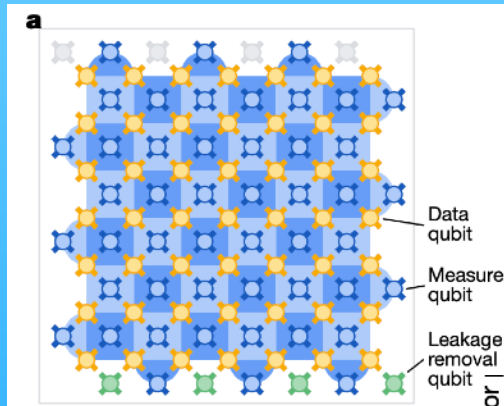


Quantum Computing

toward fault-tolerant quantum computation



surface code

stabilizers

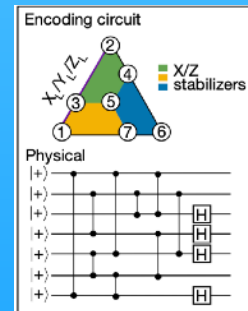
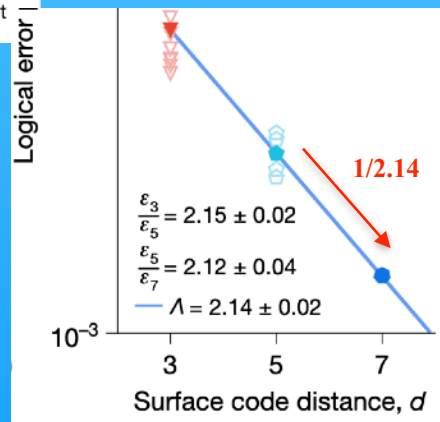
MBQC

QEC

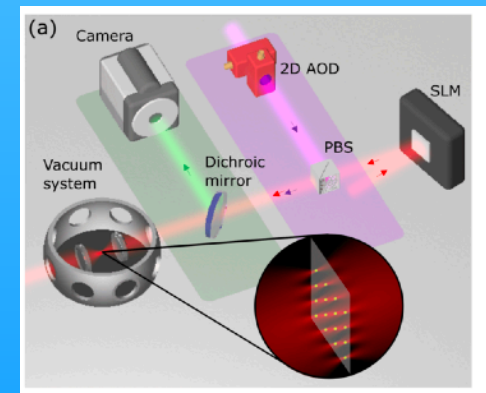
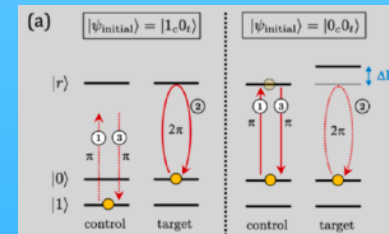
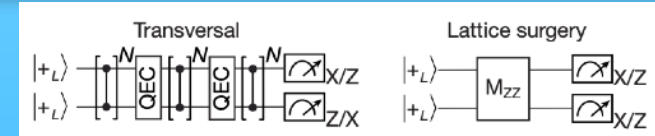
FTQC

distillation/injection

braiding / lattice surgery



^{87}Rb



Google Quantum AI and Collaborators, *Quantum error correction below the surface code threshold*, [Nature 638, 920 \(2025\)](#)

D. Bluvstein et al., *A fault-tolerant neutral-atom architecture for universal quantum computation*, [Nature 649, 39 \(2026\)](#)

A. Tamii

RCNP, the University of Osaka

v2.3 on July 20, 2026

Colloquium on July 7, 2026, the University of Osaka

Introduction

Personal Motivation

Research on the **Quantum Information** Theory and **Quantum Computation** is rapidly advancing recent years.

→ learn the essences and recent progresses

ordinary quantum mechanics $\Leftrightarrow$ modern quantum information theory

Many interesting features such as

- Languages different from fundamental quantum mechanics
→ **New Concepts**
- Challenges to realize fault tolerant quantum computation
→ **Creative Ideas**
- Realization of quantum computer hardware
→ **Technological developments**

Fault Tolerant Quantum Computation (FTQC)

Fault Tolerant Quantum Computation (FTQC)

Arbitrarily accurate quantum computation is possible when all relevant **physical error rates are below a threshold** under suitable assumptions on the locality and correlations of noise.

For realizing it, quantum information must be protected not only during storage or transmission but also during dynamical computation.

Reduction of error rate with scaling the size has been experimentally demonstrated.

Google quantum AI, [Nature 614, 676 \(2023\)](#)

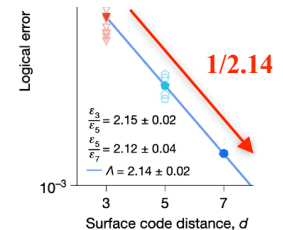
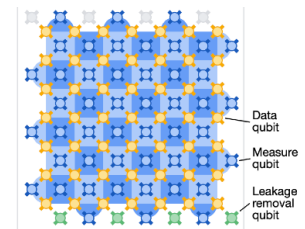
Google Quantum AI and Collaborators, [Nature 638, 920 \(2025\)](#)

Fundamental Proof of Principle



state transition

Technological Development

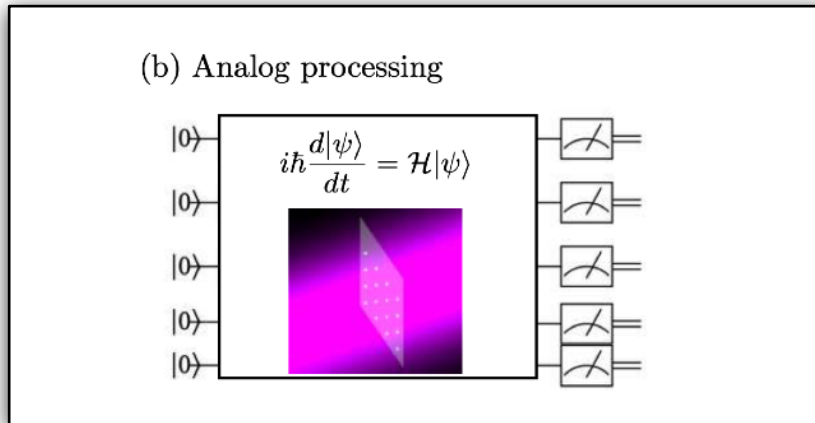


Universal Quantum Computation

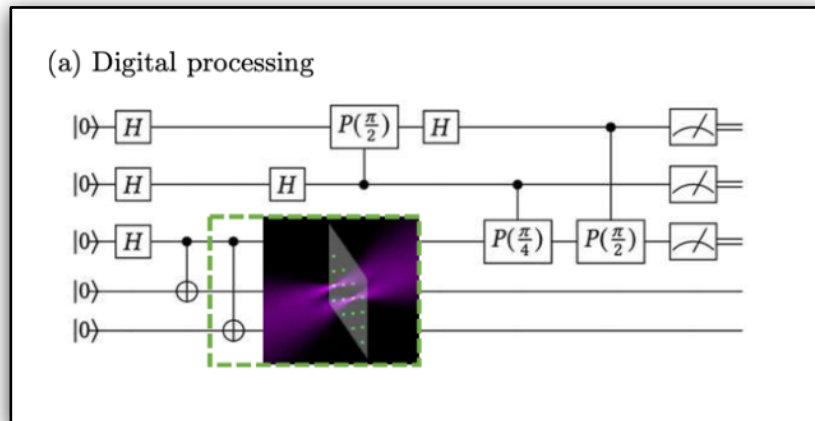
[Hen20]

Quantum Simulation and Computation

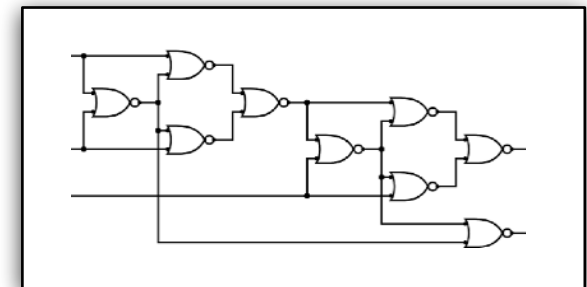
Simulation by implementing the relevant Hamiltonian



Universal Quantum Computation (quantum gates)



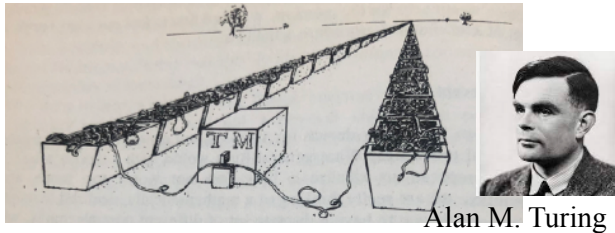
Classical Computer
(logic gates)



Quantum Supremacy (Quantum Advantage)

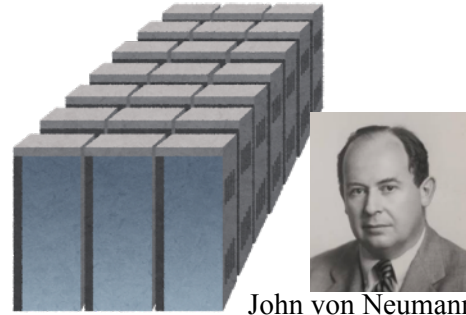
(not picked-up today)

Computability



Turing Machine (1936)

=



Classical Computer

=



Quantum Computer (1982)

Computational Complexity

Church-Turing thesis:

Any algorithmically computable function can be computed by a Turing machine

Strong form of the Church-Turing thesis

Any physically realizable computation model can be by a Turing machine with polynomial overhead.

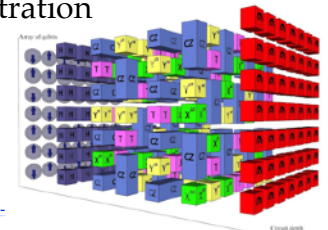
$BPP \subsetneq BQP$? ... not proved

BPP: Bounded Error Probabilistic Polynomial time

BQP: Bounded Error Quantum Polynomial time

F. Arute et al. (google), [Nature 574, 505 \(2019\)](https://doi.org/10.1038/s41586-019-1267-3)

Claimed as the first demonstration
of quantum supremacy
→ still under debating



<https://research.google/blog/the-question-of-quantum-supremacy/>

Quantum Algorithms

(not picked-up today)

Developing and applying **quantum algorithms** are essential for realizing efficient calculation than classical

applications

- Deutsch algorithm (1985)
- Deutsch-Jozsa algorithm (1992)
- Shor's algorithm & QuantumFT (1994)
- Grover's algorithm (1996)
- ...

prime factorization

unstructured search

Quantum Algorithm Catalogue → see Quantum Algorithm Zoo

<https://quantumalgorithmzoo.org/>

Quantum Computation: Hardware Implementation

[Hen23]

Present major hardware for realizing quantum computation

- **superconductors**

google, IBM, Rigetti
RIKEN+Fujitsu

very low-T, fast gate operation, low coherence time,
limited connectivity

- ion traps

IonQ, Quantinuum

high coherence, high two-qubit gate fidelity, very high connectivity
low operation speed, challenge for scaling

- **neutral atoms**

QuEra, Pasqal, Atom

long coherence time, good scalability in 2D/3D, good connectivities,
challenge for higher two-qubit gate fidelities, low operation speed

picked-up mostly today

- photons

Xanadu, PsiQuantum, NTT

good scalability, challenge to implement gate operation,

- spins in semiconductors

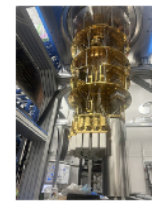
Intel, Diracq

based on semiconductor techniques, feasibility of scalability
has to be shown

google
65 qubits



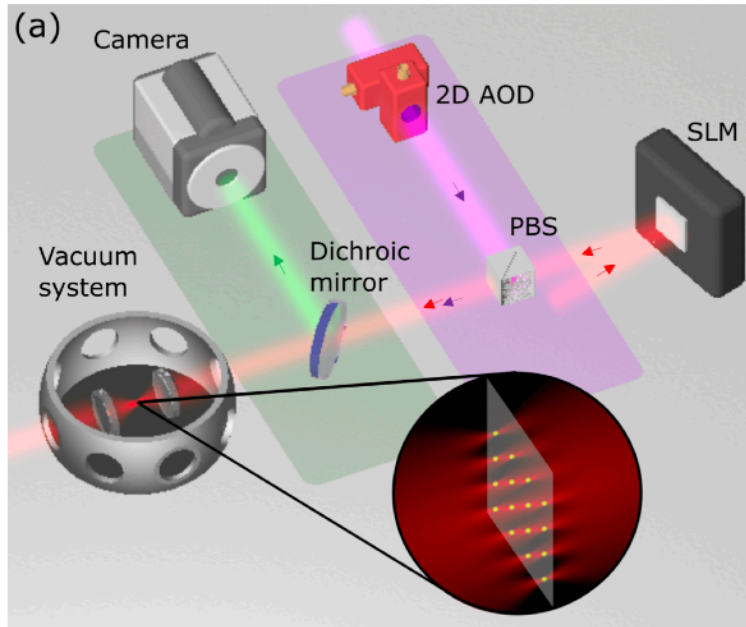
RIKEN
256 qubits



• ...

Quantum Computation using Neutral Atoms: Overview

[Hen20]



SLM: Spatial Light Modulator

AOD: Acousto-Optic Deflector

PBS: Polarizing Beam Splitter

^{87}Rb neutral atoms
in an optical lattice

$$10^6 \text{ atoms/mm}^3 = 1 / 10 \mu\text{m}^3$$

0 or 1 atoms in an optical lattice site

optical tweezer

spot size $\sim 1 \mu\text{m}$

moving success rate $> 99\%$

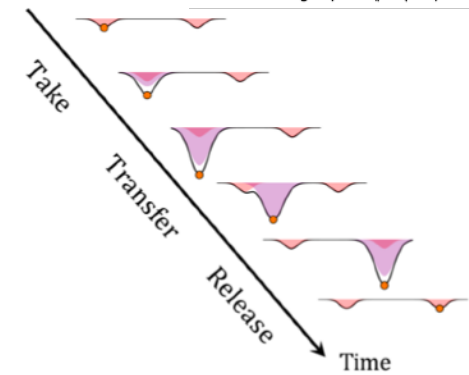
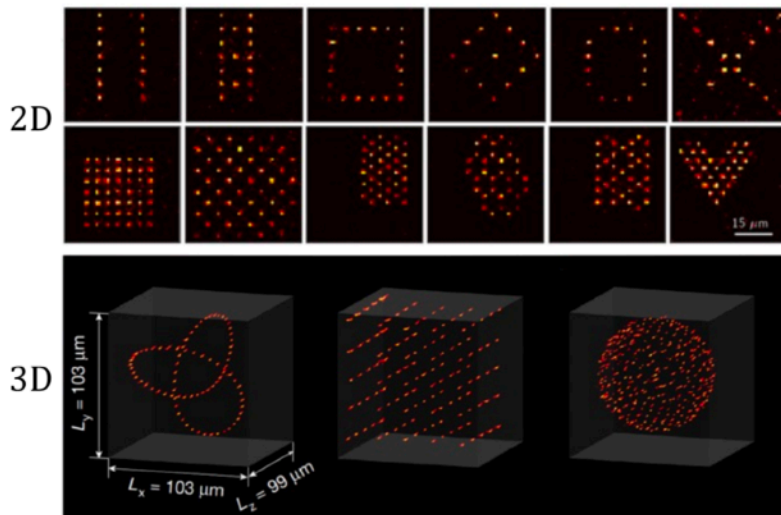
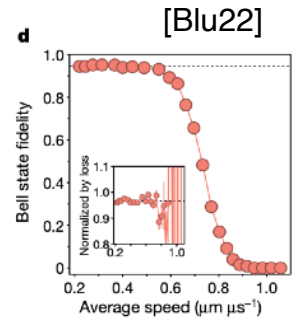
velocity $< 0.55 \mu\text{m}/\mu\text{s}$
(preventing trap loss)

[Win23]

typical distance of traps $\sim 3 \mu\text{m}$

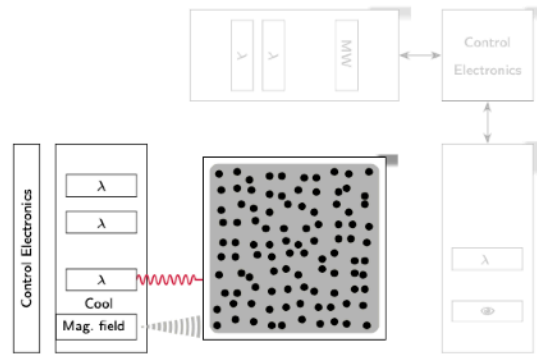
trap lifetime $\sim 10\text{-}20 \text{ sec}$

(limited by the residual vacuum pressure)

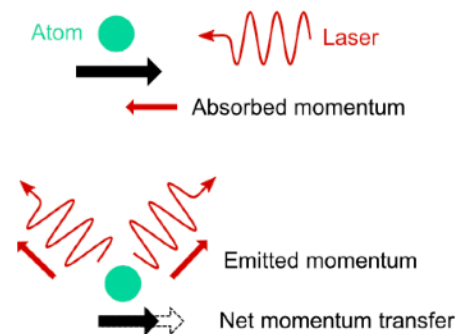


Quantum Computation using Neutral Atoms: Overview

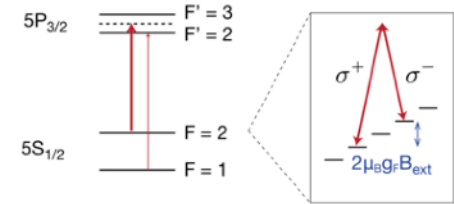
[Win23]



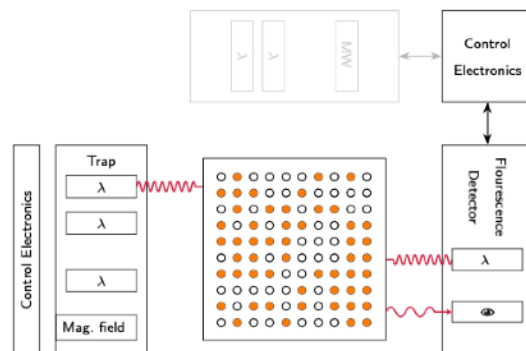
(a) Pre-cool atoms via Doppler cooling.



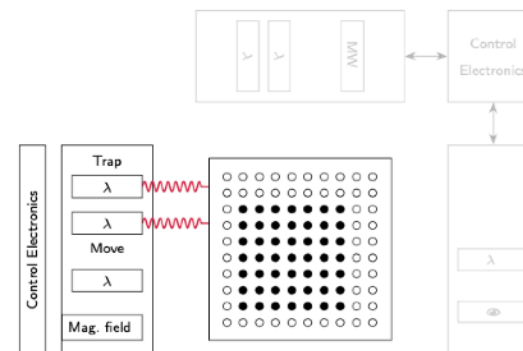
(b) Details of Doppler cooling.



cooled to a few μK

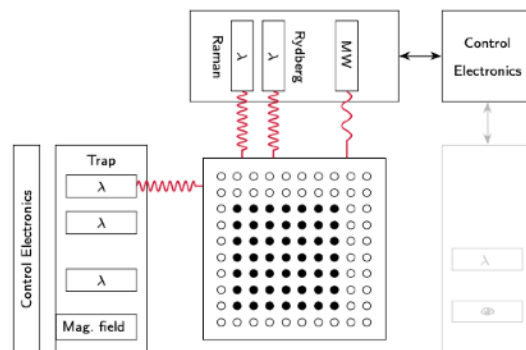


(c) Trap optically; detect filled lattice positions.

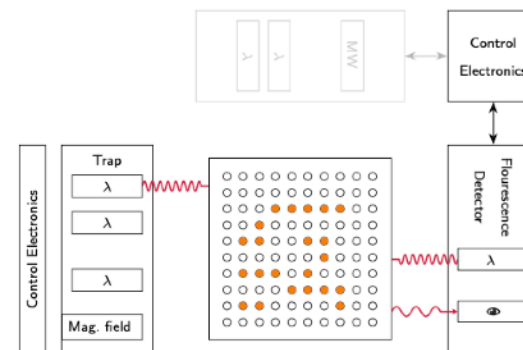


(d) Perform sub-register re-arranging.

rearranged in the lattice



(e) Apply computational gate sequence.



(f) Read results via fluorescence imaging.

Realization of a Quantum Computer

has been criticized in various ways e.g.

[Fuj20]



Rolf Landauer@IBM
(1929-1999)

information is physical

$$Q \geq k_B T \ln 2$$

Landauer was skeptical of new computer technologies, noting that most proposals gravely underestimated the difficulty in constructing a viable device. Over the past 30 years, he wrote a series of articles pointing out deficiencies in various alternative computing proposals, including quantum computation. When a researcher failed to pay adequate attention to his published admonitions, he would warn them in person.

For example, he suggested that all papers on quantum computing should carry a footnote: “This proposal, like all proposals for quantum computation, relies on speculative technology, does not in its current form take into account all possible sources of noise, unreliability and manufacturing error, and probably will not work.”

quoted from Seth Lloyd, [Nature 400, 720 \(1999\)](#)

例えば、彼(R. Landauer)は量子コンピューターに関する全ての論文について、以下のような脚注を入れるべきだと主張した。

「この提案は、量子コンピュータに関する他のすべての提案と同様に、実現性が未確立な技術を前提としている。さらに現在の理論設計では、現実の物理系に存在する多様なノイズ、不完全性、製造誤差が十分に考慮されておらず、おそらく実際に期待どおりには動作しないであろう。」

Although the statement was exaggerated, it criticized the important weak point of the quantum computer development.

Google 2023&2025 are a demonstration to answer to a part of the criticism by R. Landauer.

Fault Tolerant Quantum Computation

Steps

Quantum Error Correction (QEC) of a physical qubit



Google (2023-24)

sub-threshold scalability
of QEC protected memory

QEC protected

Encoding Information in n -qubits



logical qubits
stabilizer
surface code

Universal Computing



logical operators
magic state distillation/injection
braiding/lattice surgery

Scaling for Practical Usage

sub-threshold
concatenation
code distance

Creativity with #Qubit Mass Power

Qubit

qubit

bit = classical bit

A bit can take a value of either 0 or 1 (deterministic).

qubit = quantum bit

A qubit can take (complex) linear combination of $|0\rangle$ and $|1\rangle$.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (\text{a pure state})$$

α, β : complex numbers

- normalization: $|\alpha|^2 + |\beta|^2 = 1$
- The global phase does not have observable effect.

$$\begin{aligned} |+\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}} & |-\rangle &= \frac{|0\rangle - |1\rangle}{\sqrt{2}} \\ |+i\rangle &= \frac{|0\rangle + i|1\rangle}{\sqrt{2}} & |-i\rangle &= \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \end{aligned}$$

Bloch Sphere

Bloch sphere: qubit states mapped onto an $r = 1$ spherical surface

[Fuj15]

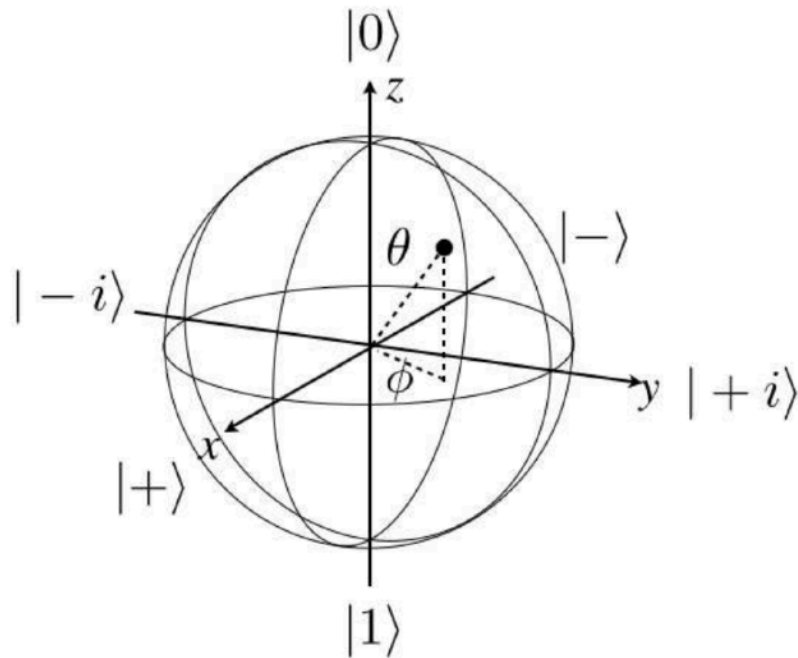


Figure 1.1: The Bloch sphere.

$$\begin{aligned} |\psi\rangle &= \alpha|0\rangle + \beta|1\rangle \\ &= \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \\ &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\ &= (\langle X\rangle, \langle Y\rangle, \langle Z\rangle) \end{aligned}$$

θ : relative amplitude between the $|0\rangle$ and $|1\rangle$ components

ϕ : relative phase between the $|0\rangle$ and $|1\rangle$ components

A qubit state is mathematically analogous to the polarization vector of a $1/2$ spin system (with taking the phase $|0\rangle$ as zero).

Density Matrix

density matrix: a more general description of quantum states
(instead of state kets)

For one qubit quantum states

1 qubit pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

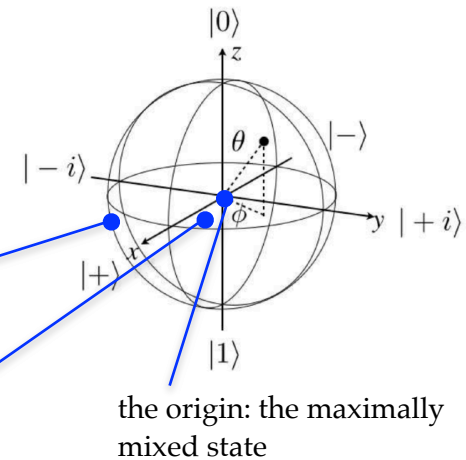
can be expressed by a ket for an appropriate basis

mapped to a point **on** the Bloch Sphere

1 qubit mixture state

= (classical) probabilistic mixture of states
(statistical ensemble of states)

mapped to a point **in** the Bloch Sphere



$$\rho = \frac{1}{2} (I + \mathbf{b} \cdot \boldsymbol{\sigma})$$

$\mathbf{b}$: Bloch vector

like a spin 1/2 polarization vector

For more general quantum states

$$\rho \equiv \sum_i p_j |\psi_j\rangle \langle \psi_j| \quad \text{for an ensemble } \{p_j, |\psi_j\rangle\}_j$$

[Nak24]

A density operator uniquely specifies a quantum state,
but its ensemble decomposition is generally not unique.

Density Matrix

[Nak24]
[Ish12]

Density Matrix (=Density Operator)

is defined as a Hermitian operator $\rho \in \mathcal{B}(\mathcal{H})$ that satisfies $\rho \geq 0$ and $\text{Tr}[\rho] = 1$.

$\rho \geq 0$: positive semidefinite

$$\langle v | \rho | v \rangle \geq 0 \text{ for } \forall v \in \mathcal{H}$$

$\text{Tr}[\rho] = 1$: normalization of total probability

Expectation value: physical observation under the operator A (Born rule on probability)

$$\langle A \rangle = \text{Tr} [A\rho]$$

Purity: $\text{Tr} [\rho^2]$

$$\frac{1}{d} \leq \text{Tr} [\rho^2] \leq 1 \quad d = \dim(\mathcal{H}): \text{dimension}$$

- The left equality holds for the completely mixed state
- The right equality holds for a pure state

pure state:

$$\Leftrightarrow \text{Tr} [\rho^2] = 1$$

$$\Leftrightarrow \rho^2 = \rho$$

$\mathcal{H}$: Hilbert space

$\mathcal{B}(\mathcal{H}) = \{A : \mathcal{H} \rightarrow \mathcal{H} \mid A \text{ is linear and bounded} \}$ 17

Density Matrix

[lsh12]

Probabilistic mixture of two density matrices: $\{p, 1 - p; \rho_1, \rho_2\}$

$$\rho := p\rho_1 + (1 - p)\rho_2 \quad (\text{convex combination})$$

^{87}Rb :

natural abundance: 27.83%

half life: 4.9×10^{10} year

g.s. spin-parity: $3/2^-$

Rb

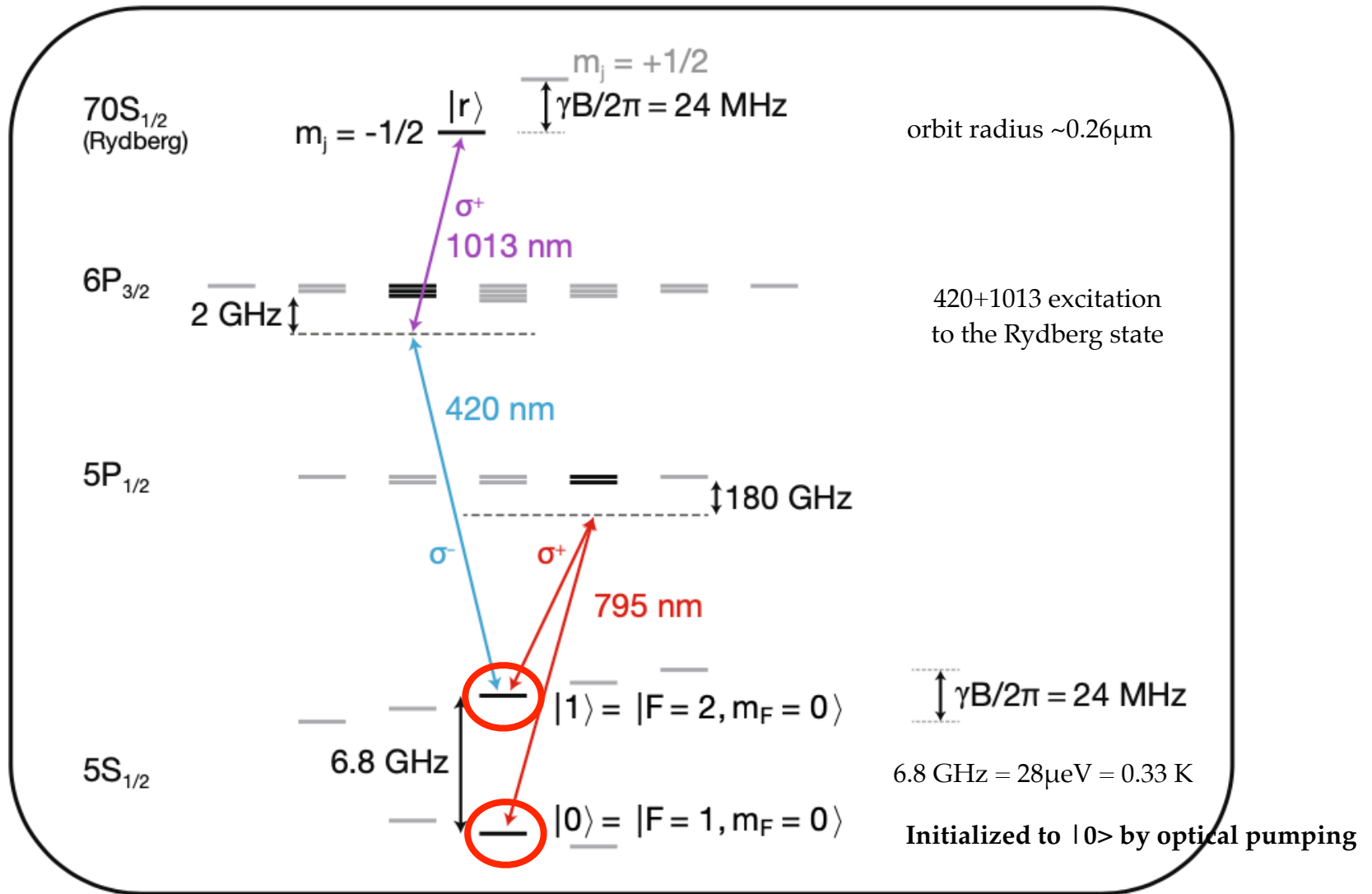
alkali metal

Periodic Table

H 1																	He 2						
Li 3	Be 4																	B 5	C 6	N 7	O 8	F 9	Ne 10
Na 11	Mg 12																	Al 13	Si 14	P 15	S 16	Cl 17	Ar 18
K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36						
Rb 37	Sr 38	Y 39	Zr 40	Nb 41	Mo 42	Tc 43	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50	Sb 51	Te 52	I 53	Xe 54						
Cs 55	Ba 56	La 57	Hf 72	Ta 73	W 74	Re 75	Os 76	Ir 77	Pt 78	Au 79	Hg 80	Tl 81	Pb 82	Bi 83	Po 84	At 85	Rn 86						
Fr 87	Ra 88	Ac 89	Rf 104	Db 105	Sg 106	Bh 107	Hs 108	Mt 109	Ds 110	Rg 111	Cn 112	Nh 113	Fl 114	Mc 115	Lv 116	Ts 117	Og 118						
La系		Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71								
Ac系		Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	No 102	Lr 103								

^{87}Rb Atom Energy Diagram

[Blu22]



^{87}Rb neutral atom Energy Diagram

1-Qubit Operations

1-qubit Operators

Pauli Operators: P

$$I \equiv \sigma_0 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$X \equiv \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y \equiv \sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z \equiv \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X^2 = Y^2 = Z^2 = I$$

$$[\sigma_i, \sigma_j] = 2i\epsilon_{i,j,k}\sigma_k \quad (i, j, k = 1, 2, 3)$$

$$\text{Hermitian: } P^\dagger = P$$

$$\text{Unitary: } P^\dagger P = I$$

1-qubit Operators

Hadamard (H)

$$H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hermitian: $H^\dagger = H$

Unitary: $H^\dagger H = I$

$$HXH^\dagger = Z$$

$$HZH^\dagger = X$$

Phase (S)

$$S \equiv \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Unitary: $S^\dagger S = I$

$$S^2 = Z$$

$$SXS^\dagger = Y$$

$$SYS^\dagger = -X$$

$$SZS^\dagger = Z$$

T ($\pi/8$) matrix (T)

$$T \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

X Operator

$$X \equiv \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Hermitian: } X^\dagger = X$$

$$\text{Unitary: } X^\dagger X = I$$

$$\text{Eigenvalues: } \pm 1$$

$$X|+\rangle = |+\rangle$$

$$X|-\rangle = -|-\rangle \quad : X \text{ eigenstates}$$

$$X|0\rangle = |1\rangle$$

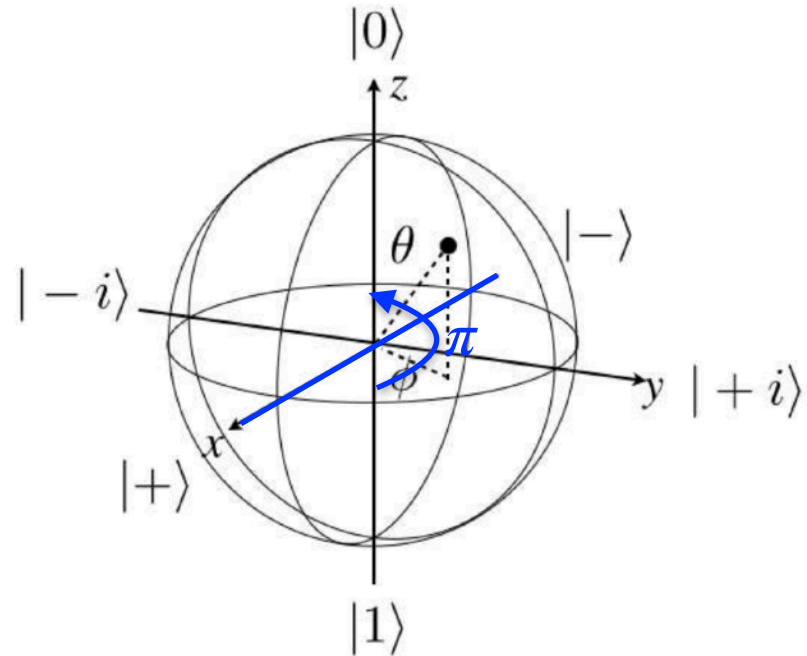
$$X|1\rangle = |0\rangle \quad : \text{bit-flip operator}$$

$$X|+i\rangle = i| -i\rangle$$

$$X| -i\rangle = -i| +i\rangle$$

$$-iX = R_x(\pi) = e^{-\frac{i\pi}{2}X}$$

$$X = HZH^\dagger = -SYS^\dagger = \frac{1}{2i}[Y, Z]$$



Z Operator

$$Z \equiv \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{Hermitian: } Z^\dagger = Z$$

$$\text{Unitary: } Z^\dagger Z = I$$

$$\text{Eigenvalues: } \pm 1$$

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

: Z eigenstates

$$Z|+\rangle = |-\rangle$$

$$Z|-\rangle = |+\rangle$$

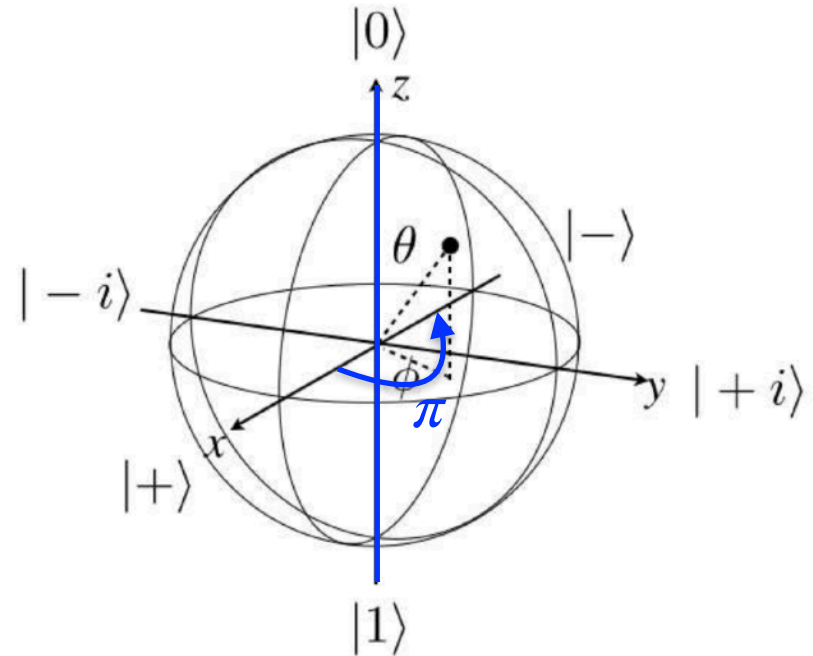
$$Z|+i\rangle = |-i\rangle$$

$$Z|-i\rangle = |+i\rangle$$

: phase-flip operator

$$-iZ = R_z(\pi) = e^{-\frac{i\pi}{2}Z}$$

$$Z = HXH^\dagger = SZS^\dagger = \frac{1}{2i}[X, Y] = S^2 = T^4$$



Hadamard Operator

$$H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{Hermitian: } H^\dagger = H$$

$$\text{Unitary: } H^\dagger H = I$$

$$\text{Eigenvalues: } \pm 1$$

$$H|0\rangle = |+\rangle$$

$$H|1\rangle = |-\rangle \quad \text{action on } X \text{ eigenstates}$$

$$H|+\rangle = |0\rangle \quad \text{action on } Z \text{ eigenstates}$$

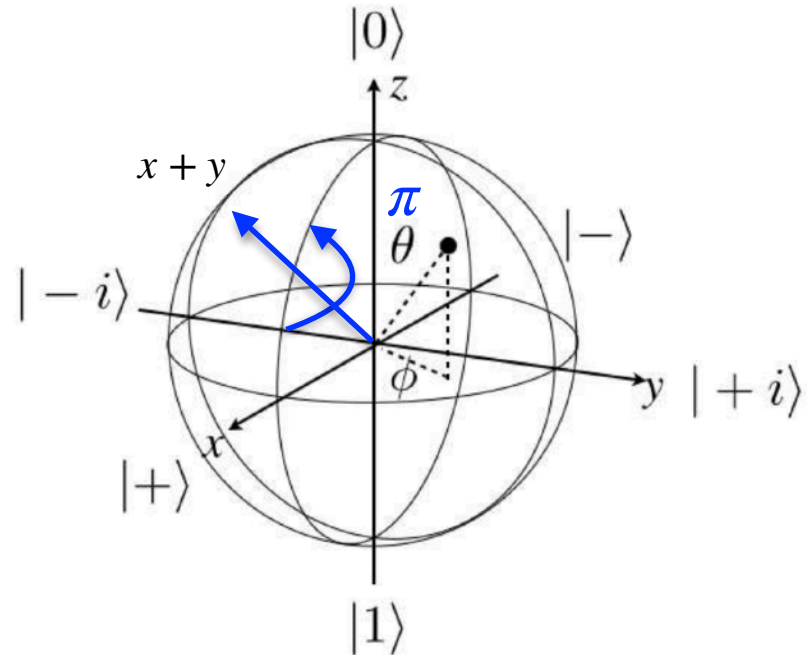
$$H|-\rangle = |1\rangle$$

$$H|+i\rangle = e^{\frac{i\pi}{4}}|-i\rangle$$

$$H|-i\rangle = e^{-\frac{i\pi}{4}}|+i\rangle$$

$$-iH = R_{\frac{(X+Y)}{\sqrt{2}}}(\pi) = e^{-\frac{i\pi}{2} \frac{(X+Y)}{\sqrt{2}}}$$

$$H = \frac{1}{\sqrt{2}}(X + Z)$$



exchange the x and z axes
(with flipping the y direction)

