Complex Scaling Method : Principles and Applications

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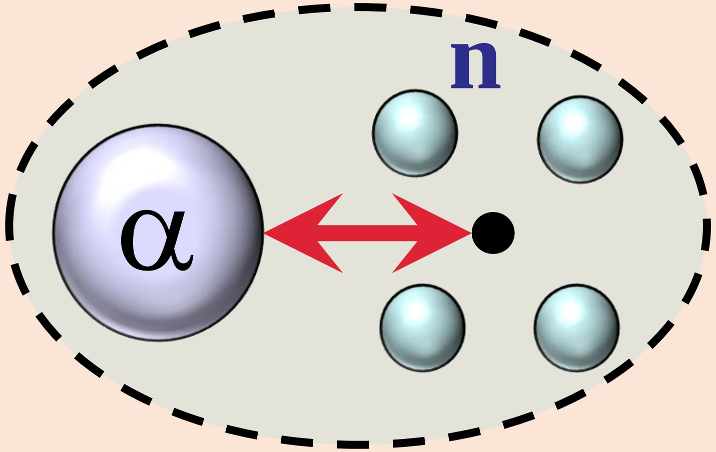
Hadron Spectroscopy Café, Nagoya Univ. 2024年5月17日

Outline

- Physics of light unstable nuclei
- Resonances and non-resonant continuum states
- Complex scaling Method (CSM)
  複素（座標）スケーリング法
- Nuclear model : Cluster model
- Basic properties of CSM
- Many-body resonances
- Level density, Green’s function
- Strength functions, breakup reactions



^{8}He

1. Resonance spectroscopy with CSM
2. Scattering states with CSM
   - Complex-scaled Green’s function

Continuum Level Density (CLD) in complex scaling

$$\Delta(E) = \rho(E) - \rho_0(E)$$
$$= -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G(E) - G_0(E)\right]\right]$$

Level density

$$\rho(E) = \sum_n \delta(E - E_n), \quad G_{(0)}(E) = \frac{1}{E - H_{(0)}}$$

$$\Delta(E) = \frac{1}{2\pi i}\mathrm{Tr}\left[S(E)^\dagger \frac{d}{dE} S(E)\right]$$

Single channel case

$$\Delta(E) = \frac{1}{\pi}\frac{d\delta_\ell}{dE}$$

S. Shlomo, NPA539('92)17

A. Kruppa, PLB431('98) 237, K. Arai and A. Kruppa, PRC60('99)064315

R. Suzuki, T. Myo and K. Kato, PTP113('05)1273.

R.D. Levine, Quantum Mechanics of Molecular Rate Processes, Oxford, 1969.

$$\frac{1}{x - x_0} = \mathrm{P}\left(\frac{1}{x - x_0}\right) - i\pi \cdot \delta(x - x_0)$$

Complex Scaling

$$\Delta(E) = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G^\theta(E) - G_0^\theta(E)\right]\right]$$

$$G^\theta(E) = \frac{1}{E - H^\theta}$$ with interaction

$$G_0^\theta(E) = \frac{1}{E - H_0^\theta}$$ Asymptotic

Continuum Level Density (CLD) in CSM using discretized continuum states

Level density $\rho(E) = \sum_n \delta(E - E_n)$

$$\Delta(E) = \rho(E) - \rho_0(E)$$

$$\rho_{(0)}^{\theta} = U(\theta)\rho_{(0)}U^{-1}(\theta)$$

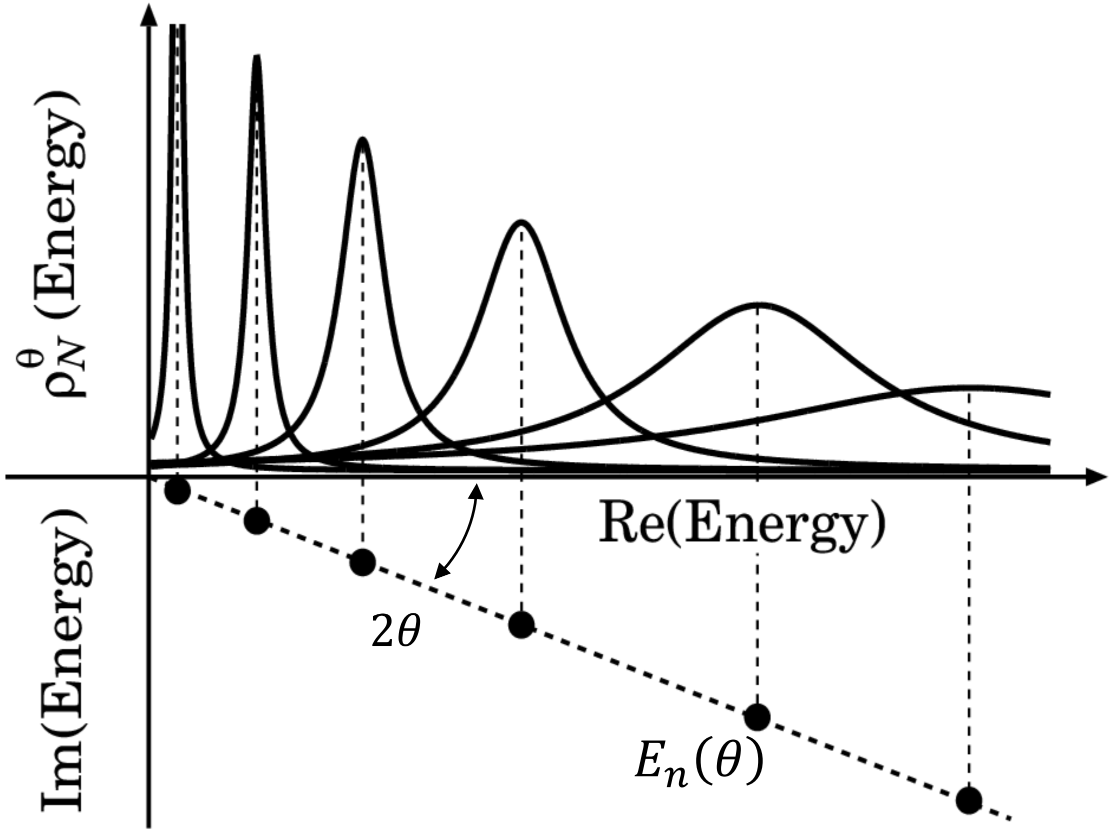
$$\rho^{\theta} \cong \rho_N^{\theta} = -\frac{1}{\pi}\mathrm{Im}\left[\sum_n^N \frac{1}{E - E_n(\theta)}\right]$$

$$= \frac{1}{\pi}\sum_n^N \frac{E_I}{(E - E_R)^2 + E_I^2}$$

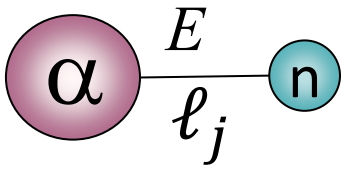
Number of discretized states

Lorentzian

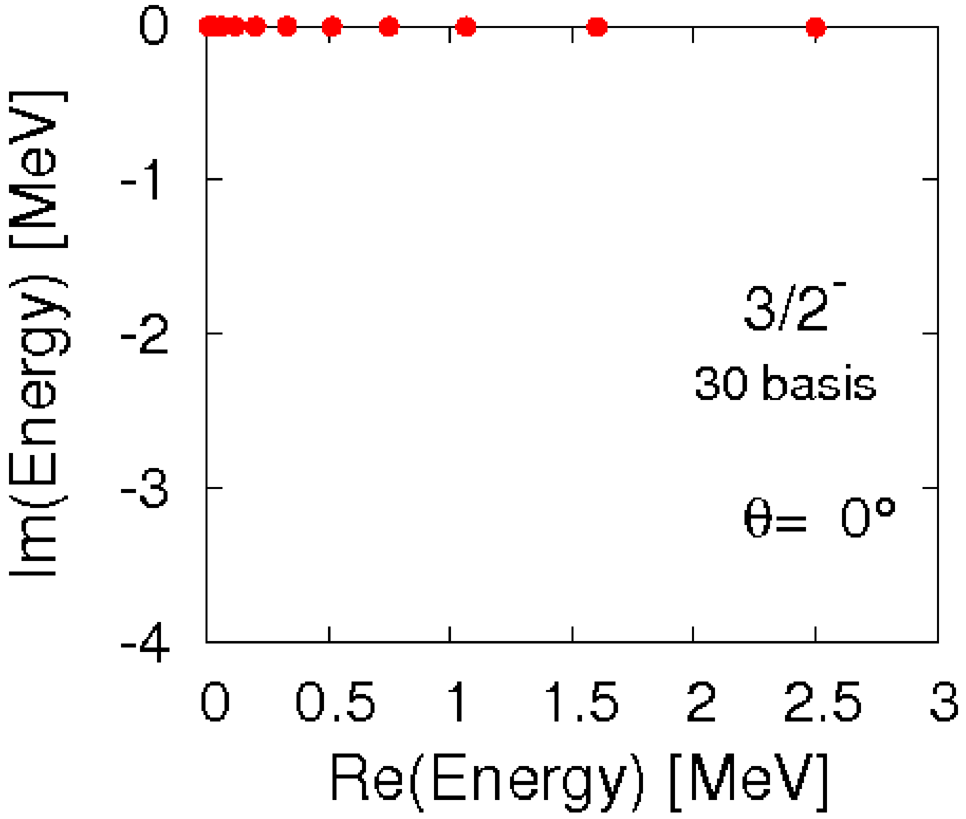
Complex-scaled energy eigenvalue $E_n(\theta) = E_R - iE_I$



α+n scattering using discretized states

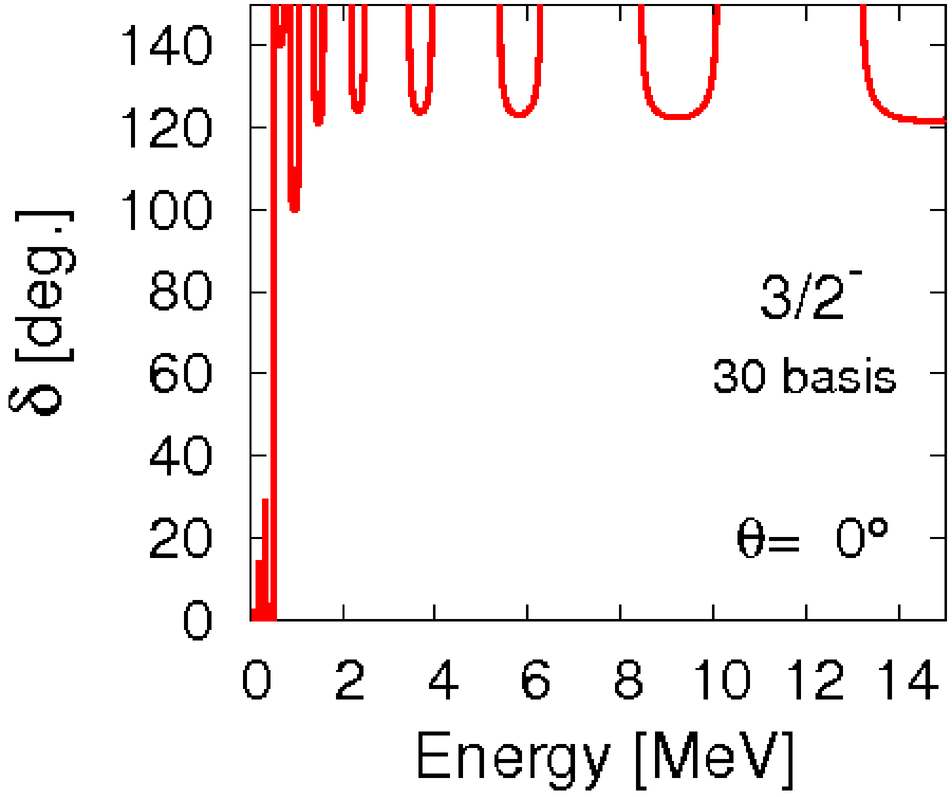


energy eigenvalues



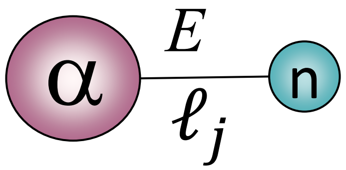
30 Gaussian basis functions

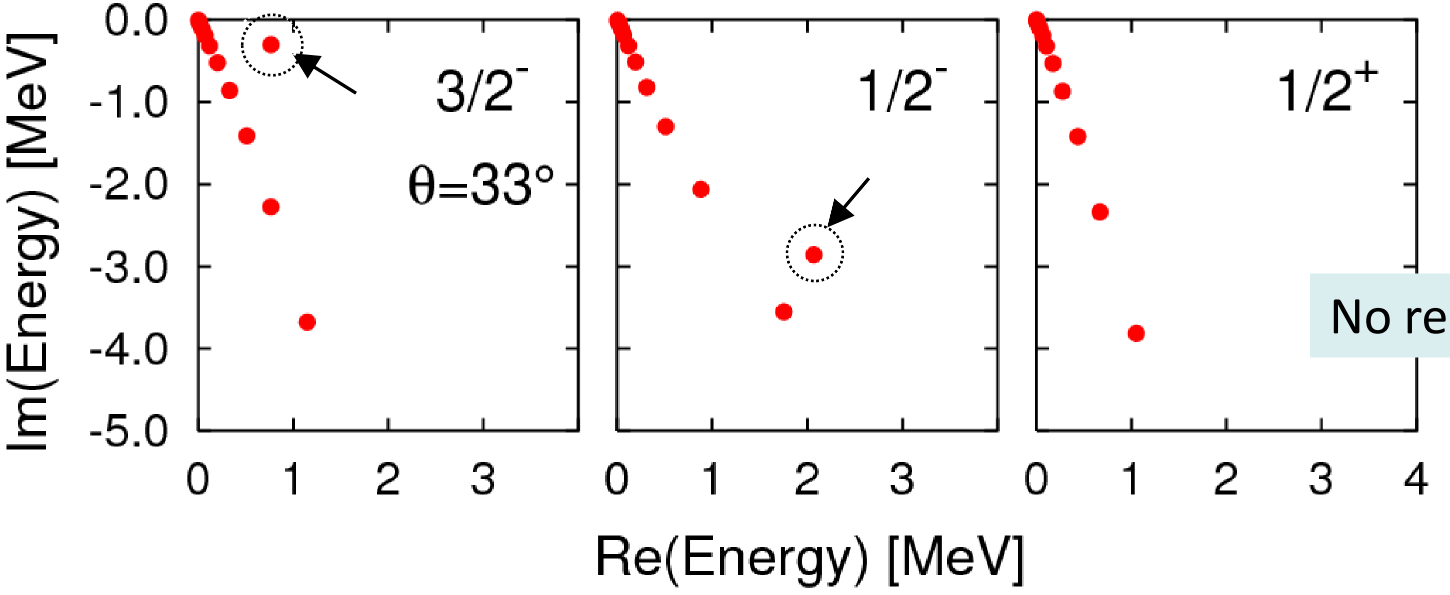
P-wave scattering phase shift



α+n scattering with discretized continuum states

Energy eigenvalues

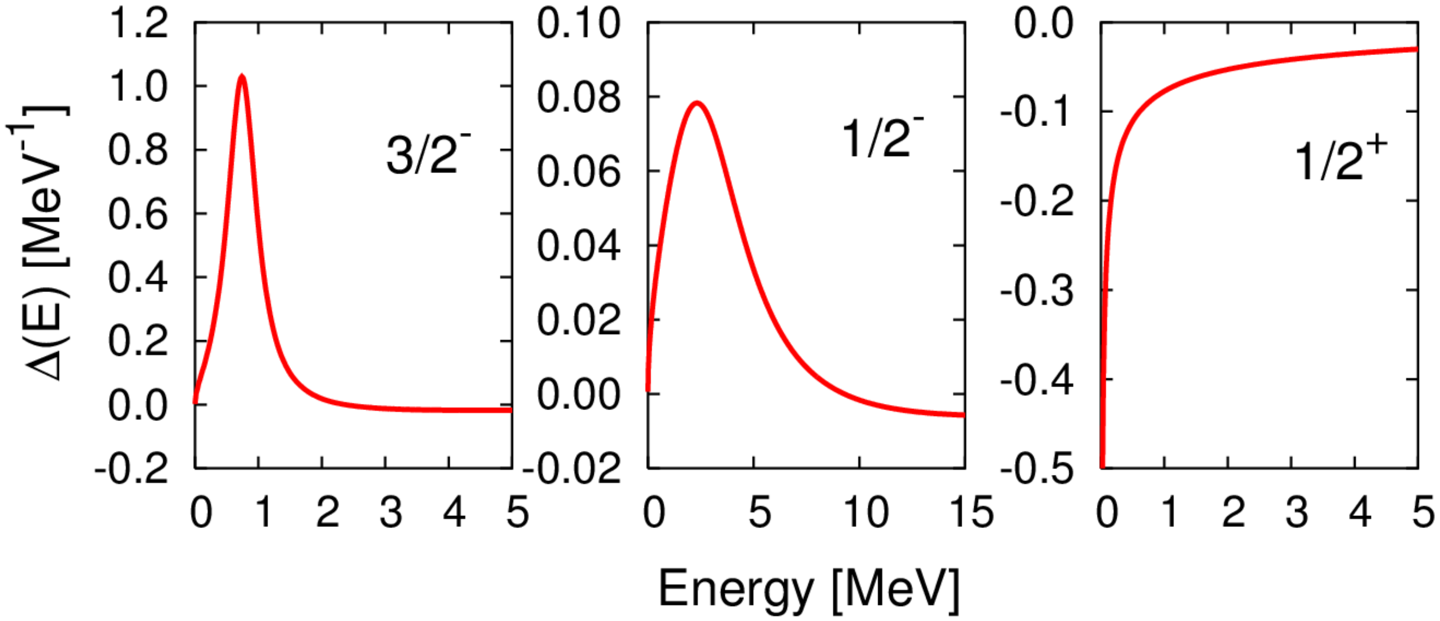




No resonance

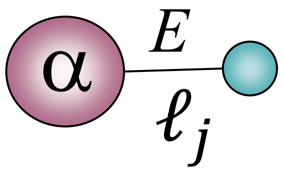
CLD Δ(E)
(s,p-waves)

