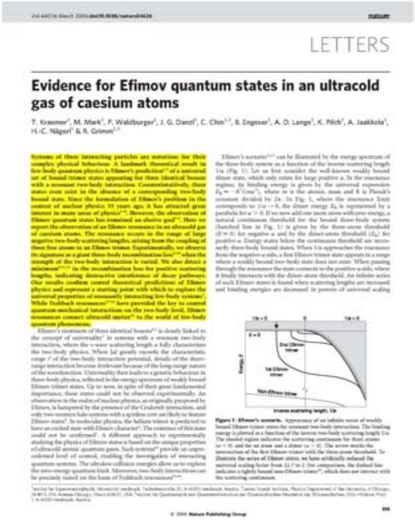
# Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer et al., Nature 440, 315-318 (2006).

reported by A. Tamii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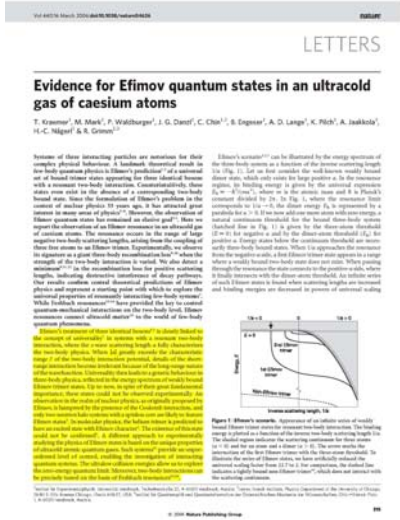
**Evidence for Efimov quantum states in an ultracold gas of caesium atoms**

abstract:

Systems of three interacting particles are notorious for their complex physical behaviour. A landmark theoretical result in few-body quantum physics is Efimov's prediction of a universal set of (infinite number of ) bound trimer states appearing for three identical bosons with a resonant two-body interaction. Counterintuitively, these states even exist in the absence of a corresponding two-body bound state. Since the formulation of Efimov's problem in the context of nuclear physics 35 years ago, it has attracted great interest in many areas of physics. However, the observation of Efimov quantum states has remained an elusive goal. Here we report the observation of an Efimov resonance in an ultracold gas of caesium atoms. The resonance occurs in the range of large negative two-body scattering lengths, arising from the coupling of three free atoms to an Efimov trimer. Experimentally, we observe its signature as a giant three-body recombination loss when the strength of the two-body interaction (scattering length 'a') is varied. We also detect a minimum in the recombination loss for positive scattering lengths, indicating destructive interference of decay pathways. Our results confirm central theoretical predictions of Efimov physics and represent a starting point with which to explore the universal properties of resonantly interacting few-body systems. While Feshbach resonances have provided the key to control quantum-mechanical interactions on the two-body level, Efimov resonances connect ultracold matter to the world of few-body quantum phenomena.

# Introduction: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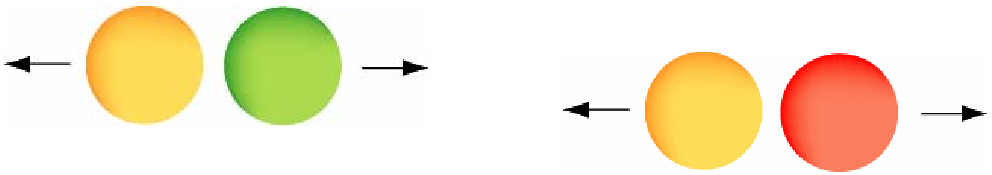
Evidence for Efimov quantum states in an ultracold gas of caesium atoms

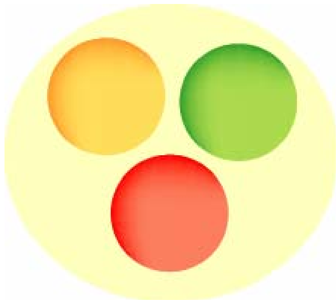
## Efimov States

[1] V. Efimov, PLB33,563(1970), 他[2,16]

3粒子のいずれの2つのペアも束縛状態を持たない場合



3粒子状態は束縛状態を持つか？



(低Energy、$|a|>>r_0$ の極限で考察)

