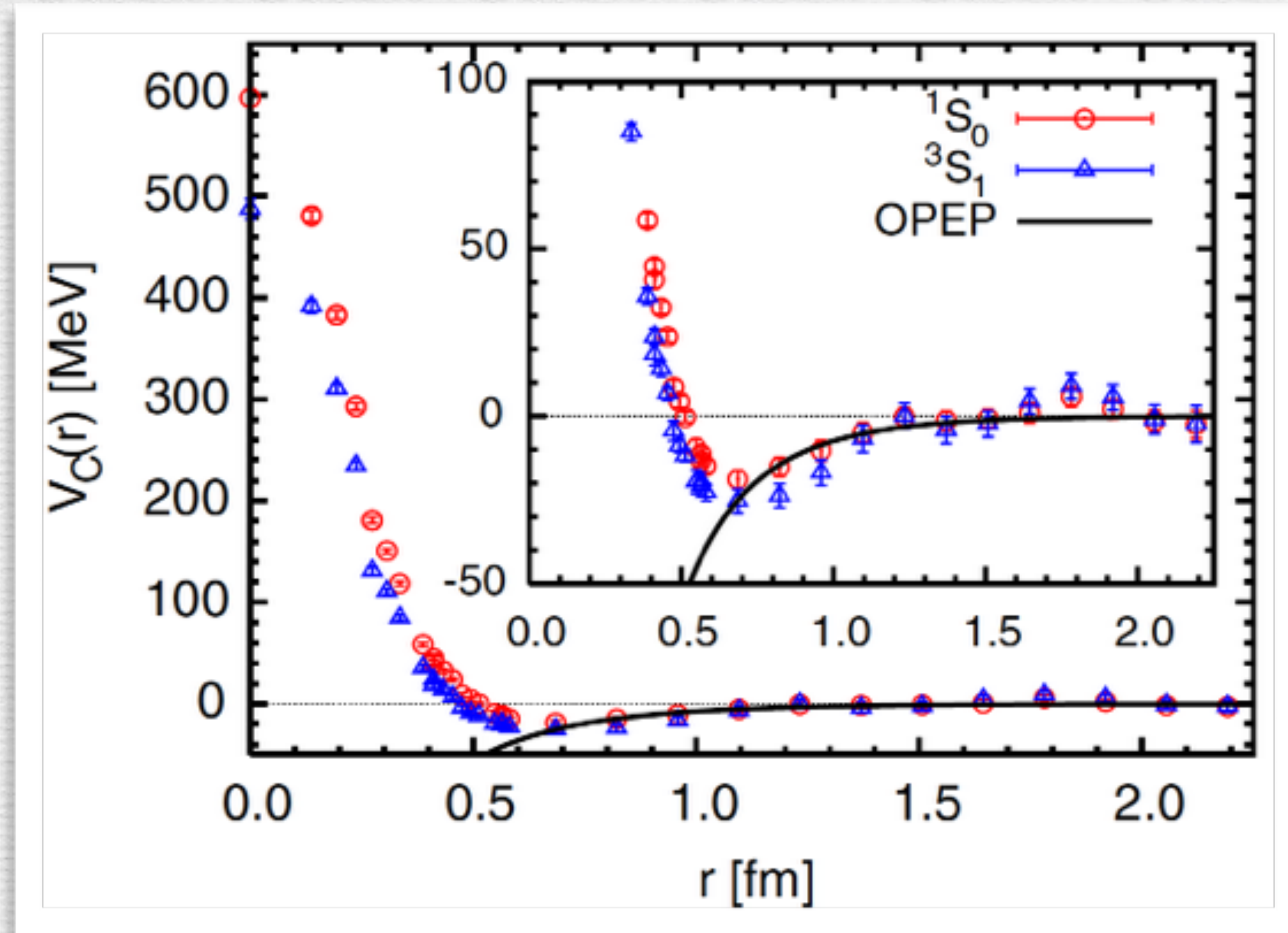
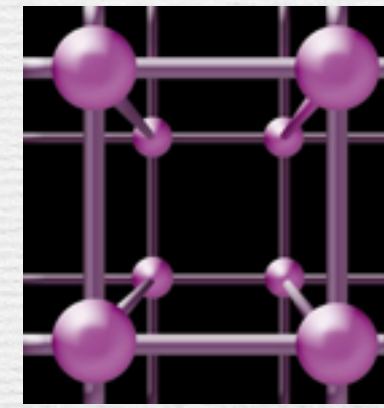
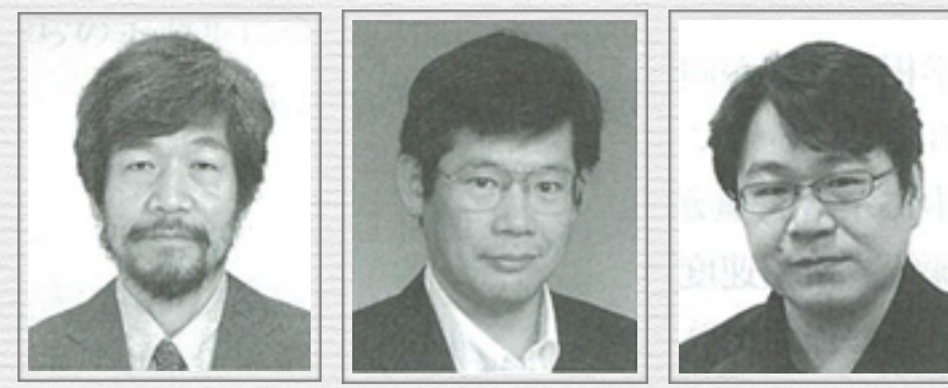
Extracting the *NN* Potential from QCD:
Recent Progresses of Lattice QCD Calculations





Awarded Nishina Prize 2012



Aoki, Hatsuda, Ishii
& HAL QCD Collaboration

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett
S. Aoki, T. Hatsuda, N. Ishii, and H. Nemura, Butsuri
S. Aoki et al., Prog. Theor. Exp. Phys.

Hadrons to Atomic nuclei HAL from Lattice QCD

RCNP核理1aグループ勉強会 2014.5.20 by A. Tamii

Abstract Phys. Rev. Lett.

Nuclear Force from Lattice QCD

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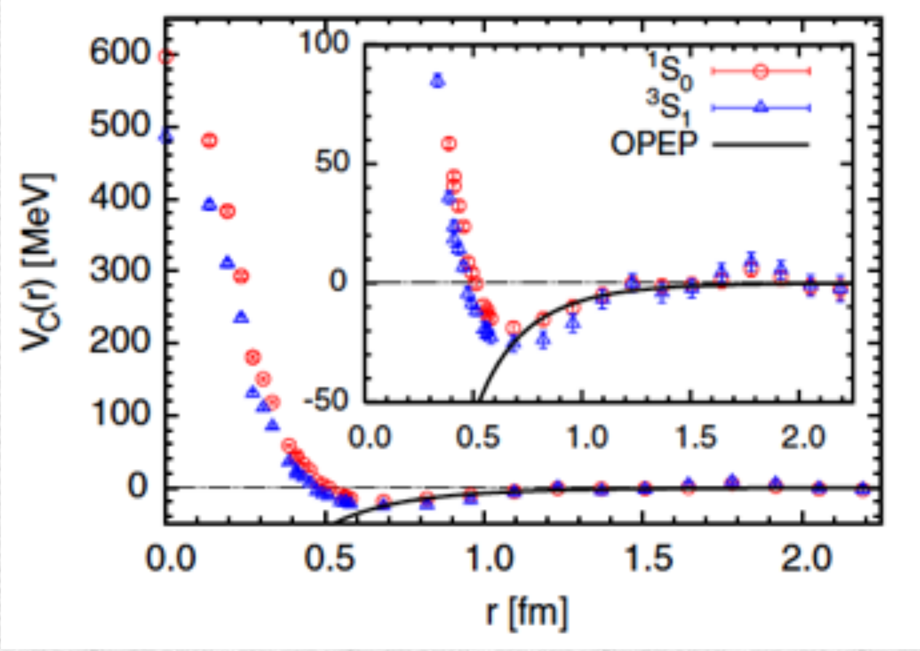
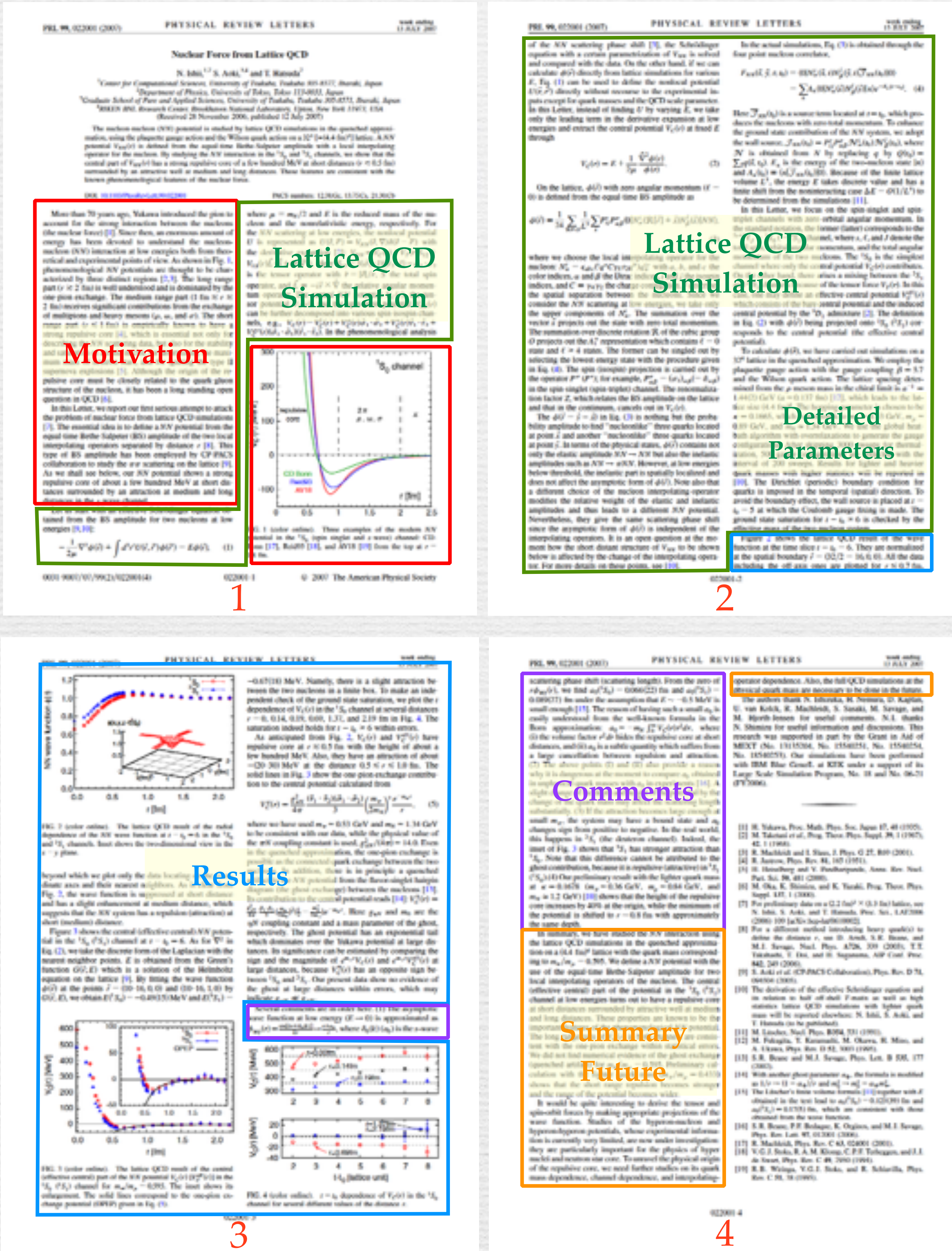
The nucleon-nucleon (NN) potential is studied by lattice QCD simulations in the quenched approximation, using the plaquette gauge action and the Wilson quark action on a 32^4 [$\simeq(4.4\ \text{fm})^4$] lattice. A NN potential $V_{NN}(r)$ is defined from the equal-time Bethe-Salpeter amplitude with a local interpolating operator for the nucleon. By studying the NN interaction in the $^1S_0$ and $^3S_1$ channels, we show that the central part of $V_{NN}(r)$ has a strong repulsive core of a few hundred MeV at short distances ($r \lesssim 0.5$ fm) surrounded by an attractive well at medium and long distances. These features are consistent with the known phenomenological features of the nuclear force.



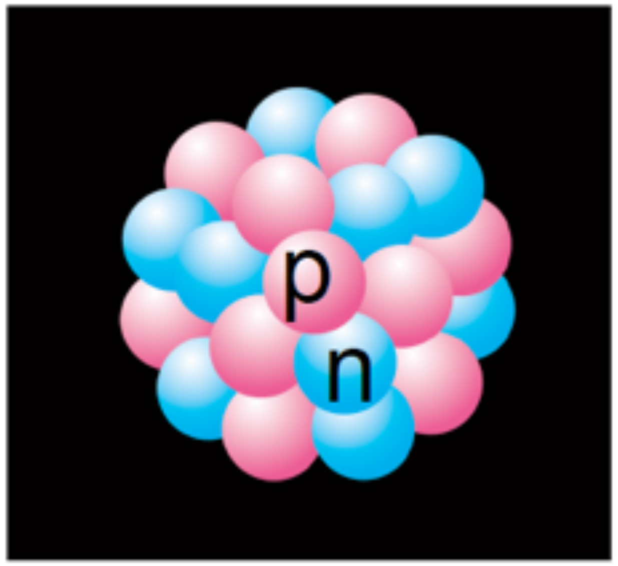
Structure of the Paper



N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett.

Motivation

What binds protons and neutrons inside a nuclei ?



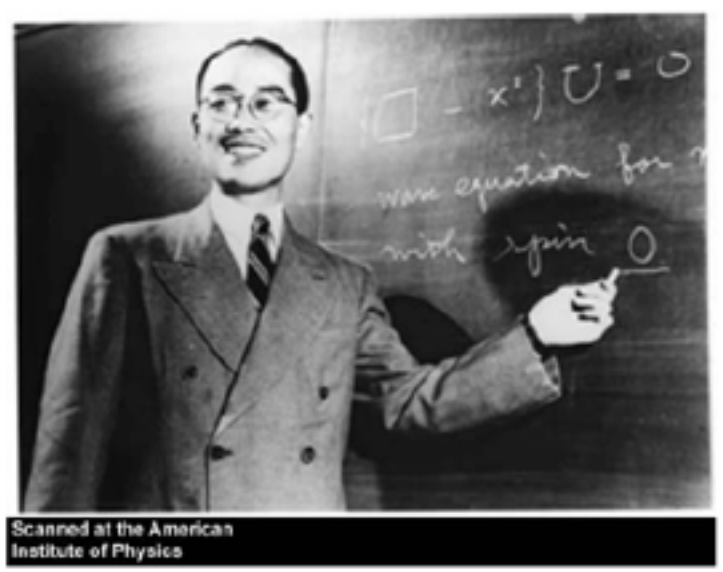
gravity: too weak

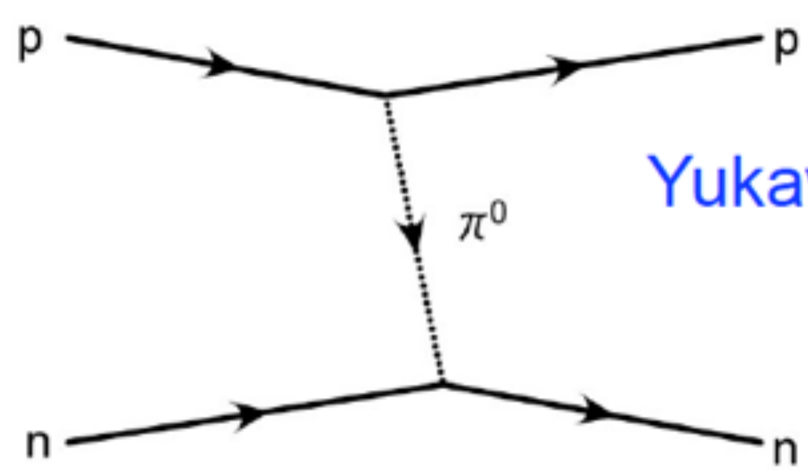
Coulomb: repulsive between pp
no force between nn, np

New force (nuclear force) ?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force