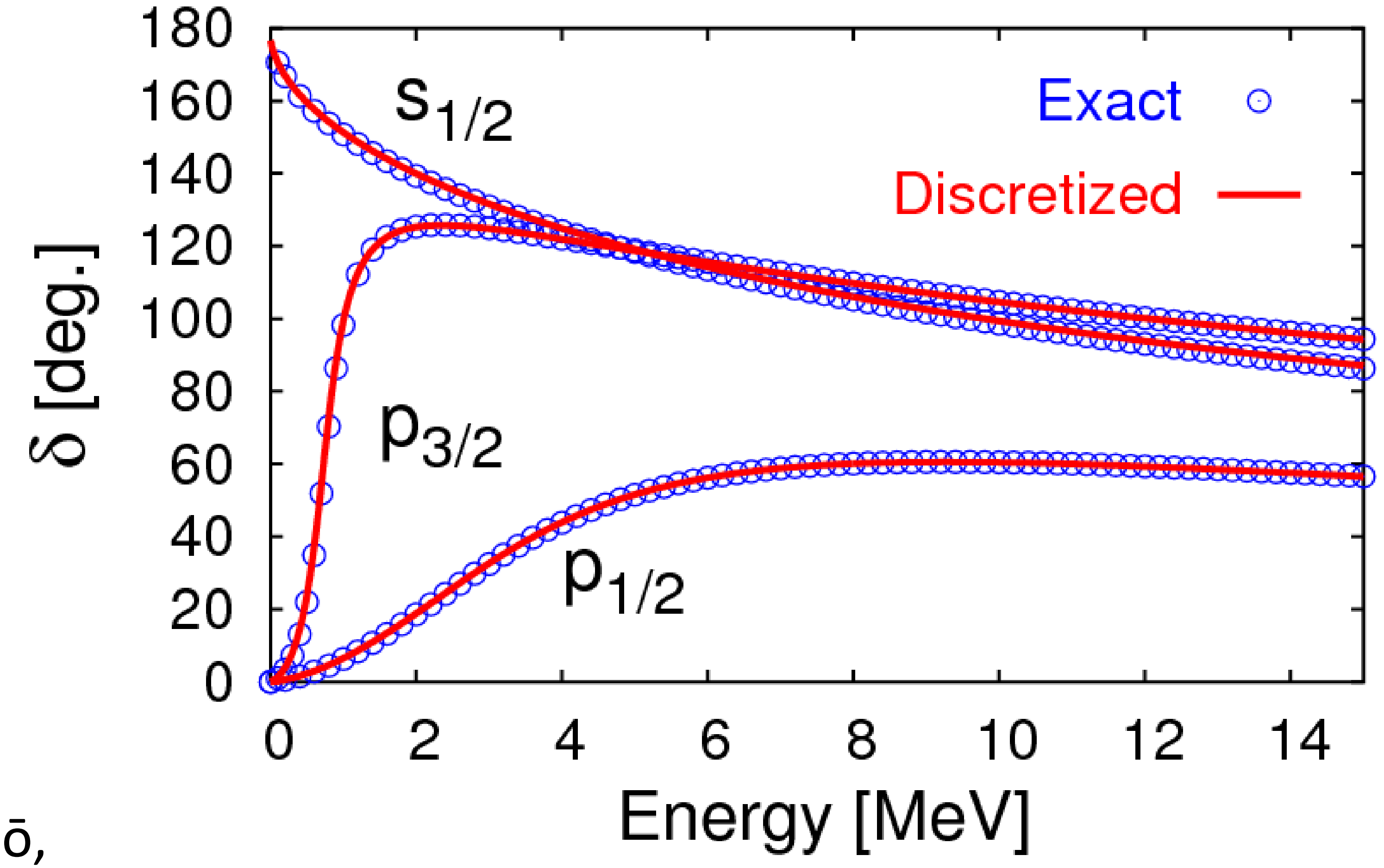
θ-independent



α+n scattering with discretized continuum states

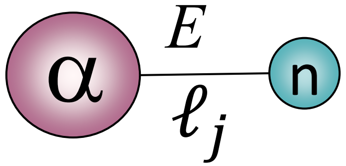
Phase shifts (s,p)





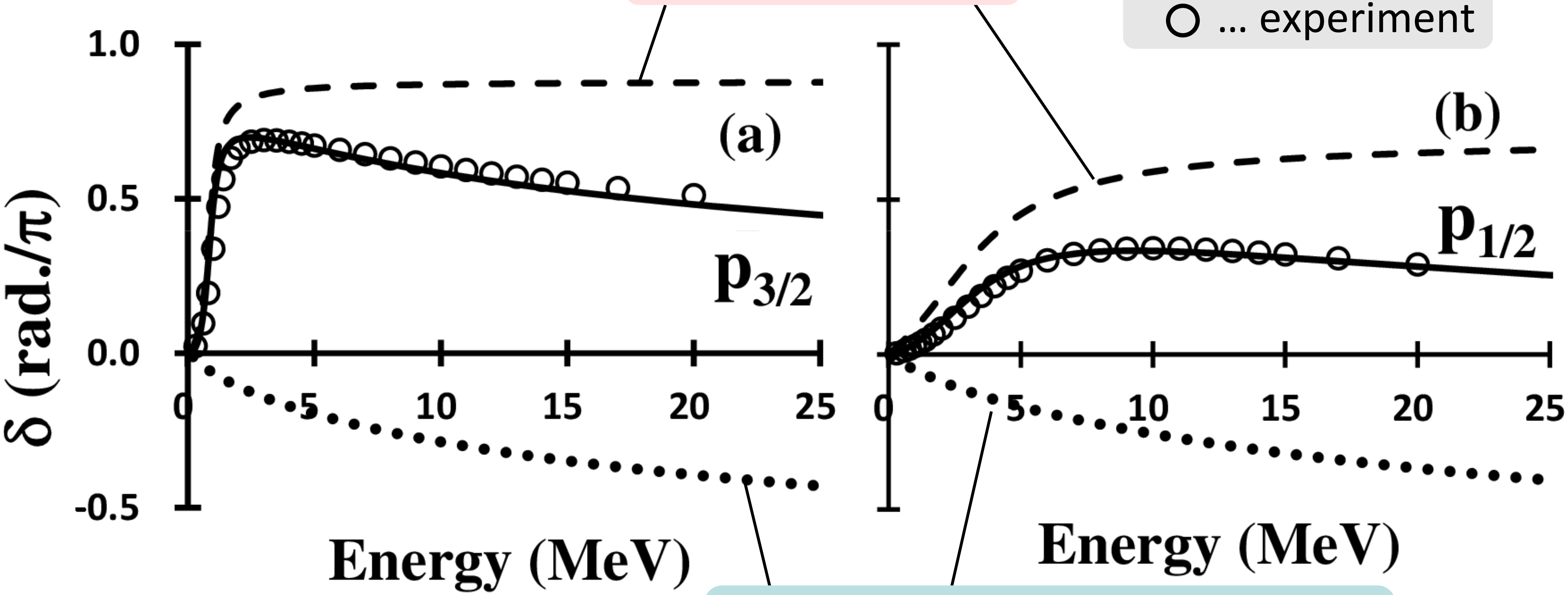
R. Suzuki, T. Myo and K. Katō,
PTP113('05)1273.

Decomposition of phase shift



Resonance term

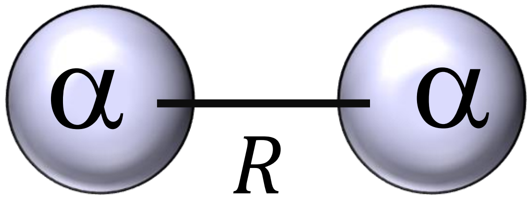
○ … experiment



Non-resonance term (Background)

Odsuren, Katō, Aikawa, Myo, Phys. Rev. C, 89, 034322 (2014).
Odsuren, Kikuchi, Myo, Khuukhenkhuu, Masui, K. Katō, Phys. Rev. C 95, 064305(2017).
Odsuren, Myo, Kikuchi, Teshigawara, Katō, Phys. Rev. C104, 014325 (2021).

^{8}Be energy

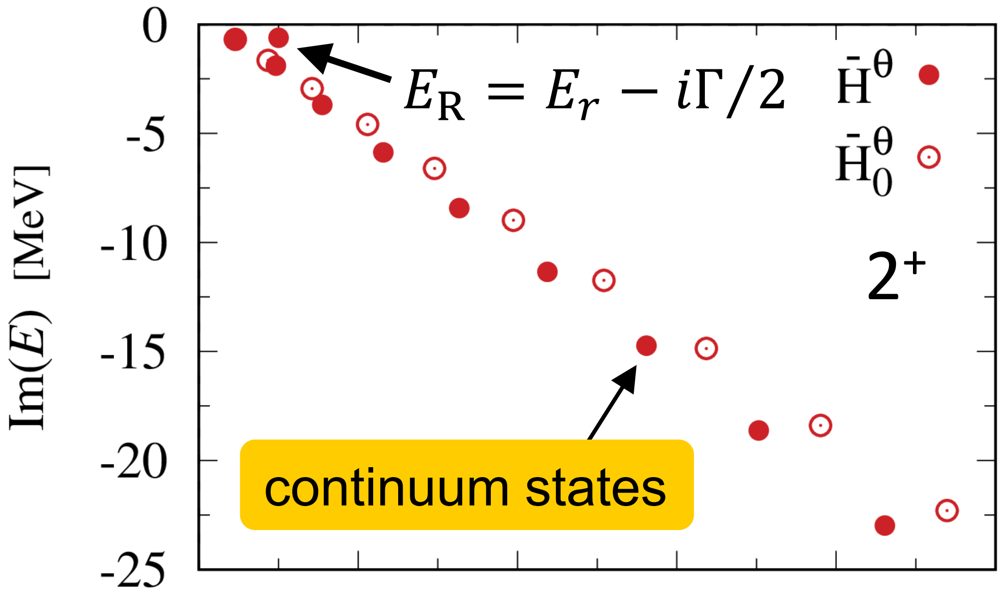


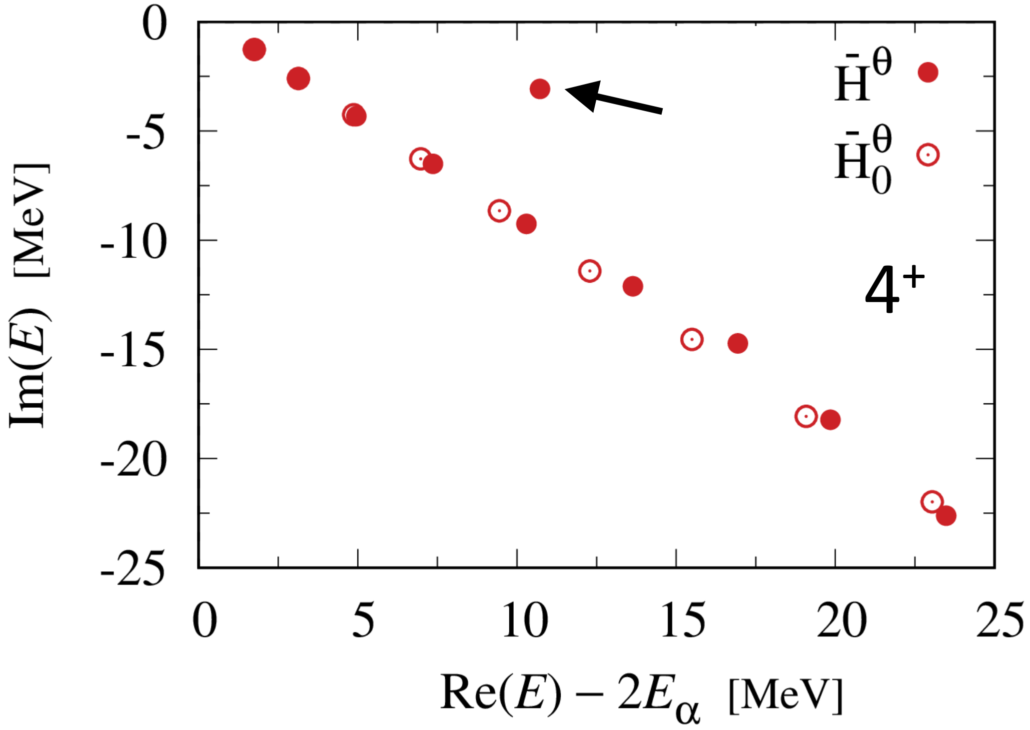
$\theta = 27°$

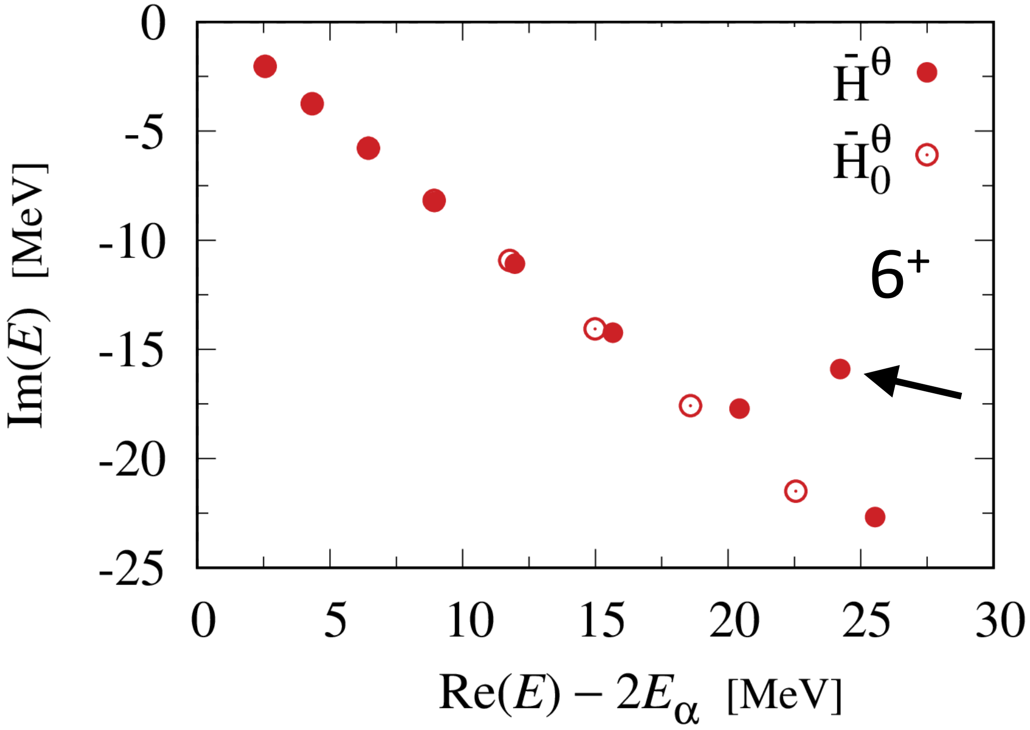
E_{ϑ} measured from $2E_{\alpha}$

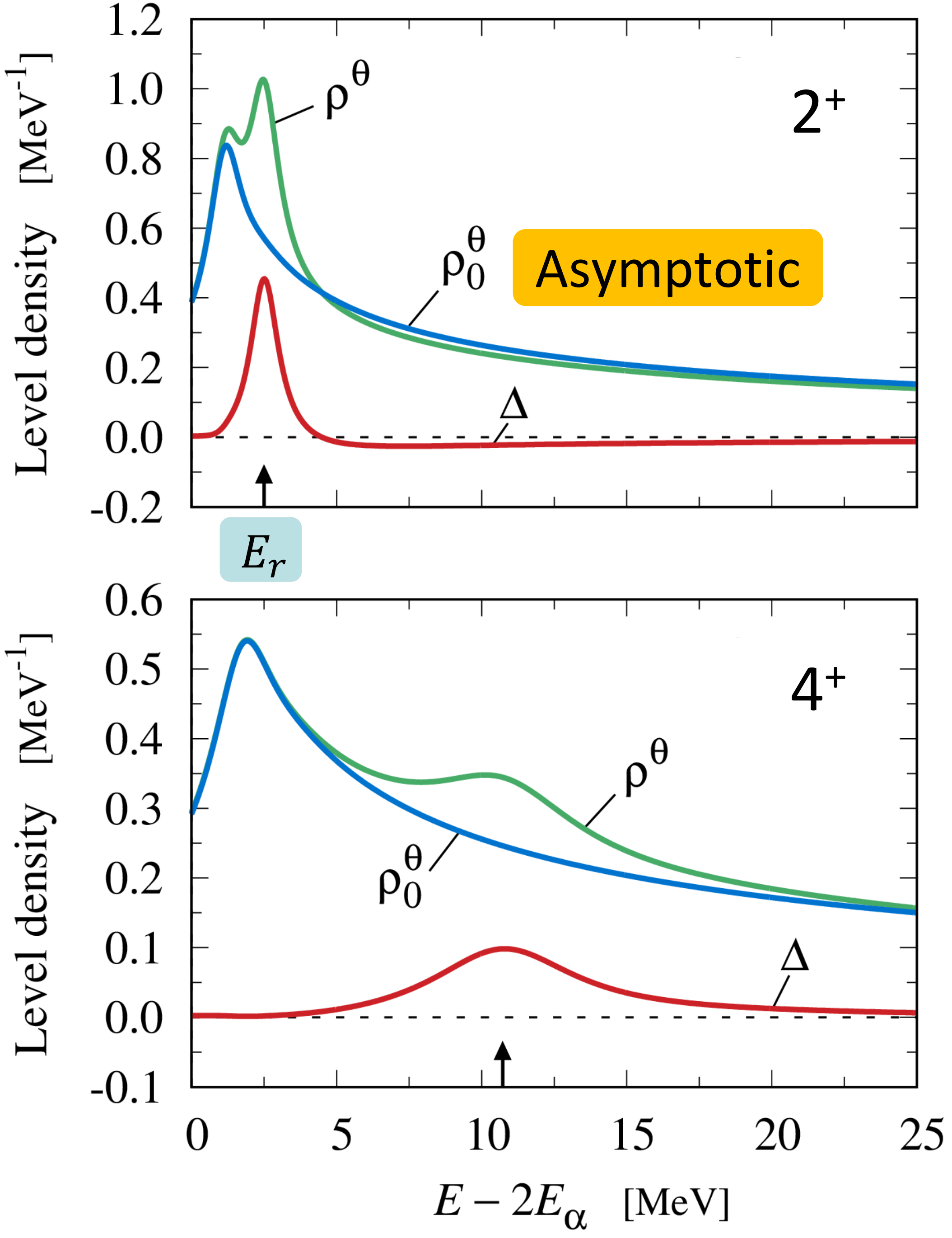
$\bar{H}^{\theta}$: Complex-scaled Hamiltonian

