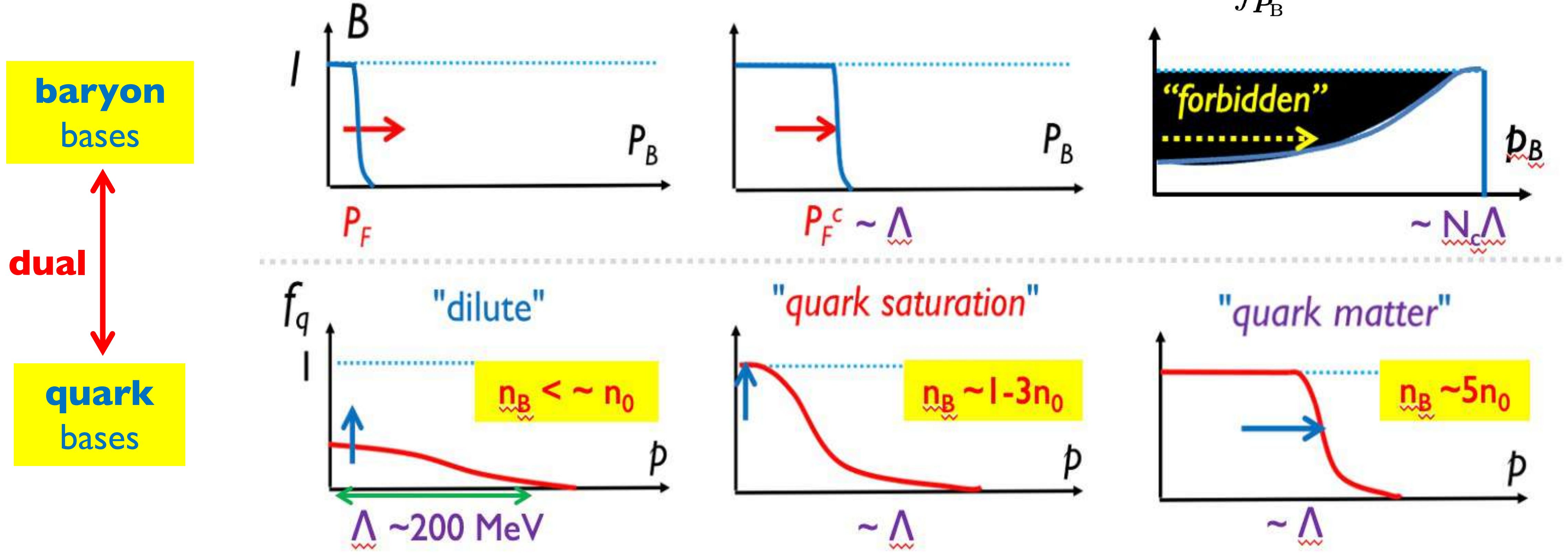
input

(quark model)



Evolution of occ. probabilities

$$f_Q(\boldsymbol{q}; n_B) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B; n_B)\varphi_Q^B(\boldsymbol{q}; \boldsymbol{P}_B)$$



"quark saturation" constraint

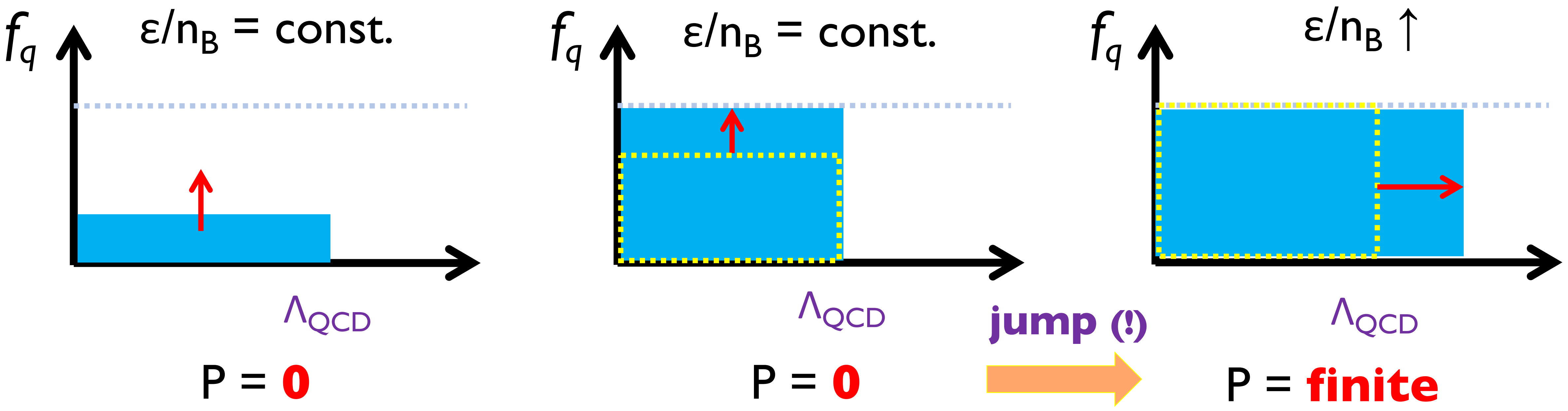
→ **relativistic baryons at low density, $n_B \sim 1\text{-}3n_0$!**

cf) McLerran-Reddy model (2019); microscopic description, TK (2021)