Yukawa potential

$$V(r) = \frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

1949 Nobel prize

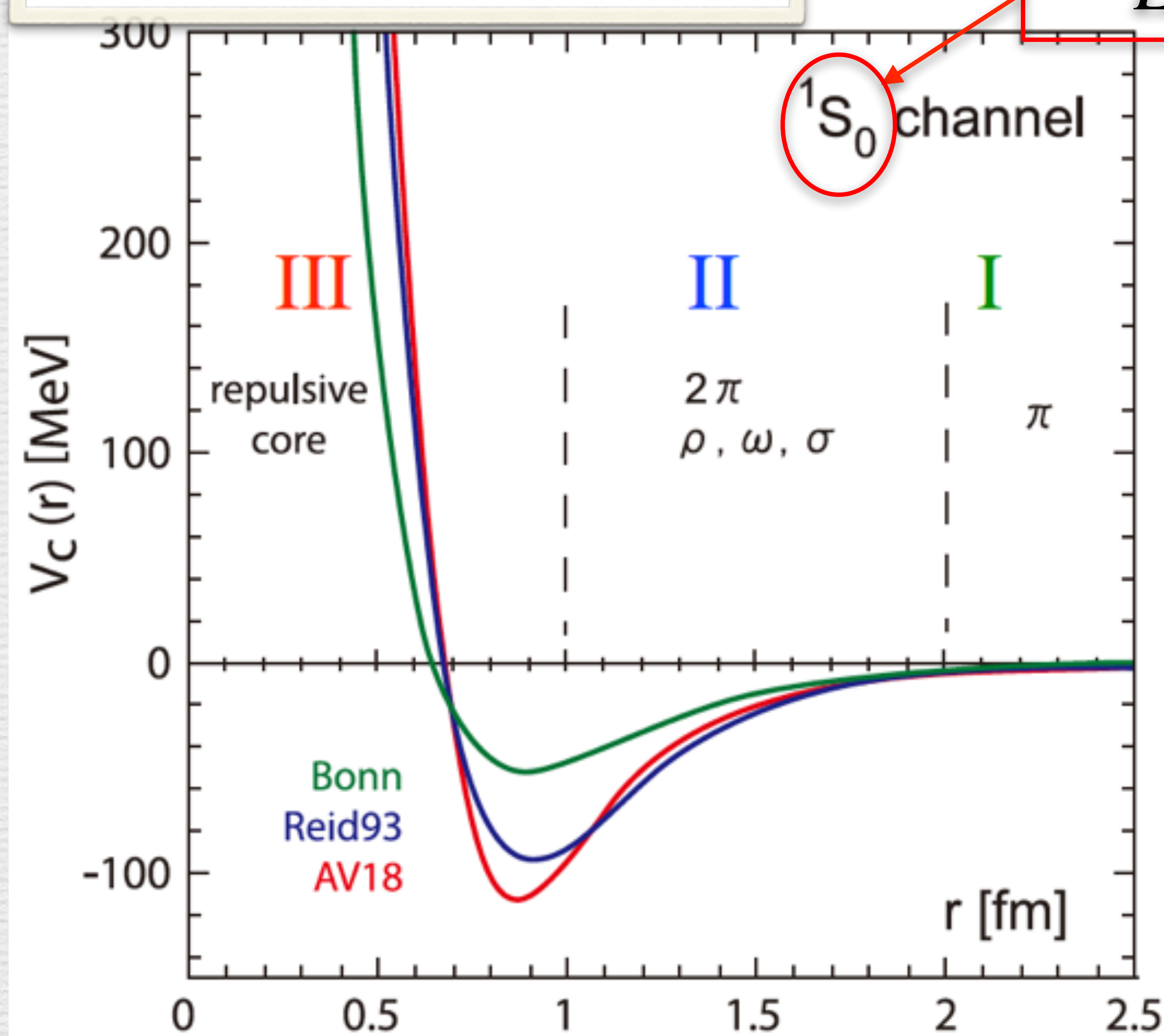
S. Aoki, Lecture Note

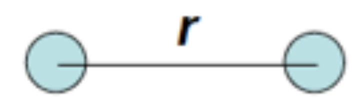
Motivation

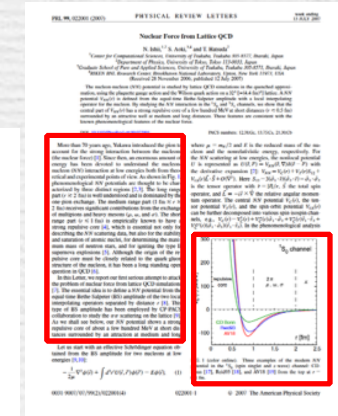
Modern nucleon-nucleon potential

Phenomenological NN Potential

$^{2S+1}L_J$







I Long range part

one pion exchange potential (OPEP)

II Medium range part

σ, ρ, ω exchange

2π exchange

III Short range part

repulsive core (RC)

R. Jastrow(1951)

quark ?

It has been a fundamental question whether the *NN* potential can be described by QCD.

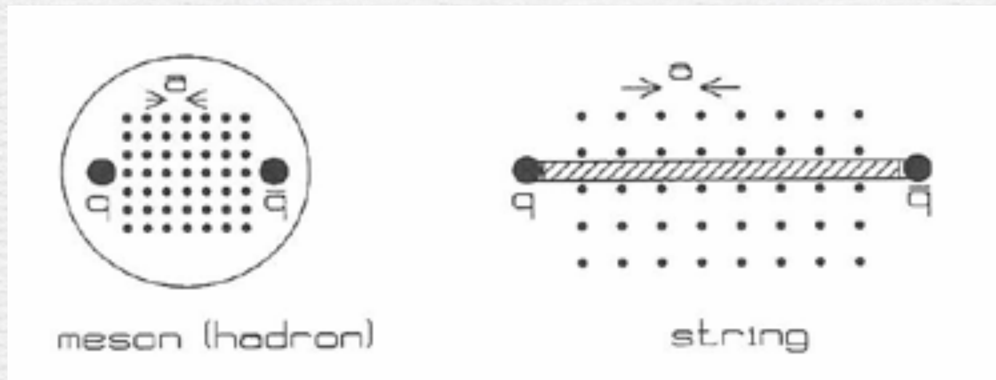
Although the origin of the repulsive core must be closely related to the quark-gluon structure of the nucleon, it has been a long-standing open question in QCD [6].

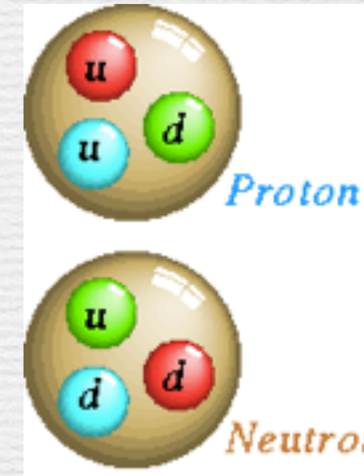
S. Aoki, Lecture Note

QCD: Quantum Chromodynamics

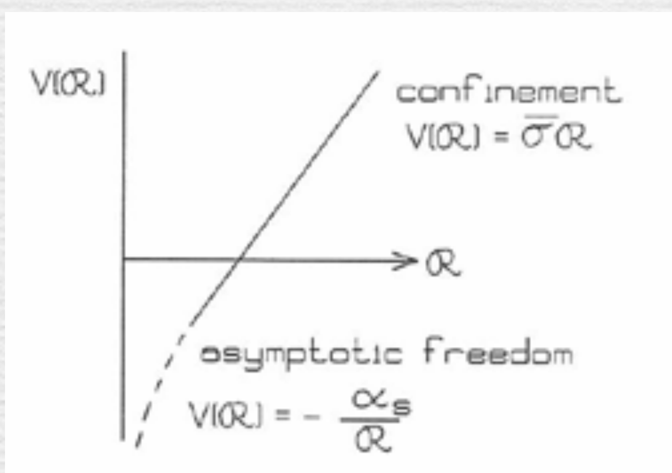
Confinement (

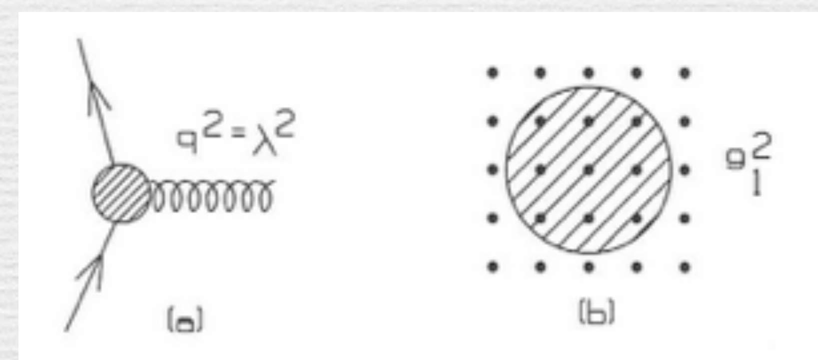
potential linearly
dependent on r





Asymptotic freedom (



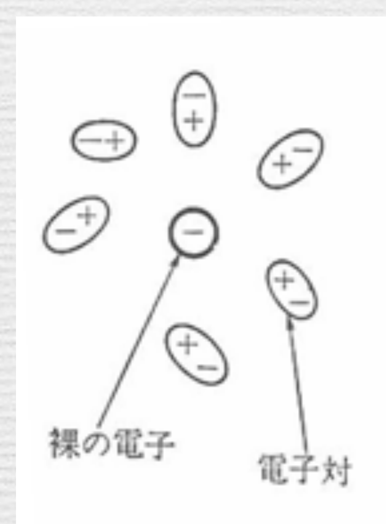


Renormalization (

also in QED

ultraviolet divergence

strong nonlinear system



The bare particle mass and charge are not observed. Instead a particle with $q\bar{q}\left(e^{+}e^{-}\right)$ clouds is observed.

Why is a QCD calc. at low energy so difficult?

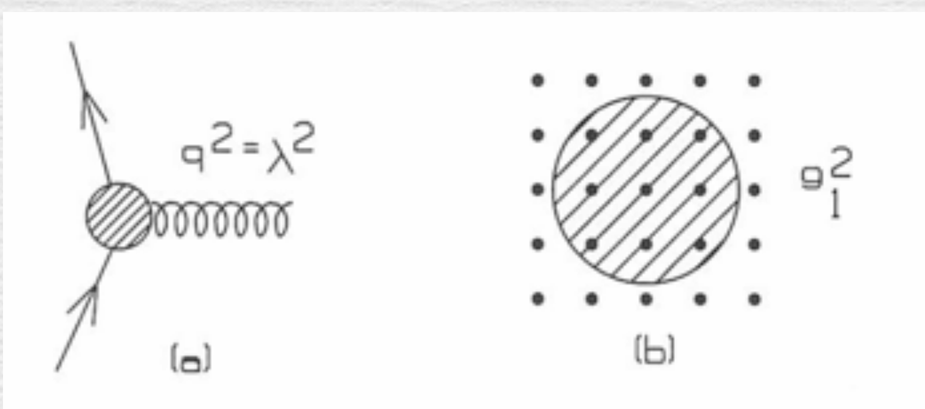
non-perturbative QCD

locally gauge invariant, non-abelian, strong-coupling field theory

Strong coupling field theory ⇔

perturbation cannot be applied

m_u = 2.15(15) MeV
m_d = 4.70(20) MeV
by Lattice QCD [PDG'12] ⇔

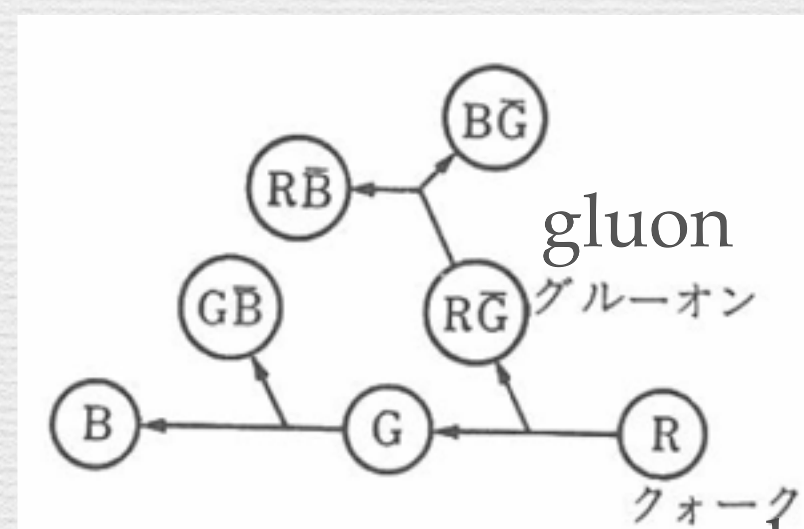


「現在でも核力の詳細を基本方程式から導くことはできない。核子自体がもう素粒子と見なされないから、いわば複雑な高分子の性質をシュレディンガー方程式から出発して決定せよというようなもので、むしろこれは無理な話である」(南部陽一郎「クォーク」(1981)

Non-abelian gauge

non-commutative

A gluon carries the color charge
can produce other gluons



quark

QCD on Lattice

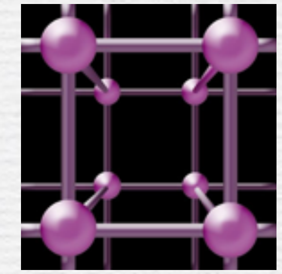
Lattice Gauge Theory (LGT)

K.G. Wilson, Phys. Rev.

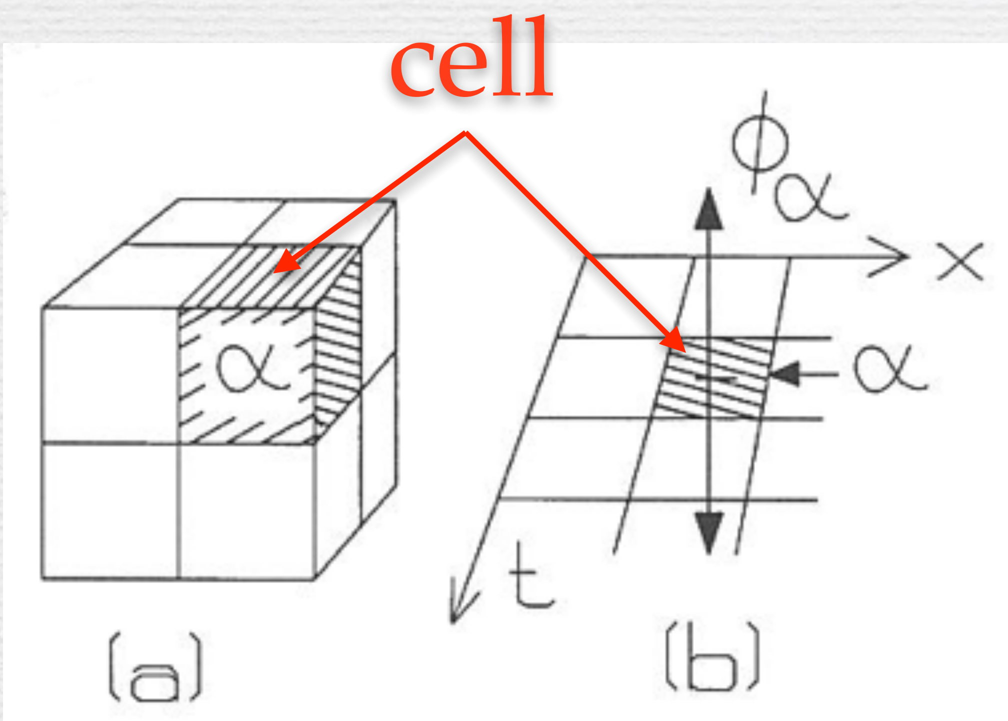
- holds local gauge invariance
- ultraviolet divergence is natural solved (ultraviolet cutoff)
- limit is taken to the infinite volume (thermodynamical limit)
- limit is taken to continuum (a
- path integral method is used to calculate amplitudes
  $\rightarrow$
- a Euclidean space-time is used with imaginary time
  $\rightarrow$
- parameters:
  - strong coupling constant
  - quark mass of each flavor
  - CP violation phase

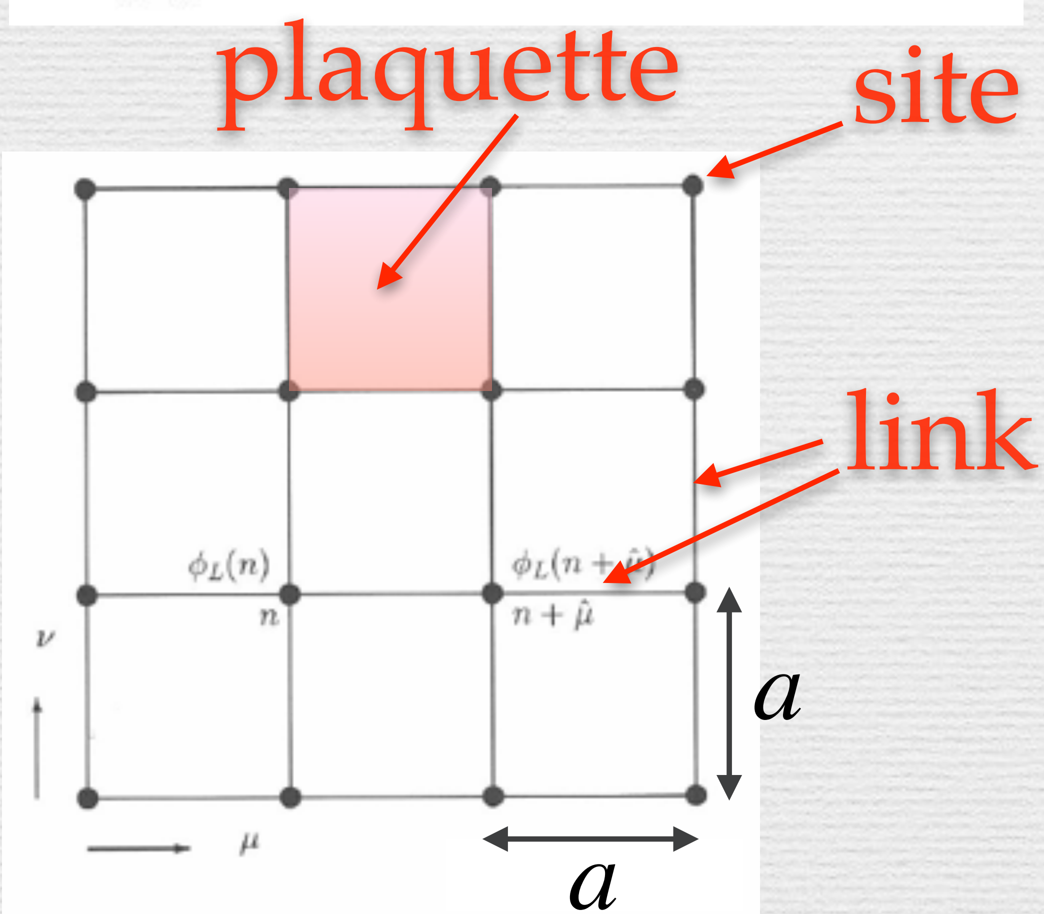
only
in the work by N. Ishii et al.

