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Exotic anti-decuplet of baryons: prediction from chiral solitons

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Abstract. We predict an exotic Z^+ baryon (having spin $1/2$, isospin 0 and strangeness +1) with a relatively low mass of about 1530 MeV and total width of less than 15 MeV. It seems that this region of masses has avoided thorough searches in the past.

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1 All light baryons are rotational excitations

The most striking success of the old Skyrme idea [1] that nucleons can be viewed as solitons of the pion (or chiral) field, is the classification of light baryons it suggests. Indeed, the minimal generalization of spherical symmetry to incorporate three isospin components of the pion field is the so-called hedgehog form,

$$\pi^a(\mathbf{x}) = \frac{x^a}{r} P(r), \tag{1}$$

where $P(r)$ is the spherically-symmetric profile of the soliton. It implies that a space rotation of the field is equivalent to that in isospace. Hence, the quantization of the soliton rotation is similar to that of a spherical top: the rotational states have isospin T equal to spin J, and their excitation energies are

$$E_{\rm rot} = \frac{J(J+1)}{2I}, \tag{2}$$

where I is the soliton moment of inertia. The rotational states are, therefore, $(2J+1)^2$-fold degenerate (in spin and isospin). For $J = 1/2$ we have the four nucleon states; for $J = 3/2$ we have the sixteen Δ-isobar states. By saying that N and Δ are different rotational states of the same object (the "classical nucleon") one gets certain relations between their characteristics which are all satisfied up to a few percent in nature [2].

Quantum Chromodynamics has shed some light into why the chiral soliton picture is correct: we know now that the spontaneous chiral symmetry breaking in QCD is, probably, the most important feature of strong interactions, determining to a great extent their dynamics (see, e.g. [3]), while the large N_c (= numbers of colours) argumentation by Witten [4] explains why the pion field inside the nucleon can be considered as a classical one, *i.e.* as a "soliton".

The generalization to hyperons [4, 5] makes the success of the chiral soliton idea even more impressive. The rotation can be now performed in the ordinary and in the flavour $SU(3)$ space. Its quantization shows [5, 6, 7, 8, 9] that the lowest baryon state is the *octet with spin* $1/2$ and the next is the *decuplet with spin* $3/2$ – exactly what we meet in reality. Again, there are numerous relations between characteristics of members of octet and decuplet which follow purely from symmetry considerations. Perhaps the most spectacular is the Guadagnini formula [5] which relates splittings inside the decuplet with those in the octet: it is satisfied to the accuracy better than one percent, see below.

What are the next rotational excitations? If one restricts oneself to only two flavours, the next state should be a (5/2, 5/2) resonance; in the three-flavour case the third rotational excitation is an *anti-decuplet with spin* $1/2$ [1]. Why do not we have any clear signal of the exotic (5/2, 5/2) resonance? The reason is that the angular momentum $J = 5/2$ is numerically comparable to $N_c = 3$. Rotations with $J \sim N_c$ cannot be considered as slow: the centrifugal forces deform considerably the spherically-symmetric profile of the soliton field [12, 13]; simultaneously at $J \sim N_c$ the radiations of pions by the rotating body makes the total width of the state comparable to its mass [12, 14]. In order to survive strong pion radiation the rotating chiral solitons with $J \geq N_c$ have to stretch into cigar-like objects; such states lie on linear Regge trajectories [12].

The situation, however, might be somewhat different in the *three*-flavour case. First, the rotation is, roughly speaking, distributed among more axes in flavour space, hence individual angular velocities are not neccessarily as large as when we consider the two-flavour case with $J = 5/2$. Actually, the $SU(2)$ baryons with $J = 5/2$ belong to a very high multiplet from the $SU(3)$ point of view. Second, the radiation by the

[1] Probably the existence of the anti-decuplet as the next $SU(3)$ rotational excitation has been first pointed out by the authors at the ITEP Winter School (February, 1984), see [6]. Other early references for the anti-decuplet include [8, 10, 11].

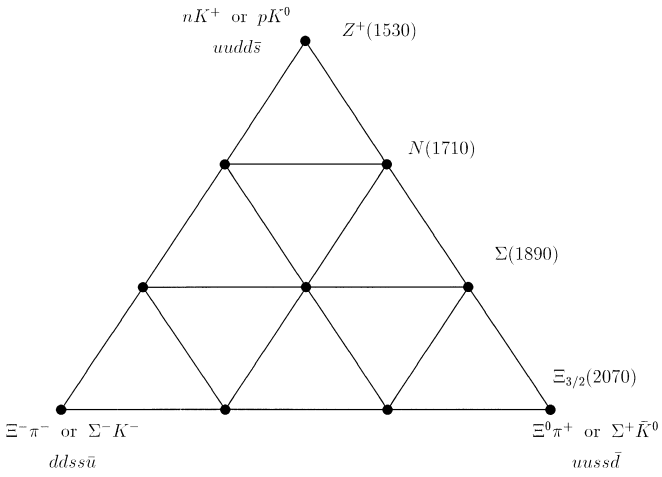

Fig. 1. The suggested anti-decuplet of baryons. The corners of this (T_3, Y) diagram are exotic. We show their quark content together with their (octet baryon+octet meson) content, as well as the predicted masses

soliton includes now K and η mesons which are substantially heavier than pions, and hence such radiation is to some extent suppressed. Therefore, the anti-decuplet baryons may not neccessarily have widths comparable to masses. And this is what we, indeed, find below.

We conclude thus that an expectaction of a relatively light and narrow anti-decuplet of baryons is theoretically motivated. Moreover, we are in a position to numerically estimate the masses and widths of the members of the anti-decuplet, and to point out possible experiments to observe them.

Let us mention that one can altogether abandon the soliton logic: the exotic anti-decuplet can be alternatively considered in a primitive way as states made of three quarks plus a quark-antiquark pair, or else, as a bound state of octet baryons with octet mesons. For example, the most interesting member of the anti-decuplet, viz. the exotic Z^+ baryon can be considered as a K^+n or K^0p bound state [2]. Unfortunately, the predictions become then to a great extent model-dependent. It is a big advantage of the chiral soliton picture that all concrete numbers (for masses and widths) do not rely upon a specific dynamical realization but follow from symmetry considerations. Actually, only one number would be useful to get from dynamics, namely a specific $SU(3)$ moment of inertia I_2 (see below), and concrete dynamical models give concrete values for this quantity. In this paper, however, we prefer to extract this quantity from experiment – by identifying the known nucleon resonance $N\left(1710, \frac{1}{2}^+\right)$ with the member of the suggested anti-decuplet [3]. We, then, are able to fix completely all the other members of the anti-decuplet together with their widths and branching ratios.

To end up this introduction we draw the $SU(3)$ diagramm for the suggested anti-decuplet in the (T_3, Y) axes, indicating its naive quark content as well as the (octet baryon + octet meson) content, see Fig. 1. In addition to the lightest Z^+ there is an exotic quadruplet of $S = -2$ baryons (we call them $\Xi_{3/2}$). However, the exotic Ξ^{--} and Ξ^+ hyperons appear to be very heavy and to have large widths, and can therefore hardly be detected. Therefore, apart from peculiar branching ratios predicted for the $N\left(1710, \frac{1}{2}^+\right)$ and the $\Sigma\left(1880, \frac{1}{2}^+\right)$ resonances, the crucial prediction is the existence of a relatively light and narrow Z^+ baryon.

[2] It is known that the KN phase shift in the $T = 0$, $J = (1/2)^+$ state corresponds to attraction [15]

[3] A possibility for such an identification has been mentioned in [16] but other members of the anti-decuplet have not been addressed there

2 Rotational states

Following Witten [4] and Guadagnini [5] we assume the self-consistent pseudoscalar field which binds up the $N_c = 3$ quarks in the "classical" baryon (i.e. the soliton field) to be of the form

$$U(\mathbf{x}) \equiv \exp\left(i\pi_A(\mathbf{x})\lambda^A/F_\pi\right) = \begin{pmatrix} \exp\left[i(\boldsymbol{n}\cdot\boldsymbol{\tau})P(r)\right] & 0 \\ 0\;0 & 1 \end{pmatrix},$$
$$\mathbf{n} = \frac{\mathbf{x}}{r}, \tag{3}$$

where the spherically-symmetric profile function $P(r)$ is defined by dynamics. We shall not need the concrete form of this function in what follows. In (3) λ^A are the eight Gell-Mann $SU(3)$ matrices, and $\boldsymbol{\tau}$ are the three Pauli $SU(2)$ matrices.

