User Notes for Relativistic EOS Table

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Abstract

This is a detailed note for users of the Relativistic Equation of State of Nuclear Matter for the use of Nuclear Astrophysics.

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1 Introduction

We write this document as a guide for the use of the relativistic equation of state (EOS) table. This EOS table is constructed in the following range

- temperature $T(MeV)$: $-1.0 \leq \log_{10}(T) \leq 2.0$; mesh of $\log_{10}(T) \sim 0.1$
- proton fraction Y_p: $-2.00 \leq \log_{10}(Y_p) \leq -0.25$; mesh of $\log_{10}(Y_p) \sim 0.025$
- baryon mass density $\rho_B(g/cm^3)$: $5.1 \leq \log_{10}(\rho_B) \leq 15.4$; mesh of $\log_{10}(\rho_B) \sim 0.1$

We also add the results for zero temperature case ($T = 0$) and pure neutron matter case ($Y_p = 0$).

We have worked out consistent calculations for uniform matter and non-uniform matter in the relativistic mean field (RMF) framework [1, 2]. We use the Thomas-Fermi approximation to describe inhomogeneous nuclear matter, which can be modeled as a mixture of free neutrons, free protons, alpha-particles, and a single species of heavy nuclei. For extremely low density and finite temperature, we approximate the nuclear matter as classical ideal gas of free neutrons, free protons, and alpha-particles. Antiparticles have some contribution when temperature is very high. The thermodynamically favorable state is the one that minimizes the free energy density in this model, and we determine the most favorable state of nuclear matter at each temperature, proton fraction, and baryon mass density.

The leptons can be considered as uniform non-interacting relativistic particles, which is relatively easy to deal with. Hence, we provide the baryon EOS without the lepton contribution in this table, users are supposed to add the lepton contribution to the baryon EOS.

This document is arranged as follows. In Sect.2, we describe the RMF theory. In Sect.3, we introduce the ideal gas approximation to be used in low density range. In Sect.4, we describe the Thomas-Fermi approximation, which is used for calculating non-uniform matter. In Sect.5, we make a detailed description of how to work out the EOS table in this model. In Sect.6, we list the definitions of the physical quantities in the EOS table. Sect.7 is devoted to suggestions and discussions for using the EOS table. For users who are not interested in the framework and calculating processes, they can only read the Sect.6 and Sect.7 for the purpose to use the EOS table correctly.

2 Relativistic mean field theory

We adopt the RMF theory with non-linear σ and ω terms to describe homogeneous nuclear matter [3]. We start with the lagrangian given by

$$\begin{aligned}\mathcal{L}_{RMF} &= \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi \\ &+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ &-\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\ &-\frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu}.\end{aligned} \tag{1}$$

Here, ψ denotes a SU(2) baryon field of mass M (proton and neutron); σ,ω^μ, and $\rho^{a\mu}$ are σ,ω, and ρ meson fields with masses m_σ,m_ω, and m_ρ, respectively. $W^{\mu\nu}$ and $R^{a\mu\nu}$ are the antisymmetric field tensors for ω^μ and $\rho^{a\mu}$, which can be written as

$$W^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu, \tag{2}$$

$$R^{a\mu\nu} = \partial^\mu\rho^{a\nu} - \partial^\nu\rho^{a\mu} + g_\rho\epsilon^{abc}\rho^{b\mu}\rho^{c\nu}. \tag{3}$$

In the lagrangian, the constants $g_\sigma, g_\omega, g_\rho$ are the coupling constants for the interactions between mesons and nucleons, the coefficients g_2 and g_3 are the self-coupling constants for σ meson field, while c_3 is the self-coupling constant for ω meson field. It is known that the inclusion of the non-linear σ terms is essential to reproduce the properties of nuclei quantitatively and a reasonable value for the incompressibility, while the non-linear ω term is added in order to reproduce the density dependence of the vector part of the self-energy of the nucleon obtained in the RBHF theory. The lagrangian contains the meson masses, the coupling constants, and the self-coupling constants as parameters. We adopt the parameter set TM1 [3], which is listed in Table 1. With the TM1 parameter set, the symmetry energy is 36.9 MeV and the incompressibility is 281 MeV. The RMF theory with the TM1 provides excellent properties of heavy nuclei including unstable nuclei [3], and it is also shown to reproduce satisfactory agreement with experimental data in the studies of the nuclei with deformed configuration [4] and the giant resonances within the RPA formalism [5]. The properties of the EOS with TM1 at high density were discussed in details in Ref.[6], and the neutron star properties were studied using this EOS in Ref.[2, 6].

Table 1: The parameter set TM1 for the lagrangian in Eq.(1)

Parameter	TM1
$M(MeV)$	938.0
$m_\sigma(MeV)$	511.19777
$m_\omega(MeV)$	783.0
$m_\rho(MeV)$	770.0
g_σ	10.02892
g_ω	12.61394
g_ρ	4.63219
$g_2(fm^{-1})$	-7.23247
g_3	0.61833
c_3	71.30747

Starting with the lagrangian, we derive a set of the Euler-Lagrange equations. The Dirac equation for the nucleon field is given by

$$\left(i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right)\psi = 0, \tag{4}$$

and the Klein-Gordon equations for the meson fields are given by

$$\partial_\nu\partial^\nu\sigma + m_\sigma^2\sigma = -g_\sigma\bar{\psi}\psi - g_2\sigma^2 - g_3\sigma^3, \tag{5}$$

$$\partial_\nu W^{\nu\mu} + m_\omega^2\omega^\mu = g_\omega\bar{\psi}\gamma^\mu\psi - c_3\left(\omega_\nu\omega^\nu\right)\omega^\mu, \tag{6}$$

$$\partial_\nu R^{a\nu\mu} + m_\rho^2\rho^{a\mu} = g_\rho\bar{\psi}\tau_a\gamma^\mu\psi + g_\rho\epsilon_{abc}\rho_\nu^b R^{c\nu\mu}. \tag{7}$$

These equations are coupled nonlinear quantum field equations, which are very difficult to solve exactly. We adopt the relativistic mean field approximation as described in Ref.[7], in which the meson fields are treated as classical fields, and the field operators σ,ω^μ, and $\rho^{a\mu}$ are replaced by their expectation values $\langle\sigma\rangle$, $\langle\omega^\mu\rangle$, and $\langle\rho^{a\mu}\rangle$. In the present study, we consider first static infinite matter so that we obtain simplified equations, where the derivative terms in the Klein-Gordon equations vanish automatically due to the translational invariance of infinite matter.