$\bar{H}_0^{\theta}$: Asymptotic Hamiltonian w/o nuclear force









^{8}Be, level density

Asymptotic

$$\Delta(E) = \rho(E) - \rho_0(E)$$

$$= \sum_i^N \delta(E - E_i) - \sum_i^N \delta\left(E - E_{0,i}\right)$$

$$= -\frac{1}{\pi}\,\mathrm{Im}\left(\sum_i^N \frac{1}{E - E_{\theta,i}}\right) + \frac{1}{\pi}\,\mathrm{Im}\left(\sum_i^N \frac{1}{E - E_{0,\theta,i}}\right)$$

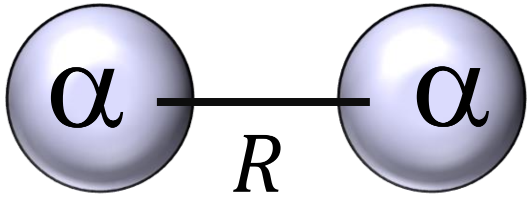
$$= \frac{1}{\pi}\frac{d\delta(E)}{dE}$$

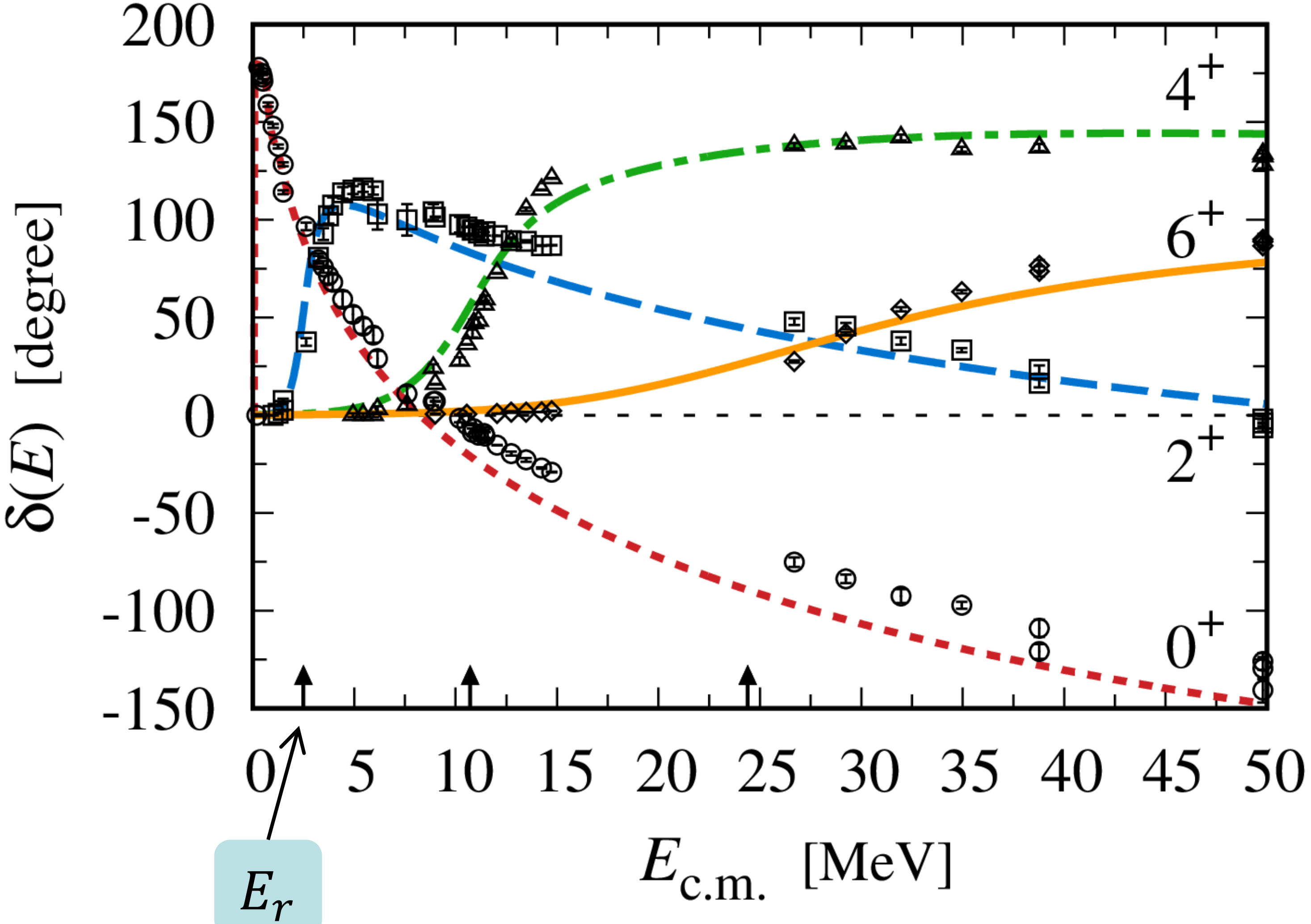
complex scaling

- Continuum level density reveals resonances
- Measured from α+α threshold energy

Myo, Kikuchi, Masui, Katō, Prog. Part. Nucl. Phys. **79**,1(2014)

α+α phase shifts





E_r

- α+α in Brink model with complex scaling $\theta = 27°$
- NO channel radius

$$\Delta(E) = \rho(E) - \rho_0(E)$$

$$= \frac{1}{\pi}\frac{d\delta(E)}{dE}$$

Myo, Takemoto, Phys. Rev. C 107 (2023) 064308

Strength function $S(E)$ in complex scaling

$$S(E) = \sum_n \langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_n \rangle \langle \tilde{\varphi}_n | \hat{O} | \Phi_0 \rangle \cdot \delta(E - E_n)$$

$\hat{O}$: Transition operator

initial state

final state

$$= -\frac{1}{\pi} \mathrm{Im}\left[R(E) \right]$$

$$R(E) = \sum_n \frac{\langle \tilde{\Phi}_0 | \hat{O}^\dagger | \varphi_n \rangle \langle \tilde{\varphi}_n | \hat{O} | \Phi_0 \rangle}{E - E_n}$$

Response function

- Complex-scaled Green's function

$$G^\theta(E) = \frac{1}{E - H_\theta} = \sum_n \frac{| \varphi_n^\theta \rangle \langle \tilde{\varphi}_n^\theta |}{E - E_n^\theta}$$

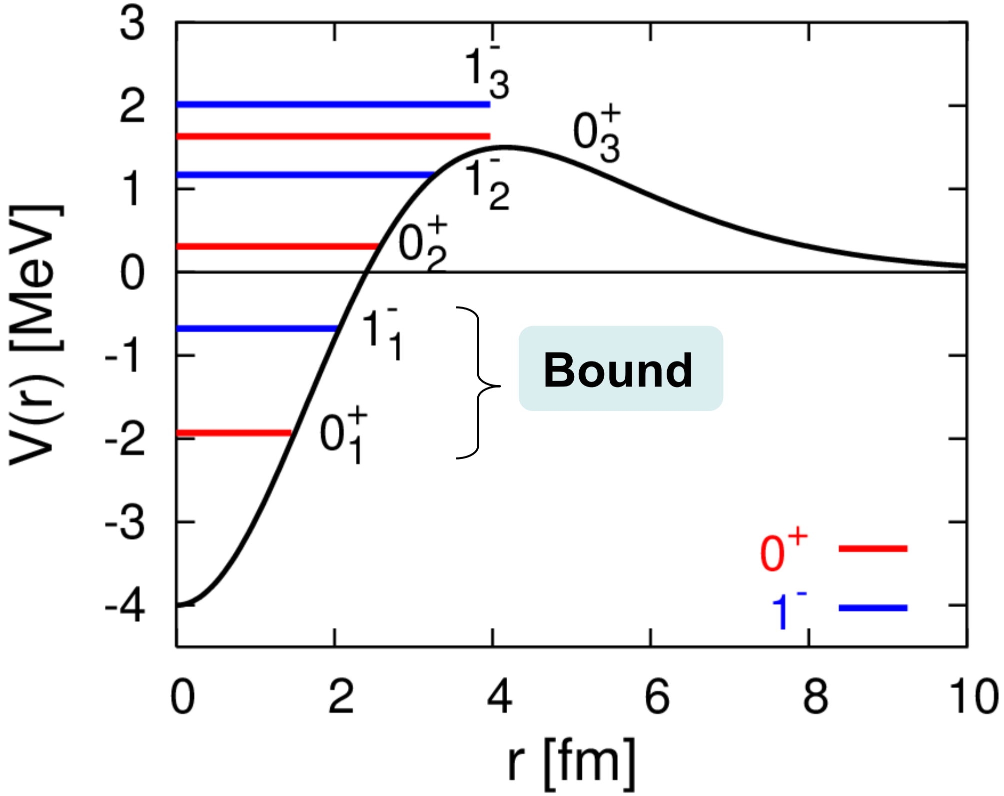
Bound+Resonance+Continuum

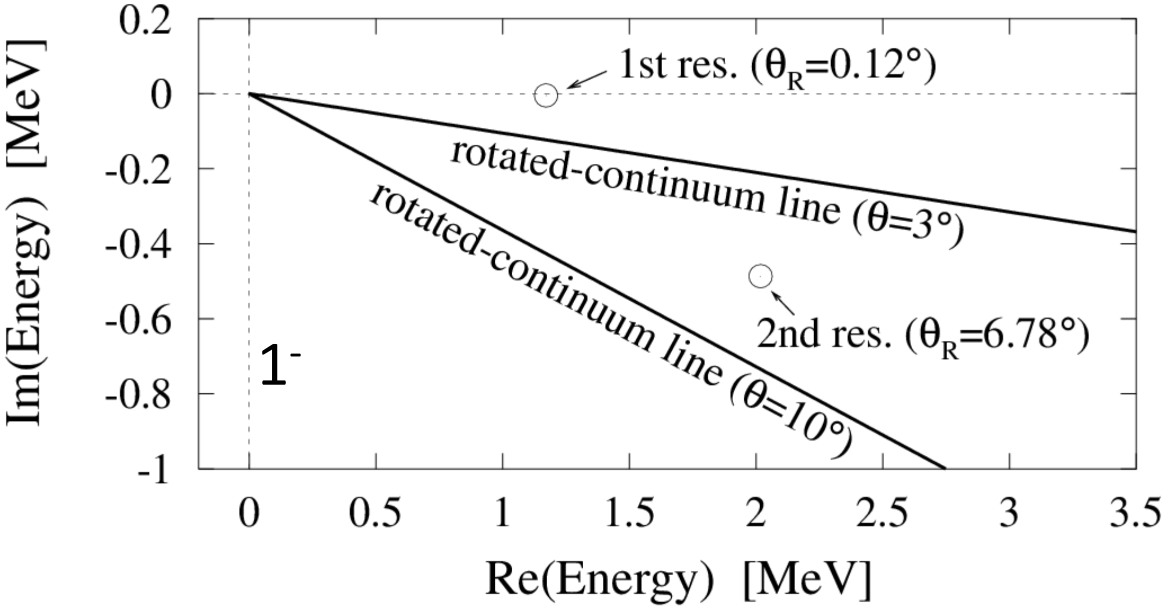
Reaction theory
- Lippmann–Schwinger-eq. (Kikuchi)
- CDCC (Matsumoto)
- Scatt. Amp. (Kruppa, Dote(K$^{\mathrm{bar}}$N))

Berggren, NPA109('68)265, Morimatsu, Yazaki, PPNP33('94),
TM, Ohnishi, Katō, PTP99('98)801

Dipole transition : Schematic potential

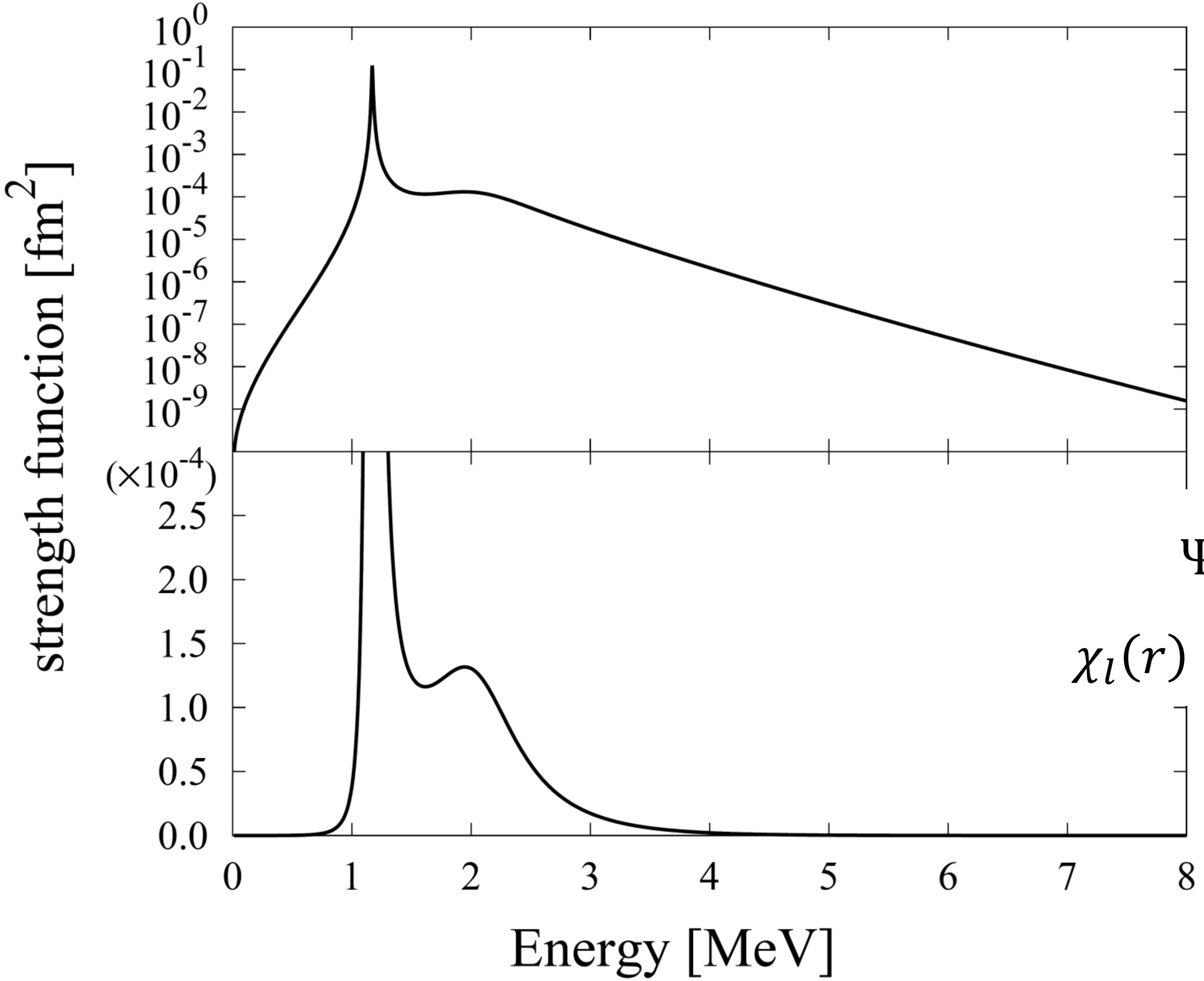
$$H = -\frac{\hbar^2\nabla^2}{2\mu} + V, \qquad \frac{\hbar^2}{\mu} = 1, \qquad V(r) = -8e^{-0.16r^2} + 4e^{-0.04r^2}$$