QCD on Lattice

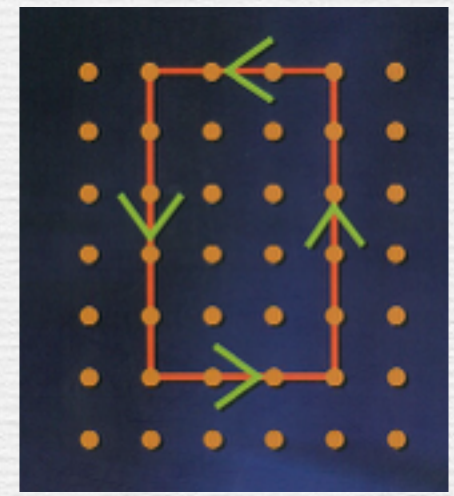


A Euclidean space-time of finite size is divided into a lattice of size "a".

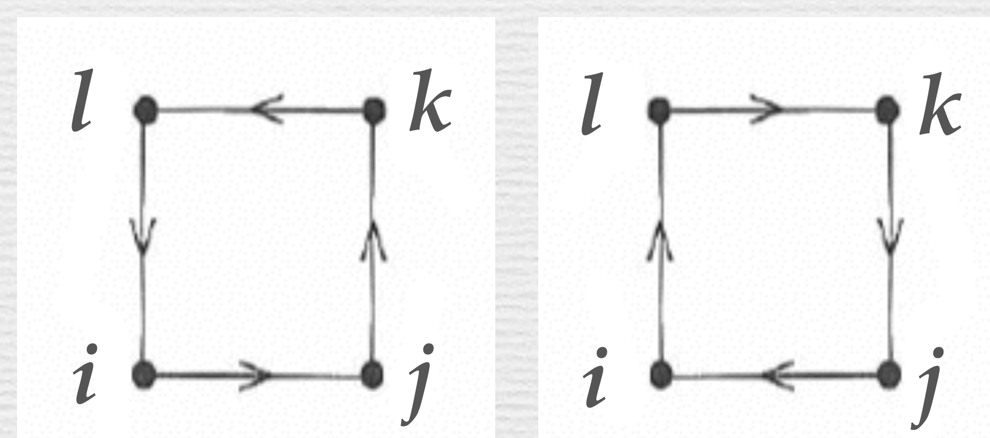




Wilson loop



contribution of action from a plaquette: plaquette action

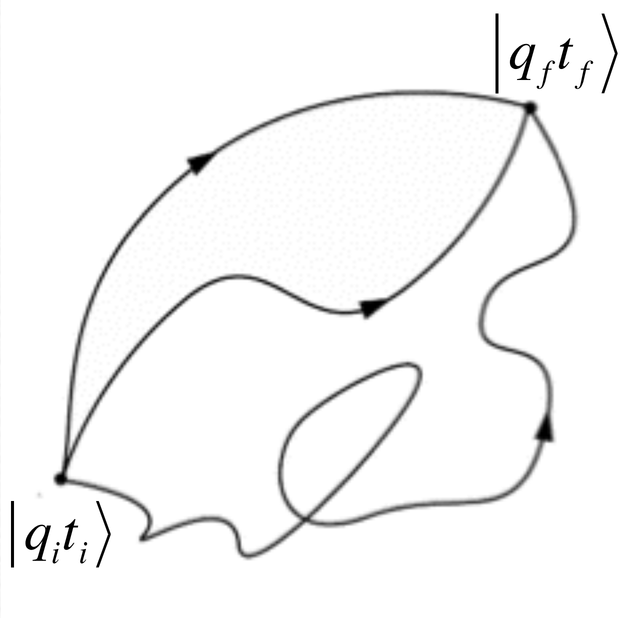


counter clockwise loop

clockwise loop

Path Integral (

N. Wiener
1933 P.A.M. Dirac
1948 R.



In analytical mechanics

Lagrangian: $L(q,\dot{q}) = T - V$

Action: $S(f,i) \equiv \int_{t_i}^{t_f} L(q,\dot{q})dt$

Principle of least action (variational principle)

$$\delta \int_{t_i}^{t_f} L(q,\dot{q})dt = 0 \quad (1)$$

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = 0 \quad (2)$$

Euler-Lagrange equation

In the path integral formulation, the
finding a particle at position

$$\langle q_f t_f | q_i t_i \rangle = \int D(q) \exp\left\{ \frac{i}{\hbar} S(f,i) \right\} \quad (3)$$

c.f. Fermat's principle for the path of the light ray

The integration should be taken for ____________________

Taking the classical limit $\hbar \to 0$

$\int D(q)$: volume element

Pass Integral and Partition Function

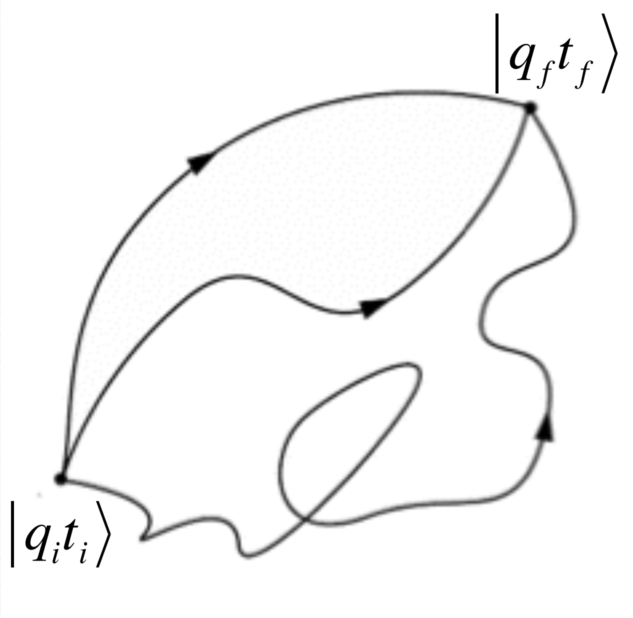
Defining the imaginary time

$$\tau \equiv it, \quad d\tau \equiv idt$$

$$\bar{S}(f,i) \equiv iS(f,i) = -\int_{\tau_i}^{\tau_f} L(q,\dot{q})\, d\tau$$

the pass integral is expressed as

$$\langle q_f t_f | q_i t_i \rangle = z = \int \bar{D}(q) \exp\left\{-\frac{1}{\hbar}\bar{S}(f,i)\right\}$$



This is the quantum mechanical representation of the partition function(

$$z = \int \bar{D}(q) \exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta, 0)\right\}$$

$$\bar{S}(\hbar\beta, 0) = \int_0^{\hbar\beta} \left[\frac{1}{2} m \left(\frac{dq}{d\tau}\right)^2 + V(q)\right] d\tau$$

In the quantum field theory, Lagrangian density

$$\bar{S}(\hbar\beta, 0) = \int_0^{\hbar\beta} d\tau \int dx \mathcal{L} = \int d^4x \mathcal{L}$$

$$z \rightarrow Z$$

micro-canonical ensamble in statistical mechanics

$$z = \sum_n \exp(-\beta E_n) = Tr\left[\exp(-\beta \hat{h})\right]$$

$$Z = z^N \qquad \beta \equiv \frac{1}{k_B T}$$

Pass Integral and Partition Function

With a periodic boundary condition in the imaginary time, the expectation value of an operator $\hat{O}$

$$\hat{\rho}_{th} = \frac{\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}}{\int \bar{D}(q)\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}} = \frac{\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}}{Z}$$

statistical operator

$$\left\langle\left\langle\hat{O}\right\rangle\right\rangle = Tr\left(\hat{O}\hat{\rho}_{th}\right) = \frac{\int \bar{D}(q)O(\phi)\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}}{\int \bar{D}(q)\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}}$$

expectation value of the operator $\hat{O}$

in statistical mechanics

$$\hat{\rho}_{th} = \frac{e^{-\beta\hat{H}}}{Tr\left(\hat{O}e^{-\beta\hat{H}}\right)} = \frac{e^{-\beta\hat{H}}}{Z}$$

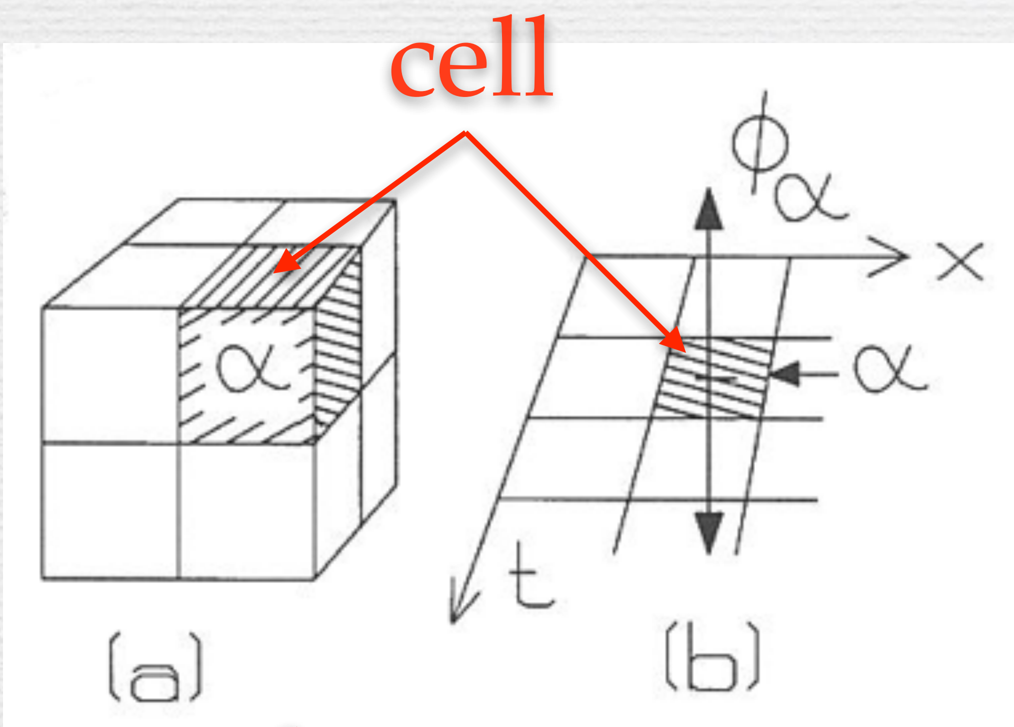
statistical operator (density matrix)

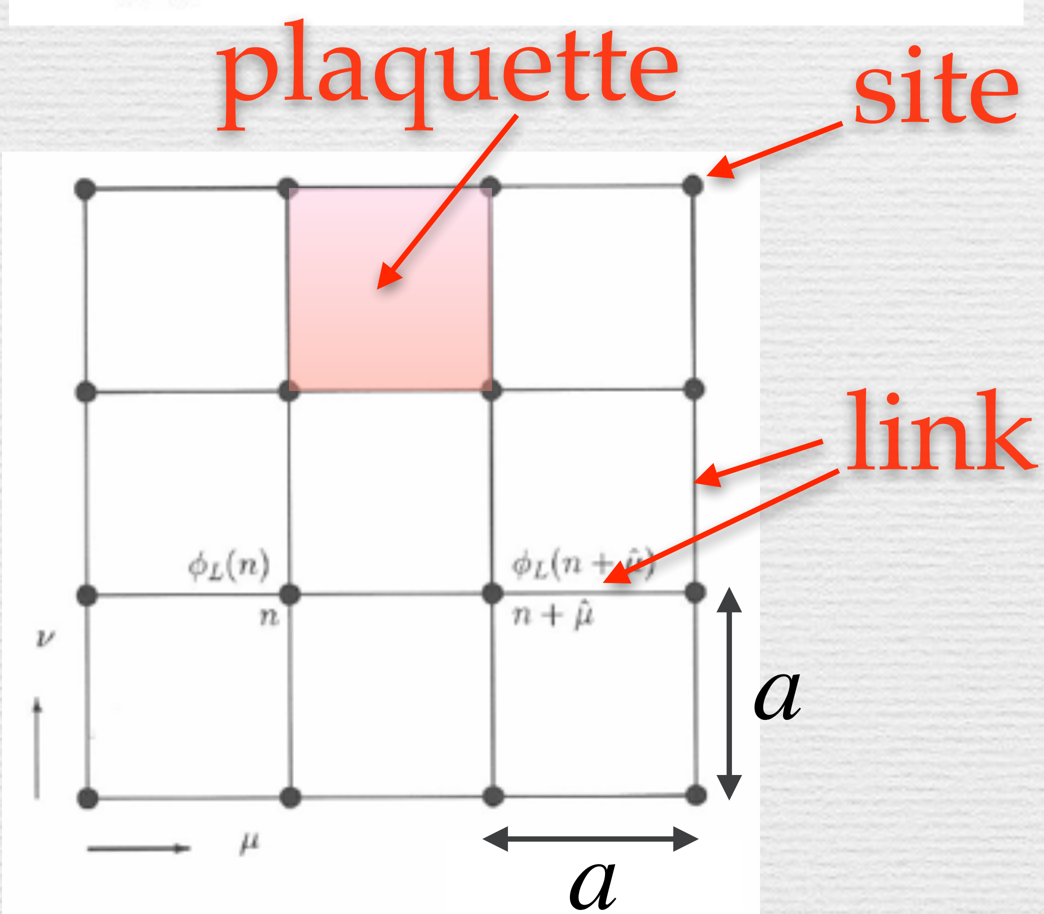
$$\left\langle\left\langle\hat{O}\right\rangle\right\rangle = \frac{Tr\left(\hat{O}e^{-\beta\hat{H}}\right)}{Z} = Tr\left(\hat{O}\hat{\rho}_{th}\right)$$

thermal average

QCD on Lattice

A Euclidean space-time of finite size is divided into a lattice of size "a".





A lattice action is not uniquely determined.