In order to provide the "classical" baryon with specific quantum numbers one has to consider a $SU(3)$-rotated pseudoscalar field,

$$\tilde{U}(\mathbf{x}, t) = R(t)U(\mathbf{x})R^+(t) \tag{4}$$

where $R(t)$ is a unitary $SU(3)$ matrix depending only on time and $U(\mathbf{x})$ is the static hedgehog field given by (3). Quantizing this rotation one gets the following rotational Hamiltonian [5],

$$H^{rot} = \frac{1}{2I_1}\sum_{A=1}^{3} J_A^2 + \frac{1}{2I_2}\sum_{A=4}^{7} J_A^2, \tag{5}$$

where J_A are the generators of the $SU(3)$ group; J_A with $A = 1, 2, 3$ are the usual angular momentum (spin) operators, and $I_{1,2}$ are the two $SU(3)$ moments of inertia, which are model-dependent. Most important is the additional quantization prescription,

$$J_8 = -\frac{N_c B}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}, \qquad Y' = -\frac{2}{\sqrt{3}}J_8 = 1, \tag{6}$$

where B is the baryon number, $B = 1$. In the Skyrme model this quantization rule follows from the Wess-Zumino term [4, 5]. In the more realistic chiral quark–soliton model [17] it arises from filling in the discrete level, i.e. from the "valence" quarks [18]. It is known to lead to the selection rule [5, 6, 7, 8, 9]: not all possible spin and $SU(3)$ multiplets are allowed as rotational excitations of the $SU(2)$ hedgehog. Equation (6) means that only those $SU(3)$ multiplets are allowed which contain particles with hypercharge $Y = 1$; if the number of particles with $Y = 1$ is denoted as $2J + 1$, the spin of the allowed $SU(3)$ multiplet is equal to J.

Therefore, the lowest allowed $SU(3)$ multiplets are:

- octet with spin 1/2 (since there are *two* baryons in the octet with $Y = 1$, the N)
- decuplet with spin 3/2 (since there are *four* baryons in the decuplet with $Y = 1$, the Δ)

– anti-decuplet with spin 1/2 (since there are *two* baryons in the anti-decuplet with $Y = 1$, the N^*)

The next are 27-plets with spin $1/2$ and $3/2$ but we do not consider them here. The appropriate rotational wave functions describing members of these multiplets are given in Appendix A.

For the representation (p, q) of the $SU(3)$ group one has

$$\sum_{A=1}^{8} J_A^2 = \frac{1}{3}[p^2 + q^2 + pq + 3(p+q)], \tag{7}$$

therefore the eigenvalues of the rotational Hamiltonian (5) are

$$E^{rot}_{(p,q)} = \frac{1}{6I_2}[p^2 + q^2 + pq + 3(p+q)] + \left(\frac{1}{2I_1} - \frac{1}{2I_2}\right) J(J+1) - \frac{(N_c B)^2}{24 I_2}. \tag{8}$$

We have the following three lowest rotational excitations:

$$(p, q) = (1, 1),\ J = 1/2 \text{ (octet)} \tag{9}$$

$$(p, q) = (3, 0),\ J = 3/2 \text{ (decuplet)} \tag{10}$$

$$(p, q) = (0, 3),\ J = 1/2 \text{ (anti-decuplet)}\,. \tag{11}$$

The splittings between the centers of these multiplets are determined by the moments of inertia, $I_{1,2}$:

$$\Delta_{10-8} = E^{rot}_{(3,0)} - E^{rot}_{(1,1)} = \frac{3}{2I_1}, \tag{12}$$

$$\Delta_{\overline{10}-8} = E^{rot}_{(0,3)} - E^{rot}_{(1,1)} = \frac{3}{2I_2}, \tag{13}$$

$$\Delta_{\overline{10}-10} = E^{rot}_{(0,3)} - E^{rot}_{(3,0)} = \frac{3}{2I_2} - \frac{3}{2I_1}. \tag{14}$$

We see that, were the moment of inertia $I_2 > I_1$, the anti-decuplet would be even lighter than the standard decuplet. Though we do not know of any strict theorem prohibiting this inequality, all dynamical models we know of give $I_1 > I_2$, so that the anti-decuplet appears to be heavier.

3 Splittings in the $SU(3)$ multiplets

We now take into account the non-vanishing strange quark mass. The effects of the non-zero m_s are of two kind: first, it splits the otherwise degenerate masses inside each $SU(3)$ multiplet; second, it leads to mixing between different $SU(3)$ multiplets. We shall systematically restrict ourselves to the linear order in m_s. In this order the phenomenologically successful Gell-Mann–Okubo and Guadagnini formulae are automatically satisfied.

Theoretically, the corrections to baryon masses due to $m_s \neq 0$ are of two types: i) leading order, $O(m_s N_c)$ and ii) subleading order, $O(m_s N_c^0)$. These corrections perturb the rotational Hamiltonian (5) (for derivation see [18]) by

$$\Delta H_m = \alpha D^{(8)}_{88} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i} J_i\,, \tag{15}$$

where $D^{(8)}_{\dots}(R)$ are the Wigner $SU(3)$ finite-rotation matrices depending on the orientation matrix of a baryon, see the Appendices. The coefficients α, β, γ are proportional to the mass of the s quark and are expressed through a combination of the soliton moments of inertia, $I_{1,2}$ and $K_{1,2}$, and the nucleon sigma term, Σ [18]:

$$\alpha = -\frac{2}{3}\frac{m_s}{m_u + m_d}\,\Sigma + m_s \frac{K_2}{I_2}\,, \tag{16}$$

$$\beta = -m_s \frac{K_2}{I_2}\,, \tag{17}$$

$$\gamma = \frac{2}{3} m_s \left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)\,, \tag{18}$$

$$\Sigma = \frac{m_u + m_d}{2}\langle N|\bar{u}u + \bar{d}d|N\rangle\,. \tag{19}$$

To get the physical splittings from (15) one has to sandwich it between the physical rotational states:

$$\Delta m_B = \langle B|\Delta H_m|B\rangle. \tag{20}$$

The mass splittings inside multiplets in terms of the coefficients α, β, γ are listed in Table 1 [4]. We call the two members of the anti-decuplet with exotic quantum numbers $T = 0,\ S = 1$ and $T = 3/2,\ S = -2$ as Z^+ and $\Xi_{3/2}$, respectively.

One observes that all splittings inside the octet and the decuplet are expressed through only two combinations of α, β and γ. This is the reason why, in the soliton picture, in addition to the standard Gell-Mann–Okubo relations,

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma, \tag{21}$$

$$m_\Delta - m_{\Sigma^*} = m_{\Sigma^*} - m_{\Xi^*} = m_{\Xi^*} - m_{\Omega^-}, \tag{22}$$

there arises a relation between the splittings inside the octet and the decuplet, the Guadagnini formula [5],

$$8(m_{\Xi^*} + m_N) + 3m_\Sigma = 11m_\Lambda + 8m_{\Sigma^*}, \tag{23}$$

which is satisfied with better than one-percent accuracy! The best fit to the splittings in the octet and the decuplet gives the following numerical values for the two combinations of the coefficients α, β and γ:

Table 1. Mass splittings within multiplets, $\Delta m_B = \langle B|\Delta H_m|B\rangle$

octet	T	Y	Δm_B
N	$1/2$	1	$(3/10)\alpha + \beta - (1/20)\gamma$
Λ	0	0	$(1/10)\alpha + (3/20)\gamma$
Σ	1	0	$-(1/10)\alpha - (3/20)\gamma$
Ξ	$1/2$	-1	$-(1/5)\alpha - \beta + (1/5)\gamma$
decuplet			
Δ	$3/2$	1	$(1/8)\alpha + \beta - (5/16)\gamma$
Σ^*	1	0	0
Ξ^*	$1/2$	-1	$-(1/8)\alpha - \beta + (5/16)\gamma$
Ω	0	-2	$-(1/4)\alpha - 2\beta + (5/8)\gamma$
antidecuplet			
Z^+	0	2	$(1/4)\alpha + 2\beta - (1/8)\gamma$
$N_{\overline{10}}$	$1/2$	1	$(1/8)\alpha + \beta - (1/16)\gamma$
$\Sigma_{\overline{10}}$	1	0	0
$\Xi_{3/2}$	$3/2$	-1	$-(1/8)\alpha - \beta + (1/16)\gamma$

[4] We take the opportunity to thank M. Praszalowicz and P. Pobylitsa who have participated in calculating this table back in 1988

$$\alpha + \frac{3}{2}\gamma = -380\,MeV,$$

$$\frac{1}{8}\alpha + \beta - \frac{5}{16}\gamma = -150\,MeV. \quad (24)$$

To learn the splittings in the anti-decuplet one needs to know the third combination of α, β and γ, which is not directly deducible from the octet and decuplet splittings. The only statement which can be immediatelly made from looking into Table 1 is that the spectrum in the anti-decuplet is equidistant, as in the normal decuplet. The third combination can be fixed, however, from the knowledge of the nucleon sigma term [19] and of the current quark mass ratio [20]:

$$\frac{m_s}{m_u + m_d} \approx 12.5, \qquad \Sigma \approx 45\,MeV. \quad (25)$$

These numbers gives for the sum:

$$\alpha + \beta = -\frac{2m_s}{3(m_u + m_d)}\Sigma \approx -375\,MeV. \quad (26)$$

Combining this knowledge with (24) we get finally all three coefficients:

$$\alpha \approx -218\,MeV, \quad \beta \approx -156\,MeV, \quad \gamma \approx -107\,MeV. \quad (27)$$

The equidistant splittings inside the anti-decuplet are thus expected to be equal to

$$\Delta m_{\overline{10}} = -\frac{\alpha}{8} - \beta + \frac{\gamma}{16} \approx 180\,MeV\,, \quad (28)$$

the lightest baryon being the exotic Z^+ resonance.

To end up this section we note that the non-zero strange quark mass leads also to the mixing of octet and anti-decuplet states with otherwise identical quantum numbers. In the linear order in m_s these mixings are derived in Appendix A and can be all expressed through one constant which we call $c_{\overline{10}}$, where

$$c_{\overline{10}} = -\frac{1}{3\sqrt{5}}\left(\alpha + \frac{1}{2}\gamma\right) I_2\,. \quad (29)$$

The true hyperon states become superpositions of the octet and anti-decuplet states:

Mainly octet baryons

$$|N\rangle = |N, 8\rangle + c_{\overline{10}}|N, \overline{10}\rangle, \quad (30)$$
$$|\Lambda\rangle = |\Lambda, 8\rangle, \quad (31)$$
$$|\Sigma\rangle = |\Sigma, 8\rangle + c_{\overline{10}}|\Sigma, \overline{10}\rangle, \quad (32)$$
$$|\Xi\rangle = |\Xi, 8\rangle; \quad (33)$$

Mainly anti-decuplet baryons

$$|Z^+\rangle = |Z^+, \overline{10}\rangle, \quad (34)$$
$$|N_{\overline{10}}\rangle = |N, \overline{10}\rangle - c_{\overline{10}}|N, 8\rangle, \quad (35)$$
$$|\Sigma_{\overline{10}}\rangle = |\Sigma, \overline{10}\rangle - c_{\overline{10}}|\Sigma, 8\rangle, \quad (36)$$
$$|\Xi_{3/2}\rangle = |\Xi_{3/2}, \overline{10}\rangle, \quad (37)$$

In the linear order in m_s the mixing does not effect the mass splittings inside the multiplets, discussed above.

Apart from α and γ, which we know now, the mixing coefficient $c_{\overline{10}}$ is proportional to the second moment of inertia I_2 which defines the shift of the anti-decuplet center (i.e. the $\Sigma_{\overline{10}}$ baryon) from the octet center, see (13). We do not know of any symmetry considerations relating this shift to that between the centers of the octet and the decuplet. The dynamical (i.e. model) predictions for the moment of inertia I_2 are rather disperse: they range from 0.43 fm in the Skyrme model [10, 11] to 0.55 fm in the chiral quark–soliton model [18]. Taken literally, the last value of I_2 would lead to a very light anti-decuplet, and in particular to a Z^+ lying below the KN threshold and thus stable against strong interactions. However, it should be mentioned that the moments of inertia have $\sim m_s$ corrections which are not computed yet. On physical grounds one can argue that the m_s corrections should be negative, since the account for non-zero quark mass makes the baryons more "tight".

In any case, we prefer to fix this fundamental quantity from identifying one of the members of the anti-decuplet, namely the one with the nucleon quantum numbers, $N_{\overline{10}}$, with the rather well established nucleon resonance $N\left(1710, \frac{1}{2}^+\right)$. Given that $N_{\overline{10}}$ is $\approx 180\,MeV$ lighter than the center of the anti-decuplet, we find

$$I_2 \approx 0.4\,\text{fm} \approx (500\,MeV)^{-1}, \quad (38)$$

and hence the octet–anti-decuplet mixing amplitude is

$$c_{\overline{10}} \approx 0.084\,, \quad (39)$$

being not a negligible quantity.

We thus arrive to the following masses of the anti-decuplet:

$$m_{Z^+} \approx 1530\,MeV,$$

$$m_{N_{\overline{10}}} \approx 1710\,MeV\text{(input)},$$

$$m_{\Sigma_{\overline{10}}} \approx 1890\,MeV,$$

$$m_{\Xi_{3/2}} \approx 2070\,MeV. \quad (40)$$

4 Baryon decays

In the non-relativistic limit for the initial and final baryons the baryon-baryon-meson coupling can be written in terms of rotational coordinates R of the baryon as [2]

$$-i\frac{3G_0}{2m_B} \cdot \frac{1}{2}\text{Tr}(R^\dagger \lambda^m R \lambda_i) \cdot p_i, \quad (41)$$

where λ^m is the Gell-Mann matrix for the emitted meson of flavour m, and p_i is the 3-momentum of the meson. To make (41) more symmetric we use for m_B in the denominator the half-sum of the initial (B_1) and final (B_2) baryon masses. Sandwiching (41) between the rotational wave functions of initial and final baryons, given by (A.1,A.2), we obtain the $B_1 \to B_2 + M$ transitions amplitude squared (averaged over the initial and summed over the final spin and isospin states) in terms of the $SU(3)$ isoscalar factors. The general formula is given in Appendix B. Using it we get for the particular modes of the $10 \to 8 + 8$ decays:

Decays of the decuplet

$$\Gamma(\Delta \to N\pi) = \frac{3G_0^2}{2\pi(M_\Delta + M_N)^2}|\mathbf{p}|^3 \frac{M_N}{M_\Delta} \cdot \frac{1}{5}$$
$$= 110\, MeV \ vs.\ 110\, MeV\ (exp.), \qquad (42)$$
$$\Gamma(\Sigma^* \to \Lambda\pi) = \frac{3G_0^2}{2\pi(M_{\Sigma^*} + M_\Lambda)^2}|\mathbf{p}|^3 \frac{M_\Lambda}{M_{\Sigma^*}} \cdot \frac{1}{10}$$
$$= 35\, MeV \ vs.\ 35\, MeV\ (exp.), \qquad (43)$$
$$\Gamma(\Sigma^* \to \Sigma\pi) = \frac{3G_0^2}{2\pi(M_{\Sigma^*} + M_\Sigma)^2}|\mathbf{p}|^3 \frac{M_\Sigma}{M_{\Sigma^*}} \cdot \frac{1}{15}$$
$$= 5.3\, MeV \ vs.\ 4.8\, MeV\ (exp.), \qquad (44)$$
$$\Gamma(\Xi^* \to \Xi\pi) = \frac{3G_0^2}{2\pi(M_{\Xi^*} + M_\Xi)^2}|\mathbf{p}|^3 \frac{M_\Xi}{M_{\Xi^*}} \cdot \frac{1}{10}$$
$$= 8.6\, MeV \ vs.\ 10\, MeV\ (exp.), \qquad (45)$$

where $|\mathbf{p}| = \sqrt{(M_1^2 - (M_2 + m)^2) \cdot (M_1^2 - (M_2 - m)^2)}/2M_1$ is the momentum of the meson of mass m. To get the concrete numbers we have used the value of the dimensionless coupling constant in (41) $G_0 \approx 19$.