TM, A. Ohnishi and K. Katō, PTP99(1998)801

Dipole transition strength, $0^+ \rightarrow 1^-$



$$\hat{O}_m(E1) = rY_{1m}(\hat{\boldsymbol{r}})$$

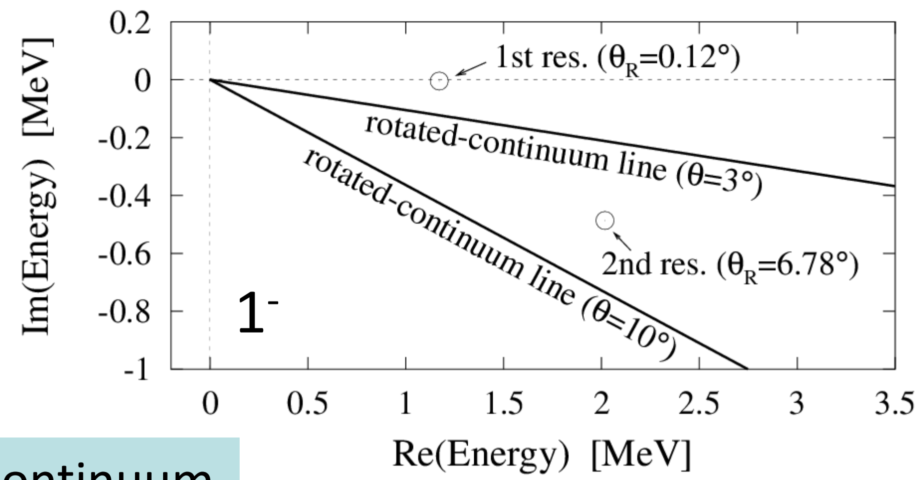
TM, A. Ohnishi and K. Katō,
PTP99(1998)801

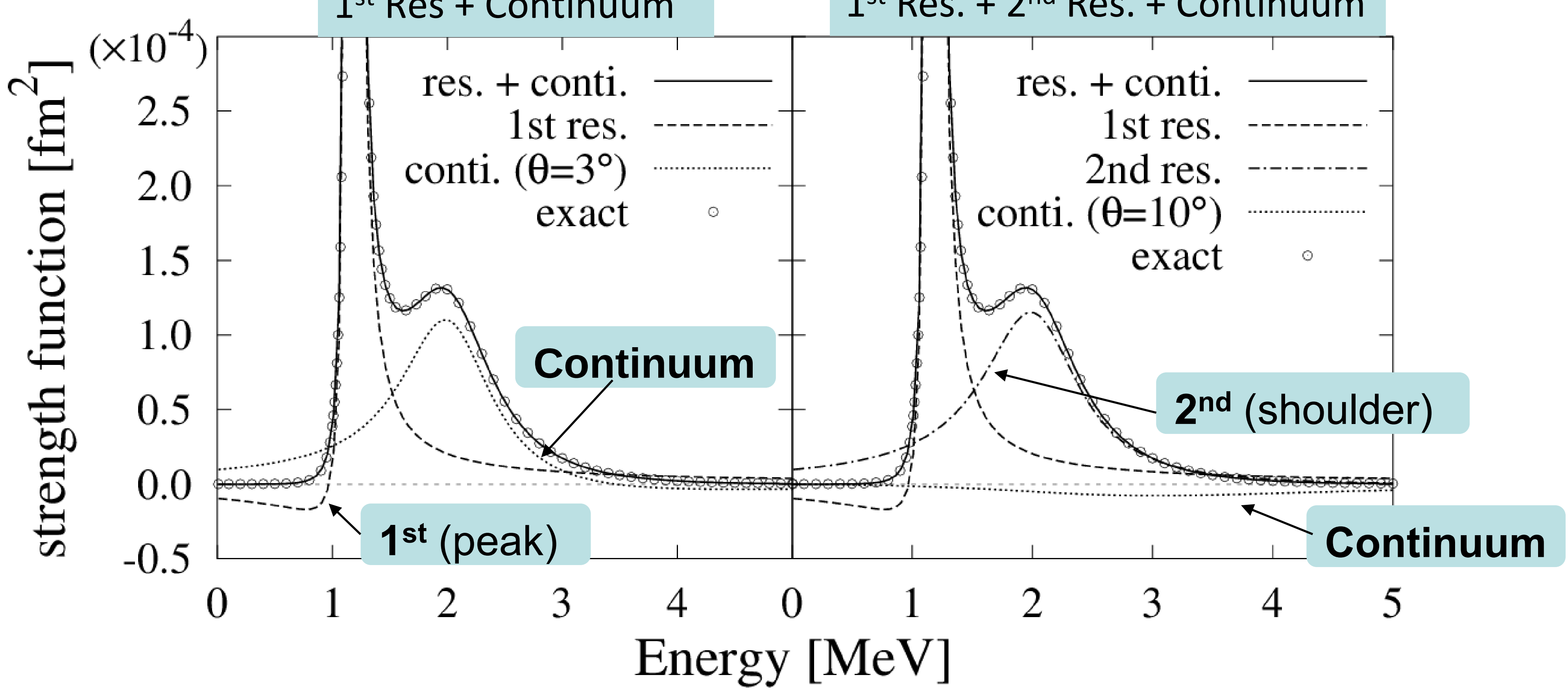
$$\Psi_{lm}(\boldsymbol{r}) = C_l(k)\frac{\chi_l(r)}{kr}Y_{lm}(\hat{\boldsymbol{r}})$$

$$\chi_l(r) \xrightarrow[r\rightarrow\infty]{} u_l^{(-)}(kr) - S_l(k)\,u_l^{(+)}(kr)$$

Solve scattering problem

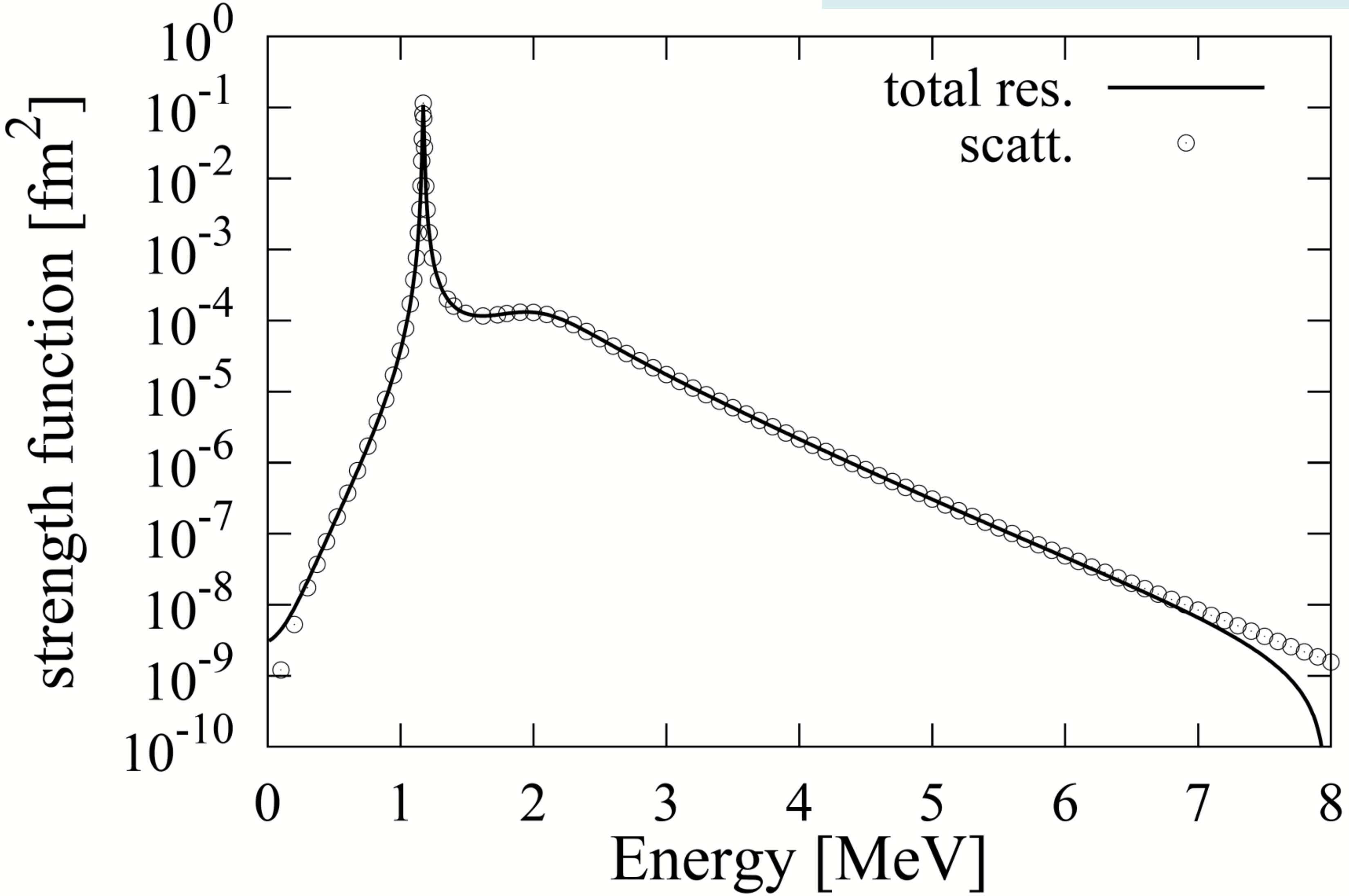
Dipole transition with extended completeness relation





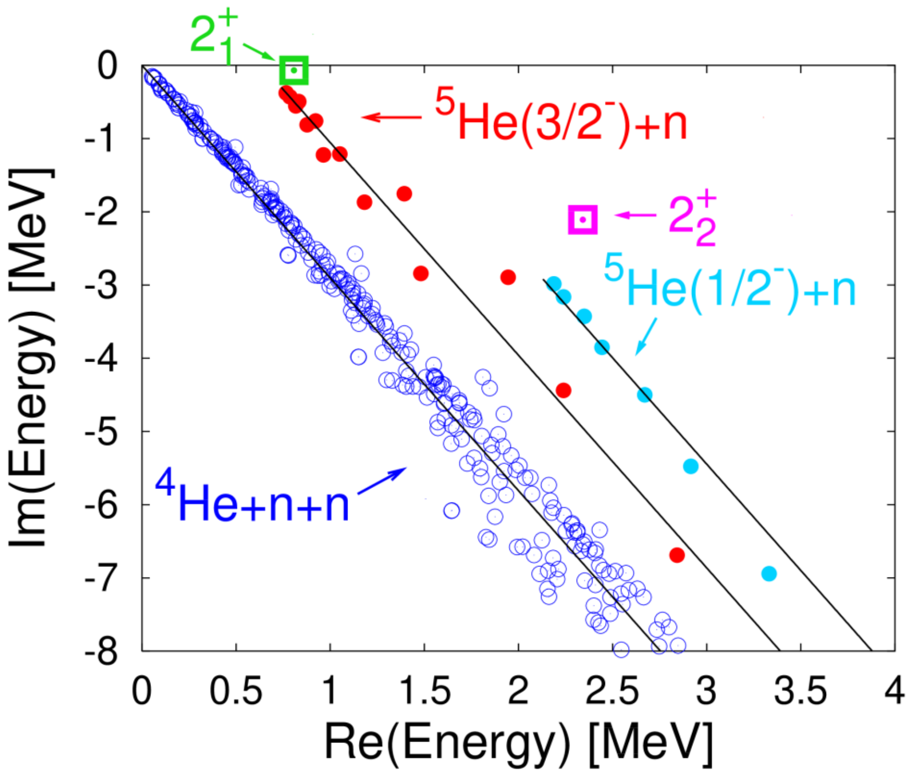
E1 strength from only resonances

9 resonances in 1^- state



Quadrupole transition of ^{6}He into α+n+n

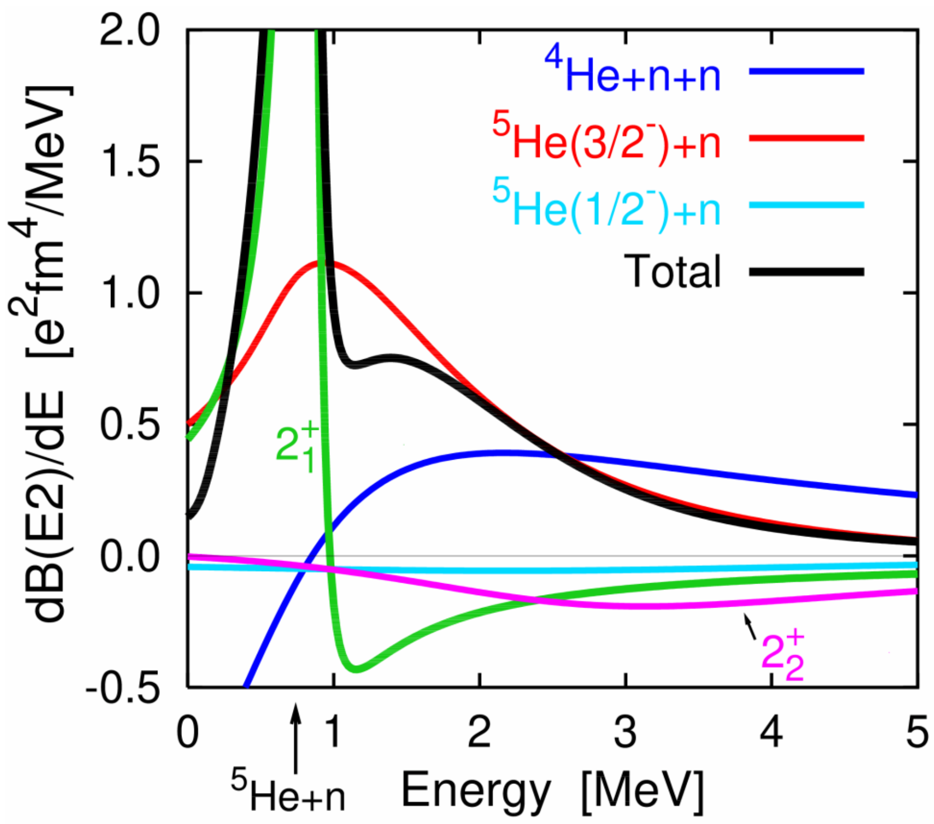
- Zero energy : α+n+n threshold



2^+

θ=35°

Transition by $er^2 Y_2(\hat{\boldsymbol{r}})$



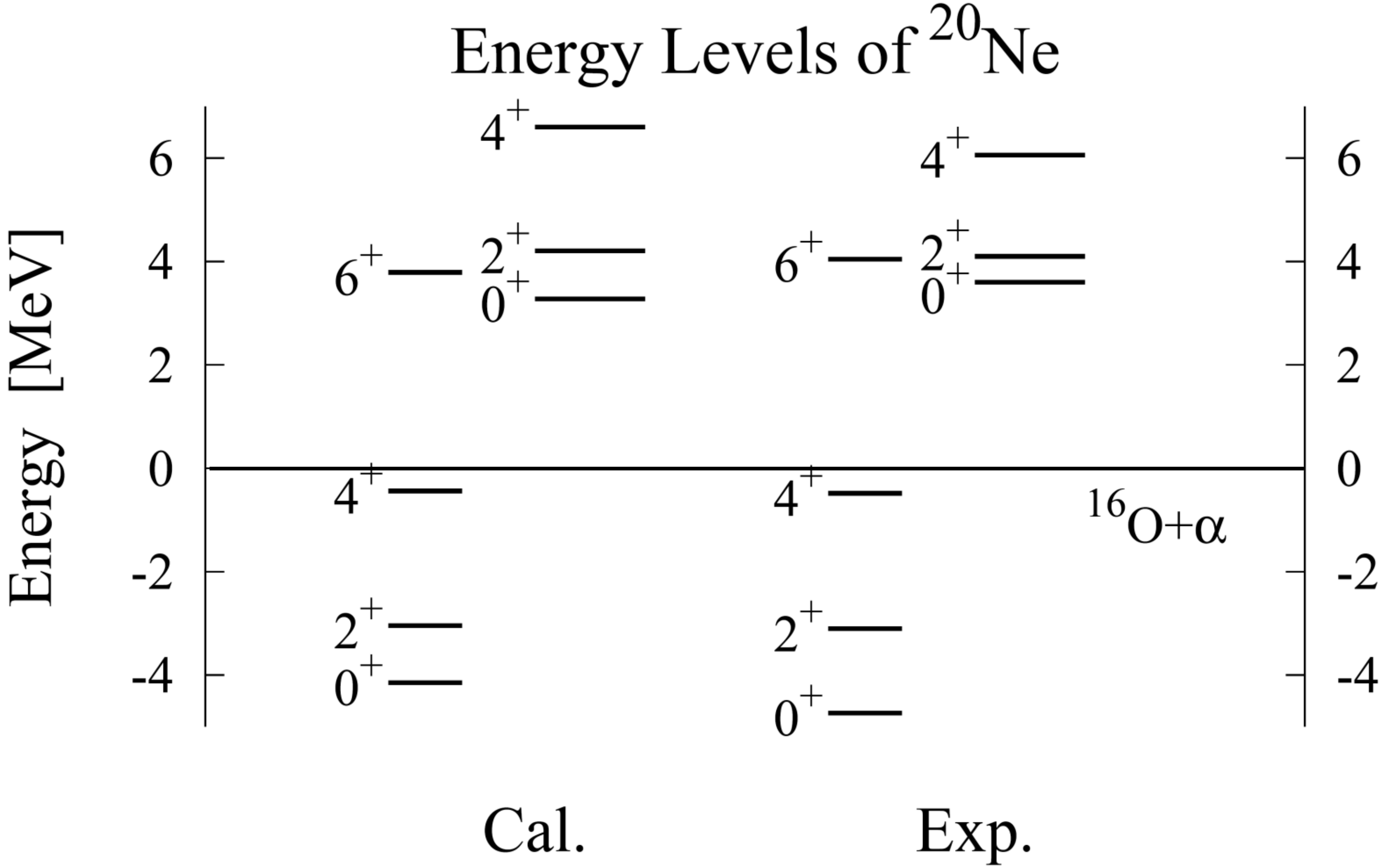
- Sharp peak : **1st resonance**
- Shoulder structure : **^{5}He+n continuum state**
- Minor effect : **2nd resonance**