答え：束縛状態を無限に持つ。

また、束縛状態のシリーズは、特徴的な定数で記述される。

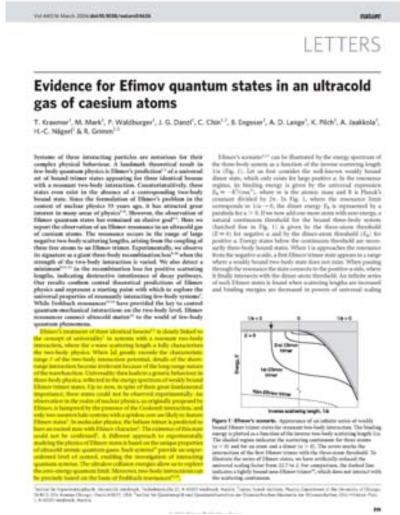
## Introduction:

‘Universality’

“properties of the many-body system that are determined (only) by the scattering length are called *universal*”

[7] E. Braaten and H.-W. Hammer, arXiv:cond-mat/0410417.

→ expansion in *ak*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**Evidence for Efimov quantum states in an ultracold gas of caesium atoms**

*c.f.*

有効距離の理論

到達距離の短い中心力相互作用によるS波の弾性散乱の振る舞いは、低エネルギーの極限では、２つの定数（$a, r_0$)により特徴づけられる。

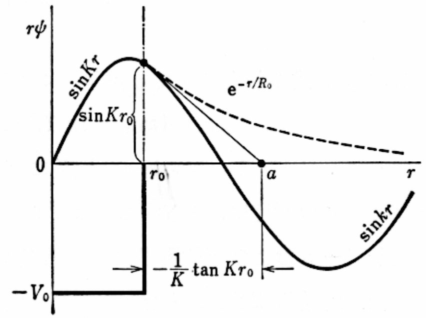
$$k\cot\delta = -\frac{1}{a} + \frac{r_0}{2}k^2$$

k: 波数、$\delta$ : 位相のずれ

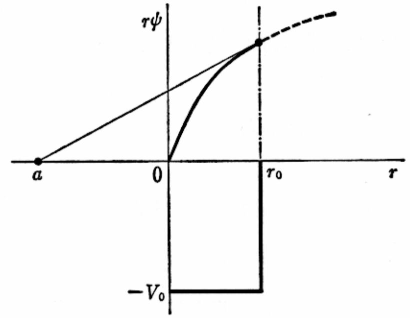
a: scattering length (散乱長、散乱半径)

$r_0$: effective length (有効距離)

※$r_0$ は論文中では $\ell$ と記載されている。




（a） 束縛状態 $a>0$




（b） 非束縛状態 $a<0$

図 32 散乱半径 $a$ とポテンシャル内部の波動関数との関係

原子核物理学 by 八木

Introduction:

Efimov States

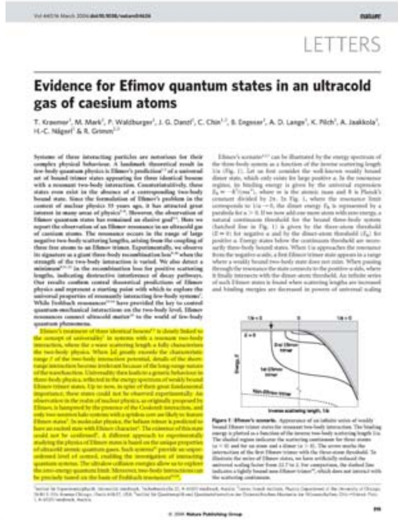
Low energy limit
Universality in the case of $|a| >> r_0$

$a \to \infty, r_0 \to 0$ の極限では、3粒子間のポテンシャルを特徴付けるスケールはどう表されるか？

→ $-\dfrac{1}{mR^2}$ の形でなければならない。

R: hyperradius (3核子の各ペア距離のrms)

3粒子状態のポテンシャル形は、electric dipole field 中の電荷のポテンシャルと同じ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Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer[1], M. Mark[1], P. Waldburger[1], J. G. Danzl[1], C. Chin[1,2], B. Engeser[1], A. D. Lange[1], K. Pilch[1], A. Jaakkola[1], H.-C. Nägerl[1] & R. Grimm[1,2]

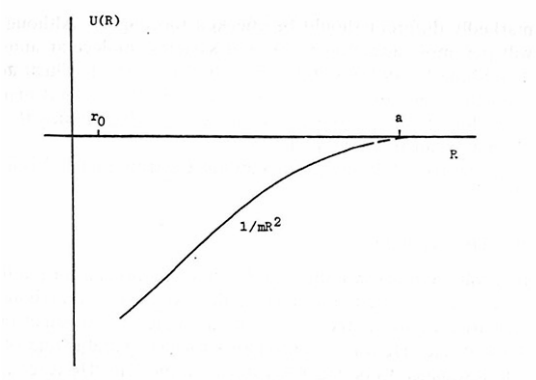


FIGURE 1 An effective three-body long-range interaction.