Jump in pressure : schematic picture in quarks

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle

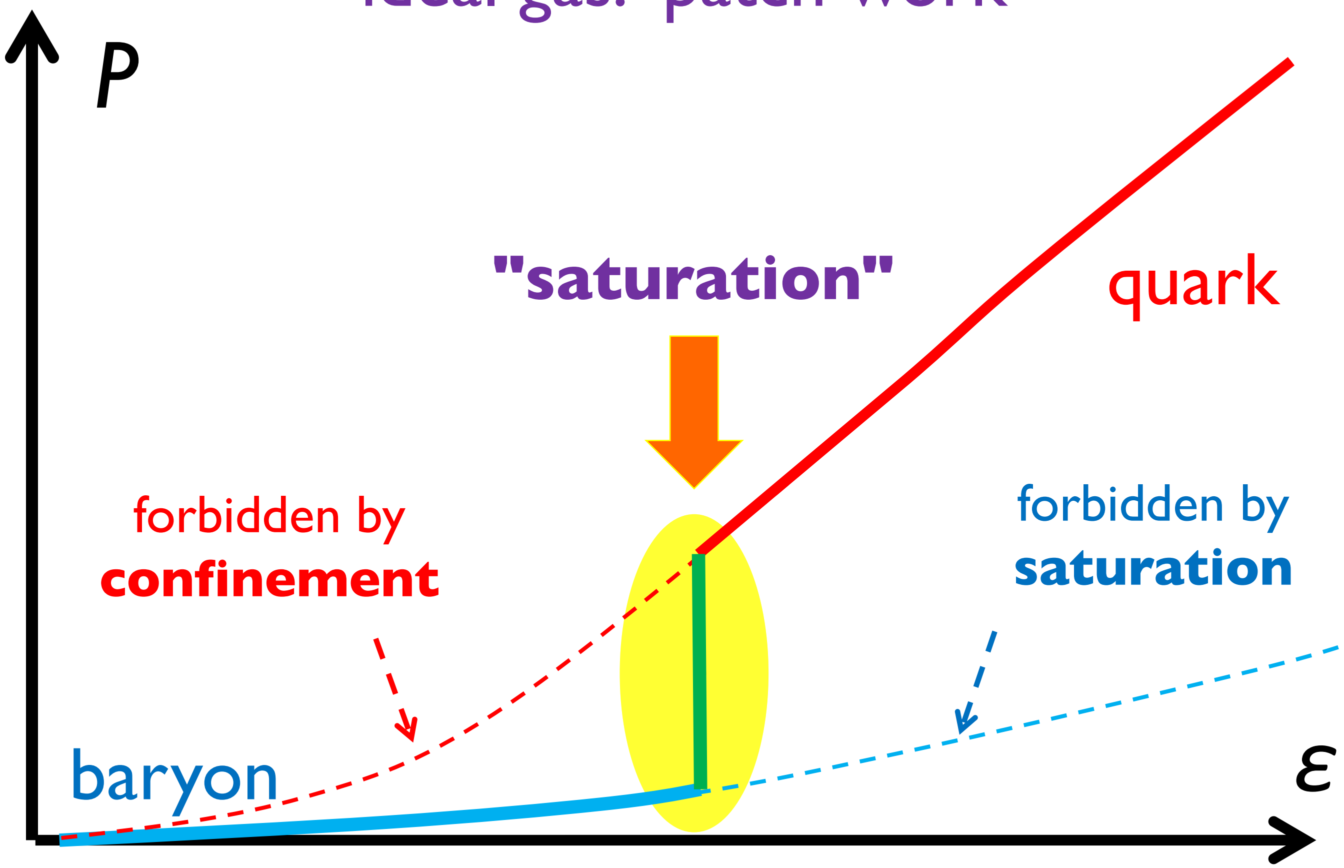


f_q continuous → ε, n_B are continuous

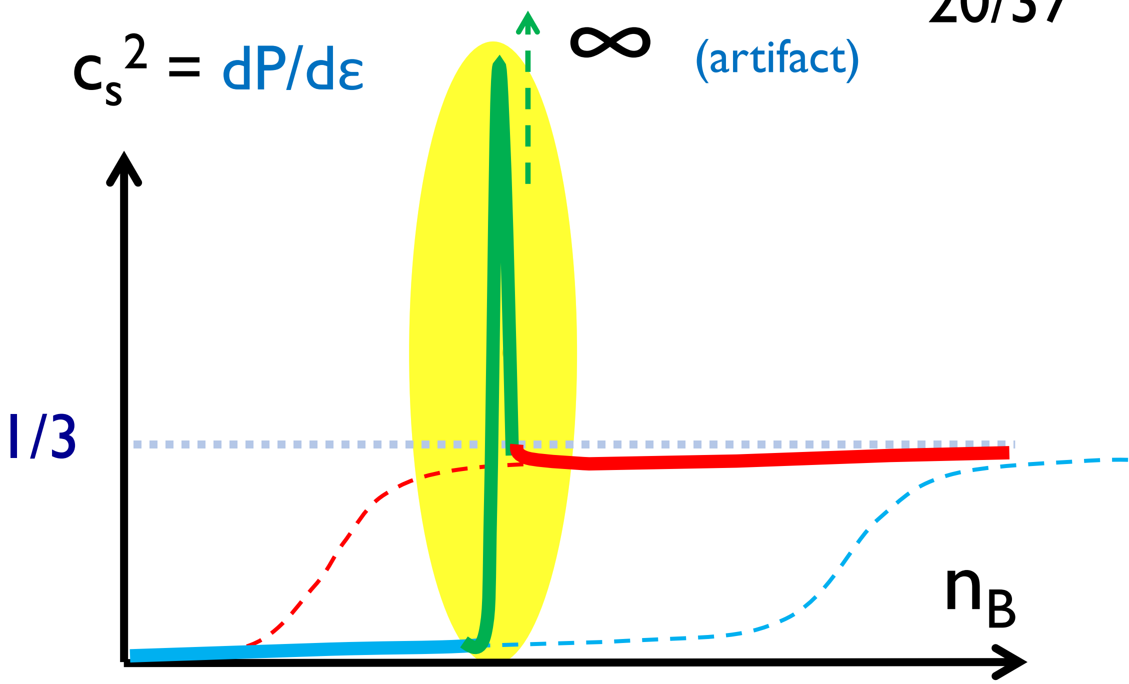
*Quarks do contribute to **ε** even before saturation; but to P only after the saturation!!*

