

Dipole Polarizability とそれを巡る状況

- ^{208}Pb の Dipole Polarizability と Neutron Skin -

核理Ia勉強会

2010.12.14

A. Tamii

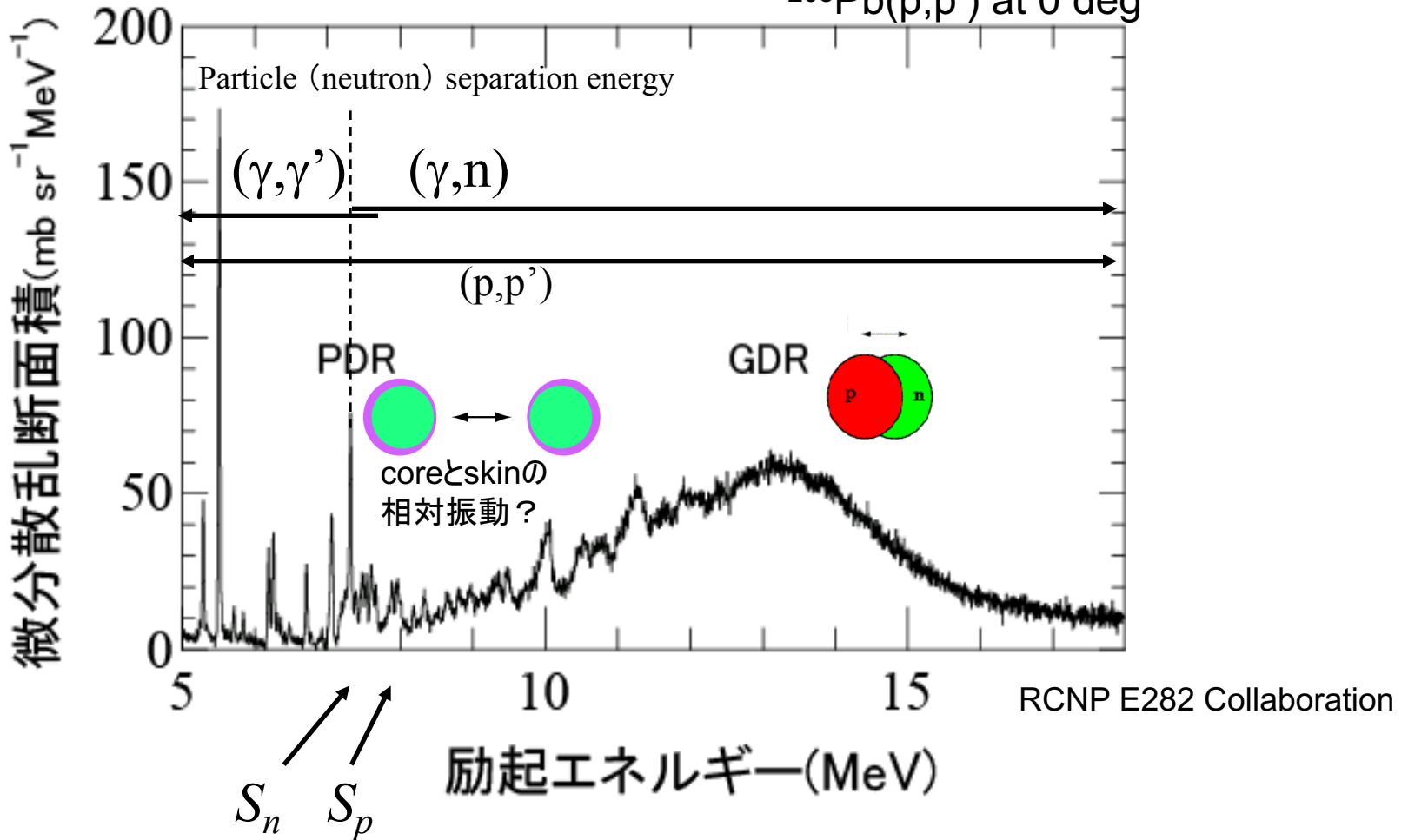
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Motivation