The spatial components of the vector meson fields vanish under the rotational symmetry. For the isovector-vector meson field $\rho^{a\mu}$, only the third isospin component has a nonvanishing value because of the charge conservation. Hence, the equations for meson fields are reduced to

$$\sigma_0 \equiv \langle\sigma\rangle = -\frac{g_\sigma}{m_\sigma^2}\langle\bar{\psi}\psi\rangle - \frac{1}{m_\sigma^2}\left(g_2\sigma_0^2 + g_3\sigma_0^3\right), \tag{8}$$

$$\omega_0 \equiv \langle\omega^0\rangle = \frac{g_\omega}{m_\omega^2}\langle\bar{\psi}\gamma^0\psi\rangle - \frac{1}{m_\omega^2}c_3\omega_0^3, \tag{9}$$

$$\rho_0 \equiv \langle\rho^{30}\rangle = \frac{g_\rho}{m_\rho^2}\langle\bar{\psi}\tau_3\gamma^0\psi\rangle. \tag{10}$$

The Dirac equation then becomes

$$\left(-i\alpha_k\nabla^k + \beta M^* + g_\omega\omega_0 + g_\rho\tau_3\rho_0\right)\psi_{is} = \varepsilon_{is}\psi_{is}, \tag{11}$$

where the index i denotes the isospin degree of freedom (proton and neutron) and s denotes the index of eigenstates of nucleon, $M^* \equiv M + g_\sigma\sigma_0$ is the effective mass, ε_{is} is the single-particle energy.

Nucleons occupy single-particle orbits with the occupation probability f_{is}. At zero temperature, $f_{is} = 1$ under Fermi surface, while $f_{is} = 0$ above Fermi surface. For finite temperature, the occupation probability is given by Fermi-Dirac distribution,

$$f_{is} = \frac{1}{1+\exp\left[\left(\varepsilon_{is}-\mu_i\right)/T\right]} = \frac{1}{1+\exp\left[\left(\sqrt{k^2+M^{*2}}-\nu_i\right)/T\right]}, \tag{12}$$

$$f_{\bar{i}s} = \frac{1}{1+\exp\left[\left(-\varepsilon_{\bar{i}s}+\mu_i\right)/T\right]} = \frac{1}{1+\exp\left[\left(\sqrt{k^2+M^{*2}}+\nu_i\right)/T\right]}. \tag{13}$$

Here the index i and $\bar{i}$ denote nucleons and antinucleons, ε_{is} and $\varepsilon_{\bar{i}s}$ are the single-particle energy of nucleons and antinucleons, respectively. The relation between the chemical potential μ_i and the kinetic part of the chemical potential ν_i is give by

$$\mu_i = \nu_i + g_\omega\omega_0 + g_\rho\tau_3\rho_0. \tag{14}$$

The chemical potential μ_i is related to the number density of nucleon n_i as

$$n_i = \frac{\gamma}{2\pi^2}\int_0^\infty dk\, k^2\, \left(f_{ik} - f_{\bar{i}k}\right), \tag{15}$$

where γ is the degeneracy factor for the spin degree of freedom ($\gamma = 2$), the quantum number s is replaced by the momentum k when we do the integration in the momentum space instead

of the summation of eigenstates. The equations of meson fields can be written as

$$\sigma_0 = -\frac{g_\sigma}{m_\sigma^2}\sum_i \frac{\gamma}{2\pi^2}\int_0^\infty dk\, k^2 \frac{M^*}{\sqrt{k^2+M^{*2}}}\left(f_{ik}+f_{\bar{i}k}\right) - \frac{1}{m_\sigma^2}\left(g_2\sigma_0^2+g_3\sigma_0^3\right), \tag{16}$$

$$\omega_0 = \frac{g_\omega}{m_\omega^2}\left(n_n+n_p\right) - \frac{c_3}{m_\omega^2}\omega_0^3, \tag{17}$$

$$\rho_0 = \frac{g_\rho}{m_\rho^2}\left(n_n-n_p\right). \tag{18}$$

Here, n_p and n_n are the proton number density and neutron number density as defined in Eq.(15), and we denote $n_B = n_p + n_n$ as the baryon number density. We solve these equations self-consistently. The thermodynamical quantities are described in Ref.[7], and we just write the expressions here. The energy density of nuclear matter is given by

$$\begin{aligned}\epsilon = & \sum_i \frac{\gamma}{2\pi^2}\int_0^\infty dk\, k^2 \sqrt{k^2+M^{*2}}\left(f_{ik}+f_{\bar{i}k}\right) + \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{3}g_2\sigma_0^3 + \frac{1}{4}g_3\sigma_0^4 \\ & + g_\omega\omega_0\left(n_n+n_p\right) - \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{4}c_3\omega_0^4 \\ & + g_\rho\rho_0\left(n_n-n_p\right) - \frac{1}{2}m_\rho^2\rho_0^2,\end{aligned} \tag{19}$$

the pressure of nuclear matter is given by

$$\begin{aligned}p = & \sum_i \frac{\gamma}{6\pi^2}\int_0^\infty dk\, k^2 \frac{k^2}{\sqrt{k^2+M^{*2}}}\left(f_{ik}+f_{\bar{i}k}\right) - \frac{1}{2}m_\sigma^2\sigma_0^2 - \frac{1}{3}g_2\sigma_0^3 - \frac{1}{4}g_3\sigma_0^4 \\ & + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{4}c_3\omega_0^4 + \frac{1}{2}m_\rho^2\rho_0^2,\end{aligned} \tag{20}$$

and the entropy density is calculated by

$$s = \sum_i \frac{\gamma}{2\pi^2}\int_0^\infty dk\, k^2 \left[-f_{ik}\ln f_{ik} - (1-f_{ik})\ln(1-f_{ik}) - f_{\bar{i}k}\ln f_{\bar{i}k} - (1-f_{\bar{i}k})\ln(1-f_{\bar{i}k})\right]. \tag{21}$$

3 Ideal gas approximation

The interaction between nucleons is negligible at low density, so that we can treat nucleons as non-interacting Boltzmann particles at low density. We use the ideal gas approximation to describe the mixed gas of neutrons, protons, and alpha-particles at low density and finite temperature. We note that the connection between the RMF results and the ideal gas approximation is smooth.

