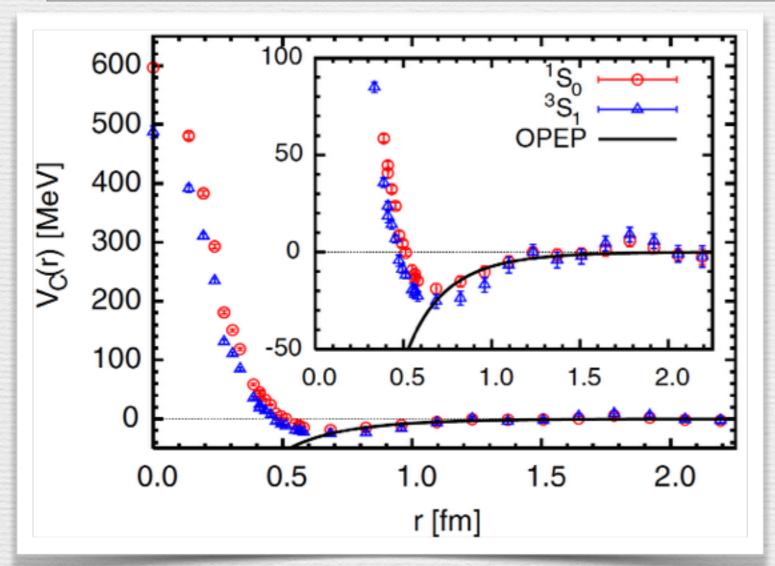
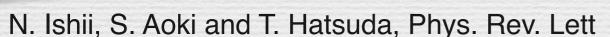
Extracting the NN Potential from QCD:

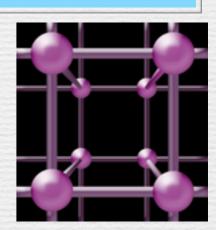
Recent Progresses of Lattice QCD Calculations





- S. Aoki, T. Hatsuda, N. Ishii, and H. Nemura, Butsuri
- S. Aoki et al., Prog. Theor. Exp. Phys.





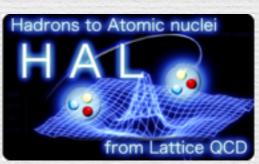
Awarded Nishina Prize 2012







Aoki, Hatsuda, Ishii & HAL QCD Collaboration





Abstract Phys. Rev. Lett.

Nuclear Force from Lattice QCD

N. Ishii, 1,2 S. Aoki, 3,4 and T. Hatsuda²

¹Center for Computational Sciences, University of Tsukuba, Tsukuba 305-8577, Ibaraki, Japan

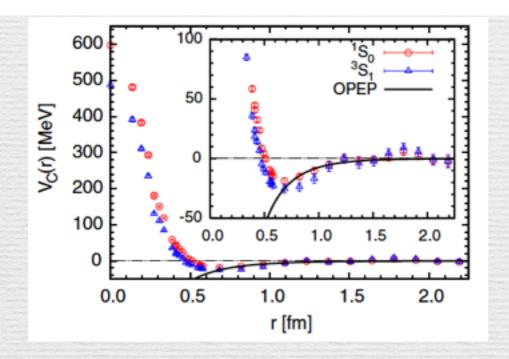
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The nucleon-nucleon (NN) potential is studied by lattice QCD simulations in the quenched approximation, using the plaquette gauge action and the Wilson quark action on a 32^4 [$\simeq (4.4 \text{ fm})^4$] lattice. A NN potential $V_{NN}(r)$ is defined from the equal-time Bethe-Salpeter amplitude with a local interpolating operator for the nucleon. By studying the NN interaction in the 1S_0 and 3S_1 channels, we show that the central part of $V_{NN}(r)$ has a strong repulsive core of a few hundred MeV at short distances $(r \leq 0.5 \text{ fm})$ surrounded by an attractive well at medium and long distances. These features are consistent with the known phenomenological features of the nuclear force.



Structure of the Paper

PRL 99, 022001 (2007)

PHYSICAL REVIEW LETTERS

N. Ishii, S. Aoki, T. Hatsuda,

Phys. Rev. Lett.

Nuclear Force from Lattice OCD

*Control for Computational Sciences, Convenity of Finducks Delastics SSS-8517, Strendt, Spons Street, Street, Convenity of Finducks, Date 153-0031, Agency Conductor Sciences of Physics, Convenity of Finducks, Delastic 153-0031, Agency Conductor Sciences of Part and Applications, Convenity of Finducks, Pacients (Appl. Street, Application Sciences Sciences Sciences Sciences Sciences Applications Sciences Sciences

The medican medican (CN) potential is standard by lattice (QC3) contribution in the quantitation code, using the plaquettic gauge action and the Wilson quark action on a 10^{12} 12

Motivation

properties regiments [3]. Although the origin of the re-polative core must be closely reliated to the quark gloven structure of the nucleons, it has bone a long standing open operation in QCD [8].

In this Learn, we report our first serious attempt to attack the problem of nuclear force from lattice QCD simulations. [7]. The essential date in to define a NVP potential from the segual since Berthe Sulpress (RS) amplitude of the two local interpolating operations separated by distance [9]. This stape of RS amplitude has bone employed by CP ENCS collaboration to midely the averantering on the lattice [9]. As we shall see below, our NV potential shows a strong regulative core of about a tow banded MeV at those dis-tances surrounded by an attraction at medium and long distances in the a surrounders.

 $-\frac{1}{2\omega}\nabla^{2}\phi(\vec{r}) + \int d^{2}r'\psi(\vec{r},\vec{r})\phi(\vec{r}) - E\phi(\vec{r}), \quad (1)$

Figure 3 shows the control (effective central) NN poten-ial in the ${}^{1}S_{\alpha}$ (${}^{2}S_{\gamma}$) channel at $r = q_{\gamma} = 6$. As for ∇^{γ} in

equation on the lattice [9]. By fitting the wave function $\phi(\vec{r})$ at the points $\vec{r} = (00\cdot 10, 0, 0)$ and $(00\cdot 10, 1, 0)$ by $G(\vec{r}, E)$, we obtain $E^{(1)}X_{\alpha}) = -0.49(35) MeV$ and $E^{(1)}X_{\alpha}) =$

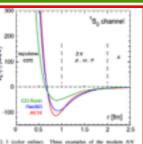
difference control part of the NN possets $V_{ij}(r)$ $(Y_{ij}^{m}(r))$ in the V_{ij} (S_{ij}) channel for $m_{ij}(n_{j} = 0.00)$. The inset shows its orthogonout. The solid lines correspond to the one-gion ex-

nal (CREST) given in Eq. (%)

80 85 10 15 20

pend to the one-pion ex-

Lattice QCD Simulation



ented in the "S_e (spin single and a wave) channel (12) on [17], Roleff [18], and AVIR [19] from the top at a :-

=0.67(10 MeV. Namely, there is a slight attraction between the two nucleons in a finite box. To make an independent check of the ground state saturation, we plot the dependence of $V_{\rm c}(s)$ in the $V_{\rm c}$ channel at soverall distance $r=0.014,\,0.15,\,0.00,\,1.37,\,$ and 2.19 fm in Fig. 4. The

7 "No. 100.0 (10.0 cm) (1.75, and 2.10 m for Fig. 2. m) automation indeed helds for 1 = g. 4 6 within sensor.
An anticipated from Fig. 2. F₁(s) and F₂⁽¹⁾(s) have applicate one at s × 0.35 fm with the height of about a few broaded MeV. Also, they have an intensition of about a few broaded MeV. Also, they have an intensition of about a color of the col

 $V_{c}^{\gamma}(s) = \frac{\Gamma_{cm}^{2}}{4\pi} \frac{(\theta_{1} \cdot \theta_{2})(\theta_{1} \cdot \theta_{2})}{3} \left(\frac{m_{\pi}}{4m_{\pi}}\right)^{2} \frac{\sigma^{-\alpha_{\pi}s}}{s}. \quad (5)$ where we have used $m_{\phi}=0.51$ GeV and $m_{\phi}=1.34$ GeV to be consistent with our date, while the physical value of the πW coupling consists is used, $g_{\phi\phi}^2/(kp)=14.0$. Even in the quantitat approximation, the one-given enchange is

\$ 44-20 - See w. Here you sail me are the

n/V coupling constant and a mass parameter of the ghost,

where emploing constant and a mean parameter of the ghost, respectively. The ghost potential has an exponential test which dominates ever the Yakawa potential at large distance. In significance, and excellentated by comparing the sign and the magnitude of $e^{a_1 A_{\alpha}}(x)$ and $e^{a_2 A_{\alpha}}(x)$ and the magnitude of $e^{a_1 A_{\alpha}}(x)$ and $e^{a_2 A_{\alpha}}(x)$ and the magnitude of $e^{a_1 A_{\alpha}}(x)$ and $e^{a_2 A_{\alpha}}(x)$ as an appoint sign between h_{α} and h_{α} . Our present data show an orderine of the ghost of large distances within course, which may be show a h_{α} and h_{α}

H_a (lettice unit)

FIG. 4 (robs collect: $z=z_0$ dependence of $V_{Z}(z)$ in the 3S_0 channel for around different values of the distance z

0001-9007/07/99(2)/022001(4)

PHYSICAL REVIEW LETTERS

ation with a certain parametrization of $V_{\rm sw}$ is solved compared with the data. On the other hand, if we can Fig. (1) can be used to define the medical potential (c, r) directly without recourse to the experimental inats except for quark masses and the QCD usale parameters is this Letter, instead of finding θ ' by varying θ , we take ely the leading term in the derivative expe

$$V_{\mathcal{L}}(s) = E + \frac{1}{2\mu} \frac{\tilde{\nabla}^2 \phi(s)}{\phi(s)}$$