V. Efimov, CNPR19,271(1990)

Schrodinger方程式の解は、無限に存在し、次の定数でスケールする。

$$E_{n+1} = \exp(-\frac{2\pi}{s_0})E_n = \frac{1}{515.03}E_n$$

$$a_{n+1} = \exp(+\frac{\pi}{s_0})a_n = 22.7a_n$$

ここで
$s_0 \approx 1.00624$
は
$$s_0 \cosh\frac{\pi s_0}{2} = \frac{8}{\sqrt{3}}\sinh\frac{\pi s_0}{6}$$
の解

# Introduction:

## Efimov States

dimerのBinding Energy は、

$$E_b = \frac{\hbar^2}{ma^2}$$

3 atoms

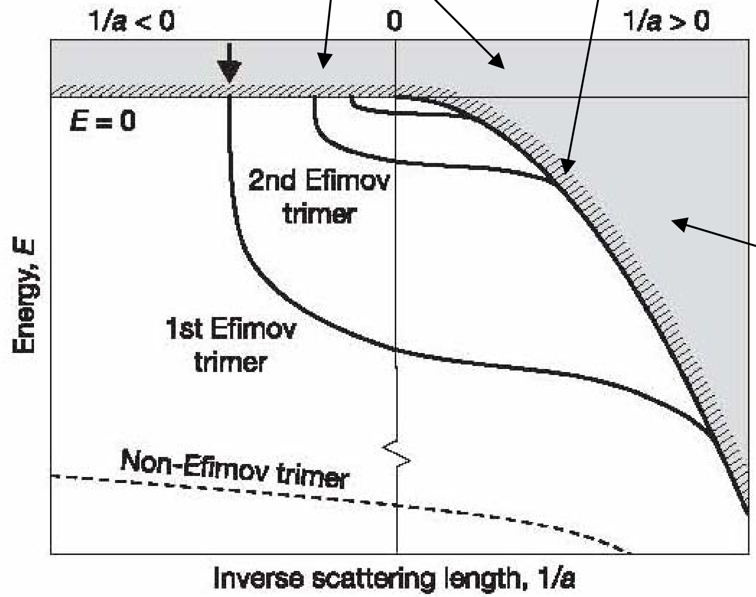
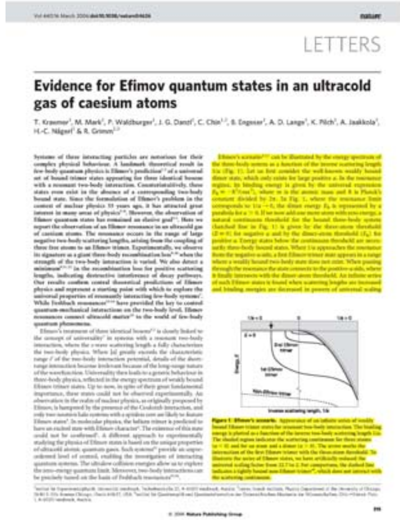



**Figure 1 | Efimov's scenario.** Appearance of an infinite series of weakly bound Efimov trimer states for resonant two-body interaction. The binding energy is plotted as a function of the inverse two-body scattering length $1/a$. The shaded region indicates the scattering continuum for three atoms ($a < 0$) and for an atom and a dimer ($a > 0$). The arrow marks the intersection of the first Efimov trimer with the three-atom threshold. To illustrate the series of Efimov states, we have artificially reduced the universal scaling factor from 22.7 to 2. For comparison, the dashed line indicates a tightly bound non-Efimov trimer[30], which does not interact with the scattering continuum.

atom+dimer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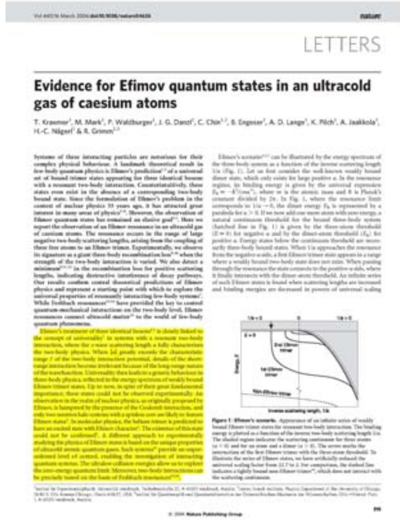
**Evidence for Efimov quantum states in an ultracold gas of caesium atoms**