For the Boltzmann particles with spin $\frac{1}{2}$, mass M, and number density n_i (i is n or p), we start with the partition function given by

$$Z_i = \left[(n_Q)^{n_i/2}/(n_i/2)!\right]^2 , \tag{22}$$

where we have used the abbreviation $n_Q = \left(\frac{MT}{2\pi\hbar^2}\right)^{3/2}$. The factor $(n_i/2)!$ comes from avoiding the double counting for $n_i/2$ indistinguishable particles of spin up or spin down. We can calculate the free energy density from the partition function as

$$f_i = -T \ln Z_i = -T\, n_i \left[\ln(2n_Q/n_i) + 1\right]. \tag{23}$$

The entropy density is given by

$$s_i = -\left(\frac{\partial f_i}{\partial T}\right)_{n_i} = n_i \left[\ln(2n_Q/n_i) + \frac{5}{2}\right], \tag{24}$$

the internal energy density can be written as

$$\epsilon_i = f_i + T s_i = \frac{3}{2} T\, n_i, \tag{25}$$

the chemical potential is obtained through the relation

$$\mu_i = \left(\frac{\partial f_i}{\partial n_i}\right)_T = -T \ln(2n_Q/n_i), \tag{26}$$

and the pressure is given by

$$p_i = \left[n_i^2 \frac{\partial\,(f_i/n_i)}{\partial n_i}\right]_T = T\, n_i. \tag{27}$$

The partition function of alpha-particles can be written as

$$Z_\alpha = \left[8 n_Q \exp\left(B_\alpha/T\right)\right]^{n_\alpha} / n_\alpha !, \tag{28}$$

where n_α is the alpha-particle number density, $B_\alpha = 28.3 MeV$ is the binding energy of an alpha-particle relative to free neutrons taken from Ref.[8], then the thermodynamical quantities of alpha-particles are given by

$$f_\alpha \;=\; -T\, n_\alpha \left[\ln(8n_Q/n_\alpha) + 1\right] - n_\alpha B_\alpha, \tag{29}$$

$$s_\alpha = n_\alpha \left[\ln(8n_Q/n_\alpha) + \frac{5}{2}\right], \tag{30}$$

$$\epsilon_\alpha = \frac{3}{2}T\, n_\alpha - n_\alpha B_\alpha, \tag{31}$$

$$\mu_\alpha = -T\ln(8n_Q/n_\alpha) - B_\alpha, \tag{32}$$

$$p_\alpha = T\, n_\alpha. \tag{33}$$

For the mixed gas with neutron number density n_n, proton number density n_p, and alpha-particle number density n_α, the free energy density is given by

$$f = f_n + f_p + f_\alpha. \tag{34}$$

We have to take into account the volume of alpha-particle, otherwise the alpha-particle fraction will become some big number at high densities, where the alpha-particles should actually disappear. After we take into account the volume excluded by the alpha-particles, the free energy densities in Eq.(34) are given by

$$f_n = -(1-u)\,T\,\tilde{n}_n\left[\ln(2n_Q/\tilde{n}_n) + 1\right], \tag{35}$$

$$f_p = -(1-u)\,T\,\tilde{n}_p\left[\ln(2n_Q/\tilde{n}_p) + 1\right], \tag{36}$$

$$f_\alpha = -(1-u)\,\left\{T\,\tilde{n}_\alpha\left[\ln(8n_Q/\tilde{n}_\alpha) + 1\right] - \tilde{n}_\alpha B_\alpha\right\}, \tag{37}$$

where $u = n_\alpha\, v_\alpha$ is the fraction of space occupied by alpha-particles, the effective volume of an alpha-particle, $v_\alpha = 24fm^{-3}$, is taken from Ref.[8]. We denote the effective number densities of neutrons, protons, and alpha-particles as $\tilde{n}_n = n_n/(1-u)$, $\tilde{n}_p = n_p/(1-u)$, and $\tilde{n}_\alpha = n_\alpha/(1-u)$. The inclusion of the volume excluded by the alpha-particles has negligible effect in the low density region, while it is necessary for the calculation at high density. Actually, we just use Eqs.(35) and (36) for the case of $\tilde{n}_n + \tilde{n}_p < 10^{-5}fm^{-3}$, we have to use the RMF theory to describe nucleons when $\tilde{n}_n + \tilde{n}_p > 10^{-5}fm^{-3}$, where the nucleon free energy density is obtained by the RMF input table. We use Eq.(37) to calculate the free energy contributed by alpha-particles for whole density range, it is considered as a reasonable approximation because the number density of alpha-particles is not so large even though the average baryon density is high, and the alpha-particle fraction tends to vanish in the high density limit.

