Comments on the experimental details of the muon g-2 measurement at BNL

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The general relativistic effects on the magnetic moment in Earth's gravity

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The magnetic moment of free fermions in the Earth's gravitational field has been studied on the basis of general relativity. Adopting the Schwarzschild metric for the background spacetime, the dipole coupling between the magnetic moment and the magnetic field has been found to be dependent on the gravity in the calculation up to the post-Newtonian order $O(1/c^2)$. The gravity dependence can be formulated by employing the effective value of the magnetic moment as a gravity-dependent quantity $\boldsymbol{\mu}_{\mathrm{m}}^{\mathrm{eff}} = (1 + 3\phi/c^2)\, \boldsymbol{\mu}_{\mathrm{m}}$ for the cases of minimal coupling, non-minimal coupling, and a mixture of the two. The gravitationally induced anomaly is found to be canceled in the experimental values of the anomalous magnetic moment measured in the Penning trap and storage ring methods.

Subject Index B30, B31, E00, E01

Reminder of the essences of the article

- **Muon** (also electron and proton) **magnetic moment** and the cyclotron frequency on earth **has a modification due to the earth's gravitational field** with the common factor of $1+\frac{3\phi}{c^2}$ ~ $1+2.8\times10^{-9}$. Here, ϕ is the gravitational potential on earth.

- The muon (g-2)/2 measured in E821 at BNL G.W. Bennett et al., PRD73, 072003(2006)

$$a_\mu(\mathrm{Expt}) = 11659208.0(6.3)\times10^{-10}(0.54\ \mathrm{ppm})$$

Deviation from the Standard model Prediction

$$\Delta a_\mu(\mathrm{Expt\text{-}SM}) = [(22.4\pm10)\ \mathrm{to}\ (26.1\pm9.4)]\times10^{-10}$$

- The **muon g-factor was measured as the ratio to the proton Larmor precession (NMR)**, thus **the gravitational effects cancel to each other**.

- However, the condition of "**magic momentum**" was used to e**liminate the effect from the electric field** (electric Q), satisfying $a_\mu - \frac{1}{\gamma^2-1} = 0$.
The **<u>gravitational effect modifies the equation and may give a sizable offset to the measured a_μ value</u>**.

→ How was the magic momentum calibrated?

BNL-E821

G.W. Bennett *et al.*, PRD**73**, 072003(2006)

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

Muon Storage Ring

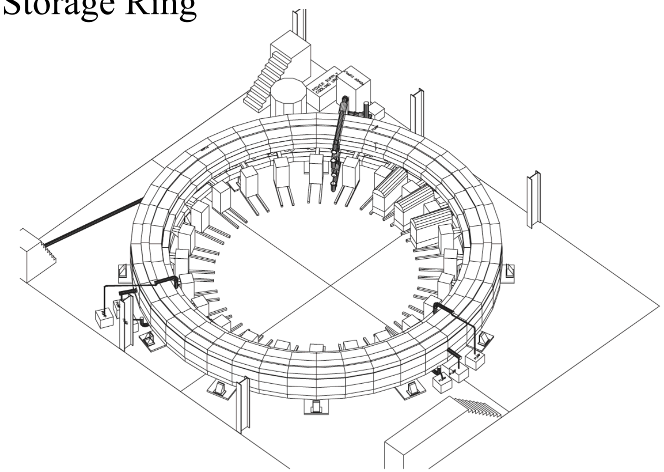

FIG. 6. A 3D engineering rendition of the E821 muon storage ring. Muons enter the back of the storage ring through a field-

“magic momentum” of 3.094 GeV/c was used to eliminate the term of the electric field by satisfying

$$a_\mu - \frac{1}{\gamma^2 - 1} = 0 \qquad \gamma = 29.3$$

The orbital length is larger (smaller) for muons with larger (smaller) momentum.

Muon velocity is 0.9994c and is practically constant.

Electric-Q

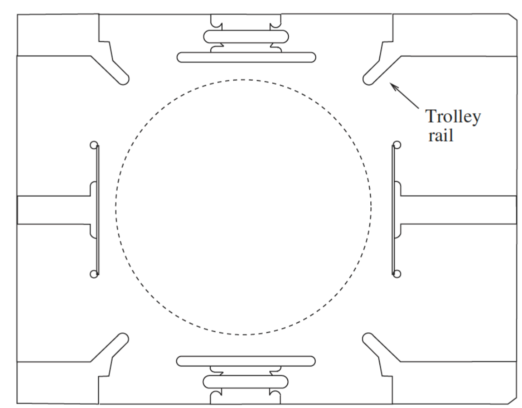

FIG. 9. Schematic view of the electrostatic quadrupoles inside the vacuum chamber. For positive muons, the top and bottom electrodes are at $\sim +24$ kV; the side electrodes are at ~ -24 kV. The NMR trolley rails can be seen between the electrodes in the $V = 0$ planes. The 90 mm diameter storage region is depicted by the dashed circle.