※見やすくするため、scaling factor の22.7　を 2 として図示している。

Introduction: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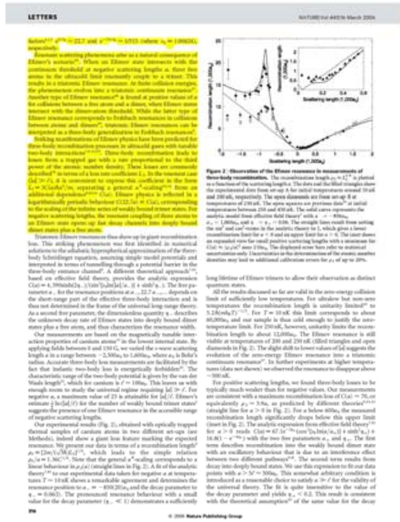
**Evidence for Efimov quantum states in an ultracold gas of caesium atoms**

Efimov States　はこれまで観測された例はない。

- [12]Cの3 $\alpha$ state ? [1,2,16] ... Coulomb 力が邪魔をする？　$|a| \sim 2r_0$
- Two nucleon halo nuclei ? [3]
- Helium trimer molecule?[4] ... Prediction could not be confirmed.[5]

↓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ultracold atomic quantum gas を使って実験する。Cs atoms。

- BECの研究を通じて発展してきた技術。
- 極低温の実現 (低Energy)
- scattering length ($a$)を自由に操ることができる。

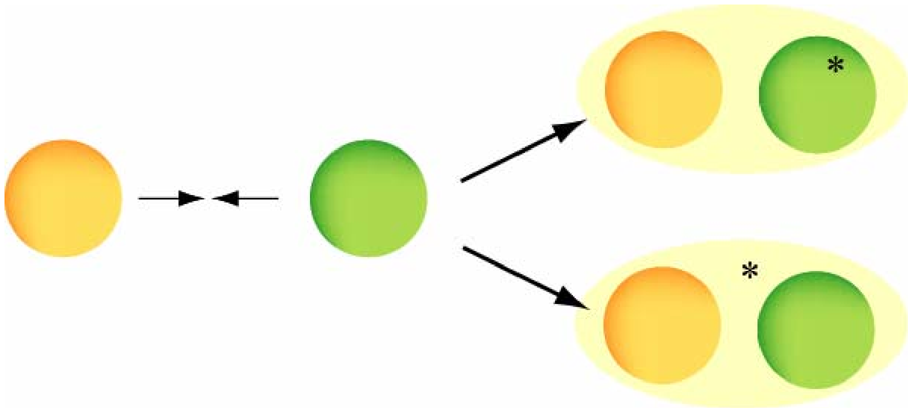
3-body recombination loss として観測する。

レートは密度の3乗に比例する。

# Experiment:

## Feshbach Resonance

2粒子の低エネルギーでの衝突で、一方の粒子が励起状態になる、もしくは分子としての励起状態に入ることでエネルギーを失い、2粒子が束縛した状態になる反応。特に初期運動と励起の自由度のカップリングが小さい場合を想定する。終状態は元に戻るため、寿命がある。




Csの2原子（$F$=3,$m_F$=3に偏極してくいる）は、衝突によりd,g状態などの分子励起状態に入るFeshbach Resonacneを起こす。励起状態のスピンは初期状態のスピンと一般には異なるため、外部磁場を加えることで励起状態とのエネルギー差を変えることができる。これにより、散乱長を(弾性散乱の散乱断面積~4$\pi$ $a^2$を)制御することができる。