For very high temperature, we need to consider the contribution from antiparticles. The alpha-particle fraction is very small at high temperature, so we can treat the matter as uniform

matter of nucleons and antinucleons. We have included the freedom of antiparticles in the RMF theory, but the RMF code meets some difficulty at extremely low density, so we need to take some approximation in order to describe the range of low density and high temperature. We take the non-relativistic approximation of the RMF theory at low density, where the effective mass $M^* \approx M$, $< mesons > \approx 0$, and $\sqrt{k^2 + M^{*2}} \approx M + \frac{k^2}{2M}$. Then the occupation probability of the Fermi-Dirac distribution become

$$f_{is} = AC_i \exp\left(-k^2/2MT\right), \tag{38}$$

$$f_{\bar{i}s} = \frac{A}{C_i} \exp\left(-k^2/2MT\right), \tag{39}$$

where $A = \exp(-M/T)$, and $C_i = \exp(\mu_i/T)$ with μ_i being the chemical potential of protons $(i = p)$ or neutrons $(i = n)$, which is related to the number density n_i as

$$n_i = 2A\left(C_i - \frac{1}{C_i}\right)\left(\frac{MT}{2\pi}\right)^{3/2}. \tag{40}$$

The thermodynamical quantities can be approximated as

$$\begin{aligned} \epsilon_i &= 2A\left(C_i + \frac{1}{C_i}\right)\left(\frac{MT}{2\pi}\right)^{3/2}\left(\frac{3}{2}T\right), \\ p_i &= 2A\left(C_i + \frac{1}{C_i}\right)\left(\frac{MT}{2\pi}\right)^{3/2} T, \\ s_i &= 2A\left[\frac{5}{2}\left(C_i + \frac{1}{C_i}\right) - C_i \ln(AC_i) - \frac{1}{C_i}\ln\left(\frac{A}{C_i}\right)\right]\left(\frac{MT}{2\pi}\right)^{3/2}. \end{aligned} \tag{41}$$

The contribution from antiparticles is negligible when $C_i \gg \frac{1}{C_i}$, where the above expressions agree with the ideal gas approximation.

4 Thomas-Fermi approximation

In the range, $T < 15MeV$ and $\rho_B < 10^{14.2} g/cm^3$, where heavy nuclei may be formed in order to lower the free energy, we perform the Thomas-Fermi calculation based on the work done by Oyamatsu [9]. In this case, the non-uniform matter can be modeled as a mixture of free neutrons, free protons, alpha-particles, and a single species of heavy nuclei, while the leptons can be treated as uniform non-interacting particles separately. For the system with fixed proton fraction, the leptons play no role in the minimization of the free energy. Hence we mainly pay

attention to baryon contribution in this calculation. Hereafter, we will not mention the leptons frequently, while we should keep in mind that there exists an uniform lepton gas existing everywhere.

We assume that each heavy spherical nucleus is located in the center of a charge-neutral cell consisting of a vapor of neutrons, protons and alpha-particles. The nuclei form the body-centered-cubic (BCC) lattice to minimize the Coulomb lattice energy. It is useful to introduce the Wigner-Seitz cell to simplify the energy of the unit cell. The Wigner-Seitz cell is a sphere whose volume is the same as the unit cell in the BCC lattice. We define the lattice constant a as the cubic root of the cell volume. Then, we have

$$V_{cell} = a^3 = N_B/n_B, \tag{42}$$

where N_B and n_B are the baryon number per cell and the average baryon number density, respectively. We define the baryon mass density as $\rho_B = m_u n_B$ with m_u being the atomic mass unit. We calculate the Coulomb energy using this Wigner-Seitz approximation and add a correction energy for the BCC lattice [9]. This correction energy is negligible unless the nuclear size is comparable to the cell size.

We assume the nucleon distribution functions $n_i(r)$ ($i = n$ for neutron, $i = p$ for proton) in the Wigner-Seitz cell as

$$n_i(r) = \begin{cases} (n_i^{in} - n_i^{out})\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out}, & 0 \le r \le R_i \\ n_i^{out}, & R_i \le r \le R_{cell} \end{cases}, \tag{43}$$

where r represents the distance from the center of the nucleus and R_{cell} is the radius of the Wigner-Seitz cell defined by the relation,

$$V_{cell} \equiv \frac{4\pi}{3} R_{cell}^3. \tag{44}$$

The density parameters n_i^{in} and n_i^{out} are the densities at $r = 0$ and $r \ge R_i$, respectively. The parameters R_i and t_i determine the boundary and the relative surface thickness of the heavy nucleus.

For the distribution function of alpha-particle $n_\alpha(r)$, which should decrease as r approaches

the center of the heavy nucleus, we assume

$$n_\alpha(r) = \begin{cases} -n_\alpha^{out}\left[1-\left(\frac{r}{R_p}\right)^{t_p}\right]^3 + n_\alpha^{out}, & 0 \le r \le R_p \\ n_\alpha^{out}, & R_p \le r \le R_{cell} \end{cases}, \tag{45}$$

which will give $n_\alpha(r=0)=0$ and $n_\alpha(r>R_p)=n_\alpha^{out}$. Here we use the same parameters R_p and t_p for both proton distribution function and alpha-particle distribution function in order to avoid too many parameters in the minimization procedure. The parameters R_n and t_n may be a little different from R_p and t_p due to the additional neutrons forming a neutron skin in the surface region. For the system with fixed temperature T, proton fraction Y_p, and baryon mass density ρ_B, there are eight independent parameters in the ten variables; $a, n_n^{in}, n_n^{out}, R_n, t_n, n_p^{in}, n_p^{out}, R_p, t_p, n_\alpha^{out}$. The thermodynamically favorable state is the one that minimizes the free energy density with respect to these eight independent parameters.

In this model the free energy density contributed by baryons is given as

$$f \;=\; F_{cell}\,/\,a^3 \;=\; (\,E_{cell} \;-\; T\,S_{cell}\,)\,/\,a^3, \tag{46}$$

where the free energy per cell F_{cell} can be written as

$$F_{cell} = (E_{bulk} + E_s + E_C) - T\,S_{cell} = F_{bulk} + E_s + E_C. \tag{47}$$

The bulk energy E_{bulk}, entropy S_{cell}, and bulk free energy F_{bulk} are calculated by

$$E_{bulk} \;=\; \int_{cell} \epsilon\,(\,n_n\,(r)\,,\,n_p\,(r)\,,\,n_\alpha\,(r)\,)\,d^3r, \tag{48}$$

$$S_{cell} \;=\; \int_{cell} s\,(\,n_n\,(r)\,,\,n_p\,(r)\,,\,n_\alpha\,(r)\,)\,d^3r, \tag{49}$$

$$F_{bulk} \;=\; \int_{cell} f\,(\,n_n\,(r)\,,\,n_p\,(r)\,,\,n_\alpha\,(r)\,)\,d^3r. \tag{50}$$

Here $\epsilon\,(\,n_n\,(r)\,,\,n_p\,(r)\,,\,n_\alpha\,(r)\,)$, $s\,(\,n_n\,(r)\,,\,n_p\,(r)\,,\,n_\alpha\,(r)\,)$, and $f\,(\,n_n\,(r)\,,\,n_p\,(r)\,,\,n_\alpha\,(r)\,)$ are the local energy density, entropy density, and free energy density at the radius r, where it can be considered as a mixed uniform matter of neutrons, protons, and alpha-particles. Note that these local densities at each radius are calculated by treating neutrons and protons in RMF theory as described in Sect.2 for the case of $n_n(r)+n_p(r) > 10^{-5} fm^{-3}$, while we can use the method described in the Sect.3 for low density case of $n_n(r)+n_p(r) < 10^{-5} fm^{-3}$.