E1 Strength Distribution in ^{208}Pb

$^{208}\text{Pb}(p,p')$ at 0 deg



E1励起強度分布と中性子スキン(Neutron Skin)、および対称エネルギー(Symmetry Energy)の表面項との関係は？

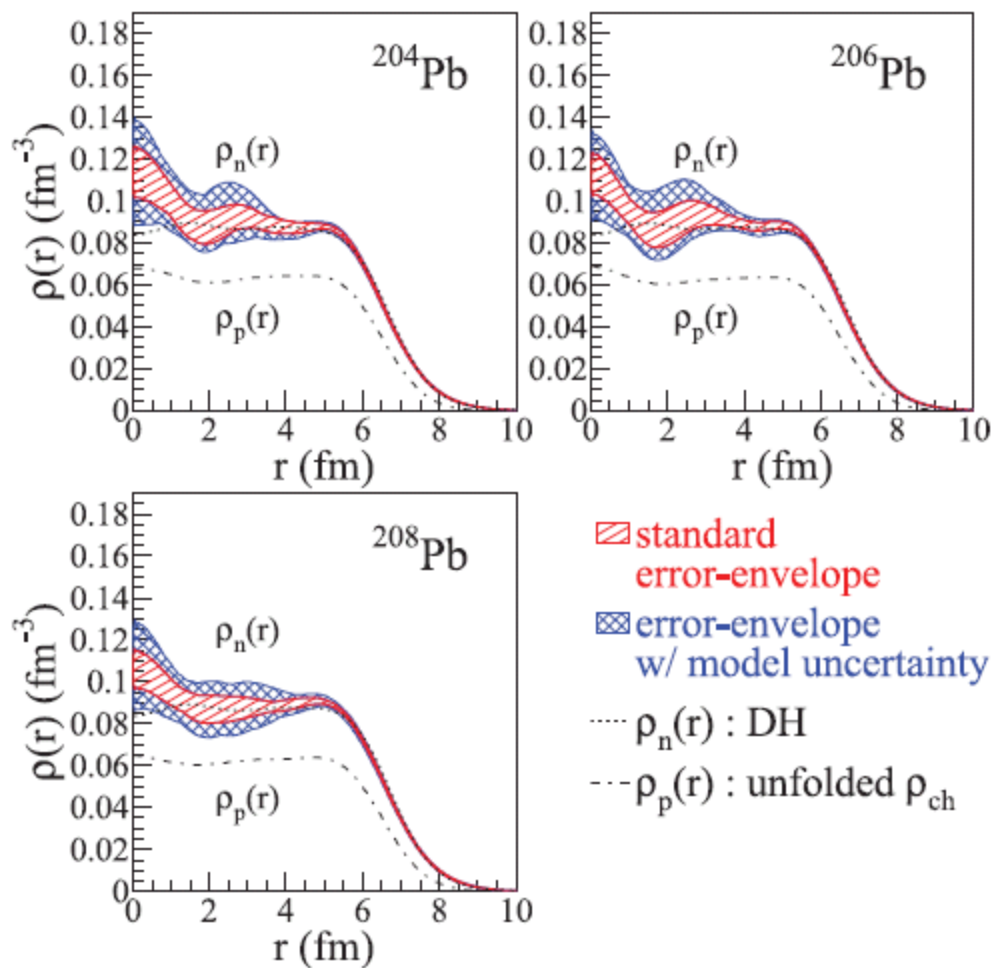


FIG. 7. (Color online) Extracted neutron densities for $^{204,206,208}\text{Pb}$ with two types of error envelopes shown together with DH neutron densities (dotted lines) and point proton densities by unfolding charge densities (dash-dotted lines). The cross-hatched and hatched error envelopes were estimated by Eqs. (8) and (6), respectively.

TABLE VI. Obtained values of r_n , r_p , and Δr_{np} for ^{208}Pb compared with several experimental and theoretical results (all in fm). Except for this work, the errors are statistical only.

	Model					Experiment					
	SkM*	SkX	NL3	DD-ME2	FSUGold	GDR ^a	PDR ^b	antiproton ^c	(p, p) at 800 MeV ^d	(p, p) at 650 MeV ^e	This work
r_p	5.45	5.44	5.46	—	—	—	—	5.44	5.45	5.46	5.442(2)
r_n	5.62	5.60	5.74	—	—	—	—	5.60	5.59(4)	5.66(4)	5.653 ^{+0.054} _{-0.063}
Δr_{np}	0.17	0.16	0.28	0.19	0.21	0.19(9)	0.18(4)	0.16(2)	0.14(4)	0.20(4)	0.211 ^{+0.054} _{-0.063}

^aThe isovector giant dipole resonance (GDR) from $^{208}\text{Pb}(\alpha, \alpha')$ at $E_\alpha = 120$ MeV [12].

^bThe measurement of “pigmy” dipole resonance (PDR) strength from $^{208}\text{Pb}(\gamma, \gamma')$ [19].

^cThe analysis of the x-ray cascade from antiprotonic atoms assuming two-parameter-Fermi distribution for both ρ_p and ρ_n [18].

^dRef. [9].

^eRef. [13].

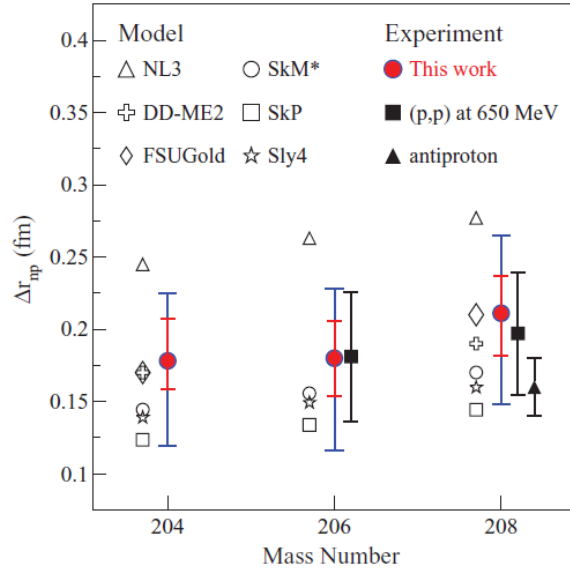


FIG. 8. (Color online) Systematic behavior of the neutron skin thicknesses for $^{204,206,208}\text{Pb}$. The filled circles are the results of this work with the two types of error bars. The filled squares and triangle are from the analysis of proton elastic scattering at 650 MeV [13] and x rays from antiprotonic atoms [18], respectively, with statistical errors only. The open triangles, crosses, and diamonds show the calculations of relativistic mean-field models with NL3 [45], DD-ME2 [46], and FSUGold [47] parametrization and the open circles, squares, and stars are from nonrelativistic mean-field models with SkM* [43], SkP [44], and Sly4 [48] parametrization.

Lead (^{208}Pb) Radius Experiment: PREX

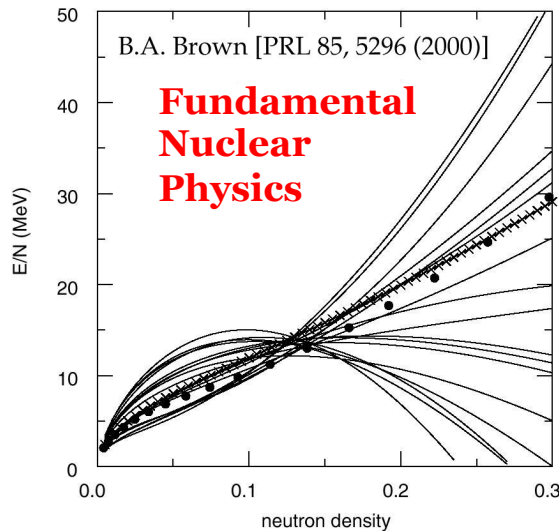


Elastic Scattering Parity-Violating Asymmetry $E = 1 \text{ GeV}$, $\theta = 5^\circ$

Z^0 : Clean Probe Couples Mainly to Neutrons

(T.W. Donnelly, J. Dubach, I Sick)

$$A = \frac{\left(\frac{d\sigma}{d\Omega}\right)_R - \left(\frac{d\sigma}{d\Omega}\right)_L}{\left(\frac{d\sigma}{d\Omega}\right)_R + \left(\frac{d\sigma}{d\Omega}\right)_L} = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}} \left[1 - 4\sin^2\theta_W - \frac{F_N(Q^2)}{F_P(Q^2)} \right] \approx 0.5 \text{ ppm}$$



PREX : precise measurement of the density-dependence of the symmetry energy.