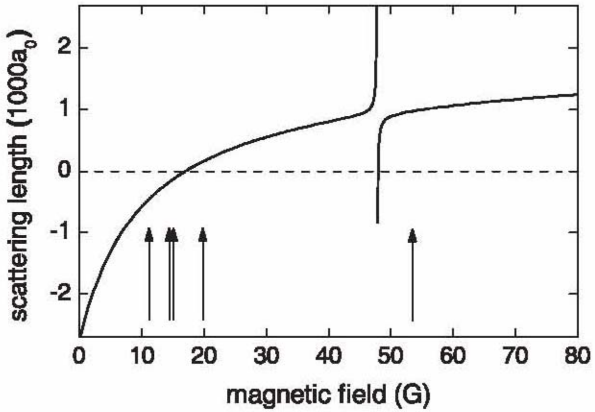
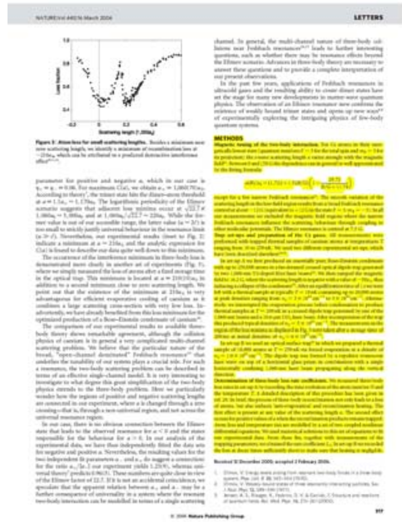



**FIGURE 1** Scattering length as a function of magnetic field for the state $F = 3$, $m_F = 3$. There is a relatively broad Feshbach resonance at 48.0 G due to coupling to a *d*-wave molecular state. The *arrows* indicate several very narrow resonances at 11.0, 14.4, 15.0, 19.9 and 53.5 G, which result from coupling to *g*-wave molecular states. The data are taken from [13]

# Experiment: Trap

T. Kraemer et al., Appl.Phys.B79, 1013(2004)



Zeeman slowed atomic beam — $3\times10^{8}$ atoms/6sec

↓

**Magneto-Optical Trap (MOT)** — τ=200s at $1\times10^{-11}$ mbar; high-power laser diode

↓

**Compression** — rampup dB/dy 7-33 G/cm in 40ms; laser 10-30 MHz

↓

laser and dB/dy off

↓

**Cooling** — degenerate Raman-sideband cooling; injection locked slave laser; 4 laser 65 mW; $4\times10^{7}$ atoms, 0.7μK, 1 mm; F=3, $m_F$=3 → 90%

↓

heated to 2μK

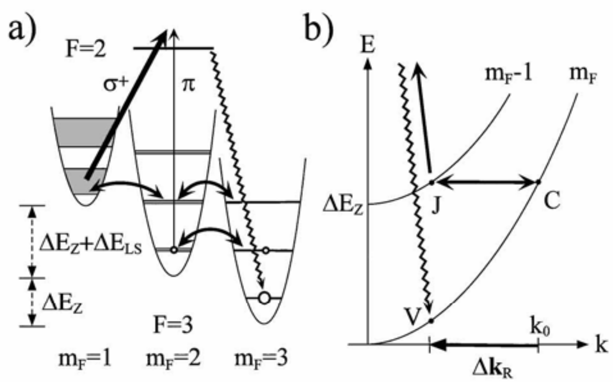



FIG. 1. (a) Vibrationally excited atoms are cooled quickly by degenerate Raman transitions (double-sided arrows) and fast $\sigma^+$ optical pumping, while the final cooling stage uses only weak $\pi$ pumping. This scheme strongly suppresses reabsorption heating. (b) Initially unbound atoms are transferred from $m_F(C)$ to $m_F - 1(J)$ when a two-photon degenerate Raman transition satisfies energy and momentum conservation. Optical pumping back to $m_F(V)$ cools the atoms by one Zeeman energy splitting $\Delta E_Z$.

A. Kerman et al., PRL84, 439 (2000).

# Experiment: Trap

↓

**Reservoir Trap**

heated to 2μK

anti-Helmholtz coil 31.3G/cm
(Magnetic levitation)
Helmholtz coil 0-200 G
$CO_2$-laser × 2 90+65W
potential depth 7μK
605μm-690μm

↓

**Plain Evapolation Cooling**

73.5 G
$1215a_0$
$4\times10^6$ atoms, 1μK, 10sec

↓

$7.8\times10^6$ atoms, 2sec

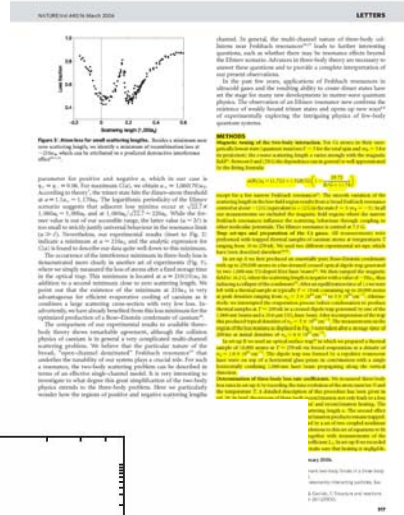
**Dimple Trap**

broadband fiber laser 1064nm
rampup 70mW-270mW/1.5sec
34μm-260μm
$300a_0$ to reduce 3-body recombination loss
$1.7\times10^6$ atoms, 2sec