T. Myo, S. Aoyama, K. Kato, K. Ikeda
PRC63 (2001) 054313

^{20}Ne as ^{16}O + ^{4}He, E2 transition

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_N + V_C, \quad V_N = V_0 e^{-a_N r^2}, \quad V_C = 8 \cdot 2 \frac{e^2}{r} \operatorname{erf}(a_C r)$$

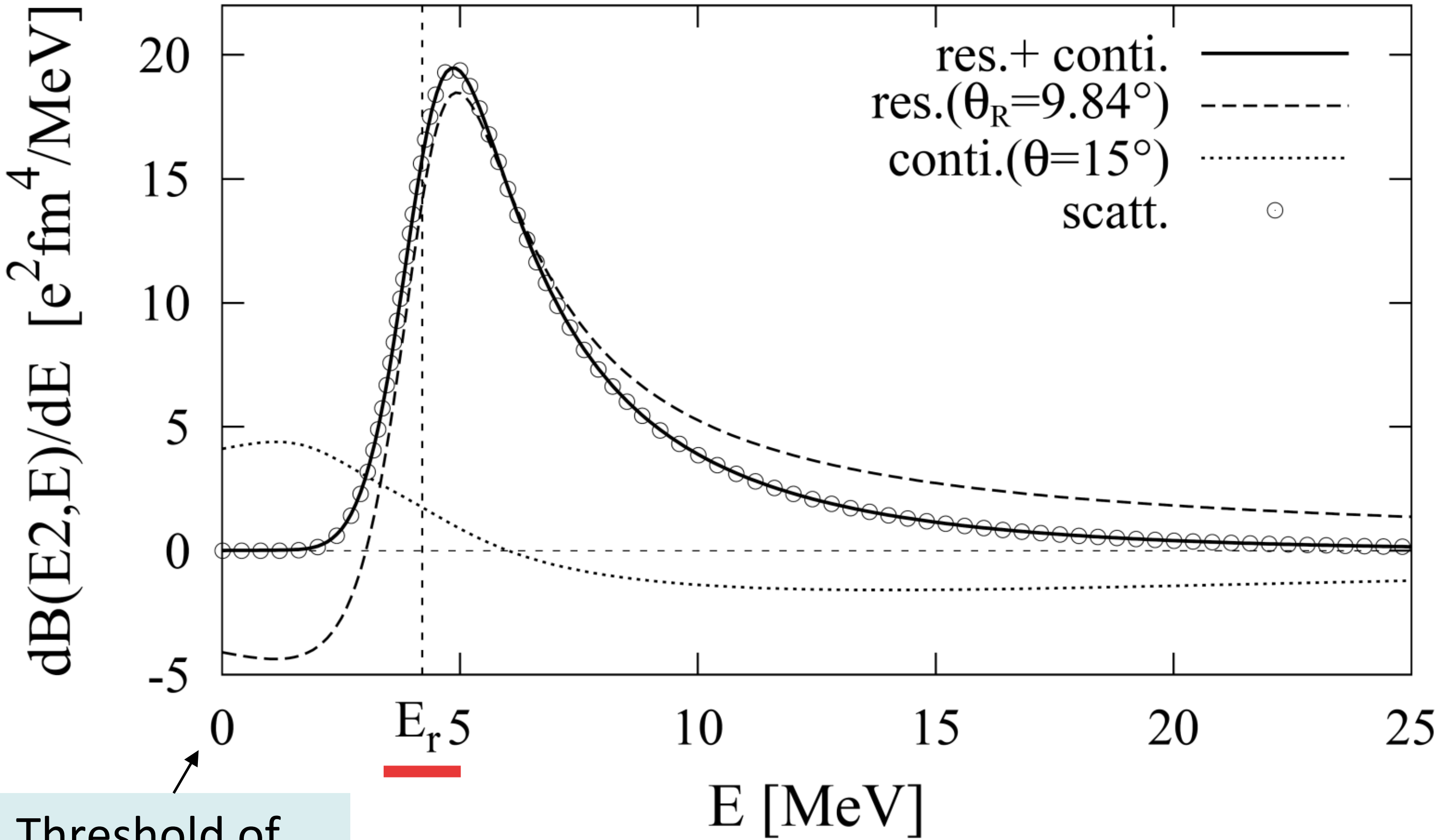
$$V_0 = -154 \text{ MeV}$$
$$a_N = 0.1102 \text{ fm}^{-2}$$
$$a_C = 0.4805 \text{ fm}^{-1}$$



TM, A. Ohnishi and K. Kato,
PTP99(1998)801

^{20}Ne = ^{16}O + ^{4}He, E2 transition

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_N + V_C, \quad V_N = V_0 e^{-a_N r^2}, \quad V_C = 8 \cdot 2\frac{e^2}{r}\mathrm{erf}(a_C r)$$



Threshold of ^{16}O + ^{4}He

Peak position is shifted from resonance energy E_r due to :

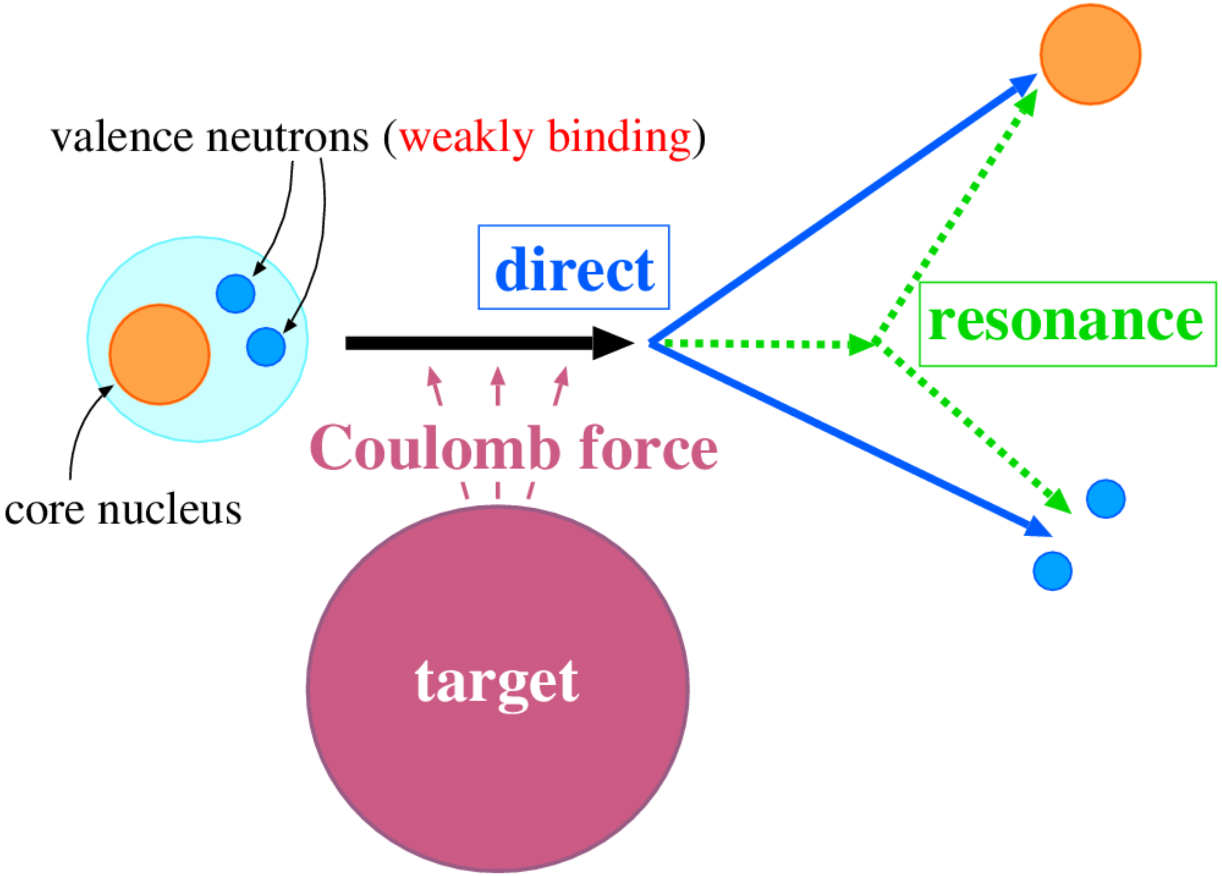
1) Imaginary part of the transition matrix element of resonance
2) Continuum component as background

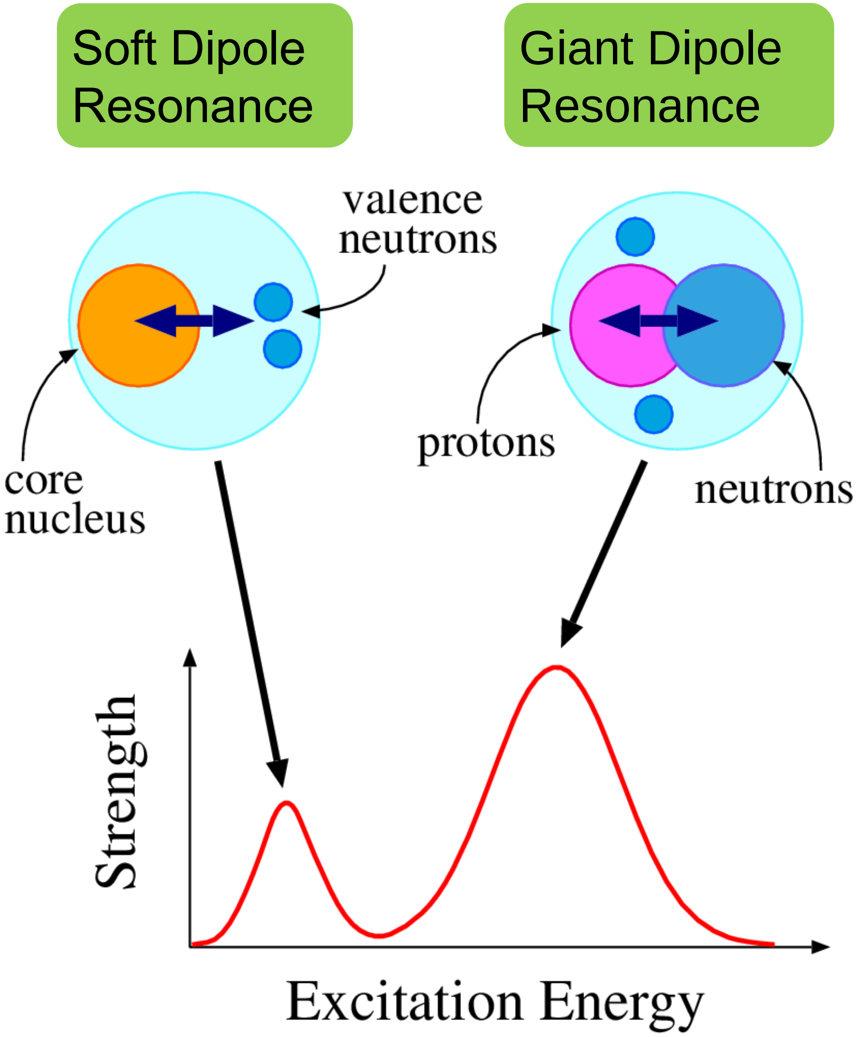
Strength function $S(E)$: properties

- $E_\alpha = E_R - iE_I$: complex energy of state α, $E_R, E_I > 0$
- $M_\alpha = \langle \Phi_\alpha | O | \Phi_0 \rangle^2 = M_R + iM_I$: transition matrix element to the state α
- $S_\alpha(E) = -\frac{1}{\pi} \mathrm{Im}\left(\frac{M_\alpha}{E-E_\alpha}\right) = -\frac{1}{\pi} \mathrm{Im}\left(\frac{M_R+iM_I}{E-E_R+iE_I}\right) = \frac{1}{\pi} \frac{M_R E_I - M_I(E-E_R)}{(E-E_R)^2+E_I^2}$
- $\int_{-\infty}^{\infty} S_\alpha(E) dE = M_R$: Strength of the distribution
- M_I : Asymmetry of the distribution measured from E_R
- $S_\alpha(E_R) = \frac{1}{\pi} \frac{M_R}{E_I}$ at central energy
- $S_\alpha(E_R \pm E_I) = \frac{1}{\pi} \frac{M_R \pm M_I}{2E_I}$: width of the distribution
- $E_{\mathrm{max/min}} = E_R + \frac{E_I}{M_I}(M_R \mp |M_\alpha|)$: Max/Min energies of $S_\alpha(E)$

Myo, Katō, PRC 107 (2023)

Coulomb breakup reactions & soft dipole resonance

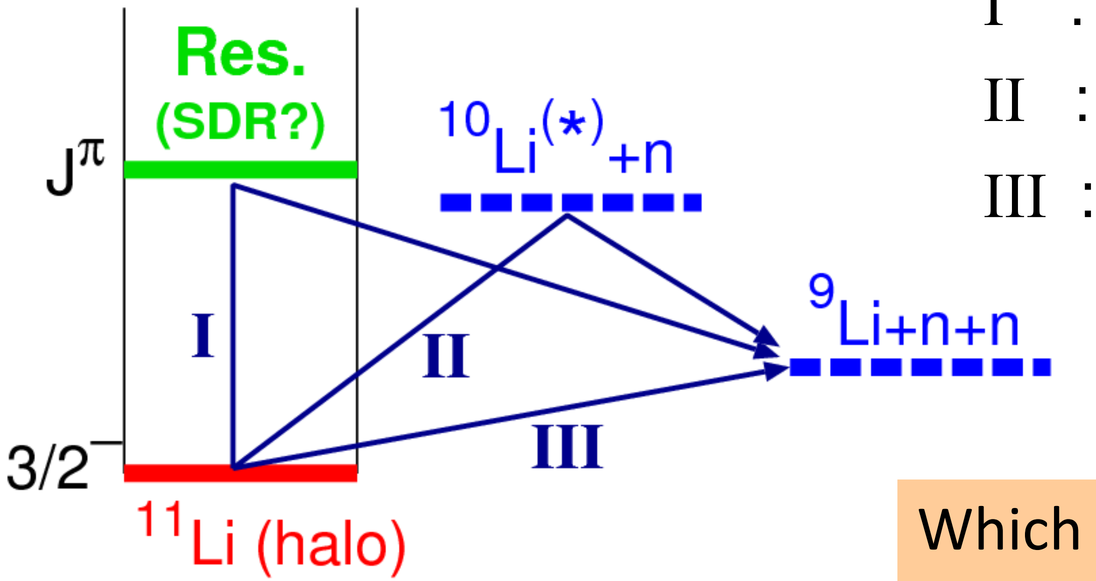




P.G. Hansen and B.Jonson, Europhys. Lett. 4(1987)409.
K. Ikeda, NPA538(1992)355c.
R. Kanungo, TM et al., PRL114 (2015) 192502

Coulomb breakup reaction

- Structure and Responses of unbound states
  - Resonance (Gamow states) and Non-resonant continuums
- Interesting excitation : soft dipole resonances?



I : resonance of ^{11}Li

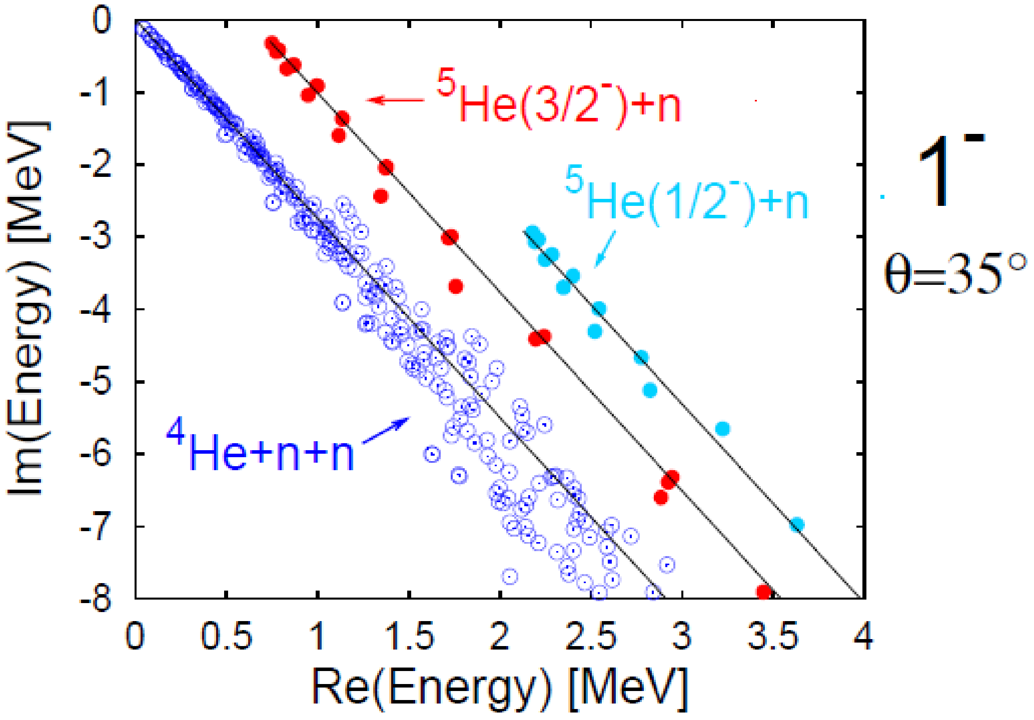
II : sequential via ^{10}Li+n

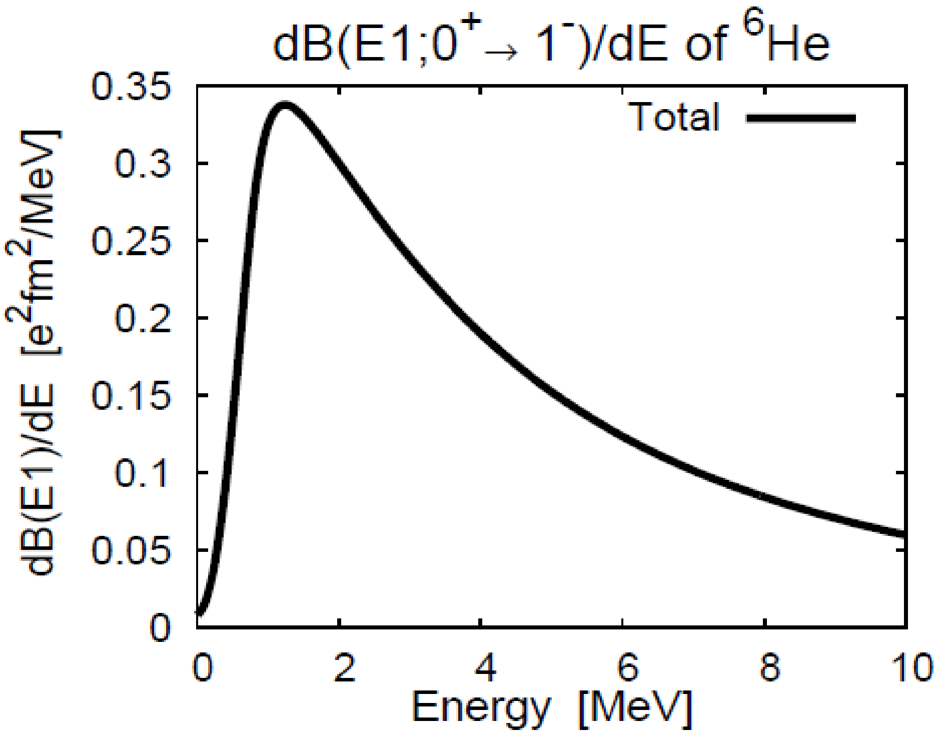
III : 3-body direct breakup

Which breakup process is preferred and constructs the structures of strength?