T ($\pi/8$) Operator

$$T \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Unitary: $T^\dagger T = I$

$$T|0\rangle = |0\rangle$$

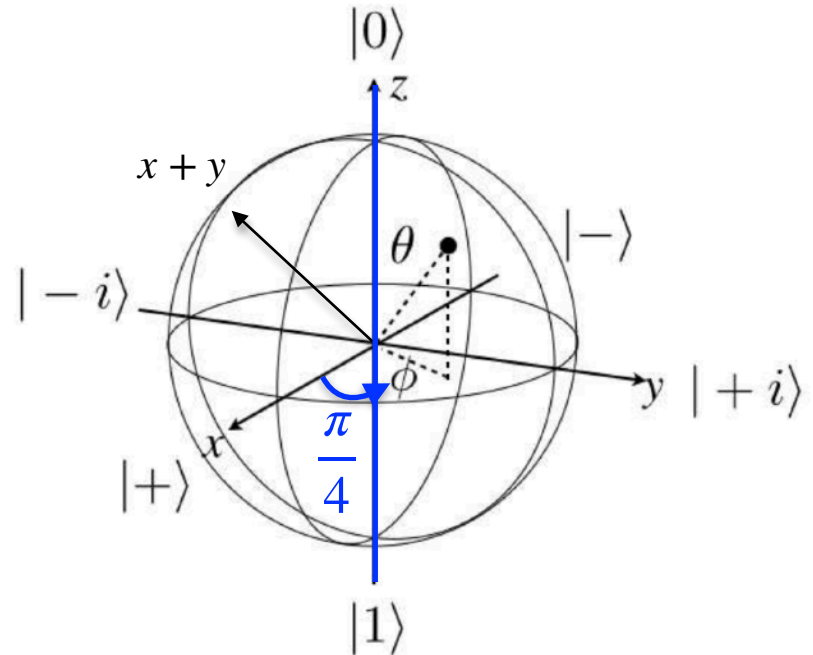
$$T|1\rangle = e^{\frac{i\pi}{4}}|1\rangle$$

$$TXT^\dagger = \frac{X+Y}{\sqrt{2}}$$

$$TYT^\dagger = \frac{Y-X}{\sqrt{2}}$$

$$TZT^\dagger = Z$$

$$T = e^{\frac{i\pi}{8}} R_z\left(\frac{\pi}{4}\right) = e^{-\frac{i\pi}{8}} Z \quad T^2 = S \quad T^4 = Z$$



1-qubit Rotation Operator

All the 1-qubit operations (unitary) can be expressed as a rotation of the Bloch sphere

Rotation around the coordinate axes

$$R_x(\theta) = e^{-\frac{i\theta}{2}X}$$

$$R_y(\theta) = e^{-\frac{i\theta}{2}Y}$$

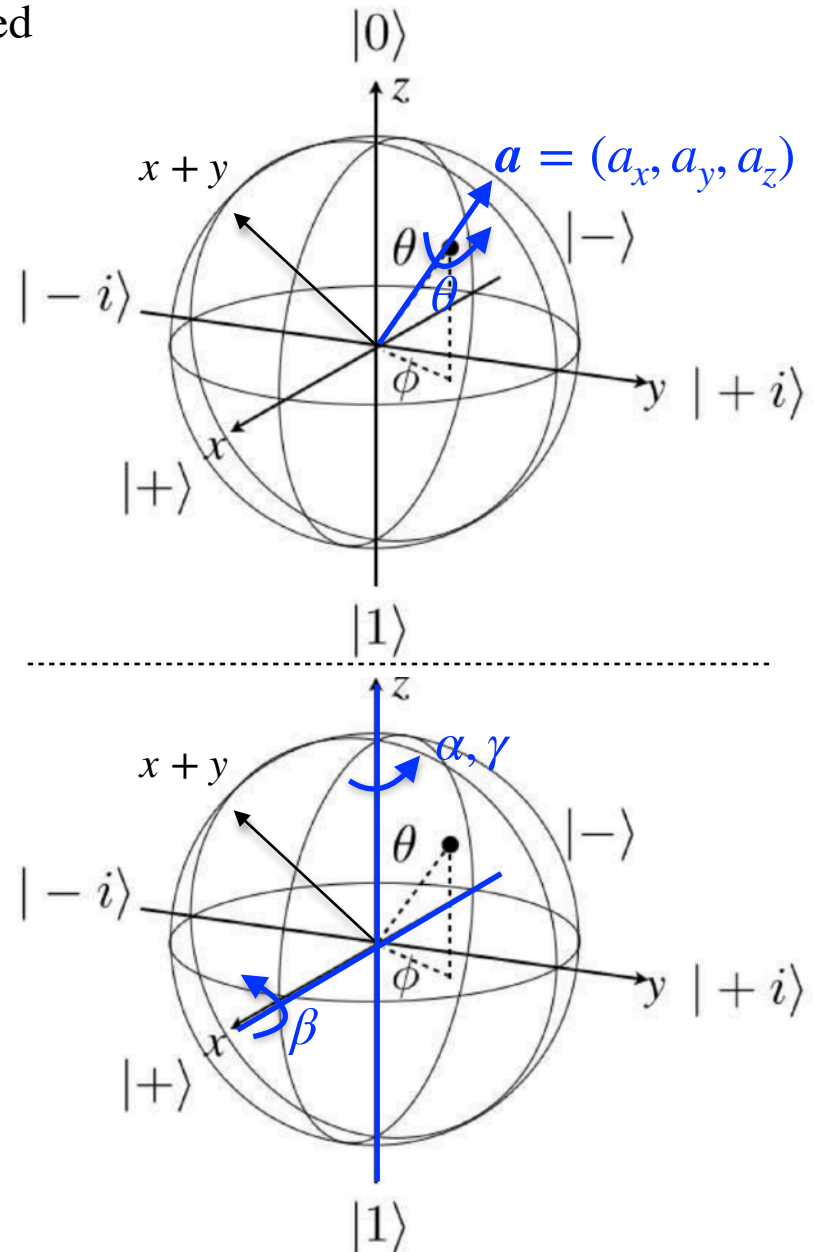
$$R_z(\theta) = e^{-\frac{i\theta}{2}Z}$$

Rotation around the $\mathbf{a} = (a_x, a_y, a_z)$ axis

$$R_{\mathbf{a}}(\theta) = e^{-\frac{i\theta}{2}(a_x X + a_y Y + a_z Z)}$$

Rotation with Euler angles

$$U = e^{i\phi} e^{-i\frac{\alpha}{2}Z} e^{-i\frac{\beta}{2}X} e^{-i\frac{\gamma}{2}Z}$$



1-qubit Basis

Z-basis states

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

$$X|0\rangle = |1\rangle$$

$$X|1\rangle = |0\rangle$$

in Z basis

Z: **phase-flip** operator

X: **bit-flip** operator

X-basis states

$$|+\rangle \equiv \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-\rangle \equiv \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$X|+\rangle = |+\rangle$$

$$X|-\rangle = -|-\rangle$$

$$Z|+\rangle = |-\rangle$$

$$Z|-\rangle = |+\rangle$$

in X basis

X: phase-flip operator

Z: bit-flip operator

conversion by H

$$H|0\rangle = |+\rangle$$

$$H|1\rangle = |-\rangle$$

$$H|+\rangle = |0\rangle$$

$$H|-\rangle = |1\rangle$$

1 qubit Operators: diagrams

$$|0\rangle \text{ --- } [Z] \text{ --- } |0\rangle$$

$$|1\rangle \text{ --- } [Z] \text{ --- } |1\rangle$$

$$|0\rangle \text{ --- } [X] \text{ --- } |1\rangle$$

$$|1\rangle \text{ --- } [X] \text{ --- } |0\rangle$$

$$|0\rangle \text{ --- } [H] \text{ --- } |+\rangle \text{ --- } [H] \text{ --- } |0\rangle$$

$$|1\rangle \text{ --- } [H] \text{ --- } |-\rangle \text{ --- } [H] \text{ --- } |1\rangle$$

$$\text{---} [S] \text{---} [S] \text{---} = \text{---} [Z] \text{---}$$

$$|\psi\rangle \text{ --- } [X] \text{---} [Y] \text{---} [Z] \text{---}$$

diagram: from left to right

$$\xleftarrow{ZYX} |\psi\rangle$$

operators: from right to left



Visit IBM Quantum Platform:

<https://quantum.cloud.ibm.com/>

for designing quantum diagrams and simulations with/without using a quantum computer

Neutral Atom: Rotation of a qubit in the Bloch Sphere

Raman transition by optical laser field (created by an overlap of two lasers) [Hen20]

rotation around the axis $\omega = (\Omega \cos \varphi, \Omega \sin \varphi, \delta)$ with the rotation vector $\tau \omega$ in the Bloch sphere

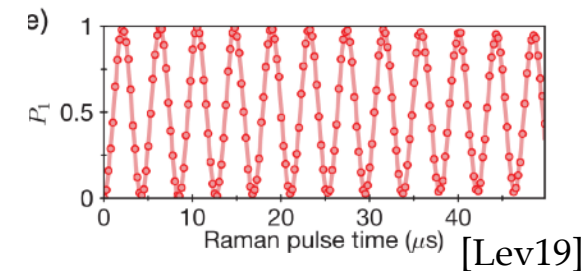
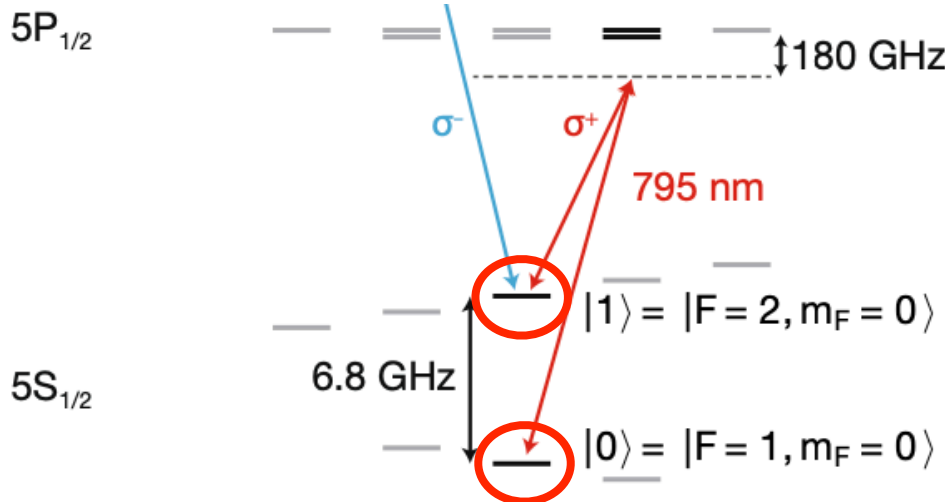
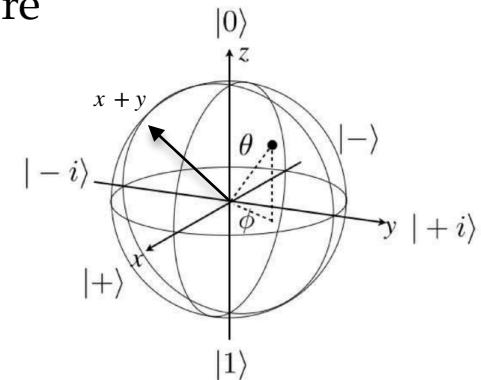
$$H = \frac{\hbar}{2} (\Omega \cos \phi X + \Omega \sin \phi Y + \delta Z)$$

φ : relative phase between the two lasers

Ω : Rabi frequency (population oscillation between $|0\rangle$ and $|1\rangle \propto \sqrt{\text{laser intensity}}$)

τ : laser pulse duration

δ : laser frequency detuning (from the resonance)



$$6.8 \text{ GHz} = 28 \mu\text{eV} = 0.33 \text{ K}$$

Initialized to $|0\rangle$ by optical pumping

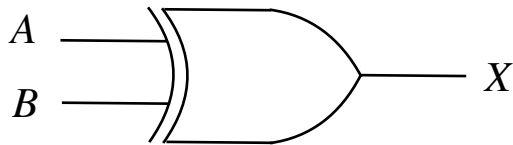
^{87}Rb atom

2- (multiple-) Qubit Operations

Two qubit operation

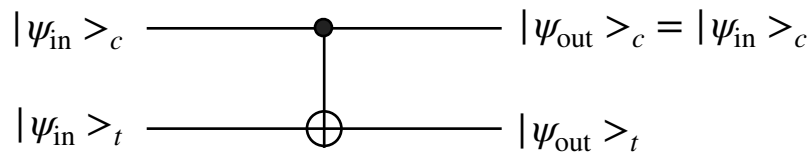
Exclusive OR (XOR)

$$X = A \oplus B$$



<i>A</i>	<i>B</i>	<i>X</i>
0	0	0
0	1	1
1	0	1
1	1	0

Controlled NOT (**CNOT**): bit-flip the target if *c* is $|1\rangle$



in	out
$ 00\rangle$	$ 00\rangle$
$ 01\rangle$	$ 01\rangle$
$ 10\rangle$	$ 11\rangle$
$ 11\rangle$	$ 10\rangle$
<i>c t</i>	<i>c t</i>

if $|\psi_{\text{in}}\rangle_c = |0\rangle_c$ then $|\psi_{\text{out}}\rangle_t = |\psi_{\text{in}}\rangle_t$

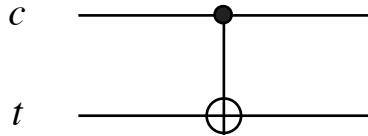
if $|\psi_{\text{in}}\rangle_c = |1\rangle_c$ then $|\psi_{\text{out}}\rangle_t = |1\rangle_t$ for $|\psi_{\text{in}}\rangle_t = |0\rangle_t$

$|\psi_{\text{out}}\rangle_t = |0\rangle_t$ for $|\psi_{\text{in}}\rangle_t = |1\rangle_t$

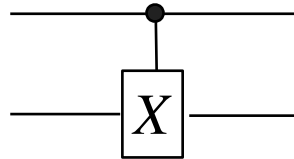
2 qubit Operators

Controlled-NOT (CNOT)

= Controlled-X (CX) = $\Lambda(X)$



=

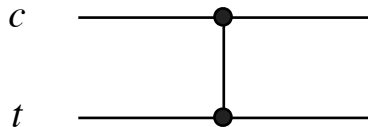


$$\Lambda_{c,t}(X) \equiv |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c X_t$$

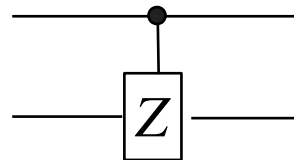
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Controlled-Z (CZ)

= $\Lambda(Z)$



=

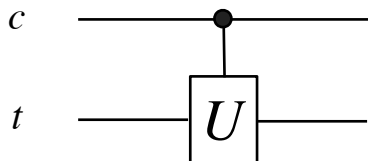


$$\Lambda_{c,t}(Z) \equiv |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c Z_t$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Controlled-U (any unitary operator)

= $\Lambda(U)$

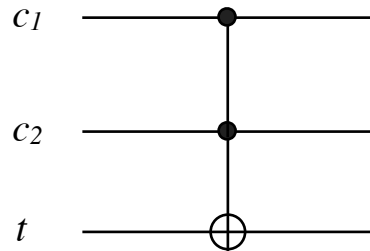


$$\Lambda_{c,t}(U) \equiv |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c U_t$$

$$= \begin{bmatrix} I_t & 0 \\ 0 & U_t \end{bmatrix}$$

3 qubit Operators

Toffoli gate (Controlled-Controlled-NOT) $= \Lambda^2(X)$

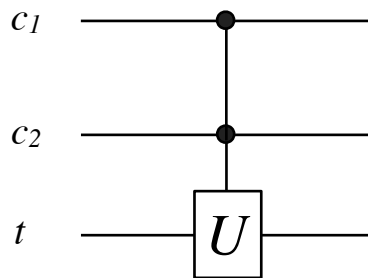


$$\Lambda_{c_1, c_2, t}^2(X) \equiv \left(I_{c_1} I_{c_2} - |1\rangle\langle 1|_{c_1} |1\rangle\langle 1|_{c_2} \right) I_t$$

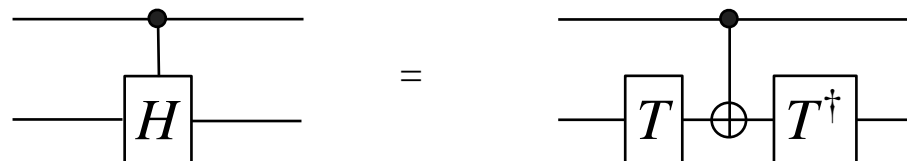
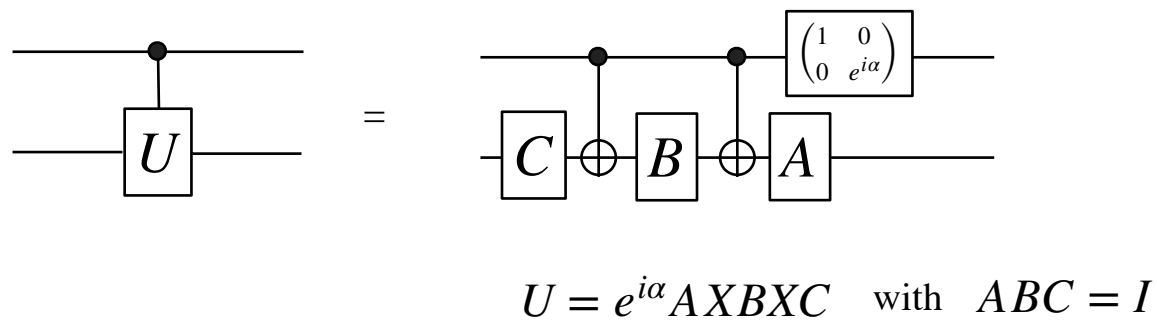
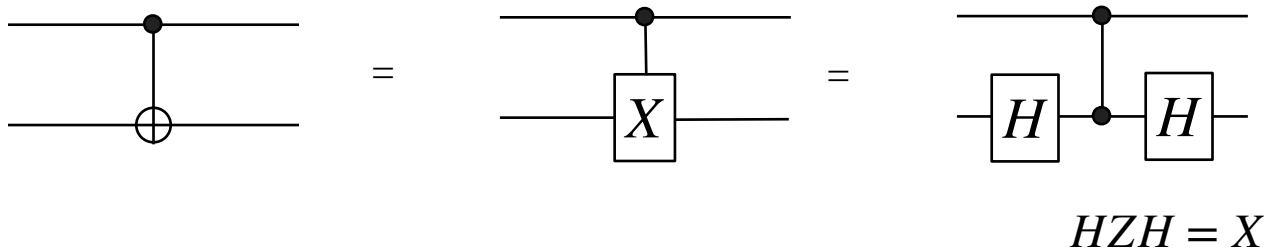
$$+ |1\rangle\langle 1|_{c_1} |1\rangle\langle 1|_{c_2} X_t |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c X_t$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Controlled-Controlled-Unitary $= \Lambda^2(U)$

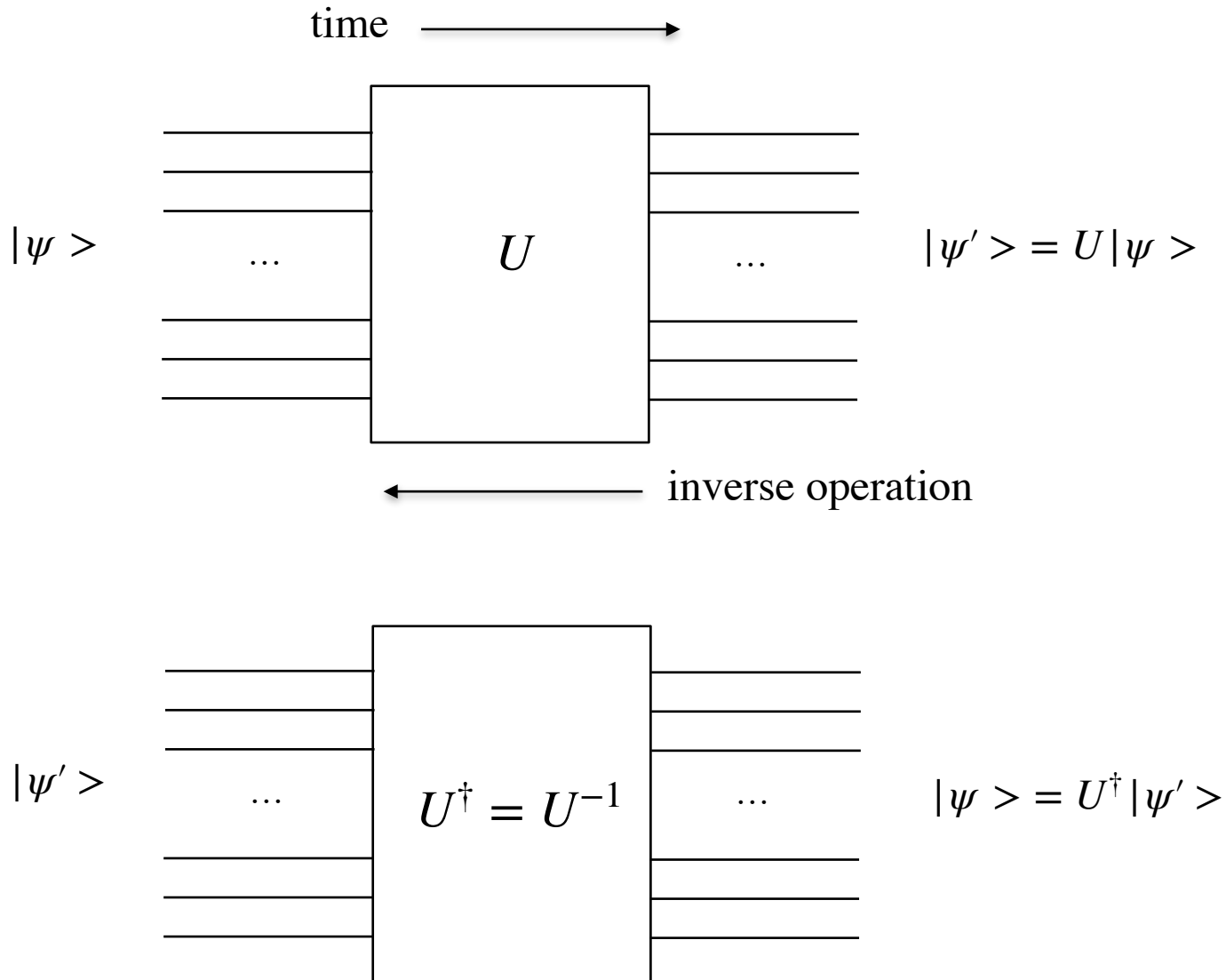


Qubit Diagram Expansions



Quantum Gates are a Unitary Process

Quantum gates are, in a closed system, a unitary process and therefore reversible.

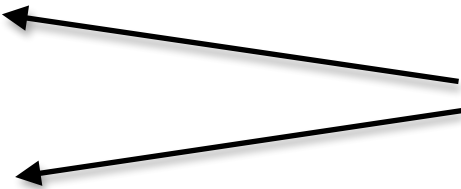


Universal Quantum Gates

Any unitary operation can be approximated to arbitrary accuracy using

[Nie00]

H
CNOT
 T

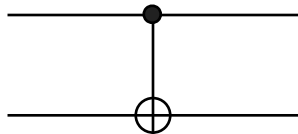


implemented as single qubit rotation

= Universal Quantum Gates

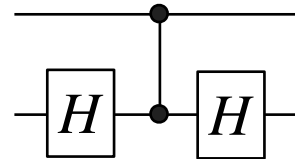
CNOT can be implemented with CZ and H

CNOT

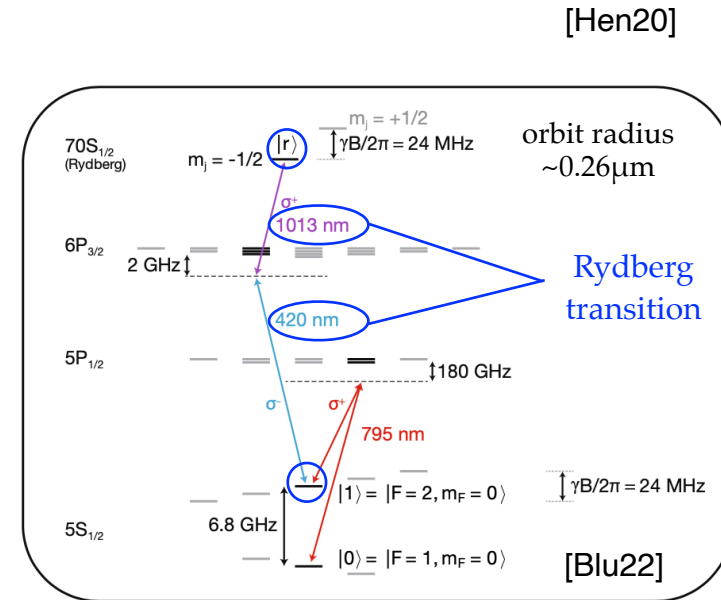
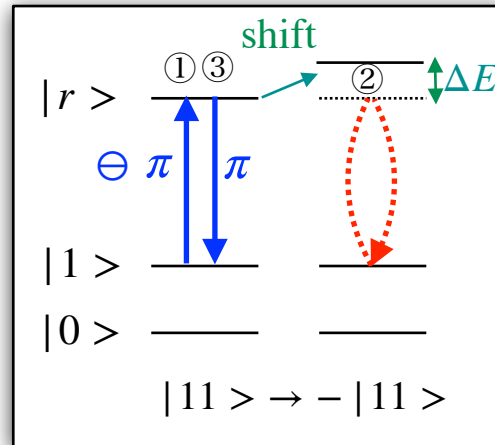
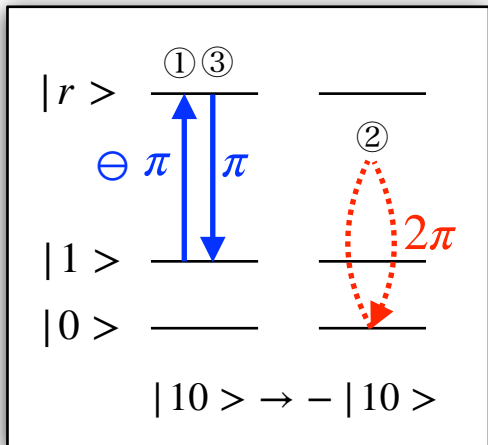
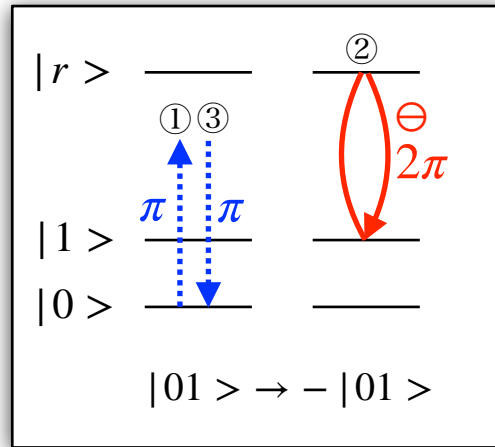
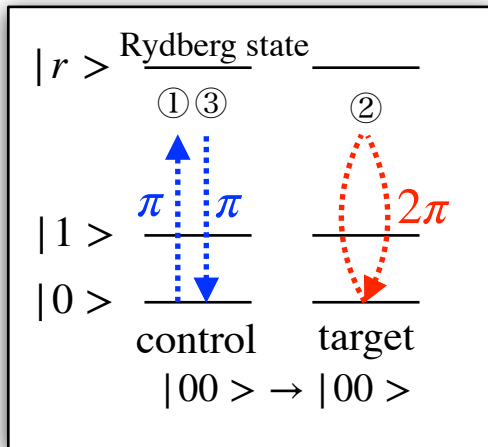


=

H, CZ, H

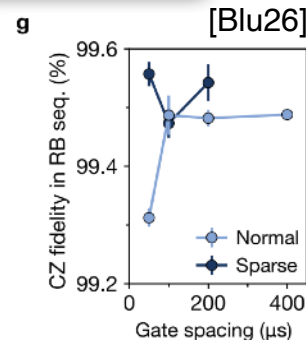


Implementation of CZ using Rydberg interaction



Highest fidelity ~ 0.995

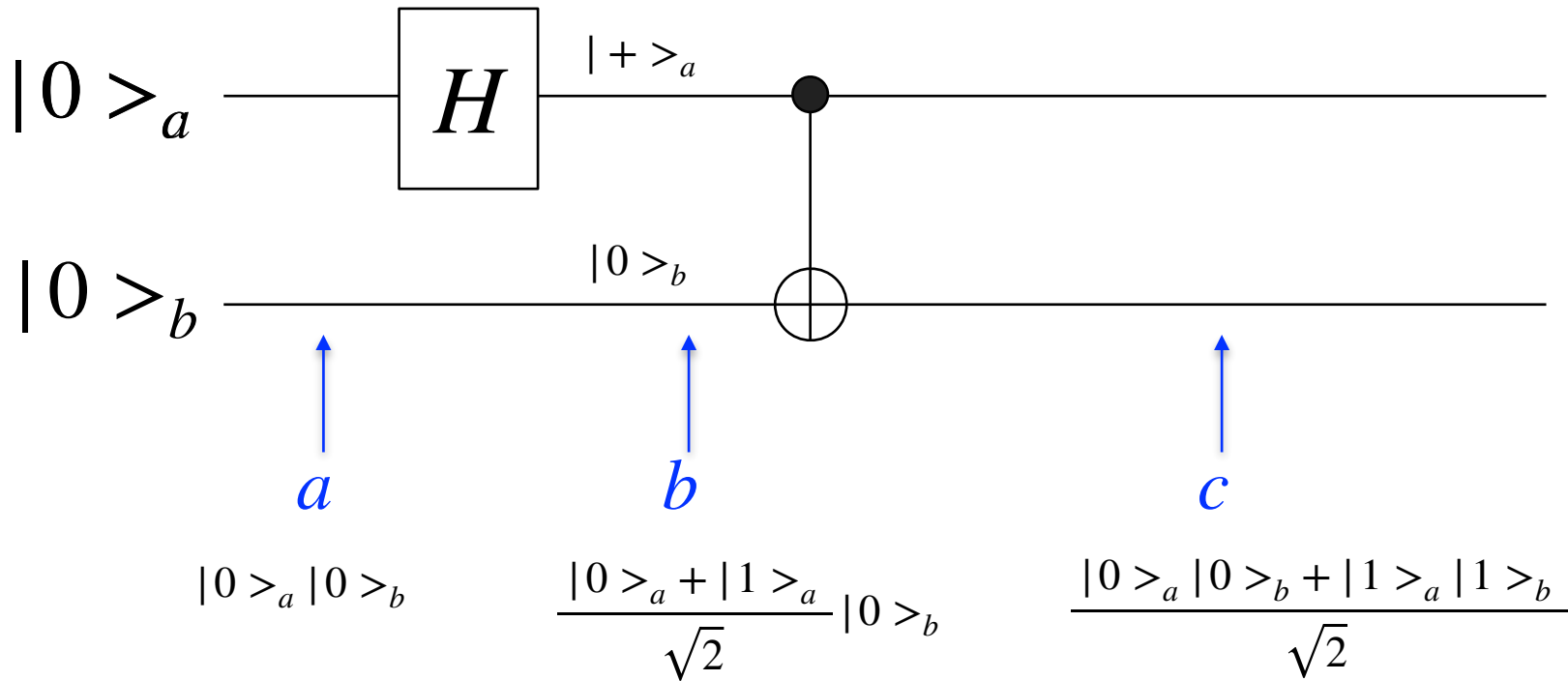
$$\overline{CZ} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = e^{i\pi} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = e^{i\pi} CZ$$



$\overline{CZ}$		CZ	
in	out	in	out
$ 00\rangle$	$- 00\rangle$	$ 00\rangle$	$ 00\rangle$
$ 01\rangle$	$- 01\rangle$	$ 01\rangle$	$ 01\rangle$
$ 10\rangle$	$- 10\rangle$	$ 10\rangle$	$ 10\rangle$
$ 11\rangle$	$ 11\rangle$	$ 11\rangle$	$- 11\rangle$

Entanglement

Bell State Generation



$$|0\rangle_a |0\rangle_b$$

$$\frac{|0\rangle_a + |1\rangle_a}{\sqrt{2}} |0\rangle_b$$

$$\frac{|0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b}{\sqrt{2}}$$

$$|00\rangle$$

$$\frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle$$

$$= |+\rangle \otimes |0\rangle$$

$$= |+0\rangle$$

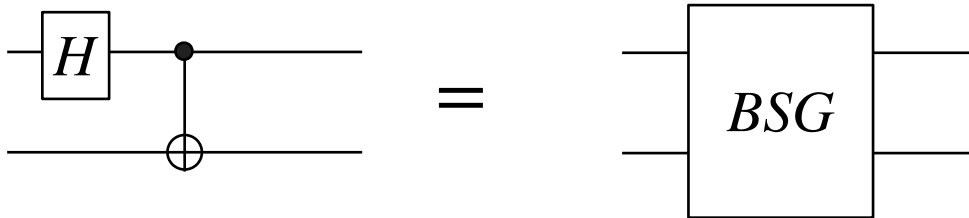
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

= a Bell State

maximally entangled
2-qubit states

Bell State Generator

[ML]



$ 0\rangle$		$\frac{ 00\rangle + 11\rangle}{\sqrt{2}}$	Bell States $\equiv \Phi^+\rangle$
$ 0\rangle$			

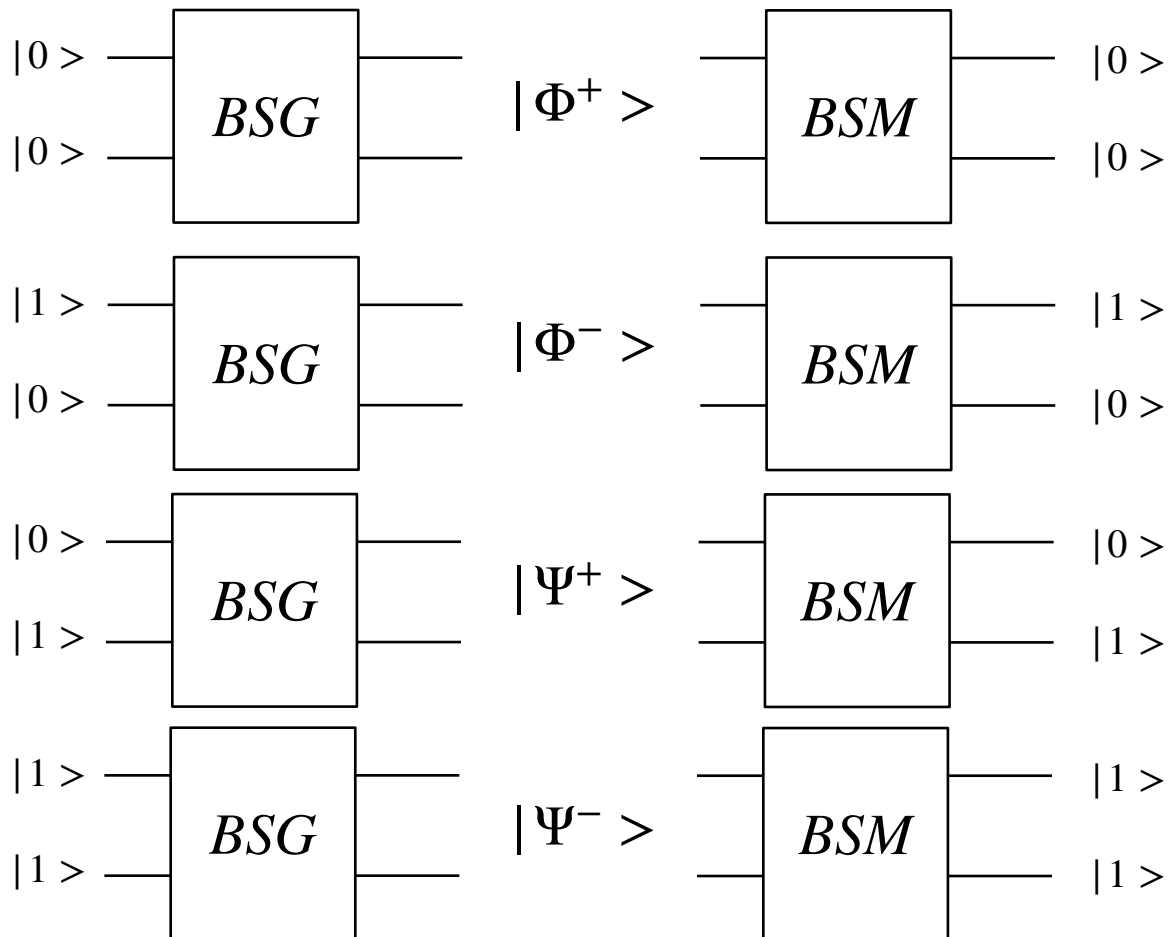
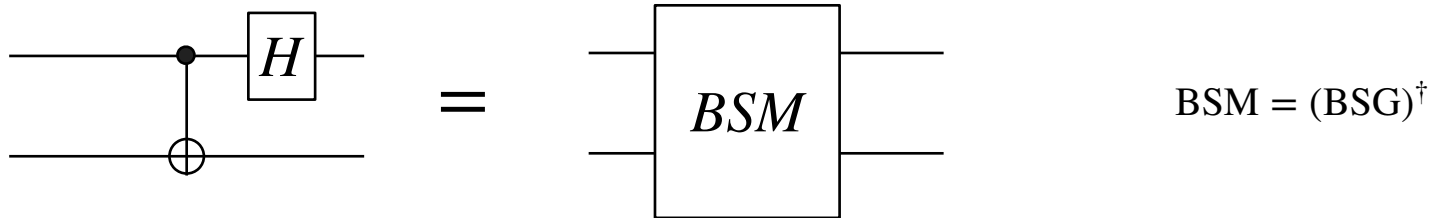
$ 1\rangle$		$\frac{ 00\rangle - 11\rangle}{\sqrt{2}}$	$\equiv \Phi^-\rangle$
$ 0\rangle$			

$ 0\rangle$		$\frac{ 01\rangle + 10\rangle}{\sqrt{2}}$	$\equiv \Psi^+\rangle$
$ 1\rangle$			

$ 1\rangle$		$\frac{ 01\rangle - 10\rangle}{\sqrt{2}}$	$\equiv \Psi^-\rangle$
$ 1\rangle$			

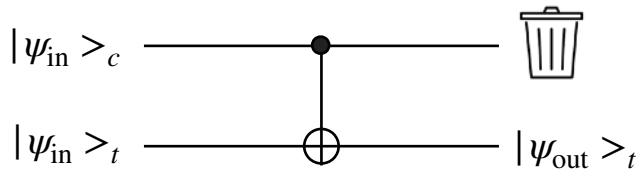
Bell State Measurement

[ML]

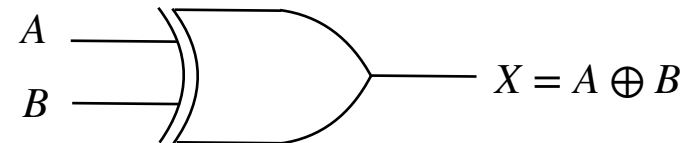


Two qubit operation

Controlled NOT (CNOT) :



Classical Exclusive OR



The control qubit after the operation looks useless. May we throw it away ?