We remind the reader that the usual $SU(3)$ symmetry would require *two* coupling constants (F and D) to determine the above widths, and of course the $SU(3)$ symmetry by itself tells nothing about the relation between decay constants for *different* multiplets. The chiral soliton models, while preserving the usual $SU(3)$ symmetry, in addition give relations between various couplings, since they all correspond to different rotation states of the same object. In particular, the chiral soliton models predict the F/D ratio to be [21]

$$\frac{F}{D} = \frac{5}{9} = 0.555... \quad vs. \quad 0.56 \pm 0.02\ (\text{exper.}), \qquad (46)$$

and the $g_{\pi NN}$ constant to be

$$g_{\pi NN} = \frac{7}{10}\, G_0 \approx 13.3 \quad vs. \quad \approx 13.6\ (\text{exper.}). \qquad (47)$$

Again, we see that the notion of 'baryons as rotational excitations' works quite satisfactory. Therefore, one would expect that *the same* coupling constant G_0 should be used for predicting the partial decay rates of the next rotational excitation, the anti-decuplet.

We remark, however, that in the particular case of the pseudoscalar couplings we expect rather large $1/N_c$ corrections which need not be universal for all multiplets. The point is, the baryon-pseudoscalar couplings are related, thanks to Goldberger–Treiman, to the baryon axial constants, g_A. Meanwhile, it is well known that the real-world ($N_c = 3$) value of the nucleon axial constant g_A differs from its large-N_c limit roughly by a factor [22] $(N_c + 2)/N_c = 5/3$, which is quite significant. This value comes from an estimate in a non-relativistic quark model and is not necessarily exactly true, however it gives an idea of the size of the $1/N_c$ corrections to the pseudoscalar couplings. Therefore, in order to perform a reliable estimate of the anti-decuplet widths we have to take into account, in addition to the leading-order (41), the $1/N_c$ corrections to that formula. The relevant $1/N_c$ corrections have been treated in ref. [23] for the $SU(2)$ case and in ref. [24] for the $SU(3)$ octet and decuplet cases; below we extend these works to the anti-decuplet couplings.

In the next-to-leading order one has to add to (41) new operators depending on the angular momentum J_a. These operators have the form [24]:

$$i\frac{3G_1}{2m_B} \cdot d_{iab} \cdot \frac{1}{2}\mathrm{Tr}(R^\dagger \lambda^m R \lambda^a) J_b \cdot p_i \ +$$
$$+ i\frac{3G_2}{2m_B\sqrt{3}} \cdot \frac{1}{2}\mathrm{Tr}(R^\dagger \lambda^m R \lambda^8) J_i \cdot p_i, \qquad (48)$$

where d_{abc} is the $SU(3)$ symmetric tensor, $a, b = 4, 5, 6, 7$, and J_a are the generators of the infinitesimal $SU(3)$ rotations. The new coupling constants $G_{1,2}$ are suppressed by $1/N_c$ relative to the leading-order coupling constant G_0, although numerically they can be sizable.

Sandwiching eqs. (41,48) between the rotational wave functions of initial and final baryons and taking into account the anti-decuplet–octet mixing represented by the coefficient $c_{\overline{10}}$ (see see (39)), we obtain the following general formula for partial widths of members of the decuplet and of the anti-decuplet:

$$\Gamma(B_1 \to B_2 M) = \frac{3G_r^2}{2\pi(M_1 + M_2)^2}|\mathbf{p}|^3 \frac{M_2}{M_1} \cdot$$
$$\cdot (C_1 + \frac{1}{\sqrt{5}} C_2 \cdot c_{\overline{10}}), \qquad (49)$$

The effective coupling constant G_r depending on the multiplet and the isoscalar factors C_1 and C_2 for various decays are listed in Table 2. The pion nucleon coupling constant $g_{\pi NN}$ and the F/D ratio can be also expressed in terms of the couplings $G_{0,1,2}$:

$$g_{\pi NN} = \frac{7}{10} \cdot (G_0 + \frac{1}{2}G_1 + \frac{1}{14}G_2), \qquad (50)$$
$$\frac{F}{D} = \frac{5}{9} \cdot \frac{G_0 + \frac{1}{2}G_1 + \frac{1}{2}G_2}{G_0 + \frac{1}{2}G_1 - \frac{1}{6}G_2}\,. \qquad (51)$$

Substituting in the last equation the experimental value of $F/D = 0.56 \pm 0.02$ we find the value of the ratio:

$$\frac{G_2}{G_0 + \frac{1}{2}G_1} = 0.01 \pm 0.05, \qquad (52)$$

which turns to be very small and will be neglected. The smallness of G_2 is not surprising as it can be related to the singlet axial constant of the nucleon,

$$G_2 = \frac{2M_N}{3F_\pi} g_A^{(0)}\,, \qquad (53)$$

Table 2. Clebsch-Gordan coefficients entering (49) for the decays of the decuplet and of the lightest members of the anti-decuplet

decay	G_r	C_1	C_2
$\Delta \to N\pi$	$G_0 + \frac{1}{2}G_1$	$\frac{1}{5}$	0
$\Sigma^* \to \Lambda\pi$	$G_0 + \frac{1}{2}G_1$	$\frac{1}{10}$	0
$\Sigma^* \to \Sigma\pi$	$G_0 + \frac{1}{2}G_1$	$\frac{1}{15}$	0
$\Xi^* \to \Xi\pi$	$G_0 + \frac{1}{2}G_1$	$\frac{1}{10}$	0
$N_{\overline{10}} \to N\pi$	$G_0 - G_1 - \frac{1}{2}G_2$	$\frac{1}{20}$	$-\frac{23}{40}$
$N_{\overline{10}} \to N\eta$	$G_0 - G_1 - \frac{1}{2}G_2$	$\frac{1}{20}$	$-\frac{1}{40}$
$N_{\overline{10}} \to \Delta\pi$	$G_0 + \frac{1}{2}G_1$	0	$\frac{4}{5}c_{\overline{10}}$
$N_{\overline{10}} \to \Lambda K$	$G_0 - G_1 - \frac{1}{2}G_2$	$\frac{1}{20}$	$\frac{2}{5}$
$N_{\overline{10}} \to \Sigma K$	$G_0 - G_1 - \frac{1}{2}G_2$	$\frac{1}{20}$	$\frac{3}{20}$
$Z^+ \to NK$	$G_0 - G_1 - \frac{1}{2}G_2$	$\frac{1}{5}$	$\frac{1}{4}$

the latter known to be small. We see here another remarkable prediction of the 'baryon as soliton' idea: the smallness of the singlet axial constant $g_A^{(0)}$ is directly related to the smallness of the deviation of the F/D ratio from 5/9.