BNL-E821

Precession frequency of the muon Anomalous magnetic moment

G.W. Bennett *et al*., PRD**73**, 072003(2006)

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

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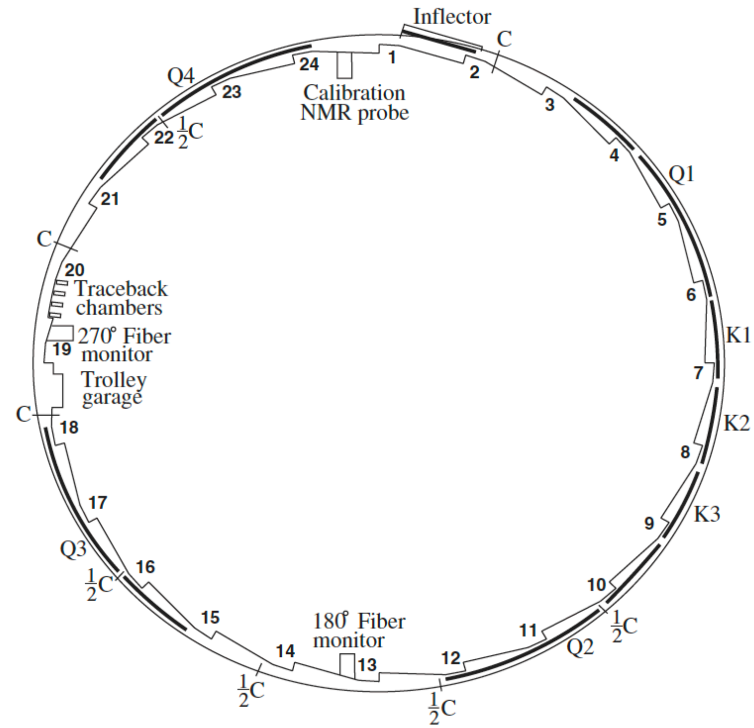
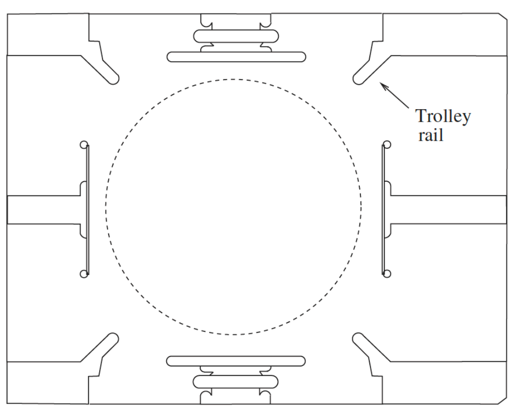


FIG. 8. The $(g-2)$ storage-ring layout. The 24 numbers represent the locations of the calorimeters immediately downstream of the scalloped vacuum chamber subsections. Inside the vacuum are four quadrupole sections (Q1–Q4), three kicker plates (K1–K3) and full-aperture (C) and half-aperture ($\frac{1}{2}$C) collimators. The traceback chambers follow a truncated scalloped vacuum chamber subsection.