 $p(r) = \frac{1}{24} \sum_{j,k} \frac{1}{\sum_{j} P_{ij}^{*} P_{ijk}^{*} \phi(|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_{j}^{*}|x|_$

Lattice QCD

underect $N_{\alpha} = \epsilon_{\alpha\beta\gamma} E_{\beta}^{\alpha} C_{YYP_{\beta\beta}}^{\alpha} / \epsilon_{\gamma}^{\beta}$ refer indices, α and β the Dirac indi-Simulation

ader the NW scattering at low energies, we take out the upper components of N_{ν} . The summation over the vector i projects out the state with zero-total momentum. The summation over discrete rotation R of the cubic group θ projects out the AT representation which contains $\theta = 0$. tate and $\ell \ge 4$ states. The former can be singled out by

of proposes over the A_i^* in expresentation which constant a = 0 areas and $a^* = a$ states. The foreser can be singled out by saliciting the lowest energy state with the procedure given in Eq. (6). The equil (singles) projection is carried out by the operator $P \cap P^*$, for example, $P^*_{ijk} = (i \cdot p_{ijk})^{-1} A_{ijk} = k_{ijk}$, the equation $P \cap P^*$ is the example, $P^*_{ijk} = (i \cdot p_{ijk})^{-1} A_{ijk} = k_{ijk}$, and the upper single repair replace dataset. The incommutation factor Z_i , which relates the EX amplitude on the lattice and that in the continuous, consolor of $P \cap P^*$. The continuous control of $P \cap P^*$ is the probability amplitude to the $P \cap P^*$ inclination $P \cap P^*$ inclinate located as point Z_i and another "inclination" three quarter located as point Z_i and another "inclination" three quarter located as point Z_i and another "inclination" three quarter located as point Z_i and another "inclination" three quarter located as point Z_i and another "inclination" $P \cap P^*$ in the element constant or only the element constant or only the element Z_i or point Z_i in the element Z_i of the element Z_i or Z_i in the element Z_i or Z_i in the element Z_i or Z_i in the articles of the element Z_i or Z_i in modifies the natural weaking to the element Z_i or possible of Z_i in the element Z_i or Z_i in Z_i i tor. For more details on those points, see \$100.

In the actual simulations, Eq. (3) is obtained through the

 $F_{NN}(\vec{k}, \vec{S}, n, t_0) = 00N_{\pi}^{*}(\vec{k}, cN_{\pi}^{*}(\vec{S}, c)\overline{T}_{NN}(t_0)00)$ $=\sum A_{\alpha}(0)N_{\alpha}^{\alpha}(\hat{g})N_{\beta}^{\beta}(\hat{g})(x)e^{-K_{\alpha}(y-x_{\beta})}, \quad (4)$

the ground state contribution of the IN system, we askyr the well-source, $\mathcal{F}_{ab}(x) = \mathcal{F}_{ab}^{-1} \mathcal{F}_{ab}(x)$, $\mathcal{F}_{ab}(x)$, where \mathcal{N} is obtained from N by neglecting q by $(0)_{ab} = \sum_{a \neq b} (1_{ab}, 1_{ab})$, where covery of the two-sourcess state $|a\rangle$, and $A_a(x_b) = (a)_{aab}^T \mathcal{F}_{ab}(b)$. Because of the finite lattice volume L^1 , the energy L takes describe value and has a finite that them the constructing case $L^2 = \mathcal{O}(1/L^2)$ to be determined from the simulations |H|. In this Lattice, we focus on the upon-singlet and spo-traject channels with zero-orbital angular monomerum. In

in Eq. (2) with $\phi(r)$ being projected onto $(T_{\phi_1}(rT_{\phi_2}))$ corresponds to the central potential (the effective central potential).

To calculate 4471, we have carried our simulations on

Detailed

Parameters

Figure 2 thinks the section order. Figure 2 thinks the latter $\Psi_{\rm c}$ is the preference of the wave function at the time dice $t=0,\dots,6$. They are normalized as the spatial boundary t=0.022=16, t, 0.0. All the data including the off-axis once are plotted for $x \approx 0.7$ fm.

contribution, because it is repulsive (attractive) in 1) general contractions of the problem of the regular of the regular or core increases by 47% at the origin, while the emissions of the potential is shifted to x = 0.8 fm with approximately

Summary Future

sergies and extract the control potential $V_c(\omega)$ at fixed E

PHYSICAL REVIEW LETTERS

No. 155055). Our simulations have been performed with IBM Blue Geneff, at KTK under a support of its Large Scale Simulation Program, No. 18 and No. 06-21 (FT/S006).

Comments

met of Fig. 3 shows that *5, has stronger attraction than *5. Note that this difference cannot be attributed to the

the name depth.

It institutes we have studied the N.V interpreses used. In minimary, we have reached the XXX interactions using the lattice QCDD immulation in the quenched approximation on a 0.6.4 Stall* lattice with the quark mass corresponding to $m_{\rm c}/m_{\rm p} \sim 0.50$. We define a XXX potential with the use of the equal-time Birthe-Sulpeter amplitude for rescharal immepotating operators of the machem. The control effectives control part of the potential in the " $\Sigma_{\rm b}$ (" $\Sigma_{\rm b}$) channel at low energies turns out to have a regulative curr

It would be quite interesting to derive the tensor as spin-orbit forces by making appropriate projections of the wave function. Studies of the hypersonactions and hyperon-legeron potentials, whose experimental informa-Ingurantegerous potentials, whose experimental informa-tion is currently very limited, are now under investigation-they are particularly important for the physics of types nuclei and neutron star core. To urranel the physical origin of the expolitive core, we need further studies on its quark-men dependence, channel dependence, and interpolating-

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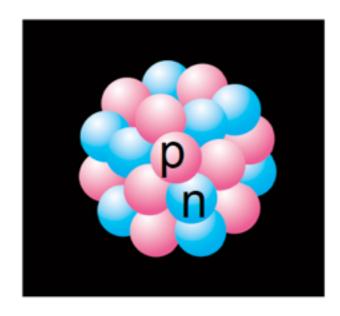
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 S. Arkis et al. CCP/BACS-Colaboration), Phys. Rev. D 73, 004400. (2008).
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(1) S. Decomposition parameter a_{2n} the formula is modified as h (n = 1) = a₂₁/2 and a₂₁ = a₂₁ = a₂₂ = a₂₁.
 (2) The Landstor from volume terminal [10] employe with E distinctly in the terminal measured, [10] employe with E distinctly in the term to lead to a₂₁/2 | − 0.2(2)(20) for and a₂₁/2 | − 0.2(2)(20) for and a₂₁/2 | − 0.2(2)(20) for an element from the term tentrino.
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Motivation

What binds protons and neutrons inside a nuclei?



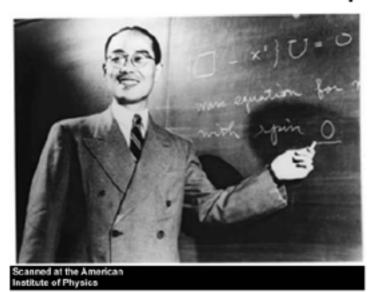
gravity: too weak

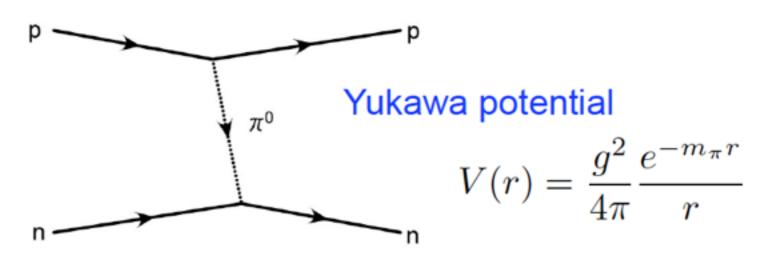
Coulomb: repulsive between pp no force between nn, np

New force (nuclear force)?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force

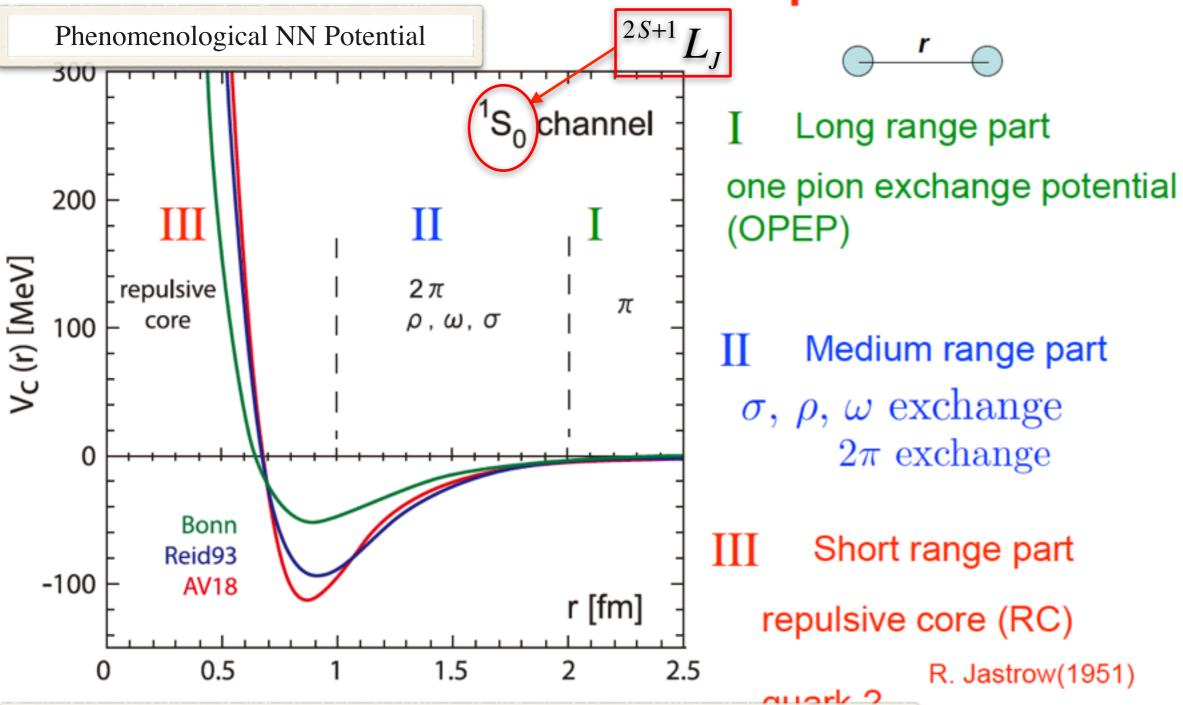




1949 Nobel prize

Motivation

Modern nucleon-nucleon potential



It has been a fundamental question whether the *NN* potential can be described by QCD.