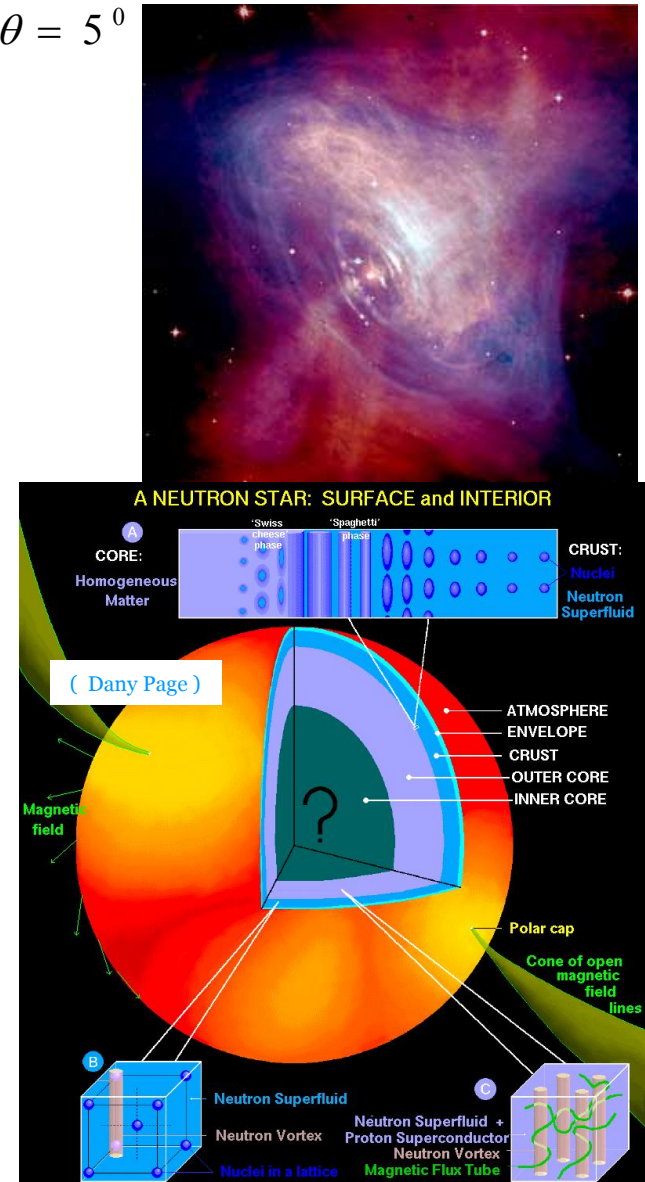
Many applications:

1. Nuclear Physics
2. Neutron Stars

Combine with astrophysics observations

Is dense Neutron-rich matter exotic?

3. Atomic Parity Violation
4. Heavy Ion Collisions



PREX at JLab

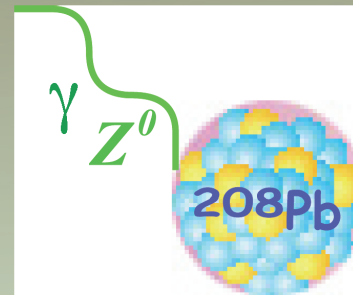
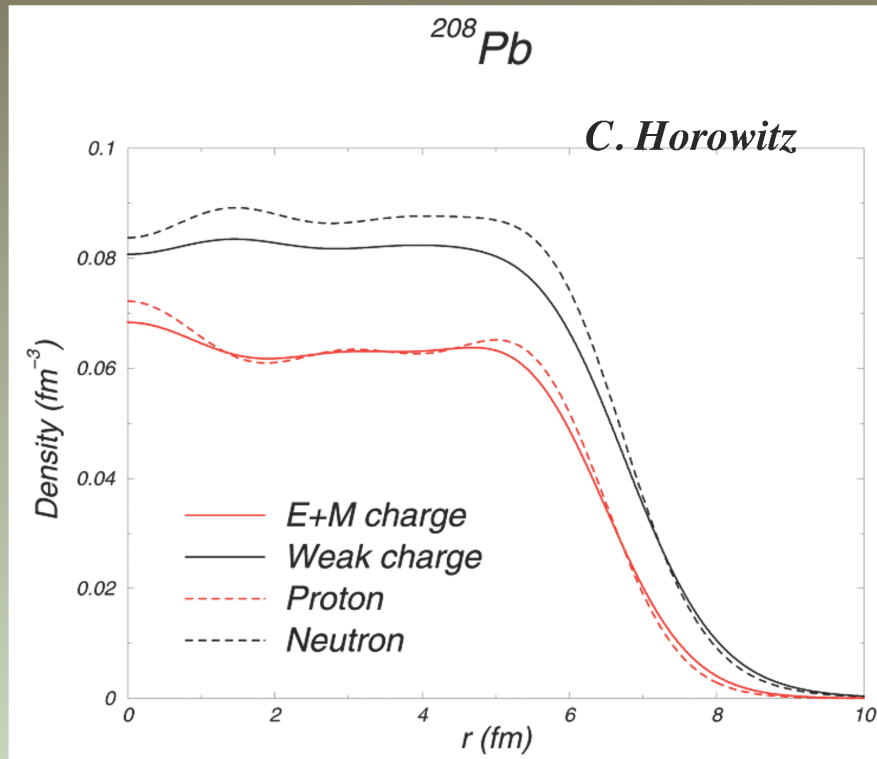
Talks by R. Michaels, G. Urciuoli

$$Q_{EM}^p \sim 1$$

$$Q_{EM}^n \sim 0$$

$$Q_W^p \sim 1 - 4\sin^2\theta_W$$

$$Q_W^n \sim 1$$



$$\delta(A_{PV}) \sim 3\%$$

$$\delta(R_p - R_n) \sim 1\%$$

$$Q^2 \sim 0.01 \text{ GeV}^2 \longrightarrow A_{PV} \sim 0.5 \text{ ppm}$$

- Exploits the large neutron coupling of the Z boson
- Neutron size sensitivity enhanced at appropriate Q

talk by I. Sick

**Data collection at JLab
Hall A in March 2010**

A technically demanding measurement:

- Rate $\sim 2 \text{ GHz}$
- Separate excited state at 2.4 MeV
- Stat. Error $\sim 15 \text{ ppb}$
- Syst. Error $\sim 1 \text{ to } 2 \%$

励起強度分布のモーメントと和則

オペレータFに対する励起強度関数 (Strength Function)