From Table 2 we see that the decuplet-octet couplings are proportional to $(G_0 + G_1/2)$ and hence (if one neglects the apparently small G_2) it can be related to the pion-nucleon coupling $g_{\pi NN}$. This observation means that the widths of the decuplet calculated in the leading $1/N_c$ order in the beginning of this section are actually not affected by the rotational $1/N_c$ corrections: in the next-to-leading order the relation of the decuplet widths to the $g_{\pi NN}$ constant is not changed. One has just to replace the G_0 of (42–47) by $G_0 + G_1/2$. Therefore, from the decuplet decay width one finds (cf. (47))

$$G_{10} = G_0 + \frac{1}{2}G_1 \approx 19,$$
$$g_{\pi NN} \approx \frac{7}{10}\left(G_0 + \frac{1}{2}G_1\right) \approx 13.3\,. \tag{54}$$

However, the situation is different for the anti-decuplet: as seen from Table 2, its effective couplings are proportional to $(G_0 - G_1)$ (again we neglect the small G_2), not to $(G_0 + \frac{1}{2}G_1)$. Therefore, with the $1/N_c$ corrections taken into account, the anti-decuplet–octet coupling is not expressed solely through $g_{\pi NN}$: to calculate the anti-decuplet decay widths one has to know the ratio G_1/G_0 as well. Unfortunately, it can not be fixed in a model-independent way – one has to resort to some model. In the chiral quark-soliton model [17] the ratio G_1/G_0 is in range from 0.4 to 0.6 [23, 24] [5]. A similar calculation of the G_2 coupling in the same model shows that it is substantially smaller than G_0 [24], in accordance with the experiment, see (52). In the estimates below we use the lower value of the ratio, $G_1/G_0 \approx 0.4$, corresponding to the value of the anti-decuplet decay constant,

$$G_{\overline{10}} \approx G_0 - G_1 \approx 0.5 \cdot G_{10} \approx 9.5\,. \tag{55}$$

It should be mentioned that the non-relativistic quark model (which, to some extent, can be used as a guiding line) predicts $G_1/G_0 = 4/5$ and $G_2/G_0 = 2/5$, which is in a qualitative agreement with a more realistic calculation in the quark soliton model. Amusingly, though, these ratios produce exactly zero $G_{\overline{10}}$. At the moment we are unable to point out the deep reason for such a cancellation; in any case the non-relativistic quark model cannot be considered as realistic as it gives also a too large value of the F/D ratio and of the singlet axial constant. However, it may indicate that (55) overestimates $G_{\overline{10}}$, and that the widths of the anti-decuplet are even more narrow than we estimate below.

We now present the decay rates of the members of the anti-decuplet using (49,55) with the Clebsch–Gordan coefficients from Table 2 which also takes into account the octet–anti-decuplet mixing discussed in Sect. 3.

$T = 0,\ S = 1$ state (the exotic Z^+ baryon)

$$\Gamma(Z^+ \to NK) = \frac{3G_{\overline{10}}^2}{2\pi(M_N + M_Z)^2}|\mathbf{p}|^3\frac{M_N}{M_Z}\cdot\frac{1}{5}\left(1 + \frac{\sqrt{5}}{4}c_{\overline{10}}\right) = 15\,MeV, \tag{56}$$

[5] M.P. is grateful to H.-C. Kim and T. Watabe for a detailed discussion of this issue

Since there are no other strong decay modes, the total width of the Z^+ coincides with the above number.

$T = \frac{1}{2},\ S = 0$ state (the N resonance)

$$\Gamma(N_{\overline{10}} \to N\pi) = \frac{3G_{\overline{10}}^2}{2\pi(M_N + M_{\overline{10}})^2}|\mathbf{p}|^3\frac{M_N}{M_{\overline{10}}} \cdot \frac{1}{20}\left(1 - \frac{23}{2\sqrt{5}}c_{\overline{10}}\right) = 5\,MeV, \tag{57}$$

$$\Gamma(N_{\overline{10}} \to N\eta) = \frac{3G_{\overline{10}}^2}{2\pi(M_N + M_{\overline{10}})^2}|\mathbf{p}|^3\frac{M_N}{M_{\overline{10}}} \cdot \frac{1}{20}\left(1 - \frac{1}{2\sqrt{5}}c_{\overline{10}}\right) = 11\,MeV, \tag{58}$$

$$\Gamma(N_{\overline{10}} \to \Delta\pi) = \frac{3G_{\overline{10}}^2}{2\pi(M_\Delta + M_{\overline{10}})^2}|\mathbf{p}|^3\frac{M_\Delta}{M_{\overline{10}}} \cdot \frac{4}{5}c_{\overline{10}}^2 = 5\,MeV, \tag{59}$$

$$\Gamma(N_{\overline{10}} \to \Lambda K) = \frac{3G_{\overline{10}}^2}{2\pi(M_\Lambda + M_{\overline{10}})^2}|\mathbf{p}|^3\frac{M_\Lambda}{M_{\overline{10}}} \cdot \frac{1}{20}\left(1 + \frac{8}{\sqrt{5}}c_{\overline{10}}\right) = 5\,MeV, \tag{60}$$

$$\Gamma(N_{\overline{10}} \to \Sigma K) = \frac{3G_{\overline{10}}^2}{2\pi(M_\Sigma + M_{\overline{10}})^2}|\mathbf{p}|^3\frac{M_\Sigma}{M_{\overline{10}}} \cdot \frac{1}{20}\left(1 + \frac{3}{\sqrt{5}}c_{\overline{10}}\right) = 0.5\,MeV, \tag{61}$$

These partial widths sum up into 27.5 MeV. However, the quantum numbers of $N(1710)$ allow decays into, say, $N\pi\pi$ states which are not fully accounted for above. Allowing a 50% branching ratio for the non-accounted decays, we estimate the full width as $\Gamma_{tot}(N_{\overline{10}}) \approx 27.5\,MeV \cdot 1.5 \approx 41\,MeV$, where from the branching ratios can be deduced, see Table 3.

$T = 1,\ S = -1$ state (the Σ resonance)

$$\Gamma(\Sigma_{\overline{10}} \to N\bar{K}) = \frac{3G_{\overline{10}}^2}{2\pi(M_N + M_{\Sigma_{\overline{10}}})^2}|\mathbf{p}|^3\frac{M_N}{M_{\Sigma_{\overline{10}}}} \cdot \frac{1}{30}\left(1 - \frac{9}{\sqrt{5}}c_{\overline{10}}\right) = 6\,MeV, \tag{62}$$

Table 3. Predictions for decay modes of $N_{\overline{10}}$ identified with $N\left(1710, \frac{1}{2}^+\right)$, confronted with the data from [25]

$N_{\overline{10}}$	prediction	data
$M_{\overline{10}}$, MeV	1710 (input)	1710
$\Gamma_{tot}(N_{\overline{10}})$, MeV	~ 40	50 to 250
$Br(N\pi)$	~ 0.13	0.10 to 0.20
$Br(N\eta)$	~ 0.28	0.16 ± 0.10
$Br(\Delta\pi)$ P-wave	~ 0.13	–
$Br(\Lambda K)$	~ 0.13	–
$Br(\Sigma K)$	~ 0.01	–
$\sqrt{Br(N\pi)Br(N\eta)}$	~ 0.19	0.30 ± 0.08
$\sqrt{Br(N\pi)Br(\Lambda K)}$	~ 0.13	0.12 to 0.18
$\sqrt{Br(N\pi)Br(\Delta\pi)}$	~ 0.12	0.16 to 0.22

$$\Gamma(\Sigma_{\overline{10}} \to \Sigma\pi) = \frac{3G^2_{\overline{10}}}{2\pi(M_\Sigma + M_{\Sigma_{\overline{10}}})^2}|\mathbf{p}|^3\frac{M_\Sigma}{M_{\Sigma_{\overline{10}}}} \cdot \frac{1}{30}\left(1-\sqrt{5}c_{\overline{10}}\right) = 10\,MeV, \quad (63)$$

$$\Gamma(\Sigma_{\overline{10}} \to \Sigma\eta) = \frac{3G^2_{\overline{10}}}{2\pi(M_\Sigma + M_{\Sigma_{\overline{10}}})^2}|\mathbf{p}|^3\frac{M_\Sigma}{M_{\Sigma_{\overline{10}}}} \cdot \frac{1}{20}\left(1+\frac{6}{\sqrt{5}}c_{\overline{10}}\right) = 9\,MeV, \quad (64)$$

$$\Gamma(\Sigma_{\overline{10}} \to \Lambda\pi) = \frac{3G^2_{\overline{10}}}{2\pi(M_\Lambda + M_{\Sigma_{\overline{10}}})^2}|\mathbf{p}|^3\frac{M_\Lambda}{M_{\Sigma_{\overline{10}}}} \cdot \frac{1}{20}\left(1-\frac{6}{\sqrt{5}}c_{\overline{10}}\right) = 17\,MeV, \quad (65)$$

$$\Gamma(\Sigma_{\overline{10}} \to \Xi K) = \frac{3G^2_{\overline{10}}}{2\pi(M_\Xi + M_{\Sigma_{\overline{10}}})^2}|\mathbf{p}|^3\frac{M_\Xi}{M_{\Sigma_{\overline{10}}}} \cdot \frac{1}{30}\left(1+\frac{19}{\sqrt{5}}c_{\overline{10}}\right) = 3\,MeV, \quad (66)$$

$$\Gamma(\Sigma_{\overline{10}} \to \Sigma^*\pi) = \frac{3G^2_{\overline{10}}}{2\pi(M_{\Sigma^*} + M_{\Sigma_{\overline{10}}})^2}|\mathbf{p}|^3\frac{M_{\Sigma^*}}{M_{\Sigma_{\overline{10}}}} \cdot \frac{2}{15}c^2_{\overline{10}} = 2\,MeV. \quad (67)$$