- Convergence to the continuum limit.

$$\lim_{a\to 0} S_{lat.} = S_{cont.}$$

- Holding the same symmetry as

Boson fields are assigned to each link

QED U(1) gauge

$$U_{ji}\left(A_\mu\right) \equiv \exp\left\{ie_0\left(x_j - x_i\right)_\mu A_\mu\left(\frac{1}{2}\left(x_j + x_i\right)\right)\right\}$$

Yang Mills SU(2) gauge

$$U_{ji}\left(\mathbf{A}_\mu\right) \equiv \exp\left\{ig_0\left(x_j - x_i\right)_\mu \frac{1}{2}\tau \bullet \mathbf{A}_\mu\left(\frac{1}{2}\left(x_j + x_i\right)\right)\right\}$$

QCD SU(3) color gauge

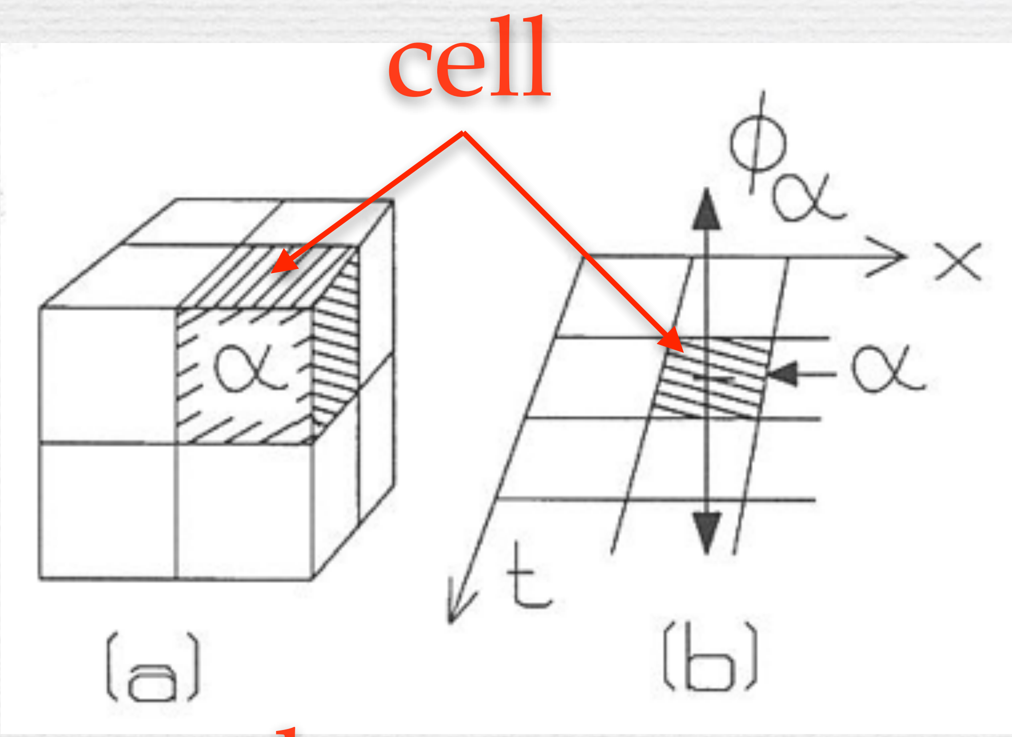
$$U_{ji}\left(A_\mu^a\right) \equiv \exp\left\{ig_0\left(x_j - x_i\right)_\mu \frac{1}{2}\lambda^a \bullet \left[A_\mu^a\left(\frac{1}{2}\left(x_j + x_i\right)\right)\right]\right\}$$

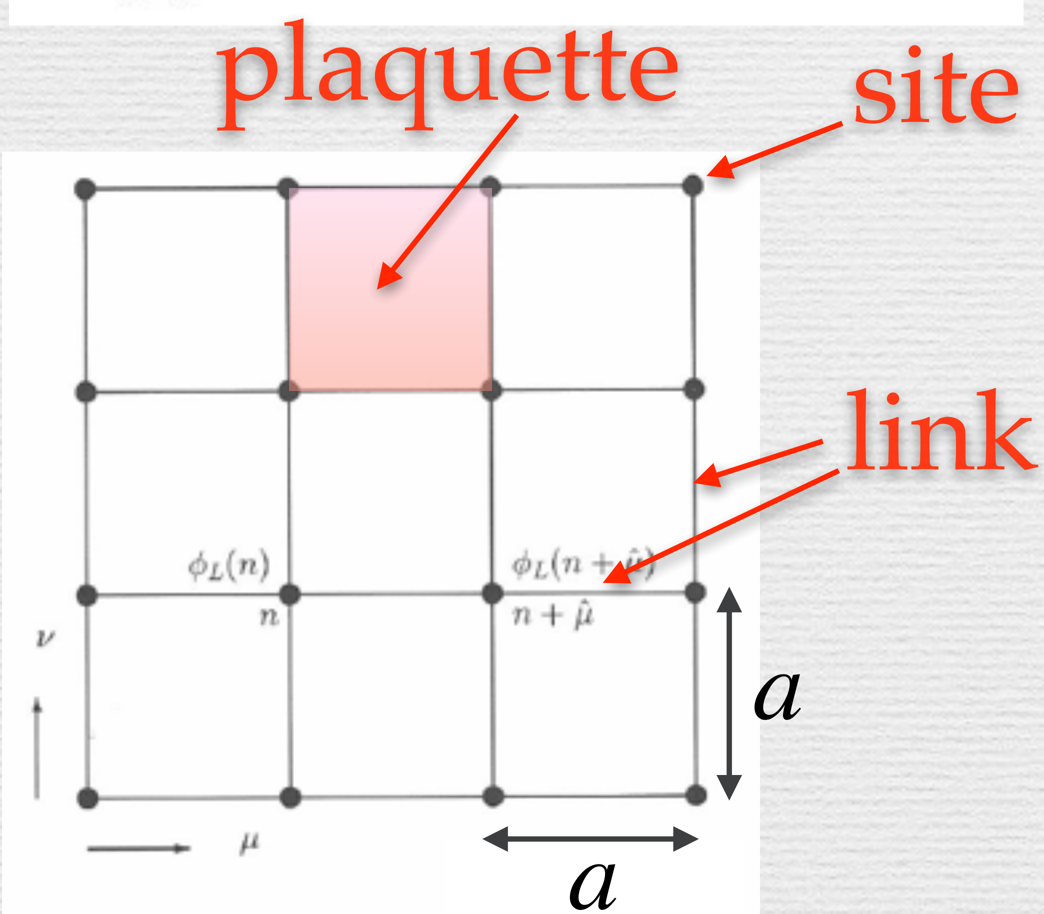
Quark fields are assigned to each site

$$S_F = \frac{1}{2}\sum_{n,\mu}\left[\bar{\psi}_n \gamma_\mu U_{n,\mu} \psi_{n+\hat{\mu}} - \bar{\psi}_{n+\hat{\mu}} \gamma_\mu U_{n,\mu}^\dagger \psi_n\right] + M\sum_n \bar{\psi}_n \psi_n$$

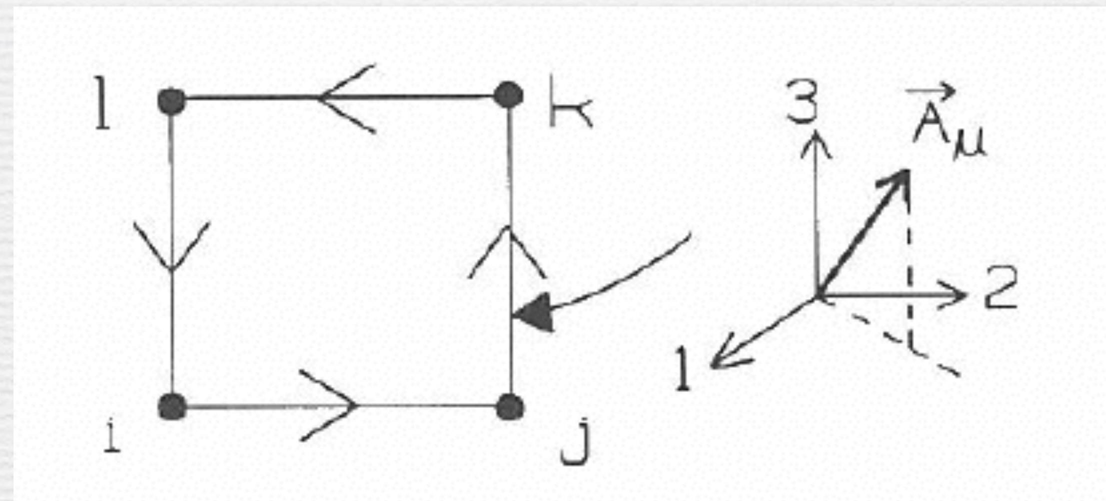
QCD on Lattice

A Euclidean space-time of finite size is divided into a lattice of size "a".





Integration of action in ALL the field dimensions.



Take the sum of the integration for all the plaquettes and sites.

$$\hat{\rho}_{th} = \frac{\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}}{\int \bar{D}(q)\exp\left\{-\frac{1}{\hbar}\bar{S}(\hbar\beta,0)\right\}}$$

$$\left\langle\left\langle\hat{O}\right\rangle\right\rangle = Tr\left(\hat{O}\hat{\rho}_{th}\right)$$

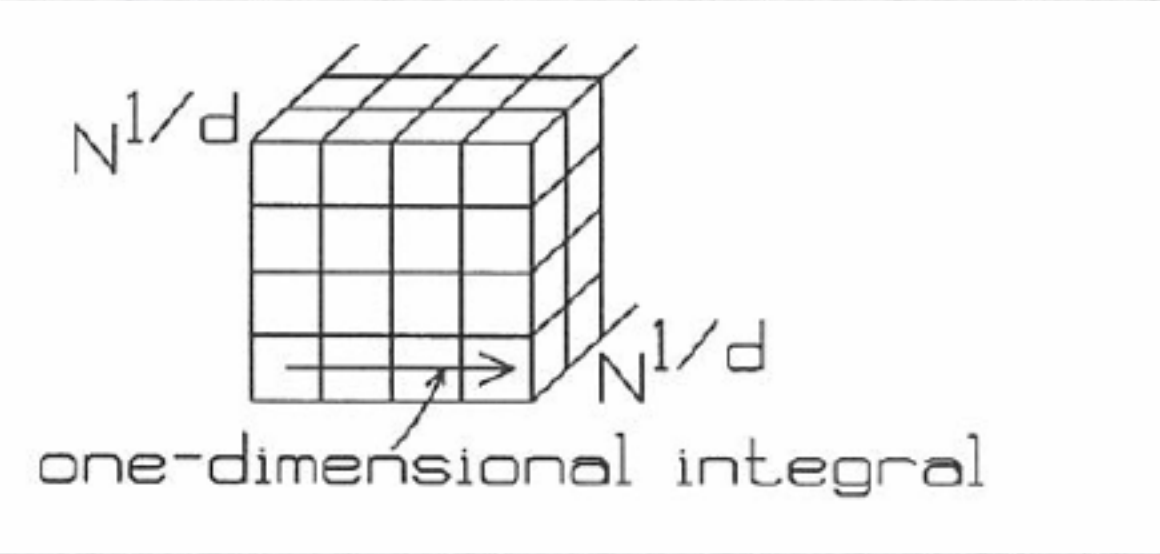
Monte-Carlo Simulations

Monte-Carlo integration

relative uncertainty $\sim \frac{1}{\sqrt{N}}$

Integration by mesh (d-dim.)

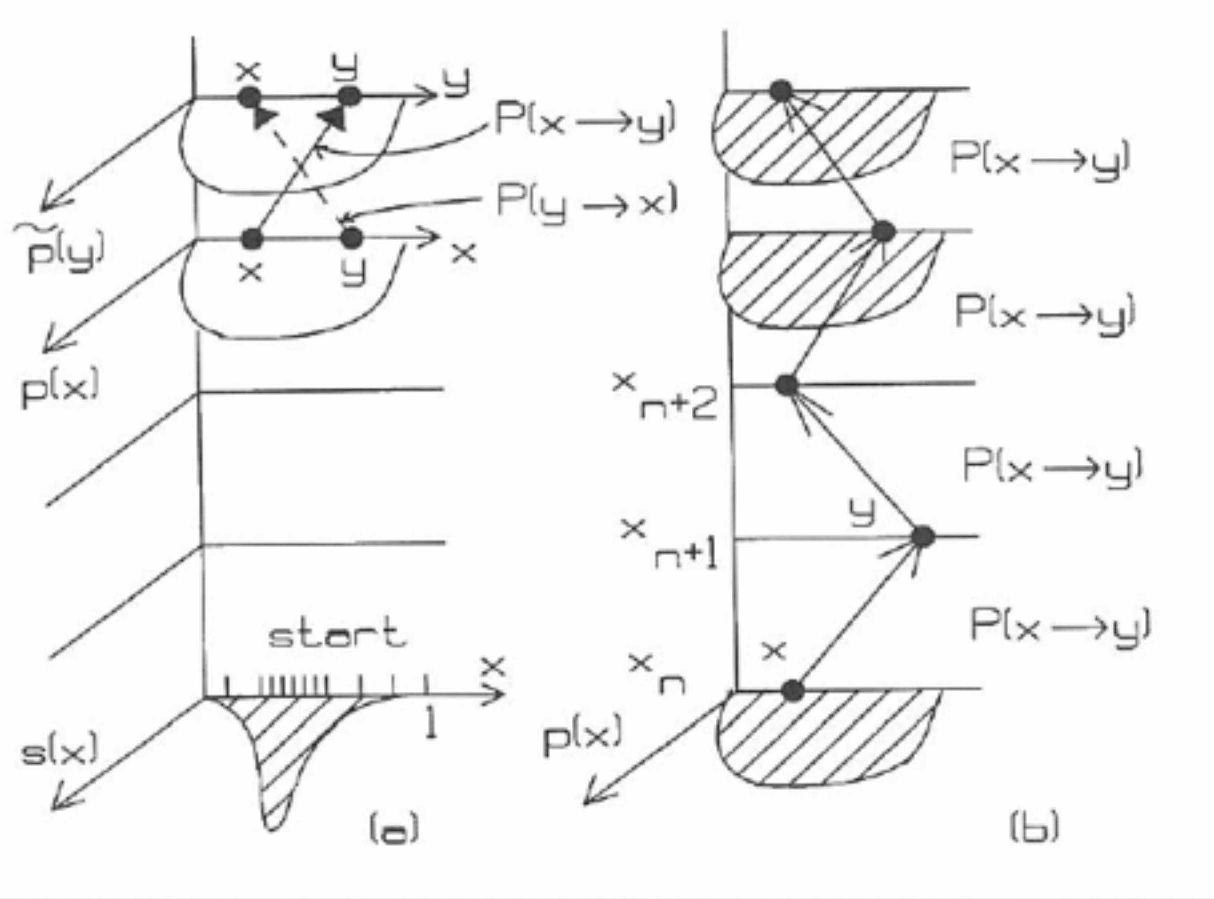
relative uncertainty $\sim \frac{1}{N^{1/d}}$



Monte-Carlo integration is faster for d>3.

Markov-Chains Monte-Carlo integration

Chain of positions are created with an appropriate weight distribution for effective integration.