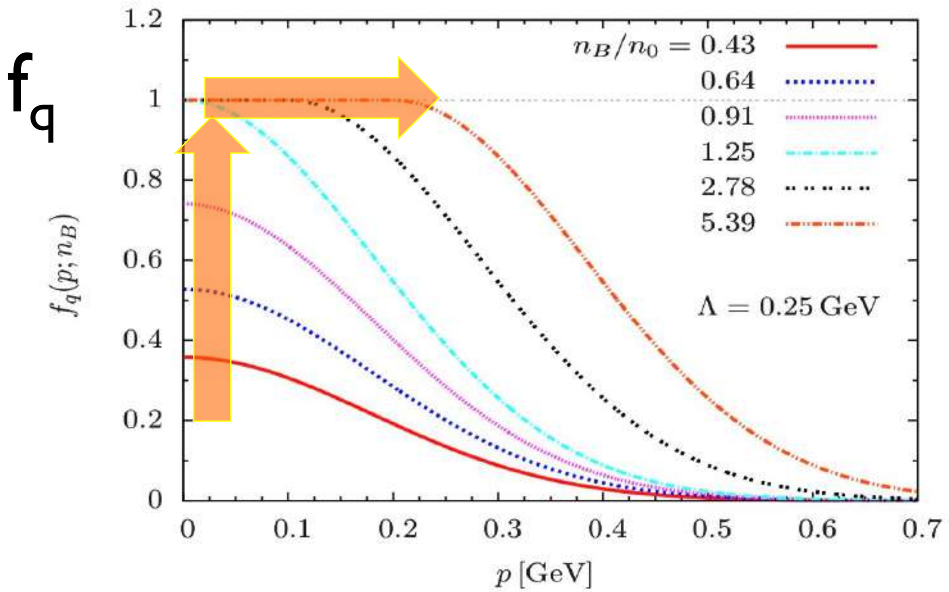
Peak in sound velocity

ideal gas: patch work

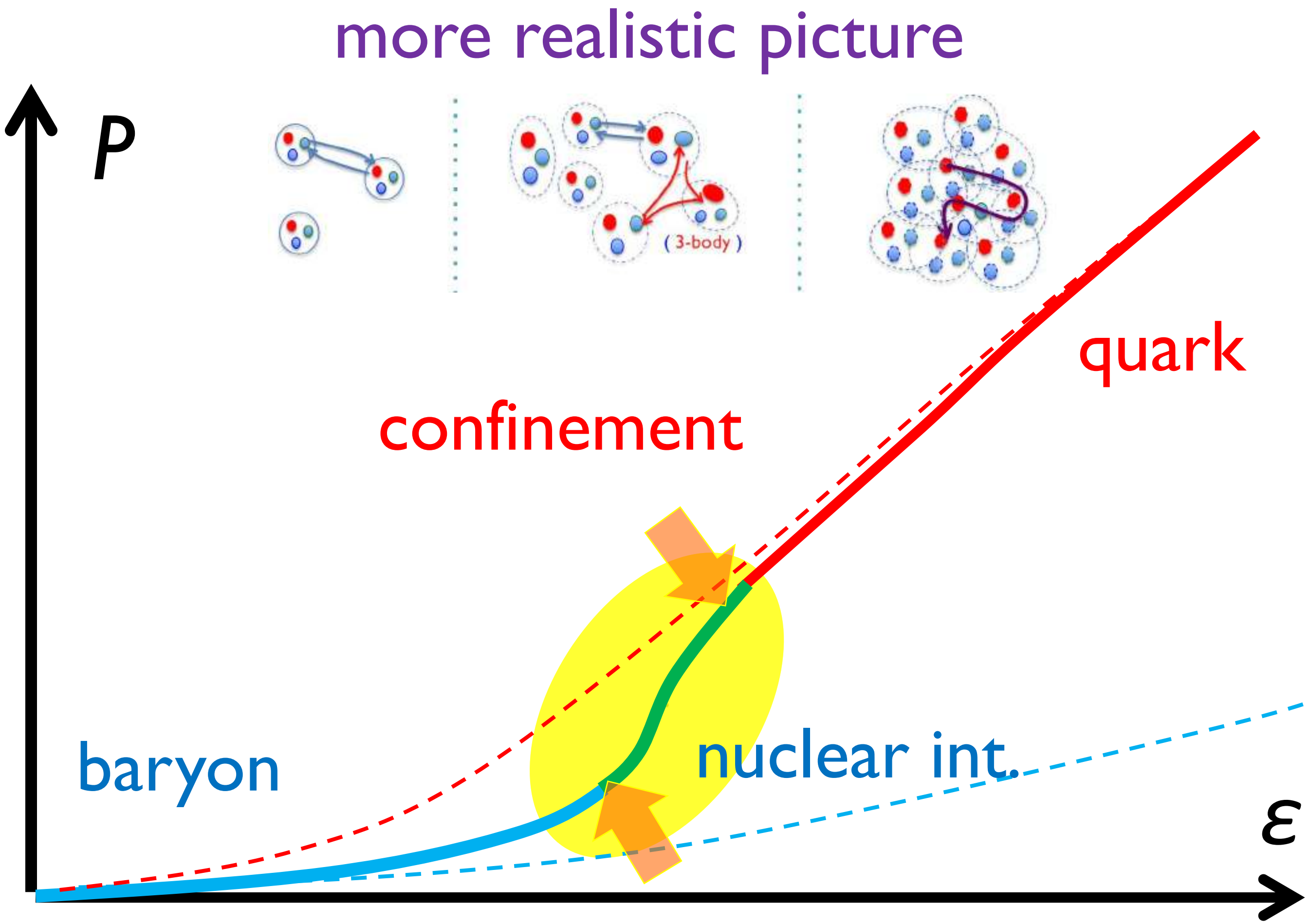


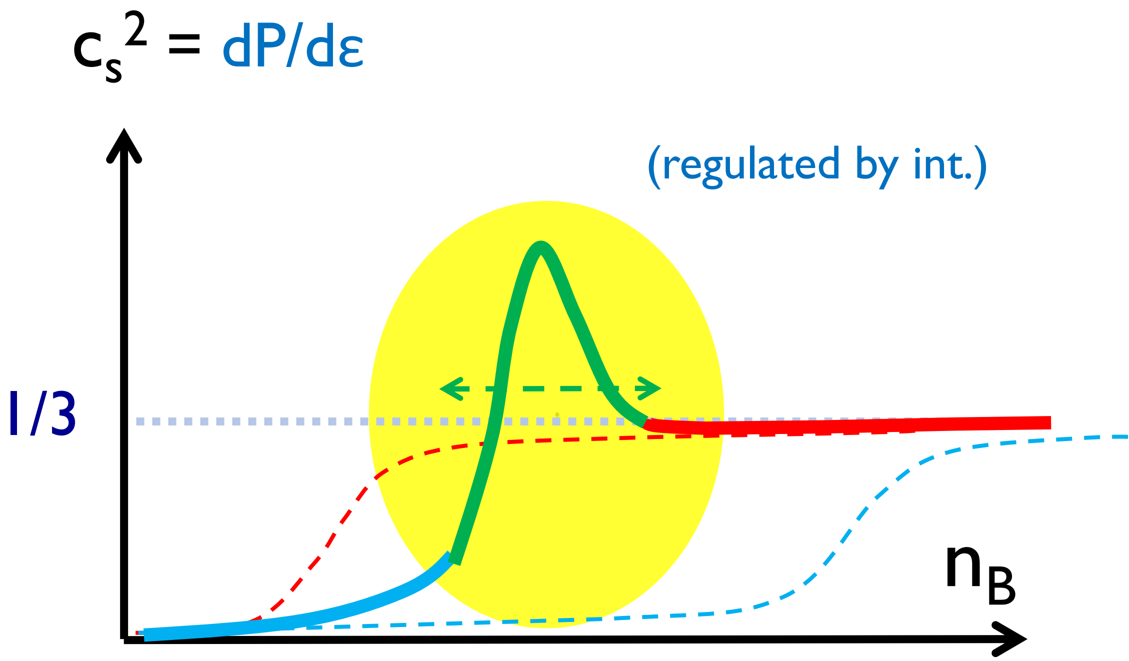
“inevitable” stiffening

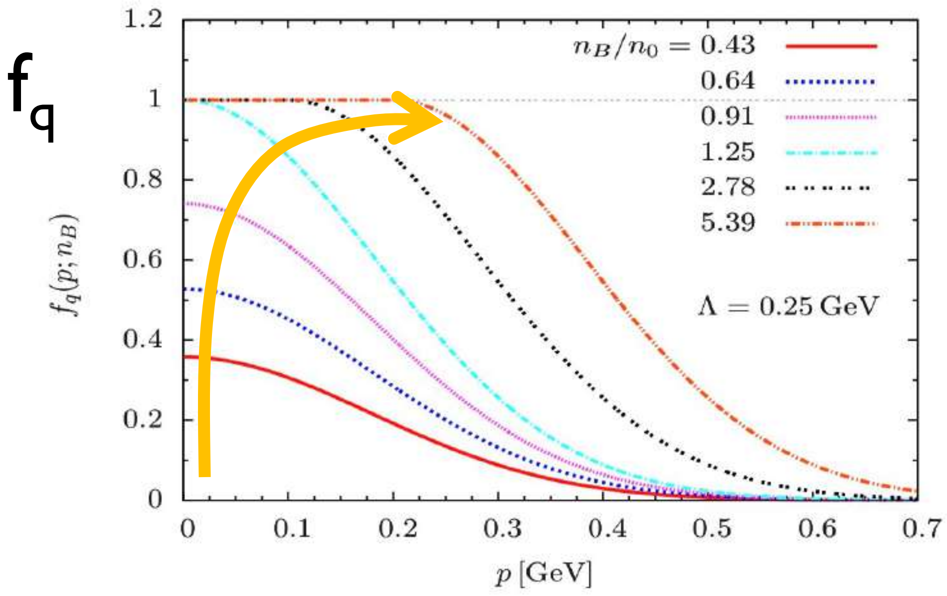




Peak in sound velocity







A solvable model

[Fujimoto-TK-McLerran, '23]

duality:

$$f_{\mathrm{Q}}(\boldsymbol{q}; n_{\mathrm{B}}) = \int_{\boldsymbol{P}_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{P}_{\mathrm{B}}; n_{\mathrm{B}}) \varphi_{\mathrm{Q}}^{\mathrm{B}}(\boldsymbol{q}; \boldsymbol{P}_{\mathrm{B}})$$

$f_B \rightarrow f_Q$ always doable, how about $f_Q \rightarrow f_B$??

global problem!

a useful model: with a specific quark distribution $\varphi_{3\mathrm{d}}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3} \frac{e^{-q/\Lambda}}{q/\Lambda}$

$$\hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p} - \boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2} \delta(\boldsymbol{p} - \boldsymbol{q})$$

local f_B from local f_Q:

$$f_{\mathrm{B}}(N_{\mathrm{c}} \boldsymbol{q}) = \frac{\Lambda^2}{N_{\mathrm{c}}^3} \hat{L}\left[f_{\mathrm{Q}}(\boldsymbol{q})\right]$$

A solvable model

[Fujimoto-TK-McLerran, '23]

2-flavor model: $$\varepsilon_B[f_B] = \underset{\text{isospin, spin}}{4} \int_k E_B(k) f_B(k)$$

IdylliQ matter (Ideal dual-lyllic Quarkyonic)

Ideal: *except* the confining forces that trap quarks, all interactions are neglected.

dual: *explicit* dual relations between baryons and quarks.