These partial widths sum up to $47\,MeV$. Multiplying it by a factor of 1.5 as in the previous case we estimate the full width of the $\Sigma_{\overline{10}}$ to be $\Gamma_{tot}(\Sigma_{\overline{10}}) \approx 70\,MeV$. See Table 4 for the calculated branching ratios and a comparison with the data.

$T = 3/2,\ S = -2$ state (the exotic $\Xi_{3/2}$ baryon)

$$\Gamma(\Xi_{3/2} \to \Sigma K) = 52\,MeV, \quad (68)$$

$$\Gamma(\Xi_{3/2} \to \Xi\pi) = 42\,MeV, \quad (69)$$

$$\Gamma_{tot}(\Xi_{3/2}) \approx 140\,MeV. \quad (70)$$

Despite the smallness of the octet–anti-decuplet mixing represented by the coefficient $c_{\overline{10}}$ (see (39)) it has a large impact on the decay widths of the anti-decuplet because the decay channels $8 \to 8+8$ and $8 \to 10+8$ are enhanced "kinematically" by large Clebsch–Gordan coefficients. For example, without taking into account this mixing, the decay $N_{\overline{10}} \to \Delta\pi$ is forbidden, however the small mixing probability, $c^2_{\overline{10}} \sim 0.007$, is amplified by a huge "kinematical" factor ~ 20.

Finally, we mention that the Z^+NK coupling correspondent to (41,48) can be written down in a relativistically-invariant form as

$$L_{int} = ig_{KNZ}\left[(\bar{p}\gamma_5 Z^+)\bar{K}^0 + (\bar{n}\gamma_5 Z^+)K^-\right]. \quad (71)$$

Comparing its non-relativistic limit with particular projections of (41,48) we find

$$g_{KNZ} = \frac{3}{\sqrt{30}}\,\frac{2m_N}{m_N+m_Z}\,(G_0 - G_1) \approx 4.1\,. \quad (72)$$

For a comparison, the ordinary Σ^+NK vertex written in the form of (71) corresponds to $g_{KN\Sigma^+} \approx 5$.

Table 4. Predictions for decay modes of $\Sigma_{\overline{10}}$, confronted with the data from [25]

$\Sigma_{\overline{10}}$	prediction	data
$M_{\Sigma_{\overline{10}}}$, MeV	1890	≈ 1880
$\Gamma_{tot}(\Sigma_{\overline{10}})$, MeV	~ 70	80 to 250
$Br(N\bar{K})$	~ 0.09	0.06 to 0.3
$\sqrt{Br(N\bar{K})Br(\Sigma\pi)}$	~ 0.11	~ 0.3
$\sqrt{Br(N\bar{K})Br(\Lambda\pi)}$	~ 0.15	0.11 to 0.25

5 Identification of members of the anti-decuplet

We see that the predicted branching ratios and total width of the $N(1710)$ are in a reasonable agreement with the data, given the large errors and inconsistencies in the data, see [25]. Our numbers should be compared also with the predictions for the $N(1710)$ decays following from the standard $SU(6)$ quark model, performed in [26]. Assuming $N(1710)$ to be a member of a normal octet the authors get, in particular, $\Gamma(N\eta) \approx \Gamma(\Lambda K) \approx 0$, which seems to contradict the data even though the errors are large. It should be mentioned, however, that a recent analysis [27] suggests that in the $\sim 1700\,MeV$ region there might be actually two nucleon resonances: one coupled stronger to pions and another to the η meson.

We conclude that the $N(1710)$ nucleon resonance is a good candidate for the $N_{\overline{10}}$ member of the anti-decuplet. Let us stress that the octet–anti-decuplet mixing is important for the analysis. It leads to a considerable reduction of the $N\pi$ branching ratio and of the total width; simultaneously a non-zero $\Delta\pi$ branching ratio appears, in accordance with the phenomenology of the $N(1710)$ decays.

There is a fair candidate for the $\Sigma_{\overline{10}}$ member of the anti-decuplet, namely the $\Sigma(1880)$ from the Particle Data Group baryon listings [25]. The resonance has only a two-star status, and its properties are not measured properly, including the mass ranging from 1826 ± 20 to 1985 ± 50 MeV. Nevertheless, we compare our predictions for the $\Sigma_{\overline{10}}$ with what is known about the resonance, see Table 4.

What can be said is that the suggested identification does not contradict the (poor) data on the $\Sigma(1880)$ resonance.

As to $\Xi_{3/2}$ which we predict at $2070\,MeV$, there are several candidates for the non-exotic components of this quadruplet in the range of masses between 1900 and 2100 MeV, however even their quantum numbers are not well established yet. In view of the estimate that our $\Xi_{3/2}$ is wider than $140\,MeV$ it would be quite difficult to pinpoint such a state, including its exotic components, Ξ^{--} and Ξ^+. Moreover, a presence of such a wide state would seriously influence the determination of the parameters of other Ξ-type resonances, were they to appear in this mass region. We sum up our predictions for the anti-decuplet in Table 5.

It should be mentioned that the masses of the anti-decuplet have been estimated in the Skyrme model with the results ranging from $M_{Z^+} \approx 1500$ MeV [10] to ≈ 1700 MeV [11]. Such an uncertainty arises in the Skyrme model since one has to make a difficult choice between having the nucleon mass correct and the F_π constant wrong, or *vice versa*. Predictions for the exotic $T = 0, S = 1$ P_{01} state in the bag model are grouped around 1750 MeV [28, 29], that is significantly higher than our estimate.

Table 5. Predictions for masses and total widths of the members of the anti-decuplet and possible candidates for these states

	T	Y	Mass in MeV	Width in MeV	Possible candidate
Z^+	0	2	1530	15	—
$N_{\overline{10}}$	1/2	1	1710 (input)	~ 40	$N(1710)P_{11}$
$\Sigma_{\overline{10}}$	1	0	1890	~ 70	$\Sigma(1880)P_{11}$
$\Xi_{3/2}$	3/2	-1	2070	> 140	$\Xi(2030)$?

Our considerations have essentially been based on the identification of the non-exotic member of the anti-decuplet with the rather well established nucleon resonance, $N\left(1710, \frac{1}{2}^+\right)$. On general grounds, we cannot exclude the possibility that the anti-decuplet as a whole lies higher. For example, the exotic Z^+ might, in principle, have a mass of more than 1750 MeV (lower masses are probably excluded by the old phase-shift analyses – see [15]). There have been claims in the past for observing such states (see [30] for a review). In this case, however, there would be difficulties in finding an appropriate candidate for the $N_{\overline{10}}$ member of the anti-decuplet. The only possibility suggested by the Particle Data baryon listings is the $N\left(2100, \frac{1}{2}^+\right)$ resonance. In this case the moment of inertia I_2 determining the shift of the anti-decuplet center in respect to the decuplet center would be very small (about ~ 0.3 fm). Such a small moment of inertia can hardly be obtained in any dynamical realization of the idea of baryons as solitons, which seems to be so successful everywhere else. Therefore, we believe that we present a good case for a relatively light and narrow exotic baryon: it probably has not been observed in the past just for these reasons.