As for the surface energy term E_s due to the inhomogeneity of nucleon distribution, we take a simple form as,

$$E_s = \int_{cell} F_0 \mid \nabla \left(n_n \left(r \right) + n_p \left(r \right) \right) \mid^2 d^3r. \tag{51}$$

The parameter $F_0 = 70\, MeV \cdot fm^5$ is determined by doing the Thomas-Fermi calculations of finite nuclei so as to reproduce the gross properties of nuclear mass, charge radii and the beta stability line as described in the appendix in Ref.[9].

The Coulomb energy per cell E_C is calculated using the Wigner-Seitz approximation and add a correction term for the BCC lattice [9]

$$E_C = \frac{1}{2} \int_{cell} e \left[n_p \left(r \right) + 2 n_\alpha \left(r \right) - n_e \right] \phi(r) d^3r \; + \; \triangle E_C, \tag{52}$$

where $\phi(r)$ stands for the electrostatic potential calculated in the Wigner-Seitz approximation, n_e is the electron number density of uniform electron gas ($n_e = Y_p \, n_B$), $\triangle E_C$ is the correction term for the BCC lattice, which can be approximated as

$$\triangle E_C = C_{BCC} \frac{(Z_{non} e)^2}{a}, \tag{53}$$

here a is the lattice constant as defined in Eq.(42), the coefficient $C_{BCC} = 0.006562$ is taken from Ref.[9], Z_{non} is the non-uniform part of the charge number per cell given by

$$Z_{non} = \int_0^{R_p} (n_p^{in} - n_p^{out} - 2 n_\alpha^{out}) \left[1 - \left(\frac{r}{R_p} \right)^{t_p} \right]^3 4\pi r^2 dr. \tag{54}$$

Because of the long-range nature of the Coulomb interaction, the Coulomb energy will depend on the lattice type. This dependence was discussed in more details in Ref.[9]. The system prefers the BCC lattice because the BCC lattice gives the lowest Coulomb energy.

5 Calculation procedure in details

We calculate the EOS table in the range mentioned in Sect.1. For each T, Y_p, and ρ_B, we perform the minimization of the free energy for non-uniform matter using the Thomas-Fermi method described in Sect.4. We also do the minimization for the uniform matter with respect to converting two protons and two neutrons into an alpha-particle, there is only one independent parameter, alpha-particle fraction X_α, in this minimization procedure. Here the phase of heavy

nuclei formed together with free nucleons and alpha-particles is referred to as non-uniform matter while the phase of nucleons mixed with alpha-particles without heavy nuclei is referred to as uniform matter. By comparing the free energies of the uniform matter and non-uniform matter, we determine the most favorable state of nuclear matter at each T, Y_p, and ρ_B. The density of the phase transition between uniform matter and non-uniform matter depends on both T and Y_p. The non-uniform matter phase exists only in low temperature range ($T < 15MeV$).

We construct the EOS table in three dimensions (T, Y_p, and ρ_B), first fix T, second fix Y_p, third fix ρ_B. We should finish the following steps in order to work out one EOS table with fixed T where the non-uniform matter phase exists.

- 1 calculate input table in RMF theory for the use of minimization
- 2 do minimization for uniform matter at low density
- 3 do minimization for non-uniform matter at medium density
- 4 calculate RMF result for uniform matter without alpha-particles at high density
- 5 determine the phase transition and connect the free energy
- 6 calculate pressure and chemical potentials from free energy
- 7 construct the finial EOS table at T

Now, we explain each step in more details.

- 1 The calculation of the minimization procedure needs the free energy per baryon as a function of n_n and n_p as input table, and this input table should be big enough in order to get good numerical accuracy in doing linear approximation. We calculate this input table in the RMF theory which describes the homogeneous matter of protons and neutrons. This input

table is covering the following range

$Y_p = \frac{n_p}{n_n+n_p}$	$from$ 0.0	to 0.5	$mesh = 0.0005$	$point = 1001$
$n_B = n_n + n_p(fm^{-3})$	$from$ 0.000010	to 0.000100	$mesh = 0.000001$	$point = 91$
	$from$ 0.000110	to 0.001000	$mesh = 0.000010$	$point = 90$
	$from$ 0.001100	to 0.060000	$mesh = 0.000100$	$point = 590$
	$from$ 0.061000	to 0.160000	$mesh = 0.001000$	$point = 100$

The density meshes are determined by the behavior of the free energy per baryon, which is necessary to get good accuracy for the minimization results. In this input table, we list the quantities of the energy per baryon E and the entropy per baryon S, then free energy per baryon is $F = E - TS$. The data at $n_n + n_p > 0.16$ is not necessary for the minimization codes, while we use ideal gas approximation to describe neutrons and protons when $n_n + n_p < 0.00001 fm^{-3}$.

• 2 For low density and finite temperature, the thermodynamically favorable state might be the uniform matter, which is a mixed gas of neutrons, protons, and alpha-particles. For each Y_p and ρ_B, we have to do minimization in order to find out the alpha-particle fraction having the minimum free energy. The free energy of the mixed gas is calculated by Eq.(34) as a function of n_n, n_p, and n_α, where the RMF input table is used instead of Eqs.(35) and (36) when $n_n + n_p > 10^{-5} fm^{-3}$. Because $n_B = n_n + n_p + 4n_\alpha$ and $Y_p = (n_p + 2n_\alpha)/n_B$ are fixed in this case, there is only one independent parameter, the alpha-particle fraction $X_\alpha = 4n_\alpha/n_B$ as the independent parameter in this minimization procedure.