(1) Case of ^{6}He : E1 transition with CSM

- ○ $\Psi(^6\mathrm{He}) = \mathcal{A}\{\ \Phi(^4\mathrm{He})\ \Phi(nn)\ \}$ with OCM, $\Phi(nn)$: Gaussian expansion (discretized continuum)
- ○ ^{4}He-n : KKNN pot. / n-n : Minnesota.



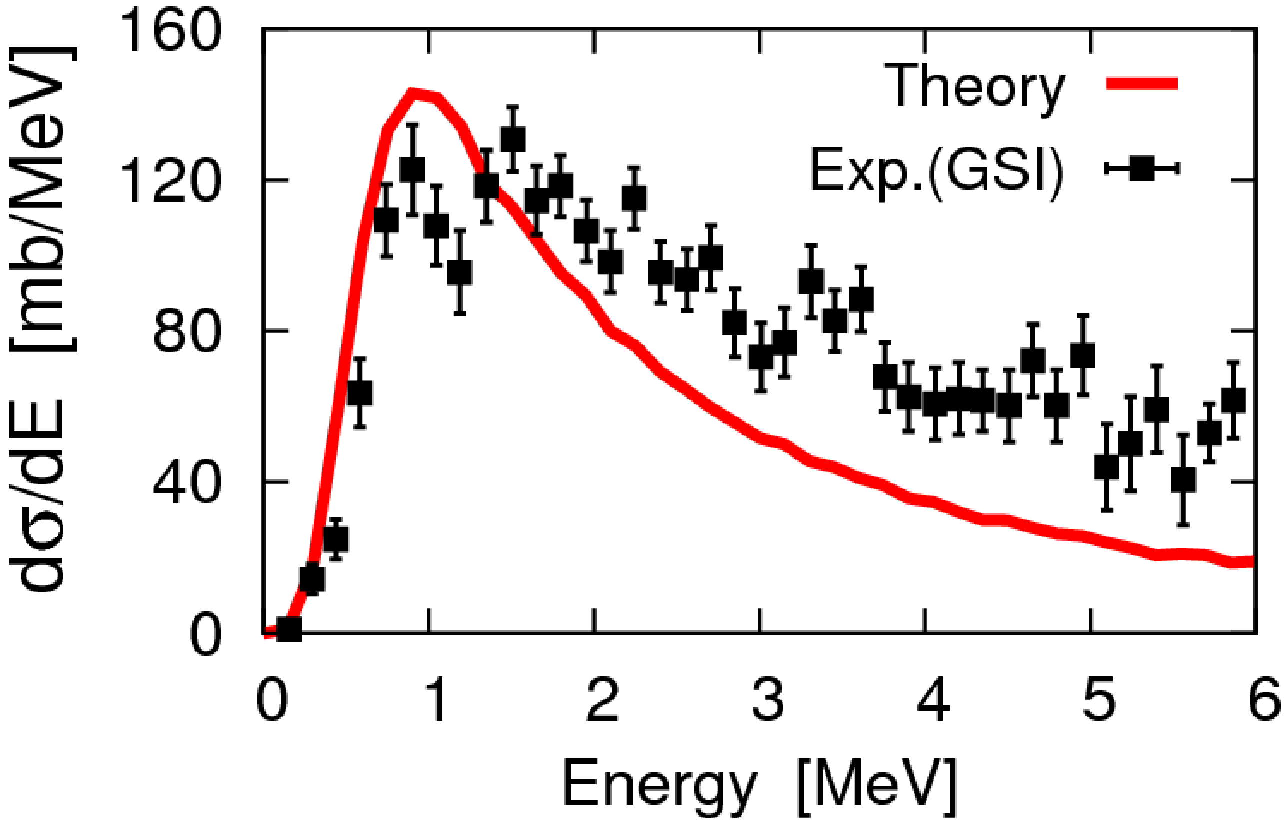


No three-body resonance

- Low energy enhancement

Coulomb breakup strength of ^{6}He

$^{6}He \rightarrow \alpha+n+n$



E1+E2 (complex scaling)
Equivalent photon method

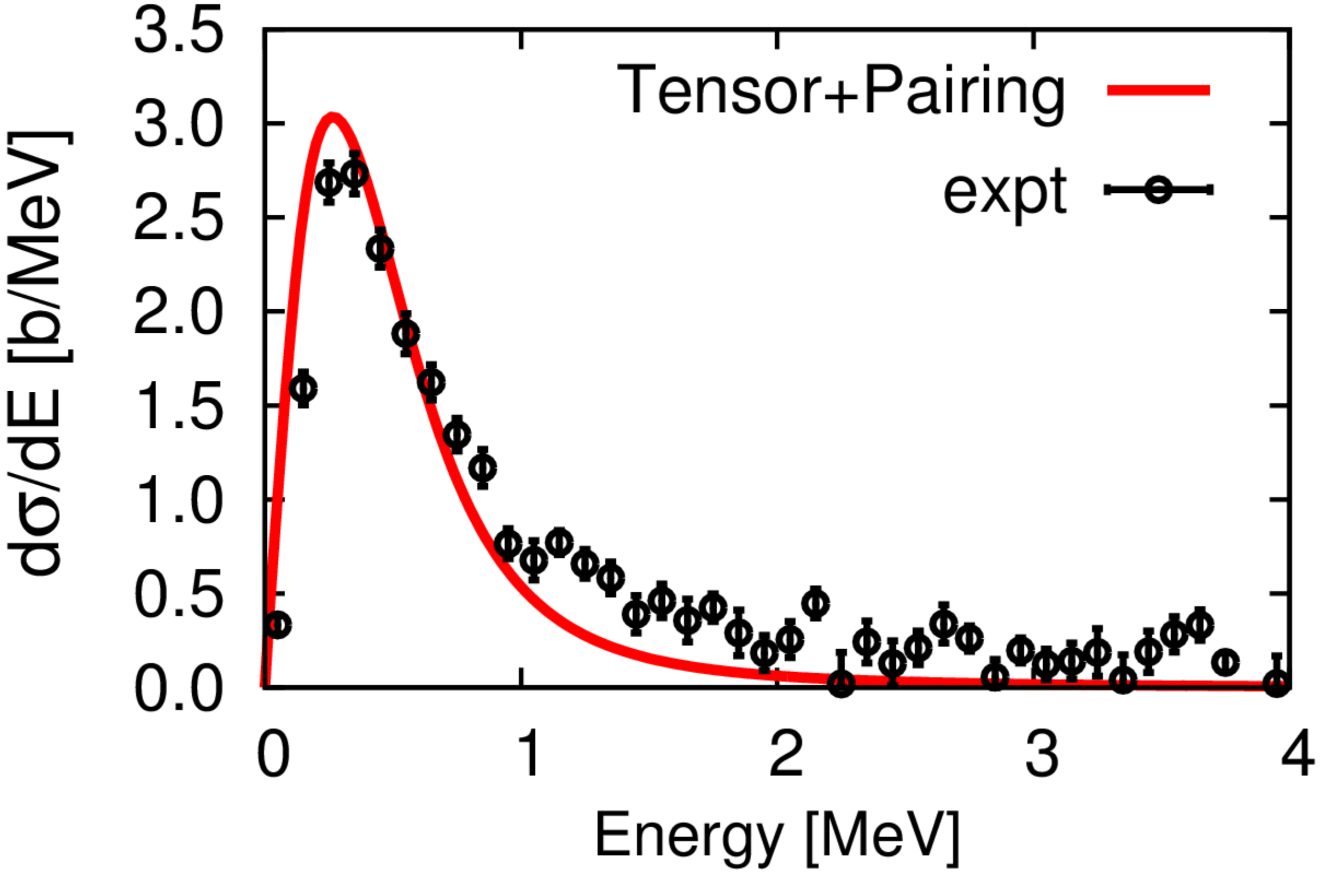
TM, K. Katō, S. Aoyama and K. Ikeda
PRC63(2001)054313.

Kikuchi, TM, Takashina, Katō, Ikeda
PTP122(2009)499
PRC81(2010)044308.
(invariant mass of α-*n* & *n*-*n*)

^{6}He : 240MeV/A, Pb Target (T. Aumann et.al, PRC59(1999)1252)

Coulomb breakup strength of halo nucleus ^{11}Li

$^{11}\text{Li} \rightarrow {}^{9}\text{Li+n+n}$



No three-body resonance

E1 strength

\+ **Complex scaling**

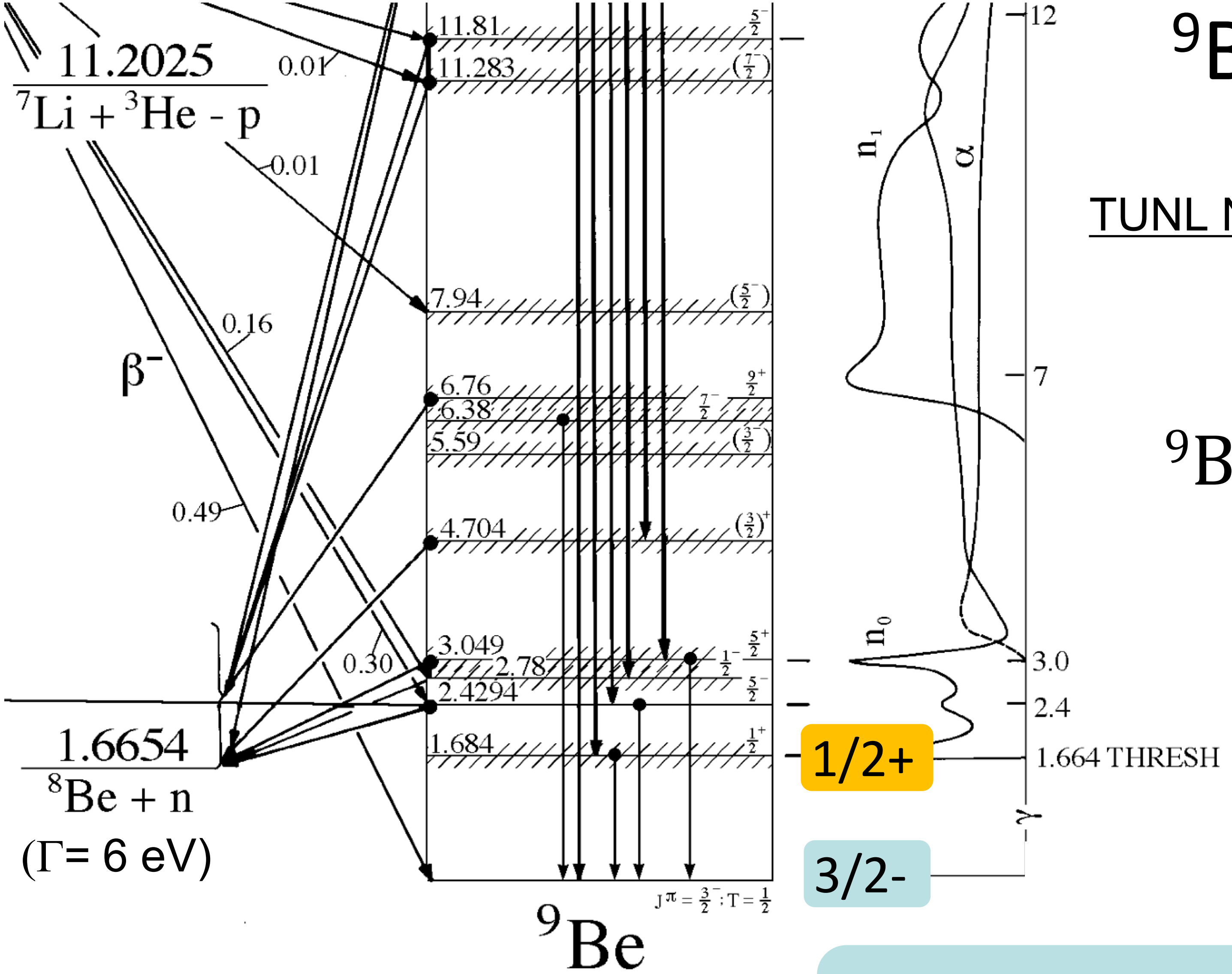
\+ Equivalent photon method

T.Myo, K.Katō, H.Toki, K.Ikeda
PRC76(2007)024305

- Expt: T. Nakamura et al. , PRL96,252502(2006)
- Energy resolution with $\sqrt{E}$ =0.17 MeV.

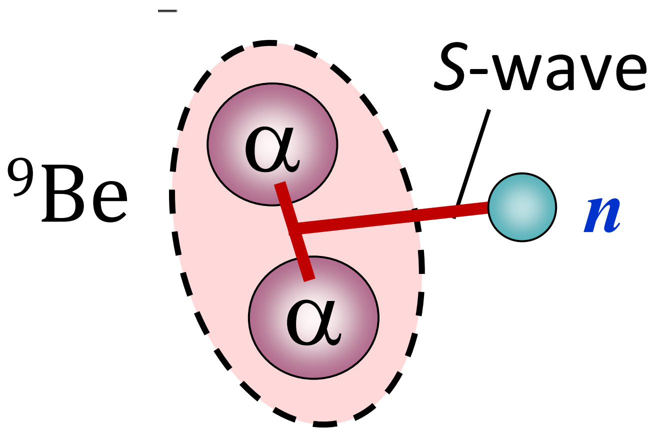
^{9}Be photodisintegration

TUNL Nuclear Data evaluation



1.5736 (MeV)

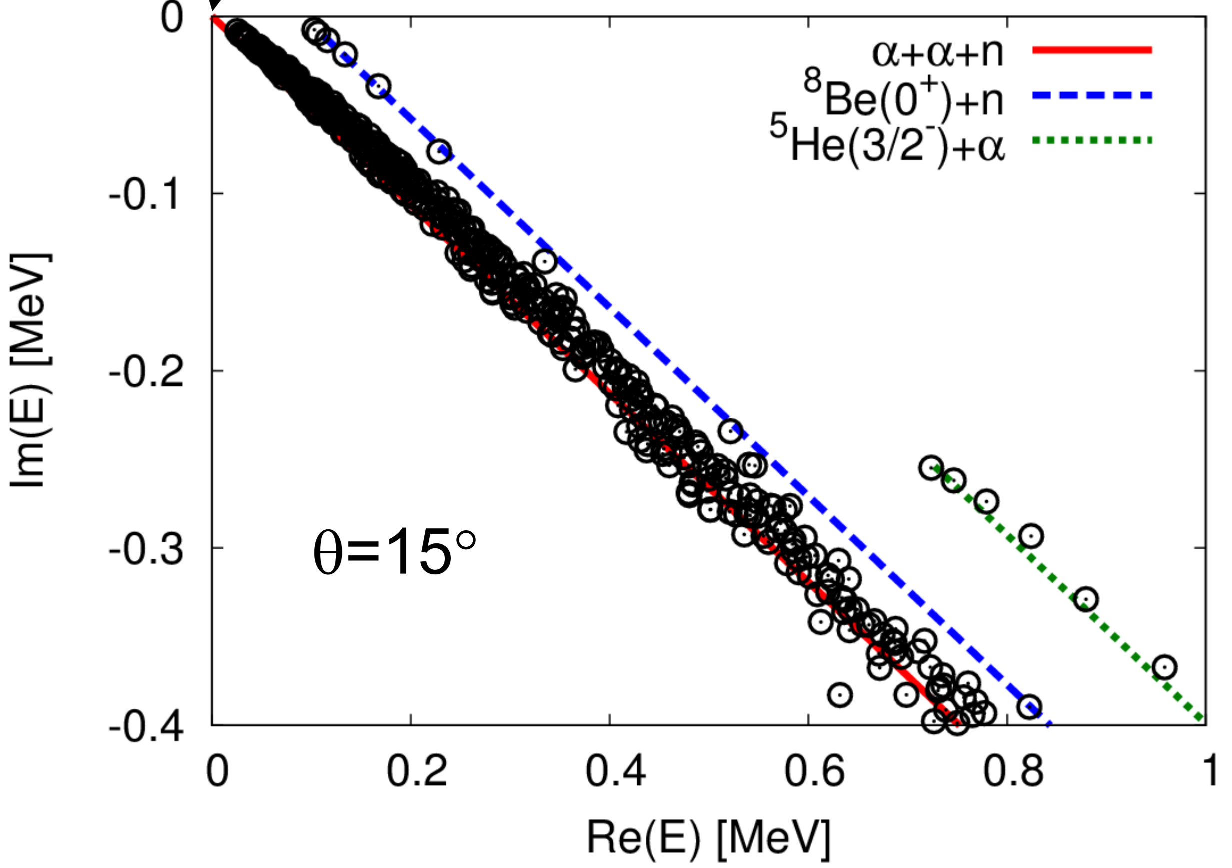
^{4}He + ^{4}He + n



- Property of 1/2^{+} S-state - resonance ? / virtual state ?
- Photodisintegration with α+α+n 3-body approach