6 Conclusions

The chiral soliton models of baryons, which correctly emphasize the important role of the spontaneous chiral symmetry breaking in the dynamics of strong interactions, are extremely successful in explaining relations between octet and decuplet baryons since in these models they all appear as various rotational excitations of the same object.

The two lowest rotational states of chiral solitons are exactly the octet with spin $\frac{1}{2}$ and the decuplet with spin $\frac{3}{2}$, and it is natural to ask oneself what is the next rotational state. The answer is [6, 8, 10, 11]: it is the anti-decuplet with spin $\frac{1}{2}$, and most of its properties can be predicted from symmetry considerations only, without entering into dynamics which is model-dependent. The only unknown parameter (a specific $SU(3)$ moment of inertia) can be fixed by identifying the nucleon-like member of the anti-decuplet with the observed $N\left(1710, \frac{1}{2}^+\right)$ resonance. Its decay modes, as well as masses and decay modes of the other members of the anti-decuplet can be then fixed unambigiously. The calculated decay modes of the $N(1710)$ are found to be in a reasonable agreement with the existing data though the data are not good enough to make a decisive conclusion. At least it seems that the standard non-relativistic $SU(6)$ description of this state as a member of an octet, is in trouble with the data: the anti-decuplet idea fits better.

In a sense, history repeats itself: there are candidates for all members of the anti-decuplet, except for its vertex – like in the early 60's when all members of the now venerable decuplet were known except the Ω^- hyperon. In our case it is the exotic Z^+ baryon, which decays into $K^+ n$ and $K^0 p$. Claims for observing such states have been made in the past (see [30] for a review) but they are all substantially higher than our prediction $m_Z \approx 1530\, MeV$, with the width lower than $\Gamma_Z \approx 15\, MeV$.

The most direct way to detect the exotic Z^+ resonance would be in the $K^0 p$ or $K^+ n$ scattering. Unfortunately, the mass range in question is too low for kaon beams and probably too high for the ϕ factory of kaons.

Another possibility to reveal the exotic Z^+ is in the collisions of non-strange particles. In comparison with direct production in KN collision, such reactions are more complicated as they involve many particles in the final states of which some are neutral and some are charged, therefore a combined detector is needed. As to the missing-mass-type experiments they seem to be vulnerable because of severe background conditions and the narrowness of the Z^+. Let us list several possibilities of the Z^+ production in reactions with non-strange particles.

- Nucleon–nucleon collisions
  $pn \to \Lambda Z^+ \to \Lambda K^+ n$ or $\Lambda K^0 p$, $p_{lab} > 2.60\ GeV/c$
  $pp \to \Sigma^+ Z^+ \to \Sigma^+ K^+ n$ or $\Sigma^+ K^0 p$, $p_{lab} > 2.8\ GeV/c$
- Photon-nucleon collisions
  $\gamma p \to \bar{K}^0 Z^+ \to \bar{K}^0 K^+ n$ or $\bar{K}^0 K^0 p$, $p_{lab} > 1.7\ GeV/c$
  $\gamma n \to K^- Z^+ \to K^- K^+ n$ or $K^- K^0 p$, $p_{lab} > 1.7\ GeV/c$
- Pion-nucleon collisions
  $\pi^- p \to K^- Z^+ \to K^- K^+ n$ or $K^- K^0 p$ $p_{lab} > 1.7\ GeV/c$
  $\pi^+ n \to \bar{K}^0 Z^+ \to \bar{K}^0 K^+ n$ or $\bar{K}^0 K^0 p$, $p_{lab} > 1.7\ GeV/c$.

One of the most promising ways to check the existence of the Z^+ baryon would be in the photon collisions with energies $> 2\, GeV$, since the photon already carries a portion of strange quarks [6]. Another possibility is to analyze the LASS data from the 11 GeV $K^+ p$ collisions [7]. In any case, a search for a light and narrow exotic Z^+ baryon seems to be a challenging task.

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Appendix A

The rotational wave functions of baryons are eigenfunctions of the collective hamiltonian

$$H = H^{rot} + \Delta H_m,$$

where the $SU(3)$-symmetric H^{rot} is given by (5) and the $SU(3)$-breaking part, ΔH_m, is given by (15). The eigenfunctions of the unperturbed hamiltonian H^{rot} are proportional to the Wigner finite-rotation matrices [31]:

[6] One of us (D.D.) is grateful to E. Paul for a conversation on this point
[7] We thank J. Bjorken and J. Napolitano for this suggestion

$$|B\rangle = |B, r\rangle = \sqrt{\dim\, r}(-1)^{J_3-1/2} D^{(\bar{r})}_{Y,T,T_3;1,J,-J_3}, \qquad (A.1)$$

where r is an irreducible representation of the $SU(3)$ group, $r = 8, 10, \overline{10}$, etc., B denotes a set of quantum numbers: Y, T, T_3 (hypercharge, isospin and its projection) and J, J_3 (spin and its projection).

In the linear approximation in m_s rotational wave functions are superpositions of different representations:

$$|B\rangle = |B, r\rangle + \sum_{r' \neq r} \frac{\langle B, r'|\Delta H_m|B, r\rangle}{E_B^{r(0)} - E_B^{r'(0)}} |B, r'\rangle. \qquad (A.2)$$

Here the unperturbed energies $E_B^{r(0)}$ are given by (8). Neglecting admixtures of the 27- and 35-plets and using the general (A.2) we obtain the wave functions (30-37).

Appendix B

In this Appendix we derive general formulae for the decay rate of a baryon B_1 with flavour quantum numbers $(YTT_3) = (Y_1T_1t_1)$ and spin $(JJ_3) = (J_1j_1)$, into a baryon B_2 with quantum numbers $(Y_2T_2t_2)$ and (J_2j_2), plus an octet pseudoscalar meson with $(YTT_3) = (Y_mT_mt_m)$. In order to obtain the amplitude one has to sandwich the meson-soliton coupling

$$-i\frac{3G_0}{2M_B} \cdot \frac{1}{2}\mathrm{Tr}(R^\dagger \lambda^m R \lambda^i) \cdot p_i, \qquad (B.1)$$

where λ^m is the Gell-Mann matrix of the correspondent meson, R is the matrix describing the orientation of the soliton, and p_i is the 3-momentum of the meson, between rotational wave functions describing the baryons B_2 and B_1. To incorporate a general situation we assume that their rotational wave functions are mixtures of certain $SU(3)$ muiltiplets r and q, so that one can write the wave functions as linear combinations of the Wigner D-functions:

$$\Psi_{B_i}(R) = (-1)^{j_i-1/2}\{\sqrt{\dim\, r_i} D^{(\bar{r}_i)}_{Y_i,T_i,t_i;1,J_i,-j_i} + A_i\sqrt{\dim\, q_i} D^{(\bar{q}_i)}_{Y_i,T_i,t_i;1,J_i,-j_i}\}. \qquad (B.2)$$

We assume the admixtures $A_{1,2}$ to be small ($\sim m_s$), and neglect systematically the A_i^2 terms.