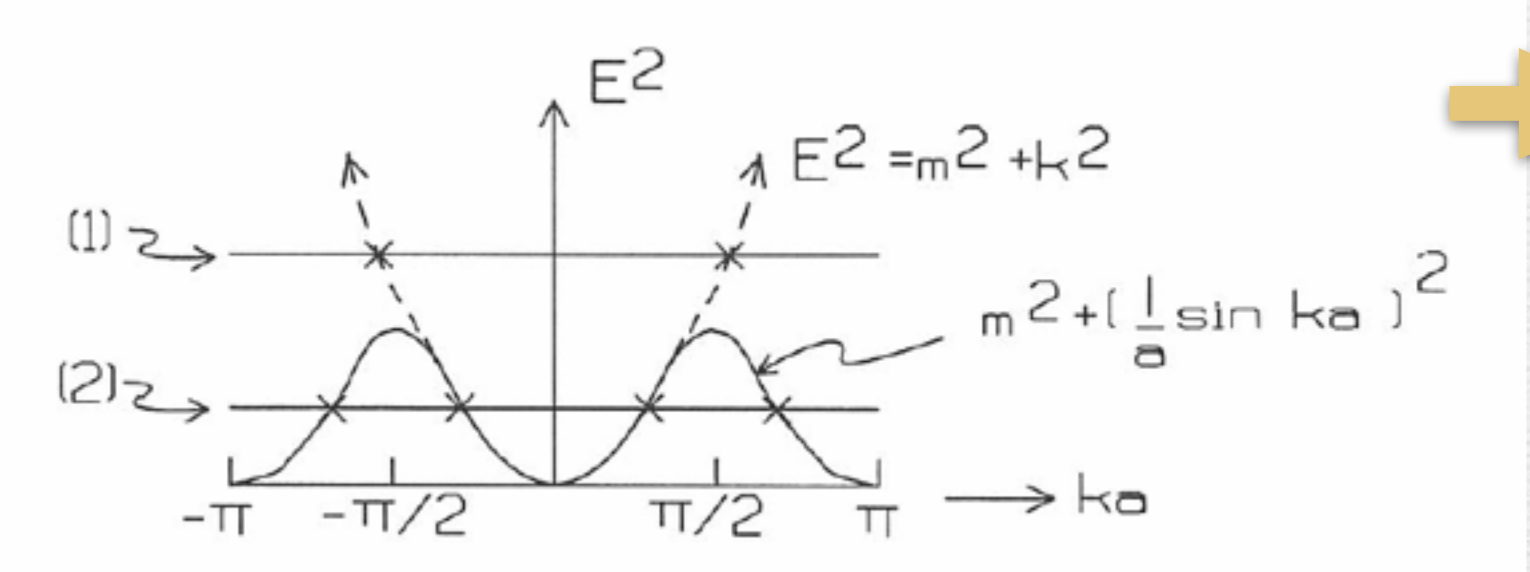


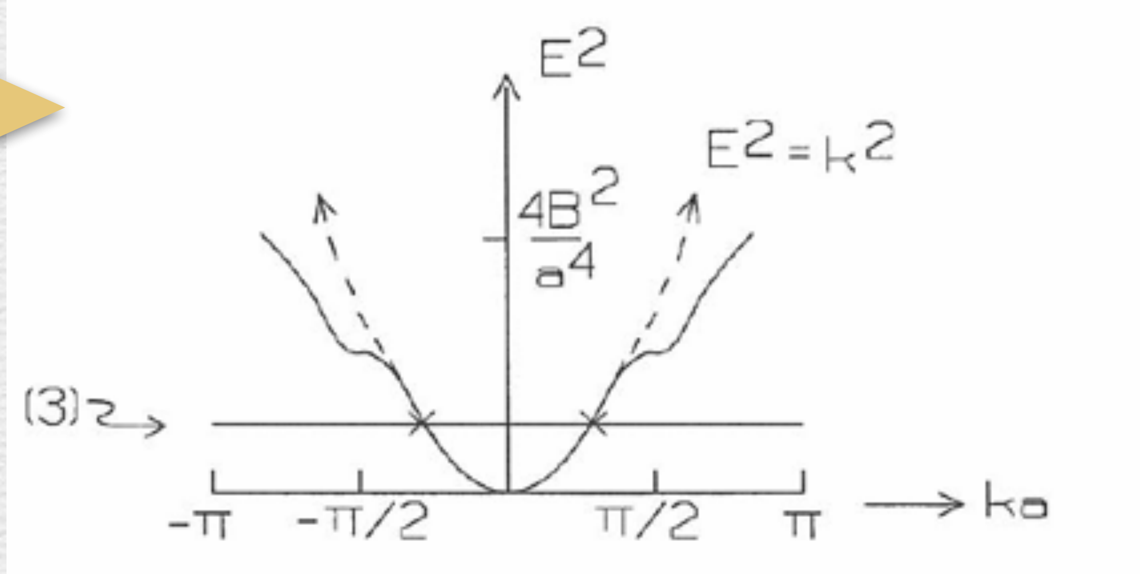
QCD on Lattice

Willson fermions

additional term

$$S_W = -ar \int d^4x \, \bar{\psi} D^2 \psi \;\rightarrow -\frac{r}{2} \sum_{n,\mu} \left[\bar{\psi}_n U_{n,\mu} \psi_{n+\hat{\mu}} + \bar{\psi}_{n+\hat{\mu}} U^\dagger_{n,\mu} \psi_n - 2\bar{\psi}_n \psi_n \right]$$

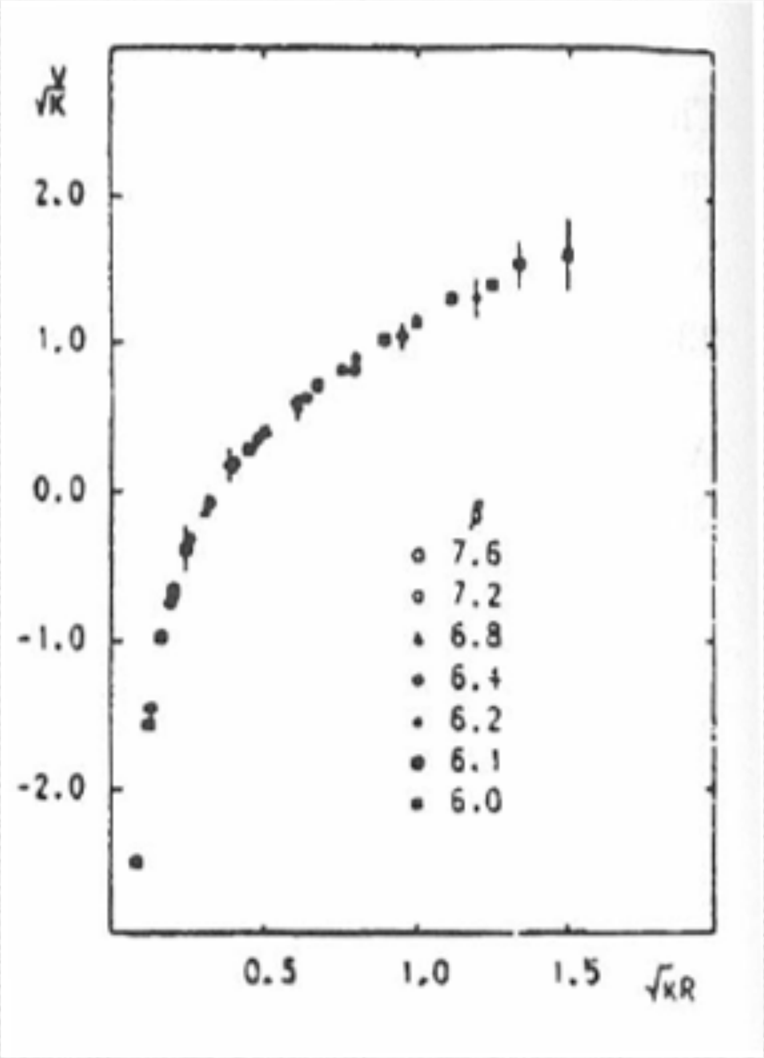


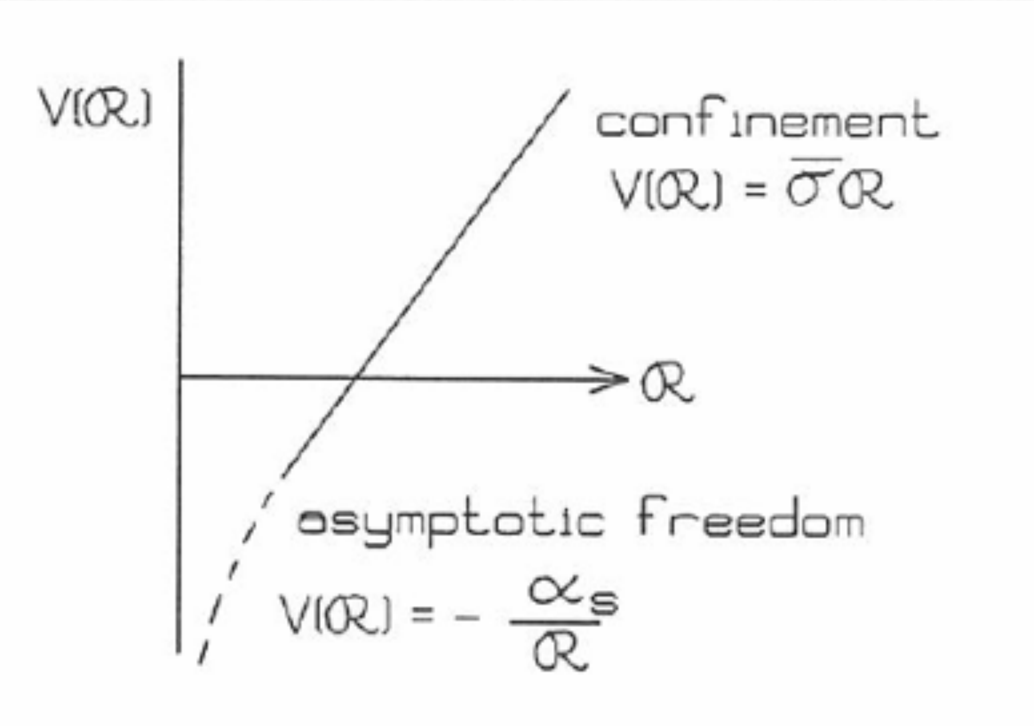


fermion doubling

Confinement: linear dependence of potential on R

S.W. Otto and J.D. Stack PRL





Lattice of size 16

Scaling with $\bar{\beta}$

$$2\sigma \equiv \bar{\beta} \equiv \frac{N}{g^2}, \quad N = 3$$

$$\lambda_1 \approx 4 \text{ MeV} \quad \sqrt{\kappa} \approx \sqrt{\bar{\sigma}} \approx 400 \text{ MeV}$$

Hadron Mass Spectrum

A.S. Kronfeld, Ann. Rev. Nucl. Part. Sci. 62, 265 (2012

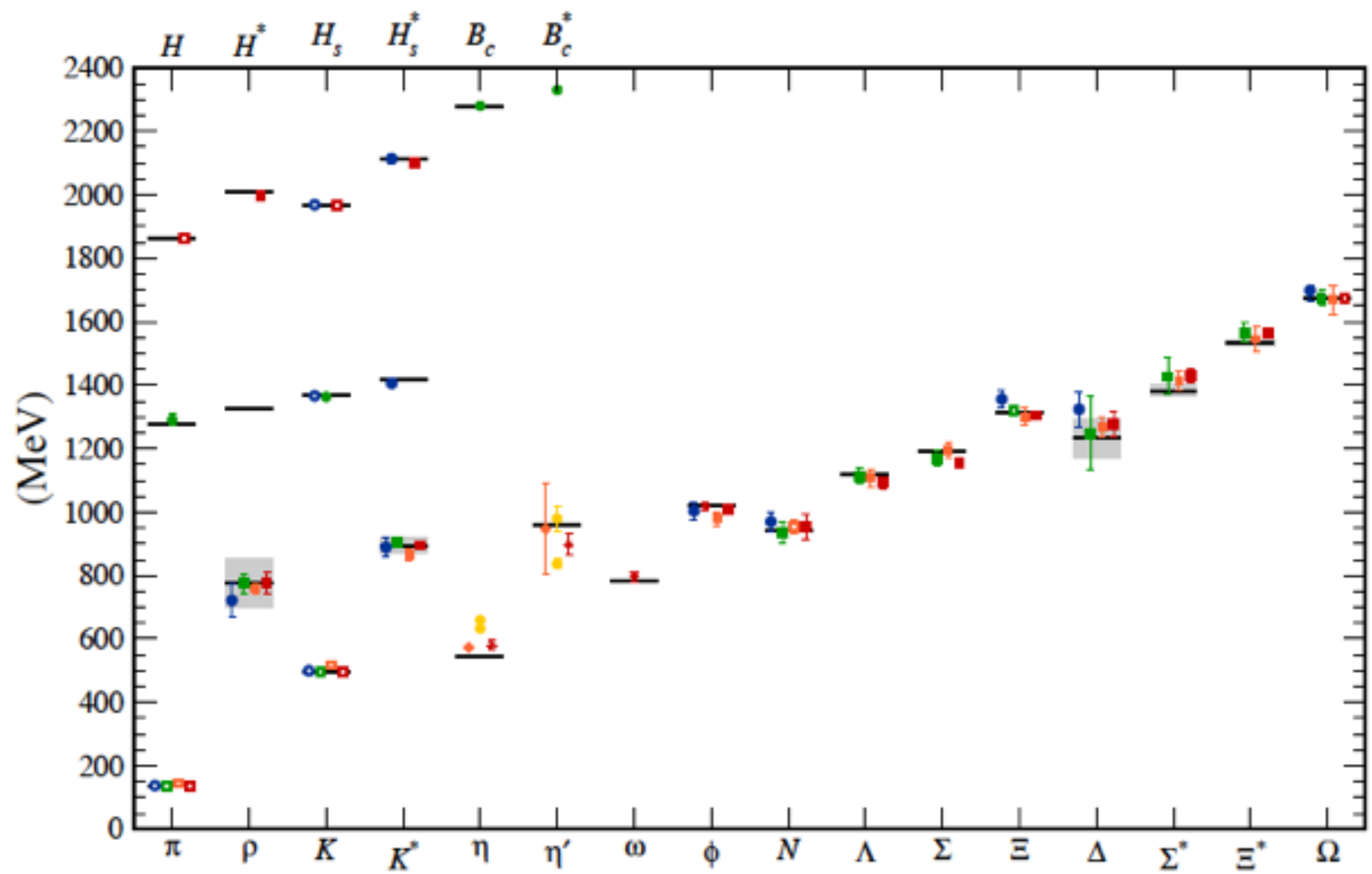


Figure 2: Hadron spectrum from lattice QCD. Comprehensive results for mesons and baryons are from MILC (27, 28), PACS-CS (29), BMW (30), and QCDSF (31). Results for η and η' are from RBC & UKQCD (32), Hadron Spectrum (33) (also the only ω mass), and UKQCD (34). Results for heavy-light hadrons from Fermilab-MILC (35), HPQCD (36), and Mohler & Woloshyn (37).

Excellent agreement among the predictions and with the experimental data.

Quark Masses

A.S. Kronfeld, Ann. Rev. Nucl. Part. Sci. 62, 265 (2012

Table 2 Quark masses from lattice quantum chromodynamics converted to the $\overline{MS}$ scheme and run to the scale indicated[a]

Flavor (scale)	Reference 28	Reference 53	Reference 54	Reference 55	Reference 56
m_u (2 GeV)	1.9 ± 0.2	2.01 ± 0.14	2.24 ± 0.35	2.15 ± 0.11	—
$\bar{m}_d$ (2 GeV)	4.6 ± 0.3	4.79 ± 0.16	4.65 ± 0.35	4.79 ± 0.14	—
$\bar{m}_s$ (2 GeV)	88 ± 5	92.4 ± 1.5	97.7 ± 6.2	95.5 ± 1.9	—
$\bar{m}_c$ (3 GeV)	—	—	—	—	986 + 10
$\bar{m}_b$ (10 GeV)	—	—	—	—	3,617 ± 25

Nambu-Bethe-Salpeter Equation

Schrödinger Equations with a Non-local Potential U(

$$-\frac{1}{2\mu}\nabla^2\phi\left(\vec{r}\right)+\int d^3r' U\left(\vec{r},\vec{r}'\right)\phi\left(\vec{r}\right)=E\phi\left(\vec{r}\right) \qquad (1)$$

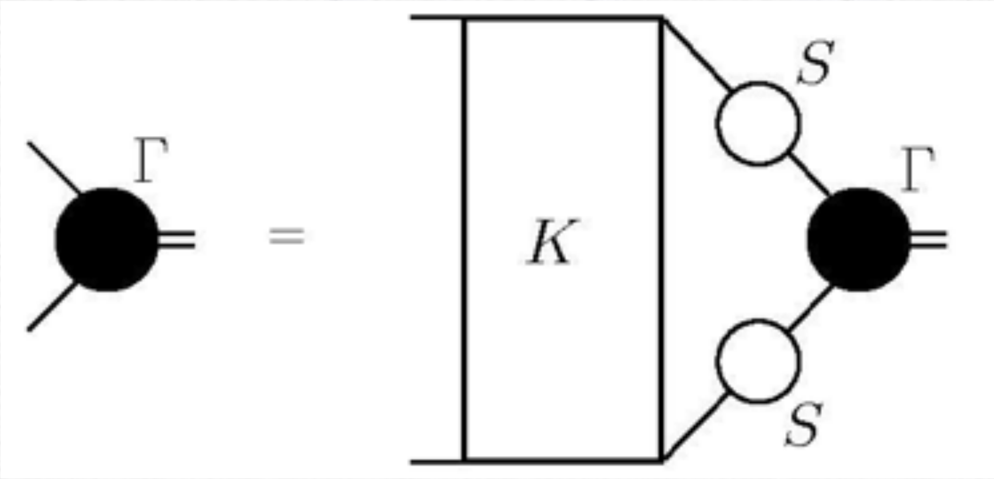
c.f. Schrödinger Equations with a local Potential U(

$$\left\{-\frac{1}{2\mu}\nabla^2+U\left(\vec{r}\right)\right\}\phi\left(\vec{r}\right)=E\phi\left(\vec{r}\right)$$

For central potential U(r)=V

$$V_C\left(r\right)=E+\frac{1}{2\mu}\frac{\vec{\nabla}^2\phi\left(r\right)}{\phi\left(r\right)} \qquad (2)$$

Nambu-Bethe-Salpeter Equation



$$\Gamma(P,p) = \int \frac{d^4k}{(2\pi)^4}\, K(P,p,k)\, S(k-\tfrac{P}{2})\, \Gamma(P,k)\, S(k+\tfrac{P}{2})$$

(Nambu-)Bethe-Salpeter equation is used to analyze and calculate properties of bound states of two constituents in relativistic quantum field theory.