Electric-Q



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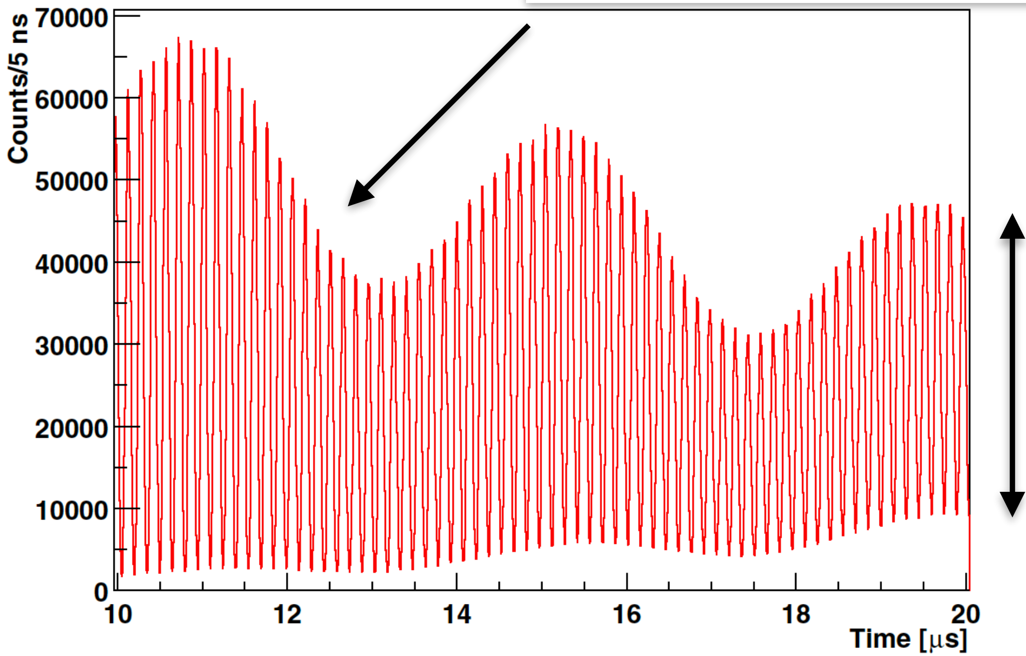
BNL-E821

"Fast rotation" analysis

G.W. Bennett *et al.*, PRD**73**, 072003(2006)

Muon-decay detections just after the muon injection.

Slower oscillation due to the muon spin rotation.



Decrease of the fast oscillation amplitude due to the de-bunching of the beam.

Analysis by modified-Fourier transformation.

FIG. 19 (color online). Intensity at a single detector station shortly after injection. The rapid modulation repeats at the cyclotron frequency as the muon bunch circles the ring. The width of the bunch grows with time because of the finite $\delta p/p$ of the stored muons. The slow variation in the maximum amplitude is at the $(g-2)$ frequency.

BNL-E821

"Fast rotation" analysis

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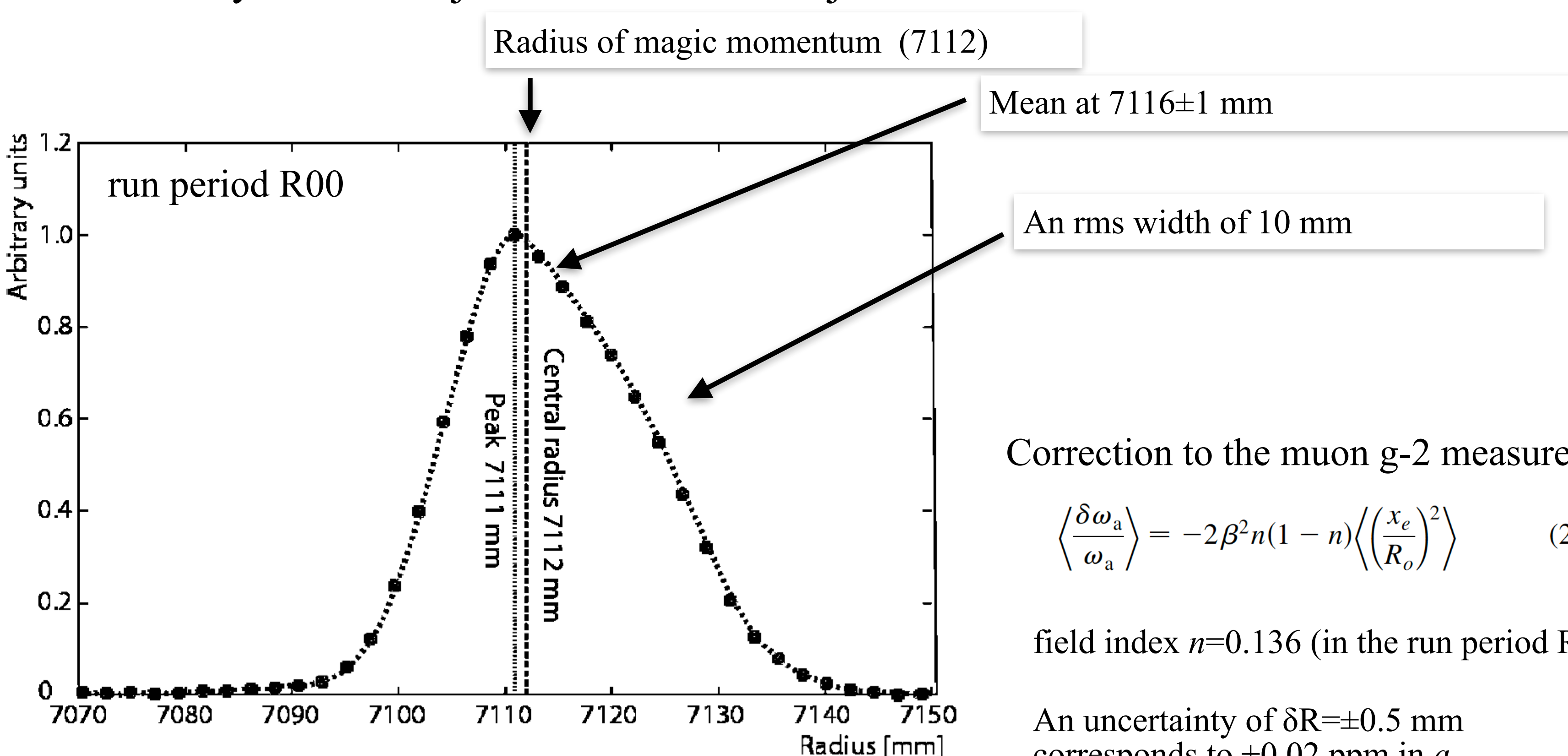