Although the origin of the repulsive core must be closely related to the quark-gluon structure of the nucleon, it has been a long-standing open question in QCD [6].

QCD: Quantum Chromodynamics

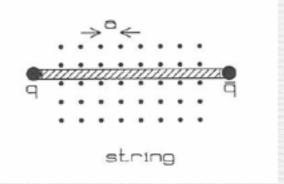


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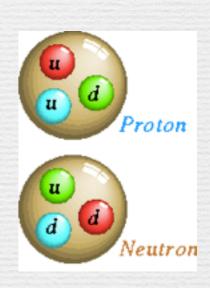




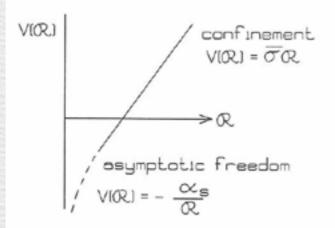
potential linearly dependent on r

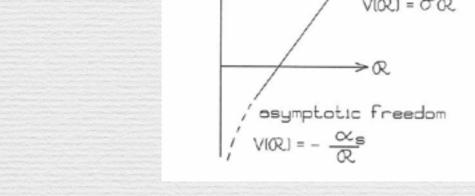






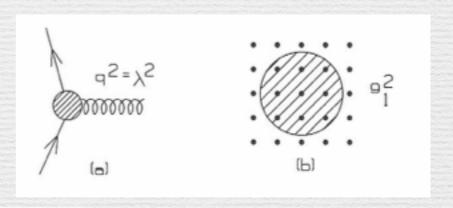
Asymptotic freedom (

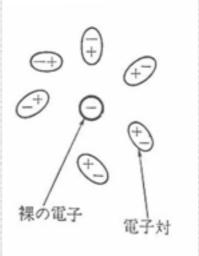






ultraviolet divergence strong nonlinear system





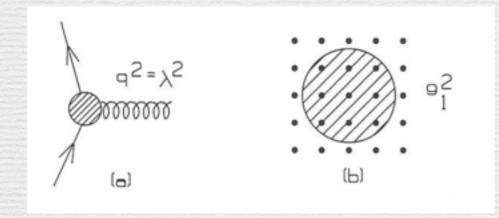
The bare particle mass and charge are not observed. Instead a particle with $q\overline{q}(e^+e^-)$ clouds is observed.

Why is a QCD calc. at low energy so difficult? non-perturbative QCD

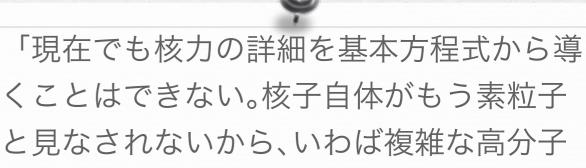
locally gauge invariant, non-abelian, strong-coupling field theory

Strong coupling field theory ⇔ perturbation cannot be applied

 $m_u = 2.15(15) \text{ MeV}$ $m_d = 4.70(20) \text{ MeV}$ by Lattice QCD [PDG'12]







の性質をシュレディンガー方程式から出発 して決定せよというようなもので、むしろ

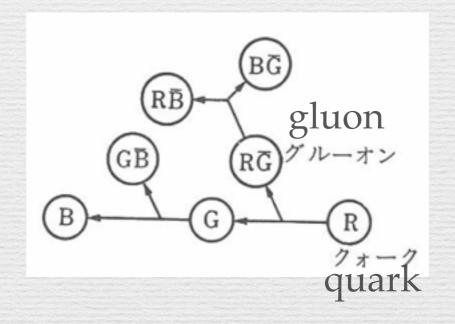
これは無理な話である」(南部陽一郎

「クォーク」(1981)

Non-abelian gauge

non-commutative

A gluon carries the color charge can produce other gluons



QCD on Lattice

Lattice Gauge Theory (LGT)

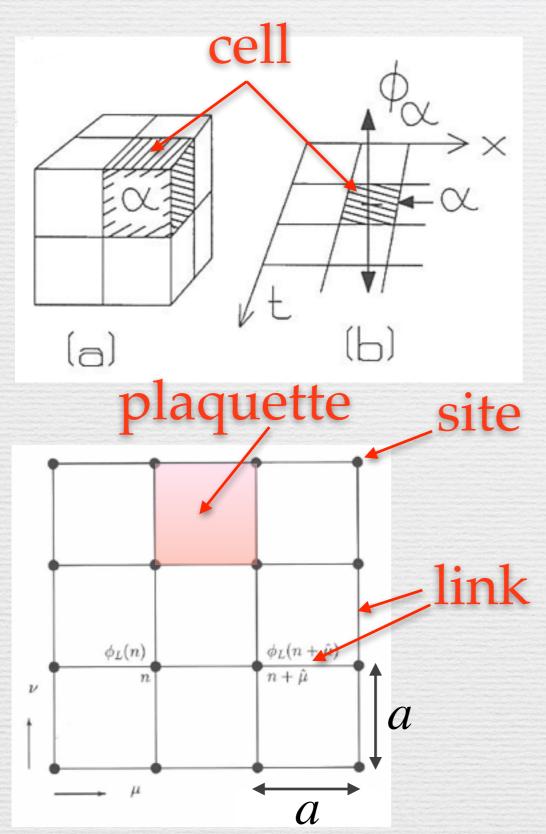
K.G. Wilson, Phys. Rev.

- holds local gauge invariance
- ultraviolet divergence is natural solved (ultraviolet cutoff)
- limit is taken to the infinite volume (thermodynamical limit)
- limit is taken to continuum (a
- path integral method is used to calculate amplitudes
- * a Euclidean space-time is used with imaginary time
- parameters:
 - strong coupling constant
 - quark mass of each flavor
 - CP violation phase

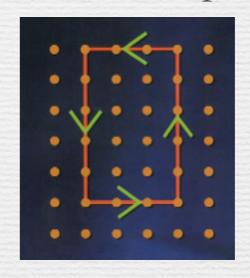
only in the work by N. Ishii et al.

QCD on Lattice

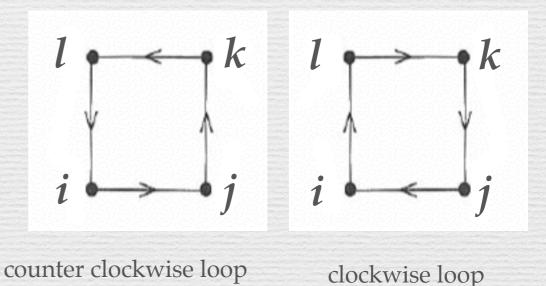
A Euclidean space-time of finite size is divided into a lattice of size "a".



Wilson loop



contribution of action from a plaquette: plaquette action



Path Integral (

N. Wiener 1933 P.A.M. Dirac 1948 R.

In analytical mechanics

Lagrangian:
$$L(q,\dot{q}) = T - V$$

Action:
$$S(f,i) \equiv \int_{t_i}^{t_f} L(q,\dot{q}) dt$$

Principle of least action (variational principle)

$$\delta \int_{t_i}^{t_f} L(q,\dot{q}) dt = 0$$

(1)

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = 0$$

(2) Euler-Lagrange equation

In the path integral formulation, the finding a particle at position

$$\langle q_f t_f | q_i t_i \rangle = \int D(q) \exp \left\{ \frac{i}{\hbar} S(f, i) \right\}$$
 (3)

c.f. Fermat's principle for the path of the light ray

 $\left|q_{f}t_{f}
ight
angle$

The integration should be taken for _____

Taking the classical limit $\hbar \rightarrow 0$

$$\int D(q)$$
: volume element

Pass Integral and Partition Function

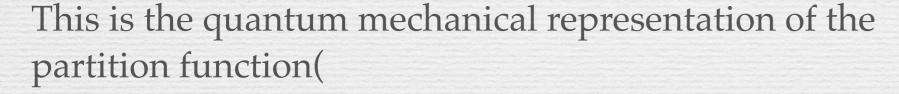
Defining the imaginary time

$$\tau \equiv it$$
, $d\tau \equiv idt$

$$\overline{S}(f,i) \equiv iS(f,i) = -\int_{\tau_i}^{\tau_f} L(q,\dot{q}) d\tau$$

the pass integral is expressed as

$$\langle q_f t_f | q_i t_i \rangle = z = \int \overline{D}(q) \exp \left\{ -\frac{1}{\hbar} \overline{S}(f,i) \right\}$$