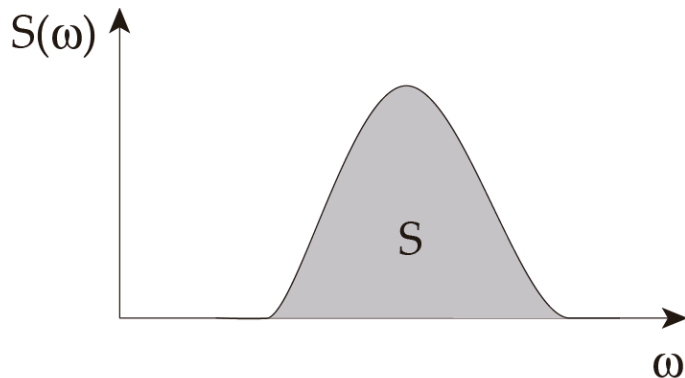
$$S(\omega) \equiv \sum_k \left| \langle k | F | 0 \rangle \right|^2 \delta(\omega - \omega_k)$$

ω : 励起エネルギー

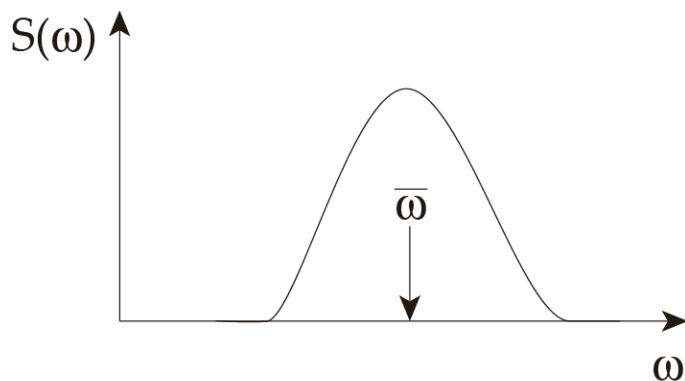
励起強度関数 $S(\omega)$ の p 次のモーメント

$$m_p \equiv \int_0^\infty S(\omega) \omega^p d\omega = \sum_k \left| \langle k | F | 0 \rangle \right|^2 \omega_k^p$$

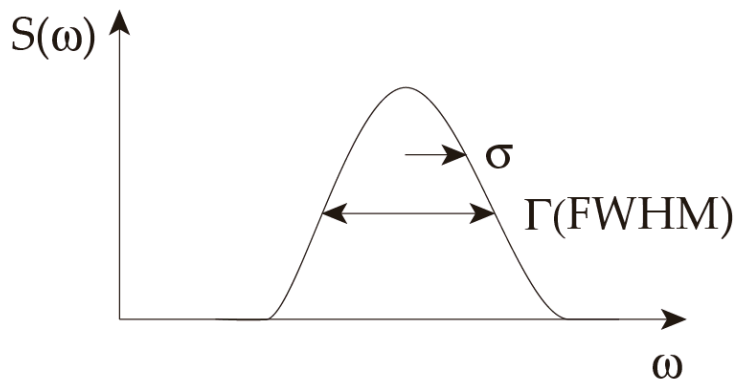
$$S = m_0 = \int_0^\infty S(\omega) d\omega = \sum_k |\langle k|F|0\rangle|^2$$



$$\bar{\omega} = \frac{m_1}{m_0} = \frac{\int_0^\infty S(\omega) \omega d\omega}{\int_0^\infty S(\omega) d\omega} = \frac{\sum_k |\langle k|F|0\rangle|^2 \omega_k}{\sum_k |\langle k|F|0\rangle|^2}$$



$$\begin{aligned} \sigma^2 &= \frac{m_2}{m_0} - \left(\frac{m_1}{m_0} \right)^2 = \frac{\int_0^\infty S(\omega) \omega^2 d\omega}{\int_0^\infty S(\omega) d\omega} - \left(\frac{\int_0^\infty S(\omega) \omega d\omega}{\int_0^\infty S(\omega) d\omega} \right)^2 \\ &= \frac{\int_0^\infty S(\omega) (\omega - \bar{\omega})^2 d\omega}{\int_0^\infty S(\omega) d\omega} \end{aligned}$$



全ての励起状態に関する積算値は、
対応するオペレータの
基底状態の期待値を表す。

$F : \text{Hermitian}$

$$F = F^\dagger$$

$$\begin{aligned} m_p &\equiv \int_0^\infty S(\omega) \omega^p d\omega = \sum_k |\langle k | F | 0 \rangle|^2 \omega_k^p \\ &= \sum_k \langle 0 | F | k \rangle \langle k | F | 0 \rangle \omega_k^p \\ &= \langle 0 | F \left(\sum_k |k\rangle \langle k| \right) \omega_k^p F | 0 \rangle \\ &= \langle 0 | F (H - E_0)^p F | 0 \rangle \end{aligned}$$

励起状態を全て調べると基底状態の性質が分かる。

対応するオペレータは、ハミルトニアン H との(反)交換関係で表すことができる(ことがある)。

$$m_0 = \frac{1}{2} \langle 0 | F^2 | 0 \rangle$$

$$m_1 = \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle$$

$$m_2 = \frac{1}{2} \langle 0 | \{[F, H], [H, F]\} | 0 \rangle$$

$$m_3 = \frac{1}{2} \langle 0 | [[F, H], [H, [H, F]]] | 0 \rangle$$

導出例:

$$\begin{aligned} m_1 &= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle \\ &= \frac{1}{2} \langle 0 | [F, HF - FH] | 0 \rangle \\ &= \frac{1}{2} \langle 0 | FHF - F^2H - HF^2 + FHF | 0 \rangle \\ &= \frac{1}{2} \langle 0 | 2FHF - F^2E_0 - E_0F^2 | 0 \rangle \\ &= \langle 0 | F(H - E_0)^\dagger F | 0 \rangle \end{aligned}$$

Dipole Polarizability

Static Dipole Polarizability
Electric Dipole Polarizability
双極子分極能

E1励起強度に対する m_{-1} 和則に対応

Dipole Polarizability

$$\alpha_D \equiv \sum_k \frac{|\langle k | D | 0 \rangle|^2}{\omega_k} \quad \text{Dipole Polarizability}$$

$$D \equiv \frac{1}{2} \sum_{k=1}^A \tau_{3,k} r_k Y_{10}(\hat{r}_k) \quad \text{Electric Dipole Operator}$$