→ No

- Qubit operations are unitary:
 - control qubit contains information to reverse the target qubit back to the initial value.
- Generally the control qubit after operation is entangled with the target qubit after operation.

Actually the control qubit after operation is quite useful.

Time Evolution of a Quantum System

in conventional quantum mechanics

[Ish12]

Time evolution of a closed quantum system is described by a **unitary operator**, and is **reversible**.

Every unitary operator can be regarded as a possible time evolution.

time evolution of a quantum system $\Leftrightarrow$ a unitary operator : 1 to 1 correspondence

Time evolution operator $U_t := \exp(-iHt)$ is a unitary operator.

time evolution of a ket : $|\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle$ $|\psi\rangle = U^\dagger|\psi'\rangle$

time evolution of a density matrix : $\rho \rightarrow \rho' = U\rho U^\dagger$ $\rho = U^\dagger\rho'U$

Measurement (Observation) of the eigenvalue of an **projection operator** P_a is **not reversible**.

$$|\psi\rangle \rightarrow |\phi_a\rangle = \frac{P_a|\psi\rangle}{|P_a|\psi\rangle|} \quad P_a^2 = P_a \quad P_a|\phi_a\rangle = a|\phi_a\rangle \quad \text{"Reduction of Wave Packet"}$$

$$\rho \rightarrow \rho' = \frac{P_a\rho P_a}{\text{Tr}[P_a\rho]} \quad a = \text{Tr}[P_a\rho]$$

$P_a = |\phi_a\rangle\langle\phi_a|$: projection operator to $|\phi_a\rangle$



Generalized Time Evolution

[lsh12]

Time evolution of the closed total system SE is not expressed by a unitary operator.

$$\rho^{SE} \rightarrow U\rho^{SE}U^\dagger$$

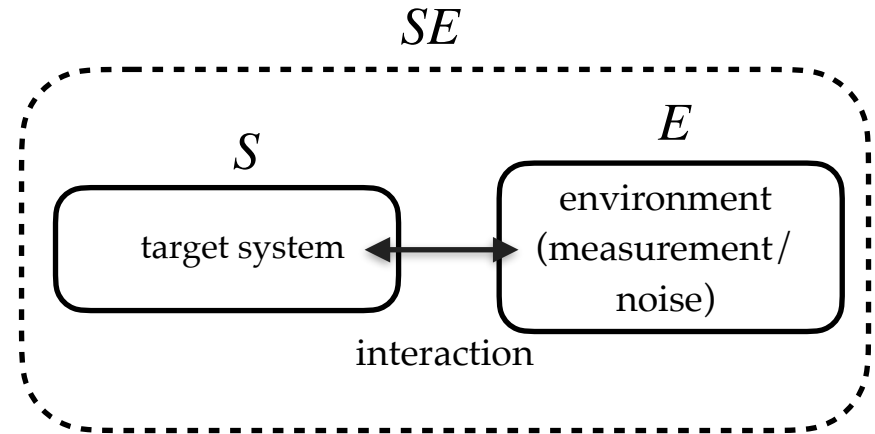
However, if we only focus on a partial space S , the time evolution is not unitary.

$$\rho^S = \text{Tr}_E[\rho^{SE}] \rightarrow \rho'^S = \text{Tr}_E[U\rho^{SE}U^\dagger]$$

Tr_E : partial trace with respect to the system E .

ρ^S : reduced density operator of ρ^{SE}

The map $\Phi : \rho^S \rightarrow \rho'^S = \Phi(\rho^S)$ is a CPTP (Complete Positive Trace Conserving) map.



Completely Positive Trace Conserving (CPTP) map

※ also called TPCP [Ish12]

A CPTP map is a map that satisfies the following three conditions.

Linearity:

$$\Phi(a\rho_1 + b\rho_2) = a\Phi(\rho_1) + b\Phi(\rho_2)$$

Complete Positive: for any ancillary system A

$$(I_A \otimes \Phi)(\rho) \geq 0 \quad \forall \rho \geq 0 \quad \text{probability = positive semidefinite}$$

Trace Preserving:

$$\text{Tr}(\Phi(\rho)) = \text{Tr}(\rho) \quad \text{probability conservation}$$



equivalent

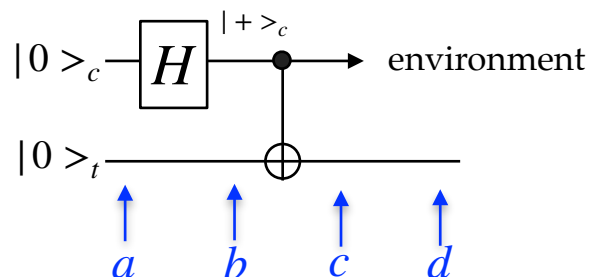
Kraus representation

There exists a set of operators that satisfies

$$\begin{aligned} \Phi(\rho) &= \sum_k E_k \rho E_k^\dagger \\ \sum_k E_k^\dagger E_k &= I \end{aligned}$$

CPTP map: an example

[lsh12]



a	b	c	d
$ 00\rangle_{ct}$	$\frac{ 00\rangle + 10\rangle}{\sqrt{2}}$	$\frac{ 00\rangle + 11\rangle}{\sqrt{2}}$	CPTP map = partial trace
$\rho^{ct} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$	$\rho^t = \text{Tr}_c[\rho^{ct}] = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

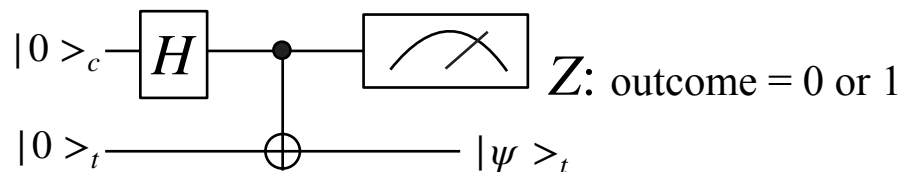
linear combination
of $|0\rangle$ and $|1\rangle$ for c

Bell state

probabilistic mixture of
 $|0\rangle$: 50% and $|1\rangle$: 50%

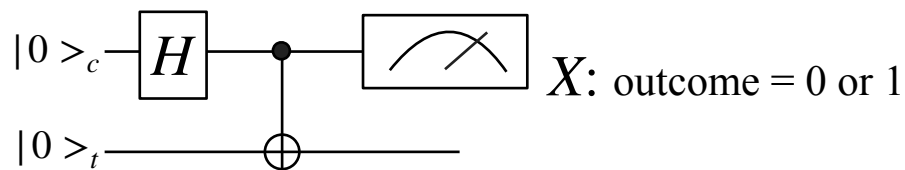
CPTP map: an example

[lsh12]



outcome = 0 (50%) $|\psi\rangle_t = |0\rangle_t$

outcome = 1 (50%) $|\psi\rangle_t = |1\rangle_t$



outcome = 0 (50%) $|\psi\rangle_t = |+\rangle_t$

outcome = 1 (50%) $|\psi\rangle_t = |-\rangle_t$

Generalized Measurement

[Nie00]

Positive Operator Value Measurement (POVM)

Collection of measurement operators $\{M_m\}$ for measurement outcome m

$$E_m = M_m^\dagger M_m$$

$$E_m \geq 0 \quad \Leftrightarrow \quad \langle \psi | E_m | \psi \rangle \geq 0 \quad \forall |\psi\rangle \quad (\text{positive semidefinite})$$

Probability of getting m by the measurement: $p(m)$

$$p(m) = \langle \psi | E_m | \psi \rangle$$

$$p(m) = \text{Tr} [E_m \rho]$$

The state after the measurement:

$$|\psi\rangle \rightarrow |\psi'\rangle = \frac{M_m |\psi\rangle}{\sqrt{\langle \psi | E_m | \psi \rangle}}$$

$$\rho \rightarrow \rho' = \frac{M_m \rho M_m^\dagger}{\text{Tr} [E_m \rho]}$$

Completeness:

$$\sum_m E_m = I \quad \Leftrightarrow \quad \sum_m p(m) = 1$$

Entanglements as Resource

Quantum Communication

Entanglements are resource for communication.

Communication is realized by **consuming entanglements**.

Quantum Computation

Entanglements are resource for computing.

Computation is realized by **consuming entanglements**.

Quantum Communication/Quantum Channel

Entanglement Distribution (ED)

Quantum Teleportation (QT)

Superdense Coding (SDC)

Entanglement Swapping

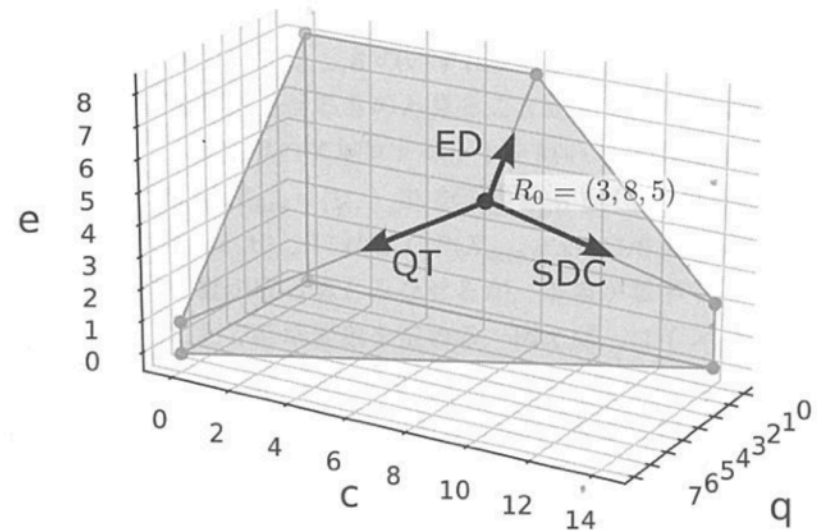


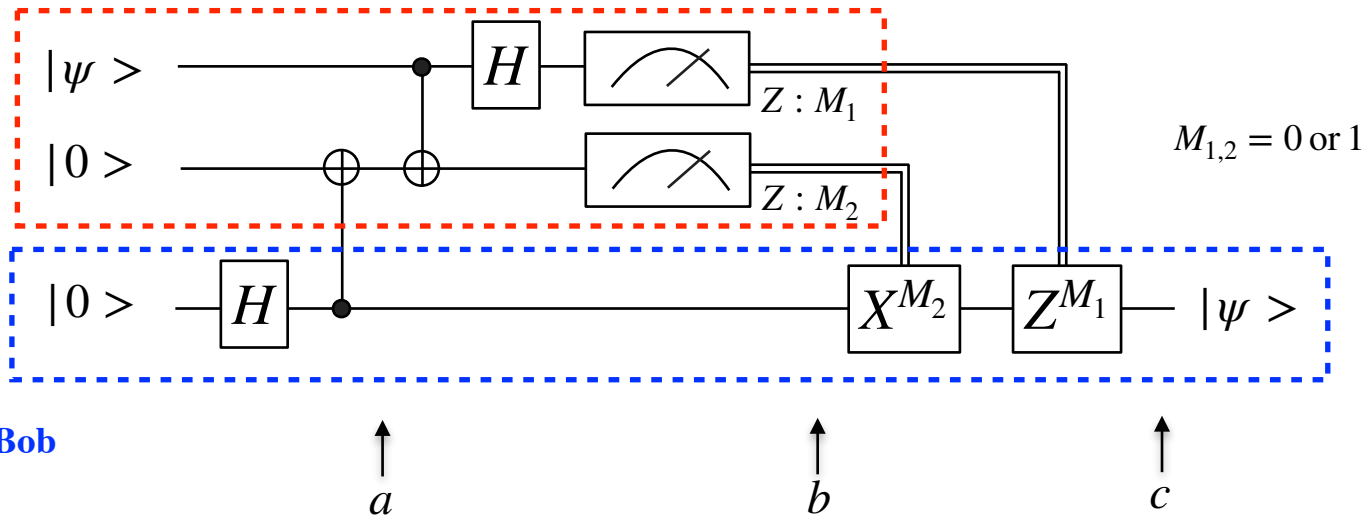
図 5.5 初期リソース $R_0 = (q_0 = 3, c_0 = 8, e_0 = 5)$ から始めて、ノイズレスな通信プロトコルで達成可能なリソース領域。黒の領域の範囲内にある点へのリソース変換は常に可能であり、領域の外にある点へのリソース変換は不可能である。ED はエンタングルメント配布を、SDC は超高密度符号を、QT は量子テレポーテーションを表す。この図を見ると、三つのユニット・プロトコルが特殊であることが一目瞭然だろう。

[Nak24]

Quantum Teleportation

Alice

[Nie00]



Bob

Quantum Teleportation of $|\psi\rangle$ from Alice to Bob.

 : measurement in Z basis

==== : classical bit communication

a: Alice and Bob shares an entanglement pair.

Alice has a qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ without knowing the α and β values.

b: Alice did operation between $|\psi\rangle$ and the entangled qubit.

Alice got the eigenvalues of M_1 and M_2 by measuring the two qubits in the Z basis.

The entanglement has been consumed.

c: Alice sent to Bob the M_1 and M_2 values with a classical bit communication.

Bob got $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ using the entangled qubit applying operations depending on M_1 and M_2 .

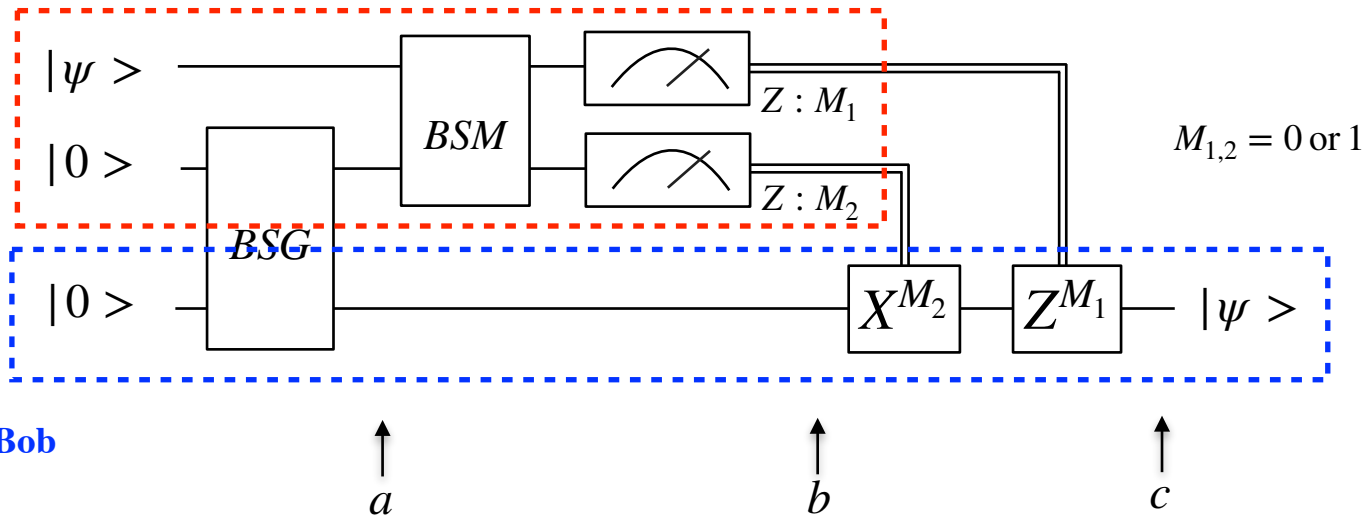
Bob has $|\psi\rangle$ but does not know the α and β values.

LOCC: Local Operations and Classical Communication

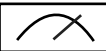
Quantum Teleportation

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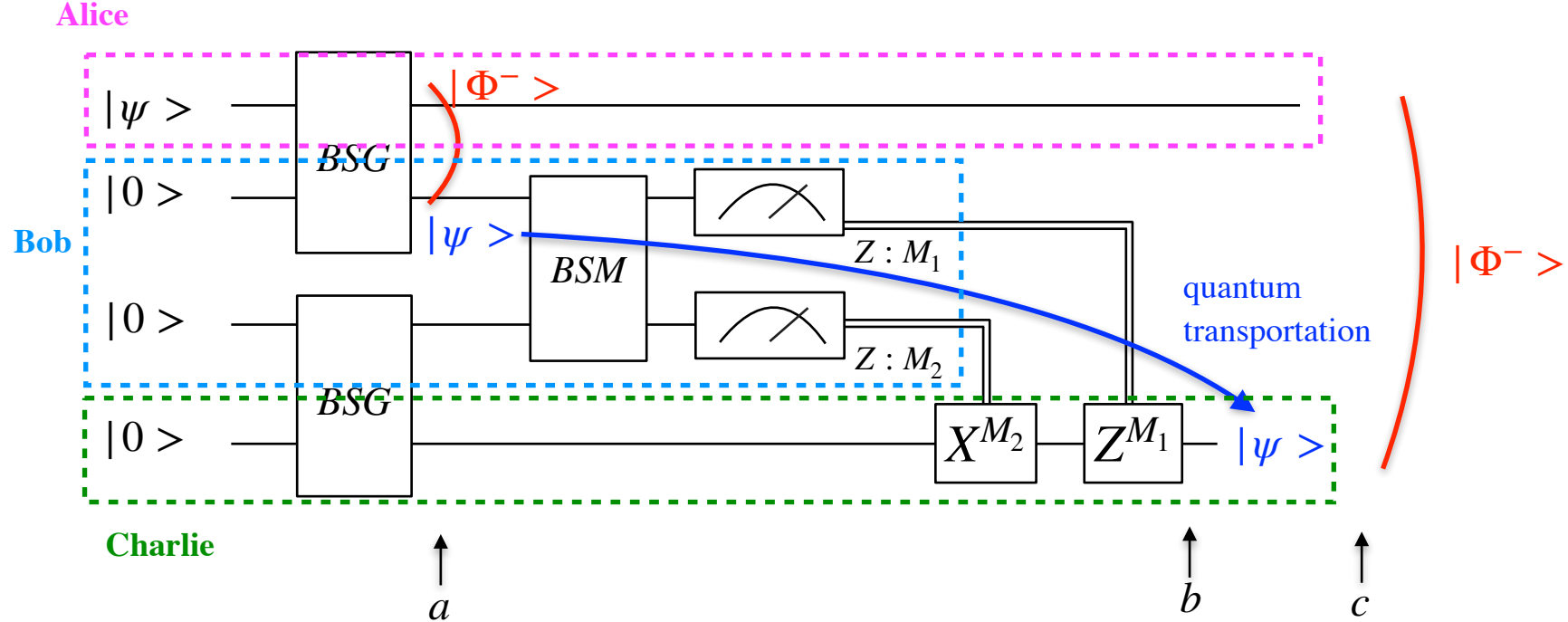
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Bob has $|\psi\rangle$ but does not know the α and β values.

LOCC: Local Operations and Classical Communication

Entanglement Swapping

[Nie00]



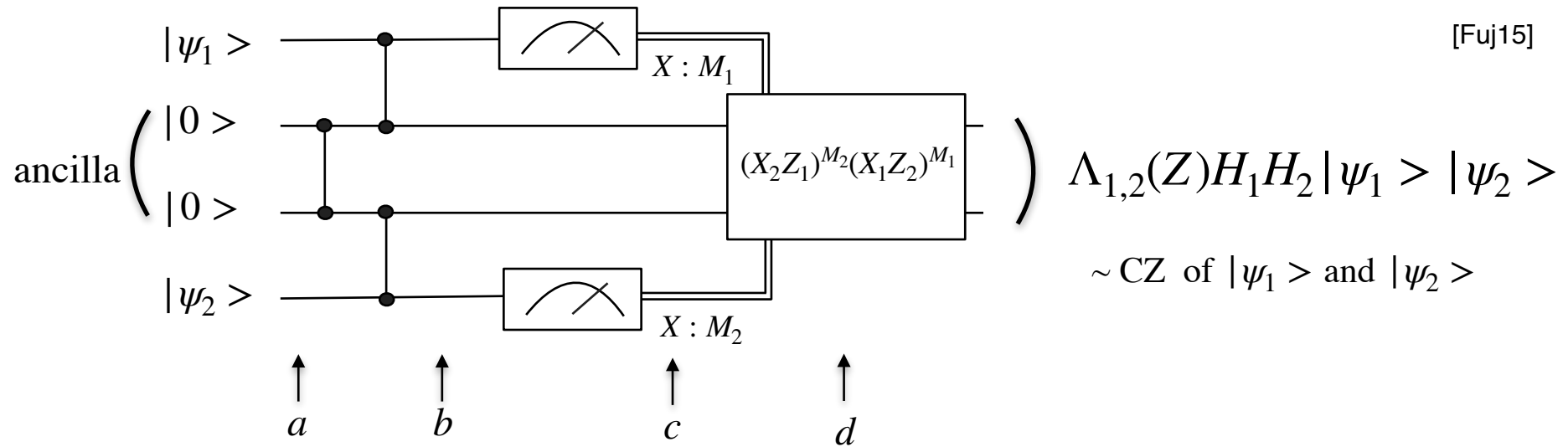
Swapping entanglement between Alice and Bob to Alice and Charlie (by Bob and Charlie).

- a: Alice and Bob shares an entanglement pair $|\Phi^-\rangle$.
- b: Bob sent to Charlie his entangled qubit by quantum teleportation.
- c: Alice's qubit and Charlie's qubit are entangled even though Alice did nothing.

Even operation by gates can be teleported by the quantum teleportation.

Measurement-Based Quantum Computation (QBQC)

[Fuj15]



2-qubit operation with ancilla qubits and measurements

- a: Prepare 2 ancilla qubits in addition to $|\psi_1\rangle$ and $|\psi_2\rangle$.
- b: Produce entanglements (operation in space).
- c: Measure qubits: consumption of entanglement.
- d: Get results applying the measured outcomes (feed-forward).

Even operation by gates can be teleported by the quantum teleportation.

Calculation in Space	$\Leftrightarrow$	Calculation in Time
quantum computation		classical computation

Quantum Error Correction

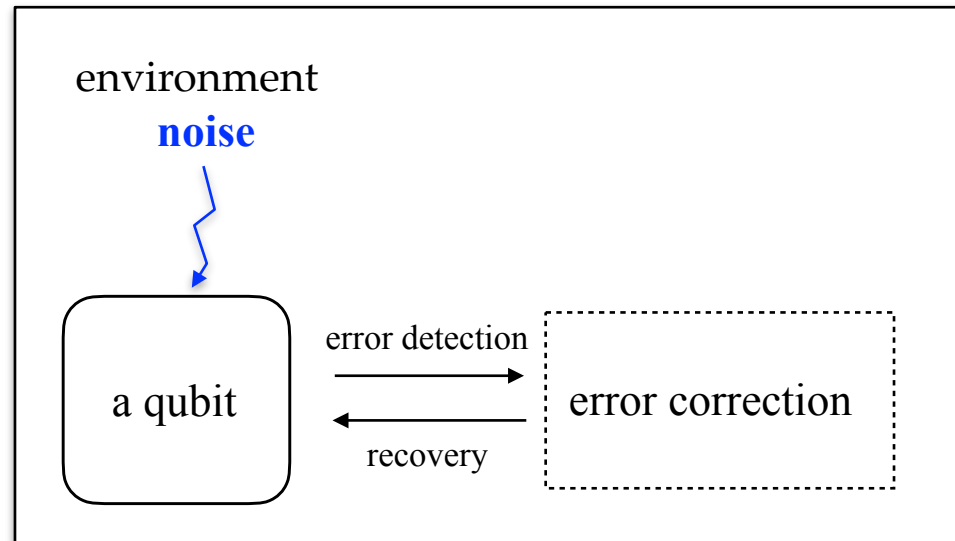
Quantum Error Correction (QEC)

A qubit is intrinsically an “analogue” object. It is fragile for noises originating from interaction with environment.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Error correction is necessary to realize quantum computation:

→ Quantum Error Correction (QEC) 量子誤り訂正



Concept of Error Correction

QEC: Difficulties

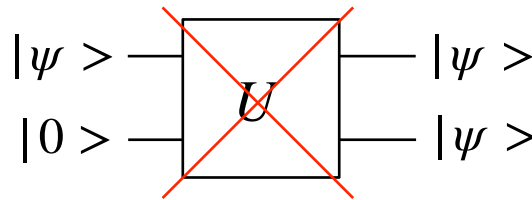
[Nie00]

Chap. 10

QEC Difficulties at a level of preserving one qubit information

- No cloning theory

It is impossible to “clone” a qubit information (strictly forbidden by quantum mechanics)



difficulty in using *majority vote* among clones for correcting errors

- Errors are continuous

requires very (infinitesimally) precise detection and correction.

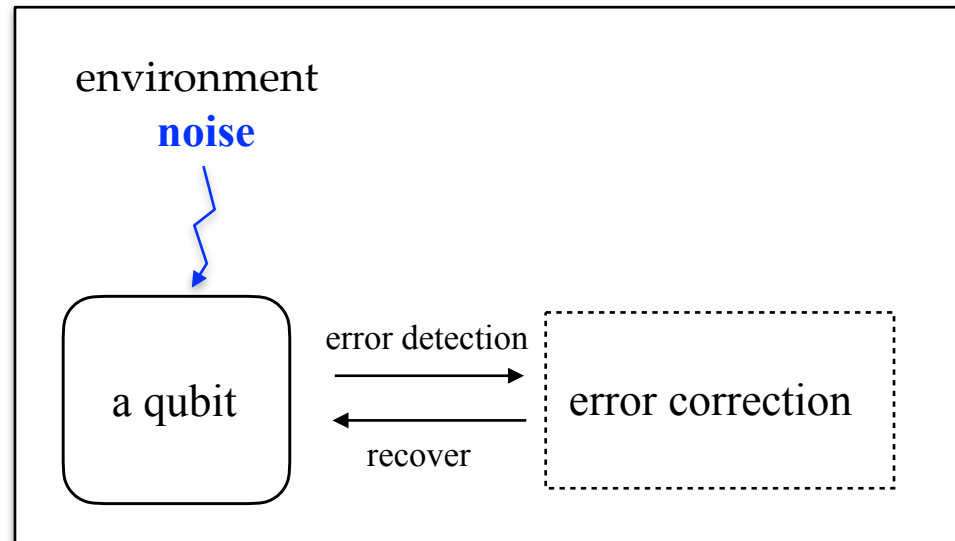
- Measurement destroys quantum information

difficulty in detection and correction of an error

Despite the difficulties, QEC is realizable exploiting quantum mechanical effects!

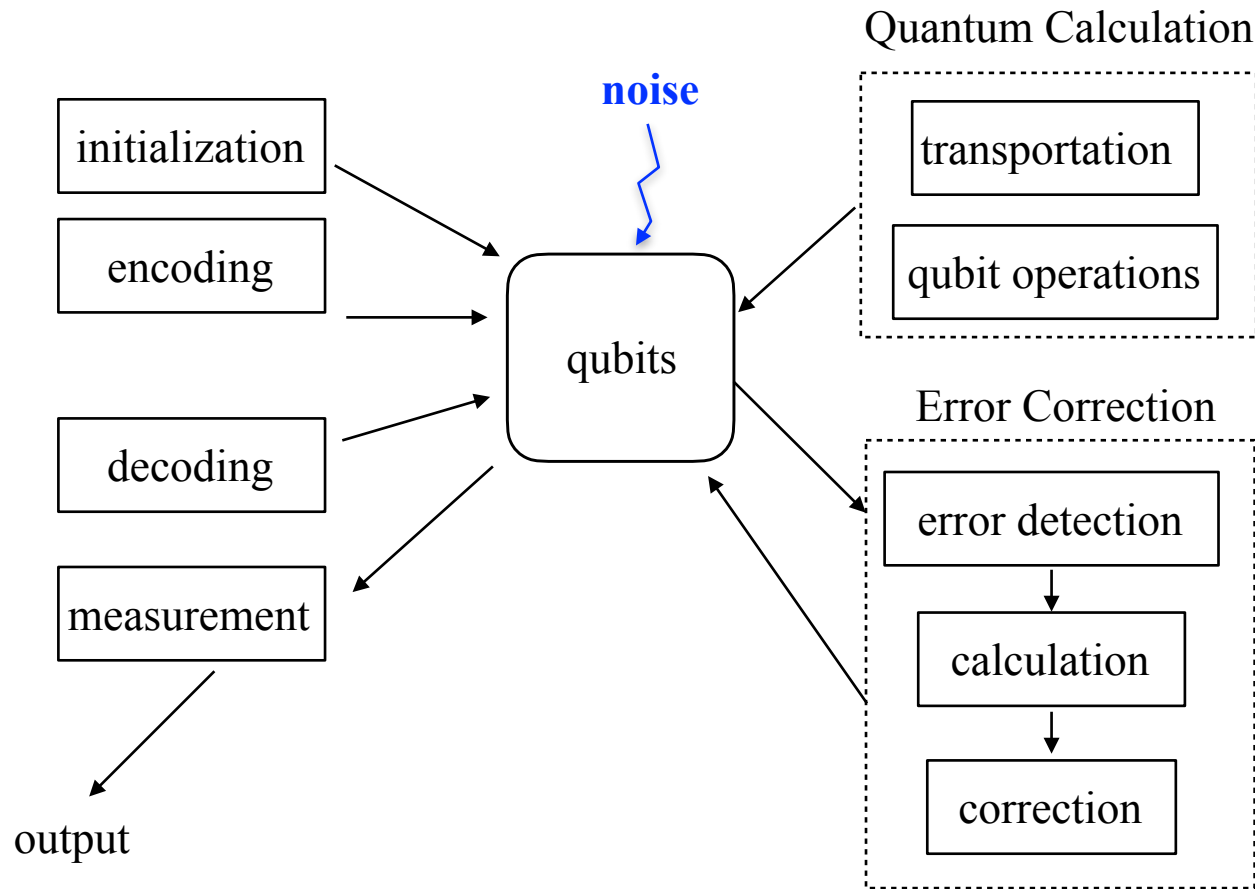
Fault Tolerant Quantum Computation

- A qubit is an “analogue” object and is fragile for noises (interaction with environments)
The noise effect is intrinsically “analogue”. $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- The noise can take place at various part of the quantum circuit including the part of QEC and measurement.
- The qubit operation must proceed with protecting the qubit information by QEC.
- Does the error rate really decrease with scaling the system size (scalability) ?

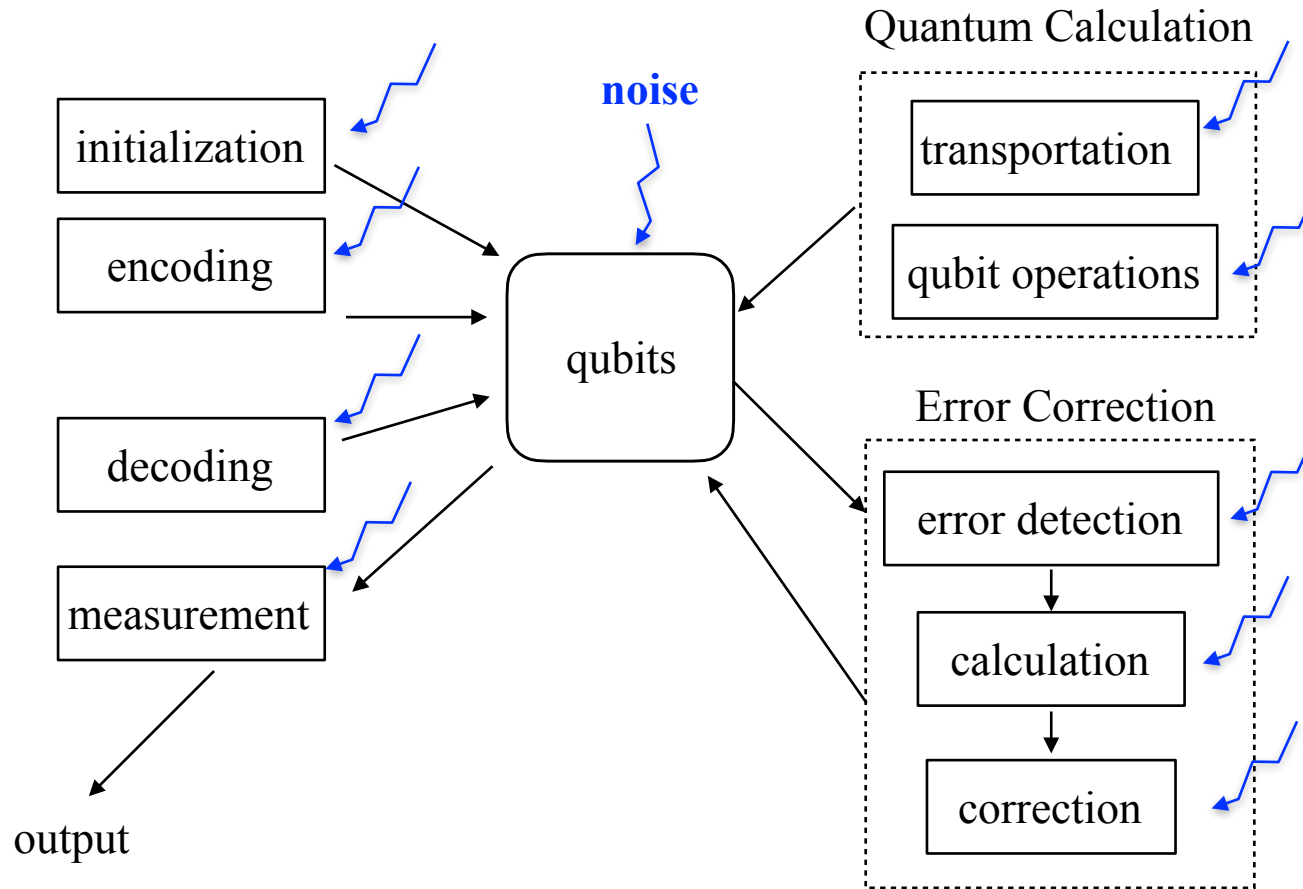


Concept of Error Correction

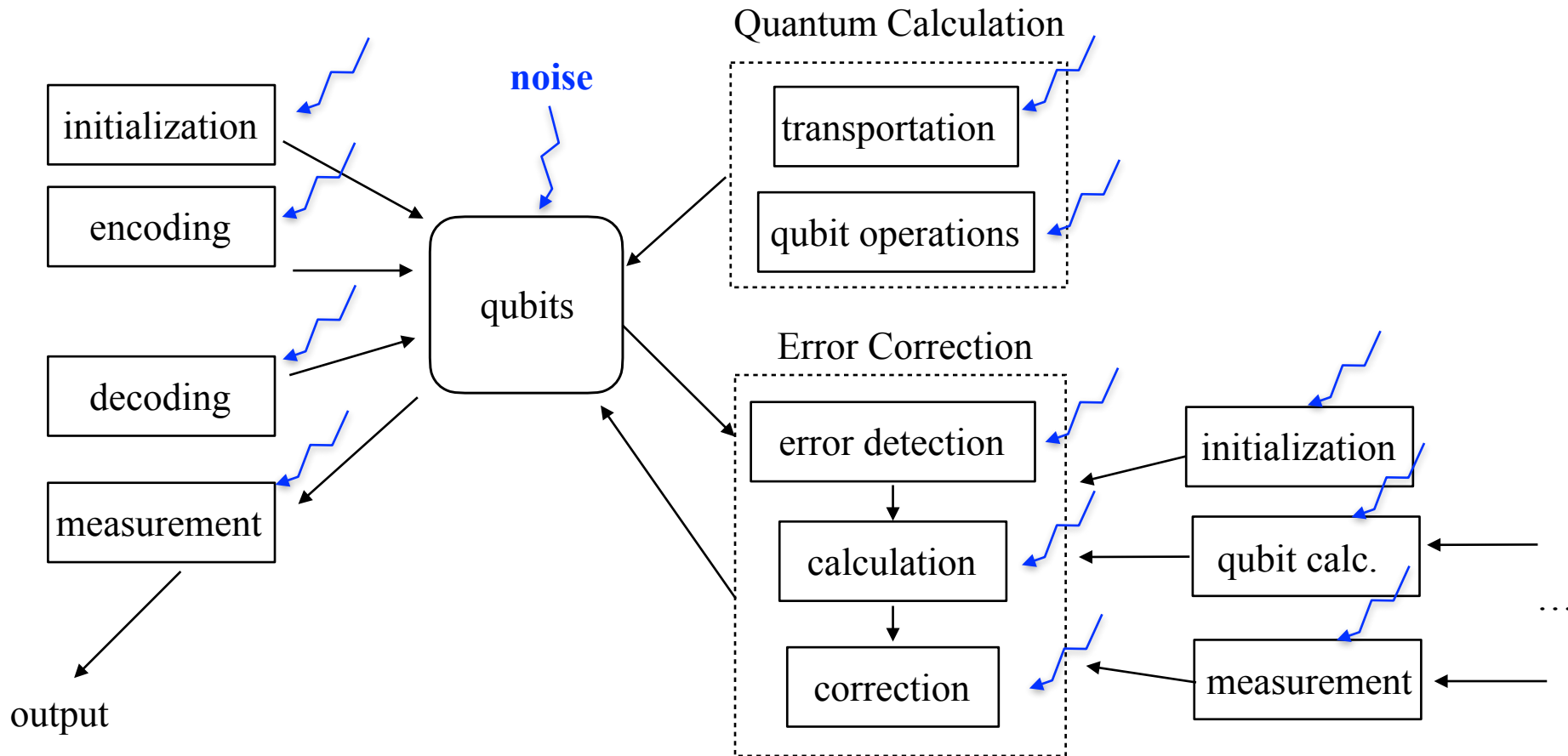
Fault Tolerant Quantum Computation



Fault Tolerant Quantum Computation



Fault Tolerant Quantum Computation



Realization of reliable quantum computation and scalability!