Quarkyonic: quark matter with non-perturbative (*confining*) gluons.

Variational problem

[Fujimoto-TK-McLerran, '23]

$$E_B(k) = \sqrt{M_B^2 + k^2}$$

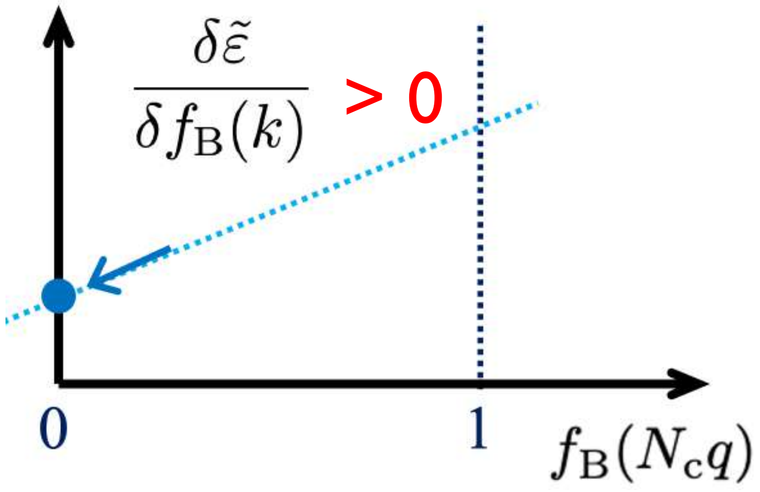
$$n_B = 4 \int_k f_B(k)$$

Lagrange multiplier

$$\tilde{\varepsilon} = \varepsilon_B[f_B] - \lambda_B n_B$$

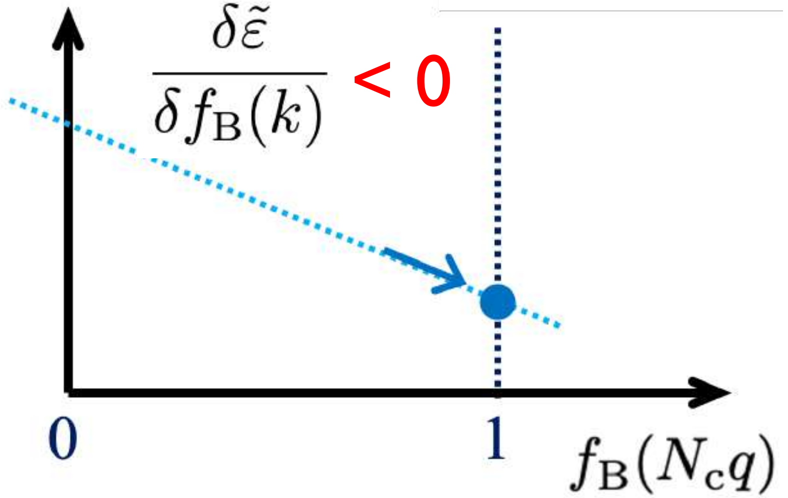
optimization: $$\frac{\delta \tilde{\varepsilon}}{\delta f_B(k)} = E_B(k) - \lambda_B$$ *at a given k*