Using the fact that

$$\frac{1}{2}\mathrm{Tr}(R^\dagger \lambda^m R \lambda^i) = D^{(8)}_{m,i}(R), \qquad (B.3)$$

and the general formula,

$$\int dR D^{r*}_{\nu\nu'}(R) D^{r_1}_{\nu_1\nu_1'}(R) D^{r_2}_{\nu_2\nu_2'}(R) = \frac{1}{dim(r)} \sum_{\mu'} \delta_{rr'} \begin{pmatrix} r_1 & r_2 & r' \\ \nu_1 & \nu_2 & \nu \end{pmatrix} \begin{pmatrix} r_1 & r_2 & r' \\ \nu_1' & \nu_2' & \nu', \end{pmatrix}, \qquad (B.4)$$

where the sum goes over all occurrences of the representation r in the product of representations r_1 and r_2, one gets for the decay amplitude $B_1 \to B_2 + M$ the following expression in terms of the $SU(3)$ Clebsch–Gordan coefficients :

$$\mathcal{M}(B_1 \to B_2 M) = \frac{3g}{M_2 + M_1} p^i \left\{ \sqrt{\frac{r_2}{r_1}} \sum_{r_1'} \begin{pmatrix} r_2 & 8 & r_1' \\ Y_2T_2t_2 & Y_mT_mt_m & Y_1T_1t_1 \end{pmatrix} \begin{pmatrix} r_2 & 8 & r_1' \\ 1J_2j_2 & 01i & 1J_1j_1 \end{pmatrix} + A_1\sqrt{\frac{r_2}{q_1}} \sum_{q_1'} \begin{pmatrix} r_2 & 8 & q_1' \\ Y_2T_2t_2 & Y_mT_mt_m & Y_1T_1t_1 \end{pmatrix} \begin{pmatrix} r_2 & 8 & q_1' \\ 1J_2j_2 & 01i & 1J_1j_1 \end{pmatrix} + A_2\sqrt{\frac{q_2}{r_1}} \sum_{r_1'} \begin{pmatrix} q_2 & 8 & r_1' \\ Y_2T_2t_2 & Y_mT_mt_m & Y_1T_1t_1 \end{pmatrix} \begin{pmatrix} q_2 & 8 & r_1' \\ 1J_2j_2 & 01i & 1J_1j_1 \end{pmatrix} \right\}.$$

Before we square this amplitude let us factorize out the dependence on the $SU(2)$ quantum numbers (referring both to spin and isospin) using the relation between the $SU(3)$ and the $SU(2)$ Clebsch–Gordan coefficients [31] (the proportionality coefficient is called the isoscalar factor):

$$\begin{pmatrix} r_1 & r_2 & r_3 \\ Y_1T_1t_1 & Y_2T_2t_2 & Y_3T_3t_3 \end{pmatrix} = C^{T_3t_3}_{T_1t_1;T_2t_2} \left(\begin{array}{cc|c} r_1 & r_2 & r_3 \\ Y_1T_1 & Y_2T_2 & Y_3T_3 \end{array} \right). \qquad (B.5)$$

Making use of the above relation one gets for the amplitude squared:

$$|\mathcal{M}|^2 = \frac{9G_0^2}{(M_2 + M_1)^2} p^i p^j |C^{T_1t_1}_{T_2,t_2;T_m,t_m}|^2 C^{J_1j_1}_{J_2,j_2;1i} C^{J_1j_1}_{J_2,j_2;1j} \times \left\{ \frac{r_2}{r_1} \left| \sum_{r_1'} \left(\begin{array}{cc|c} r_2 & 8 & r_1' \\ Y_2T_2 & Y_mT_m & Y_1T_1 \end{array} \right) \left(\begin{array}{cc|c} r_2 & 8 & r_1' \\ 1J_2 & 01 & 1J_1 \end{array} \right) \right|^2 + 2A_1 \frac{r_2}{\sqrt{q_1 r_1}} \sum_{r_1'} \left(\begin{array}{cc|c} r_2 & 8 & r_1' \\ Y_2T_2 & Y_mT_m & Y_1T_1 \end{array} \right) \left(\begin{array}{cc|c} r_2 & 8 & r_1' \\ 1J_2 & 01 & 1J_1 \end{array} \right) \times \sum_{q_1'} \left(\begin{array}{cc|c} r_2 & 8 & q_1' \\ Y_2T_2 & Y_mT_m & Y_1T_1 \end{array} \right) \left(\begin{array}{cc|c} r_2 & 8 & q_1' \\ 1J_2 & 01 & 1J_1 \end{array} \right) + 2A_2 \frac{\sqrt{r_2 q_2}}{r_1} \sum_{r_1'} \left(\begin{array}{cc|c} r_2 & 8 & r_1' \\ Y_2T_2 & Y_mT_m & Y_1T_1 \end{array} \right) \left(\begin{array}{cc|c} r_2 & 8 & r_1' \\ 1J_2 & 01 & 1J_1 \end{array} \right) \times \sum_{r_1'} \left(\begin{array}{cc|c} q_2 & 8 & r_1' \\ Y_2T_2 & Y_mT_m & Y_1T_1 \end{array} \right) \left(\begin{array}{cc|c} q_2 & 8 & q_1' \\ 1J_2 & 01 & 1J_1 \end{array} \right) \right\}. \qquad (B.6)$$

To get the decay width one needs to average the amplitude squared $|\mathcal{M}|^2$ over the initial and to sum over the final spin and isospin states:

$$\overline{\mathcal{M}^2} = \frac{1}{(2T_1 + 1)(2J_1 + 1)} \sum_{t_2 t_m j_2} \sum_{t_1 j_1} |\mathcal{M}|^2. \qquad (B.7)$$

In (B.6) the dependence on spin and isospin projections is factored out, hence one can perform the summation over final and initial spin and isospin states with the help of the orthogonality relations for the $SU(2)$ Clebsh–Gordan coefficients:

$$\sum_{j_2 j_1} C^{J_1j_1}_{J_2,j_2;1i} C^{J_1j_1}_{J_2,j_2;1j} = \frac{2J_1 + 1}{3} \delta_{ij}, \qquad (B.8)$$

$$\sum_{t_2 t_1 t_m} |C^{T_1t_1}_{T_2,t_2;T_m,t_m}|^2 = 2T_1 + 1. \qquad (B.9)$$

Multiplying $\overline{\mathcal{M}^2}$ by the phase volume we get the final result for the decay rate of $B_1 \to B_2 + M$ in terms of the $SU(3)$

isoscalar factors:

$$\Gamma(B_1 \to B_2 + M) = \frac{3G_0^2}{2\pi(M_2+M_1)^2}|p|^3\frac{M_2}{M_1}$$
$$\times\left\{\frac{r_2}{r_1}\left|\sum_{r_1'}\left(\begin{array}{ccc} r_2 & 8 & \\ Y_2T_2 & Y_mT_m & \end{array}\middle|\begin{array}{c} r_1' \\ Y_1T_1 \end{array}\right)\left(\begin{array}{ccc} r_2 & 8 & \\ 1J_2 & 01 & \end{array}\middle|\begin{array}{c} r_1' \\ 1J_1 \end{array}\right)\right|^2 +\right.$$
$$2A_1\frac{r_2}{\sqrt{q_1r_1}}\sum_{r_1'}\left(\begin{array}{ccc} r_2 & 8 & \\ Y_2T_2 & Y_mT_m & \end{array}\middle|\begin{array}{c} r_1' \\ Y_1T_1 \end{array}\right)\left(\begin{array}{ccc} r_2 & 8 & \\ 1J_2 & 01 & \end{array}\middle|\begin{array}{c} r_1' \\ 1J_1 \end{array}\right)$$
$$\times\sum_{q_1'}\left(\begin{array}{ccc} r_2 & 8 & \\ Y_2T_2 & Y_mT_m & \end{array}\middle|\begin{array}{c} q_1' \\ Y_1T_1 \end{array}\right)\left(\begin{array}{ccc} r_2 & 8 & \\ 1J_2 & 01 & \end{array}\middle|\begin{array}{c} q_1' \\ 1J_1 \end{array}\right) + \quad \text{(B.10)}$$
$$2A_2\frac{\sqrt{r_2q_2}}{r_1}\sum_{r_1'}\left(\begin{array}{ccc} r_2 & 8 & \\ Y_2T_2 & Y_mT_m & \end{array}\middle|\begin{array}{c} r_1' \\ Y_1T_1 \end{array}\right)\left(\begin{array}{ccc} r_2 & 8 & \\ 1J_2 & 01 & \end{array}\middle|\begin{array}{c} r_1' \\ 1J_1 \end{array}\right)$$
$$\times\left.\sum_{r_1'}\left(\begin{array}{ccc} q_2 & 8 & \\ Y_2T_2 & Y_mT_m & \end{array}\middle|\begin{array}{c} r_1' \\ Y_1T_1 \end{array}\right)\left(\begin{array}{ccc} q_2 & 8 & \\ 1J_2 & 01 & \end{array}\middle|\begin{array}{c} q_1' \\ 1J_1 \end{array}\right)\right\}.$$

These $SU(3)$ isoscalar factors can be found in ref. [31].

Similar derivation can be performed for the next-to-leading pseudoscalar couplinigs eqs. (48). The results are summarized in Table 2.

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