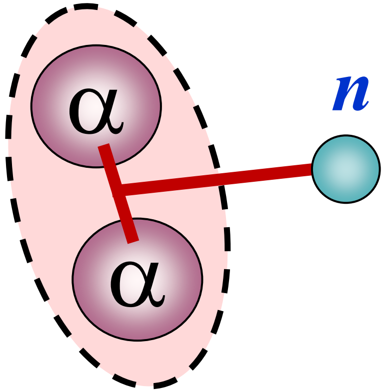
^{9}Be : Property of $1/2^+$ state

α+α+n threshold energy



α+α+n 3-body model (OCM+CSM)

No 3-body resonance

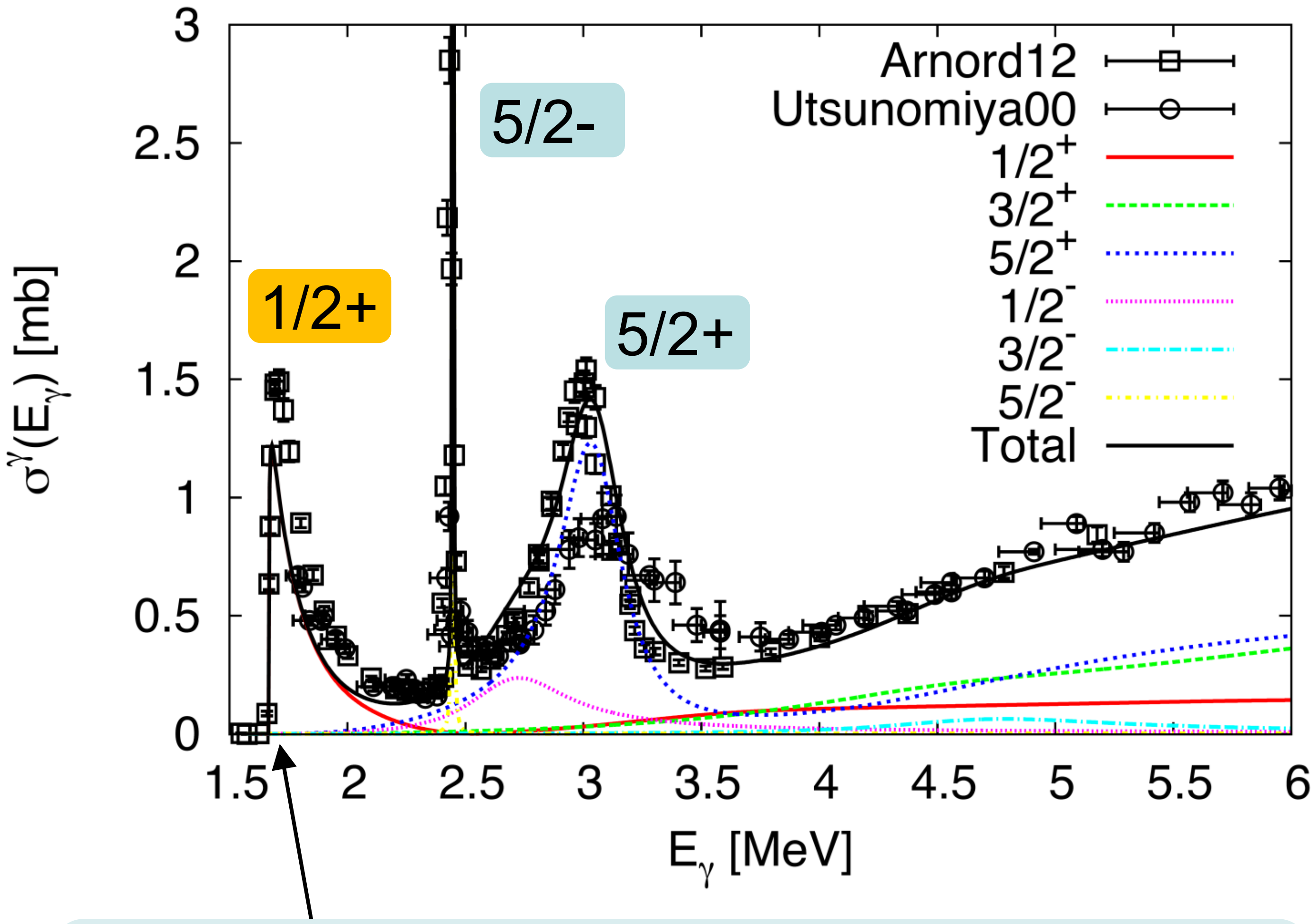


$$H\left({}^9\mathrm{Be}\right) = T_R + T_r + V_{\alpha_1 n} + V_{\alpha_2 n} + V_{\alpha\alpha} + V_{\mathrm{PF}} + V_3$$

3-body potential
(1 range Gaussian
with hyper-radius)

M. Odsuren, Y. Kikuchi, T. Myo, M. Aikawa and K. Kato, Physical Review C (2015) 014322

^{9}Be : Photodisintegration to α+α+n

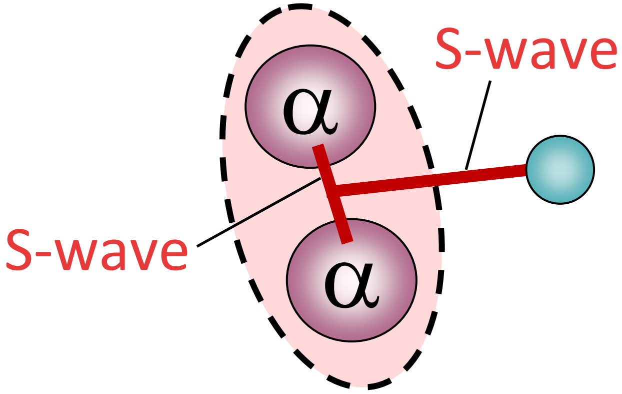


Low-lying 1/2$^+$ enhancement suggests the ^{8}Be+n **S-wave virtual state** above the α+α+n threshold.

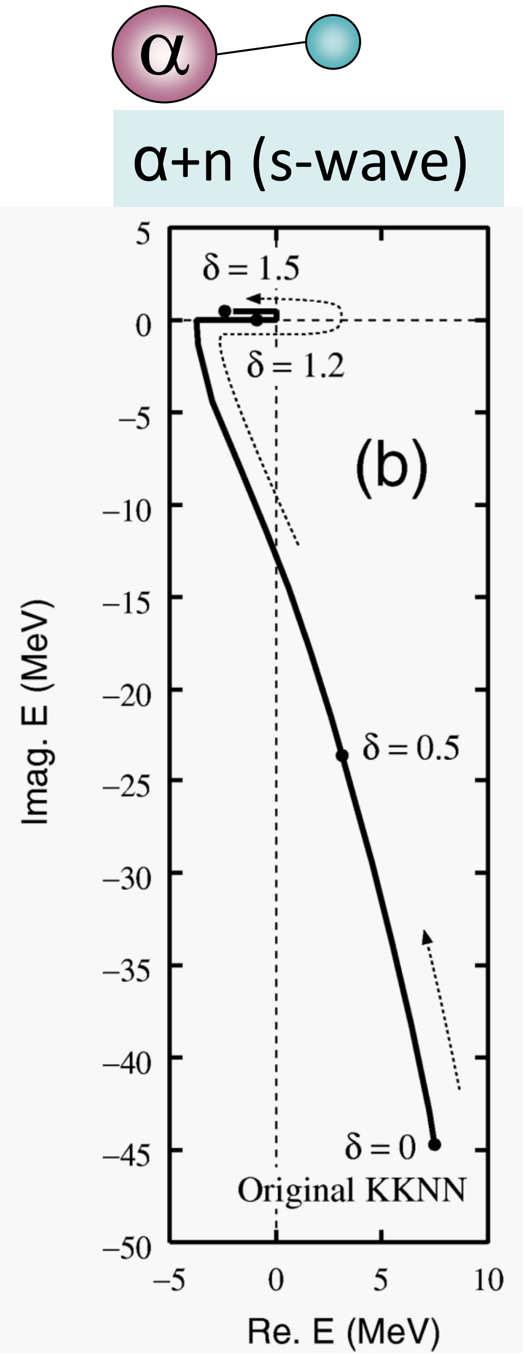
Reaction rate of α(αn,γ)^{9}Be in supernova

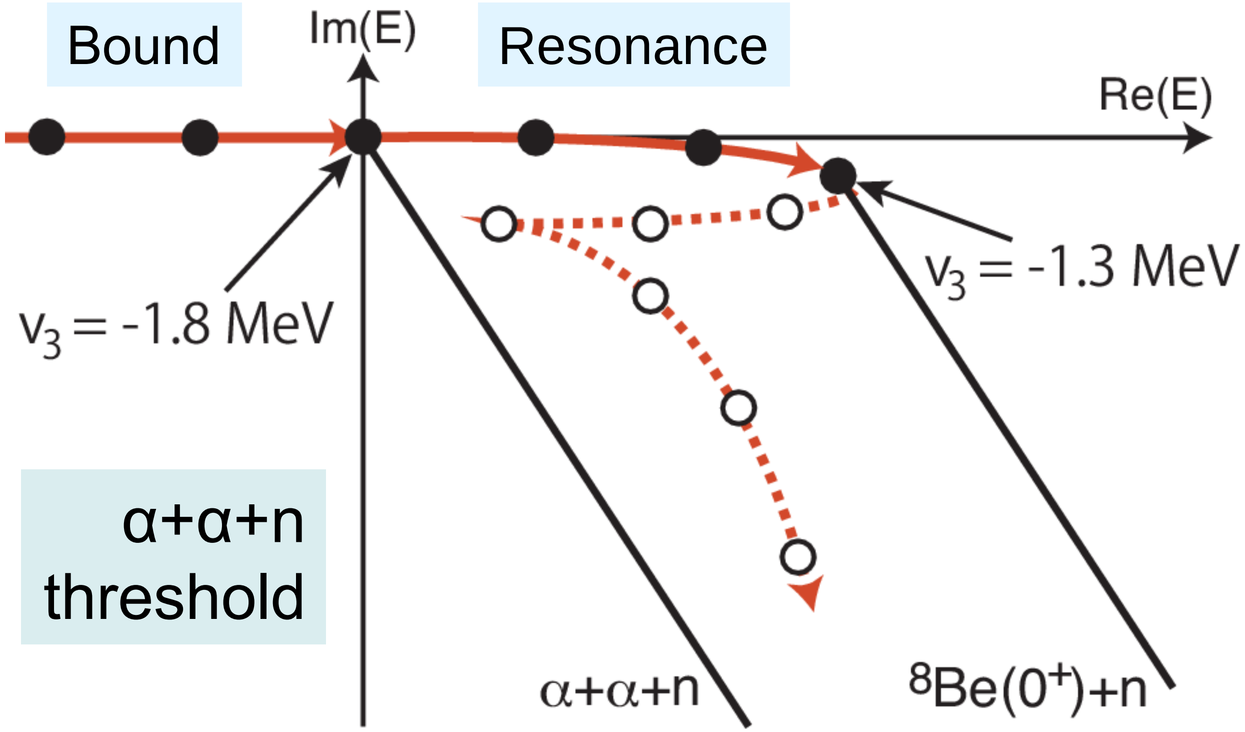
α+α+n OCM

Kikuchi, Katō, Odsuren, T.Myo, Aikawa
PRC 93 (2016) 054605



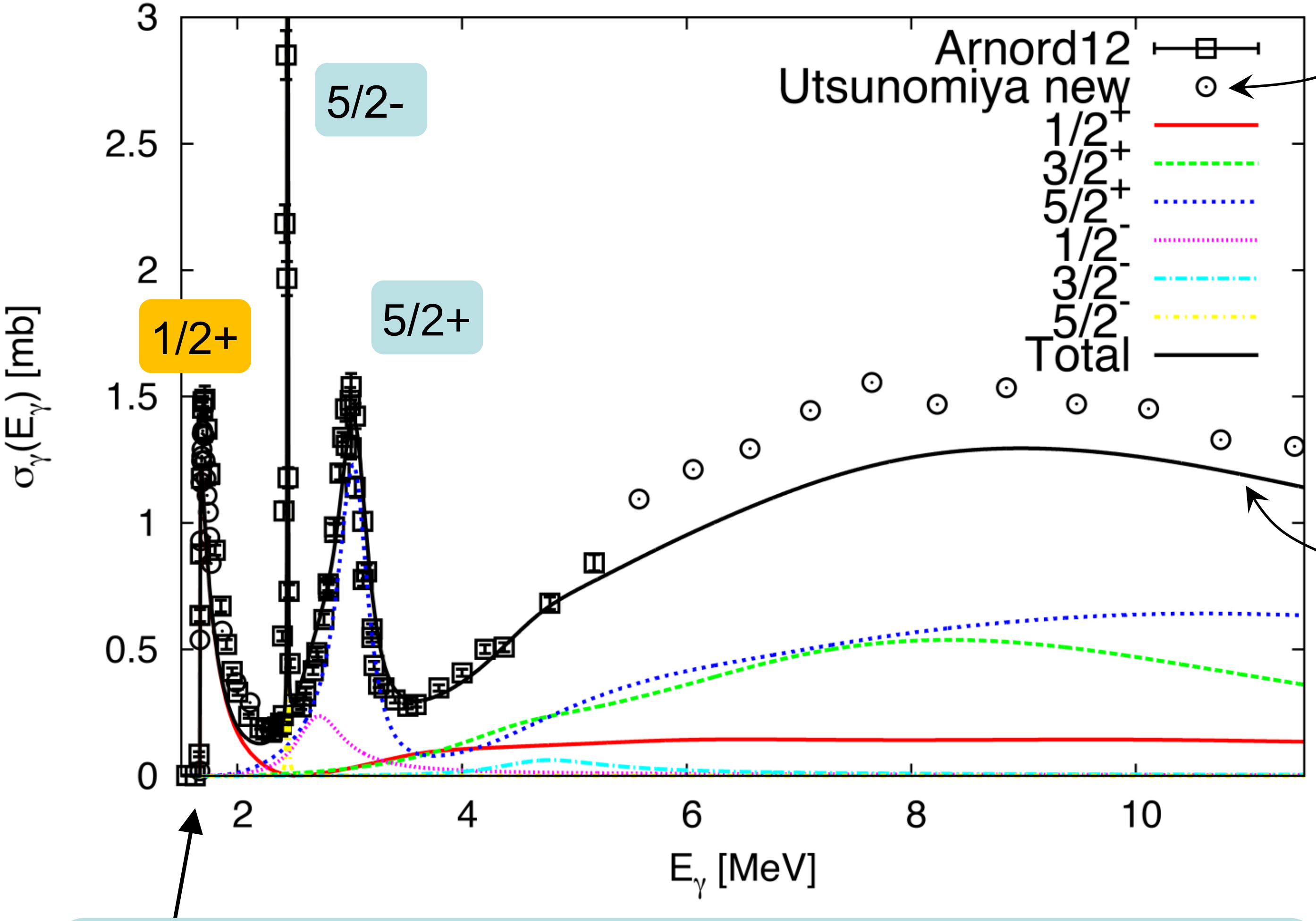
Pole trajectory of ^{9}Be 1/2$^+$ by analytical continuation of interaction





S-wave pole in α+n, ^{9}Li+n with Jost function method
H. Masui, S. Aoyama, T. Myo, K. Kato, K. Ikeda, Nucl. Phys. A 673 (2000) 207.

^{9}Be : Photodisintegration to α+α+n



PRC**92**(2015)064323

α+α+n OCM

Kikuchi, Katō, Odsuren, TM, Aikawa
PRC **93** (2016) 054605

Continuum contribution
makes bump structure

GDR E_γ ~ 30 MeV

Low-lying $1/2^+$ enhancement suggests the ^{8}Be+n **S-wave virtual state** above the α+α+n threshold.