It contains an infinite series of interactions in the shape of ladder diagram in Feynman diagrams. (cf Lipman-Schwinger equation).

$$\varphi_E(\mathbf{r},t) = \langle 0 | N(\mathbf{x},t) N(\mathbf{y},t) | 2N, E \rangle \qquad N(\mathbf{r},t) \sim q(\mathbf{r},t)^3$$

of the equal-time NBS equation ~ two nucleon amplitude (below m

Extract the "Potential" from the simulated "Amplitude".

Nucleon-Nucleon (*NN*) Interaction

Expansion

$$V = V_0(r) + V_\sigma(r)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_\tau(r)\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + V_{\sigma\tau}(r)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Central Terms

$$+ V_{LS0}(r)\mathbf{L} \cdot \mathbf{S} + V_{LS\tau}(r)\mathbf{L} \cdot \mathbf{S}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Spin–Orbit Terms

$$+ V_{T0}(r)S_{12} + V_{T\tau}(r)S_{12}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Tensor Terms

$$S_{12} \equiv 3\frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$$

Tensor Interaction

Lattice QCD Parameters

- Lattice 32
- quenched approximation
- plaquette gauge action, gauge coupling β
- Wilson quark action
- a=0.137 fm,
- hopping parameter κ=0.1665

  m_π

  ⇔

- global heat bath algorithm
- Dirichlet (periodic) boundary condition
- wall source is placed at

Lattice QCD Result

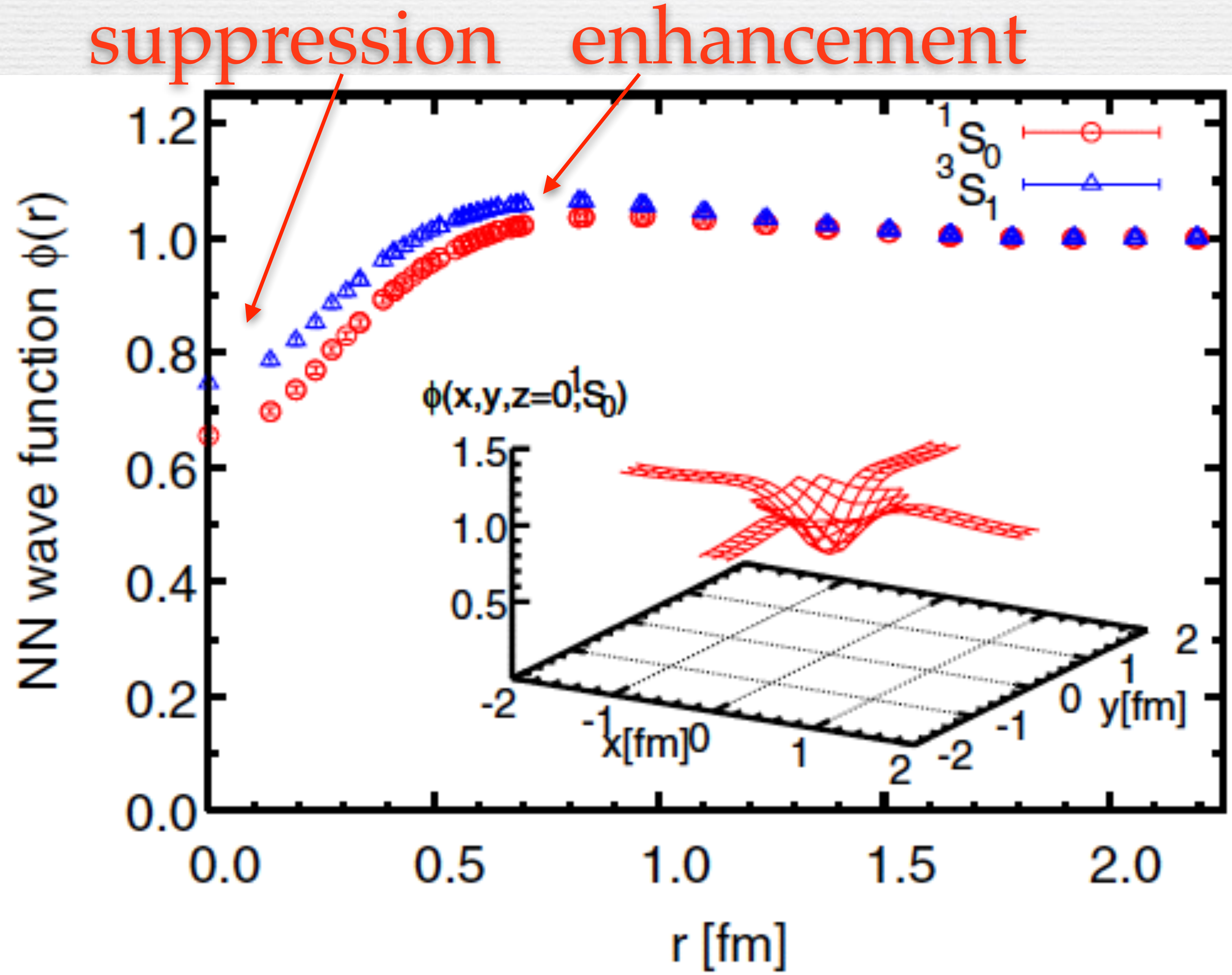
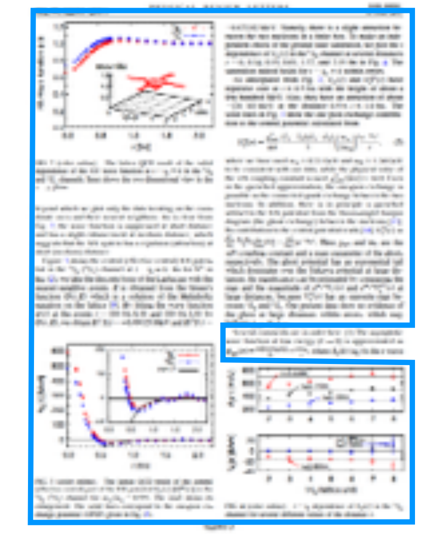


FIG. 2 (color online). The lattice QCD result of the radial dependence of the NN wave function at $t - t_0 = 6$ in the ${}^1S_0$ and ${}^3S_1$ channels. Inset shows the two-dimensional view in the $x - y$ plane.



Lattice QCD Result

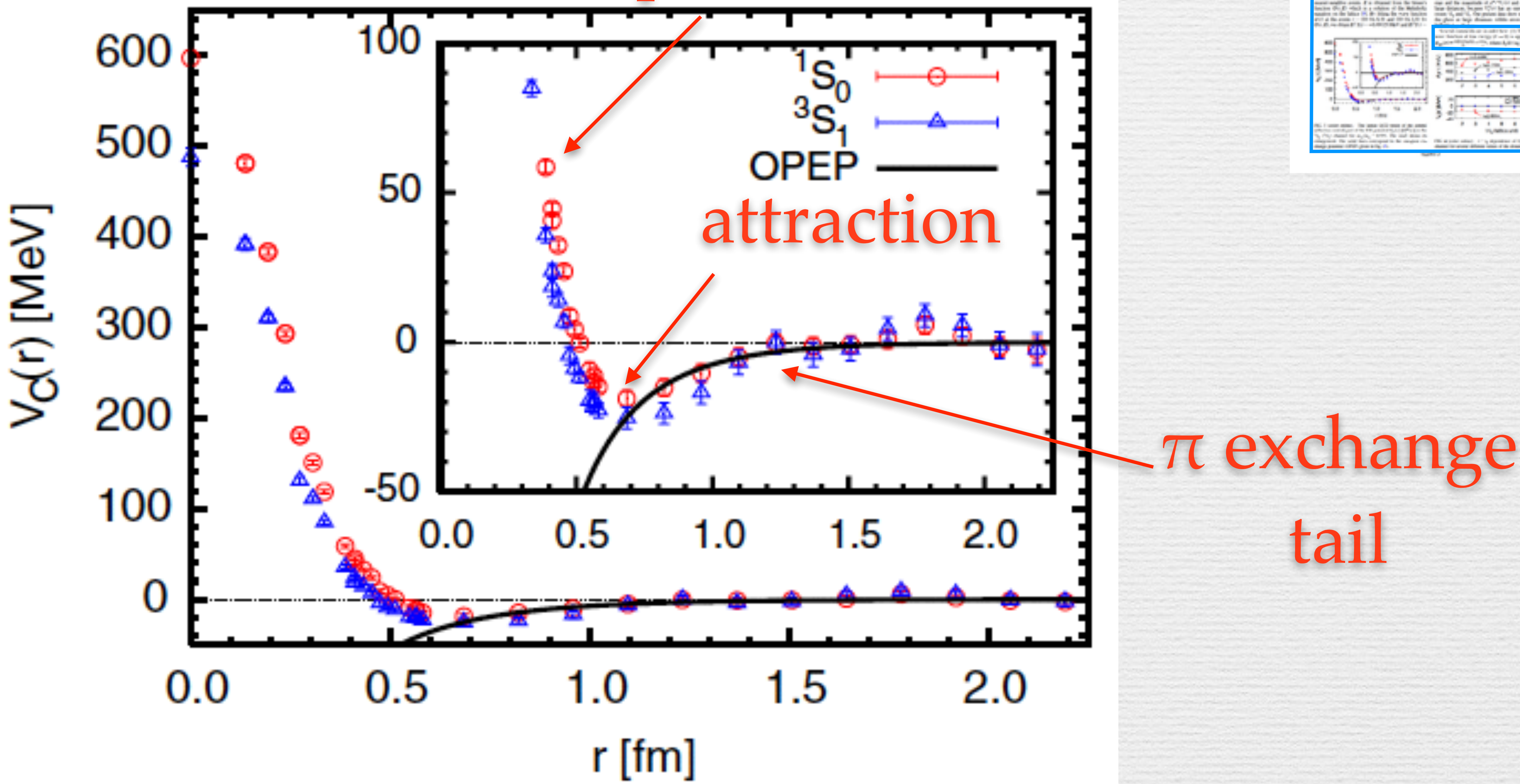
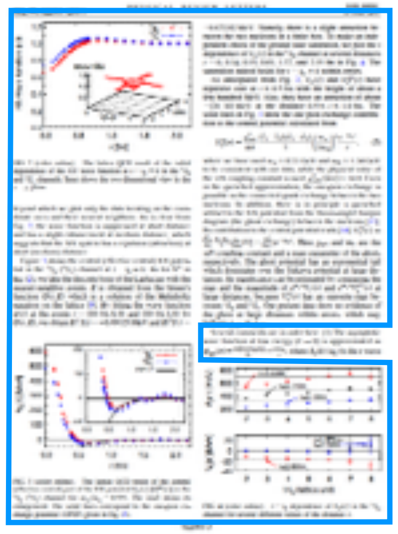


FIG. 3 (color online). The lattice QCD result of the central (effective central) part of the NN potential $V_C(r)$ [$V_C^{\text{eff}}(r)$] in the $^1S_0$ ($^3S_1$) channel for $m_\pi/m_\rho = 0.595$. The inset shows its enlargement. The solid lines correspond to the one-pion exchange potential (OPEP) given in Eq. (5).



One Pion Exchange Potential (OPEP)

Schrödinger Equations with a Non-local Potential U(

$$V_C^{\pi}(r) = \frac{g_{\pi N}^2}{4\pi} \frac{(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)}{3} \left(\frac{m_\pi}{2m_N}\right)^2 \frac{e^{-m_\pi r}}{r}, \qquad (5)$$

(1)

with

Lattice QCD Result

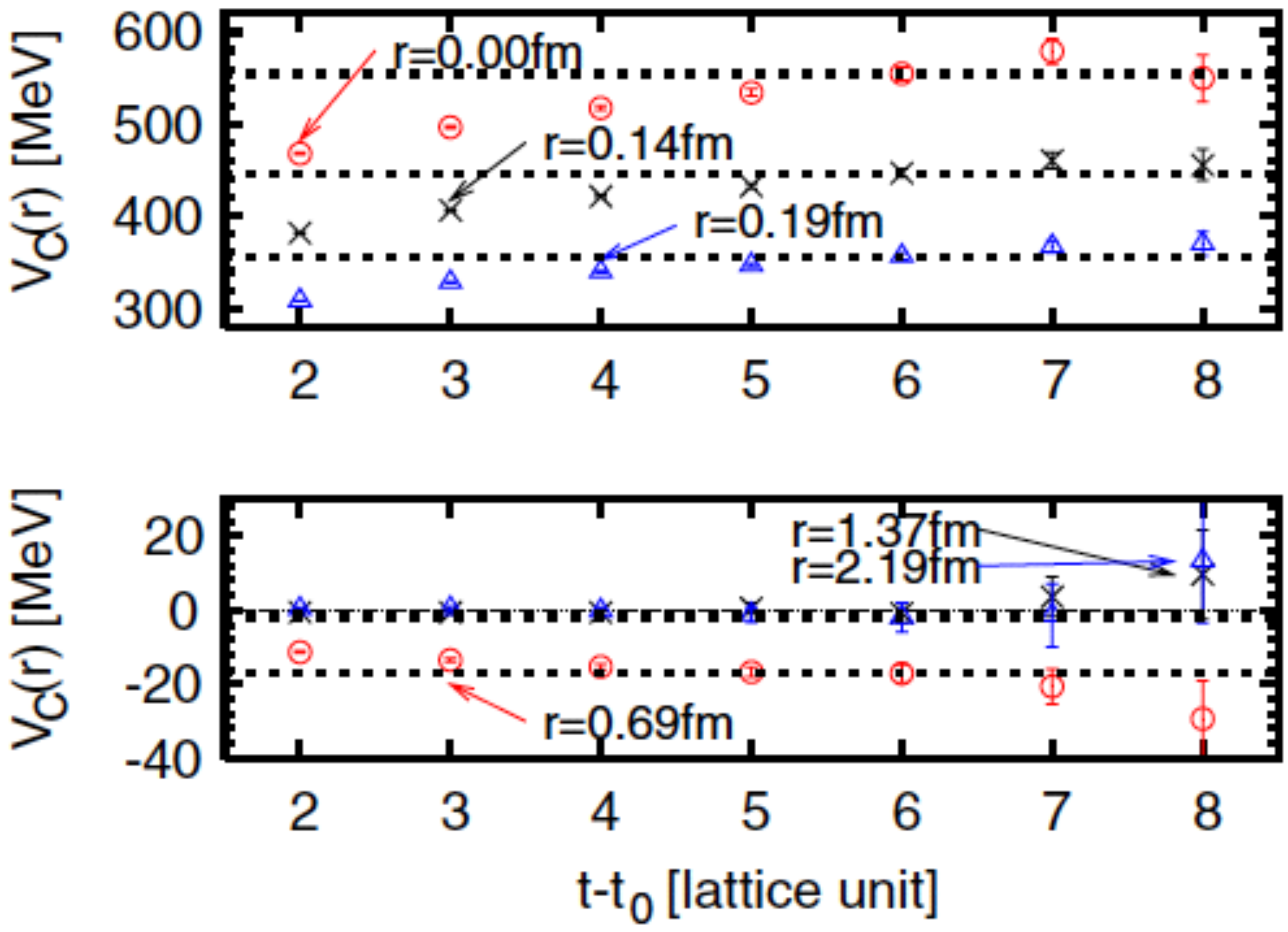
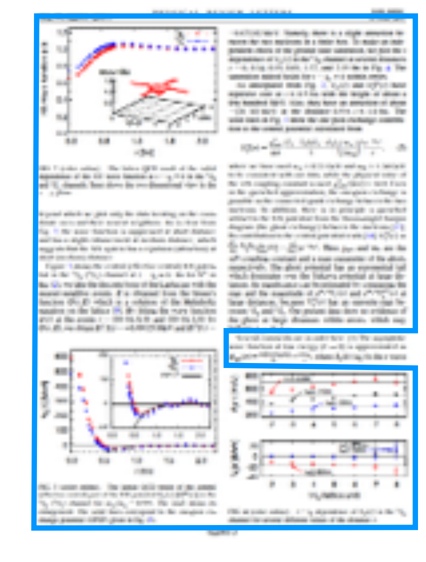


FIG. 4 (color online). $t - t_0$ dependence of $V_C(r)$ in the $^1S_0$ channel for several different values of the distance r.