Encoding

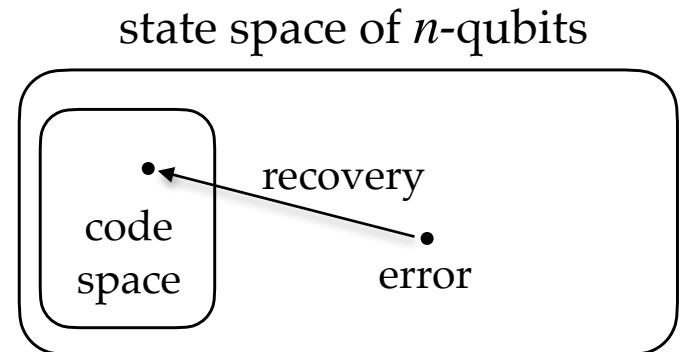
Encoding

Information is embedded in qubits according to a certain rule.

$\Leftrightarrow$ decoding

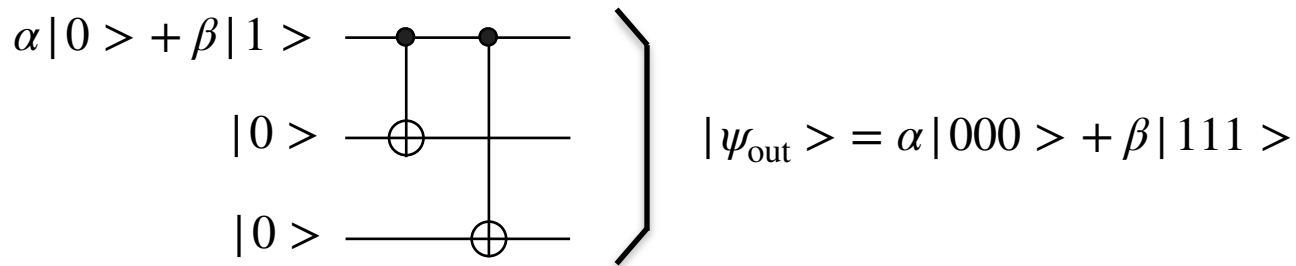
A **subspace** (code space) of the entire state space of qubits is used for the encoding.

$\rightarrow$ error detection and recovery (correction).



example code: 3-qubit bit flip

3-qubit bit flip code



$$|\psi_{\text{in}}\rangle = \alpha|000\rangle + \beta|100\rangle$$



https://nazology.kusuguru.co.jp/archives/107875#google_vignette

in code space: $|000\rangle, |111\rangle$

“cat” states

out of code space: $|100\rangle, |101\rangle, |110\rangle, |001\rangle, |010\rangle, |011\rangle$

By a bit flip error X_i the state changes to

	X_1	X_2	X_3
$ 000\rangle \rightarrow$	$ 100\rangle$	$ 010\rangle$	$ 001\rangle$
$ 111\rangle \rightarrow$	$ 011\rangle$	$ 101\rangle$	$ 110\rangle$

Syndrome Measurement

By a bit flip error X_j

		eigenvalues		recoverable by
		Z_1Z_2	Z_2Z_3	
$ 000\rangle$		+1	+1	I
$X_1 \rightarrow$	$ 100\rangle$	-1	+1	X_1
$X_2 \rightarrow$	$ 010\rangle$	-1	-1	X_2
$X_3 \rightarrow$	$ 001\rangle$	+1	-1	X_3
$ 111\rangle$		+1	+1	I
$X_1 \rightarrow$	$ 011\rangle$	-1	+1	X_1
$X_2 \rightarrow$	$ 101\rangle$	-1	-1	X_2
$X_3 \rightarrow$	$ 110\rangle$	+1	-1	X_3

measurement by Z_1Z_2 and Z_2Z_3 :

Syndrome Measurements

eigenvalue of Z_1Z_2 and Z_2Z_3 :

Error Syndrome

A single bit flip **error** is **detected** by the syndrome measurements.

The operator for the **recovery** is **identified** from the obtained error syndrome.

The initial state is perfectly (not approximately) recoverable by the identified operator.

Syndrome Measurement

For a continuous error by X_j

$$\begin{aligned} |\psi\rangle &\rightarrow \exp^{i\frac{\theta}{2}X_j}|\psi\rangle \\ &= \cos\frac{\theta}{2}I_j|\psi\rangle + i\sin\frac{\theta}{2}X_j|\psi\rangle \end{aligned}$$

by measuring the syndrome operators including Z_j

$$\begin{aligned} \text{outcome}=0 &\rightarrow |\psi\rangle \quad \text{with the probability: } \cos^2\frac{\theta}{2} \\ \text{outcome}=1 &\rightarrow X_j|\psi\rangle \quad \text{with the probability: } \sin^2\frac{\theta}{2} \end{aligned}$$

thus

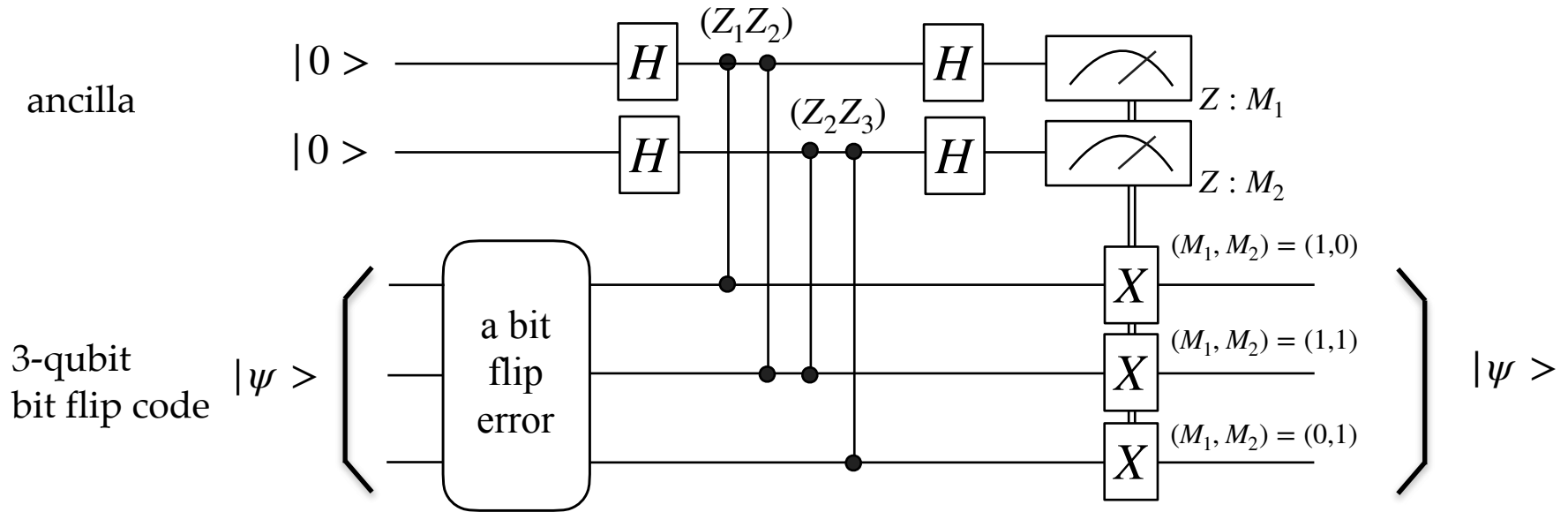
if the outcome is 0, the state after measurement is $|\psi\rangle$

if the outcome is 1, the state after measurement is
and after applying X_j , $|\psi\rangle$ is obtained.

A continuous error has been perfectly (not approximately) corrected!

The procedure exploits the property that the state after measurement is projected to an eigenstate of the Syndrome operator.

Syndrome Measurement: Recovery

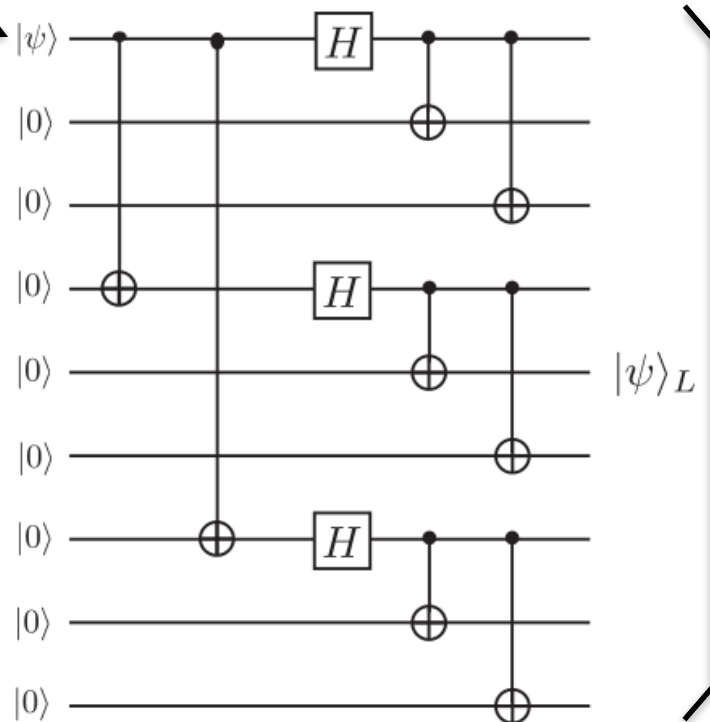


The eigenvalue for the stabilizer operators Z_1Z_2 and Z_2Z_3 are extracted by the [Hadamard Test](#).

Shor 9- qubit Code

Peter Shor 1995

qubit having information
to be encoded



qubits containing
the encoded information

encoding for
bit flip X_j

encoding for
phase flip Z_j

recoverable for all the 1-bit error of X , Z , and XZ

Quantum Error Correction

Interaction of a qubit with noise

[lsh12]

$$\rho^S = \text{Tr}_E[\rho^{SE}] \rightarrow \rho'^S = \text{Tr}_E[U\rho^{SE}U^\dagger]$$

CPTP map $\Phi : \rho^S \rightarrow \rho'^S = \Phi(\rho^S)$

assuming an initially uncorrelated state

$$\rho_{SE} = \rho_S \otimes \rho_E$$

Kraus representation

$$\Phi(\rho) = \sum_k E_k \rho E_k^\dagger$$
$$\sum_k E_k^\dagger E_k = I$$

Each E_i (1-qubit error) is expanded as

$$E_i = \varepsilon_{i0}I + \varepsilon_{i1}X + \varepsilon_{i2}Z + \varepsilon_{i3}XZ$$

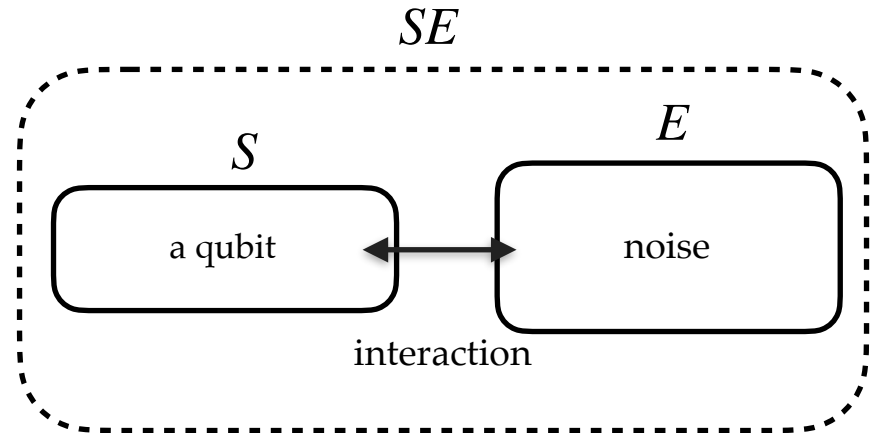
$$\because XZ = -iY$$

An error is expanded to a set of discrete errors: X , Z , and XZ .

X : bit-flip error

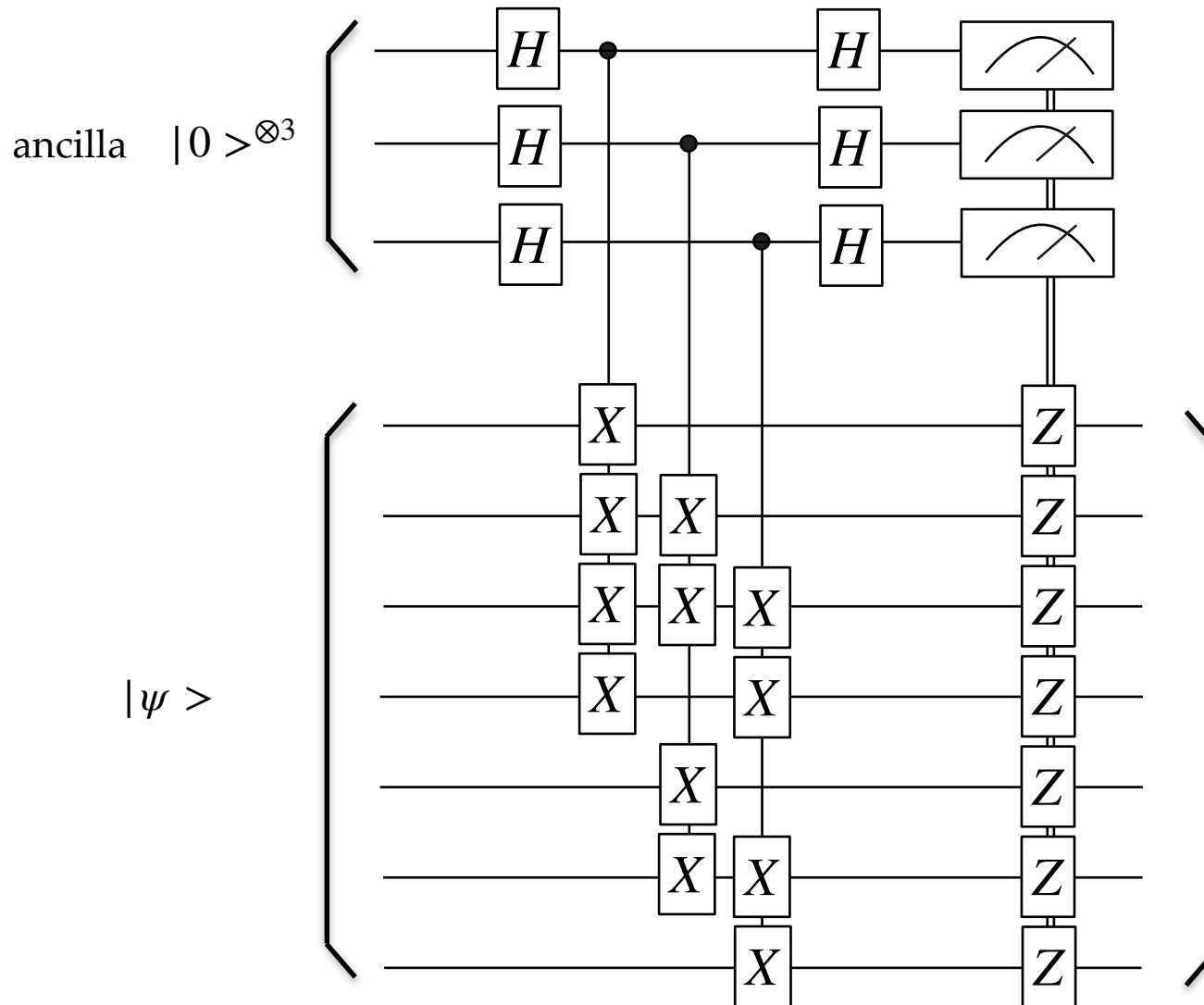
Z : phase-flip error

XZ : both

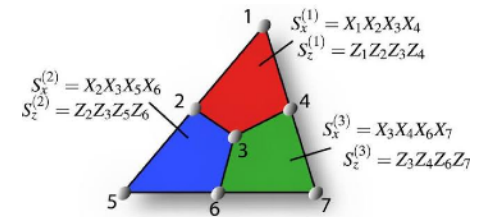


Steane 7-qubit Code

Andrew Steane 1996



Steane Code



~ classical hamming code
geometrical representation

$|\psi\rangle$

Stabilizer

Stabilizer Formalism

[Got97] Gottesmann'97

<https://arxiv.org/abs/quant-ph/9705052>

For n -qubit Pauli group $\mathcal{P}_n$:

$$\mathcal{P}_n := \{\pm 1, \pm i\} \times \{I, X, Y, Z\}^{\otimes n}$$

A Stabilizer group $\mathcal{S}$ is an Abelian subgroup of the n -qubit Pauli group:

$$\mathcal{S} := \left\{ S_i \in \mathcal{P}_n \mid -I \notin \mathcal{S} \text{ and } \forall S_i, S_j, [S_i, S_j] = 0 \right\}$$

A Stabilizer generator: $\mathcal{S}_g$

$$\mathcal{S} = \langle \mathcal{S}_g \rangle$$

A stabilizer code is the simultaneous +1 eigenspace of an Abelian subgroup $\mathcal{S}$.

$$\forall S_i \in \mathcal{S}, S_i |\psi_L\rangle = |\psi_L\rangle$$

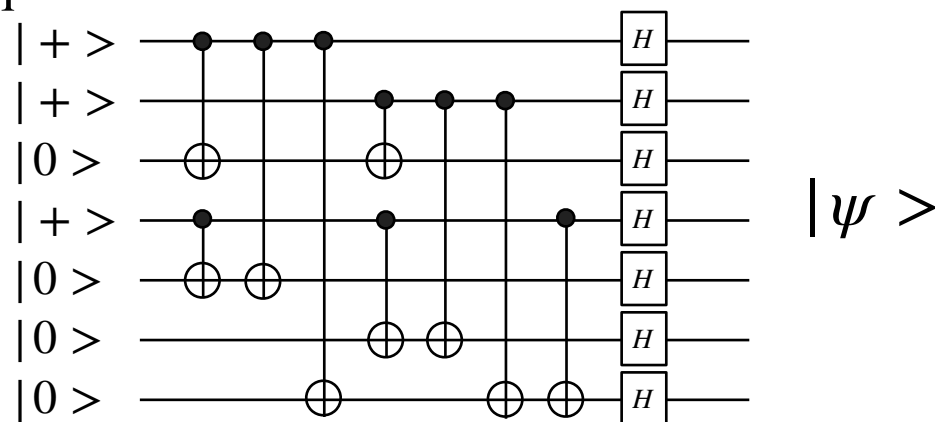
$$\Leftrightarrow \forall S_i \in \mathcal{S}_g, S_i |\psi_L\rangle = |\psi_L\rangle$$

A stabilizer state is uniquely specified by n independent stabilizers.

Stabilizer: Example

[Fuj15]

Example state: $|\psi\rangle$ with 7 qubits



Ket representation of the state $|\psi\rangle$

$$\begin{aligned}
 |\psi\rangle = & (|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle \\
 & + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle \\
 & + |1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle \\
 & + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle) / 4
 \end{aligned}$$

$\sim 2^n$ kets for large n

Stabilizer representation of the state $|\psi\rangle$

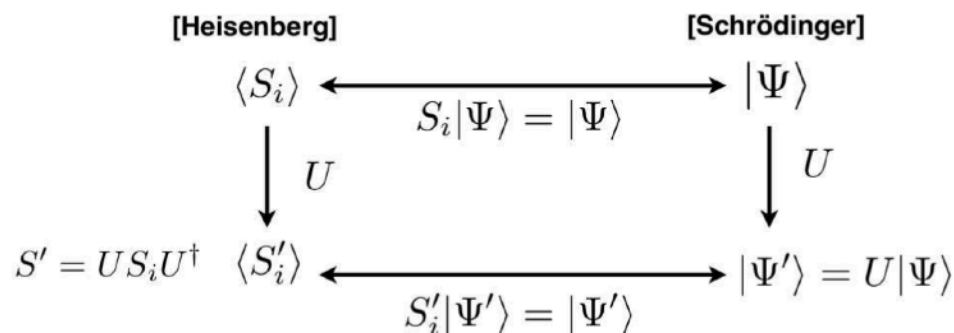
$$|\psi\rangle: \{ZIZIZIZ, IZZIIZZ, IIIZZZZ, XXXIIII, XXIXXII, IXIXIXI, XIIXII X\}$$

n independent stabilizer generators uniquely specify an n -qubit stabilizer state.
much more efficient!

※The stabilizer group is unique, but its choice of generators is not unique.

Stabilizer Formalism

[Fuj15]



analogous to “Schrödinger representation”

Time evolution: applying a unitary operator to kets

$$|\psi'\rangle = U|\psi\rangle$$

analogous to “Heisenberg representation”

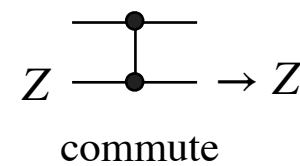
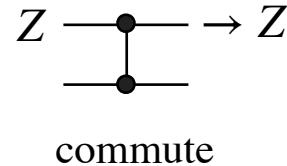
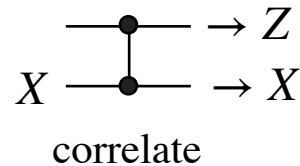
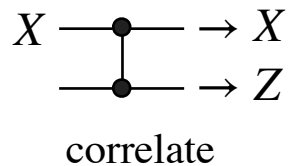
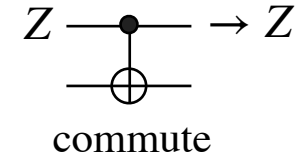
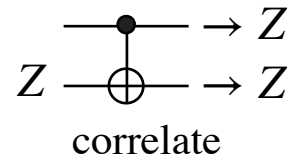
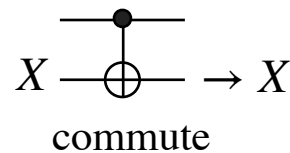
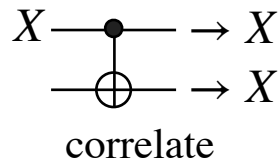
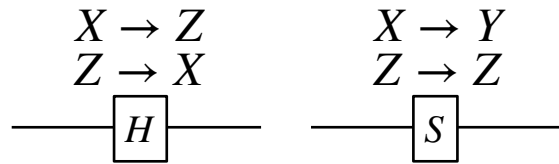
Time evolution: applying a unitary operator to stabilizers

$$S'_j = US_jU^\dagger$$

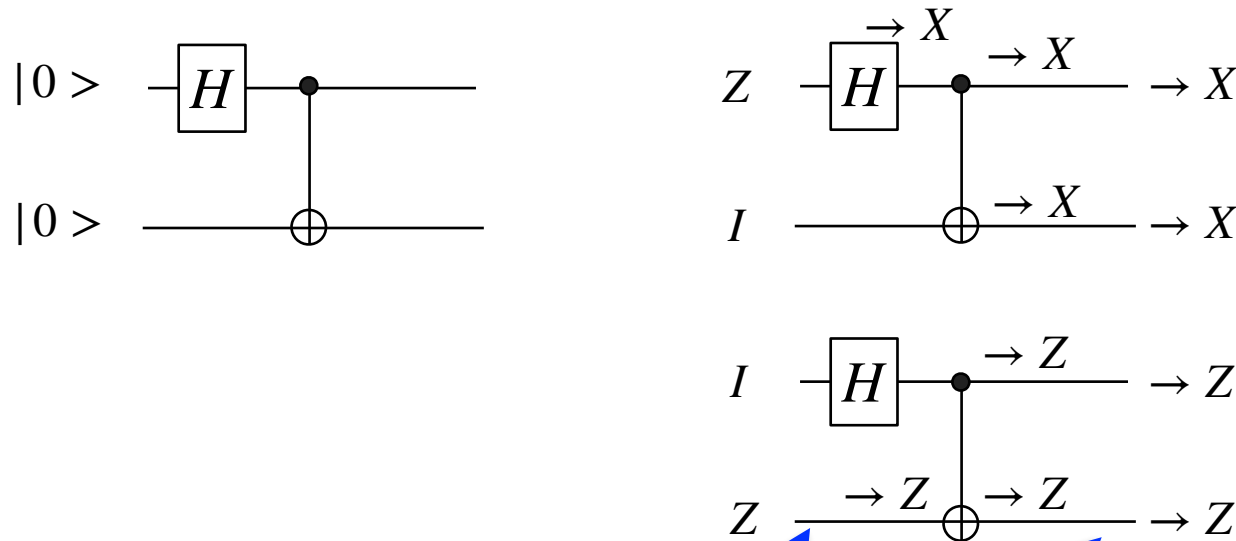
Stabilizer: Propagation Rules

[Fuj15]

Stabilizer Propagation Rules



Stabilizer: Propagation Example



Initial State Stabilizer = $\{ZI, IZ\}$

Final State Stabilizer = $\{XX, ZZ\}$

$$|\psi_{\text{in}}\rangle = |00\rangle$$

$$|\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$XX|\psi_{\text{out}}\rangle = |\psi_{\text{out}}\rangle$$

$$ZZ|\psi_{\text{out}}\rangle = |\psi_{\text{out}}\rangle$$

Stabilizer: Pauli Measurement Rule

[Fuj15]

Pauli operator (P) measurement of a stabilizer state:

(i) if P commutes with all the stabilizer generators

Either P or $-P$ is an element in the stabilizer group.

If $P(-P)$ is an element, eigenvalue is $1(-1)$ with unit probability.

The stabilizers group do not change after the measurement.

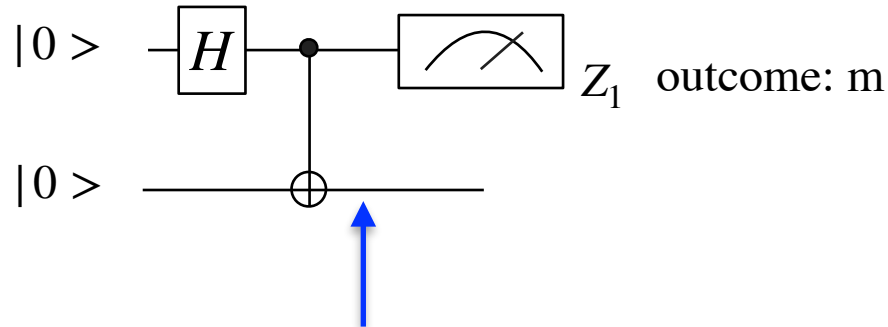
(ii) if P anti-commutes with one or more stabilizer generators

The measurement outcome is $m = \pm 1$ with the equal probability.

Remove the stabilizers that anti-commutes with P .

Add $(-1)^m P$ depending on the measurement outcome m .

Stabilizer: Pauli Measurement Example



stabilizers $\{X_1X_2, Z_1Z_2\}$

$$\longrightarrow \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

commutation of Z_1 with stabilizers

$$(X_1X_2)Z_1 = -Z_1(X_1X_2)$$

$$(Z_1Z_2)Z_1 = Z_1(Z_1Z_2)$$

by Z_1 measurement,

outcome:

50% $m=+1$

50% $m=-1$

stabilizer after measurement:

$$\{Z_1, Z_1Z_2\}$$

$$\{-Z_1, Z_1Z_2\}$$



$$|00\rangle$$



$$|11\rangle$$

Logical Operator

[Got97] Gottesmann'97

<https://arxiv.org/abs/quant-ph/9705052>

Logical Operator L

L is a n -qubit Pauli operator that commutes with all the stabilizers but not contained in the stabilizer group.

$$L; L \in \mathcal{P}_n, L \notin \mathcal{S}, [L, S] = 0 \quad \forall S \in \mathcal{S}$$

$k = n - r$ pairs of independent logical operators can be defined for a n -qubit system with r independent stabilizers.

For example, if $n - 1$ stabilizers are defined,

A pair of logical operators, X_L and Z_L , can be defined satisfying

$$X_L Z_L = -Z_L X_L$$

with corresponding logical states $|0_L\rangle$ and $|1_L\rangle$

$$Z_L |0_L\rangle = |0_L\rangle \quad X_L |0_L\rangle = |1_L\rangle$$

$$Z_L |1_L\rangle = -|1_L\rangle \quad X_L |1_L\rangle = |0_L\rangle$$

Surface Code

Fault Tolerant Quantum Computation

Steps

Quantum Error Correction (QEC) of a physical qubit



Google (2023-24)

sub-threshold scalability
of QEC protected memory

QEC protected

Encoding Information in n -qubits



logical qubits
stabilizer
surface code

Universal Computing



logical operators
magic state distillation/injection
braiding/lattice surgery

Scaling for Practical Usage

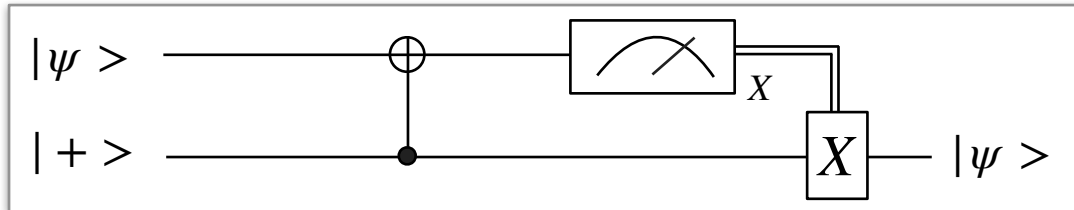
sub-threshold
concatenation
code distance

Magic State Injection

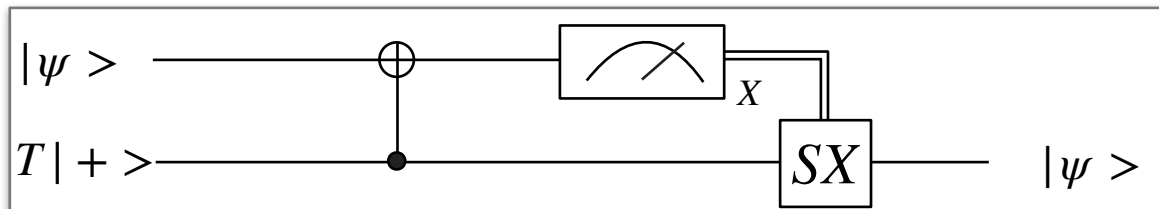
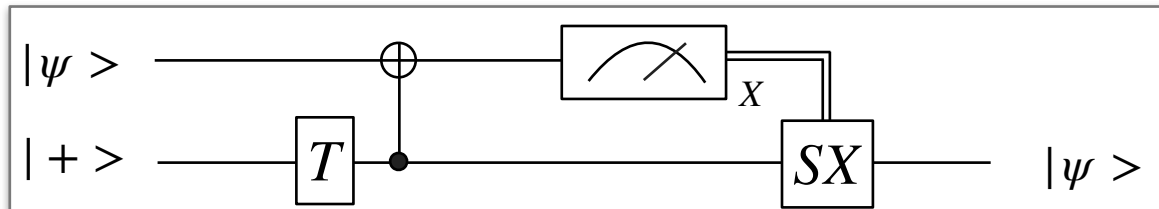
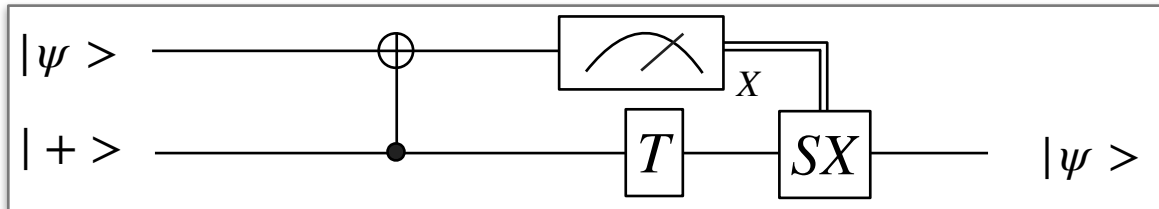
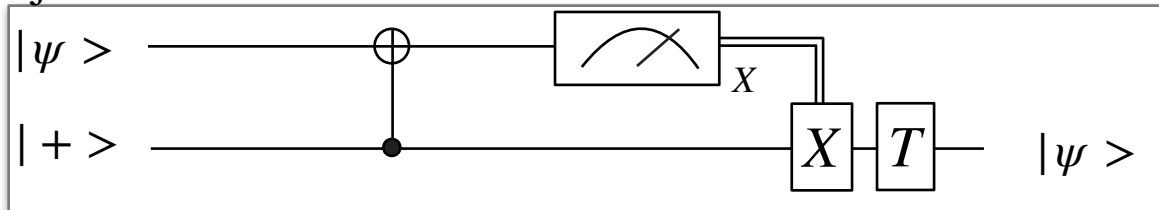
1-bt quantum teleportation

by Gate Teleportation

[Fuj20]



T injection



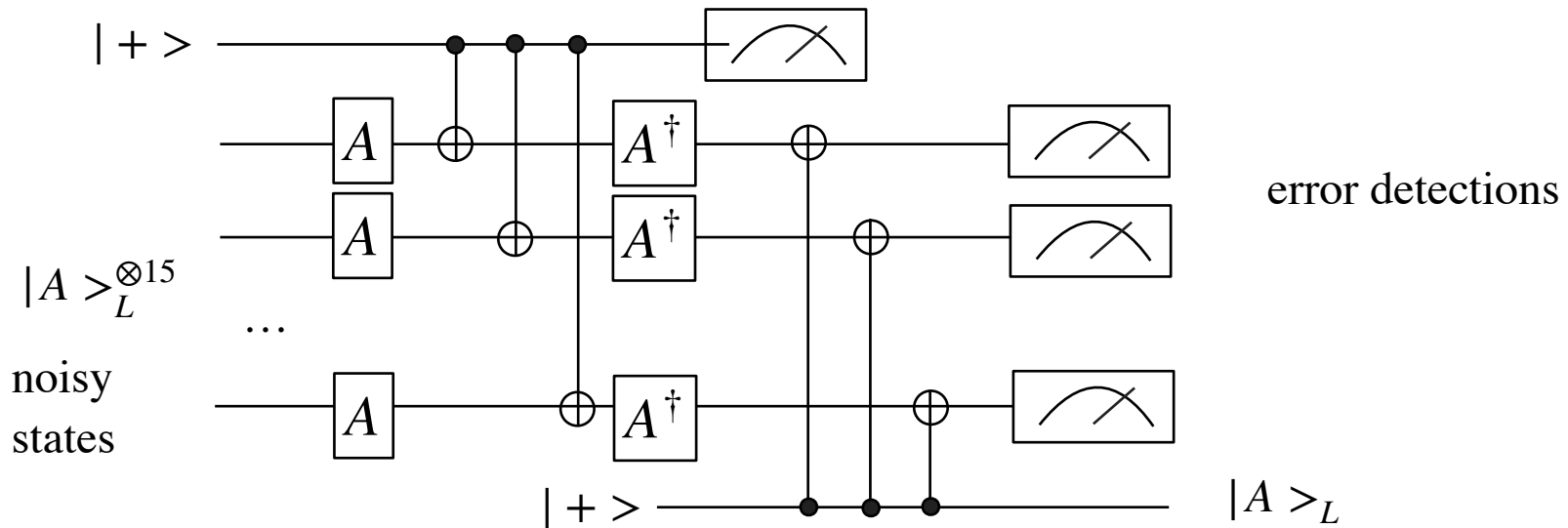
requires a clean $T|+>$ state
= magic states

Magic State Distillation

For getting a pure magic state

$$A \equiv e^{-i\frac{\pi}{8}Y}$$

$$|A\rangle \equiv A|+\rangle$$

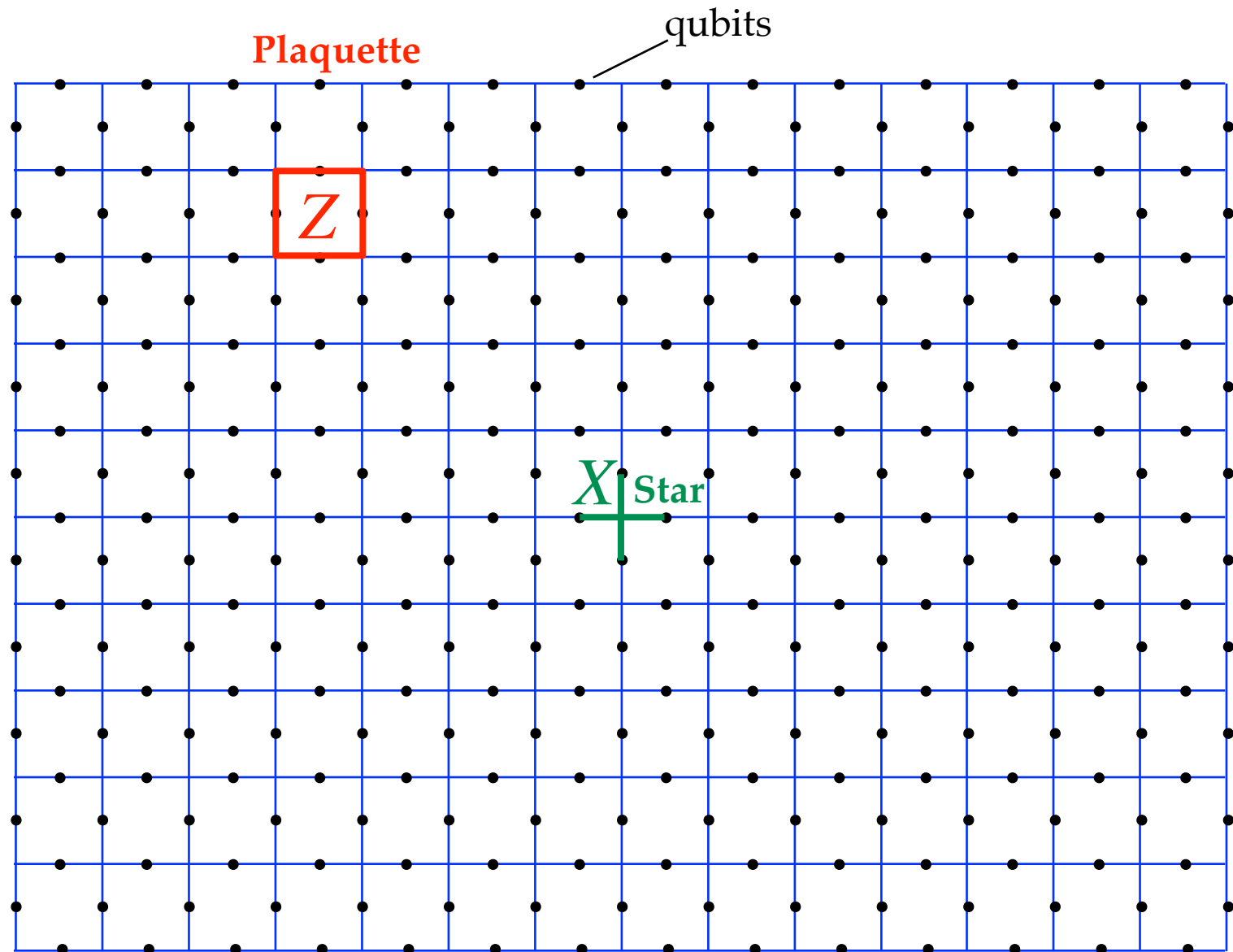


The process is repeated hierarchically
(p^2 reduction by a layer)

Production of distilled state for T requires large qubit resources (70~95%)

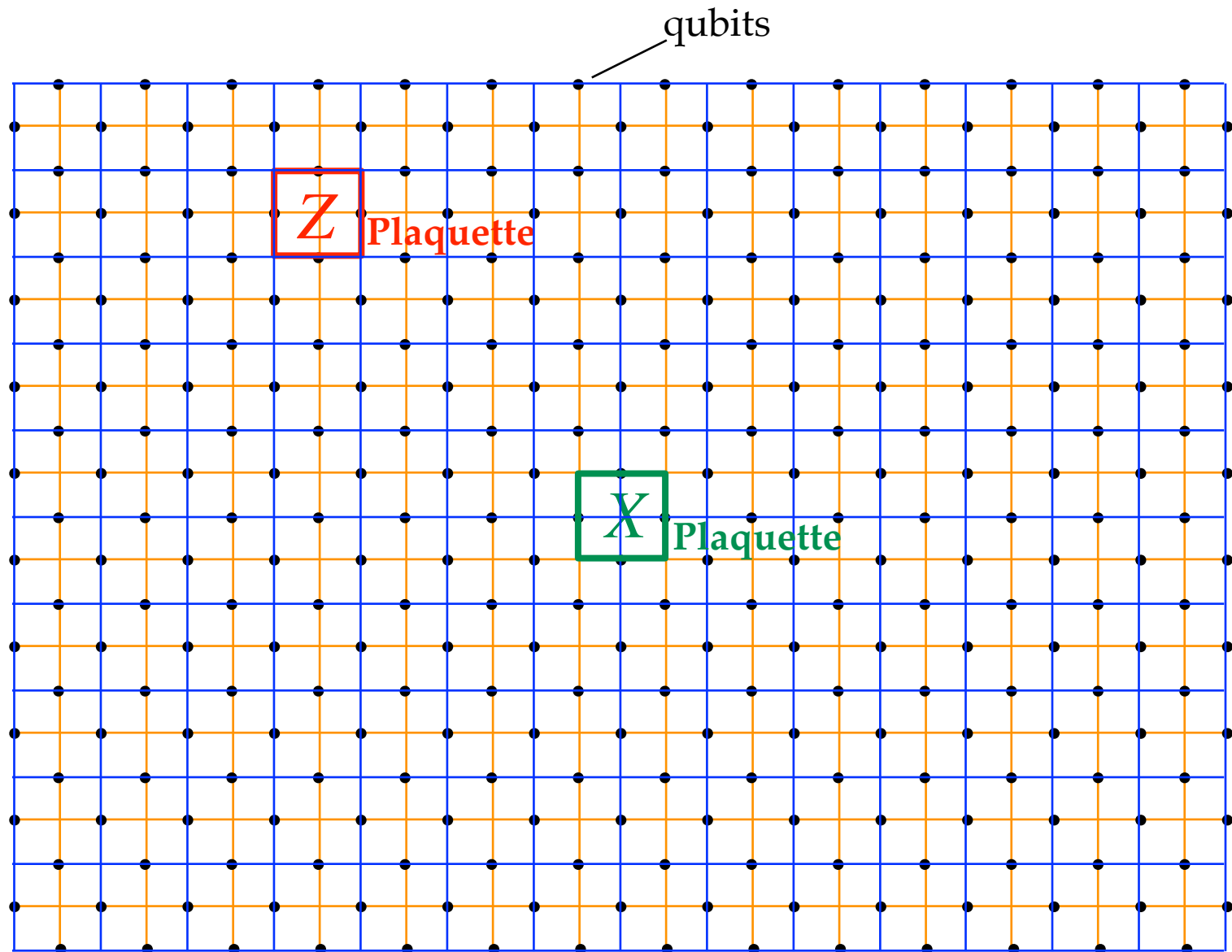
→ T factory (next problem to be solved for scaling)

Surface Code



Schematic bulk picture; boundary stabilizers are omitted.