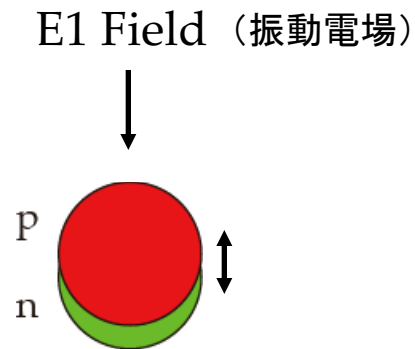
$\frac{1}{2} \sum (1 - \tau_3) r Y_{10}$ から重心移動項
を除いて符号反転したもの

$$\alpha_D = m_{-1} = \frac{1}{2} \langle 0 | [[X^\dagger, H], X] | 0 \rangle$$

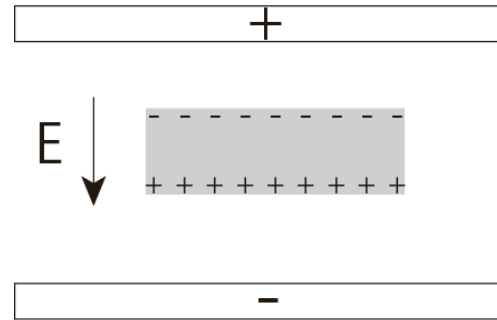
X: solution of $[H, X] = D$

Dipole Polarizability

Dipole Polarizability は、外部電場に対する誘電分極に対応する。



E1 response of a nucleus



誘電分極

E1励起強度と和則

A. Migdal (1944)

1-(および2+)の平均励起エネルギーを和則により求めようとした。

$$\overline{\omega}^{E1} \equiv \sqrt{\frac{m_1^{E1}}{m_{-1}^{E1}}}$$

$$m_1^{E1} \equiv \sum_k |\langle k | D | 0 \rangle|^2 \omega_k$$

$$m_{-1}^{E1} \equiv \sum_k \frac{|\langle k | D | 0 \rangle|^2}{\omega_k}$$

E1励起強度と和則

1次のモーメント

$$m_1^{E1} = \frac{1}{2} \langle 0 | [D, [H, D]] | 0 \rangle$$

$$H = \sum_{k=1}^A \frac{p_k^2}{2M} + V(r_1, r_2, \dots, r_A)$$

$$[D, V] = 0$$

$$m_1^{E1} = \frac{3A}{32\pi M}$$

Thomas-Reich-Kuhn (TRK) Sum-Rule

E1励起強度と和則

-1次のモーメント

原子核を外場 $H_{\text{ext}} = D$ の中に置く。全系のハミルトニアン H は

$$H = H_0 + D$$

無摂動のハミルトニアン H_0 に対する波動関数を Φ_0 と置くと、 H に対する波動関数 Φ は1次の摂動で、

$$\Phi = \Phi_0 - \sum_n \frac{\langle n | D | 0 \rangle}{E_n - E_0} | n \rangle$$

これを用いると D の期待値は、2次の項を無視して

$$\langle \Phi | D | \Phi \rangle = -2 \sum_n \frac{|\langle n | D | 0 \rangle|^2}{E_n - E_0} = -2m_{-1}^{E1}$$

左辺は、E1外場中での陽子と中性子の密度分布差で記述でき

$$\langle \Phi | D | \Phi \rangle = -\frac{1}{2} \int (\rho_n(r) - \rho_p(r)) r Y_{10} d\mathbf{r}$$

E1励起強度と和則

-1次のモーメント

$$m_{-1}^{E1} = \frac{1}{4} \int (\rho_n(r) - \rho_p(r)) r Y_{10} d\mathbf{r}$$

Weizsaecker-Bete の質量公式

$$E_0 = AM + \alpha A + \gamma A^{2/3} + \beta \frac{(N-Z)^2}{A} + \frac{3}{5} \frac{Z^2 e^2}{R_c}$$

の対称エネルギー(Symmetry Energy)項 β と関連付ける。

Migdalは、pとnの分布を同じ半径Rの球とし、外場中での平衡状態のエネルギーを最低とする変分条件から、1次のモーメントを求めた

$$m_{-1}^{E1} = \frac{3AR^2}{320\pi\beta} \qquad \frac{3}{5}R^2 = \frac{1}{A} \int r^2 (\rho_n + \rho_p) d\mathbf{r} \quad \text{: 平均自乗半径}$$

m.s. radius

$$\bar{\omega}^{E1} \equiv \sqrt{\frac{m_1^{E1}}{m_{-1}^{E1}}} = \frac{1}{R} \sqrt{\frac{10\beta}{M}} \approx 85 A^{-1/3} \text{ (MeV)} \quad \text{核全体の効果(Volume)}$$

参考文献[5]

E1励起強度と和則

Goldhaber-Teller(1948)

原子核を陽子と中性子からなる非圧縮性の流体とし、E1励起状態は陽子と中性子の流体が逆位相に振動する状態であるとし、調和振動として近似した。

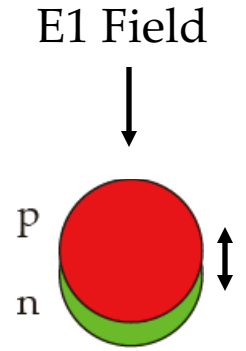
$$H = \frac{1}{2B} \Pi_v^2 + \frac{C}{2} v^2$$

Bは陽子流体と中性子流体の換算質量でN=Zの場合 $B=AM/4$ 。

Cは復元力で表面積 $4\pi R^2$ に比例すると考えられる。

E1の平均励起エネルギーは、

$$\bar{\omega}^{E1} \equiv \sqrt{\frac{C}{B}} \propto \sqrt{\frac{R^2}{A}} \propto A^{-1/6}$$



核表面の効果(Surface)