Saturation of potential in time coordinate

Discussions



- Fig. 2: The wave function is suppressed at short distance, which suggests that the NN system has a repulsion (attraction) at short (medium) distance.
- Fig. 3: system energy (in the finite box)
  E(^{1}
  E(^{3}
  slight attraction in the

Mixing with the
interaction is effectively incorporated in the
central channel.

- The central potentials have
  - repulsive core at $r<0.5$ fm: a few hundred MeV
  - attraction at $0.5<r<1.0$ fm: -(20-30) MeV

- quenched artifact from the flavor-singlet hairpin diagram (the ghost exchange)

Comments

1. scattering length,
   - $a_0(^1$
   - $a_0(^3$

small positive value

2. It is dangerous to compare masses with
3. If the attraction is large enough, the system may have a bound state.

   →

   The

Current values:

NN	a (fm)	r_0 (fm)
nn	-18.9 ± 0.4	2.75 ± 0.11
np	-23.740 ± 0.020	2.77 ± 0.05
pp	-17.3 ± 0.4	2.85 ± 0.04

4. Preliminary result with lighter quark mass at κ=0.1657

   m_π
   - 40% higher repulsive core
   - same potential minimum at r~0.8 fm with the same depth

Summary

1. The central (effective central) potential in the channel at low energy have a repulsive core surrounded by attractive well at medium and long distances.
2. These properties are known to be the important features of the phenomenological
3. The long range tail of with the one-pion exchange.
4. It would be quite interesting to derive
   - the tensor and spin-orbit forces
   - hyperon-nucleon and hyperon-hyperon potentials
   - physics origin of the repulsive core

Recent Progresses

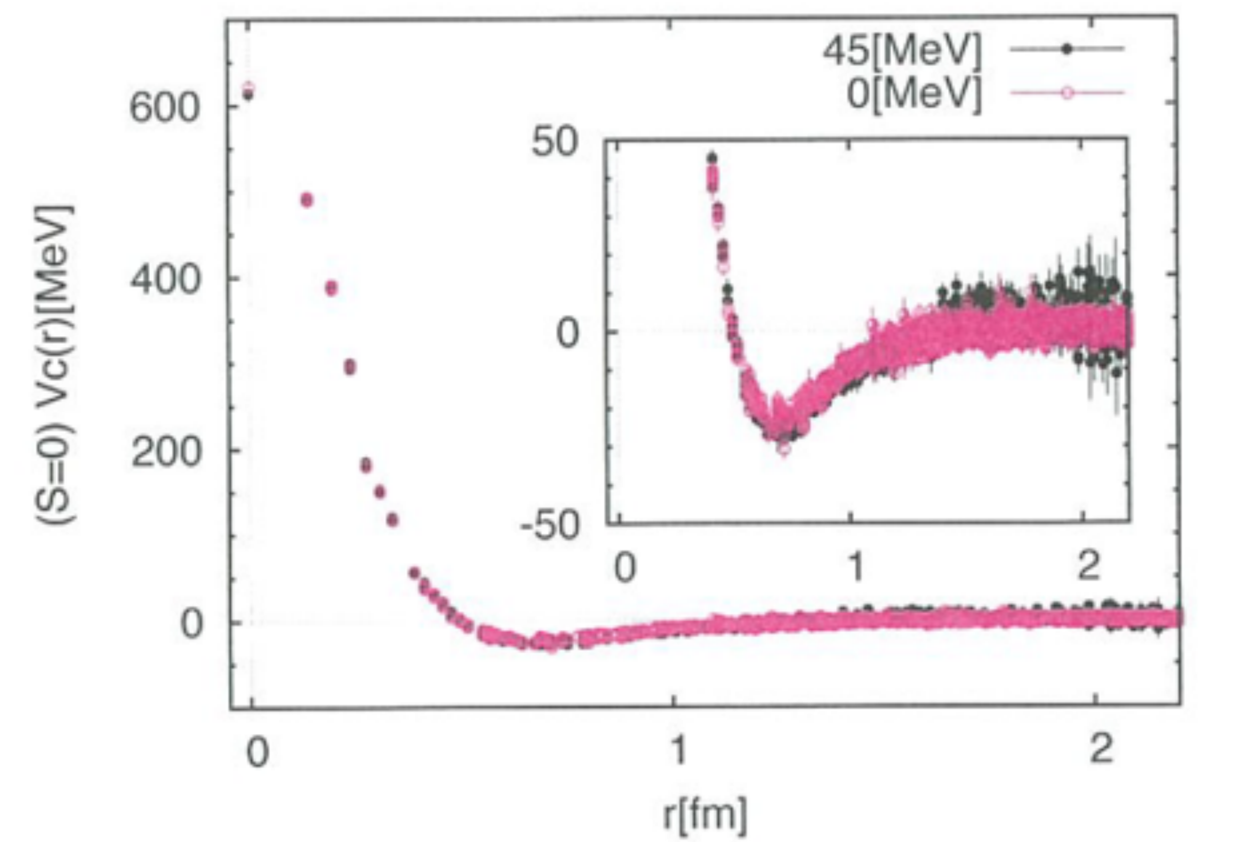


図3　異なったエネルギーの波動関数から求められた$S=0$の状態の核力ポテンシャルの比較．$e_k \simeq 0$ MeV（黒），$\simeq 45$ MeV（赤）である．計算の他の条件は図2と同じ．

Comparison of potentials derived from wave functions at two different energies.

hyperon-nucleon potential

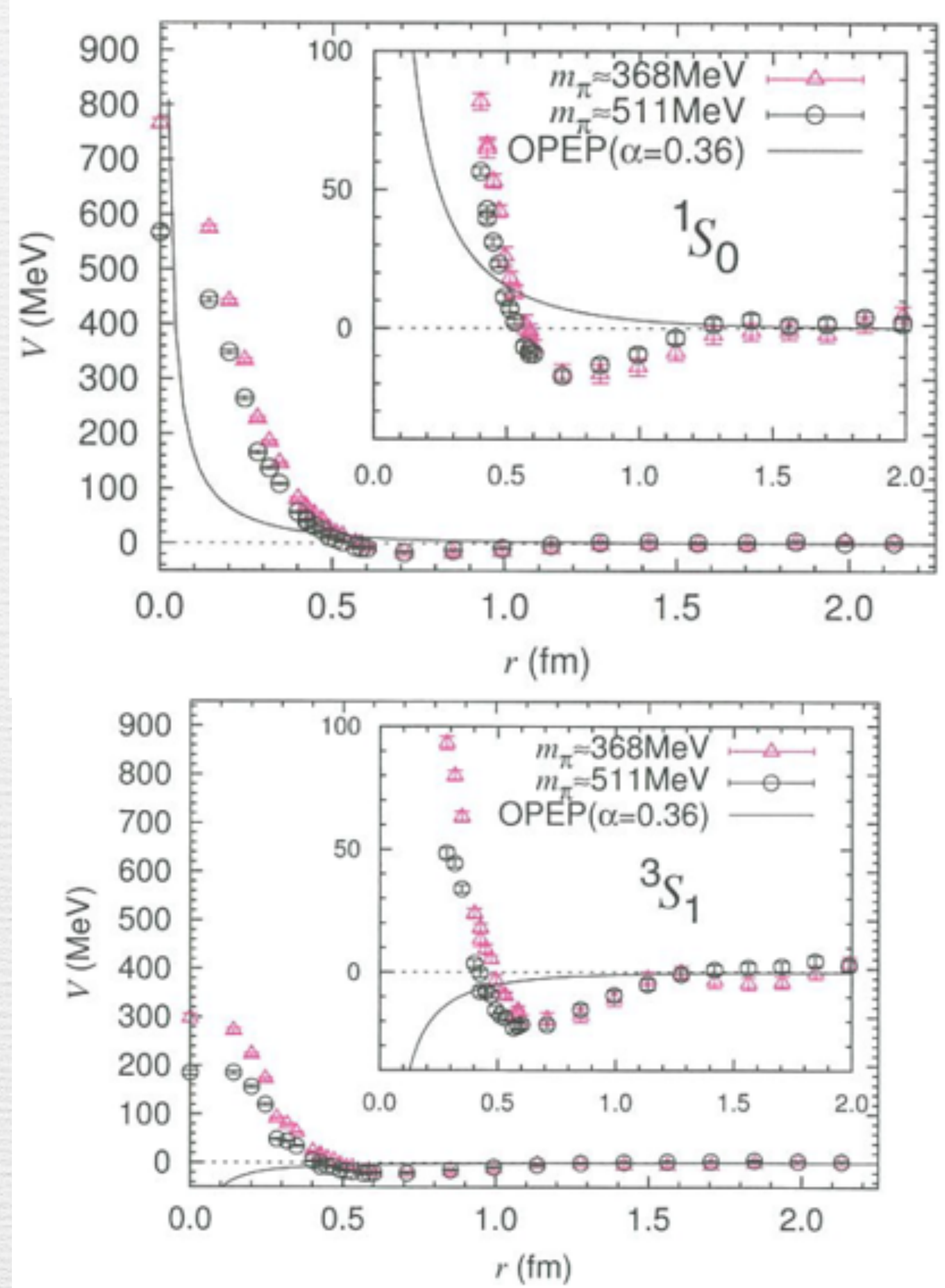


図4　$\Xi^0 p$ポテンシャル．微分展開の最低次の結果．丸が$m_\pi \simeq 510$ MeV，三角が$m_\pi \simeq 370$ MeVのデータ．計算の他の条件は図2と同じ．実線は1つのパイオンを交換した場合のポテンシャル（OPEP）である．（上）全スピンがゼロの場合．（下）全スピンが1の場合．

S. Aoki, T. Hatsuda, N. Ishii, and H. Nemura, Butsuri

Recent Progresses

Central potential π

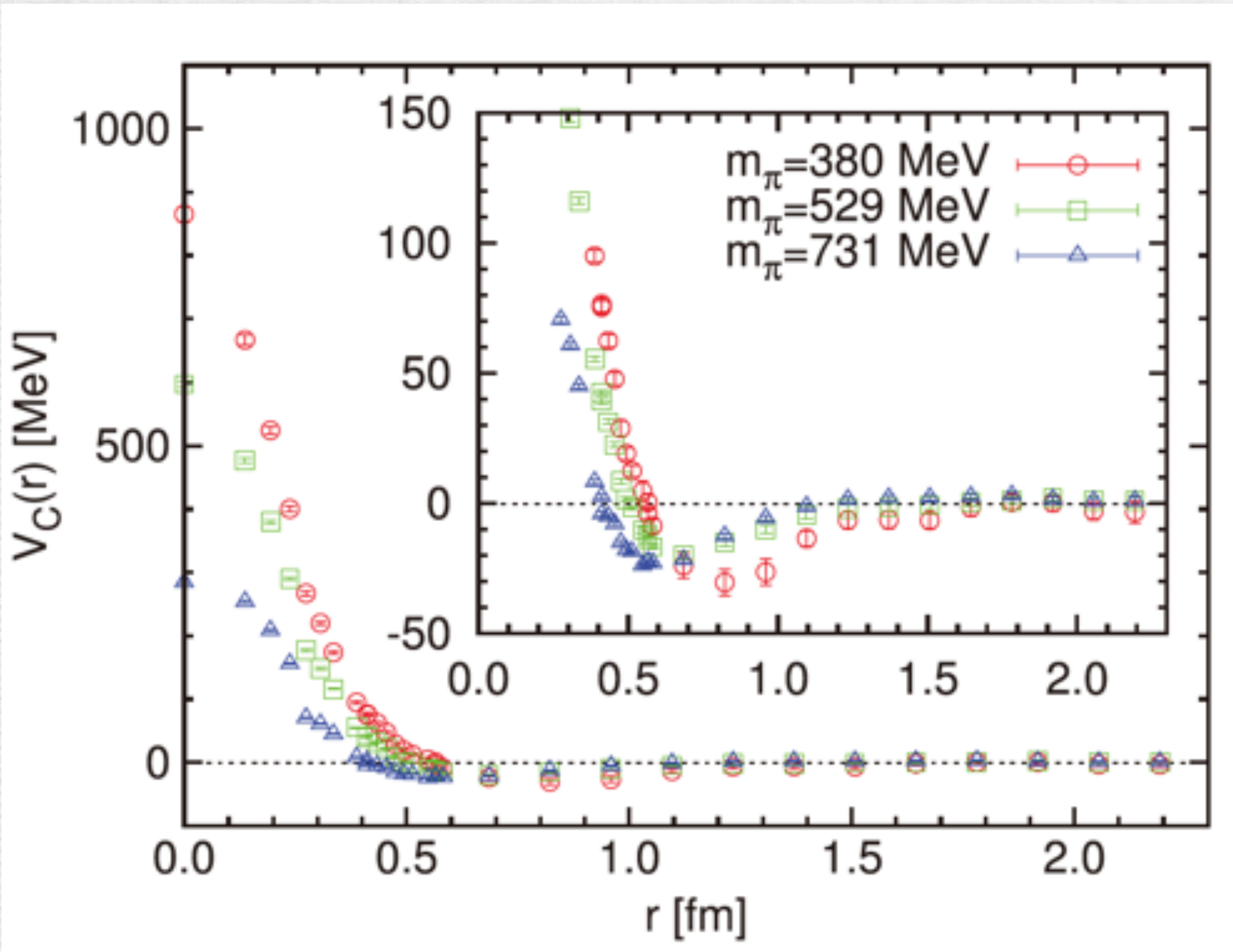


Fig. 3. The central potentials for the spin-singlet channel from the
orbital A
QCD. Taken from Ref. [