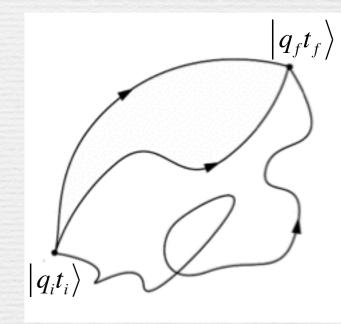


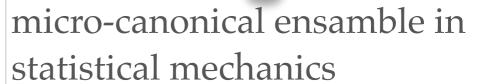
$$z = \int \overline{D}(q) \exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}$$
$$\overline{S}(\hbar\beta,0) = \int_0^{\hbar\beta} \left[\frac{1}{2}m\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]d\tau$$

In the quantum field theory, Lagrangian density

$$\overline{S}(\hbar\beta,0) = \int_0^{\hbar\beta} d\tau \int dx \mathcal{L} = \int d^4x \mathcal{L}$$

$$z \to Z$$





$$z = \sum_{n} \exp(-\beta E_n) = Tr \left[\exp(-\beta \hat{h}) \right]$$

$$Z = z^{N} \qquad \beta \equiv \frac{1}{k_{B}T}$$

Pass Integral and Partition Function

With a periodic boundary condition in the imaginary time, the expectation value of an operator \hat{O}

$$\hat{\rho}_{th} = \frac{\exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}}{\int \overline{D}(q) \exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}} = \frac{\exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}}{Z}$$
 statistical operator

$$\left\langle \left\langle \hat{O} \right\rangle \right\rangle = Tr(\hat{O}\hat{\rho}_{th}) = \frac{\int \overline{D}(q)O(\phi)\exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}}{\int \overline{D}(q)\exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}}$$

expectation value of the oper@or



in statistical mechanics

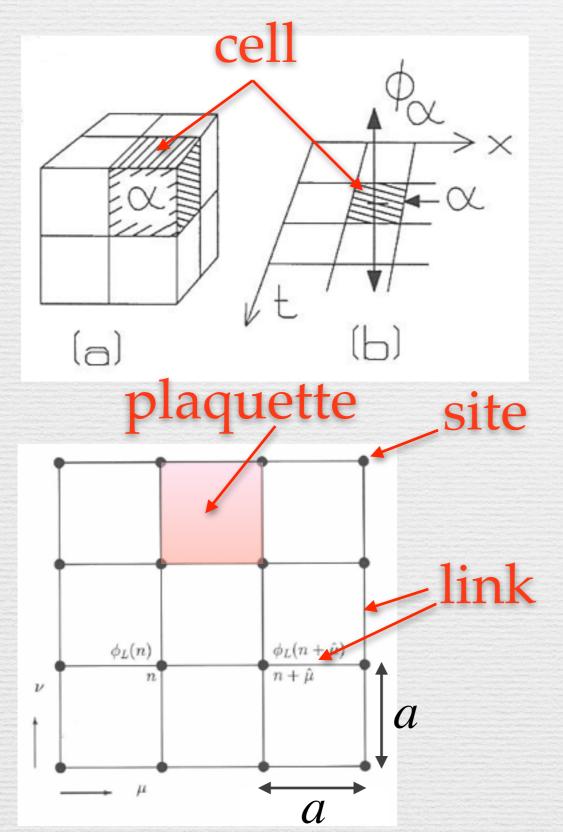
$$\hat{\rho}_{th} = \frac{e^{-\beta \hat{H}}}{Tr(\hat{O}e^{-\beta \hat{H}})} = \frac{e^{-\beta \hat{H}}}{Z} \quad \text{statistical operator}$$

$$(\text{density matrix})$$

$$\langle \langle \hat{O} \rangle \rangle = \frac{Tr(\hat{O}e^{-\beta\hat{H}})}{Z} = Tr(\hat{O}\hat{\rho}_{th})$$
 thermal average

QCD on Lattice

A Euclidean space-time of finite size is divided into a lattice of size "a".



A lattice action is not uniquely determined.

Convergence to the continuum limit.

$$\lim_{a\to 0} S_{lat.} = S_{cont.}$$

Holding the same symmetry as

Boson fields are assigned to each link

QED U(1) gauge

$$U_{ji}(A_{\mu}) \equiv \exp\left\{ie_0(x_j - x_i)_{\mu} A_{\mu}\left(\frac{1}{2}(x_j + x_i)\right)\right\}$$

Yang Mills SU(2) gauge

$$U_{ji}(\mathbf{A}_{\mu}) \equiv \exp\left\{ig_0(x_j - x_i)_{\mu} \frac{1}{2} \boldsymbol{\tau} \cdot \mathbf{A}_{\mu} \left(\frac{1}{2}(x_j + x_i)\right)\right\}$$

QCD SU(3) color gauge

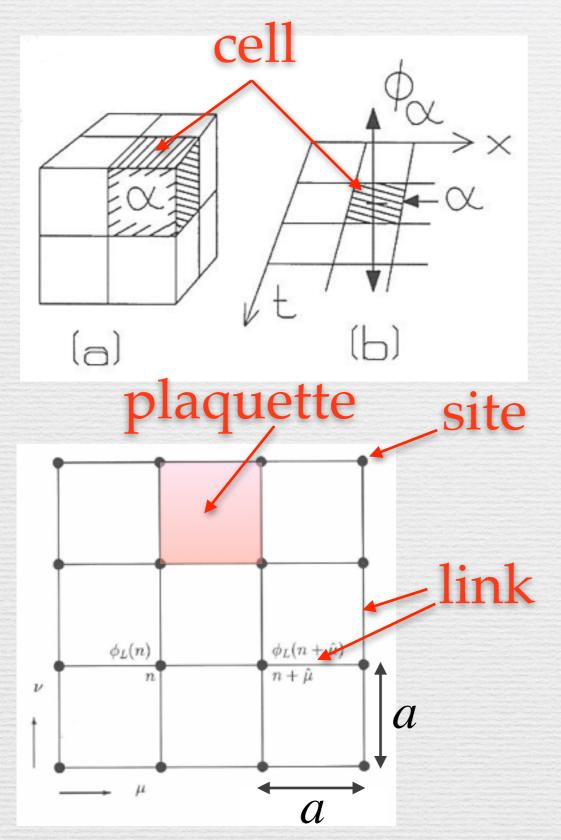
$$U_{ji}\left(A_{\mu}^{a}\right) \equiv \exp\left\{ig_{0}\left(x_{j}-x_{i}\right)_{\mu}\frac{1}{2}\lambda^{a} \bullet \left[A_{\mu}^{a}\left(\frac{1}{2}\left(x_{j}+x_{i}\right)\right)\right]\right\}$$

Quark fields are assigned to each site

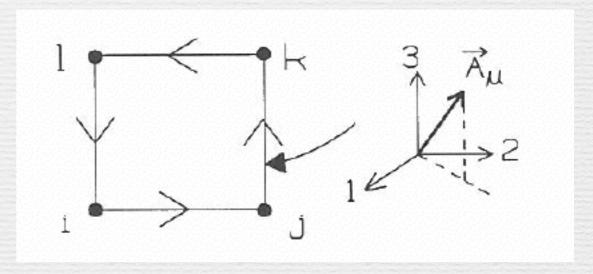
$$S_{F} = \frac{1}{2} \sum_{n,\mu} \left[\overline{\psi}_{n} \gamma_{\mu} U_{n,\mu} \psi_{n+\hat{\mu}} - \overline{\psi}_{n+\hat{\mu}} \gamma_{\mu} U_{n,\mu}^{\dagger} \psi_{n} \right] + M \sum_{n} \overline{\psi}_{n} \psi_{n}$$

QCD on Lattice

A Euclidean space-time of finite size is divided into a lattice of size "a".



Integration of action in ALL the field dimensions.

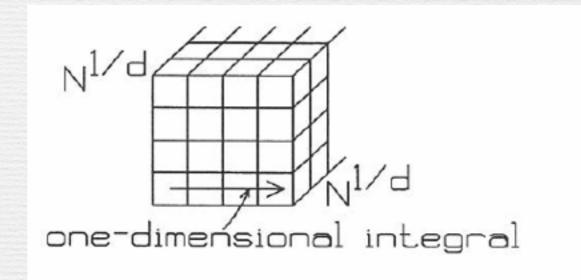


Take the sum of the integration for all the plaquettes and sites.

$$\hat{\rho}_{th} = \frac{\exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}}{\int \overline{D}(q) \exp\left\{-\frac{1}{\hbar}\overline{S}(\hbar\beta,0)\right\}}$$
$$\left\langle \left\langle \hat{O} \right\rangle \right\rangle = Tr\left(\hat{O}\hat{\rho}_{th}\right)$$

Monte-Carlo Simulations

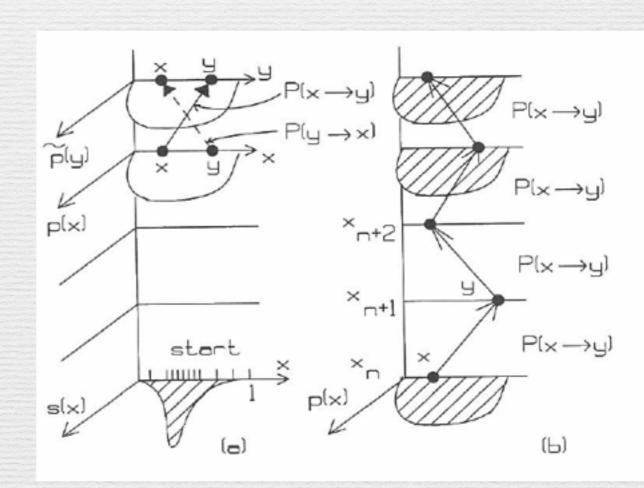
Monte-Carlo integration relative uncertainty $\sim \frac{1}{\sqrt{N}}$ Integration by mesh (d-dim.) relative uncertainty $\sim \frac{1}{N^{1/d}}$



Monte-Carlo integration is faster for d>3.