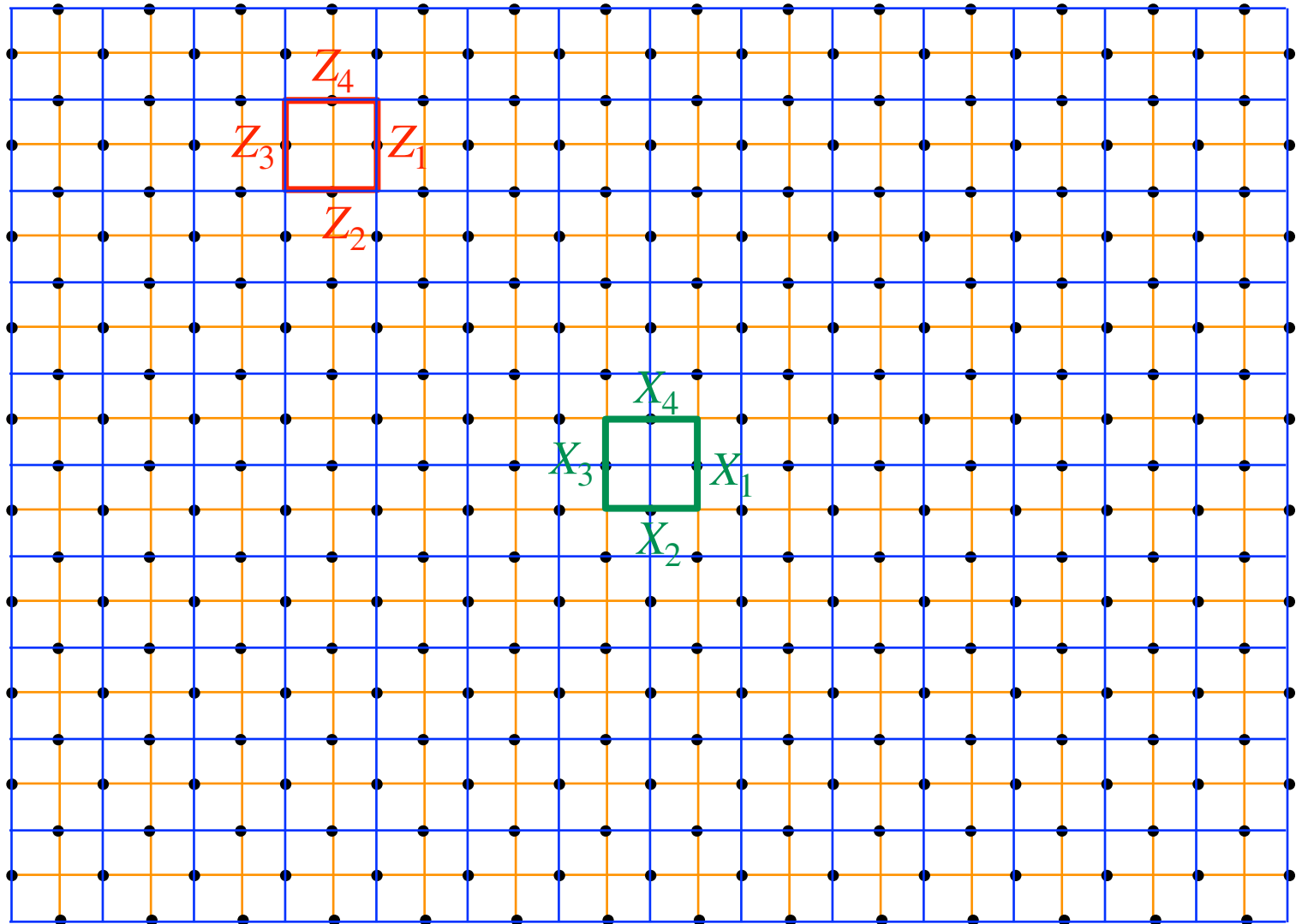
Surface Code



Schematic bulk picture; boundary stabilizers are omitted.

Surface Code

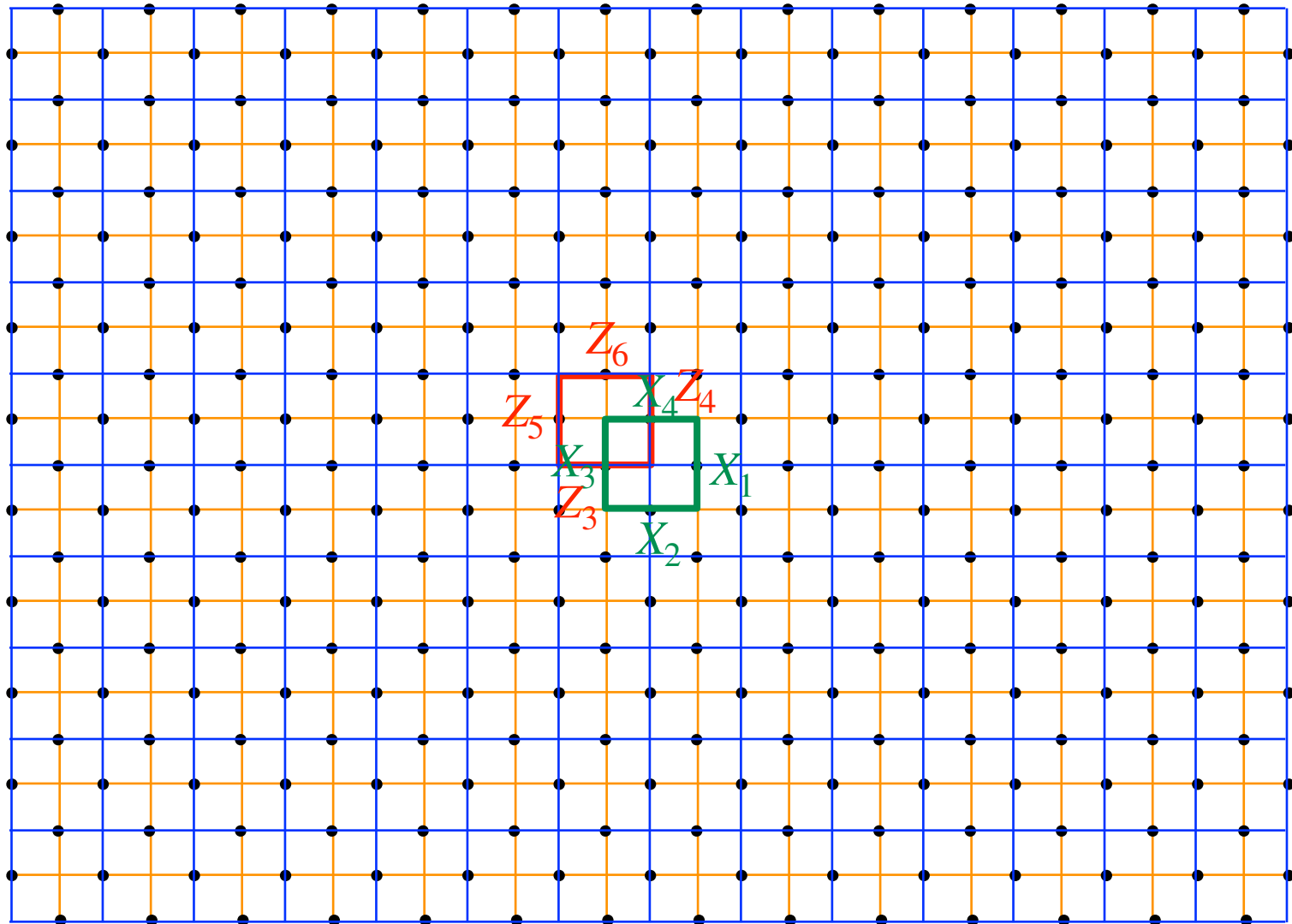
Stabilizer: $X_1X_2X_3X_4, Z_1Z_2Z_3Z_4, \dots$



Schematic bulk picture; boundary stabilizers are omitted.

Surface Code

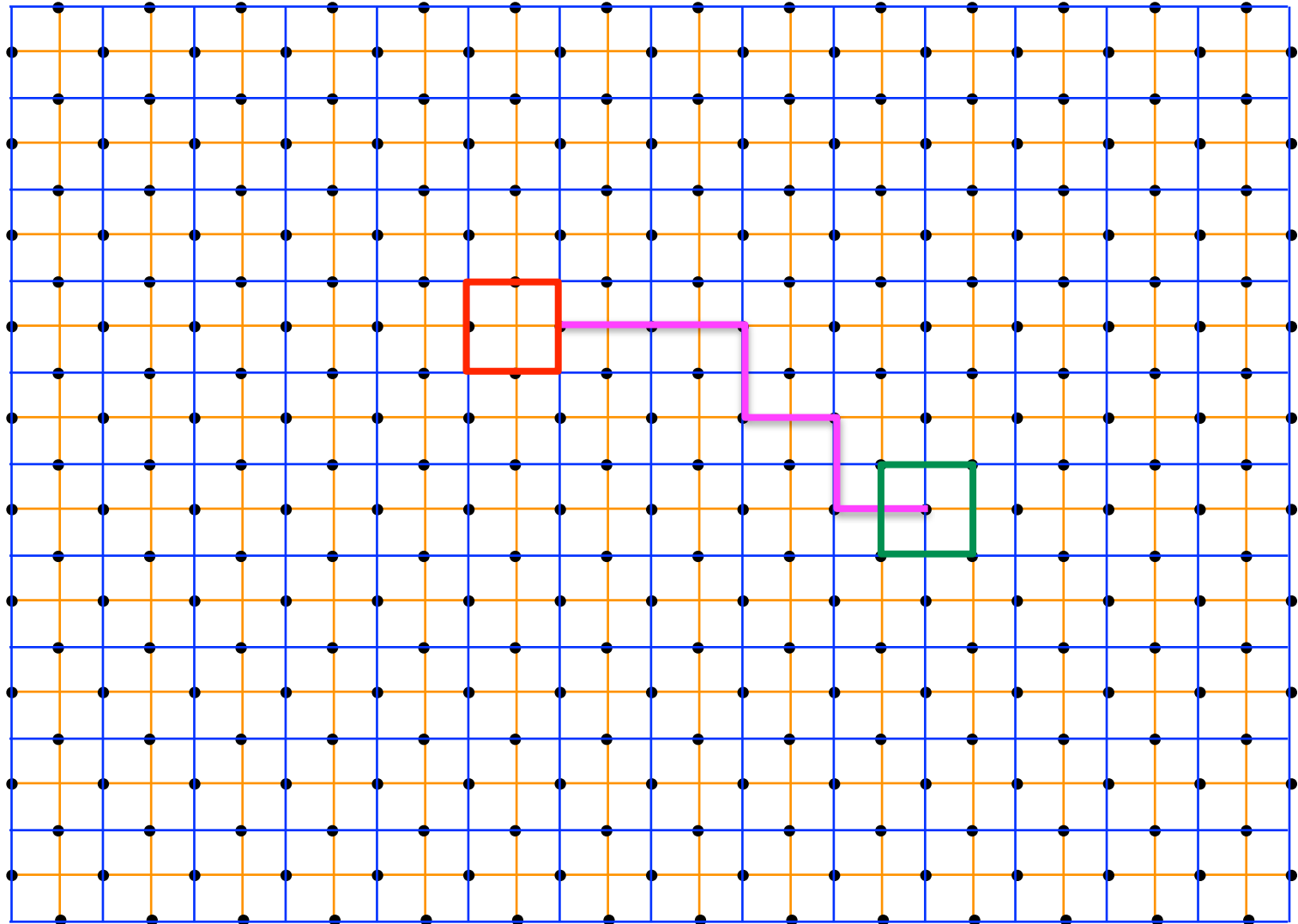
$$[X_1 X_2 X_3 X_4, Z_3 Z_4 Z_5 Z_6] = 0$$



Schematic bulk picture; boundary stabilizers are omitted.

Surface Code

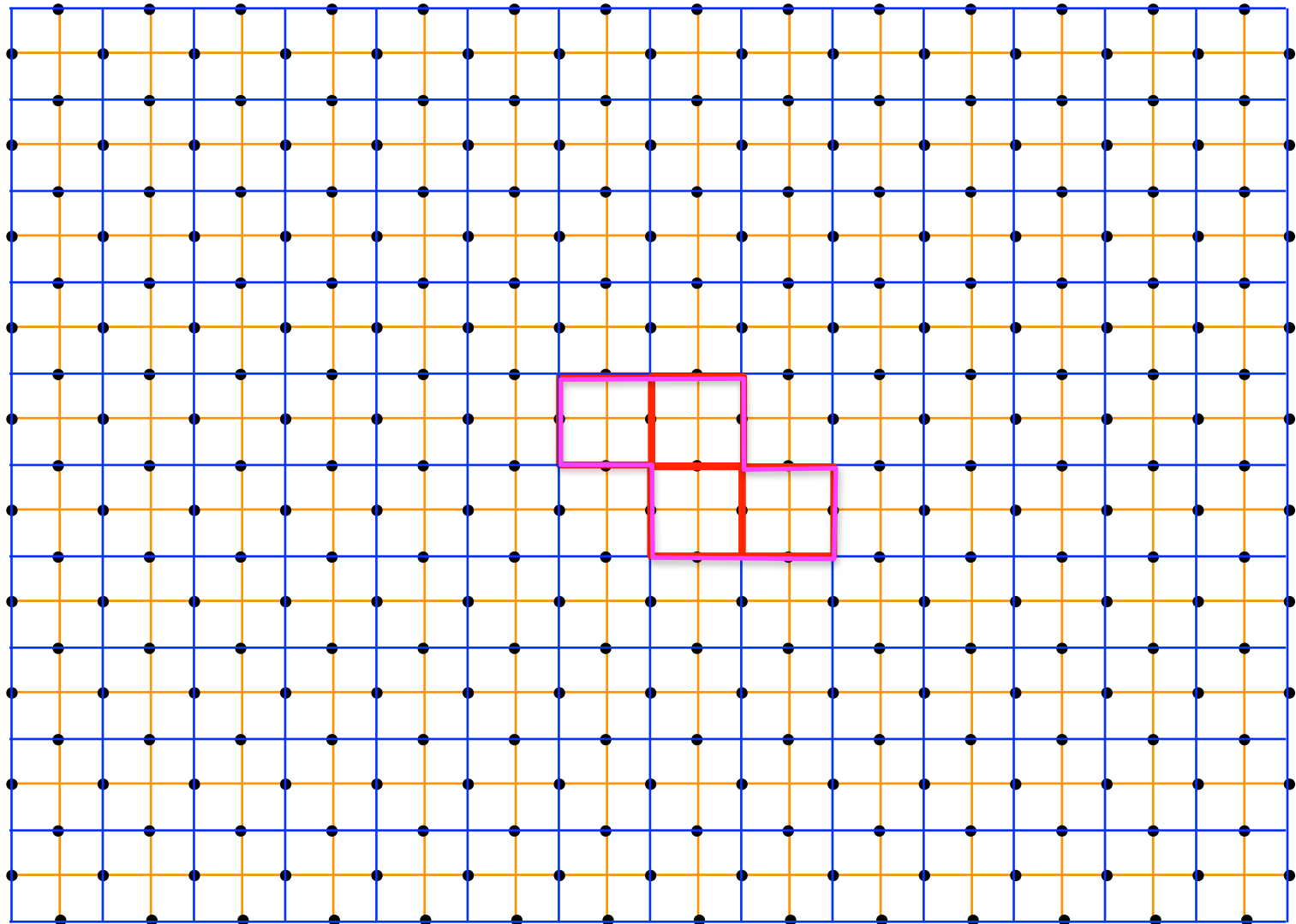
$$[S', S_i] \neq 0$$



Schematic bulk picture; boundary stabilizers are omitted.

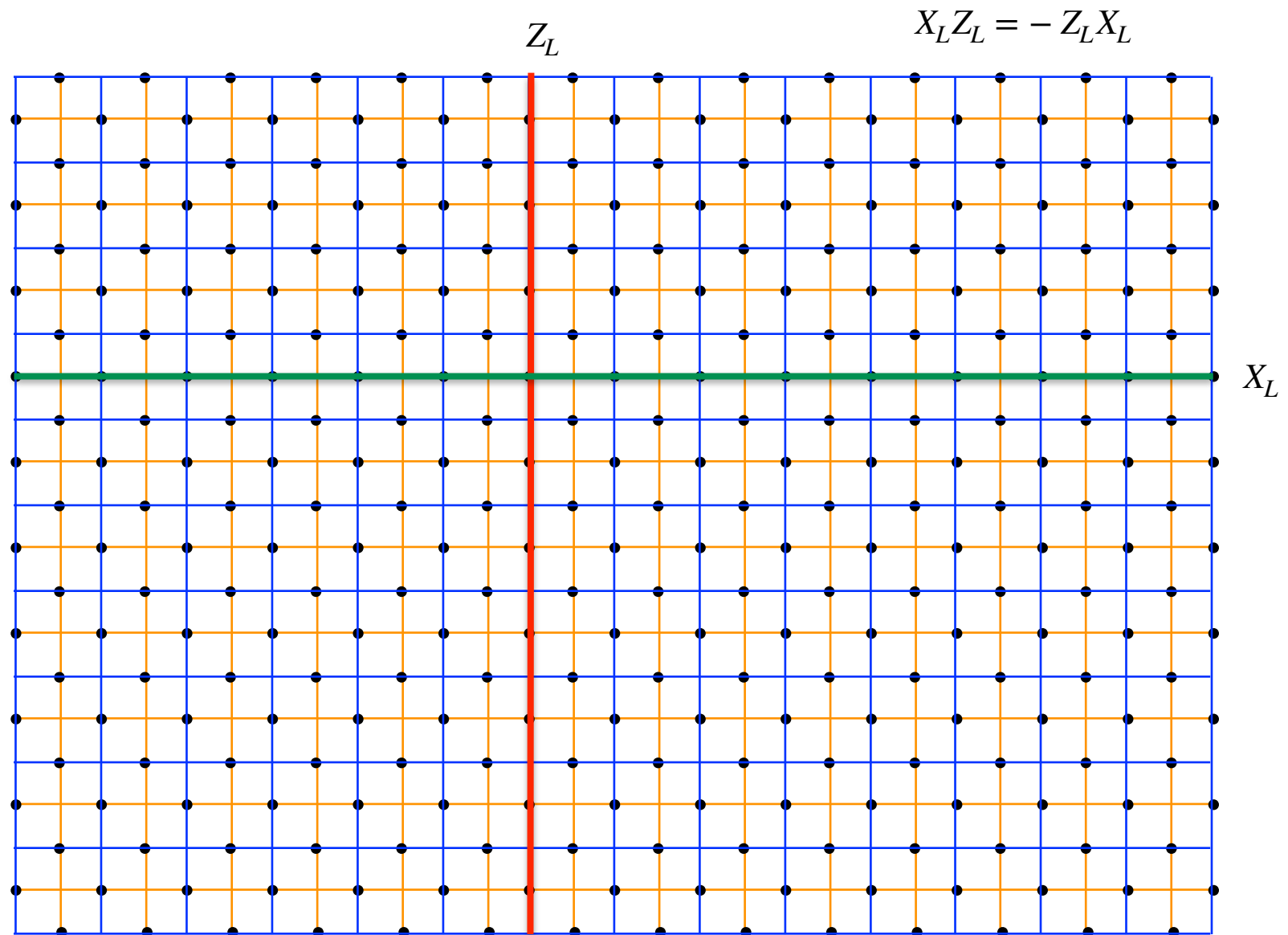
Surface Code

$$S' \in \langle S_g \rangle$$



Schematic bulk picture; boundary stabilizers are omitted.

Surface Code

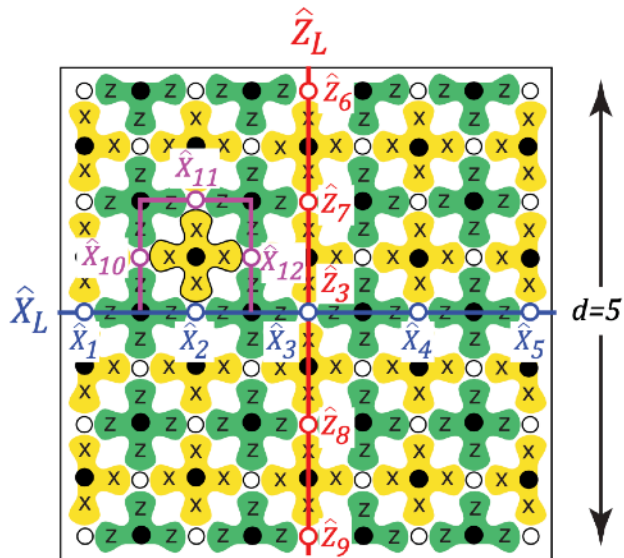


Schematic bulk picture; boundary stabilizers are omitted.

Code Distance

Code distance

Minimum number of Pauli operators contained in all the possible representations of logical operators.



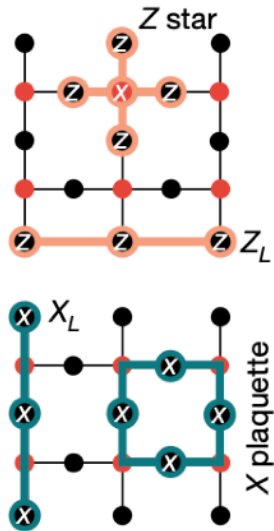
Toric Code

Toric Code: Alexei Kitaev 1997

[Blu22]

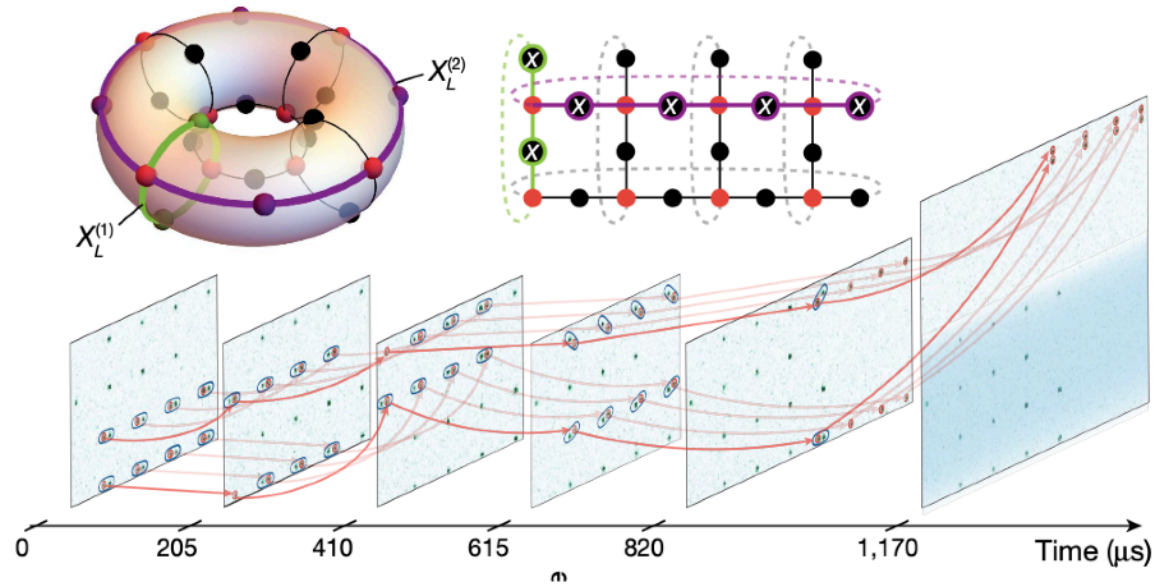
Surface code

- Data qubit
- Ancilla



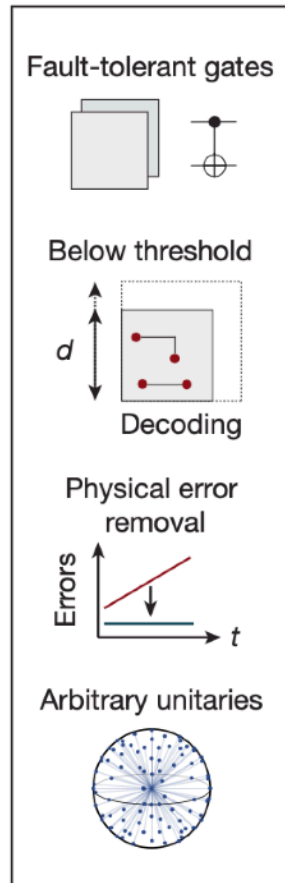
c

Toric code (on a torus)

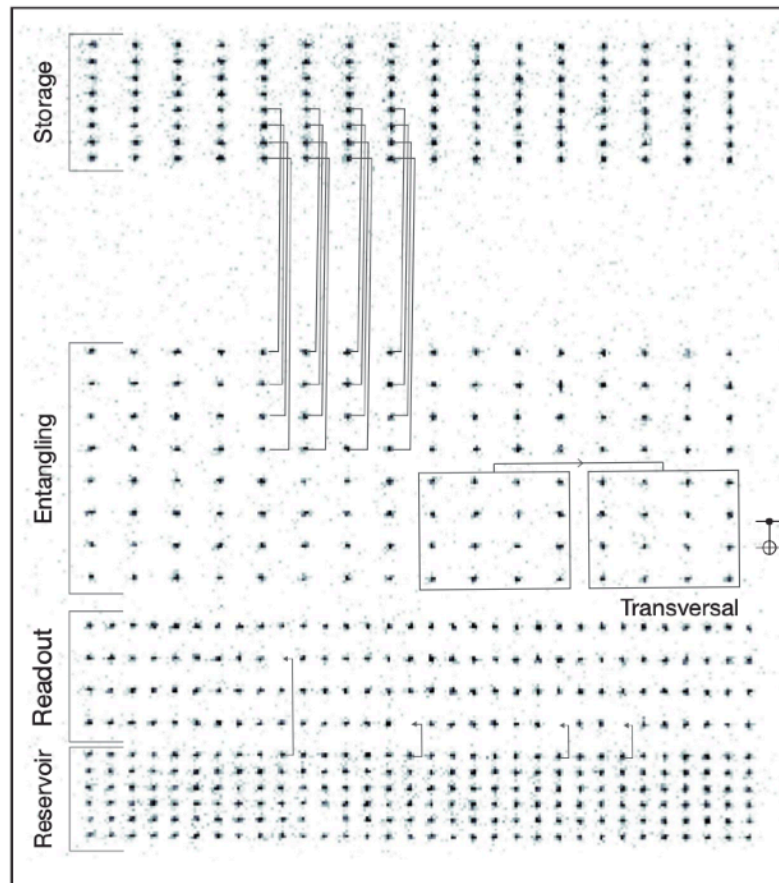


Neutral Atom Architecture

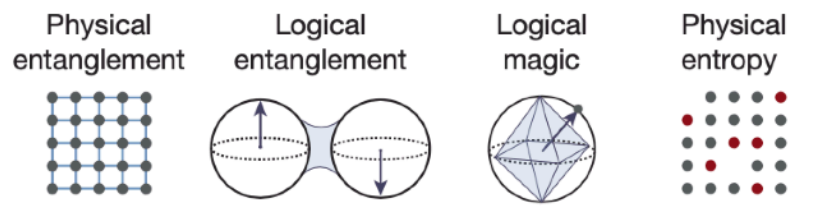
a Fault-tolerant processing



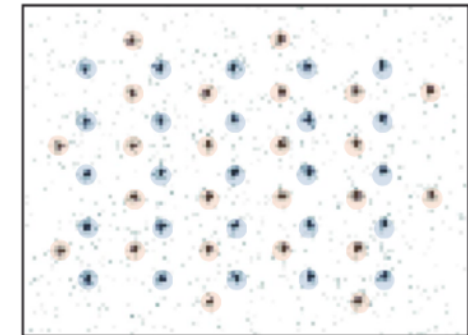
Architecture



Underlying mechanisms

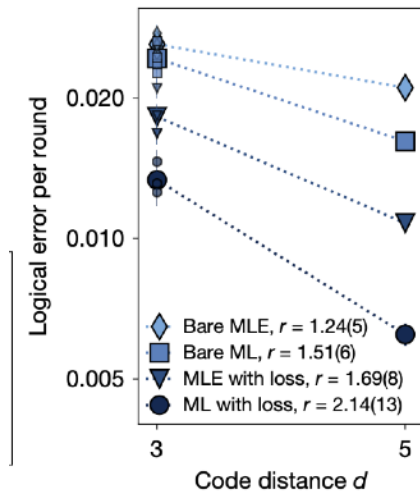


[Blu25]



$d = 5$ surface code ● Data ● Ancilla

d

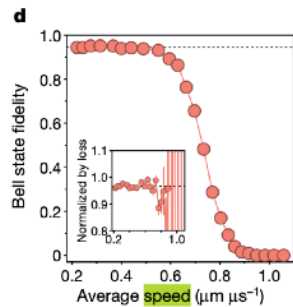
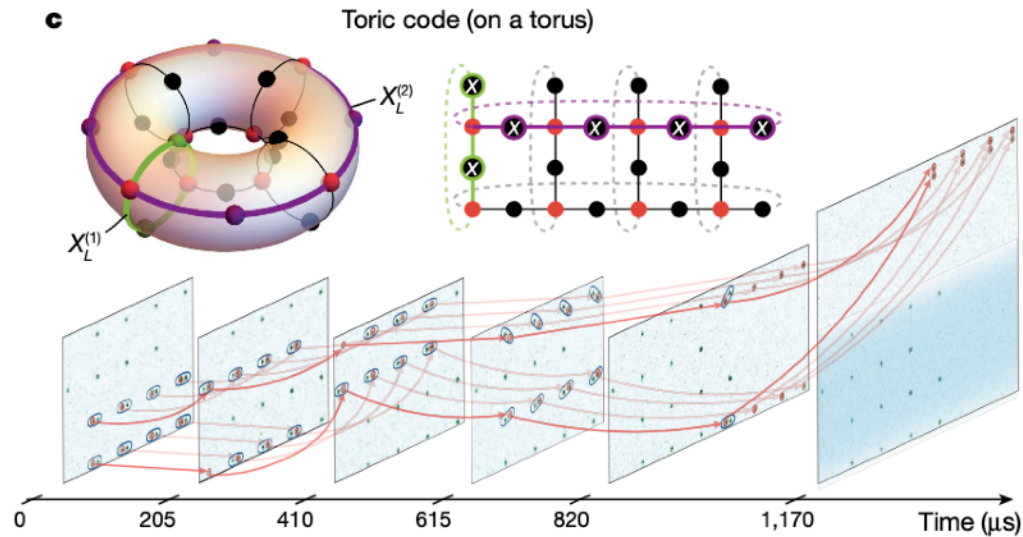


most likely (MLE) machine learning (ML)

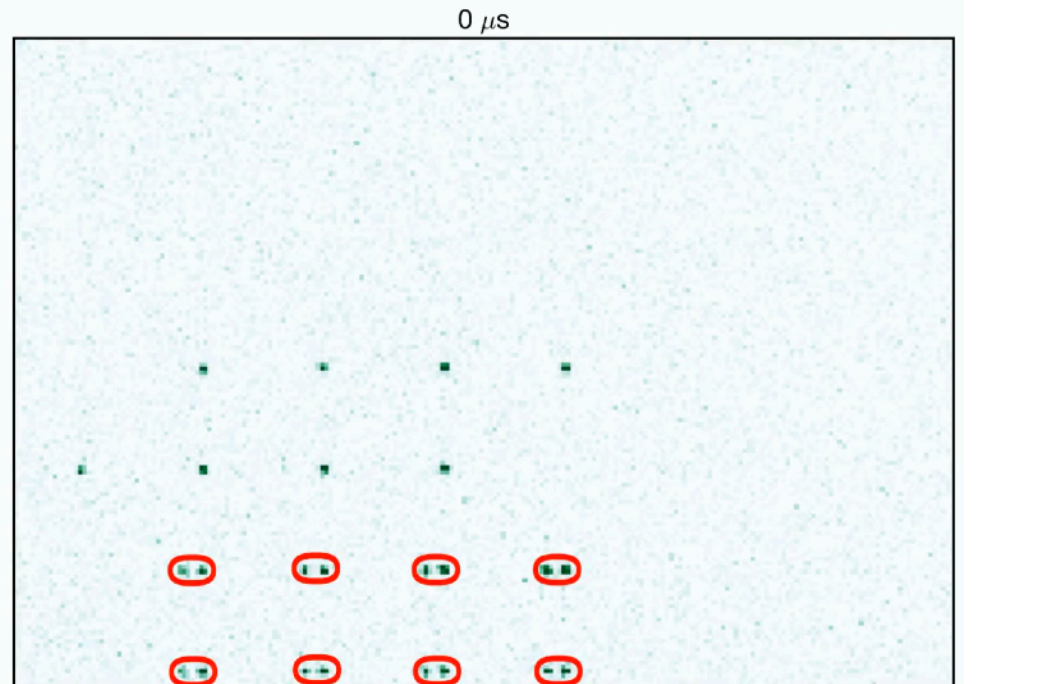
loss: correction with atom loss information 94

Construction of a Toric code

[Blu22]



velocity $< 0.55 \mu\text{m}/\mu\text{s}$
(due to trap loss)



Typical Values

[Win23]

Parameter	Typical values today (near Future)
Qubits	
Amount	~ 100 [20, 21, 22, 23] (~ 1000 for 2024 [44, 45])
Connectivity	10 : 1 – 20 : 1 [20], (50 : 1 – 100 : 1 possible in principle)
Multiple states (i.e., qudit)	In principle possible
Lifetimes and Decoherence times	
Trap lifetime	10 – 60 s [29, 22, 30, 23] (up to 6000 s with cryostat [39])
Decoherence times (electronic spin)	$T_1 \sim 4$ s, $T_2 \sim 1$ s, $T_2^* \sim 4$ ms [21, 22]
Decoherence times (nuclear spin)	$T_1 \gg 5$ s, $T_2 \sim 40$ s, $T_2^* \gg 3$ s [23]
Native gates (<i>gg</i>-qubits)	
List of gates	Single-qubit rotations, $CZ \rightarrow CNOT$ [20, 38], $SWAP$, $CPHASE$ [37]
>2-qubit gates:	$CCZ \rightarrow Toffoli / CCNOT$ [20, 38], $C_k Z$, generalisation to k control qubits [37]
Parallelism	Apply the same gate on multiple Qubits simultaneously: Single-qubit rotations, CZ [21] (Multiple gates on multiple Qubits)
Fidelities of operations	
1-qubit gate	0.996 – 0.999 [21, 22]
2-qubit gate	0.955 – 0.995 [38, 22, 42]
Readout	$\gtrsim 0.95$ [46]
Preparation	Trap occupation probability (after rearrangement): 0.988 [47] Success probability for defect-free array ~ 0.75 , depending on size of array [47]
Execution times	
1-qubit gate	$\sim 2 \mu\text{s}$ (π -pulse) [21]
2-qubit gate	~ 400 ns (CZ) [21, 38]
Preparation (incl. rearrangement)	~ 400 ms [47]
Readout	~ 10 ms for fluorescence imaging [46], ($\sim 6 \mu\text{s}$ using a collective readout scheme) [46]
Installation and operation	
Required infrastructure	Vacuum cell and pumps, lasers, optical elements, microwave sources, signal generators and modulators, magnetic field coils. Cooling the setup with a cryostat can improve vacuum quality and increase trap lifetimes [39]. (Rack-level implementation possible [48])
Calibration	Rearrangement at beginning, no calibration of individual qubits necessary
Specificity	Shuttling operations [21]
Access	Via the cloud [49, 50] and on-premise
Quantum computing paradigm	Gate-based (digital) quantum computing, digital-analogue quantum computing, quantum annealing, analogue quantum simulation

Threshold Theorem

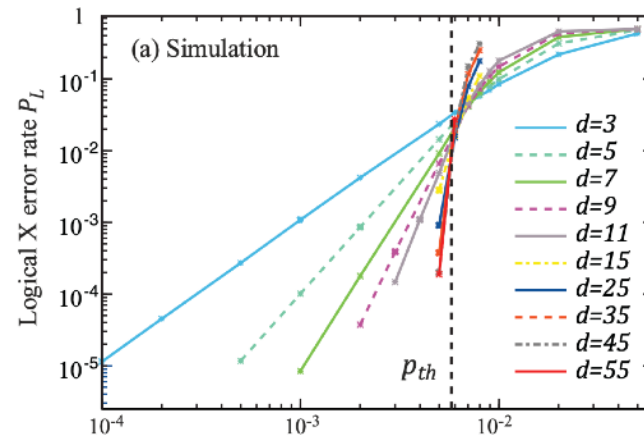
$$\varepsilon_d \propto \left(\frac{p}{p_{\text{thr}}} \right)^{\frac{d+1}{2}}$$