↓

**Forced Evapolation Cooling**

rampdown laser beams
A2→off
A1→0, B1→5mW, B2→220mW/5.5sec
21G, $210a_0$
$5\times10^5$ atoms at 200nK
$1.1\times10^5$ atoms at 10nK



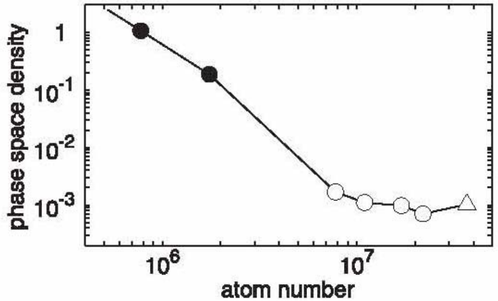



FIGURE 3 Peak phase space density as function of atom number. The path of evaporation proceeds from right to left. The *triangle* shows the atomic ensemble immediately after lattice cooling. The *open circles* show the ensemble in the reservoir trap after 0.08, 0.22, 0.64, and 2.0 s. The *filled circles* correspond to the sample in dimple trap right after loading and after 1.5 s of evaporation. The phase transition occurs after 2 s of forced evaporation with $\sim 5\times10^5$ atoms left in the dimple trap

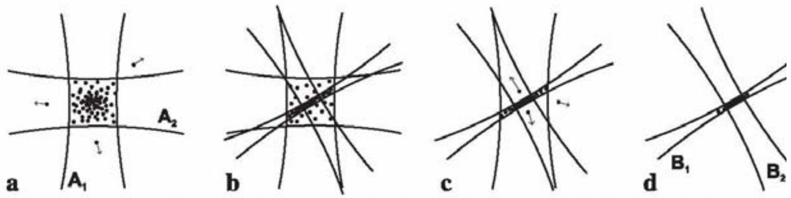
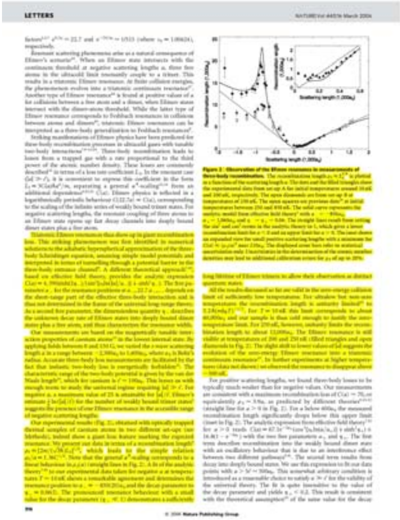



FIGURE 2 Illustration of the various stages of trap loading and evaporative cooling as seen from above. (a) Plain evaporation in a crossed $CO_2$-laser trap generated by beams $A_1$ and $A_2$ at a scattering length of $a = 1215a_0$. (b) 1.5 s of ramping and collisional loading into a crossed 1064-nm fibre laser trap generated by beams $B_1$ and $B_2$ with a final scattering length $a = 210a_0$. (c) Forced evaporative cooling after switching off $CO_2$-laser beam $A_2$. The power of all remaining lasers is ramped down, and the power in $CO_2$-laser beam $A_1$ is reduced to zero. (d) Final configuration of the crossed 1064-nm trap. Imaging is done in the horizontal plane at an angle of 30° with respect to the long axis of the cigar-shaped atomic cloud

060907: Evidence for Efimov quantum states ....