$E_B(k) > \lambda_B$

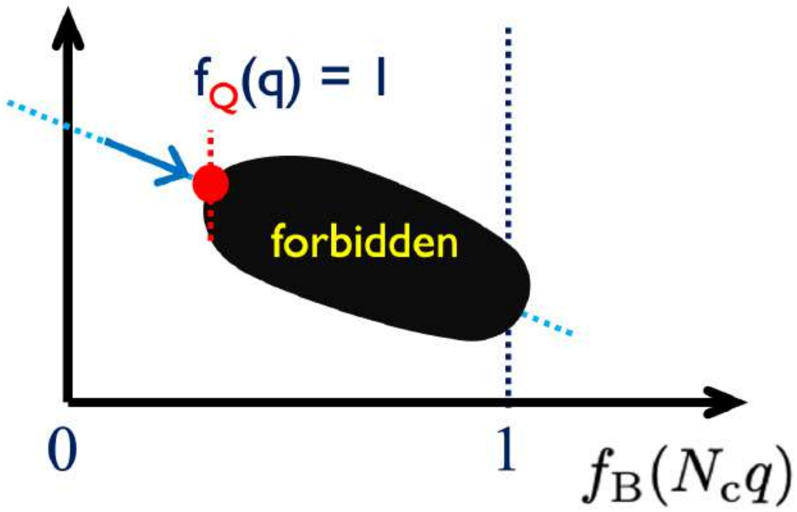


$E_B(k) < \lambda_B$

before saturation

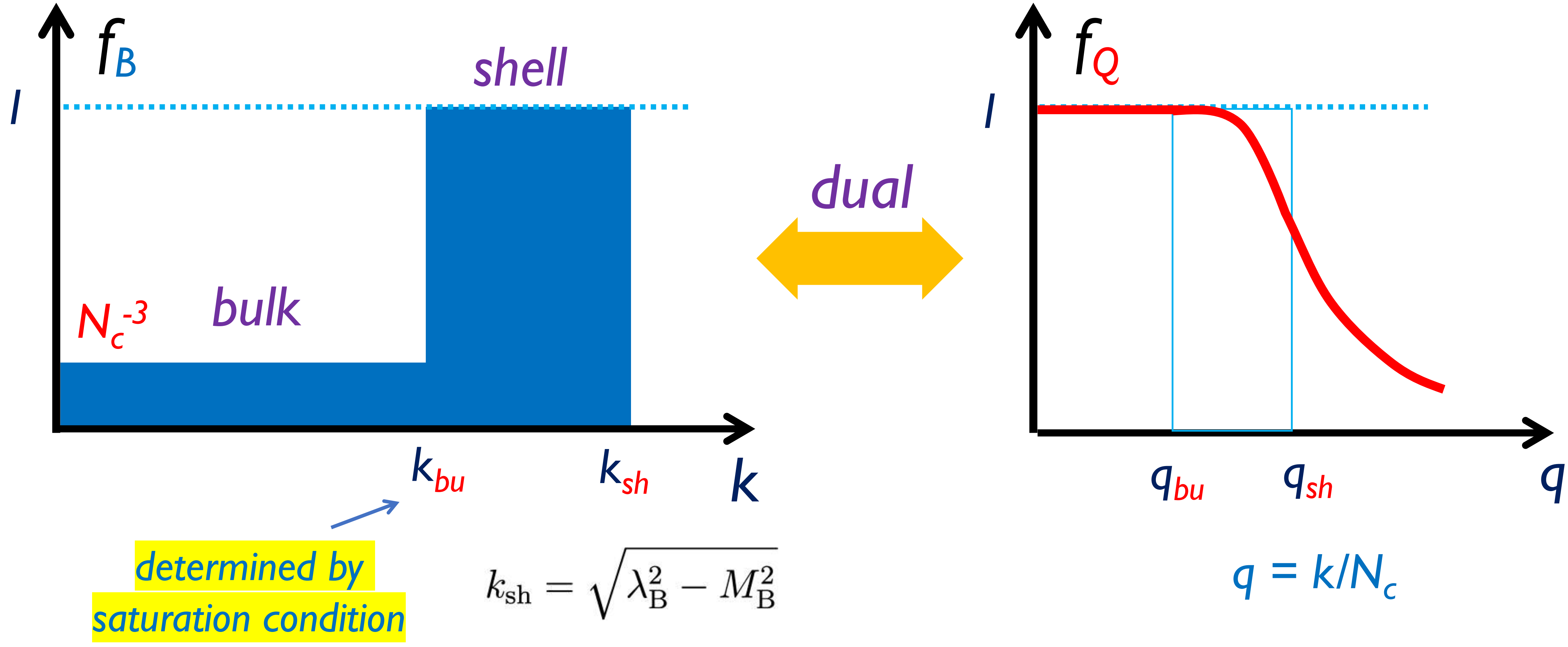


after saturation



Solution (post saturation)

[Fujimoto-TK-McLerran, '23]



$$k_{\rm sh} = \sqrt{\lambda_{\rm B}^2 - M_{\rm B}^2}$$

$q = k/N_c$

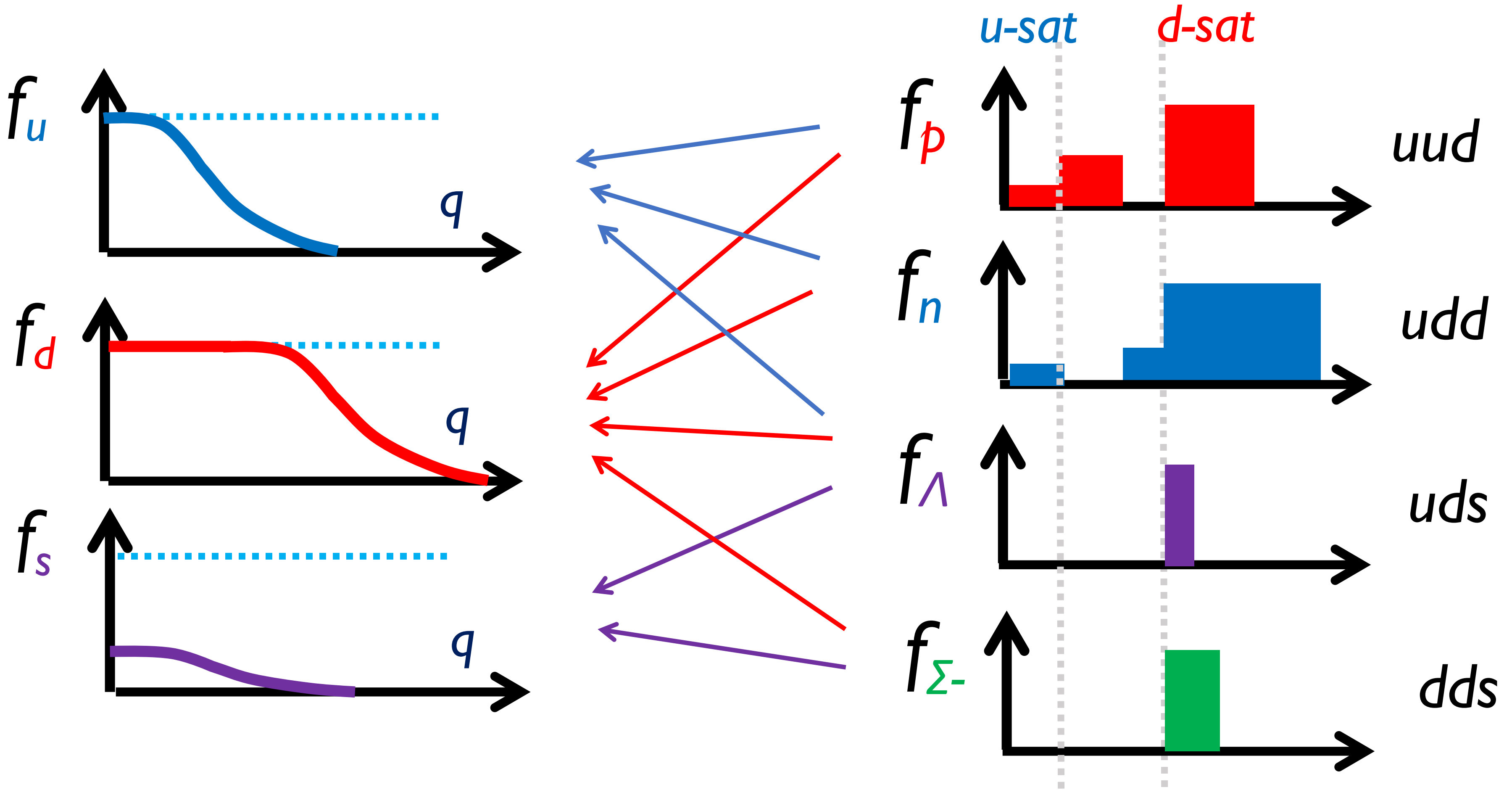
The ***most compact*** *distributions* ***compatible with*** *the saturation*

Multi-flavor extension

[Fujimoto-TK-McLerran, '23, to appear soon]

$$f_{\mathrm{Q}}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\mathrm{Q}}^{\mathrm{B}} \int_{\boldsymbol{k}} f_{\mathrm{B}}(\boldsymbol{k}) \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right)$$

Q = u, d, s



and so on…

saturation constraints

flavor correlations

Contents

Stiff quark matter

The appearance of c_s^2 peak is characteristic in the QHC scenarios, but is not sufficient condition for ~ 2.1-2.3$M_\odot$ NS.

[see detailed studies in QHC series (3-window model); Baym+'17, '19 and TK+ '21]

Just after the crossover, quarks are not fully relativistic.

Can the chiral restoration makes quarks massless and stiffens EOS?

Unlikely: it adds "*the bag constant*" to the energy density! (look at Dirac sea!)

→ ε increases & P decreases: **significant** softening!

Now, we consider interactions on top of IdylliQ models.