ε_d : logical error rate

p : error rate in a physical operation

p_{thr} : threshold error rate (depending on the code)

d : code distance



[Fow12]

[Goo24]

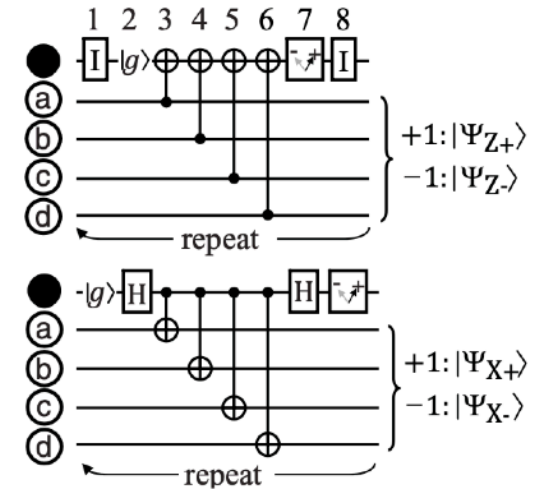
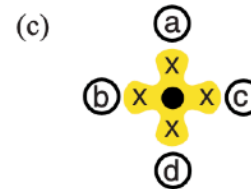
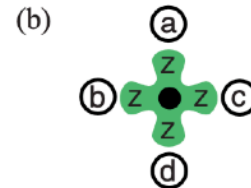
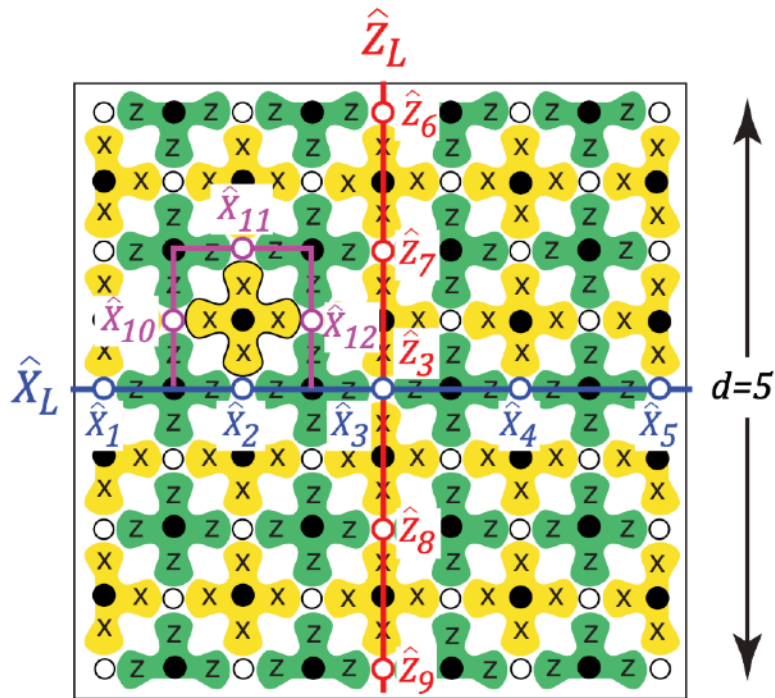
The logical error rate can be exponentially decreased by increasing the code distance, provided a physical error rate below a certain threshold.

target logical error rate $\sim 10^{-12}$

$$\Lambda \equiv \frac{\varepsilon_{d-2}}{\varepsilon_d} = \frac{p_{\text{thr}}}{p}$$

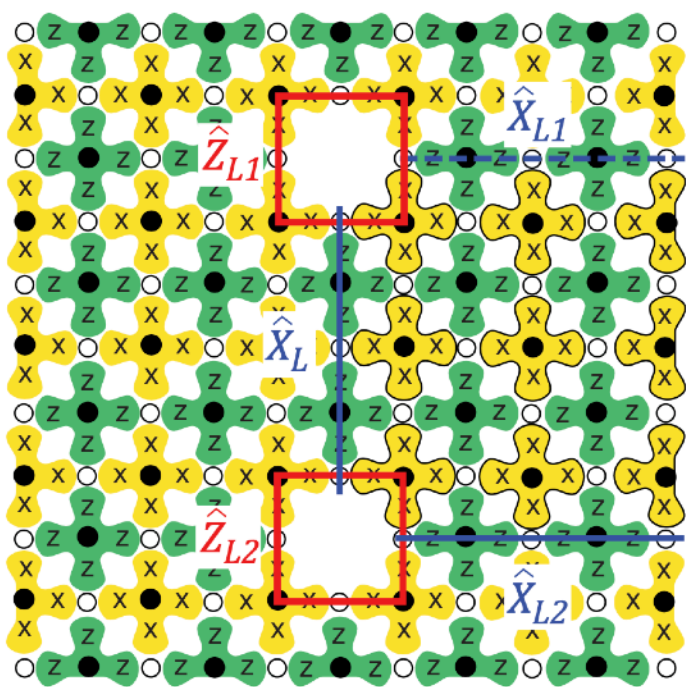
Surface Code

[Fow12]



Surface Code

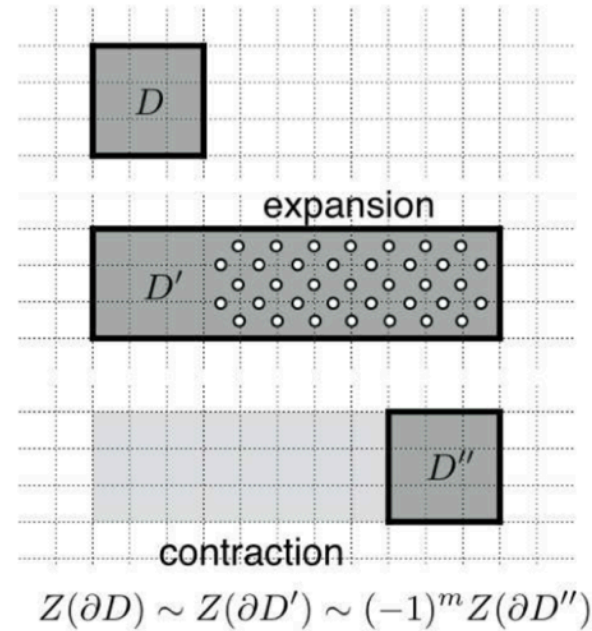
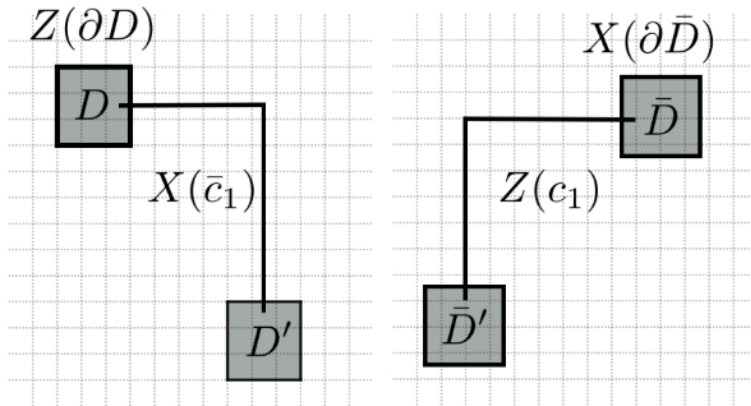
[Fow12]



similarity with “Go”?

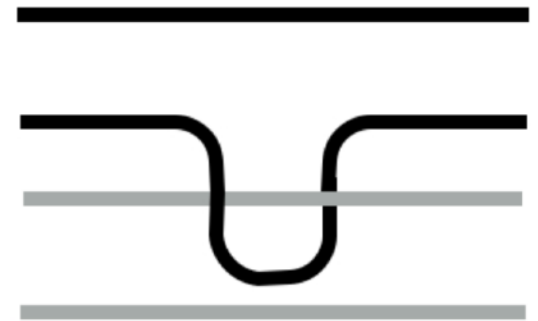
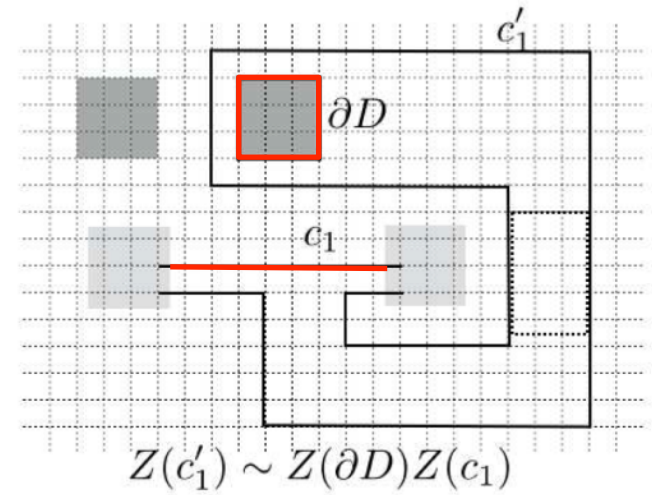
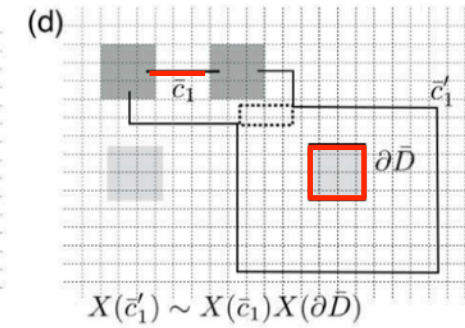
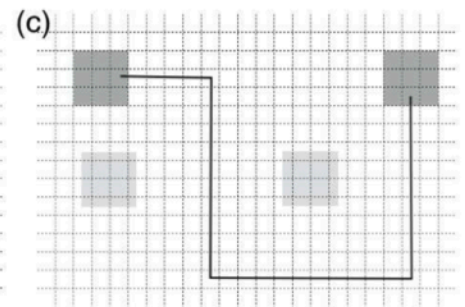
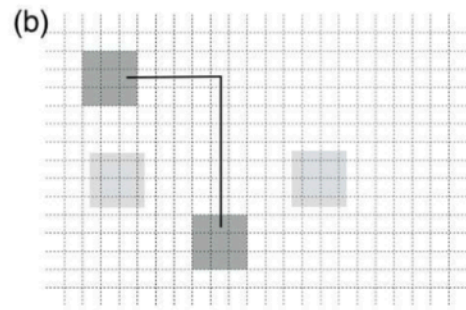
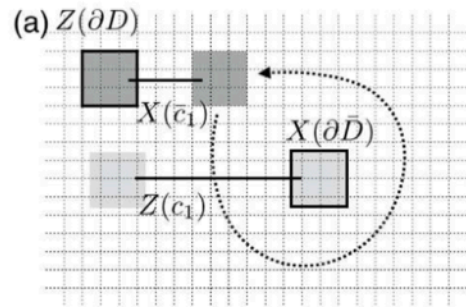
Surface Code

[Fuj15]



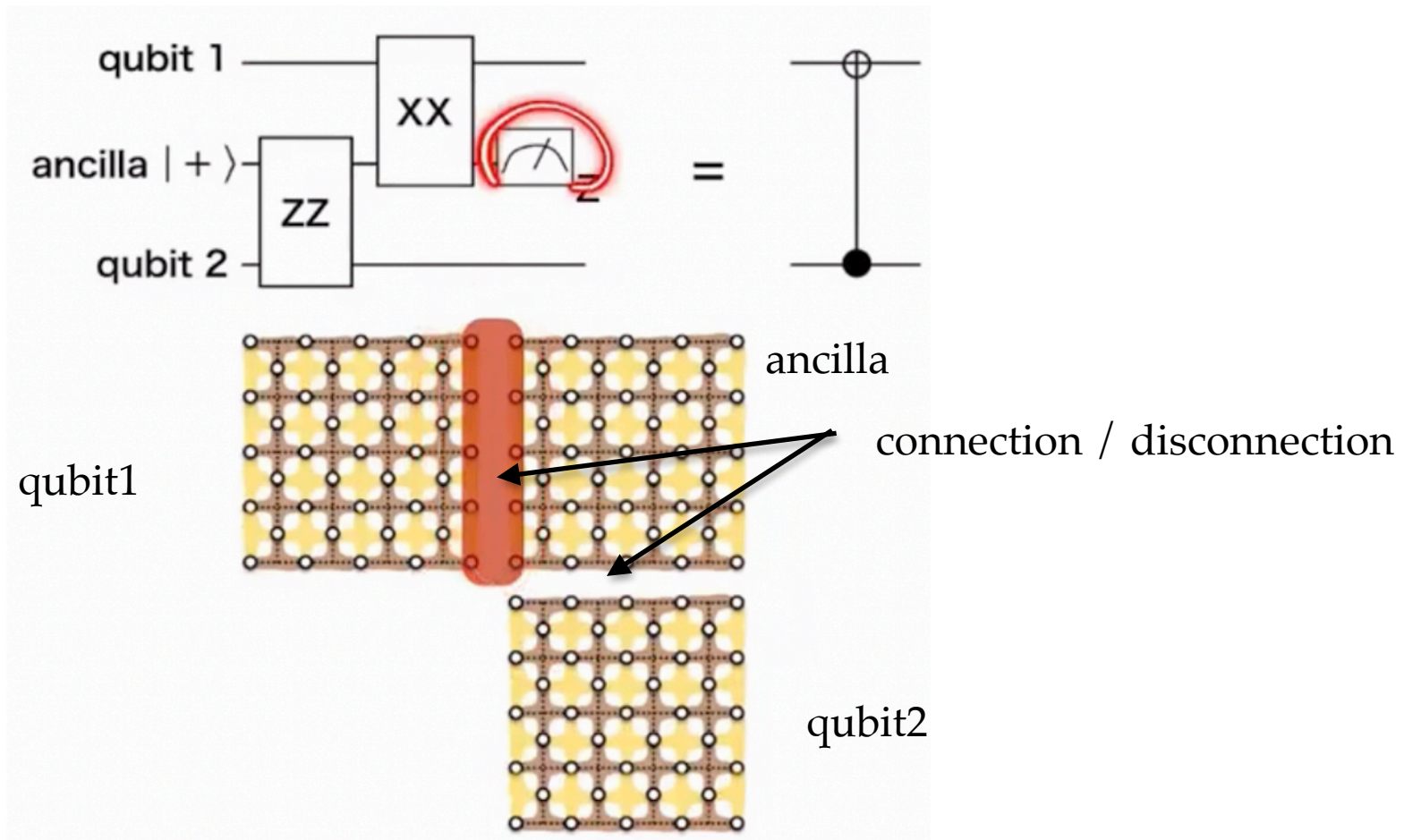
Braiding

[Fuj15]



Lattice Surgery

[Dom13]



Lattice Surgery

[Lit19]

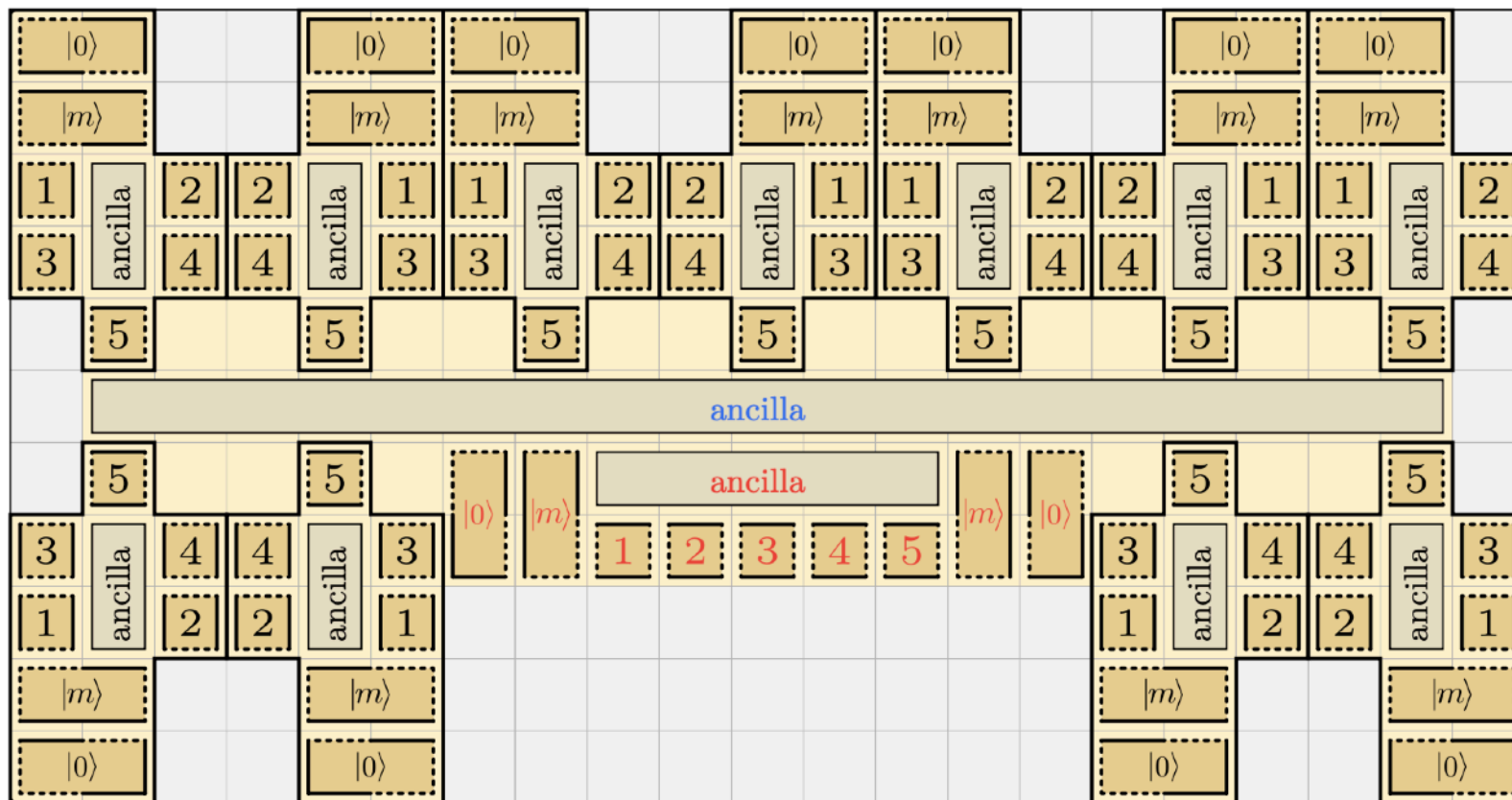
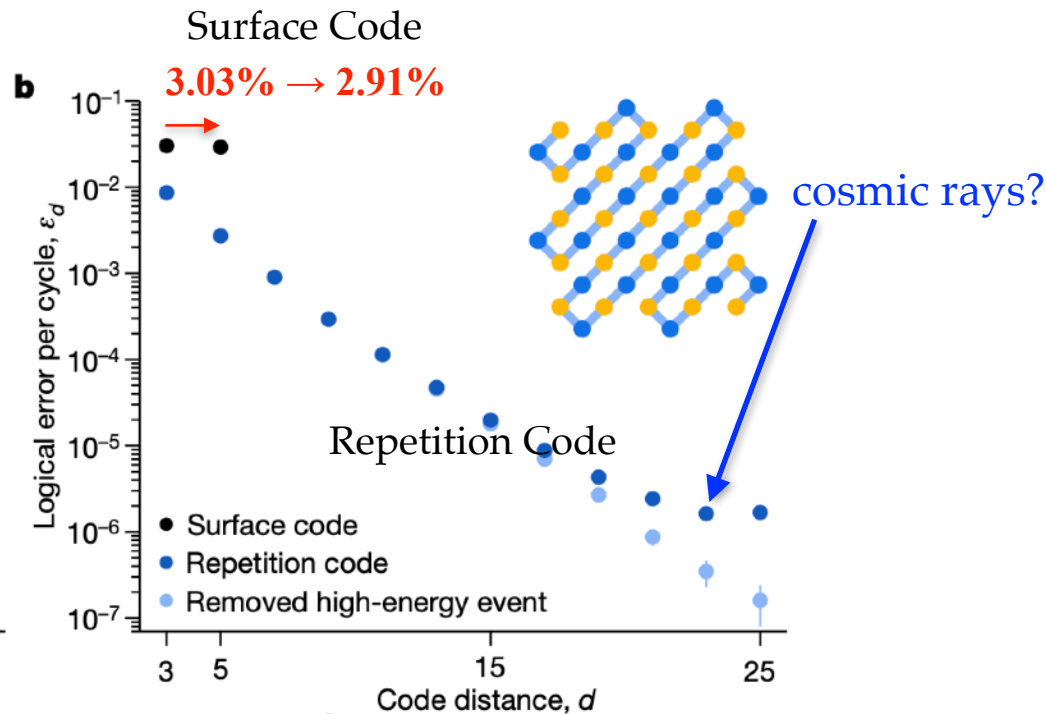
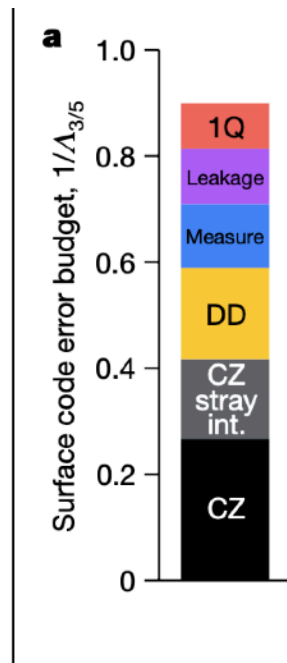
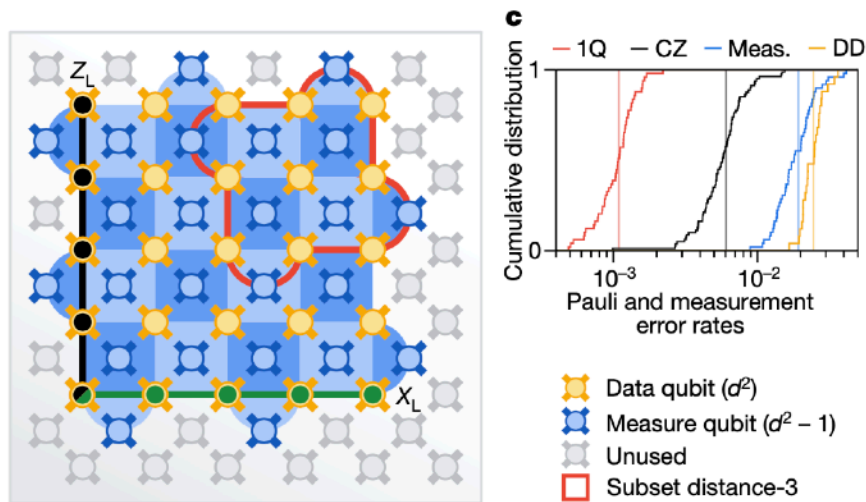
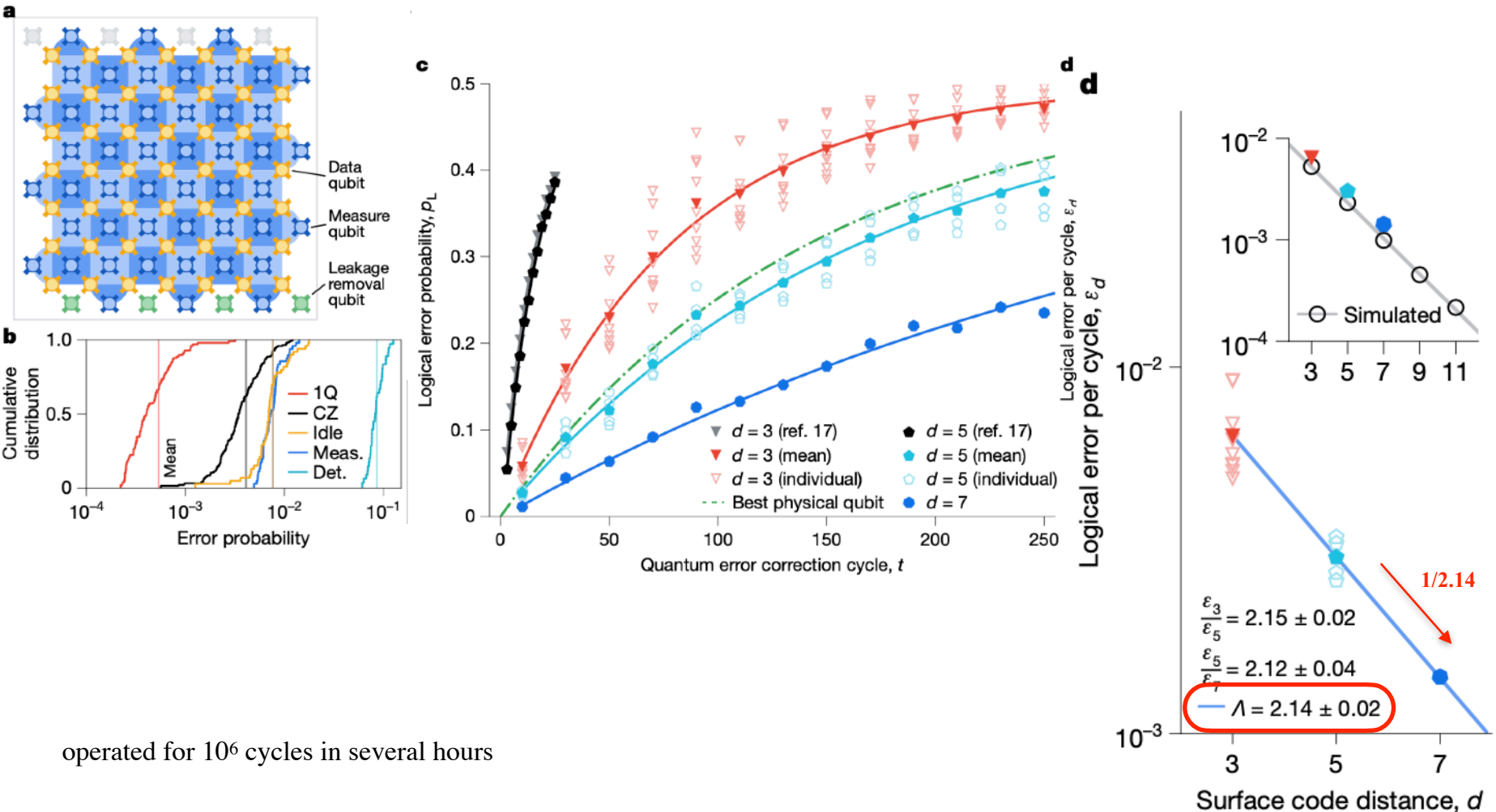


Figure 19: 176-tile block that can be used for 225-to-1 distillation. The qubits highlighted in red are used for the second level of the distillation protocol. The blue ancilla is used to move level-1 magic states into the two $|m\rangle$ - $|0\rangle$ blocks of the level-2 distillation.

Results

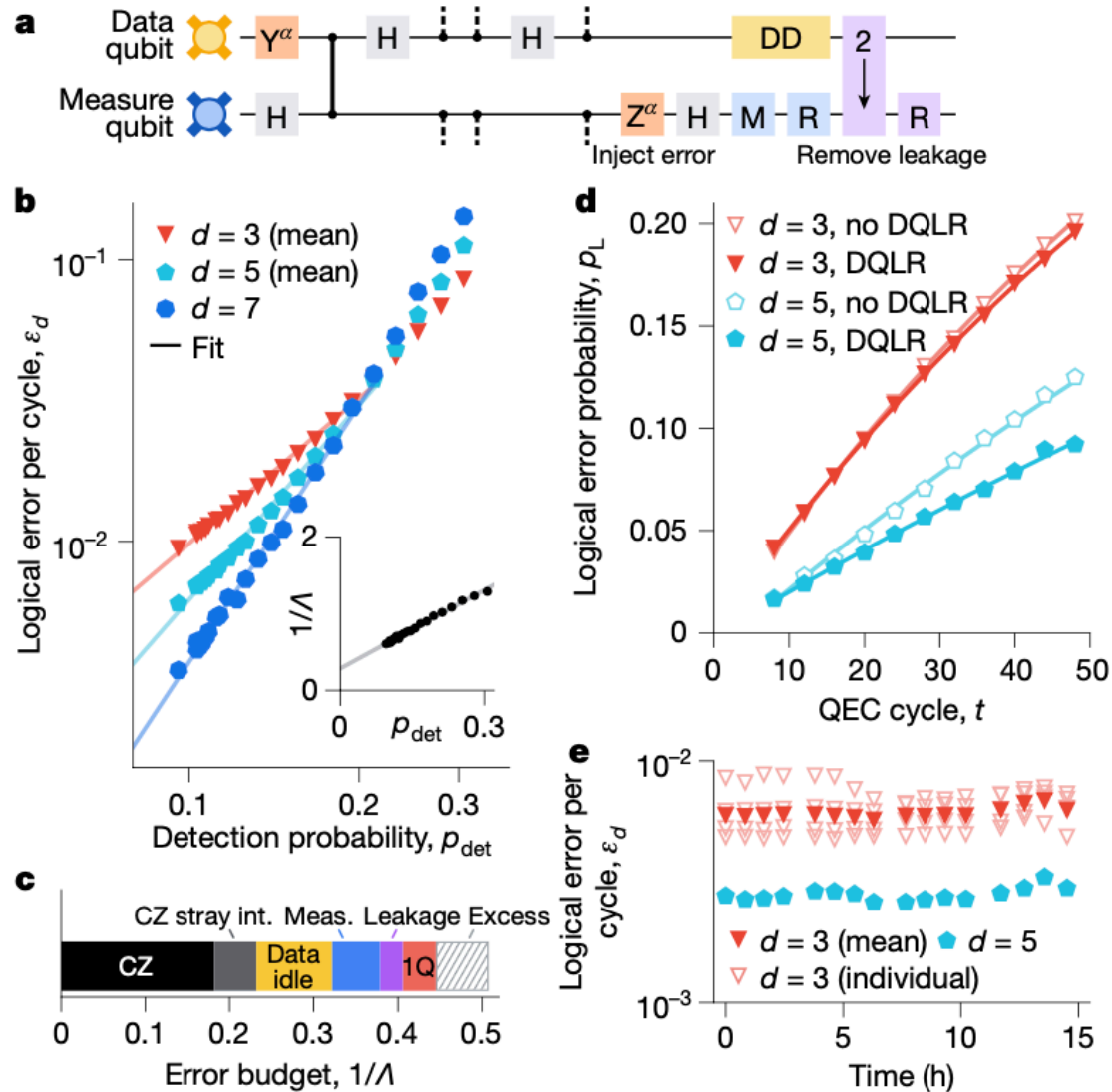




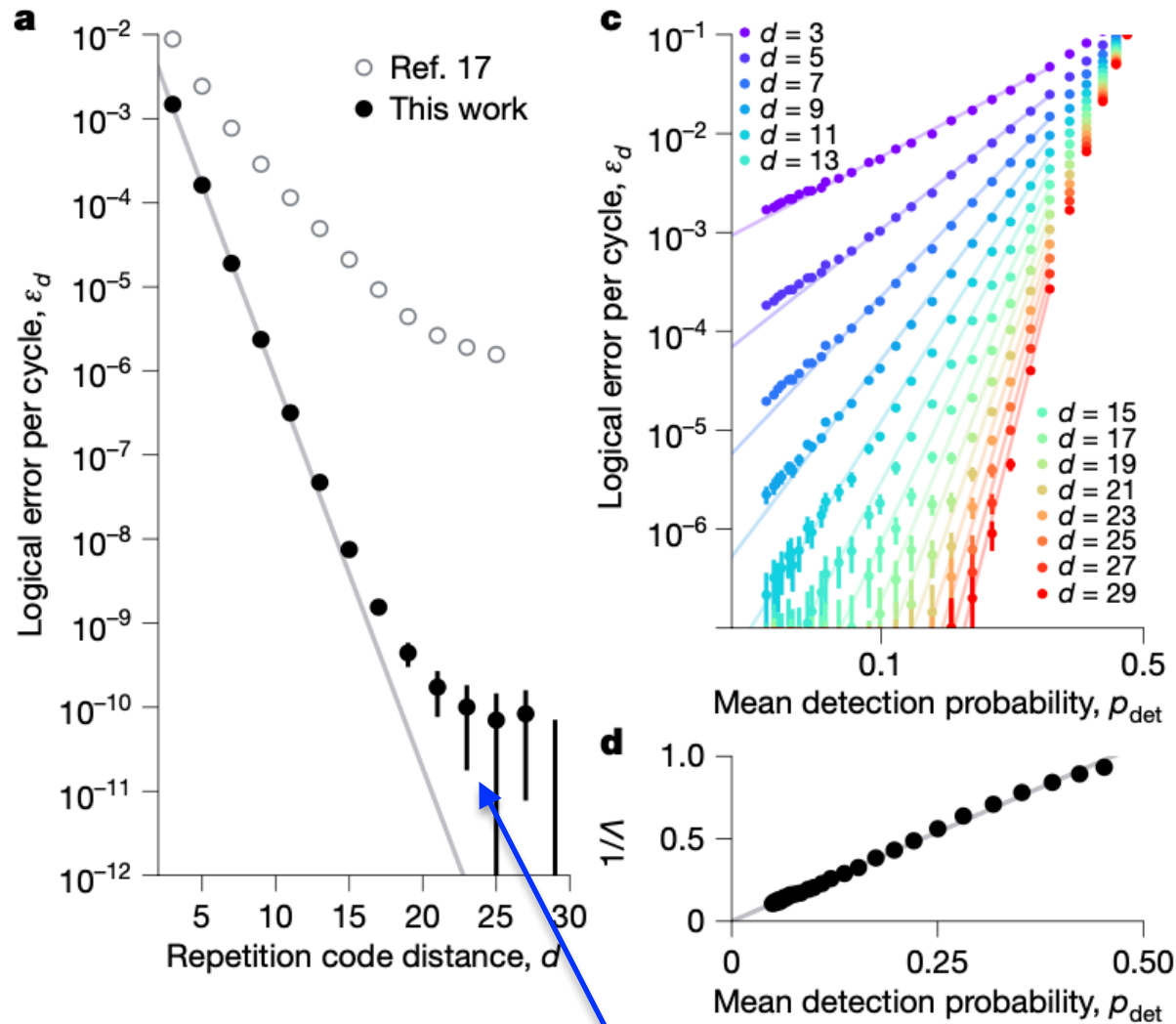
operated for 10^6 cycles in several hours

mean coherence time T_1 of 68 μs and T_2 of 89 μs ,

realtime decoding $\sim 24 \mu\text{s}$ by neural network, requiring $\sim 1.1 \mu\text{s}$



Comparison with a simulation using an error model



Repetition Code

saturation caused by
cosmic rays and radioactivity?

RSA2048 Encryption

Craig Gidney and Martin Eker
[arXiv:1905.09749](https://arxiv.org/abs/1905.09749) [quant-ph]

How to factor 2048 bit RSA integer in 8 hours using 2×10^7 noisy qubits

How many qubits for computation:	14,238
How many qubits for QEC:	22,325,184
Cost of a single entangling gate:	2 qubits and ~ 100 ns
Cost of a single entangling gate with QEC:	2,925 qubits and $\sim 27,000$ ns
	39,339 qubit initialization
	39,339 measurements
	153,090 CNOTS

Assumptions: time of a logical joint measurements with surface code
with $d = 27$ and time per measurement round $\sim 1\mu\text{sec}$.

.....

required number of T gate $\sim 10^{10}$

Connection to Other Fields

AdS/CFT and Holography

Holographic QCD

Quantum Gravity

Topological insulator

Tensor network

Computational Complexity

$MIP^*=RE$ Theorem (2020)

Personal Statements as a Summary

- Realization of FTQC for practical usage is still far away.
- Nevertheless, the conceptual and technical developments are very interesting.

For the nuclear physics field....

- introduction of the concept of generalized time evolution, generalized measurement, and stabilizer, ...
- adaption of nuclei to quantum computing (would be difficult)
- measuring nuclear excited states for finding answers to specific calculation? (provided precise Hamiltonian and physics models)
analogy to observing stellar objects for astrophysics

Much more to learn

Thank you for your attention!

References

- [Blu26] D. Bluvstein et al., *A fault-tolerant neutral-atom architecture for universal quantum computation*, [Nature 649, 39 \(2026\)](#).
- [Blu22] D. Bluvstein et al., *A quantum processor based on coherent transport of entangled atom arrays*, [Nature 604, 451 \(2022\)](#).
- [Blu23] D. Bluvstein et al., *Logical quantum processor based on reconfigurable atom arrays*, [Nature 626, 58 \(2023\)](#).
- [Bri09] H.J. Briegel et al., *Measurement-based quantum computation*, [Nature Physics 5, 19 \(2009\)](#).
- [Dom13] Dominic Horsman, *Surface code quantum computing by lattice surgery* [arXiv:1808.02892 \[quant-ph\]](#) (2013)
- [Fow12] A.G. Fowler et al., *Surface codes: Towards practical large-scale quantum computation*, [Phys. Rev. A 86, 032324 \(2012\)](#).
- [Fuj15] K. Fujii, *Quantum Computation with Topological Codes — from qubit to topological fault-tolerance —*, [arXiv:1504.01444 \[quant-ph\]](#) (2015).
- [Fuj20] Keisuke Fujii, 量子コンピューティングコース, [藤井研究室@Youtube](#), (2020).
- [Goo23] Google Quantum AI, *Suppressing quantum errors by scaling a surface code logical qubit*, [Nature 614, 676 \(2023\)](#).
- [Goo25] Google Quantum AI and Collaborators, *Quantum error correction below the surface code threshold*, [Nature 638, 920 \(2025\)](#).
- [Got97] Daniel Gottesman, PhD thesis@CIT, *Stabilizer Codes and Quantum Error Correction*, <https://arxiv.org/abs/quant-ph/9705052> (1997).
- [Hei06] M. Hein, *Multi-party entanglement in graph states*, [arXiv:quant-ph/0602096v1](#) (2006).
- [Hen20] L. Henriette et al., *Quantum Computing with Neutral Atoms*, [arXiv:2006.12326v2](#) (2020).
- [Ish12] 石坂、小川、河内、木村、林 「量子情報科学入門第2版」 (共立出版, 2012)。
- [Joz05] R. Joz, *An introduction to measurement based quantum computation*, [arXiv:quant-ph/0508124v2](#) (2005).
- Daniel Litinski, *A Game of Surface Codes: Large-Scale Quantum Computing with Lattice Surgery*, [arXiv:1808.02892v3 \[quant-ph\]](#) (2019)
- [ML} MaruLabo, [maruyamalecture@Youtube](#), <https://www.marulabo.net/>.
- [Moh26] M. Mohseni et al., *How to Build a Quantum Supercomputer: Scaling from Hundreds to Millions of Qubits*, [arXiv:2411.10406v3 \[quant-ph\]](#) (2026).
- [Nak24] 中田芳史 「量子情報理論 - 情報から物理現象の理解まで」 (朝倉書店、2024)。
- [Nie00] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
- [Saf16] M. Saffman, *Quantum computing with atomic qubits and Rydberg interactions: progress and challenges*, [J. Phys. B. At. Mol. Opt Phys. 4, 2020019 \(2016\)](#).
- [Wei21] Tzu-Chieh Wei, *Measurement-based quantum computation*, [arXiv:2109.10111 \(2021\)](#).
- [Win23] K. Wintersperger et al., *Neutral Atom Quantum Computing Hardware: Performance and End-User Perspective*, [arXiv:2304.14360v3](#) (2023).