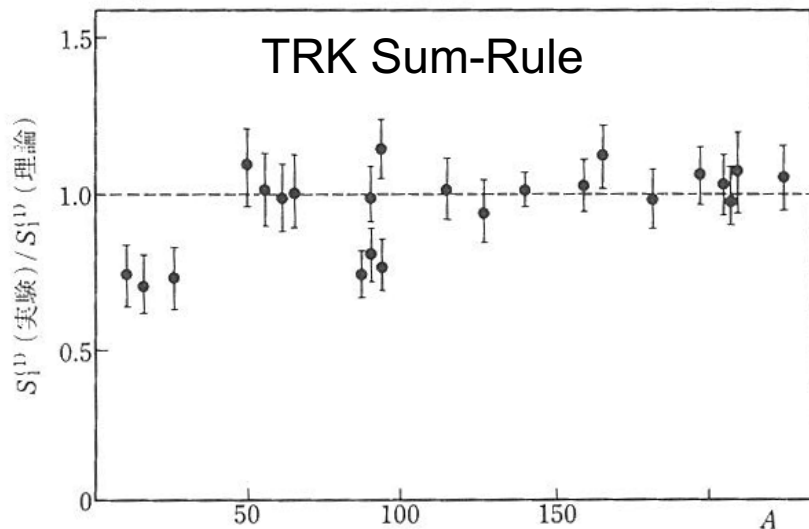


図 3.1 1^- 状態励起に対する和則値の理論値と実験値の比⁸⁾
実験値は励起エネルギー30 MeV までの和. A は原子核の質量数

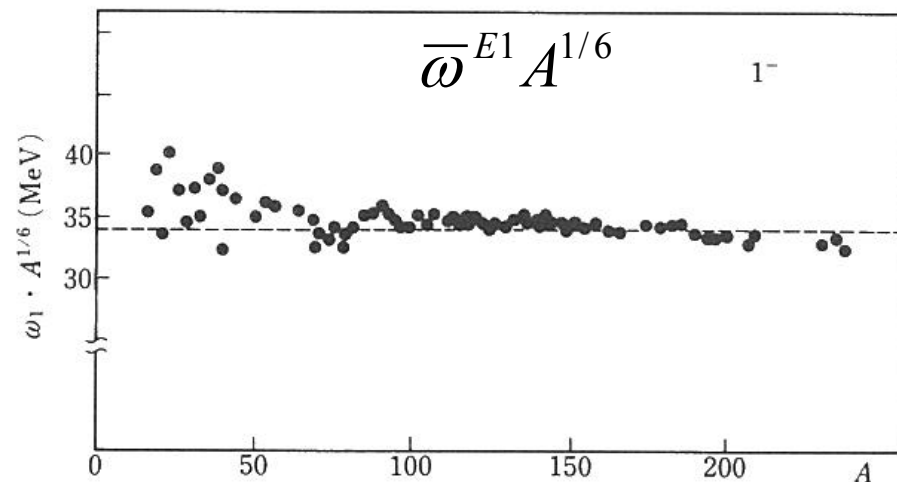


図 3.3 1^- 巨大共鳴状態のエネルギー ω_1 の質量数 (A) 依存⁹⁾ (実験値 (●) は $\omega_1 = 34/A^{1/6}$ (MeV) でよく再現される)

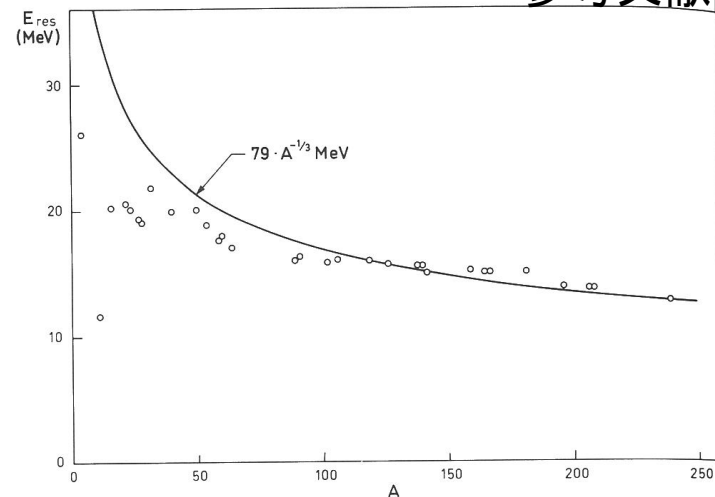


Figure 6-19 Systematics of dipole resonance frequency. The experimental data are taken from the review article by E. Hayward (*Nuclear Structure and Electromagnetic Interactions*, p. 141, ed. N. MacDonald, Oliver and Boyd, Edinburgh and London, 1965), except for ^4He , for which the resonance frequency is that given in the survey article by W. E. Meyerhof and T. A. Tombrello, *Nuclear Phys.* **A109**, 1 (1968). In the case of the deformed nuclei, which exhibit two resonance maxima, the energy represents a weighted mean of the two resonance energies. The solid curve represents the estimate based on the liquid-drop model (see Eq. (6A-65)).

Symmetry Energy に、Volumeの効果とSurfaceの効果を両方とりいれる必要がある。

Surface効果には中性子スキンが大きく寄与する。

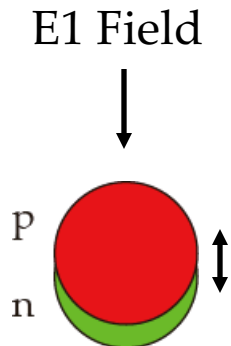
Hydro-dynamical Approach

• E. Lipparini and s. Stringari, Phys. Lett. 112B, 421 (1982).

$$E_{sym} = \frac{1}{2} \frac{(N-Z)^2}{A^2} (b_{vol} - b_{surf} A^{-1/3})$$

$$\alpha_D = \alpha_D^M \left(1 + \frac{5}{3} \frac{b_{surf}}{b_{vol}} A^{-1/3} \right)$$

$$\alpha_D^M = \frac{1}{12} \frac{A \langle r^2 \rangle}{b_{vol}}$$



Symmetry Energy に、**Volume**の効果と
Surfaceの効果を両方とりいれる必要がある。

Surface効果には中性子スキンが大きく寄与する。

P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 81, 051303(R) (2010).