• 3 The non-uniform matter phase exists in some middle density range where the heavy nuclei can be formed in order to lower the free energy. The starting density of the non-uniform matter phase depends on the temperature strongly, while the ending density is nearly independent of the temperature. Both of them have some weak dependence on Y_p. So we should first try to find out the starting density at T, then calculate the non-uniform matter results in this range by using the minimization codes with respect to some independent parameters as described in Sect.4. Actually, we have to use a few codes to do this calculation because of the numerical difficulty in minimization procedure. As the density ρ_B increases with fixed T and Y_p, the heavy nucleus fraction increases, while the nucleon fractions and alpha-particle fraction decrease. When one of them decreases to some small number like 10^{-5}, it will bring

Table 2: The minimization codes for non-uniform matter phase

code name	compositions	of non-uniform	matter		parameter number
npah	free neutrons,	free protons,	alpha particles,	heavy nuclei	8
pah		free protons,	alpha particles,	heavy nuclei	7
nah	free neutrons,		alpha particles,	heavy nuclei	7
nph	free neutrons,	free protons,		heavy nuclei	7
nh	free neutrons,			heavy nuclei	6
ph		free protons,		heavy nuclei	6
ah			alpha particles,	heavy nuclei	6
h				heavy nuclei	5

some difficulty for the minimization code, and lose good accuracy in the results. Then we have to use another code which take this composition out by deleting one independent parameter. It may run into difficulty again as density increases, then we need to delete one more parameter. We list all codes used in this procedure in table 2. We determine which code we should use by comparing the free energies, we choose the one with lowest free energy. Generally speaking, the alpha-particles will disappear first, then free neutrons ($Y_p > 0.45$) or free protons ($Y_p < 0.45$) disappear, at the end only heavy nuclei are formed without free particles outside (this pure nucleus phase doesn't occur at $Y_p < 0.3$). When $T \geq 5MeV$, the calculation becomes relatively easy because the favorable state is always the one including all compositions. As temperature increases, the heavy nucleus fraction decreases, and it will disappear completely when $T \geq 15MeV$.

• 4 As density ρ_B increases beyond $10^{14.2} g/cm^3$, the non-uniform matter phase disappears, it becomes homogeneous matter of neutrons and protons with less alpha-particles. We switch off alpha-particle freedom when $X_\alpha < 10^{-4}$, then we can use RMF code to do the calculation in this high density range.

• 5 We judge the phase transitions in whole density range at each Y_p with fixed T by

comparing the free energies calculated with the uniform matter codes and non-uniform matter codes in table 2, all the coming results in step 2, 3, and 4 will be the input files for this judging code. We get two output files from it, eos.fe and eos.tab, eos.fe includes only free energy per particle F which will be used for calculating pressure and chemical potentials, eos.tab includes all quantities which are needed for the finial EOS table.

• 6 We calculate pressure and chemical potentials through the following thermodynamical relations

$$p = \left[n_B^2 (\partial F/\partial n_B) \right]_{T,Y_p}, \tag{55}$$

$$\mu_n = \left[\partial(n_B F)/\partial n_n \right]_{T,n_p}, \tag{56}$$

$$\mu_p = \left[\partial(n_B F)/\partial n_p \right]_{T,n_n}, \tag{57}$$

where F is the free energy per baryon from the file eos.fe obtained in step 5, n_B is the baryon number density related to the baryon mass density ρ_B as $\rho_B = m_u n_B$ with $m_u = 931.49432 MeV$ being the atomic mass unit, $n_n = (1-Y_p)n_B$ and $n_p = Y_p n_B$ are the neutron number density and proton number density. The numerical derivatives are calculated by five point differentiation method, where the five points F around ρ_B and Y_p are obtained by spline method. For pressure, we use one dimension spline method because the differentiation in Eq.(55) is under the condition of fixed Y_p. We have to use two dimension spline method to get the five point F for calculating chemical potential which is under the condition of fixed n_p or fixed n_n. In high density range where we switch off alpha-particles freedom, we take the exact results calculated in the RMF theory by Eq.(20) and Eq.(14) instead of the numerical differentiations. The output files of this step are pre.tab for pressure and che.tab for chemical potentials.

• 7 We combine the files eos.tab, pre.tab, and che.tab in order to get the finial EOS table at T. Because of the use of many codes in table 2 for non-uniform matter phase, some fractions might be equal to zero at some densities in the file eos.tab, this is due to some compositions taken out for keeping good accuracy. However, we know the fraction should be some finite number at finite temperature if the chemical potential is finite. In this case, we calculate the fraction X_i by chemical potential μ_i and write it in the finial EOS table instead of $X_i = 0$ in eos.tab

$$X_i = (n_i^{out} V^{out})/(n_B V_{cell}), \tag{58}$$

where V_{cell} is the cell volume as defined in Eq.(42), $V^{out} = V_{cell} - \frac{4\pi}{3}R_A^3$ is the volume outside the heavy nucleus with R_A being the maximum between R_p and R_n considered as the boundary of the heavy nucleus. n_i^{out} is the free neutron number density ($i = n$) or free proton number density ($i = p$). For alpha-particle ($i = \alpha$), n_i^{out} should be 4 times of the alpha-particle number density outside, because there are 2 protons and 2 neutrons inside one alpha-particle. The number densities outside the heavy nucleus can be obtained through the relations,

$$n_n = 2\left(\frac{MT}{2\pi\hbar^2}\right)^{3/2}\exp(\frac{\mu_n}{T}), \tag{59}$$

$$n_p = 2\left(\frac{MT}{2\pi\hbar^2}\right)^{3/2}\exp(\frac{\mu_p}{T}), \tag{60}$$

$$n_\alpha = 8\left(\frac{MT}{2\pi\hbar^2}\right)^{3/2}\exp(\frac{\mu_\alpha + B_\alpha}{T}), \tag{61}$$

here the alpha-particle chemical potential is given by the equilibrium condition, $\mu_\alpha = 2\mu_n + 2\mu_p$. The finial EOS table at this T is the output file of this step.