S. Aoki et al., Prog. Theor. Exp. Phys.

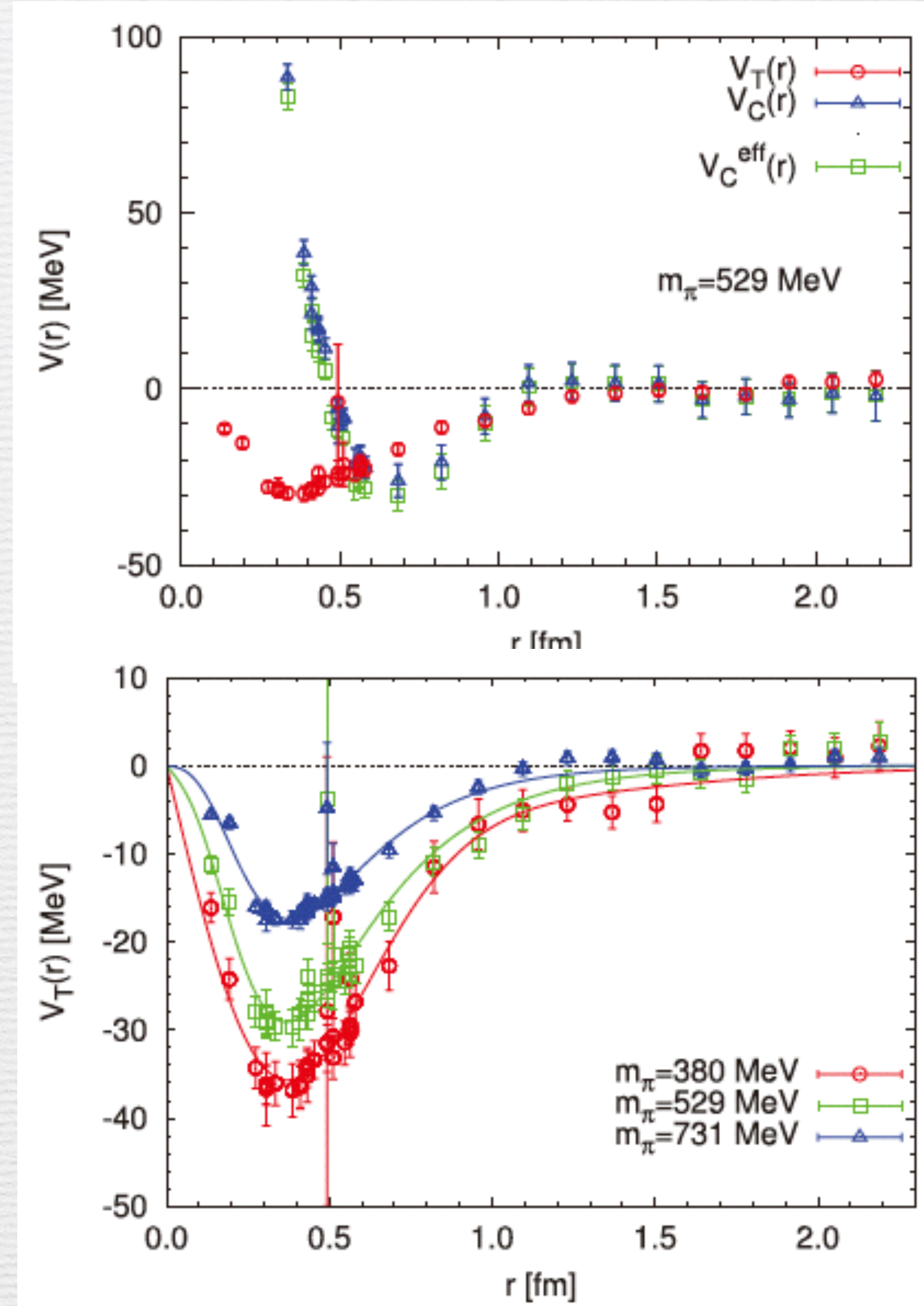
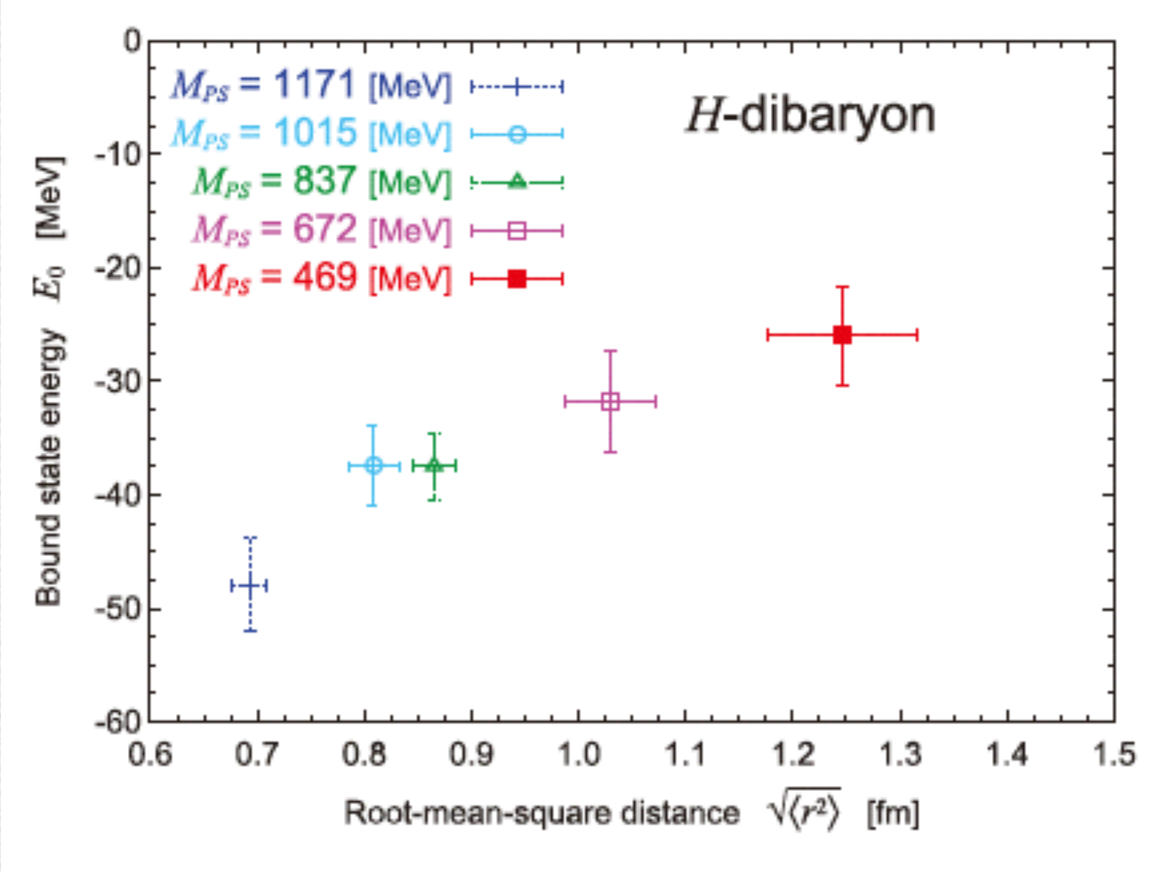


Fig. 5. (Top) The central potential V
NBS wave function, together with the effective central potential V
(Bottom) Pion mass dependence of the tensor potential. The lines show the four-parameter fit using
one-pion-exchange

Recent Progresses

Binding energy of H-dibaryon



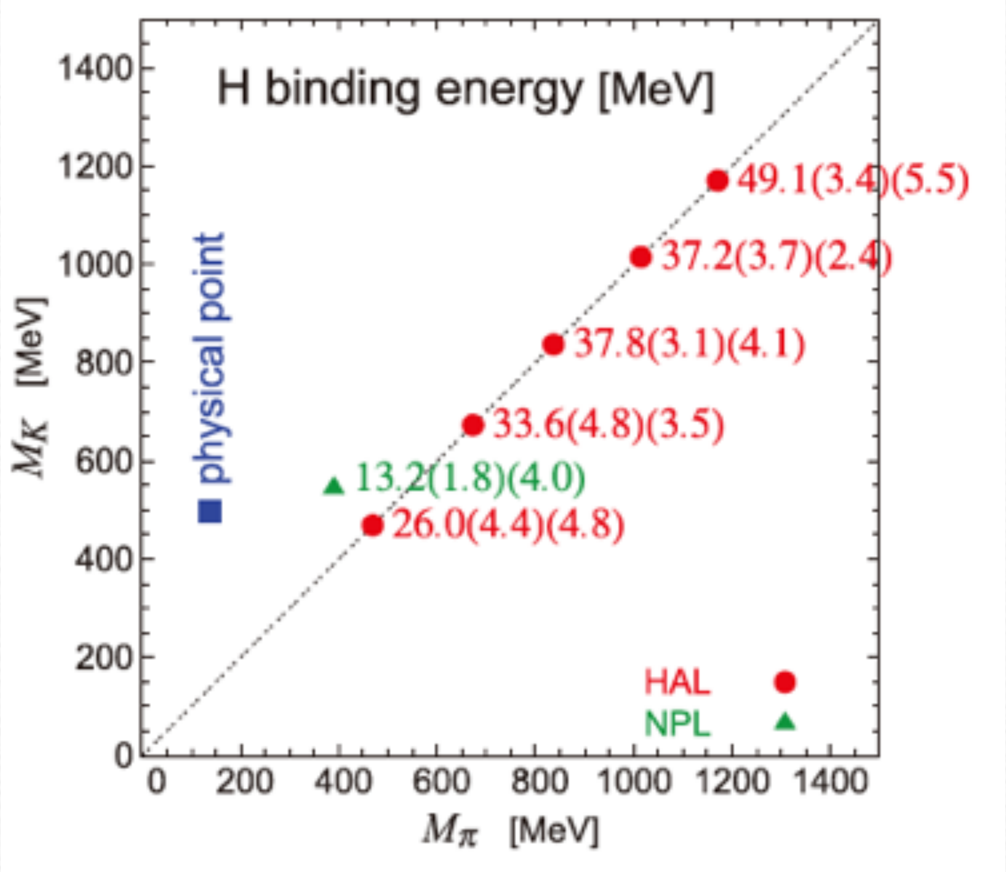


Fig. 3. The central potentials for the spin-singlet channel from the orbital A
QCD. Taken from Ref. [

S. Aoki et al., Prog. Theor. Exp. Phys.

Recent Progresses

Three Nucleon Force

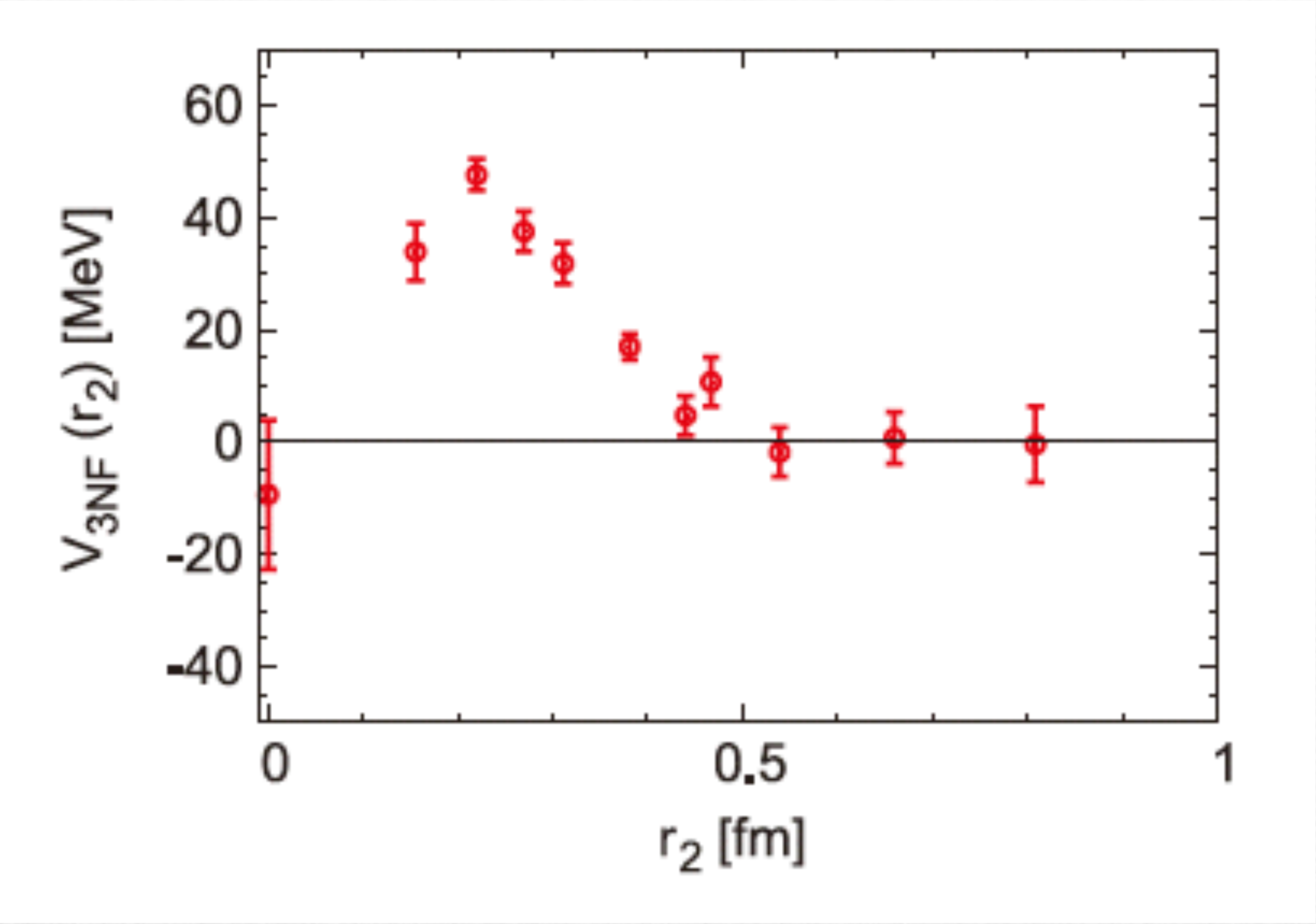


Fig. 23. The effective scalar–isoscalar 3NF in the triton channel with the linear setup obtained at
center and edge in the linear setup.

S. Aoki et al., Prog. Theor. Exp. Phys.

Work in Progress

Calculation at the real pion mass with the K super-computer is in progress.

- Baryon-Baryon interaction
- Precise determination of quark bare masses
- three nucleon force
- quark gluon plasma
- etc…

Seminar by Prof. N. Ishii

LS force and anti-symmetric LS force from lattice QCD

Noriyoshi Ishii (RCNP,Osaka University)

Please Join it!

Thursday 29 May 2014, 14:00

Lecture Room 1, RCNP 6F Osaka University

Abstract:s

We will present our recent results on LS force (NN) and anti-symmetric LS force (hyperon interaction) from lattice QCD. We begin with a brief review of our strategy of determining inter-baryon potentials. The potentials are obtained from Nambu-Bethe-Salpeter (NBS) wave functions [HAL QCD method], which has been applied to many systems, such as NN, YN, YY, NNN, etc. These studies are restricted to local potentials (potentials which do not involve any derivatives) in the parity-even sector. The restriction is due to source functions which these calculations employ, i.e., these calculations employ source functions with A1 representation of the cubic group which roughly corresponds to s-wave. By using a momentum wall source with T1 representation of the cubic group which roughly corresponds to p-wave, we study NN potentials in the parity-odd sector as well as the LS potentials. A strong attractive LS force with a weak repulsive central force in spin triplet P-wave channels lead to an attraction in the 3P2 channel, which is related to the P-wave neutron pairing in neutron stars. We extend this method to the hyperon sector, and consider a phenomenologically expected cancellation between the symmetric and the anti-symmetric LS potentials in the flavor SU(3) symmetric limit.

References

- John Dirk Walecka,
- 青木慎也
- 南部陽一郎「クォーク第
- I.J.R.Aitchison and A.J.G. Hey,
- F. Halzen and A.D. Martin,
- J.D. Bjorken and S.D. Drell,
- R.P. Feynman,

Thank you

for your attention

Dirac Equation

$$\left(-i\alpha \bullet \nabla + \beta m\right)\psi\left(\mathbf{x},t\right) = i\frac{\partial}{\partial t}\psi\left(\mathbf{x},t\right)$$

$$\alpha_i \equiv \begin{pmatrix} \mathbf{0} & \sigma_i \\ \sigma_i & \mathbf{0} \end{pmatrix} \qquad \beta \equiv \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$$

$$\left(i\gamma^\mu \partial_\mu - m\right)\psi = 0$$

$$\gamma^\mu \equiv \left(\gamma^0, \gamma^i\right)$$

$$\gamma^0 \equiv \beta, \gamma^i \equiv \beta\alpha_i, \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\gamma^i = \begin{pmatrix} \mathbf{0} & \sigma_i \\ -\sigma_i & \mathbf{0} \end{pmatrix} \quad \gamma^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

$$\not\nabla \equiv \gamma^\mu \partial_\mu$$

Gell-Mann matrices SU(3)

like Pauli matrices in the isospin space

$$\left[\frac{1}{2}\lambda^a, \frac{1}{2}\lambda^b\right] = i\frac{1}{2} f^{abc}\lambda^c$$

f^{abc} :structure function

$$\lambda^1 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} & & -i \\ & & \\ i & & \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix} \quad \lambda^8 = \begin{pmatrix} 1/\sqrt{3} & & \\ & 1/\sqrt{3} & \\ & & -2/\sqrt{3} \end{pmatrix}$$

EM U(1) gauge

covariant derivative

$$D^{\mu} \equiv \partial^{\mu} - ie_0 A^{\mu}$$

field tensor

$$\mathbf{F}^{\mu\nu} \equiv \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

gauge transformation

$$A^{\mu} \to A'^{\mu} = A^{\mu} + \frac{1}{e_0}\partial^{\mu}\Lambda$$

$$\overline{\psi} \to \overline{\psi}' = \exp(-i\Lambda)\overline{\psi}$$

Yang-Mills SU(2) gauge

covariant derivative

$$\frac{D}{Dx_\mu} \equiv \partial^\mu - igT^a A_\mu^a(x)$$

field tensor

$$\mathbf{F}_{\mu\nu} = \partial^\mu A_\nu - \partial^\nu A_\mu + gA_\mu \times A_\nu$$

infinitesimal local gauge transformation $\theta \rightarrow 0$

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{g_0}\partial^\mu \theta + \theta \times A_\mu$$

QCD SU(3) gauge

covariant derivative

$$\frac{D}{Dx_\mu} \equiv \partial^\mu - \frac{i}{2} g \lambda^a A_\mu^a(x)$$

field tensor

$$\mathbf{F}_{\mu\nu}^a = \partial^\mu A_\nu^a - \partial^\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

infinitesimal local gauge transformation $\theta^a \to 0$

$$A_\mu^a \to A'^a_\mu = A_\mu^a - \frac{1}{g} \partial^\mu \theta^a + f^{abc} \theta^b A_\mu^c$$

$$\mathbf{F}_{\mu\nu}^a \to \mathbf{F}'^a_{\mu\nu} = \mathbf{F}_{\mu\nu}^a + f^{abc} \theta^b \mathbf{F}_{\mu\nu}^c$$

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\overline{\psi} \left\{ \gamma_\mu \left[\partial^\mu - \frac{i}{2} g \lambda^a A_\mu^a(x) \right] + M \right\} \psi - \frac{1}{4} \mathbf{F}_{\mu\nu}^a \mathbf{F}_{\mu\nu}^a$$