Underlying picture (guess)

- *Gluons remain non-perturbative at 5-10n_0*

  (see, e.g., lattice results for 2-color & isospin QCD)

- *Chiral restoration occurs mildly*

} implicitly included in IdylliQ type models

- *Continuity*: interactions in quark matter should have *natural counterpart* in hadron physics

Short range correlations in a baryon:

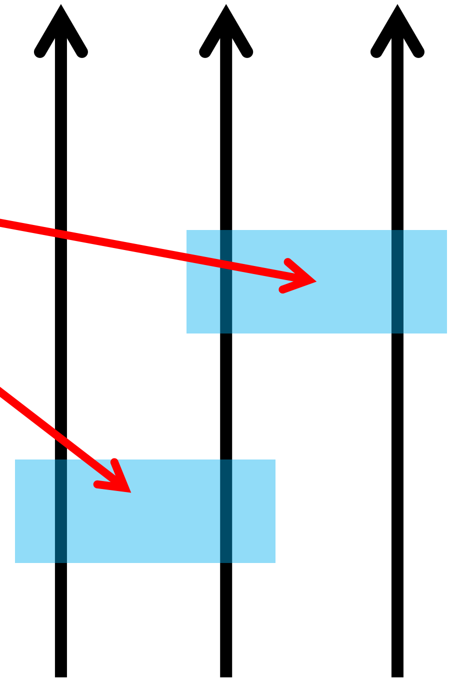
my favorite: *color-electric* & *magnetic interactions*

a baryon in dilute regime

(color-singlet)

(always) color-**antisymmetric**

(attractive **electric** int.)



e.g., **nucleons**

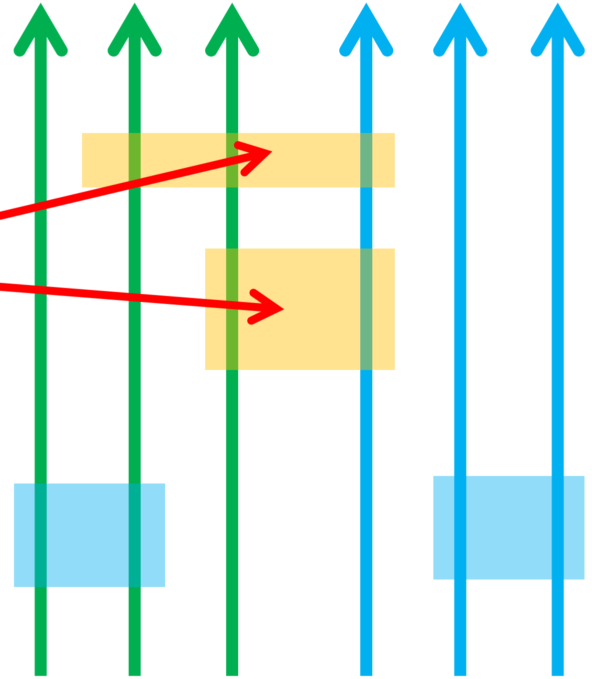
$M_N \sim 3M_{q.} + \text{kin.} + \text{color-EM}$

~ 940MeV ~ 1100MeV ~ −150-200MeV

in dense regime

sometimes color-**symmetric**

(repulsive)



more chances to feel repulsion

EoS with interactions

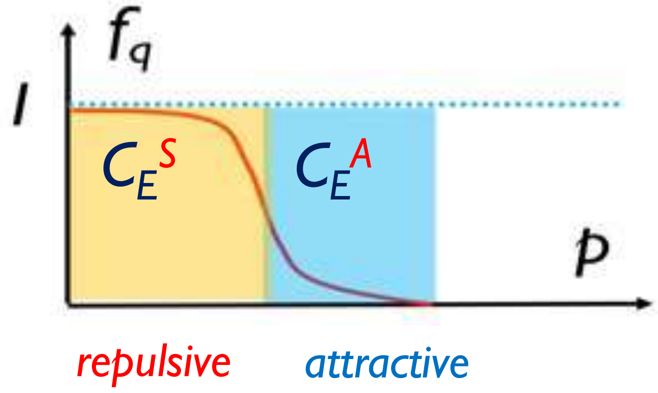
cf) [TK '21, TK-Suenaga '21]

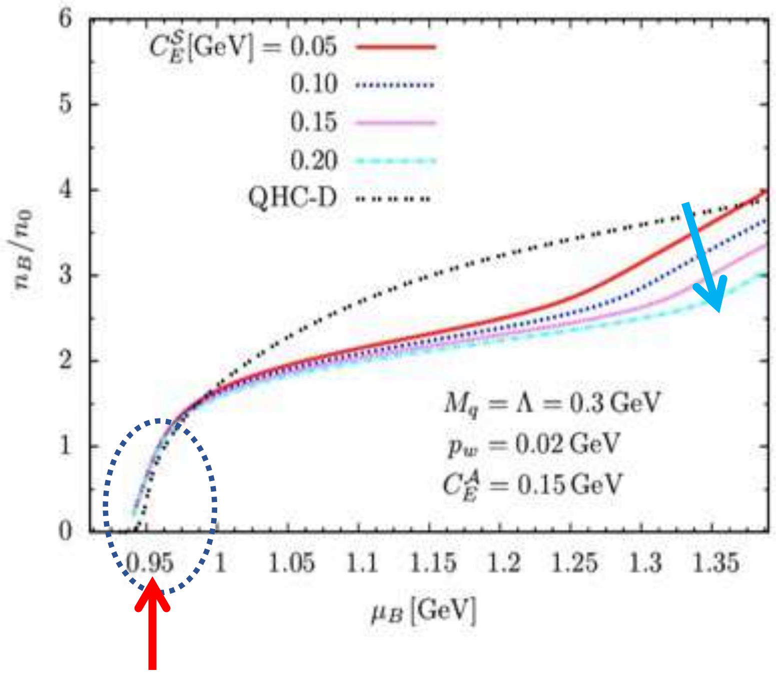
e.g.,

$$\mathcal{V}[f_{\mathrm{Q}}] = -C_E^{\mathcal{A}}\left[1-(f_{\mathrm{Q}})^n\right] + C_E^{\mathcal{S}}(f_{\mathrm{Q}})^n$$

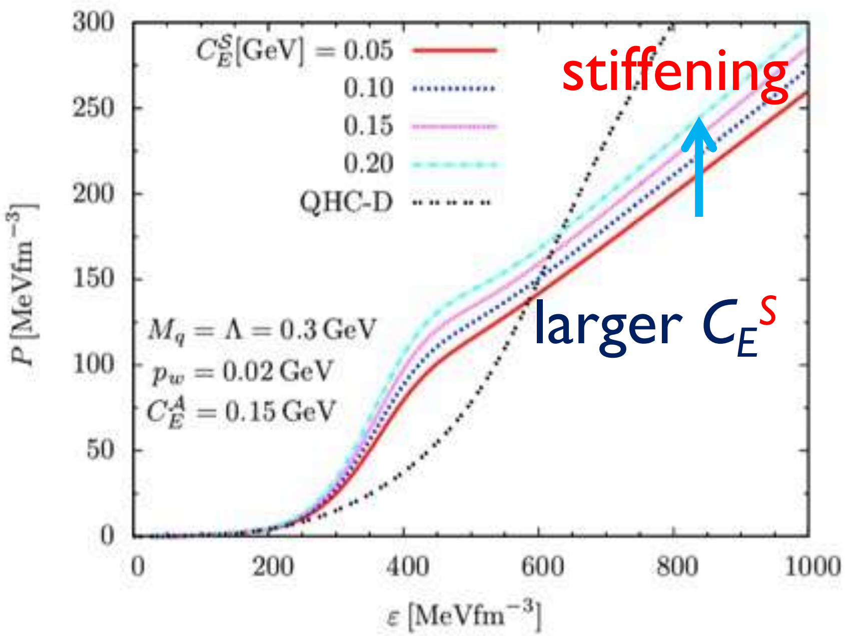
→ 1 (dilute) → 0 (dense)

→ 0 (dilute) → 1 (dense)