Self-Consistent Mean Field Theory

平均場計算の入力として用いる軌道 (Single Particle Orbit) と、
計算によって得られた平均場の作る軌道が Consistent である
平均場計算

(Nuclear) Energy Density Functional Theory: エネルギー密度汎関数理論

原子核の全ての物理量を、エネルギー密度分布関数の関数として記述
しようとする理論的試み。

Skym Force

主に重い原子核を中心として、原子核の平均場的性質をうまく記述する
核内有効相互作用を、使いやすい形にパラメータ化して現象論的に決めたもの。

δ 型の相互作用で密度依存性があり、2体力と3体力を含む。

数多くの種類がある。

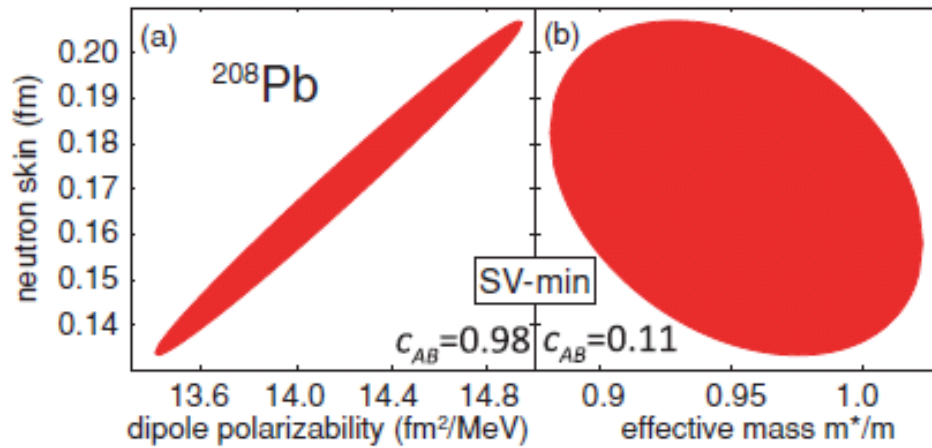


FIG. 1. (Color online) The covariance ellipsoids for two pairs of observables as indicated. The filled area shows the region of reasonable domain \mathbf{p} . (Left) Neutron skin and isovector dipole polarizability in ²⁰⁸Pb. (Right) Neutron skin in ²⁰⁸Pb and effective nucleon mass m^*/m in symmetric nuclear matter.

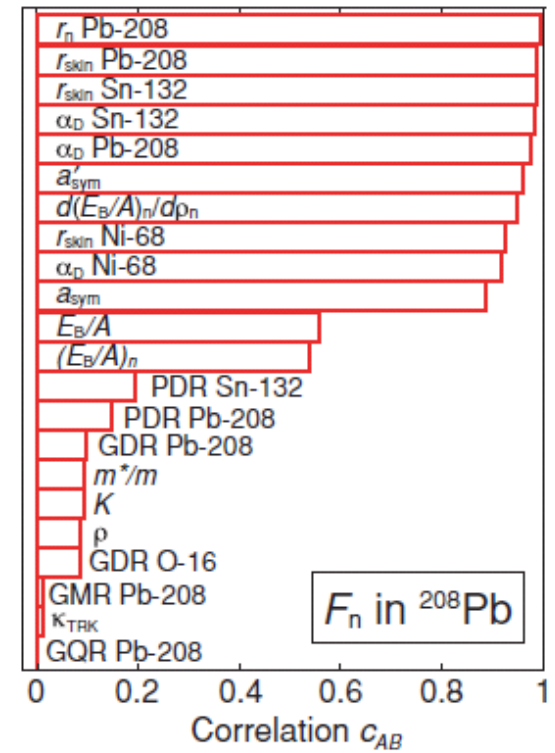


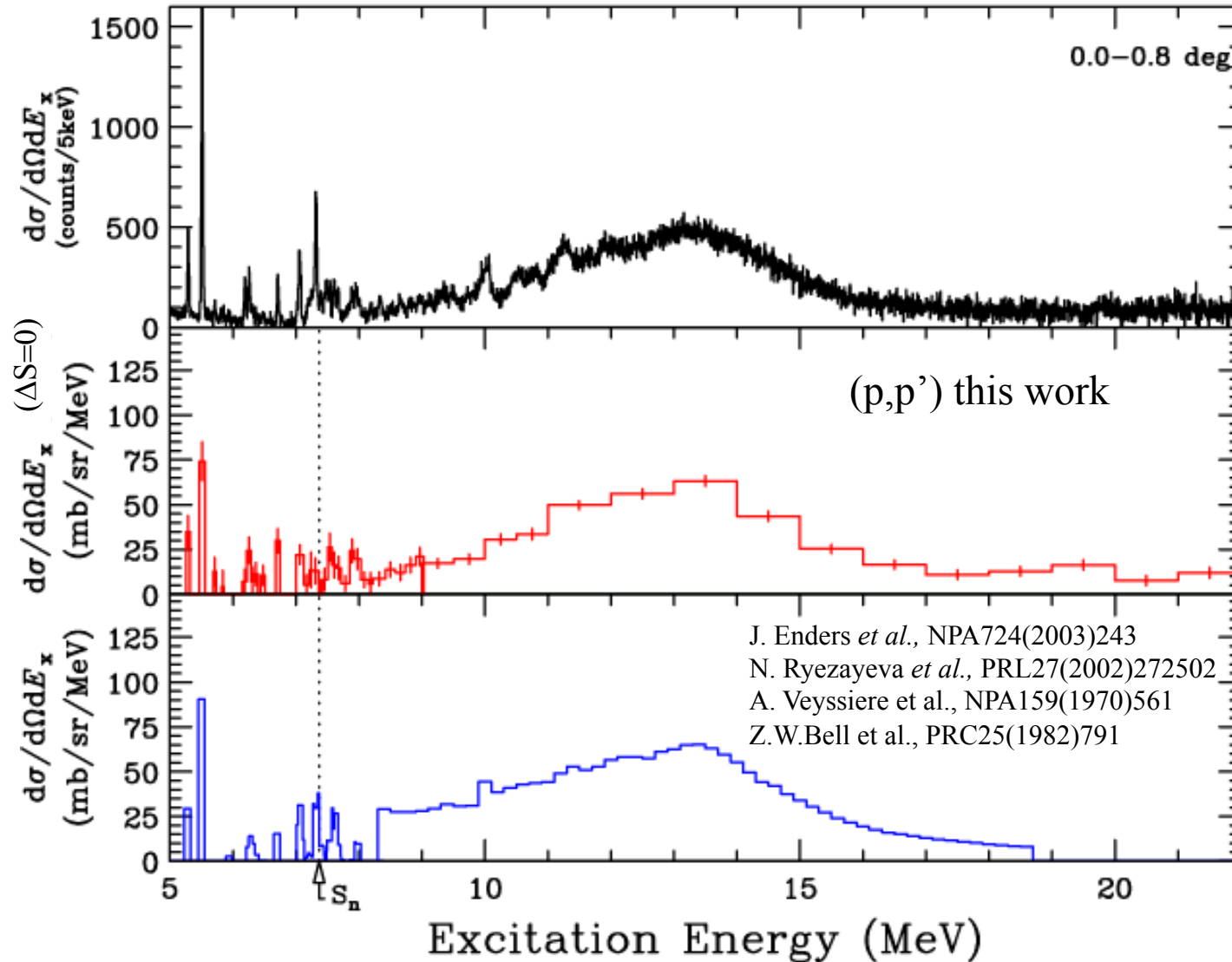
FIG. 2. (Color online) Correlation (6) of various observables with the neutron form factor F_n ($q = 0.45 \text{ fm}^{-1}$) in ²⁰⁸Pb.