Markov-Chains Monte-Carlo integration

Chain of positions are created with an appropriate weight distribution for effective integration.

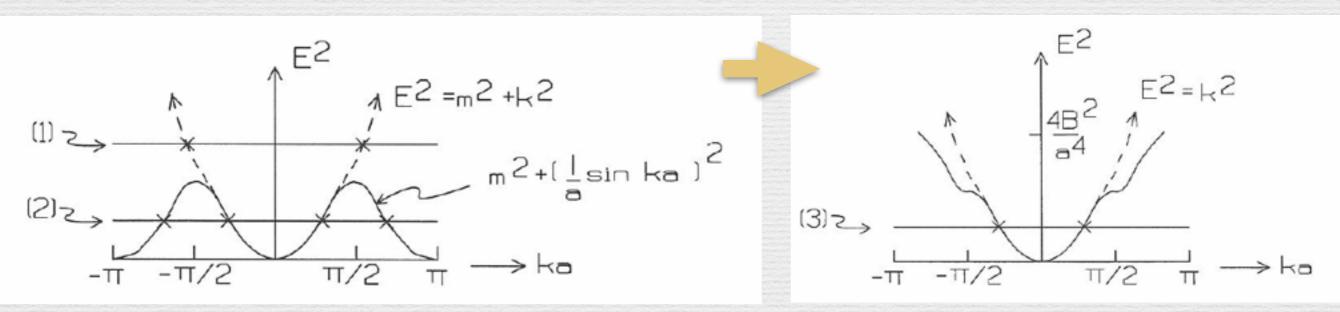


QCD on Lattice

Willson fermions

additional term

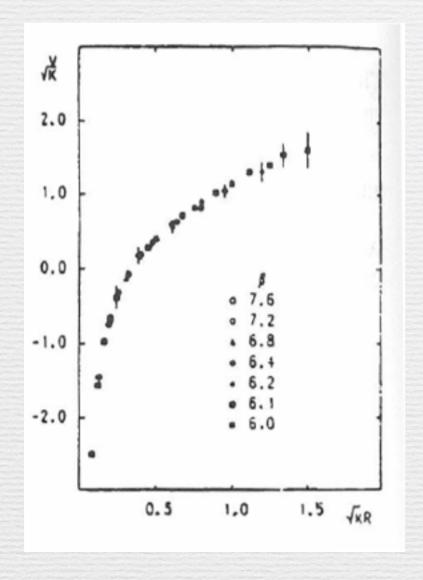
$$S_W = -ar \int d^4x \, \bar{\psi} D^2 \psi \quad \rightarrow -\frac{r}{2} \sum_{n,\mu} \left[\bar{\psi}_n U_{n,\mu} \psi_{n+\hat{\mu}} + \bar{\psi}_{n+\hat{\mu}} U_{n,\mu}^{\dagger} \psi_n - 2\bar{\psi}_n \psi_n \right]$$

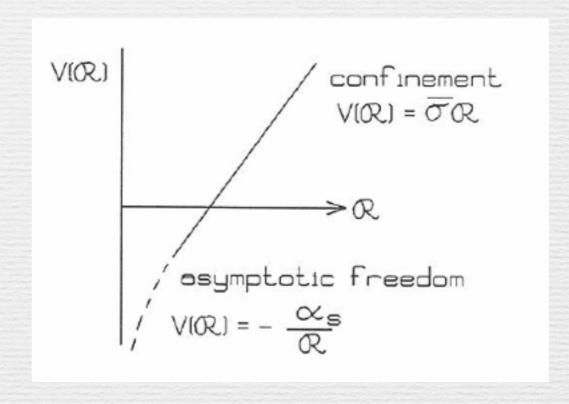


fermion doubling

Confinement: linear dependence of potential on R

S.W. Otto and J.D. Stack PRL





Lattice of size 16
Scaling with
$$\overline{\beta}$$

$$2\sigma \equiv \overline{\beta} \equiv \frac{N}{g^2}, \quad N = 3$$

$$\lambda_1 \approx 4 \text{ MeV} \quad \sqrt{\kappa} \approx \sqrt{\overline{\sigma}} \approx 400 \text{ MeV}$$

Hadron Mass Spectrum

A.S. Kronfeld, Ann. Rev. Nucl. Part. Sci. 62, 265 (2012)

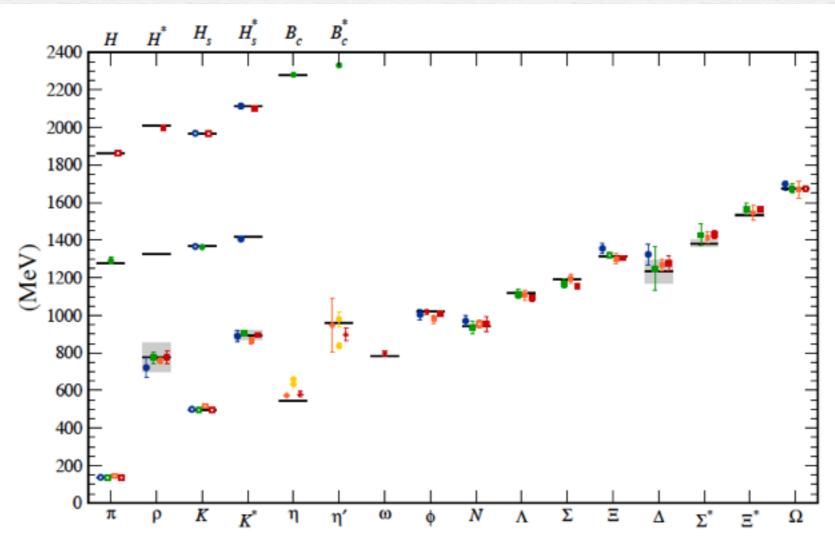


Figure 2: Hadron spectrum from lattice QCD. Comprehensive results for mesons and baryons are from MILC (27, 28), PACS-CS (29), BMW (30), and QCDSF (31). Results for η and η' are from RBC & UKQCD (32), Hadron Spectrum (33) (also the only ω mass), and UKQCD (34). Results for heavy-light hadrons from Fermilab-MILC (35), HPQCD (36), and Mohler & Woloshyn (37).

Excellent agreement among the predictions and with the experimental data.

Quark Masses

A.S. Kronfeld, Ann. Rev. Nucl. Part. Sci. 62, 265 (2012

 $Table \ 2 \quad Quark \ masses \ from \ lattice \ quantum \ chromodynamics \ converted \ to \ the \ \overline{MS} \ scheme \ and \ run \ to \ the \ scale \ indicated^a$

Flavor (scale)	Reference 28	Reference 53	Reference 54	Reference 55	Reference 56
m_u (2 GeV)	1.9 ± 0.2	2.01 ± 0.14	2.24 ± 0.35	2.15 ± 0.11	_
\bar{m}_d (2 GeV)	4.6 ± 0.3	4.79 ± 0.16	4.65 ± 0.35	4.79 ± 0.14	_
\bar{m}_s (2 GeV)	88 ± 5	92.4 ± 1.5	97.7 ± 6.2	95.5 ± 1.9	_
\bar{m}_c (3 GeV)	_	_	_	_	986 + 10
\bar{m}_b (10 GeV)	_	_	_	_	$3,617 \pm 25$

Nambu-Bethe-Salpeter Equation

Schrödinger Equations with a Non-local Potential U(

$$-\frac{1}{2\mu}\nabla^2\phi(\vec{r}) + \int d^3r' U(\vec{r},\vec{r}')\phi(\vec{r}) = E\phi(\vec{r}) \qquad (1)$$

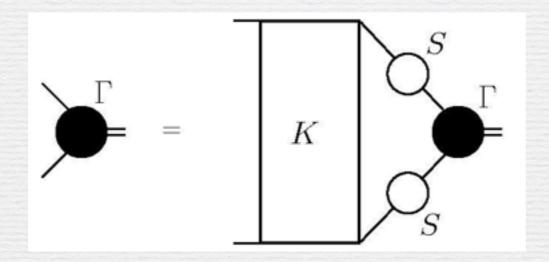
c.f. Schrödinger Equations with a local Potential U(

$$\left\{-\frac{1}{2\mu}\nabla^2 + U(\vec{r})\right\}\phi(\vec{r}) = E\phi(\vec{r})$$

For central potential U(r)=V

$$V_C(r) = E + \frac{1}{2\mu} \frac{\overrightarrow{\nabla}^2 \phi(r)}{\phi(r)}$$
 (2)

Nambu-Bethe-Salpeter Equation



$$\Gamma(P,p) = \int \frac{d^4k}{(2\pi)^4} \ K(P,p,k) \, S(k-\frac{P}{2}) \, \Gamma(P,k) \, S(k+\frac{P}{2}) \label{eq:gamma}$$

(Nambu-)Bethe-Salpeter equation is used to analyze and calculate properties of bound states of two constituents in relativistic quantum field theory.

It contains an infinite series of interactions in the shape of ladder diagram in Feynman diagrams. (cf Lipman-Schwinger equation).

$$\varphi_E(\mathbf{r},t) = \langle 0|N(\mathbf{x},t)N(\mathbf{y},t)|2N,E\rangle$$
 $N(\mathbf{r},t) \sim q(\mathbf{r},t)^3$

of the equal-time NBS equation ~ two nucleon amplitude (below m

Extract the "Potential" from the simulated "Amplitude".