We have to go through these steps for each T where non-uniform matter phase exists. We also need to add the $T = 0$ case, which is calculated by similar procedure as the above steps. There are two difference between $T = 0$ and $T \neq 0$ cases. (1) There are no free protons and alpha-particles outside the heavy nucleus at $T = 0$, so we needn't calculate so many codes in table 2. The starting density of the non-uniform matter phase is below $10^5 g/cm^3$ at $T \leq 0.4MeV$ and $T = 0$, so we can skip the step 2 in this case (2) The ideal gas approximation is not available for $T = 0$, we use a function Ck_f^2 to express the non-relativistic kinetic energy at low density instead of $\frac{3}{2}T$ in ideal gas approximation, note that the free energy at $T = 0$ is equal to the internal energy, the coefficient C is fixed by the first point in the RMF table in order to get smooth connection.

When the non-uniform matter phase disappears at high temperature, the calculation becomes relatively easy, the step 3 is skipped which is the most extensive calculation. As temperature increases, the alpha-particle contribution gets less and less, we actually do this uniform matter calculation dependent on T. For $T = 15, 20MeV$, we still use the large RMF input table as described in step 1 to calculate the mixed gas phase including alpha-particles. When we write the finial EOS table, we switch off the alpha-particle contribution in the range of

$X_\alpha < 10^{-4}$ due to accuracy requirement. For $T = 25, 32, 40 MeV$, we use a small RMF input table for the calculation of the fractions (X_n, X_p, and X_α) without alpha-particle contribution for other quantities, actually the alpha-particle contribution is really small in this high temperature range. As for $T = 50, 63, 80, 100 MeV$, we only use a small RMF input table to calculate X_α, while all other quantities are coming from the ideal gas approximation (including antiparticles) or the RMF theory. The connection between ideal gas approximation and RMF theory is also dependent on T, and we try to find out the density where they can be connected with each other smoothly.

At the end, we should add the pure neutron matter results ($Y_p = 0$), there is only one composition in this case, we use the RMF theory to do this calculation in high density range, while adopt ideal gas approximation for low density range, then combine them to get finial EOS table for $Y_p = 0$.

6 Resulting EOS table

In this section, we will explain the resulting EOS table and list the definitions of the quantities tabulated. We give the resulting EOS by three tables, which are named

(1) eos.tab (main EOS table, size: 54.5MB)

- temperature $T(MeV)$: $-1.0 \leq \log_{10}(T) \leq 2.0$; mesh of $\log_{10}(T) \sim 0.1$
- proton fraction Y_p: $-2.00 \leq \log_{10}(Y_p) \leq -0.25$; mesh of $\log_{10}(Y_p) \sim 0.025$
- baryon mass density $\rho_B(g/cm^3)$: $5.1 \leq \log_{10}(\rho_B) \leq 15.4$; mesh of $\log_{10}(\rho_B) \sim 0.1$

(2) eos.t00 (EOS at $T = 0$, same range of Y_p and ρ_B, size: 1.76MB)

(3) eos.yp0 (EOS at $Y_p = 0$, same range of T and ρ_B, size: 0.79MB)

We putted them in the website after compressed by 'gzip'

http://www-server.rcnp.osaka-u.ac.jp/~shen/table/

In the model used in this calculation, we consider four compositions which are free neutrons, free protons, alpha-particles, and a single species of heavy nuclei. Generally speaking, there are

two phases existing in the range covered by this table, we call the phase where the heavy nuclei are formed as non-uniform matter phase, and call the phase without heavy nuclei as uniform matter phase. The phase transition can be found by the variation of the heavy nucleus fraction between $X_A = 0$ and $X_A \neq 0$.

We write the table in the following order, first fix T which is noted at the top of each block, second fix Y_p, third fix ρ_B. The blocks with different T are divided by line 'ccccccc'. For each T, Y_p, and ρ_B, we tabulate all the quantities in one line in the order

- (1) logarithm of baryon mass density: $\log_{10}(\rho_B)$ $[g/cm^3]$

- (2) baryon number density: n_B $[fm^{-3}]$

  The baryon number density is related to the baryon mass density as $\rho_B = m_u n_B$ with $m_u = 931.49432 MeV$ being the atomic mass unit taken from Ref.[10].

- (3) logarithm of proton fraction: $\log_{10}(Y_p)$

  The proton fraction Y_p of uniform matter is defined by

$$Y_p = \frac{n_p + 2n_\alpha}{n_B} = \frac{n_p + 2n_\alpha}{n_n + n_p + 4n_\alpha} \tag{62}$$

  where n_p is the proton number density, n_n is the neutron number density, n_α is the alpha-particle number density, and n_B is the baryon number density.

  For non-uniform matter case, Y_p is the average proton fraction defined by

$$Y_p = \frac{N_p}{N_B} \tag{63}$$

  where N_p is the proton number per cell, N_B is the baryon number per cell;

$$N_p = \int_{cell} [\, n_p(r) + 2n_\alpha(r) \,] \, d^3r, \tag{64}$$

$$N_B = \int_{cell} [\, n_n(r) + n_p(r) + 4n_\alpha(r) \,] \, d^3r, \tag{65}$$

  $n_p(r)$ and $n_n(r)$ are the proton and neutron density distribution function given in Eq.(43), $n_\alpha(r)$ is the alpha-particle density distribution function given in Eq.(45).