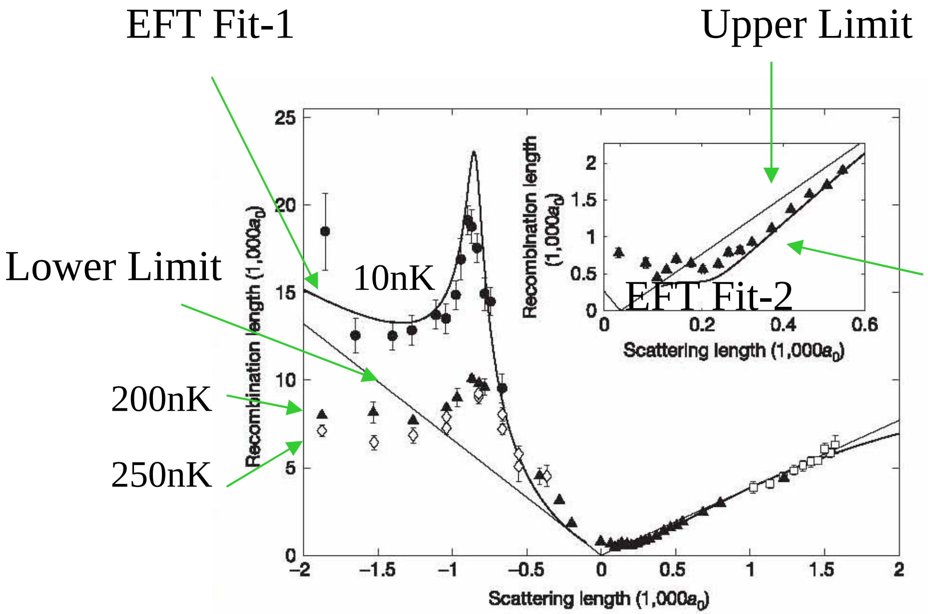



**Figure 2 | Observation of the Efimov resonance in measurements of three-body recombination.** The recombination length $\rho_3 \propto L_3^{1/4}$ is plotted as a function of the scattering length $a$. The dots and the filled triangles show the experimental data from set-up A for initial temperatures around 10 nK and 200 nK, respectively. The open diamonds are from set-up B at temperatures of 250 nK. The open squares are previous data[20] at initial temperatures between 250 and 450 nK. The solid curve represents the analytic model from effective field theory[7] with $a_- = -850a_0$, $a_+ = 1{,}060a_0$, and $\eta_- = \eta_+ = 0.06$. The straight lines result from setting the $\sin^2$ and $\cos^2$-terms in the analytic theory to 1, which gives a lower recombination limit for $a < 0$ and an upper limit for $a > 0$. The inset shows an expanded view for small positive scattering lengths with a minimum for $C(a) \propto (\rho_3/a)^4$ near $210a_0$. The displayed error bars refer to statistical uncertainties only. Uncertainties in the determination of the atomic number densities may lead to additional calibration errors for $\rho_3$ of up to 20%.

Effective Field Theory:

低EもしくはThreshold近傍で、適切な相互作用レンジ内の詳細を無視してパラメータフィットし、特徴的なスケールパラメータ$E/E_0$にて展開することにより、高次のパラメータを順次決定していく近似計算手法

## EFT Fit

for $a$<0

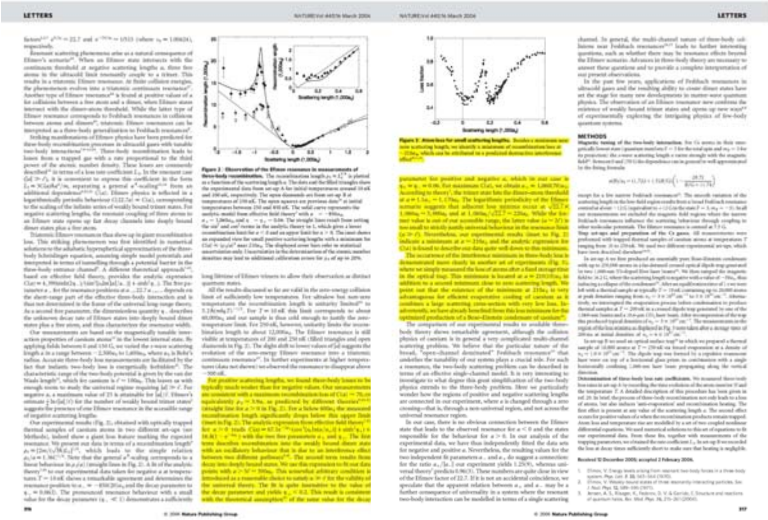
$$a_- = -850(20)a_0,\ \eta_- = 0.06(1)$$

$$C(a) = 4590\sinh(2\eta_-)/\left(\sin^2[s_0 \ln(|a|/a_-)] + \sinh^2\eta_-\right)$$