The excellent correlation between the neutron skin and dipole polarizability is not surprising as $r_{\text{skin}} \propto \alpha_D a_{\text{sym}}$ [32]. The experimental value of α_D for ^{208}Pb is $13.3 \pm 1.4 \text{ fm}^2/\text{MeV}$ [33] while the value obtained by the Lorentz fit to the total experimental photodisintegration cross section is $13.6 \text{ fm}^2/\text{MeV}$ [31]. As seen in Fig. 1, both values are consistent with the SV-min predictions for r_{skin} . However, a 10% experimental uncertainty due to statistical and photon-beam calibration errors makes it impossible to use the current best value of α_D as an independent check on neutron skin.

such as symmetry energy and pressure. An indicator that is particularly attractive, as it can be measured in finite nuclei, is dipole polarizability. Unfortunately, the current best experimental value of α_D in ^{208}Pb is not known precisely enough to offer an independent check on the neutron skin or to provide a quality constraint on EDF. We also demonstrate

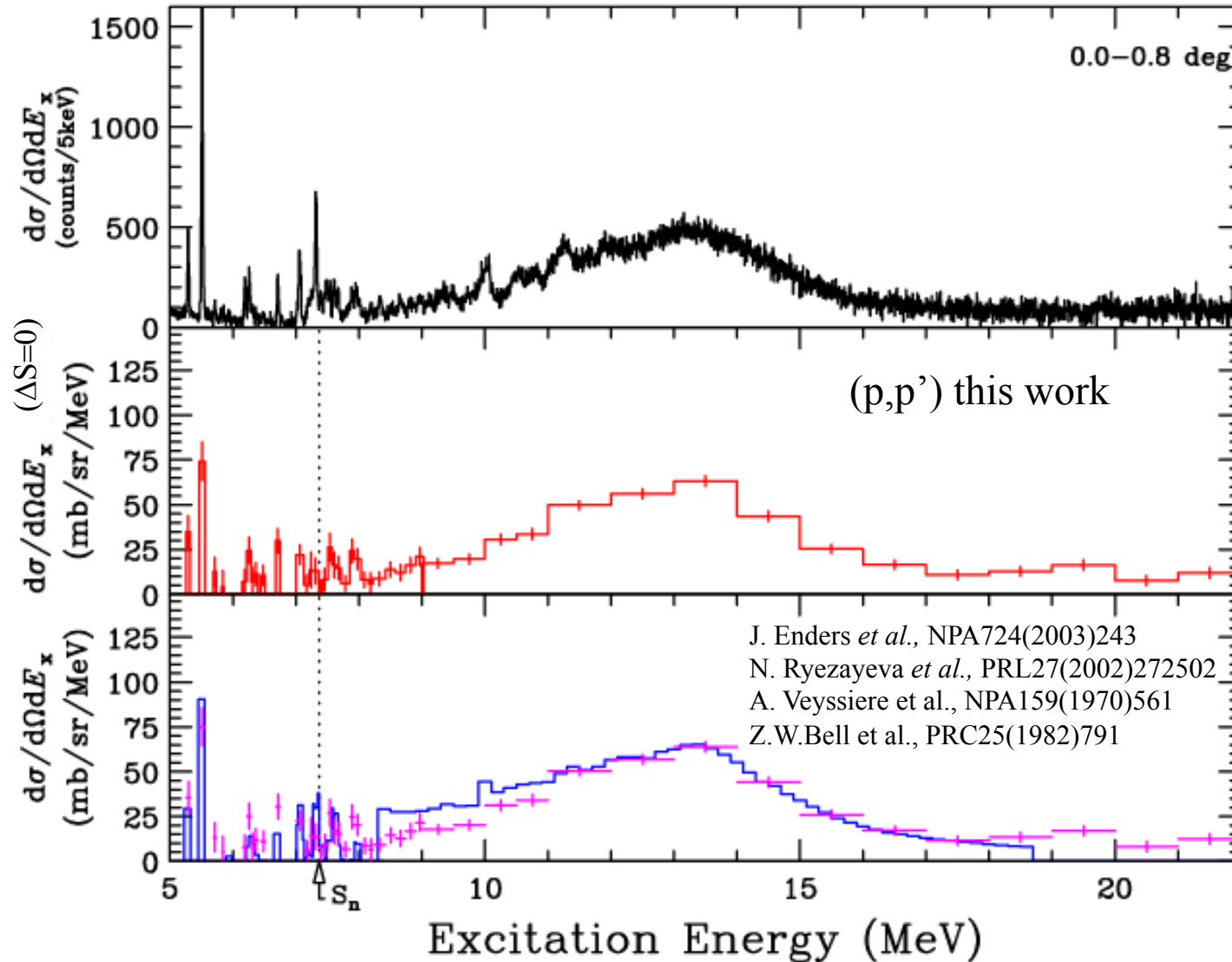
RCNP-E282実験

$^{208}\text{Pb}(p,p')$ at $E_p=295$ MeV



RCNP-E282実験

$^{208}\text{Pb}(p,p')$ at $E_p=295$ MeV



実験結果

我々の実験データから定量的な値を出す作業が間に合いませんでした。

申し訳ない。

References:

- [1] P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 81, 051303(R) (2010).
- [2] E. Lipparini and s. Stringari, Phys. Lett. 112B, 421 (1982).
- [3] O. Bohigas, N. van Giai, and D. Vautherin, Phys. Lett. 102B, 105 (1981).
- [4] J. Zenihiro et al., Phys. Rev. C 82, 044611 (2010).

E1励起状態とSum-Ruleについて

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- [6] A. Bohr and B.R. Mottelson, Nuclear Structure Vol II, pp.474-493.

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- [7] M. Bender, P.-H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75, 121 (2003).

Sum-Rules

- [8] E. Lipparini and S. Stringari, Phys. Rep. 175, 103 (1989).