FIG. 20. The distribution of equilibrium radii dN/dx_e, as determined from the fast-rotation analysis. The dashed vertical line is at 7112 mm, the magic radius; the dotted line is at 7111 mm. The solid circles are from a debunching model fit to the data, and the dashed curve is obtained from a modified Fourier analysis.

Correction to the muon g-2 measurement

$$\left\langle \frac{\delta\omega_a}{\omega_a} \right\rangle = -2\beta^2 n(1-n)\left\langle \left(\frac{x_e}{R_o}\right)^2 \right\rangle \qquad (21)$$

field index n=0.136 (in the run period R00)

An uncertainty of δR=±0.5 mm corresponds to ±0.02 ppm in a_μ

My **personal opinion** on the correction of the BNL-E821 result due to the Earth's gravity

- The "**magic momentum**" in BNL-E821 **was calibrated** to the orbital radius and thus **to the cyclotron frequency of the muons**.
- The cyclotron frequency of the muons is modified by the Earth's gravitational field by 2.8×10^{-9}. However, the **known muon cyclotron frequency was measured on Earth**. Thus **no correction is required**.
- **Even if the correction is required**, a correction of 2.8×10^{-9} to the cyclotron frequency introduces the correction to the orbital radius with the same factor thus results **in a very small correction of 8×10^{-7} ppm to the measured g-2 values compared to the experimental uncertainty of 0.54 ppm.**
- **Nevertheless, the general relativistic effect on the fundamental physical quantities is interesting and can be important in other special cases, such as astrophysical reactions in Big-Bang or in a neutron star.**

$0.02 \text{ ppm} / (0.5 / 7116) \times 2.8\times10^{-9} = 8\times10^{-7} \text{ ppm}$

However I still don’t understand yet

- Shimizu-san understands that the gravitational effect appears when the muon is falling down in the magnetic field that is fixed to the Earth’s ground, and does not when both the muon and the magnetic field are fixed to the Earth’s ground.

  How can this small difference (~1μm fall in the measurement time) make sizable effect on the result though the beam profile is much larger?

Thank you

for your attention

BNL-E821

G.W. Bennett *et al.*, PRD**73**, 072003(2006)

a_μ(expt)=11659208.0(6.3)×10^{-10} 0.54 ppm

a_μ(expt-SM)=[(22.4±10) to (26.1±9.4)]×10^{-10}

$$\vec{\omega}_c = -\frac{q\vec{B}}{m\gamma}, \qquad \vec{\omega}_s = -\frac{gq\vec{B}}{2m} - (1-\gamma)\frac{q\vec{B}}{\gamma m}$$

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\left(\frac{g-2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}$$

Muon anomalous precession frequency

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

including the effect of the electric field

"magic momentum" of 3.094 GeV/c was used to have $a_\mu - \frac{1}{\gamma^2 - 1} = 0$

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\tilde{\omega}_p}{\omega_L/\tilde{\omega}_p - \omega_a/\tilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

Muon anomalous magnetic moment is measured relative to the proton Larmor precession (NMR).