Appendices

Operations

Hermitian / Projection / Unitary Operators

[Nak24]

Hermitian Operators

An operator $O \in \mathcal{B}(\mathcal{H})$ that satisfies $O^\dagger = O$

(Orthogonal) Projection Operators

A Hermitian operator $\Pi \in \mathcal{B}(\mathcal{H})$ that satisfies $\Pi^2 = \Pi$

Unitary Operators

An operator $U \in \mathcal{B}(\mathcal{H})$ that satisfies $U^\dagger U = U U^\dagger = I$

Unitary Group: $U(\mathcal{H}) = \left\{ U : \mathcal{H} \rightarrow \mathcal{H} \mid U^\dagger U = U U^\dagger = I \right\}$

$\mathcal{H}$: Hilbert space

$\mathcal{B}(\mathcal{H}) = \{A : \mathcal{H} \rightarrow \mathcal{H} \mid A \text{ is linear and bounded}\}$

1,n-Pauli Operators / Groups

1-qubit Pauli Operators

[Nak24]

Provided a normalized orthogonal basis $\{ |0\rangle, |1\rangle \}$ on a two dimensional Hilbert space $\mathbb{C}^2$, the following operators

$$\sigma_0 = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$\sigma_1 = X = |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$\sigma_2 = Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0|$$

$$\sigma_3 = Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

are called 1-qubit Pauli operators.

In the $\{ |0\rangle, |1\rangle \}$ representation

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1-qubit Pauli Group (P_1): the group generated* by 1-qubit Pauli operators.

n-qubit Pauli operators: $\sigma_j \equiv \bigotimes_{m=1}^n \sigma_{j_m}$ for $j = j_1 \dots j_n \in \{0,1,2,3\}^n$

n-qubit Pauli Group (P_n): the group generated by n = qubit Pauli operators.

* all the operators that are represented by any number of products of the generator operators.

***n*-qubit Clifford Group**

A group defined as

$$C_n := \{ C \in U(\mathbb{C}^2)^{\otimes n} : CPC^\dagger \in P_n, \forall P \in P_n \}$$

A group of unitary operators that preserves the *n*-qubit Pauli group

1 Qubit Operations

1 qubit Operators

Pauli Operators: P

$$I \equiv \sigma_0 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$X \equiv \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y \equiv \sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z \equiv \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Hermitian: $P^\dagger = P$

Unitary: $P^\dagger P = I$

$$X^2 = Y^2 = Z^2 = I$$

$$[\sigma_i, \sigma_j] = 2i\epsilon_{i,j,k}\sigma_k \quad (i, j, k = 1, 2, 3)$$

1 qubit Operators

Hadamard (H)

$$H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$HXH^\dagger = Z$$

$$HZH^\dagger = X$$

$$\text{Hermitian: } H^\dagger = H$$

$$\text{Unitary: } H^\dagger H = I$$

Phase (S)

$$S \equiv \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$S^2 = Z$$

$$SXS^\dagger = Y$$

$$SYS^\dagger = -X$$

$$SZS^\dagger = Z$$

$$\text{Unitary: } S^\dagger S = I$$

1 qubit Operators

$T(\pi/8)$ matrix (T)

$$T \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Unitary: $T^\dagger T = I$

$$TXT^\dagger = \frac{X + Y}{\sqrt{2}}$$

$$TYT^\dagger = \frac{Y - X}{\sqrt{2}}$$

$$TZT^\dagger = Z$$

$$T^2 = S$$

$$T^4 = Z$$

X Operator

$$X \equiv \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Hermitian: } X^\dagger = X$$

$$\text{Unitary: } X^\dagger X = I$$

$$\text{Eigenvalues: } \pm 1$$

$$X|+\rangle = |+\rangle$$

$$X|-\rangle = -|-\rangle \quad : X \text{ eigenstates}$$

$$X|0\rangle = |1\rangle$$

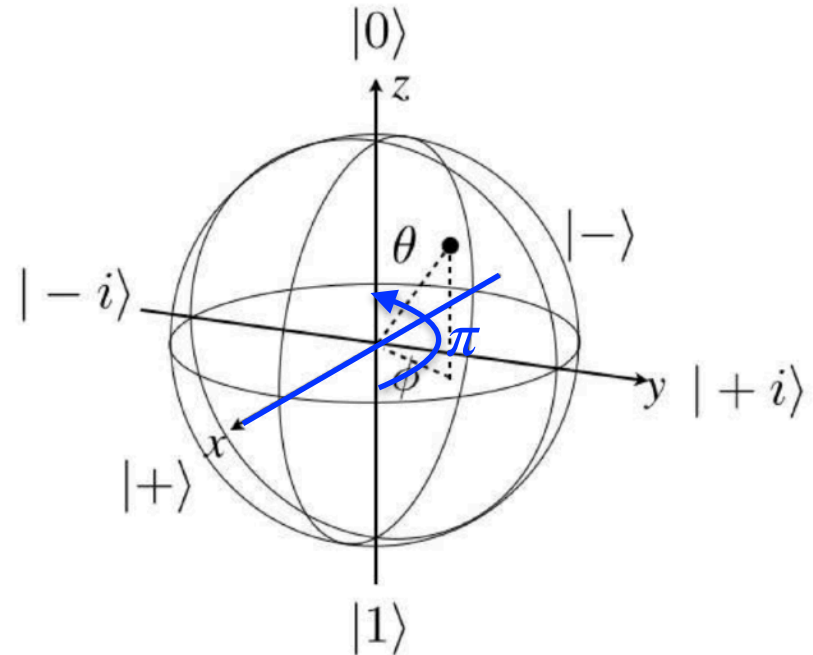
$$X|1\rangle = |0\rangle \quad : \text{bit-flip operator}$$

$$X|+i\rangle = i| -i\rangle$$

$$X| -i\rangle = -i| +i\rangle$$

$$-iX = R_x(\pi) = e^{-\frac{i\pi}{2}X}$$

$$X = HZH^\dagger = -SYS^\dagger = \frac{1}{2i}[Y, Z]$$



Y Operator

$$Y \equiv \sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Hermitian: $Y^\dagger = Y$

Unitary: $Y^\dagger Y = I$

Eigenvalues: ± 1

$$Y|+i\rangle = |+i\rangle$$

$$Y|-i\rangle = -|-i\rangle \quad : Y \text{ eigenstates}$$

$$Y|0\rangle = i|1\rangle$$

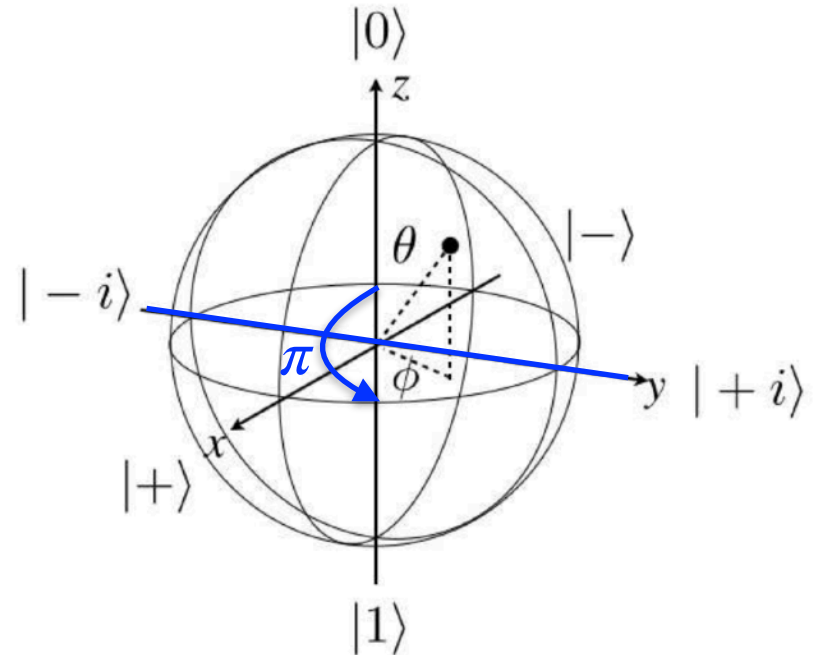
$$Y|1\rangle = -i|0\rangle$$

$$Y|+\rangle = -i|-\rangle$$

$$Y|-\rangle = i|+\rangle$$

$$-iY = R_y(\pi) = e^{-\frac{i\pi}{2}Y}$$

$$Y = -HYH^\dagger = SXS^\dagger = \frac{1}{2i}[Z, X]$$



Z Operator

$$Z \equiv \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\text{Hermitian: } Z^\dagger = Z$$

$$\text{Unitary: } Z^\dagger Z = I$$

$$\text{Eigenvalues: } \pm 1$$

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

: Z eigenstates

$$Z|+\rangle = |-\rangle$$

$$Z|-\rangle = |+\rangle$$

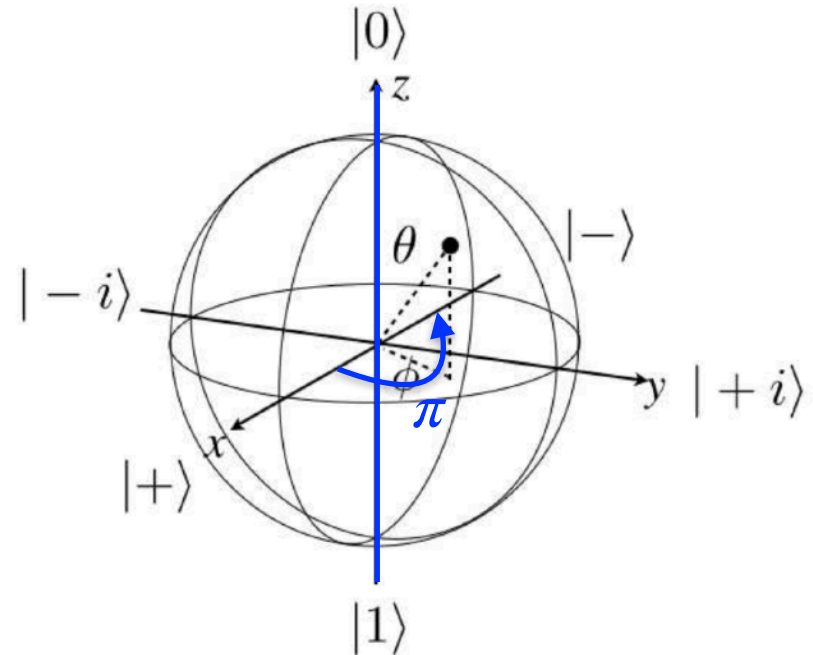
$$Z|+i\rangle = |-i\rangle$$

$$Z|-i\rangle = |+i\rangle$$

: phase-flip operator

$$-iZ = R_z(\pi) = e^{-\frac{i\pi}{2}Z}$$

$$Z = HXH^\dagger = SZS^\dagger = \frac{1}{2i}[X, Y] = S^2 = T^4$$



Hadamard Operator

$$H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hermitian: $H^\dagger = H$

Unitary: $H^\dagger H = I$

Eigenvalues: ± 1

$$H|0\rangle = |+\rangle$$

$$H|1\rangle = |-\rangle \quad \text{action on } X \text{ eigenstates}$$

$$H|+\rangle = |0\rangle \quad \text{action on } Z \text{ eigenstates}$$

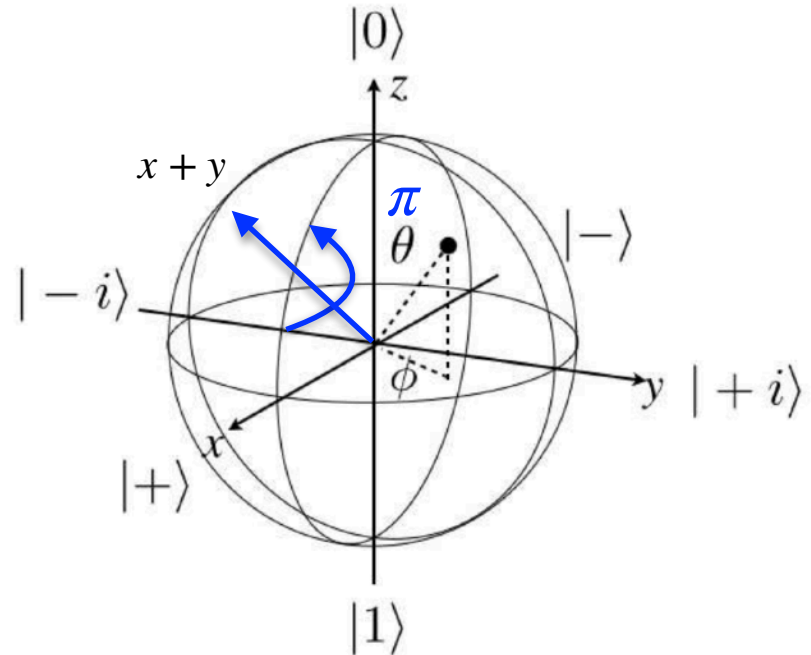
$$H|-\rangle = |1\rangle$$

$$H|+i\rangle = e^{\frac{i\pi}{4}}|-i\rangle$$

$$H|-i\rangle = e^{-\frac{i\pi}{4}}|+i\rangle$$

$$-iH = R_{\frac{(X+Y)}{\sqrt{2}}}(\pi) = e^{-\frac{i\pi}{2} \frac{(X+Y)}{\sqrt{2}}}$$

$$H = \frac{1}{\sqrt{2}}(X + Z)$$



exchange the x and z axes
(with flipping the y direction)

S Operator

$$S \equiv \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

Unitary: $S^\dagger S = I$

$$S|0\rangle = |0\rangle$$

$$S|1\rangle = i|1\rangle$$

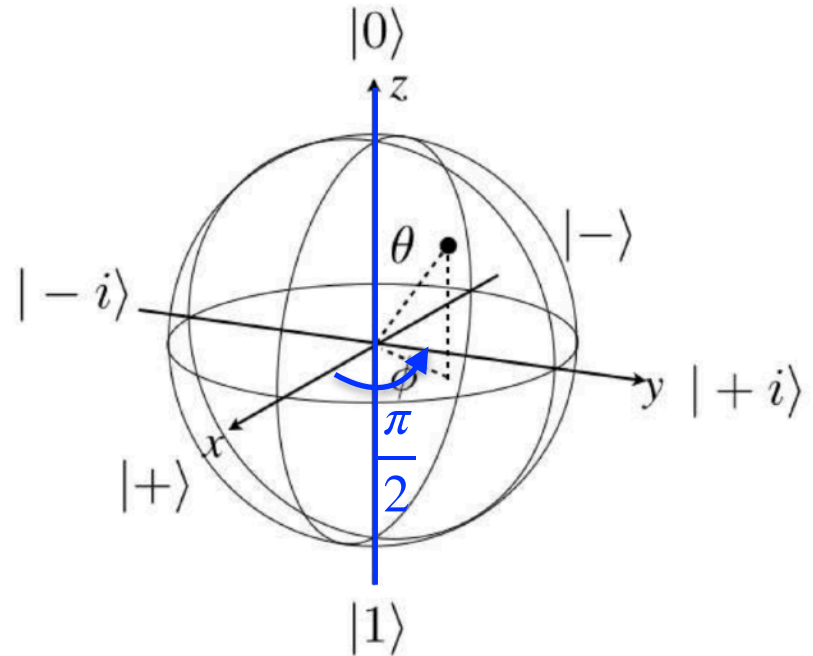
$$S|+\rangle = |+i\rangle$$

$$S|-\rangle = |-i\rangle$$

$$S|+i\rangle = |-\rangle$$

$$S|-i\rangle = |+\rangle$$

$$S = e^{\frac{i\pi}{4}} R_z\left(\frac{\pi}{2}\right) = e^{-\frac{i\pi}{4}} Z$$



T ($\pi/8$) Operator

$$T \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

$$\text{Unitary: } T^\dagger T = I$$

$$T|0\rangle = |0\rangle$$

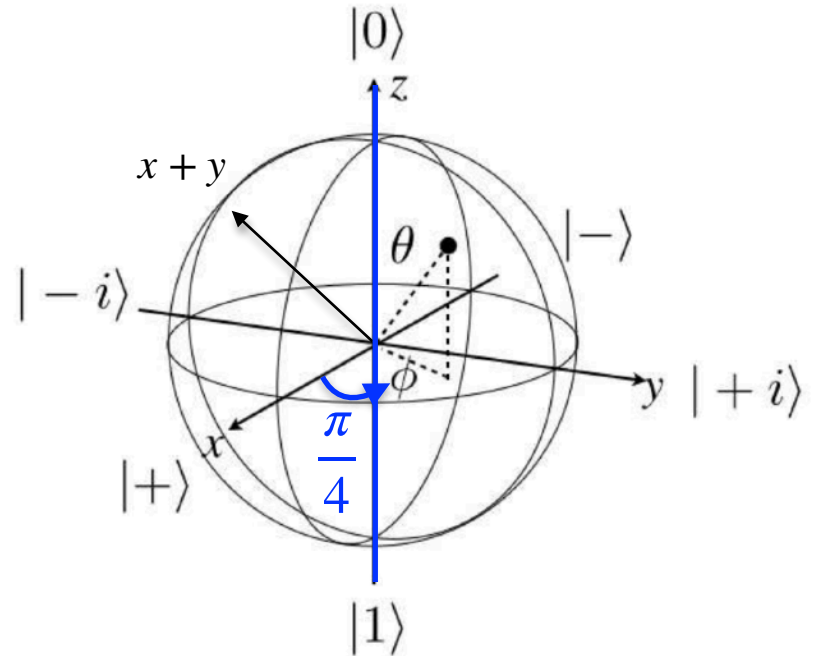
$$T|1\rangle = e^{\frac{i\pi}{4}}|1\rangle$$

$$TXT^\dagger = \frac{X+Y}{\sqrt{2}}$$

$$TYT^\dagger = \frac{Y-X}{\sqrt{2}}$$

$$TZT^\dagger = Z$$

$$T = e^{\frac{i\pi}{8}} R_z\left(\frac{\pi}{4}\right) = e^{-\frac{i\pi}{8}} Z \quad T^2 = S \quad T^4 = Z$$



1 qubit Rotation Operator

Rotation around the coordinate axes

$$R_x(\theta) = e^{-\frac{i\theta}{2}X}$$

$$R_y(\theta) = e^{-\frac{i\theta}{2}Y}$$

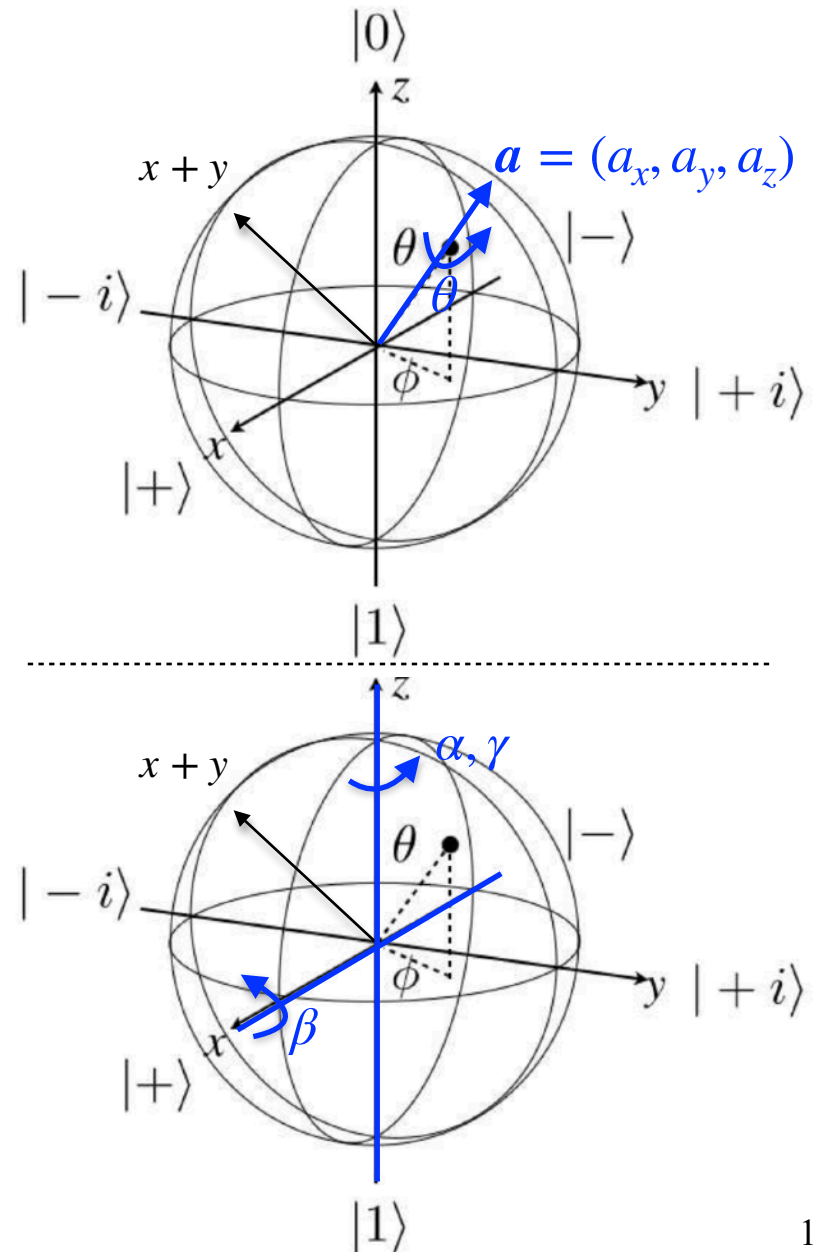
$$R_z(\theta) = e^{-\frac{i\theta}{2}Z}$$

Rotation around the $\mathbf{a} = (a_x, a_y, a_z)$ axis

$$R_{\mathbf{a}}(\theta) = e^{-\frac{i\theta}{2}(a_x X + a_y Y + a_z Z)}$$

Rotation with Euler angles

$$U = e^{i\phi} e^{-i\frac{\alpha}{2}Z} e^{-i\frac{\beta}{2}X} e^{-i\frac{\gamma}{2}Z}$$



1 qubit Operators: definition

Z-basis states

$$Z|0\rangle = |0\rangle \quad Z|1\rangle = -|1\rangle$$

$$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$$

$$Y|0\rangle = i|1\rangle \quad Y|1\rangle = -i|0\rangle$$

X-basis states

$$|+\rangle \equiv \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad |-\rangle \equiv \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$X|+\rangle = |+\rangle \quad X|-\rangle = -|-\rangle$$

Y-basis states

$$|+i\rangle \equiv \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \quad |-i\rangle \equiv \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$

$$Y|+i\rangle = |+i\rangle \quad Y|-i\rangle = -|-i\rangle$$

conversion by H and S

$$H|0\rangle = |+\rangle \quad H|1\rangle = |-\rangle$$

$$S|+\rangle = |+i\rangle \quad S|-\rangle = |-i\rangle$$

1 qubit Operators: Equations

$$X^2 = Y^2 = Z^2 = I$$

$$S^2 = Z$$

$$HXH = Z$$

$$HZH = X$$

1 qubit Operators: diagrams

$$|0\rangle \text{ --- } \boxed{Z} \text{ --- } |0\rangle$$

$$|1\rangle \text{ --- } \boxed{Z} \text{ --- } |1\rangle$$

$$|0\rangle \text{ --- } \boxed{X} \text{ --- } |1\rangle$$

$$|1\rangle \text{ --- } \boxed{X} \text{ --- } |0\rangle$$

$$|0\rangle \text{ --- } \boxed{H} \text{ --- } |+\rangle \text{ --- } \boxed{H} \text{ --- } |0\rangle$$

$$|1\rangle \text{ --- } \boxed{H} \text{ --- } |-\rangle \text{ --- } \boxed{H} \text{ --- } |1\rangle$$

$$\text{---} \boxed{S} \text{---} \boxed{S} \text{---} = \text{---} \boxed{Z} \text{---}$$

$$|\psi\rangle \text{ --- } \boxed{X} \text{---} \boxed{Y} \text{---} \boxed{Z} \text{---}$$

diagram: from left to right

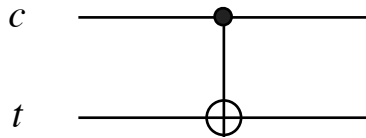
$$\overleftarrow{ZYX} |\psi\rangle$$

operators: from right to left

2 and more Qubit Operations

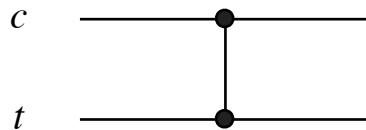
2 qubit Operators

Controlled-NOT (CNOT)



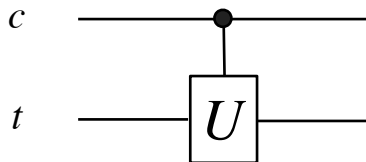
$$\Lambda_{c,t}(X) \equiv |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c X_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Controlled-Z (CZ)



$$\Lambda_{c,t}(Z) \equiv |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c Z_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

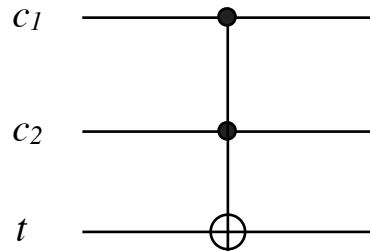
Controlled-U (any unitary operator)



$$\Lambda_{c,t}(U) \equiv |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c U_t = \begin{bmatrix} I_t & 0 \\ 0 & U_t \end{bmatrix}$$

3 qubit Operators

Toffoli gate (Controlled-Controlled-NOT)

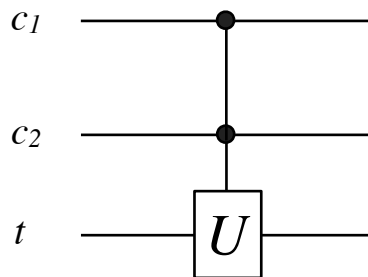


$$\Lambda_{c_1, c_2, t}^2(X) \equiv \left(I_{c_1} I_{c_2} - |1\rangle\langle 1|_{c_1} |1\rangle\langle 1|_{c_2} \right) I_t$$

$$+ |1\rangle\langle 1|_{c_1} |1\rangle\langle 1|_{c_2} X_t |0\rangle\langle 0|_c I_t + |1\rangle\langle 1|_c X_t$$

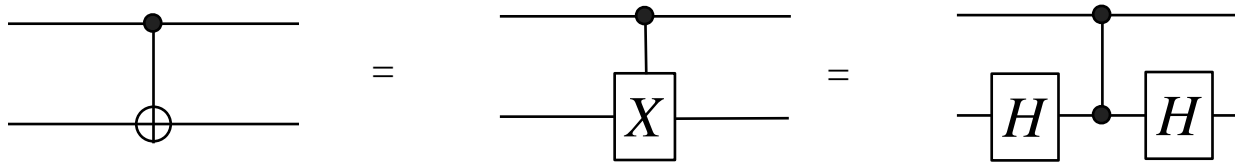
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Controlled-Controlled-Unitary

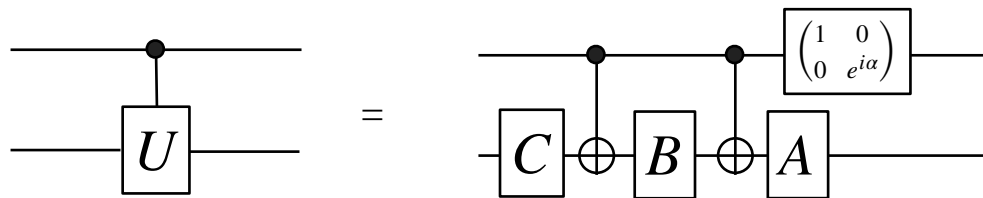


$$\Lambda^2(U)$$

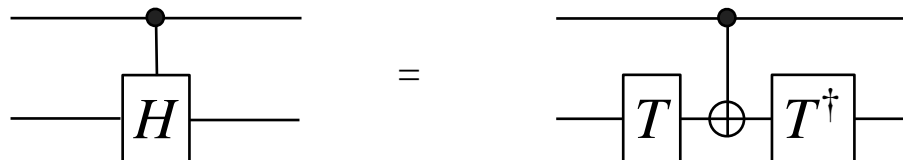
Qubit Diagram Expansions



$$HZH = X$$

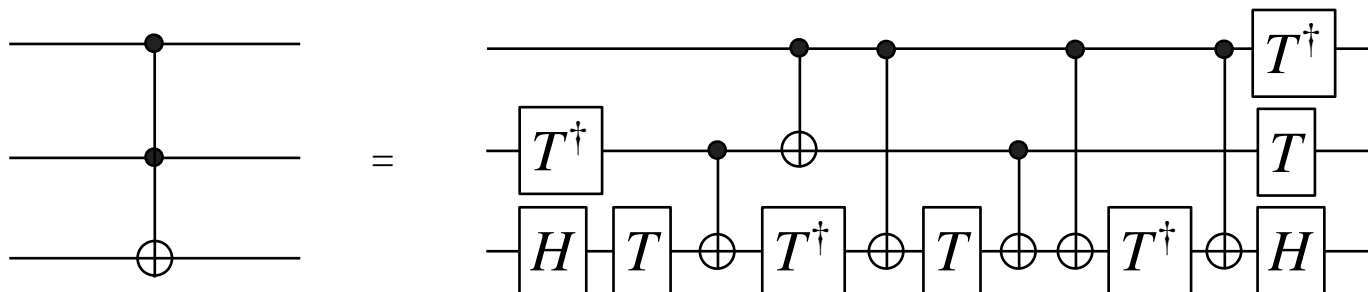
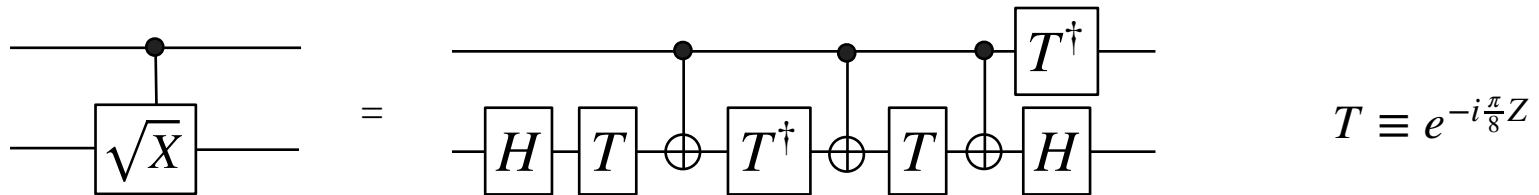
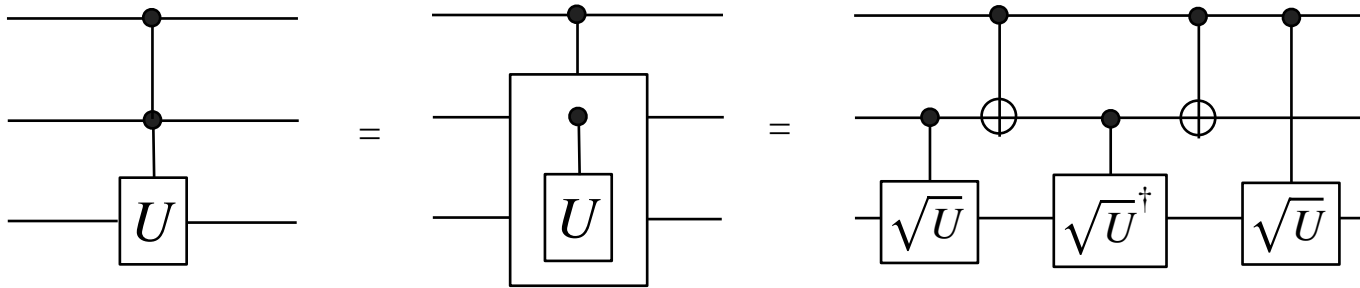


$$U = e^{i\alpha} A X B X C \quad \text{with} \quad ABC = I$$



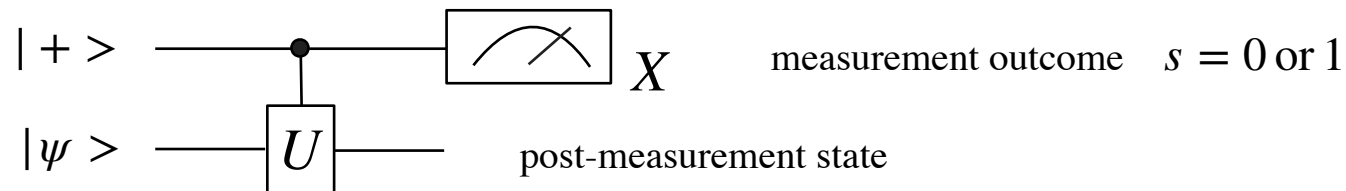
$$T \equiv e^{-i\frac{\pi}{8}Z}$$

Qubit Diagram Expansions



Quantum Calculations

Indirect Measurement

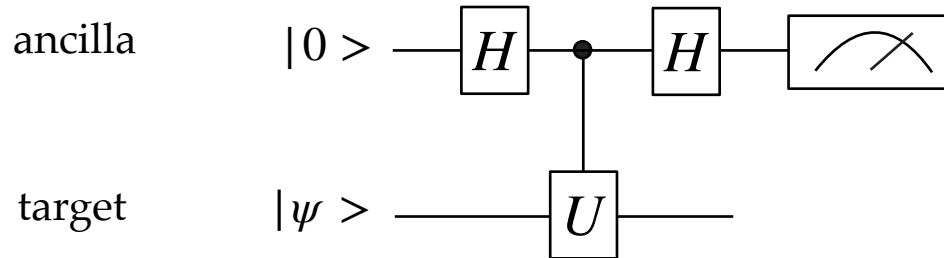


$$f(s)|\psi\rangle / \sqrt{\text{Tr}[f(s)|\psi\rangle\langle\psi|]}$$

$$f(s) \equiv \frac{I + (-1)^s A}{2} \quad \text{phase kick-back}$$

Hadamard Test

Hadamard Test



$$P(0) = \frac{1}{2} (1 + \text{Re}\langle\psi|U|\psi\rangle)$$

$$P(1) = \frac{1}{2} (1 - \text{Re}\langle\psi|U|\psi\rangle)$$

$$P(0) - P(1) = \text{Re}\langle\psi|U|\psi\rangle$$

The real part of $\langle\psi|U|\psi\rangle$ is measured.

If $|\psi\rangle$ is an eigenstate of U , the phase

Put a S after the first H of ancilla to get the Imaginary part.

Quantum Information

The Postulates of Quantum Mechanics

[Nie00]

Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (this is, a Hilbert space) known as the *state space* of the system. The system is completely described by its *state vector*, which is a unit vector in the system's state space

Postulate 2: The evolution of a *closed* quantum system is described by a *unitary transformation*. This is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2 ,

$$|\psi'\rangle = U|\psi\rangle.$$

Postulate 2': The time evolution of the state of a closed quantum system is described by the *Schrödinger equation*

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle.$$

In this equation, $\hbar$ is a physical constant known as *Plank's constant* whose value must be experimentally determined. The exact value is not important to us. In practice, it is common to absorb the factor $\hbar$ into H , effectively setting $\hbar = 1$. H is a fixed Hermitian operator known as the *Hamiltonian* of the closed system.

The Postulates of Quantum Mechanics

[Nie00]

Postulate 3: Quantum measurements are described by a collection $\{M_m\}$ of *measurement operators*. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by

$$p(m) = \langle \psi | M_m^\dagger M_m | \psi \rangle ,$$

and the state of the system after the measurement is

$$\frac{M_m |\psi\rangle}{\sqrt{\langle \psi | M_m^\dagger M_m | \psi \rangle}} .$$

The measurement operators satisfy the *completeness equation*,

$$\sum_m M_m^\dagger M_m = I .$$

The completeness equation expresses the fact that probabilities sum to one:

$$1 = \sum_m p(m) = \sum_m \langle \psi | M_m^\dagger M_m | \psi \rangle .$$

Postulate 4: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n , and system number i is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$.

Density Matrix

density matrix: a more general description of quantum states
(instead of state kets)

For one qubit quantum states

1 qubit pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

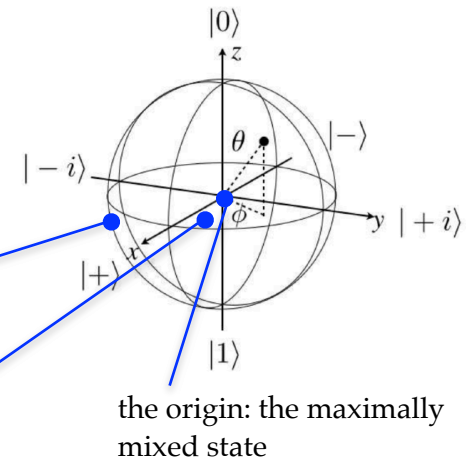
can be expressed by a ket for an appropriate basis

mapped to a point **on** the Bloch Sphere

1 qubit mixture state

= (classical) probabilistic mixture of states
(statistical ensemble of states)

mapped to a point **in** the Bloch Sphere



$$\rho = \frac{1}{2} (I + \mathbf{b} \cdot \boldsymbol{\sigma})$$

$\mathbf{b}$: Bloch vector

like a spin 1/2 polarization vector

For more general quantum states

$$\rho \equiv \sum_i p_j |\psi_j\rangle \langle \psi_j| \quad \text{for an ensemble } \{p_j, |\psi_j\rangle\}_j$$

[Nak24]

A density operator uniquely specifies a quantum state,
but its ensemble decomposition is generally not unique.

Density Matrix

[Nak24]

[Ish12]

Density Matrix (=Density Operator)

is defined as a Hermitian operator $\rho \in \mathcal{B}(\mathcal{H})$ that satisfies $\rho \geq 0$ and $\text{Tr}[\rho] = 1$.

$\rho \geq 0$: positive semidefinite

$$\langle v | \rho | v \rangle \geq 0 \text{ for } \forall v \in \mathcal{H}$$

$\text{Tr}[\rho] = 1$: normalization of total probability

Expectation value: physical observation under the operator A (Born rule on probability)

$$\langle A \rangle = \text{Tr} [A\rho]$$

Purity: $\text{Tr} [\rho^2]$

$$\frac{1}{d} \leq \text{Tr} [\rho^2] \leq 1 \quad d = \dim(\mathcal{H}): \text{dimension}$$

- The left equality holds for the completely mixed state

- The right equality holds for a pure state

pure state:

$$\Leftrightarrow \text{Tr} [\rho^2] = 1$$

$$\Leftrightarrow \rho^2 = \rho$$

$\mathcal{H}$: Hilbert space

$\mathcal{B}(\mathcal{H}) = \{A : \mathcal{H} \rightarrow \mathcal{H} \mid A \text{ is linear and bounded} \}$ 144

Density Matrix

[lsh12]

Probabilistic mixture of two density matrices: $\{p, 1 - p; \rho_1, \rho_2\}$

$$\rho := p\rho_1 + (1 - p)\rho_2 \quad (\text{convex combination})$$

Density Matrix

density matrix: a more general description of quantum states

density matrix (1 qubit)

1 qubit pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $\rho = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$

mapped to a point **on** the Bloch Sphere

$$\text{Tr}\rho^2 = 1$$

1 qubit mixture state

$$\rho = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1| = \begin{pmatrix} p_0 & 0 \\ 0 & p_1 \end{pmatrix}$$

= classical probabilistic mixture

$$p_0 + p_1 = 1$$

mapped to a point **in** the Bloch Sphere

$$\frac{1}{d} < \text{Tr}\rho^2 < 1$$

More generally

$$\rho = \sum_i p_i |i\rangle\langle i| \quad \sum_i p_i = 1 \quad \text{for an orthogonal complete set of states } |i\rangle \text{ (basis)}$$

expectation value of an operator A

$$\langle A \rangle = \text{Tr}[A\rho]$$

similar to a statistical ensemble of spin 1/2 states

Completely Positive Trace Conserving (CPTP) map

※ also called TPCP [Ish12]

A CPTP map is a map that satisfies the following three conditions.