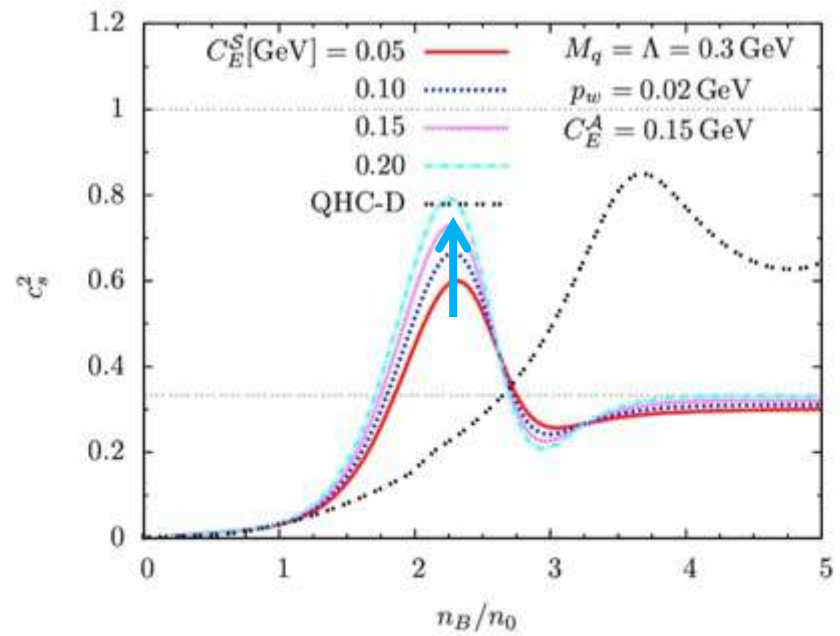




adjust C_E^A (fit M_B = 939 MeV)



high density stiffening



stronger peak in c_s

Simple parametric analyses

[TK-Powell-Song-Baym, '14]

rela. kin. energy interactions

$$\varepsilon(n) = a n^{4/3} + b n^{\alpha}$$

(n: quark density)

ideal interactions

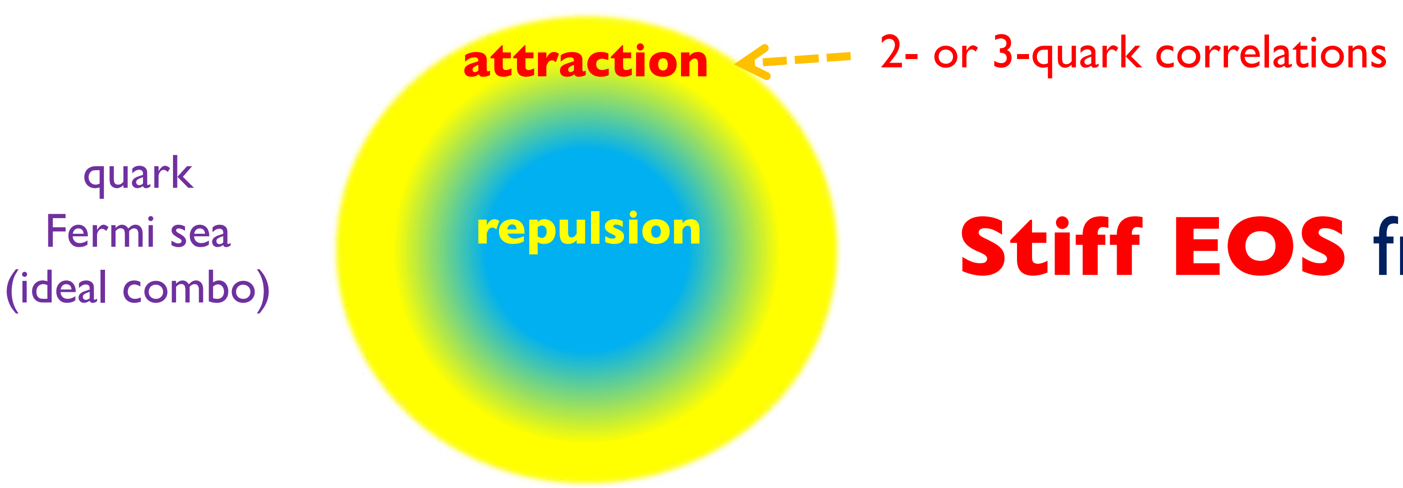
$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right) n^{\alpha}$$

For stiff EOS:

(for large P)

for $\alpha > 4/3$: $b > 0$ (e.g. bulk repulsion, ~ $+ n_B^2/\Lambda^2$)

for $\alpha < 4/3$: $b < 0$ (e.g. surface pairings, ~ $-\Lambda^2 n_B^{2/3}$)

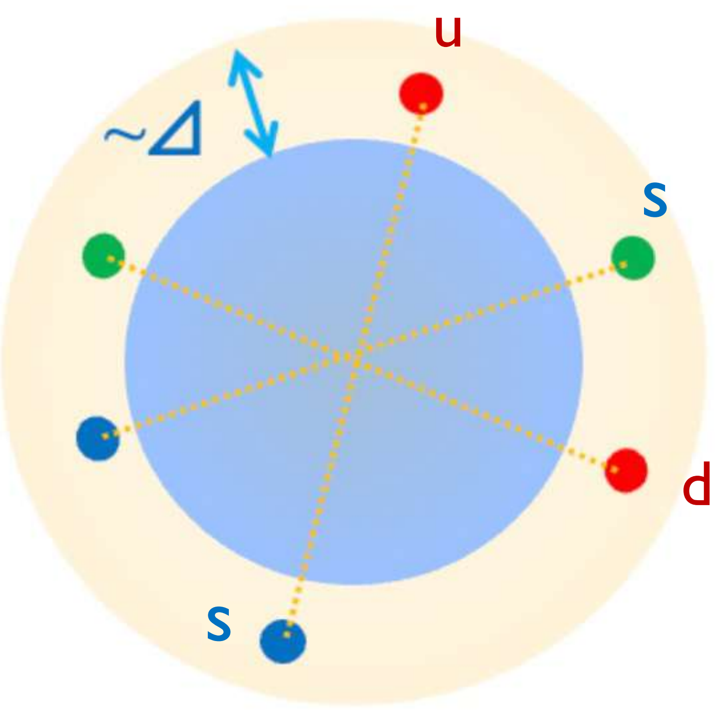


Stiff EOS from attractive forces

New look at the Fermi surface physics

2-particle correlation

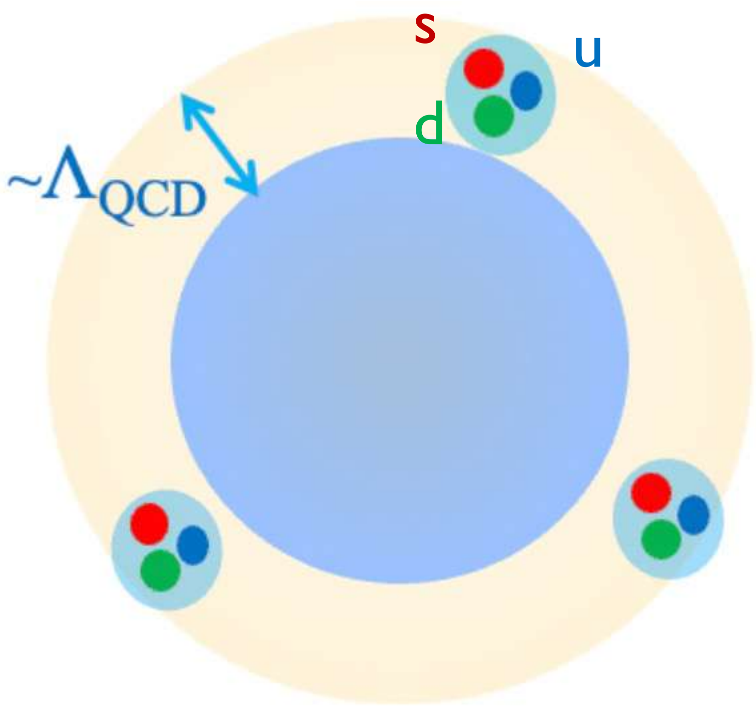
e.g., diquark pairing



color-superconductor (CSC)

[Bailin-Love, Alford, Rajagopal, Wilczek, ...]