Soft dipole resonance in ^{8}He and ^{8}C

- Physical Review C104 (2021) 044306 Energy spectra of ^{8}He and ^{8}C
- Physical Review C106 (2022) L021302 Soft dipole mode of ^{8}He (letter)
- Prog. Theor. Exp. Phys. 2022 (2022) 103D01 Soft dipole resonance in ^{8}He (full paper)
- Physical Review C107 (2023) 034305 Soft dipole resonance in ^{8}C

Ikeda’s idea on Soft Dipole Resonance

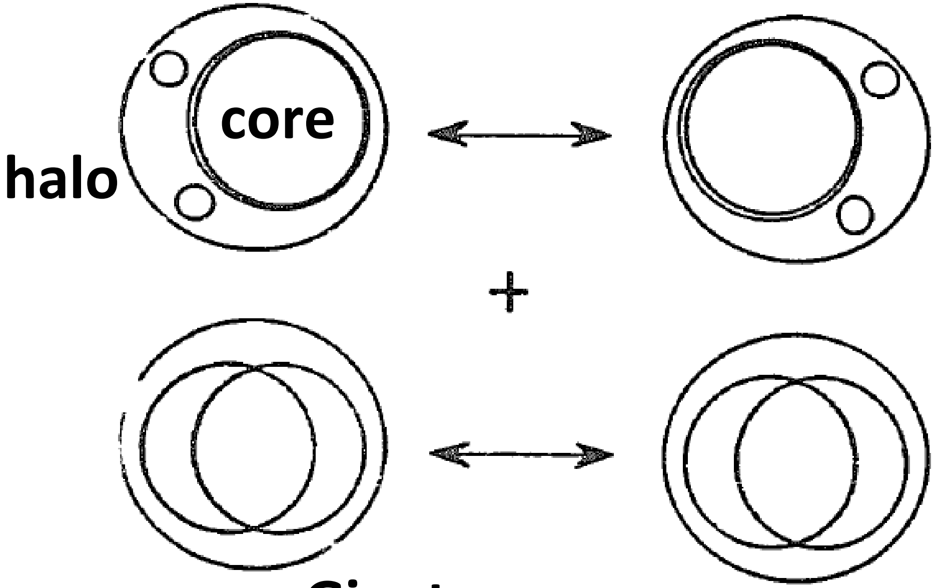
STRUCTURE OF NEUTRON RICH NUCLEI
—A Typical Example of the Nucleus ^{11}Li—

Kiyomi IKEDA 池田 清美

NPA538 (1992) 355

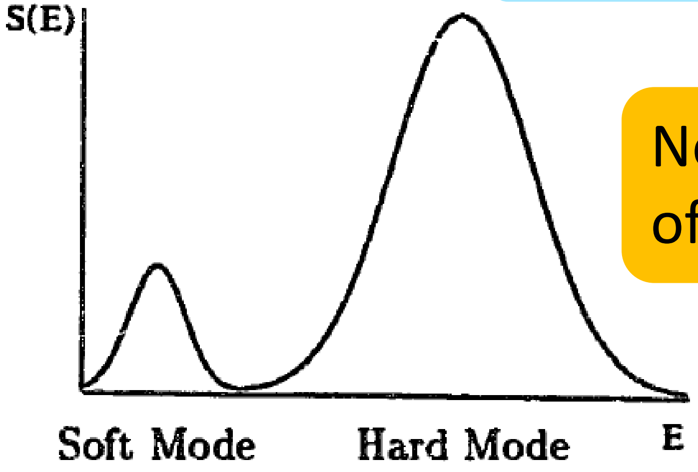
Dipole oscillation of valence neutrons against core

P.G. Hansen and B. Jonson, Europhys. Lett. **4** (1987) 409.
^{11}Li=^{9}Li+dineutron



Giant resonance

E1 strength

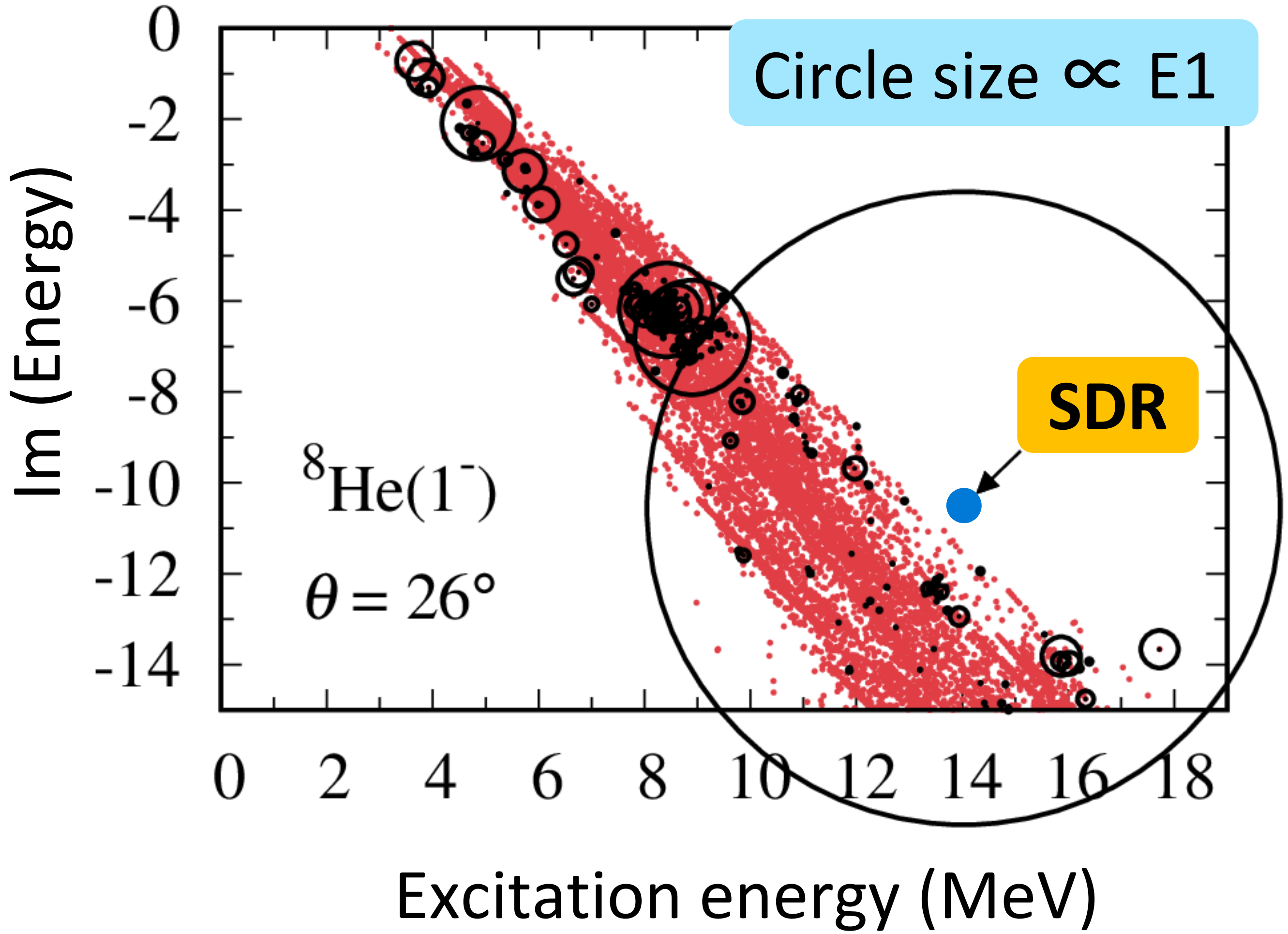


New collective excitation of multi-neutrons

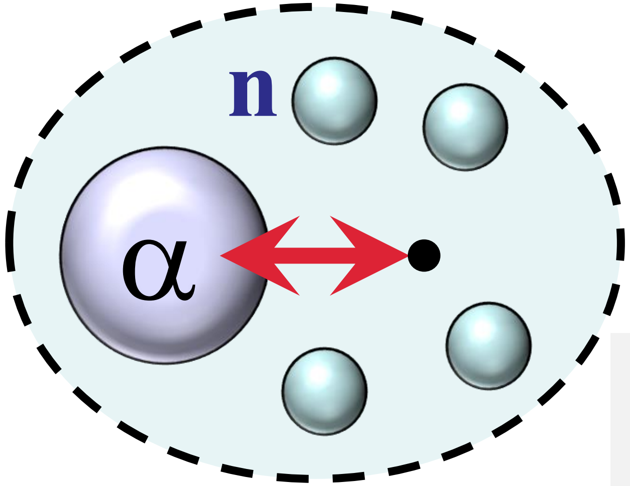
^{8}He : Candidate of Soft Dipole Resonance

complex scaling

E1 transition (pole) = 0.55 (e^2fm^2)



- Cluster Sum-Rule Value $B_c(E1) = 1.01\ e^2\mathrm{fm}^2$
- $(E_x, \Gamma) = (14, 21)$ MeV
- α-4n distance : extended to 2.7 fm
- Possibility of SDR of $^5_\Lambda$He + 4n



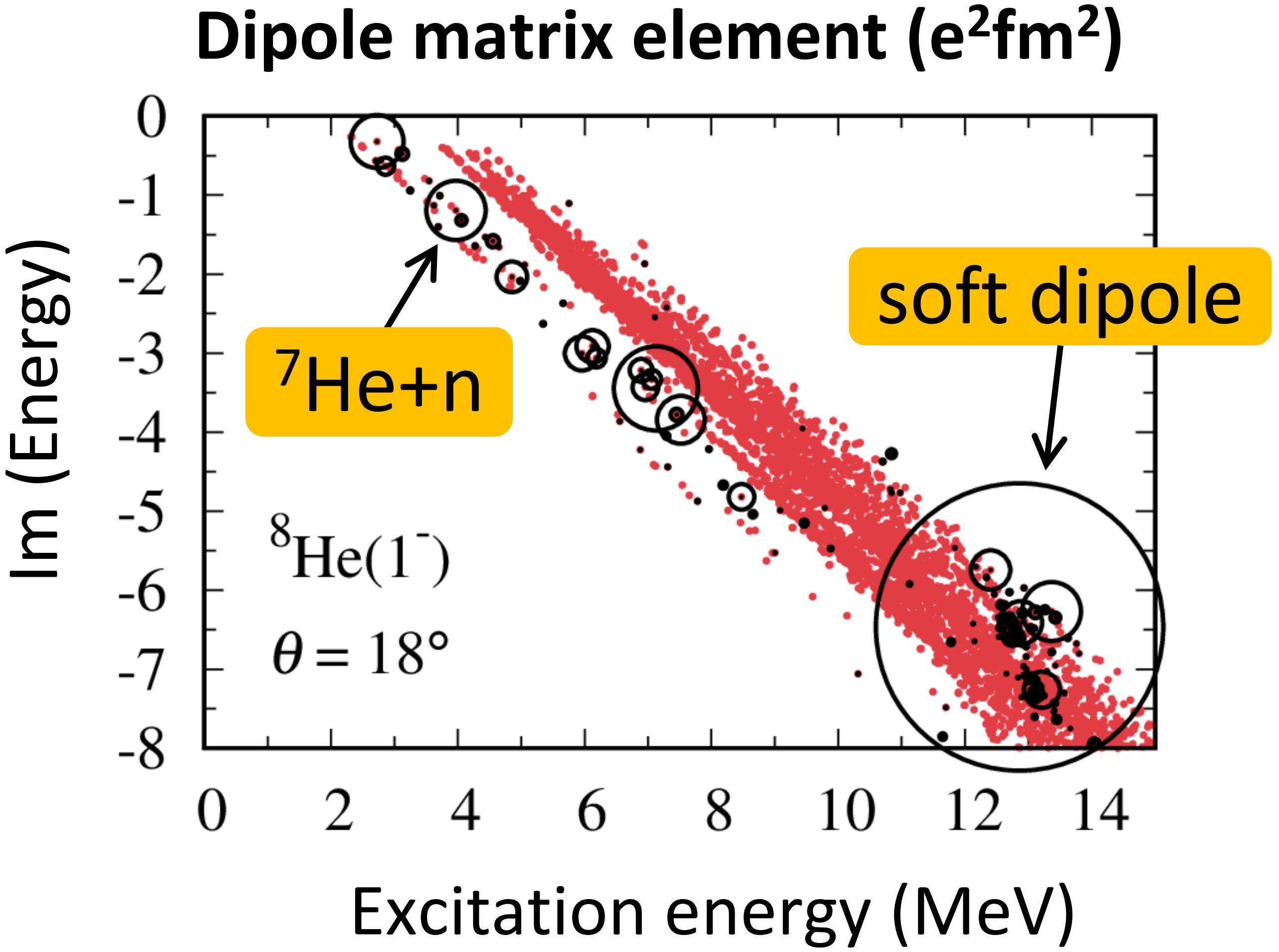
Myo, Odsuren, Katō, PTEP2023

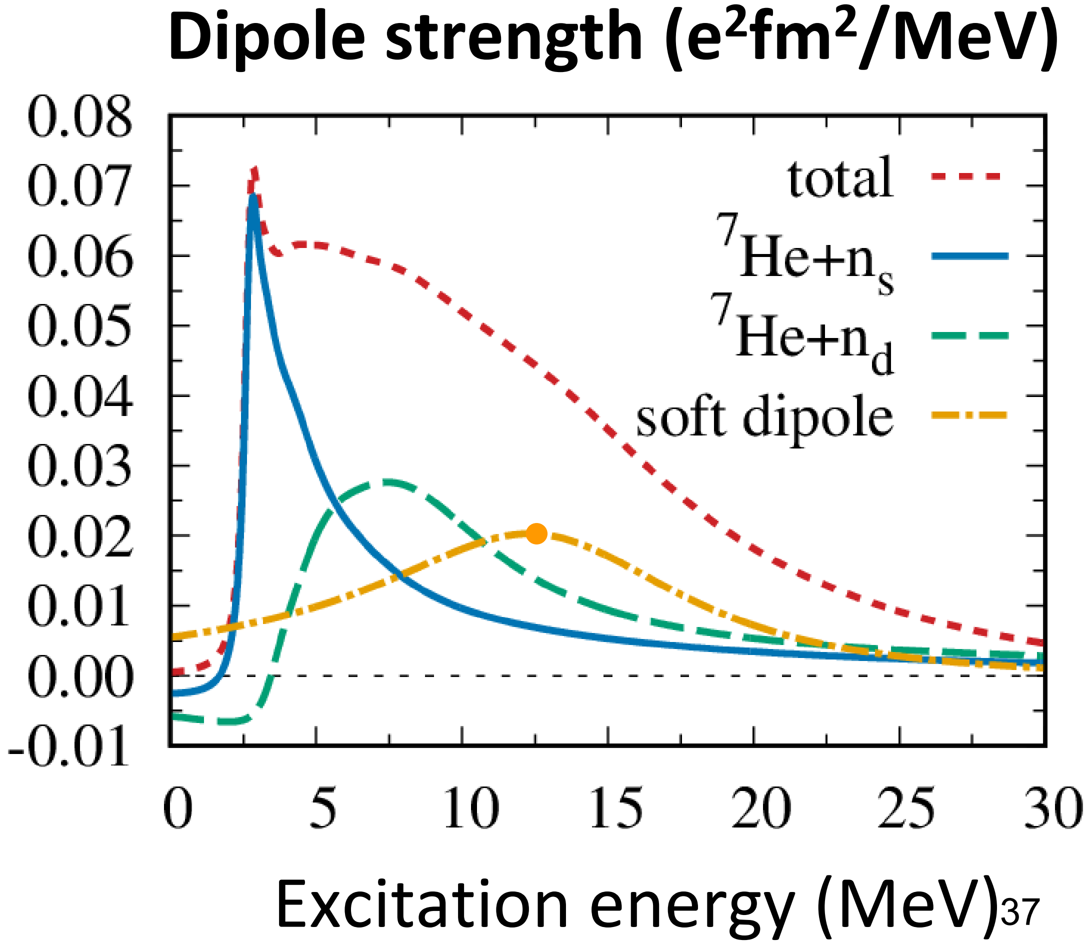
Dipole strength function of ^{8}He

- Zero energy : $^8\mathrm{He}_{g.s.}$

Circle size ∝ E1

complex scaling





PhD Thesis by Christopher Lehr (TU Darmstadt)

- Title : Low-energy dipole response of the halo nuclei ^{6,8}He
- Coulomb breakup experiment of ^{8}He with Pb target with the help of C target (SAMURAI 37)

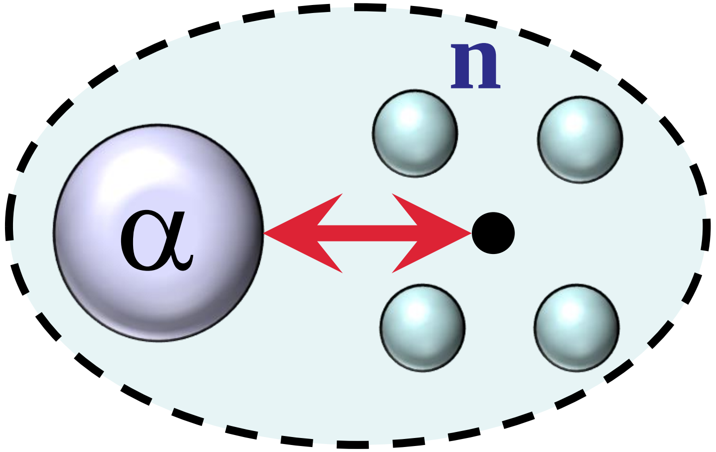
Radius of SDR in ^{8}He

Burgers, Rost, J. Phys. B 29 (1996)
Dote, Inoue, Myo, PLB784 (2018)
Myo, Katō, PTEP2020 (2020)
Myo, Katō, PRC107 (2023)

(fm)

	G.S.	SDR
Matter	2.53	3.11 + i0.86
Proton	1.81	1.97 + i0.26
Neutron	2.73	3.41 + i0.99
α-4n	**2.05**	**2.67** + i0.84
4n	**2.91**	**3.71** + i1.14

complex scaling



- α-4n : extend by **0.6 fm**
- 4n : expand by **0.8 fm**

Interpretation of imaginary part : Myo, Katō, PRC 107 (2023)

Summary

- Complex scaling not only for resonance, but also for level density
- Natural extension of completeness relation
- Asymptotically damping condition for resonances
- Apply complex scaling to few-body approach of nuclei
- Many-body unbound states are obtained in unstable nuclei.
- Remaining Problems
  - Many-body virtual state
  - Partial decay width