Linearity:

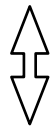
$$\Phi(a\rho_1 + b\rho_2) = a\Phi(\rho_1) + b\Phi(\rho_2)$$

Complete Positive: for any ancillary system A

$$(I_A \otimes \Phi)(\rho) \geq 0 \quad \forall \rho \geq 0 \quad \text{probability = positive semidefinite}$$

Trace Preserving:

$$\text{Tr}(\Phi(\rho)) = \text{Tr}(\rho) \quad \text{probability conservation}$$



equivalent

Kraus representation

There exists a set of operators that satisfies

$$\begin{aligned} \Phi(\rho) &= \sum_k E_k \rho E_k^\dagger \\ \sum_k E_k^\dagger E_k &= I \end{aligned}$$

POVM vs Projection Operator

[Nie00]

POVM

$$E_m = M_m^\dagger M_m$$

$$E_m \geq 0$$

$$p(m) = \langle \psi | E_m | \psi \rangle$$

$$p(m) = \text{Tr} [E_m \rho]$$

$$|\psi\rangle \rightarrow |\psi'\rangle = \frac{M_m |\psi\rangle}{\sqrt{p(m)}}$$

$$\rho \rightarrow \rho' = \frac{M_m \rho M_m^\dagger}{\text{Tr} [E_m \rho]}$$

$$\sum_m E_m = I \quad \sum_m p(m) = 1$$

Orthogonal Projection Operators

$$P_m = P_m^\dagger P_m = P_m^2$$

$$P_m \geq 0$$

$$p(m) = \langle \psi | P_m | \psi \rangle$$

$$p(m) = \text{Tr} [P_m \rho]$$

$$|\psi\rangle \rightarrow |\phi_m\rangle = \frac{P_m |\psi\rangle}{\sqrt{p(m)}}$$

$$\rho \rightarrow \rho' = \frac{P_m \rho P_m}{\text{Tr} [P_m \rho]}$$

$$\sum_m P_m = I \quad \sum_m p(m) = 1$$

$$P_m = |\phi_m\rangle \langle \phi_m|$$

$$P_i P_j = \delta_{i,j} P_i$$

Indirect Measurement

[Nak24]
pp. 32

Naimark extension: existence of an isometry $V^{A \rightarrow AE}$ that satisfies

$$M_j^A = (V^{A \rightarrow AE})^\dagger \left(I^A \otimes |e_j\rangle \langle e_j|^E \right) V^{A \rightarrow AE}$$

With expressing the isometry $V^{A \rightarrow AE}$ by U^{AE} , an unitary in the system AE , and $|0\rangle^E$, a pure state in the system E , as

$$V^{A \rightarrow AE} = U^{AE} (I^A \otimes |0\rangle^E)$$

the probabilities of the outcomes by the measurement POVM $\{M_j^A\}_{j=0}^{J-1}$ in the system A is

$$\begin{aligned} \text{Tr} [M_j^A \rho^A] &= \text{Tr} \left[(V^{A \rightarrow AE})^\dagger \left(I^A \otimes |e_j\rangle \langle e_j|^E \right) V^{A \rightarrow AE} \rho^A \right] \\ &= \text{Tr} \left[\left(I^A \otimes |e_j\rangle \langle e_j|^E \right) U^{AE} \left(\rho^A \otimes |0\rangle \langle 0|^E \right) U^{AE\dagger} \right] \\ &= \text{Tr} \left[|e_j\rangle \langle e_j|^E \text{Tr}_A \left[U^{AE} \left(\rho^A \otimes |0\rangle \langle 0|^E \right) U^{AE\dagger} \right] \right] \end{aligned}$$

M_j of system A can be measured without directly measuring A by adding a system E to A , time evolving by U^{AE} , tracing out A , and measure in E .

Schmidt Decomposition

[Nak24]
pp. 17

Schmidt Decomposition

For any quantum state $|\Psi\rangle^{AB}$ is the combined system AB , there exist normalized orthogonal bases $\left\{ |\alpha_j\rangle^A \right\}_{j=0}^{d_A-1}$ and $\left\{ |\alpha_j\rangle^B \right\}_{j=0}^{d_B-1}$ for the system A and B , respectively,

with which $|\Psi\rangle^{AB}$ can be decomposed as

$$|\Psi\rangle^{AB} = \sum_{j=0}^{d_{\min}-1} \sqrt{\lambda_j} |a_j\rangle^A |b_j\rangle^B ,$$

where $d_{A(B)}$ is the dimension of the system $A(B)$, $d_{\min} = \min \{d_A, d_B\}$, $\lambda_j \in \mathcal{R}_{\geq}$ and $\sum_j \lambda_j = 1$.

$\sqrt{\lambda_j}$: Schmidt coefficients

$|a_j\rangle, |b_j\rangle$: Schmidt basis

$\#\{j; \lambda_j \neq 0\}$: Schmidt rank (number of $\lambda_j \neq 0$)

Gottesman-Knill Theorem

[Nie00]

Computation with

Initialization

H , S , CNOT, Pauli-Operators

Pauli-measurements

may be efficiently simulated by a classical computer.

Neutral Atom Quantum Computing

Neutral Atom Logical Processor

448 atoms, hyperfine clock state of ^{87}Rb

two-dimensional acousto-optic deflector

non-destructive, spin-resolved qubit readout using a 1D optical lattice in the readout zone

spin-to-loss conversion $\rightarrow$ spin-to-position conversion

repeated error correction using distance-5 surface code logic cubit

Implementation of CNOT using Rydberg interaction

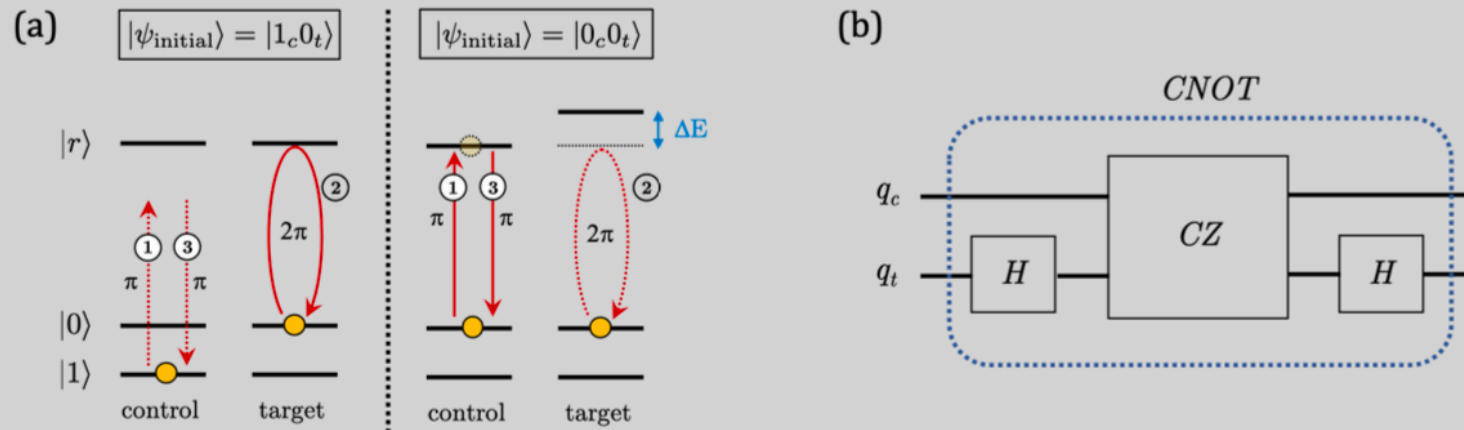
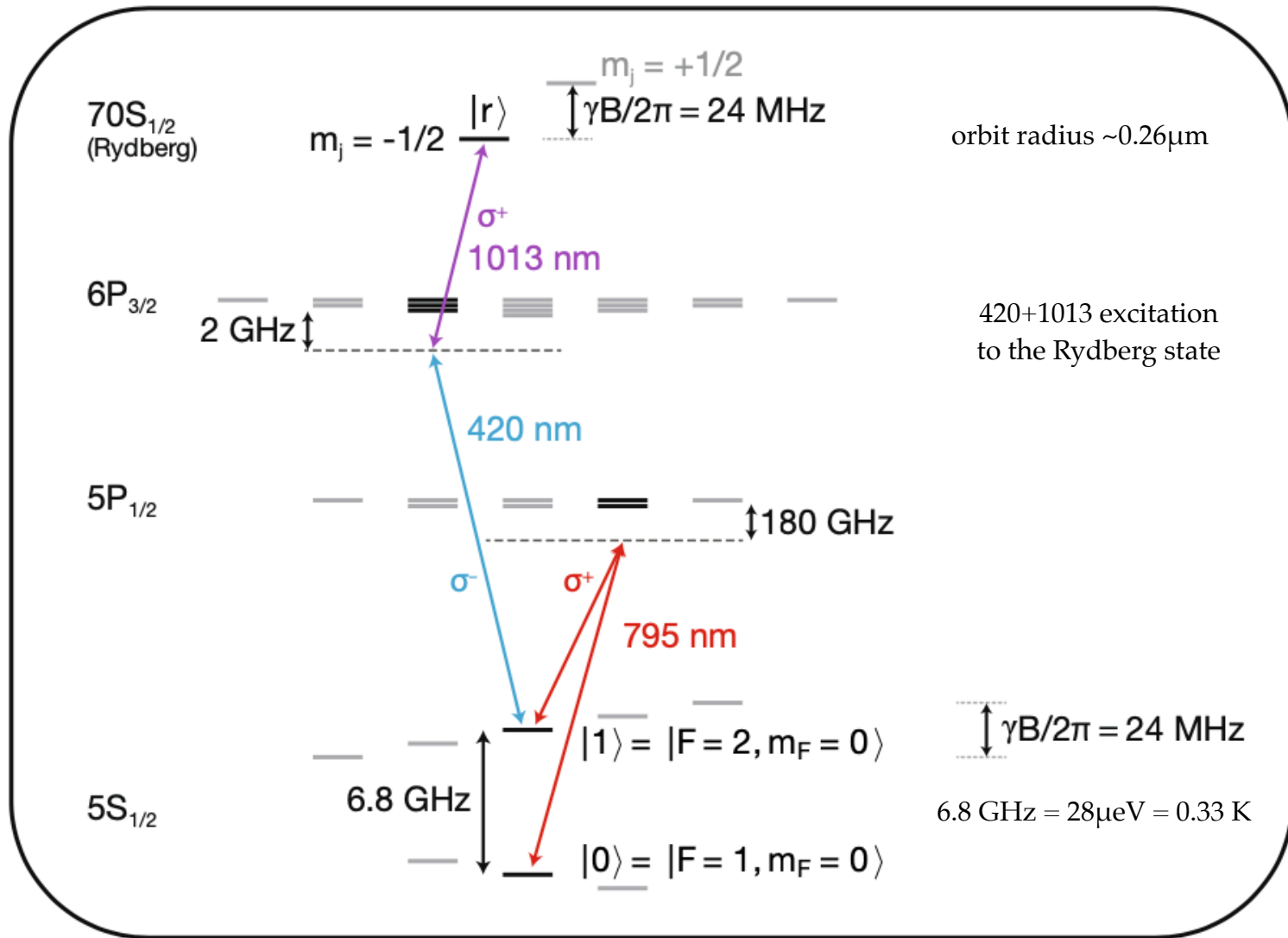


Figure B1.1 | Implementation of the CNOT gate using Rydberg interactions. (a) Principle of the controlled-Z gate based on dipolar Rydberg interaction. First a π pulse is applied on the control atom, then a 2π pulse on the target atom, and finally another π pulse on the control one. (b) Realization of a CNOT gate using a CZ gate and two Hadamard gates.

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = e^{i\pi} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = e^{i\pi} CZ$$

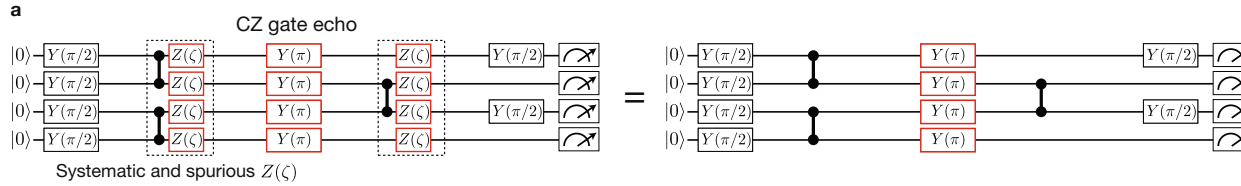
87Rb Atom Energy Diagram

[Blu22]

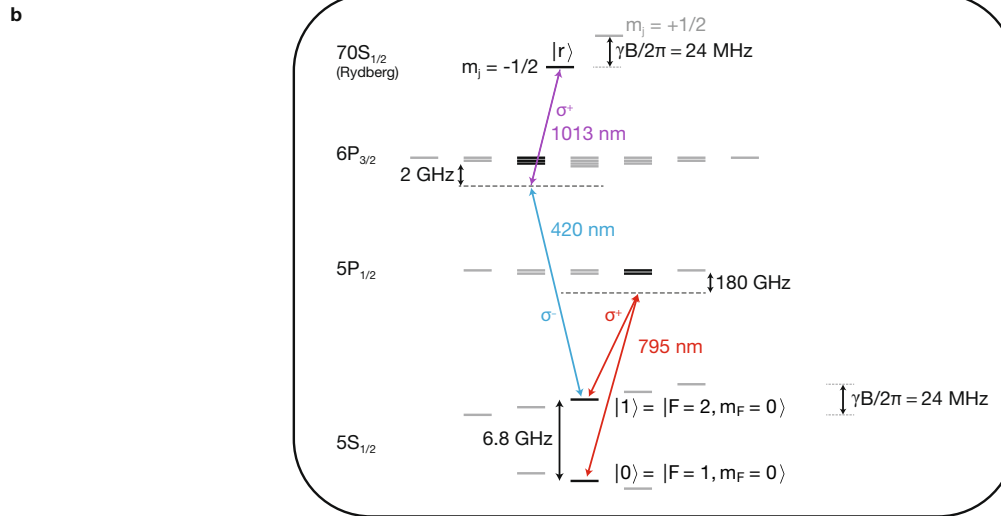


^{87}Rb neutral atom

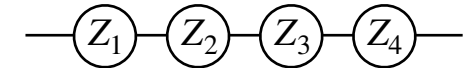
Quantum Operation with ^{87}Rb by Lasers



[Blu22]

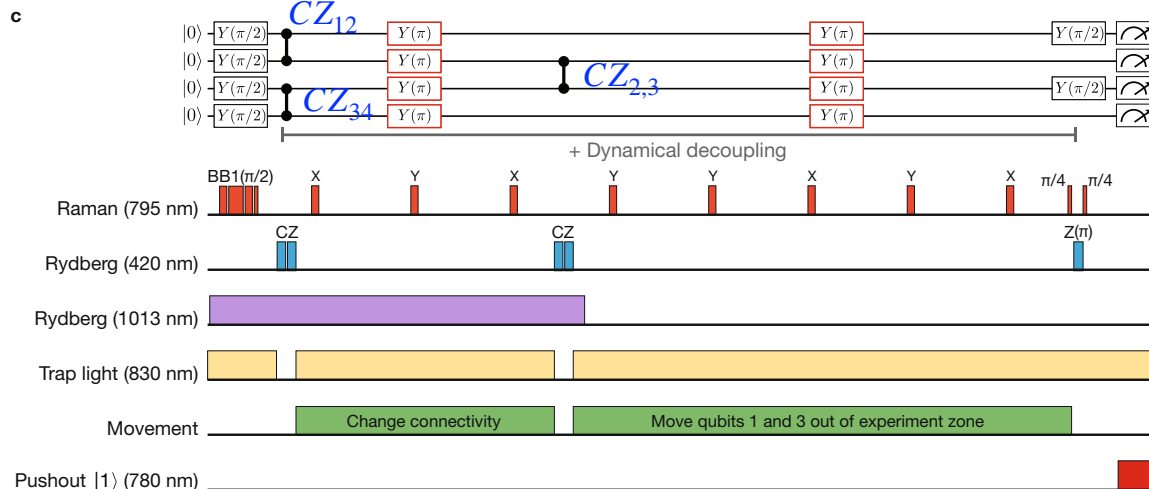


created graph-state



$$|G\rangle = \prod_{i,j \in E} \Lambda(Z)_{i,i+1} |+\rangle^{\otimes |V|}$$

$$\Lambda(Z)_{i,i+1} \equiv CZ_{i,i+1}$$



BB1: Broadband1 (Stephen Wimperis 1994)

establishes a high-precision control of rotation $R_X(\theta)$
by applying subsequent correction pulses $R_\phi(\pi) R_{3\phi}(2\pi) R_\phi(\pi)$

$$R_X(\theta) R_\phi(\pi) R_{3\phi}(2\pi) R_\phi(\pi) \quad \phi = \arccos\left(-\frac{\theta}{4\pi}\right)$$

up to the order $O(\varepsilon^6)$ for $R_X(\theta + \varepsilon)$

$XYXYXYX$ sequence: Dynamic Decoupling

cancels phase noises from magnetic field fluctuation, laser phase fluctuation and AC Stark shift

CZ-Y(π)-CZ sequence: CZ Gate Echo

cancel spurious $Z(\zeta)$ noise using spin echo (phase compensation).

$$Y(\pi) Z Y(\pi)^\dagger = -Z$$

※ They are originated from NMR technology.

$$R_y\left(\frac{\pi}{2}\right) \text{ for } Z \rightarrow X \text{ base conversion } (|0,1\rangle \rightarrow |\pm\rangle)$$

Rydberg Blockade

[Win23]

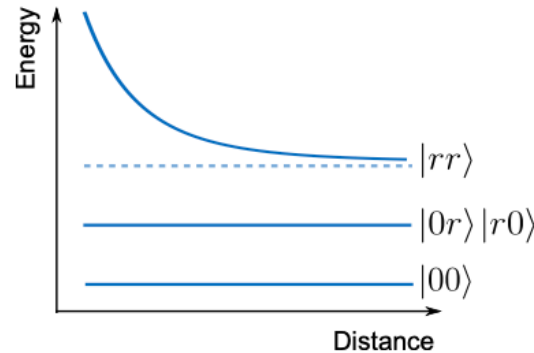


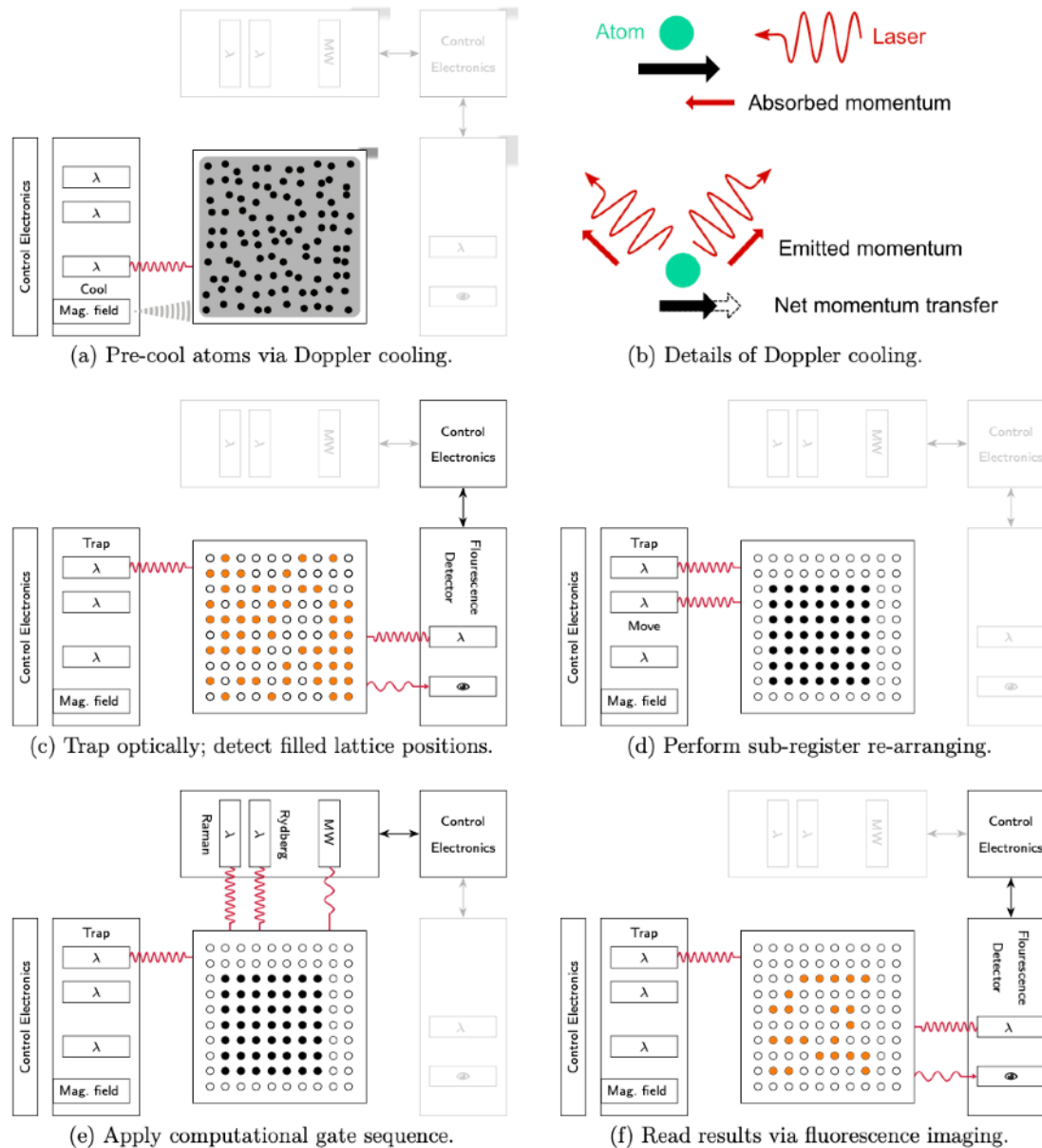
Figure 4: Illustration of Rydberg blockade: The energy of two atoms being excited to the Rydberg state $|r\rangle$ diverges with their inverse distance.

blockade radius $\sim$ distance of 2-3 lattice sites

CZ operation time $\sim 400\text{ns}$ (two pulses) $\sim 1\mu\text{s}$ (three pulses)

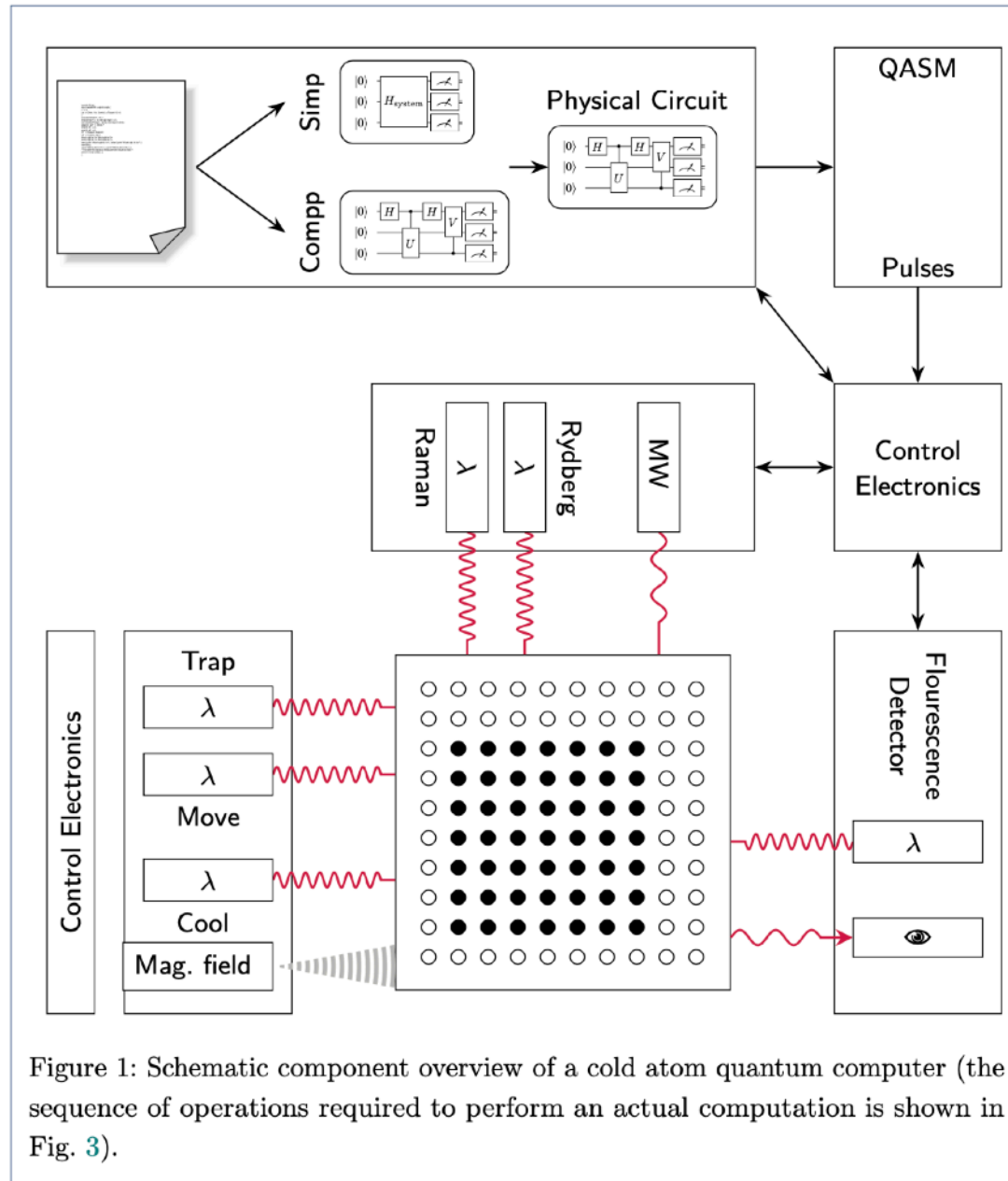
Quantum Operation with Neutral Atoms: Overall Steps

[Win23]



Quantum Operation with Neutral Atoms: The System

[Win23]



$^{87}\text{Rb}:$

natural abundance: 27.83%

half life: 4.9×10^{10} year

g.s. spin-parity: $3/2^-$

Rb

alkali metal

Periodic Table

H 1																	He 2						
Li 3	Be 4																	B 5	C 6	N 7	O 8	F 9	Ne 10
Na 11	Mg 12																	Al 13	Si 14	P 15	S 16	Cl 17	Ar 18
K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36						
Rb 37	Sr 38	Y 39	Zr 40	Nb 41	Mo 42	Tc 43	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50	Sb 51	Te 52	I 53	Xe 54						
Cs 55	Ba 56	La 57	Hf 72	Ta 73	W 74	Re 75	Os 76	Ir 77	Pt 78	Au 79	Hg 80	Tl 81	Pb 82	Bi 83	Po 84	At 85	Rn 86						
Fr 87	Ra 88	Ac 89	Rf 104	Db 105	Sg 106	Bh 107	Hs 108	Mt 109	Ds 110	Rg 111	Cn 112	Nh 113	Fl 114	Mc 115	Lv 116	Ts 117	Og 118						
La系		Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71								
Ac系		Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	No 102	Lr 103								

Hardware Setup

[Hen20]

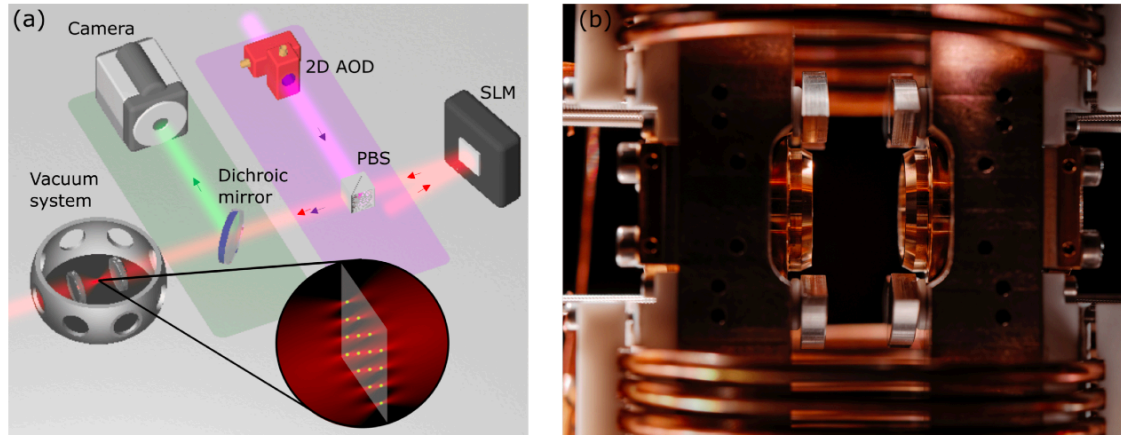
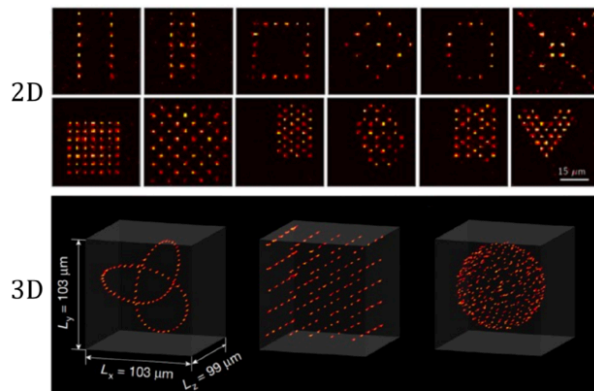


Figure 3: (a) Overview of the main hardware components constituting a quantum processor. The trapping laser light (in red) is shaped by the spatial light modulator (SLM) to produce multiple microtraps at the focal plane of the lens (see inset). The moving tweezers (in purple), dedicated to rearranging the atoms in the register, are controlled by a 2D acousto-optic laser beam deflector (AOD) and superimposed on the main trapping beam with a polarizing beam-splitter (PBS). The fluorescence light (in green) emitted by the atoms is split from the trapping laser light by a dichroic mirror and collected onto a camera. (b) Photography of the heart of a neutral-atom quantum co-processor. The register is prepared at the center of this setup.



^{87}Rb neutral atoms
in optical lattices

$$10^6 \text{ atoms/mm}^3 = 1 / 10 \mu\text{m}$$

$$\text{spot size} \sim 1 \mu\text{m}$$

0 or 1 atoms in an optical lattice site

[Win23]

typical distance of traps $\sim 3 \mu\text{m}$

trap lifetime $\sim 10\text{-}20 \text{ sec}$
(limited by the residual vacuum pressure)

Figure 4: Examples of tweezer arrays in 2D or 3D geometries, extracted from Ref [12].

Trap lifetime 10-60 sec at room temperature, 6000 sec at 4K, limited by residual gas.

Trap depth $\Leftrightarrow$ T1 decoherence trade-off, optimized by the trap laser detuning.

Rydberg state lifetime $\sim 150\mu\text{sec}$, limited by spontaneous decay and surrounding black-body radiation.

Fidelity of the two-qubit gate operations (highest value is ~ 0.995)

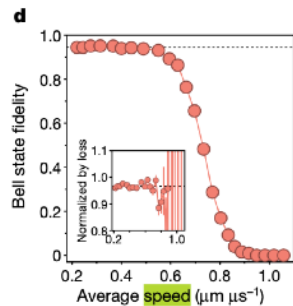
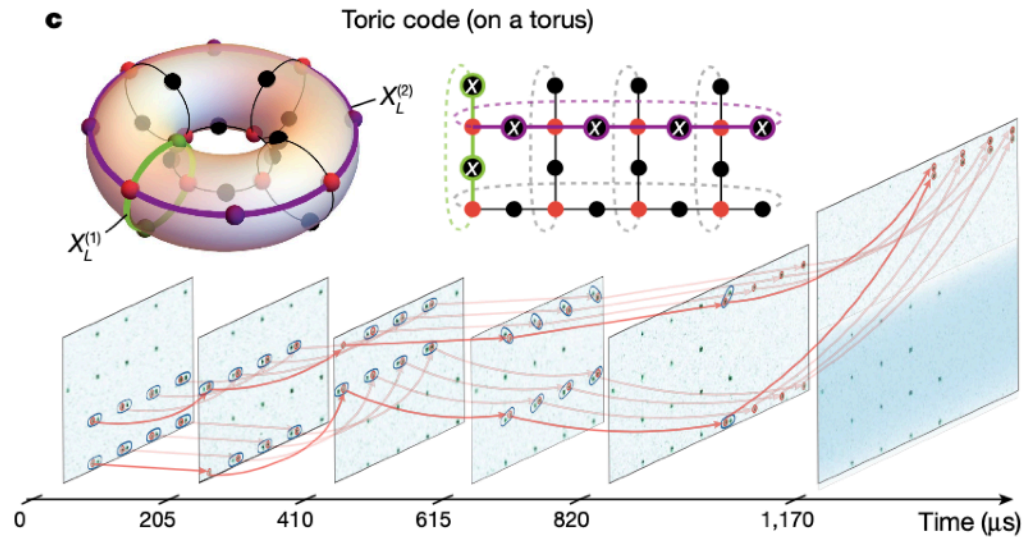
Typical Values

[Win23]

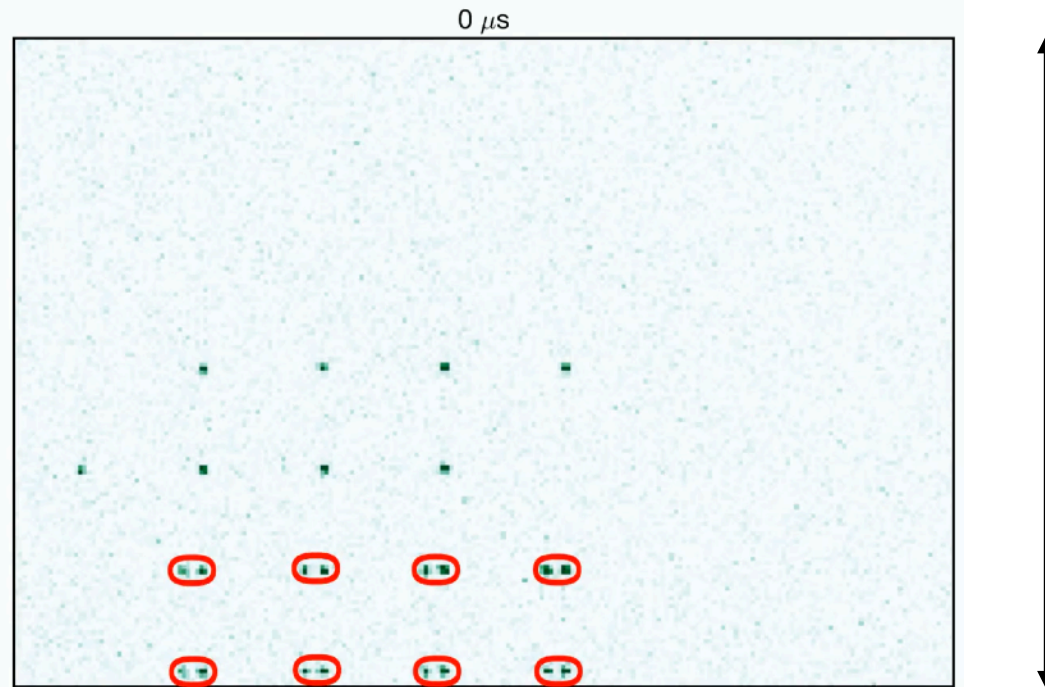
Parameter	Typical values today (near Future)
Qubits	
Amount	~ 100 [20, 21, 22, 23] (~ 1000 for 2024 [44, 45])
Connectivity	$10 : 1 - 20 : 1$ [20], ($50 : 1 - 100 : 1$ possible in principle)
Multiple states (i.e., qudit)	In principle possible
Lifetimes and Decoherence times	
Trap lifetime	$10 - 60$ s [29, 22, 30, 23] (up to 6000 s with cryostat [39])
Decoherence times (electronic spin)	$T_1 \sim 4$ s, $T_2 \sim 1$ s, $T_2^* \sim 4$ ms [21, 22]
Decoherence times (nuclear spin)	$T_1 \gg 5$ s, $T_2 \sim 40$ s, $T_2^* \gg 3$ s [23]
Native gates (gg-qubits)	
List of gates	Single-qubit rotations, $CZ \rightarrow CNOT$ [20, 38], $SWAP$, $CPHASE$ [37]
>2-qubit gates:	$CCZ \rightarrow Toffoli / CCNOT$ [20, 38], $C_k Z$, generalisation to k control qubits [37]
Parallelism	Apply the same gate on multiple Qubits simultaneously: Single-qubit rotations, CZ [21] (Multiple gates on multiple Qubits)
Fidelities of operations	
1-qubit gate	$0.996 - 0.999$ [21, 22]
2-qubit gate	$0.955 - 0.995$ [38, 22, 42]
Readout	$\gtrsim 0.95$ [46]
Preparation	Trap occupation probability (after rearrangement): 0.988 [47] Success probability for defect-free array ~ 0.75 , depending on size of array [47]
Execution times	
1-qubit gate	$\sim 2 \mu\text{s}$ (π -pulse) [21]
2-qubit gate	~ 400 ns (CZ) [21, 38]
Preparation (incl. rearrangement)	~ 400 ms [47]
Readout	~ 10 ms for fluorescence imaging [46], ($\sim 6 \mu\text{s}$ using a collective readout scheme) [46]
Installation and operation	
Required infrastructure	Vacuum cell and pumps, lasers, optical elements, microwave sources, signal generators and modulators, magnetic field coils. Cooling the setup with a cryostat can improve vacuum quality and increase trap lifetimes [39]. (Rack-level implementation possible [48])
Calibration	Rearrangement at beginning, no calibration of individual qubits necessary
Specificity	Shuttling operations [21]
Access	Via the cloud [49, 50] and on-premise
Quantum computing paradigm	Gate-based (digital) quantum computing, digital-analogue quantum computing, quantum annealing, analogue quantum simulation

Construction of a Toric code

[Blu22]



velocity $< 0.55 \mu\text{m}/\mu\text{s}$
(due to trap loss)



Surface Code

Toric code

Alexei Kitaev 1997

toric code は「計算機」ではなく「メモリ（量子情報の保存場所）」として提案されたものです。

一言でまとめると toric code はトーラス上の格子に定義されたトポロジカル量子誤り訂正コードであり surface code の原型である。

そしてその本質は、論理情報を局所的な量子ビットではなく、トーラスを一周する非局所的ループに保存することにあります。

局所的なノイズでは論理状態を壊しにくく、toric code は「トポロジカルに保護された量子メモリ」として提案されました。

Kitaev は toric code を提案したとき、単に量子メモリだけではなく、anyon を操作することで量子計算ができることを示しました。

Gottesman-Knill theorem

Theorem 1 Any Clifford operations, applied to the input state $|0\rangle^{\otimes n}$ followed by the Z measurements, can be simulated efficiently in the strong sense.

Any Clifford operations, applied to any product states of convex mixtures of the Pauli basis states, followed by Z measurements can be efficiently simulated in the weak sense.

Graph States

Theorem 1