3-particle correlation



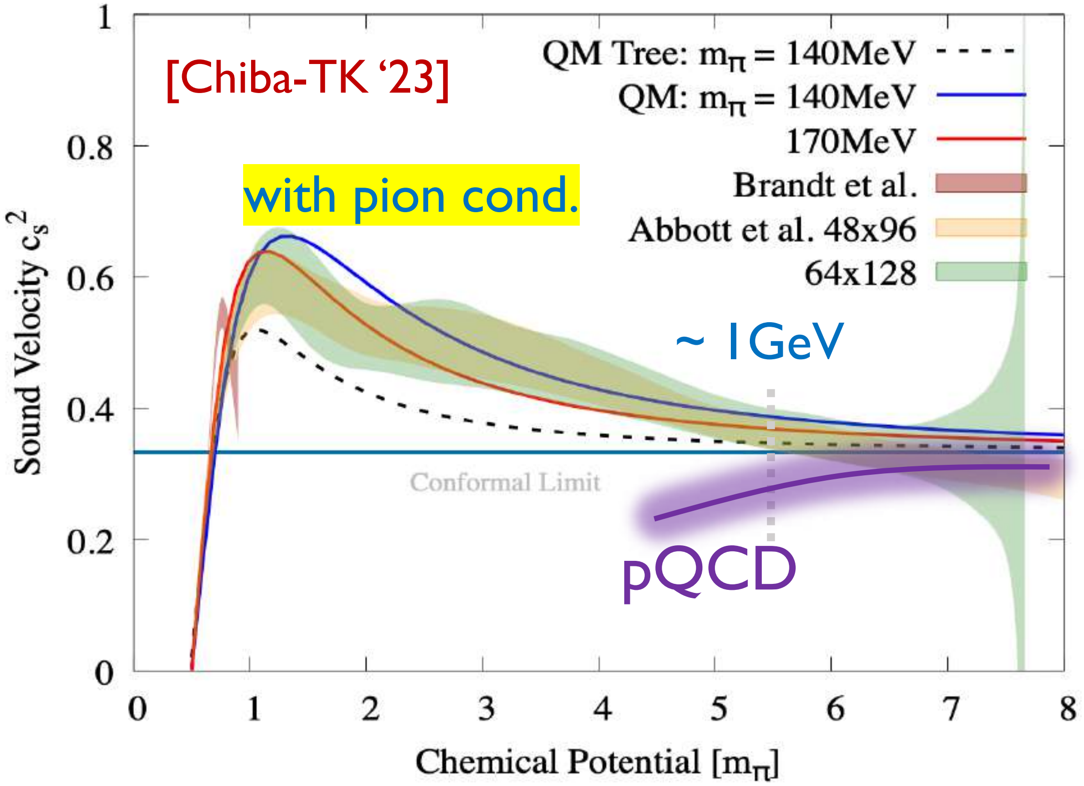
quarkyonic matter

[McLerran-Pisarski '07, ...]

Phase structure: Not mere academic questions, but have very practical meaning

Hints for non-pert. physics at large n_B

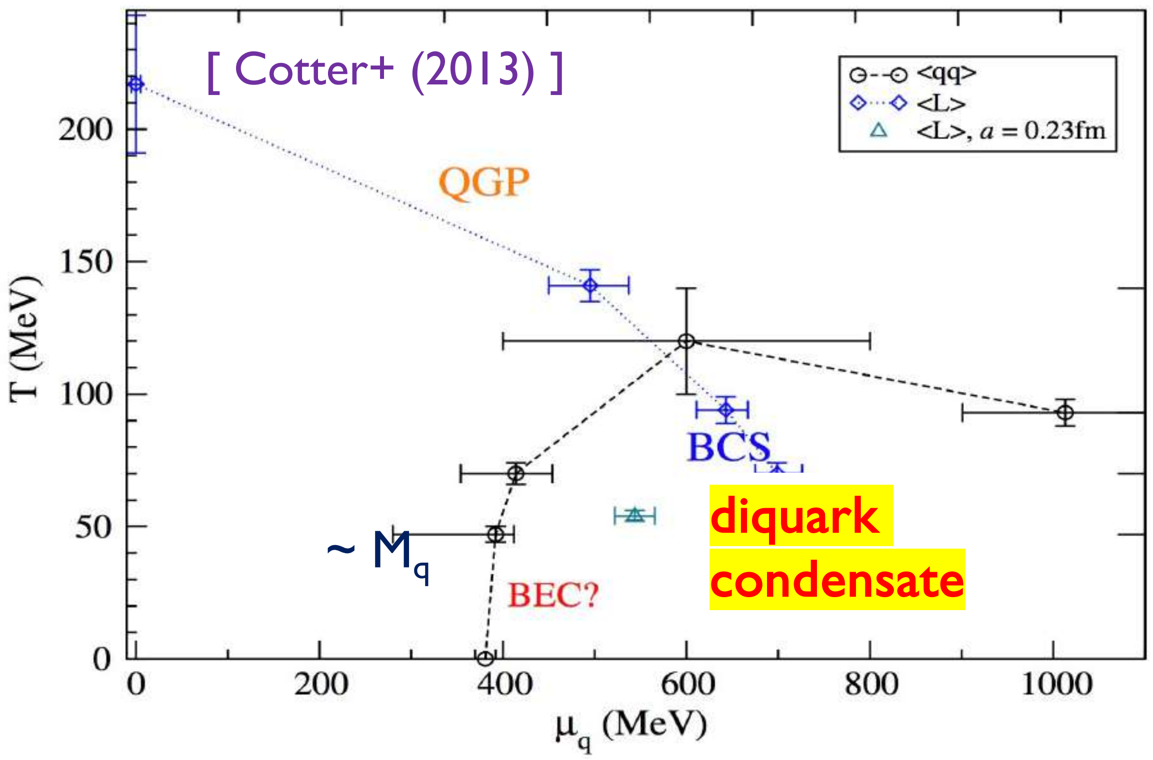
isospin QCD [lattice vs model]



EOS & c_s^2 in 2-color QCD

→ Itou-san's talk

2-color QCD phase diagram [lattice]



→ Δ ~ 1.75 T_c ~ **175 - 210** MeV

at μ_q ~ **1GeV** or n_B ~ **40** n_0

Summary

- For soft-to-stiff EOS: QHC is a good baseline
- Quark saturation effects: likely occur at ~1-3n_0
- Saturation triggers rapid stiffening (sound velocity peak)
- Hyperons are not independent; highly suppressed by saturation

[Fujimoto-TK-McLerran, to appear]

- Bulk repulsion and Fermi surface attraction → stiffening of EOS

Quarks are important for NS physics in multiple contexts