Nucleon-Nucleon (NN) Interaction

Expansion

$$\begin{split} V &= V_0(r) + V_{\sigma}(r) \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2 + V_{\tau}(r) \mathbf{\tau}_1 \cdot \mathbf{\tau}_2 + V_{\sigma\tau}(r) \mathbf{\sigma}_1 \cdot \mathbf{\sigma}_2 \ \mathbf{\tau}_1 \cdot \mathbf{\tau}_2 \quad \text{Central Terms} \\ &+ V_{LS0}(r) \mathbf{L} \cdot \mathbf{S} + V_{LS\tau}(r) \mathbf{L} \cdot \mathbf{S} \mathbf{\tau}_1 \cdot \mathbf{\tau}_2 \quad \text{Spin-Orbit Terms} \\ &+ V_{T0}(r) S_{12} + V_{T\tau}(r) S_{12} \mathbf{\tau}_1 \cdot \mathbf{\tau}_2 \quad \text{Tensor Terms} \end{split}$$

$$S_{12} = 3 \frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - \sigma_1 \sigma_2$$

Tensor Interaction

Lattice QCD Parameters

- Lattice 32
- quenched approximation
- plaquette gauge action, gauge coupling β
- Wilson quark action
- a=0.137 fm,
- hopping parameter κ =0.1665 m_{π}



- global heat bath algorithm
- Dirichlet (periodic) boundary condition
- wall source is placed at



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 $\label{eq:control_entrol_entrol} w_{i,j} = \frac{1}{i} \sum_{j} \sum_{i} \sum_{j} \hat{x}_i (x_i \hat{x}_i) w_i(y_i(y_i) - y_i(y_i) x_i} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_i(y_i) - y_j(y_i) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_i(y_i) - y_j(y_i) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_i(y_i) - y_j(y_i) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_i) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_i) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_j) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \sum_{j} \hat{x}_j (x_j \hat{x}_j) w_j(y_j) x_j} \\ e^{-i \frac{1}{i} \sum_{j} \sum_{$

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Lattice QCD Result

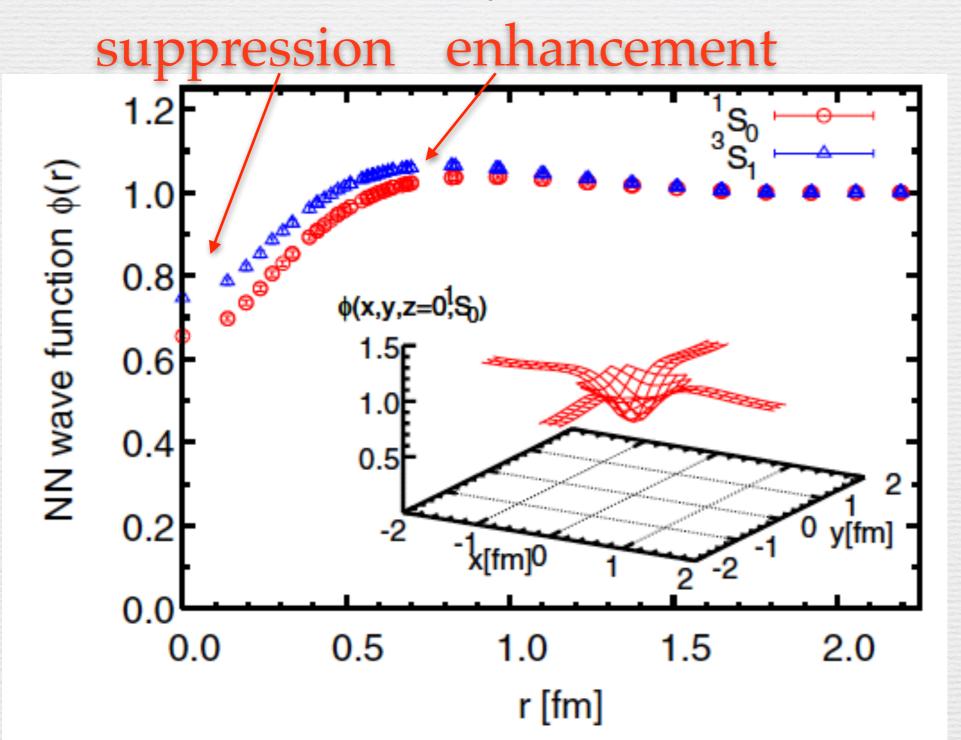
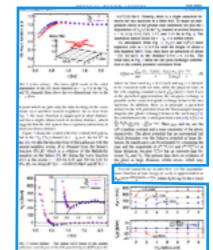


FIG. 2 (color online). The lattice QCD result of the radial dependence of the NN wave function at $t - t_0 = 6$ in the 1S_0 and 3S_1 channels. Inset shows the two-dimensional view in the x - y plane.



Lattice QCD Result



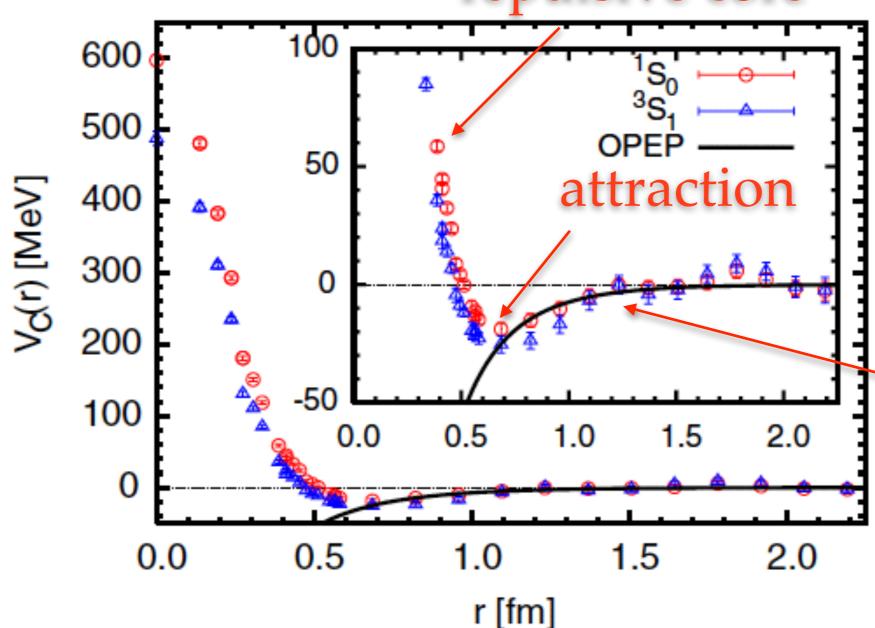
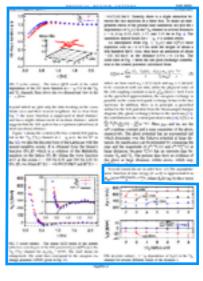


FIG. 3 (color online). The lattice QCD result of the central (effective central) part of the NN potential $V_C(r)$ [$V_C^{\rm eff}(r)$] in the 1S_0 (3S_1) channel for $m_\pi/m_\rho=0.595$. The inset shows its enlargement. The solid lines correspond to the one-pion exchange potential (OPEP) given in Eq. (5).



π exchange tail

One Pion Exchange Potential (OPEP)

Schrödinger Equations with a Non-local Potential U(

$$V_C^{\pi}(r) = \frac{g_{\pi N}^2}{4\pi} \frac{(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)}{3} \left(\frac{m_{\pi}}{2m_N}\right)^2 \frac{e^{-m_{\pi}r}}{r}, \quad (5)$$

with

Lattice QCD Result

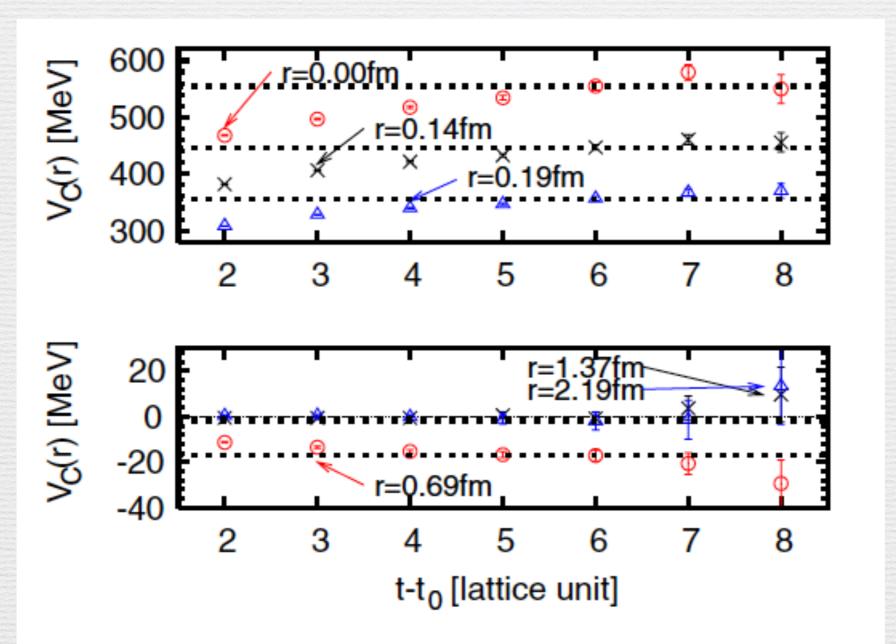


FIG. 4 (color online). $t - t_0$ dependence of $V_C(r)$ in the 1S_0 channel for several different values of the distance r.