- (4) proton fraction: Y_p

- (5) free energy per baryon: $F\ [MeV]$

  The free energy per baryon is defined as relative to the free nucleon mass $M = 938.0MeV$ in TM1 parameter set as

$$F = \frac{f}{n_B} - M. \tag{66}$$

- (6) internal energy per baryon: $E_{int}\ [MeV]$

  The internal energy per baryon is defined as relative to the atomic mass unit m_u as

$$E_{int} = \frac{\epsilon}{n_B} - m_u. \tag{67}$$

- (7) entropy per baryon: $S\ [k_B]$

  The entropy per baryon is related to the entropy density via

$$S = \frac{s}{n_B}. \tag{68}$$

- (8) mass number of heavy nucleus: A

  The mass number of heavy nucleus is defined by

$$A = \int_0^{R_A} [\, n_n\,(r) + n_p\,(r)\,]\, 4\pi r^2 dr, \tag{69}$$

  where $n_n(r)$ and $n_p(r)$ are the density distribution functions in the Thomas-Fermi approximation as given in Eq.(43), R_A is the maximum between R_p and R_n, which is considered as the boundary of the heavy nucleus.

- (9) charge number of heavy nucleus: Z

  The charge number of heavy nucleus is defined by

$$Z = \int_0^{R_A} n_p\,(r)\ 4\pi r^2 dr, \tag{70}$$

- (10) effective mass: $M^*\ [MeV]$

  The effective mass is obtained in the RMF theory for uniform matter. In non-uniform matter case, the effective mass is a function of space due to inhomogeneity of nucleon

distribution, so it is meaningless to list the effective mass for non-uniform matter. We replace the effective mass M^* by the free nucleon mass M in the non-uniform matter phase.

- (11) free neutron fraction: X_n

  The free neutron fraction is given by

$$X_n = (n_n^{out} V^{out})/(n_B V_{cell}) \tag{71}$$

  where V_{cell} is the cell volume as defined in Eq.(42), $V^{out} = V_{cell} - \frac{4\pi}{3} R_A^3$ is the volume outside heavy nucleus, n_n^{out} is the free neutron number density outside heavy nucleus, while n_B is the average baryon number density.

- (12) free proton fraction: X_p

  The free proton fraction is given by

$$X_p = (n_p^{out} V^{out})/(n_B V_{cell}) \tag{72}$$

  with n_p^{out} is the free proton number density outside heavy nucleus.

- (13) alpha-particle fraction: X_α

  The alpha-particle fraction is defined by

$$X_\alpha = 4N_\alpha/(n_B V_{cell}) \tag{73}$$

  where N_α is the alpha-particle number per cell obtained by

$$N_\alpha = \int_{cell} n_\alpha(r)\, d^3 r, \tag{74}$$

  $n_\alpha(r)$ is the alpha-particle density distribution function given in Eq.(45).

- (14) heavy nucleus fraction: X_A

  The heavy nucleus fraction is defined by

$$X_A = A/(n_B V_{cell}) \tag{75}$$

  where A is the mass number of heavy nucleus as defined in Eq.(69).

- (15) pressure: $p\ [MeV/fm^3]$

  The pressure is calculated through the following thermodynamical relation

$$p = \left[n_B^2 (\partial F/\partial n_B) \right]_{T,Y_p} \tag{76}$$

- (16) chemical potential of neutron: $\mu_n\ [MeV]$

  The chemical potential of neutron relative to the free nucleon mass M is calculated through the thermodynamical relation

$$\mu_n = \left[\partial(n_B F)/\partial n_n \right]_{T,n_p} \tag{77}$$

  here $n_n = (1 - Y_p)\, n_B$.

- (17) chemical potential of proton: $\mu_p\ [MeV]$

  The chemical potential of proton relative to the free nucleon mass M is calculated through the thermodynamical relation

$$\mu_p = \left[\partial(n_B F)/\partial n_p \right]_{T,n_n} \tag{78}$$

  here $n_p = Y_p\, n_B$.

We have done the following check for the EOS table,

(1) the consistency of the thermodynamical quantities,

$$F + \frac{p}{n_B} = \mu_n(1 - Y_p) + \mu_p Y_p, \tag{79}$$

(2) the consistency of the fractions,

$$X_n + X_p + X_\alpha + X_A = 1, \tag{80}$$

(3) the consistency of the relation between F, E_{int}, and S,

$$F = E_{int} - TS + m_u - M. \tag{81}$$

Generally these consistency relations can be satisfied within the accuracy of 0.001. Physical constants to convert units are taken from Ref.[10].

We also put the minimization resulting parameters in one table named par.tab (size: 33.5MB) in the website

http://www-server.rcnp.osaka-u.ac.jp/~shen/parameter/

We write the parameters in the following order, $(1)dnn = n_n^{in} - n_n^{out}$, $(2)t_n$, $(3)R_n$, $(4)dnp = n_p^{in} - n_p^{out}$, $(5)t_p$, $(6)R_p$, $(7)a$, $(8)n_\alpha^{out}$, $(9)n_n^{out}$, $(10)n_p^{out}$. This file was written in the same range of Y_p and ρ_B, but T is below $15MeV$ where the non-uniform matter phase exists. The parameters, $(1)dnn = n_n^{in} - n_n^{out}$, $(2)t_n$, $(3)R_n$, $(4)dnp = n_p^{in} - n_p^{out}$, $(5)t_p$, $(6)R_p$, $(7)a$, are written as zero in uniform matter phase.

7 Suggestions for using the EOS table

This relativistic EOS table is designed for the use of supernova simulations, so we do the calculation at each T, Y_p, and ρ_B. We note here that the meshes of $\log_{10}(\rho_B)$ and $\log_{10}(T)$ are not exactly equal meshes in the EOS table because of the difficulty in numerical calculation.

In order to perform the minimization of the free energy for non-uniform matter, we have to parametrize the nucleon distributions, so that some quantities in the EOS table like A, Z, X_n, X_p, X_α, and X_A are dependent on this parametrization method. One should keep in mind their definitions when these quantities are used. We suggest users pay more attention to the thermodynamical quantities like F, E_{int}, S, P, μ_n, and μ_p, which are supposed not to be sensitive to the parametrization method. The thermodynamical quantities are found more smooth in the resulting EOS, while the use of many codes in table 2 may bring some fluctuations for the fractions, especially in the range of $2MeV < T < 5MeV$, where the connection is the most complicated.

We hope this relativistic EOS table can be used in your calculation successfully, and comparing with other EOS to see the relativistic effect. We emphasize again that this EOS table includes only baryon contribution, you have to add the lepton EOS before you use it.

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