for $a$>0

$$a_+ = 1060(70)a_0,\ \eta_+ < 0.2$$

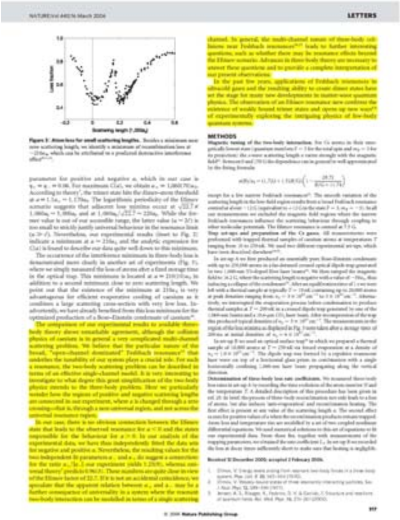
$$C(a) = 67.1e^{-2\eta_+}\left(\cos^2[s_0 \ln(|a|/a_+)] + \sinh^2\eta_+\right)$$



Efimov State を発見した：

- $a<0$ の共鳴が、EFT-Fit でよく記述できる。
- $a<0$ と $a>0$ の Fit値はよくあっている。$a>0$ に対応する Minimum は $220a_0$と予想され、実験値 $210a_0$とよくあっている。

（但し、$220a_0$は、$|a|>>r_0$の仮定があまりよく成り立たない領域)



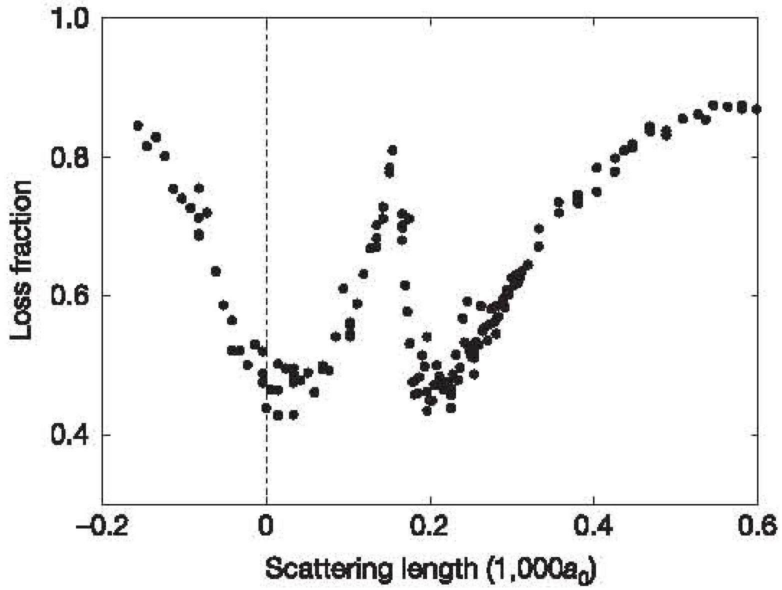
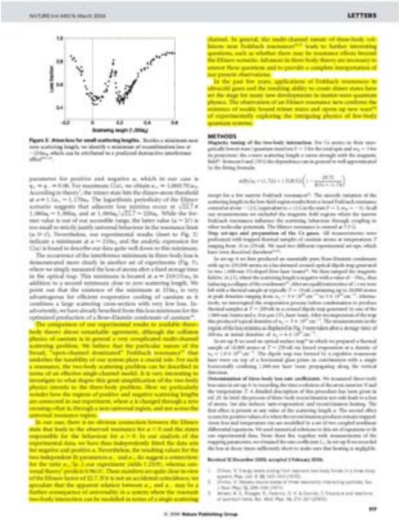



**Figure 3 | Atom loss for small scattering lengths.** Besides a minimum near zero scattering length, we identify a minimum of recombination loss at $\sim 210 a_0$, which can be attributed to a predicted destructive interference effect[9,11,12].

Minimum の実験値 $210a_0$は、単純なロスレートをプロットした図3にもよく現れている。

Minimumは、2つの3-body decay channel の干渉から生じる。



Conclusion:

- Efimov state を発見したと言えるであろう。
- scattering length a を調整可能にするシステムが果たした役割は大きい。
- a<0とa>0に見られた構造に、明確な connection はないが、同じ Effimov state に由来している可能性が高い。
- 3体計算の理論発展が必要。