Saturation of potential in time coordinate

Discussions

- Fig. 2: The wave function is suppressed at short distance, which suggests that the NN system has a repulsion (attraction) at short (medium) distance.
- Fig. 3: system energy (in the finite box)

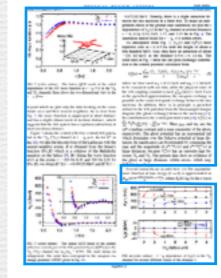
E(1)

E(3)

slight attraction in the

Mixing with the interaction is effectively incorporated in the central channel.

- The central potentials have
 - * repulsive core at r<0.5 fm: a few hundred MeV
 - + attraction at 0.5<r<1.0 fm: -(20-30) MeV
- quenched artifact from the flavor-singlet hairpin diagram (the ghost exchange)



Comments

- 1. scattering length,
 - $a_0(1)$
 - $a_0(3)$

small positive value

- 2. It is dangerous to compare masses with
- 3. If the attraction is large enough, the system may have a bound state.

The

Current values: *a* (fm) r_0 (fm) NN -18.9 ± 0.4 2.75 ± 0.11 nn

 -23.740 ± 0.020 2.77 ± 0.05 np -17.3 ± 0.4 2.85 ± 0.04 pp

- 4. Preliminary result with lighter quark mass at κ =0.1657 m_{π}
 - 40% higher repulsive core
 - same potential minimum at r~0.8 fm with the same depth

Summary

- 1. The central (effective central) potential in the channel at low energy have a repulsive core surrounded by attractive well at medium and long distances.
- 2. These properties are known to be the important features of the phenomenological
- 3. The long range tail of with the one-pion exchange.
- 4. It would be quite interesting to derive
 - the tensor and spin-orbit forces
 - hyperon-nucleon and hyperon-hyperon potentials
 - physics origin of the repulsive core

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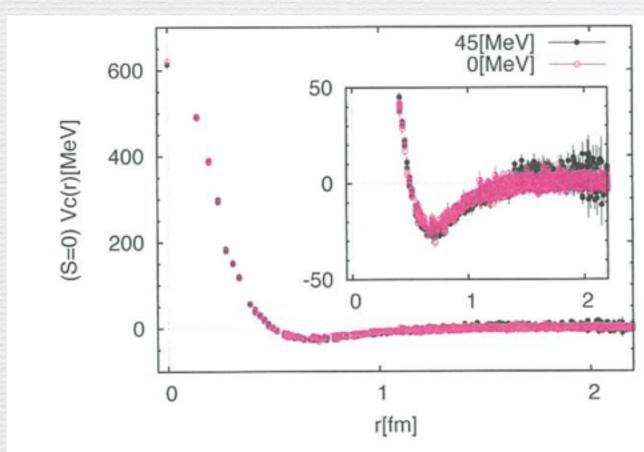


図3 異なったエネルギーの波動関数から求められたS=0の状態の核力ポテンシャルの比較. $e_k \approx 0$ MeV(黒), ≈ 45 MeV(赤)である. 計算の他の条件は図2と同じ.

Comparison of potentials derived from wave functions at two different energies.

hyperon-nucleon potential

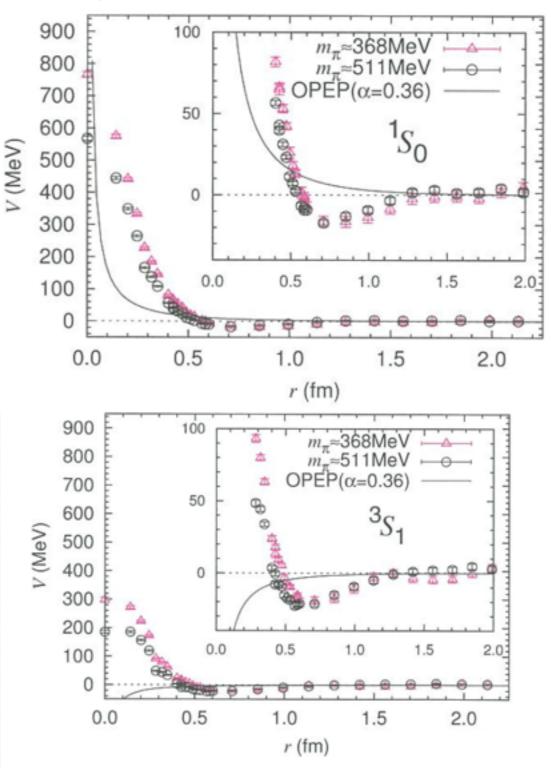


図4 $\Xi^0 p$ ポテンシャル. 微分展開の最低次の結果. 丸が $m_\pi \approx 510$ MeV, 三角が $m_\pi \approx 370$ MeV のデータ. 計算の他の条件は図2と同じ. 実線は1つのパイオシを交換した場合のポテンシャル (OPEP) である. (上) 全スピンがゼロの場合. (下) 全スピンが1の場合.

Central potential π

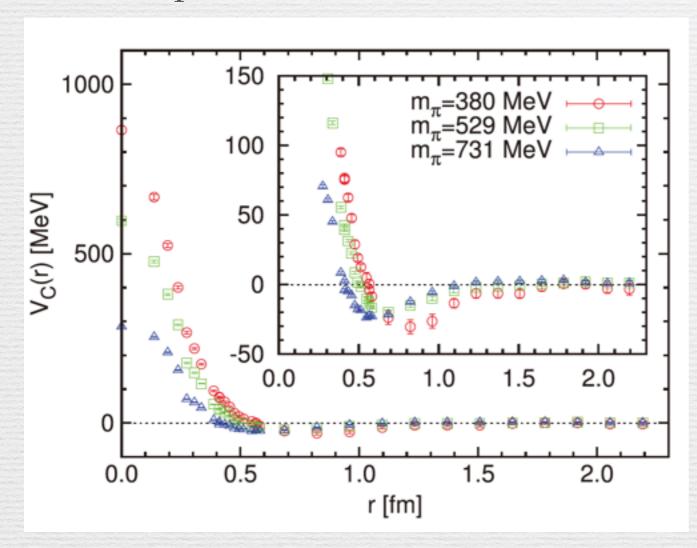


Fig. 3. The central potentials for the spin-singlet channel from the orbital A

QCD. Taken from Ref. [

S. Aoki et al., Prog. Theor. Exp. Phys.

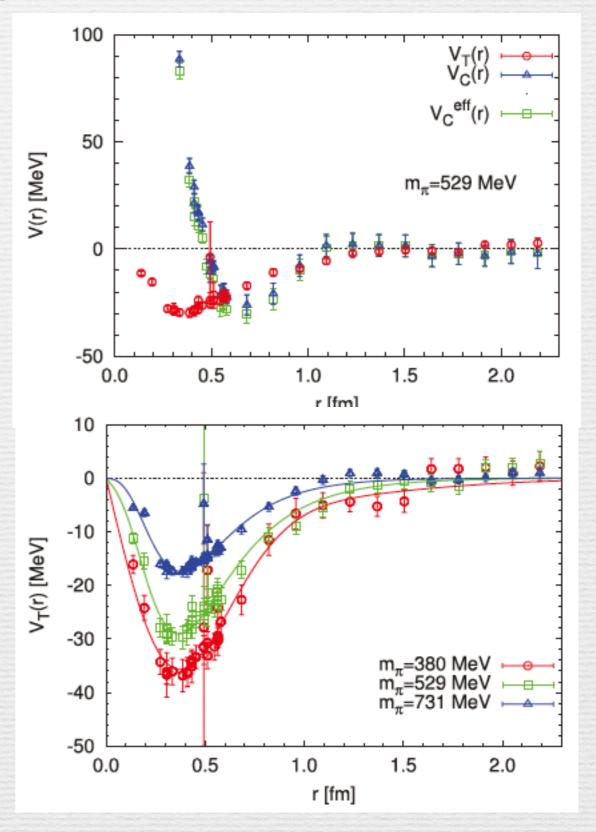


Fig. 5. (Top) The central potential V
NBS wave function, together with the effective central potential V
(Bottom) Pion mass dependence of the tensor potential. The lines show the four-parameter fit using one-pion-exchange

Binding energy of H-dibaryon

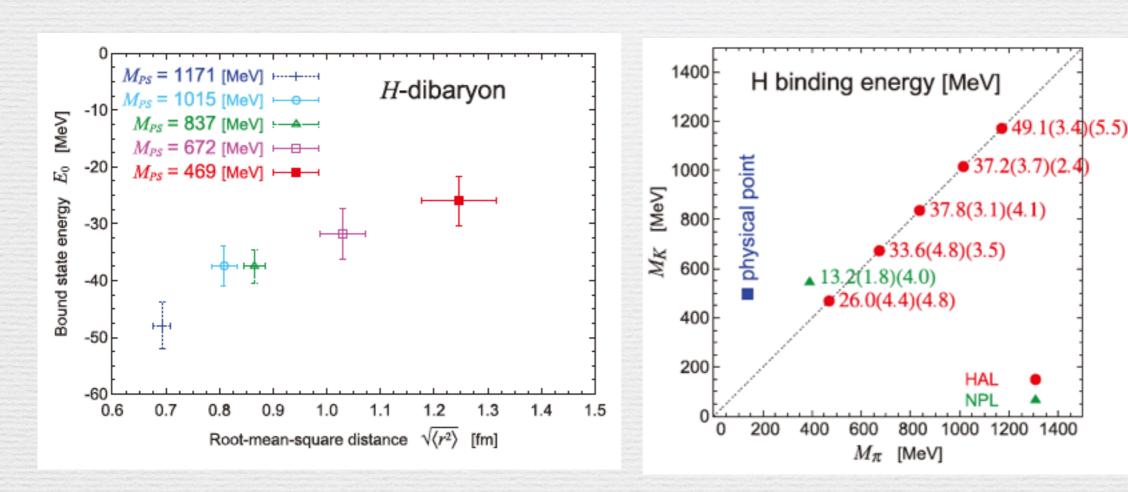


Fig. 3. The central potentials for the spin-singlet channel from the orbital A QCD. Taken from Ref. [

Three Nucleon Force

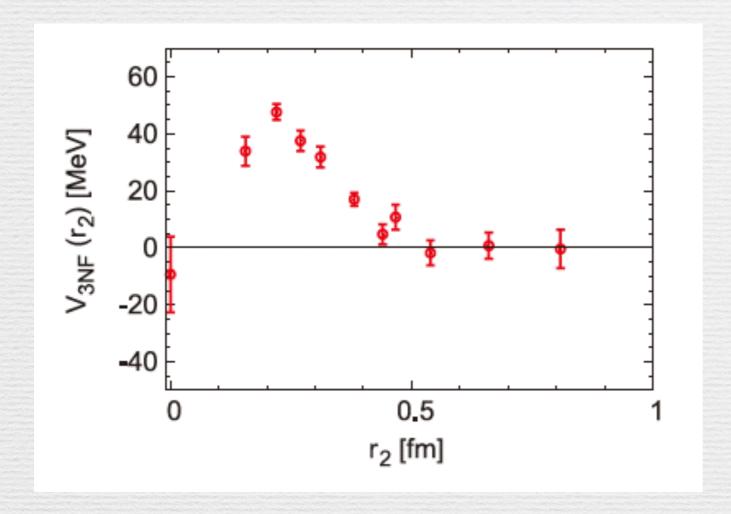
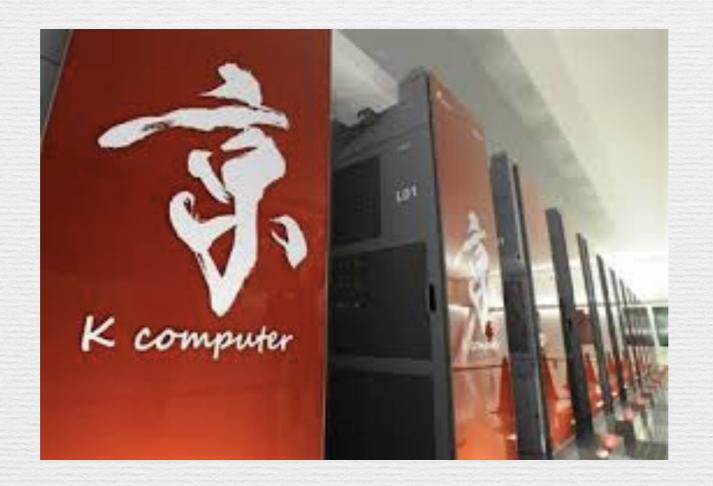


Fig. 23. The effective scalar–isoscalar 3NF in the triton channel with the linear setup obtained at center and edge in the linear setup.

S. Aoki et al., Prog. Theor. Exp. Phys.

Work in Progress

Calculation at the real pion mass with the K super-computer is in progress.



- Baryon-Baryon interaction
- Precise determination of quark bare masses
- three nucleon force
- quark gluon plasma
- etc...

Seminar by Prof. N. Ishii

LS force and anti-symmetric LS force from lattice QCD

Noriyoshi Ishii (RCNP,Osaka University)

Thursday 29 May 2014, 14:00

Please Join it!

Lecture Room 1, RCNP 6F Osaka University

Abstract:s

We will present our recent results on LS force (NN) and anti-symmetric LS force (hyperon interaction) from lattice QCD. We begin with a brief review of our strategy of determining interbaryon potentials. The potentials are obtained from Nambu-Bethe-Salpeter (NBS) wave functions [HAL QCD method], which has been applied to many systems, such as NN, YN, YY, NNN, etc. These studies are restricted to local potentials (potentials which do not involve any derivatives) in the parity-even sector. The restriction is due to source functions which these calculations employ, i.e., these calculations employ source functions with A1 representation of the cubic group which roughly corresponds to s-wave. By using a momentum wall source with T1 representation of the cubic group which roughly corresponds to p-wave, we study NN potentials in the parity-odd sector as well as the LS potentials. A strong attractive LS force with a weak repulsive central force in spin triplet P-wave channels lead to an attraction in the 3P2 channel, which is related to the P-wave neutron pairing in neutron stars. We extend this method to the hyperon sector, and consider a phenomenologically expected cancellation between the symmetric and the anti-symmetric LS potentials in the flavor SU(3) symmetric limit.

References

- John Dirk Walecka,
- 青木慎也
- ・ 南部陽一郎「クォーク第
- I.J.R.Aitchison and A.J.G. Hey,
- F. Halzen and A.D. Martin,
- J.D. Bjorken and S.D. Drell,
- R.P. Feynman,

Thank you for your attention

Dirac Equation

$$(-i\alpha \cdot \nabla + \beta m)\psi(\mathbf{x},t) = i\frac{\partial}{\partial t}\psi(\mathbf{x},t)$$

$$\alpha_i \equiv \left(\begin{array}{cc} \mathbf{0} & \sigma_i \\ \sigma_i & \mathbf{0} \end{array} \right) \qquad \beta \equiv \left(\begin{array}{cc} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{array} \right)$$

$$\left(i\gamma^{\mu}\,\partial_{\mu}-m\right)\psi=0$$

$$\gamma^{\mu} \equiv (\gamma^{0}, \gamma^{i})$$

$$\gamma^{0} \equiv \beta, \gamma^{i} \equiv \beta \alpha_{i}, \gamma^{5} \equiv i \gamma^{0} \gamma^{1} \gamma^{2} \gamma^{3}$$

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}$$

$$\nabla \!\!\!/ \equiv \gamma^\mu \, \partial_\mu$$

$$\boldsymbol{\gamma}^{i} = \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_{i} \\ -\boldsymbol{\sigma}_{i} & \mathbf{0} \end{pmatrix} \quad \boldsymbol{\gamma}^{0} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}$$

Gell-Mann matrices SU(3)

like Pauli matrices in the isospin space

$$\left[\frac{1}{2}\lambda^a, \frac{1}{2}\lambda^b\right] = i\frac{1}{2}f^{abc}\lambda^c$$

 f^{abc} :structure function

$$\lambda^{1} = \begin{pmatrix} 1 & 1 \\ 1 & \end{pmatrix} \qquad \lambda^{2} = \begin{pmatrix} i & -i \\ i & \end{pmatrix} \qquad \lambda^{3} = \begin{pmatrix} 1 & \\ & -1 & \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} & & -i \\ & & \\ i & & \end{pmatrix} \qquad \lambda^6 = \begin{pmatrix} & & \\ & & 1 \\ & & \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} & & \\ & i & \end{pmatrix} \quad \lambda^8 = \begin{pmatrix} 1/\sqrt{3} & & \\ & 1/\sqrt{3} & \\ & & -2/\sqrt{3} \end{pmatrix}$$

EM U(1) gauge

covariant derivative

$$D^{\mu} \equiv \partial^{\mu} - ie_0 A^{\mu}$$

field tensor

$$\mathbf{F}^{\mu\nu} \equiv \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

gauge transformation

$$A^{\mu} \to A^{\mu} = A^{\mu} + \frac{1}{e_0} \partial^{\mu} \Lambda$$
$$\overline{\psi} \to \overline{\psi}' = \exp(-i\Lambda) \overline{\psi}$$

Yang-Mills SU(2) gauge

covariant derivative

$$\frac{D}{Dx_{\mu}} \equiv \partial^{\mu} - igT^{a}A_{\mu}^{a}(x)$$

field tensor

$$\mathbf{F}_{\mu\nu} = \partial^{\mu} A_{\nu} - \partial^{\nu} A_{\mu} + g A_{\mu} \times A_{\nu}$$

infinitesimal local gauge transformation $\theta \rightarrow 0$

$$A_{\mu} \to A'_{\mu} = A_{\mu} - \frac{1}{g_0} \partial^{\mu} \theta + \theta \times A_{\mu}$$

QCD SU(3) gauge

covariant derivative

$$\frac{D}{Dx_{\mu}} \equiv \partial^{\mu} - \frac{i}{2} g \lambda^{a} A_{\mu}^{a}(x)$$

field tensor

$$\mathbf{F}_{\mu\nu}^{a} = \partial^{\mu} A_{\nu}^{a} - \partial^{\nu} A_{\mu}^{a} + g f^{abc} A_{\mu}^{b} A_{\nu}^{c}$$

infinitesimal local gauge transformation $\theta^a \rightarrow 0$

$$A^{a}_{\mu} \to A^{\prime a}_{\mu} = A^{a}_{\mu} - \frac{1}{g} \partial^{\mu} \theta^{a} + f^{abc} \theta^{b} A^{c}_{\mu}$$
$$\mathbf{F}^{a}_{\mu\nu} \to \mathbf{F}^{\prime a}_{\mu\nu} = \mathbf{F}^{a}_{\mu\nu} + f^{abc} \theta^{b} \mathbf{F}^{c}_{\mu\nu}$$

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\overline{\psi} \left\{ \gamma_{\mu} \left[\partial^{\mu} - \frac{i}{2} g \lambda^{a} A_{\mu}^{a}(x) \right] + M \right\} \psi - \frac{1}{4} \mathbf{F}_{\mu\nu}^{a} \mathbf{F}_{\mu\nu}^{a}$$