Lattice Study of Glue-Dynamics
- Gauge Dependent Objects -

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Quark Confinement and the Hadron Spectrum VI
21 September,2004, Tanka Village

Plan of the Talk

1. Introduction
   – Gauge Fixing
2. Gluon Propagators
   – Zero temperature and Finite temperature
   – Landau Gauge and Coulomb gauge
3. Polyakov Loop Correlations
   – Finite Temperature
   – Zero Temperature
      • Finite Length Polyakov Loops

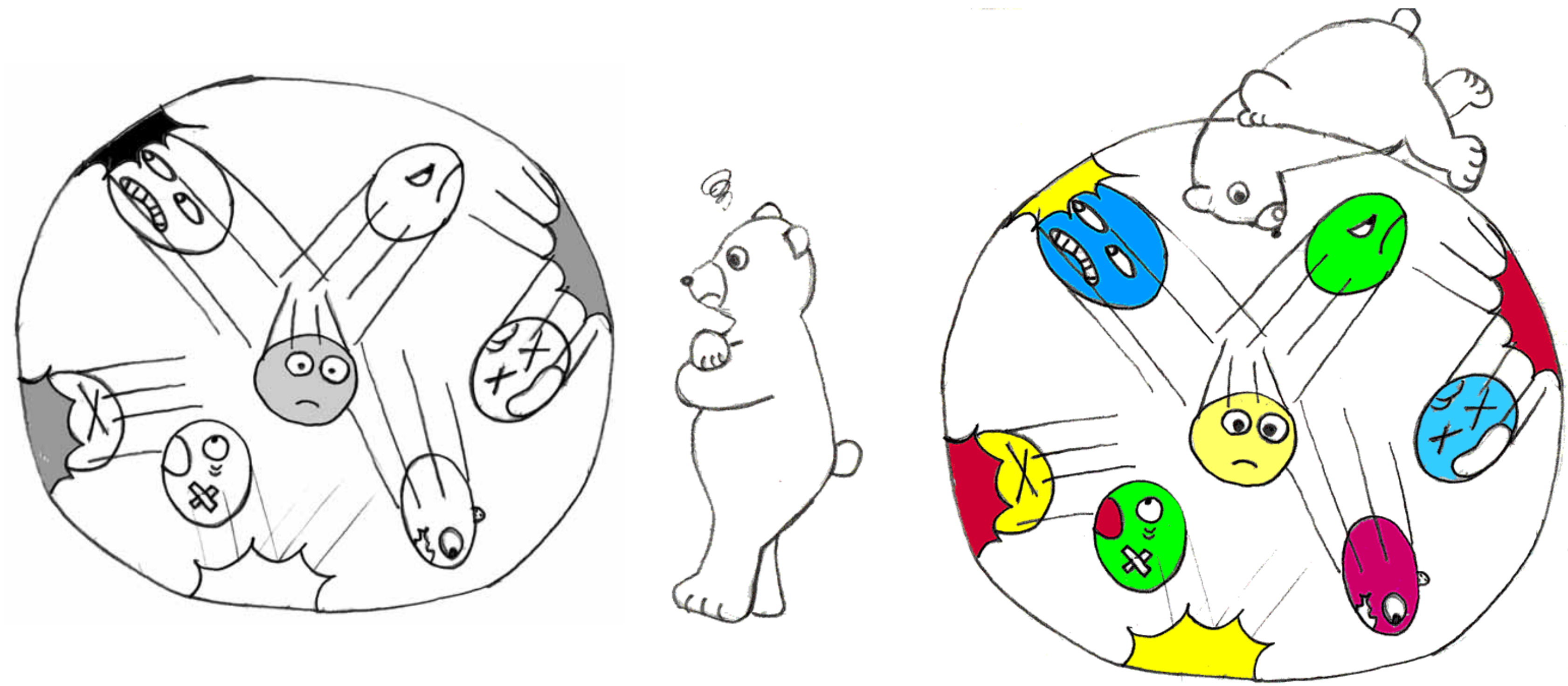
Preliminary

QCD Vacuum=Quarks+Gluons

Quarks and Gluons are confined.

QCD World is Colorful, but it looked Colorless.

Can we see Colorful Inside ?

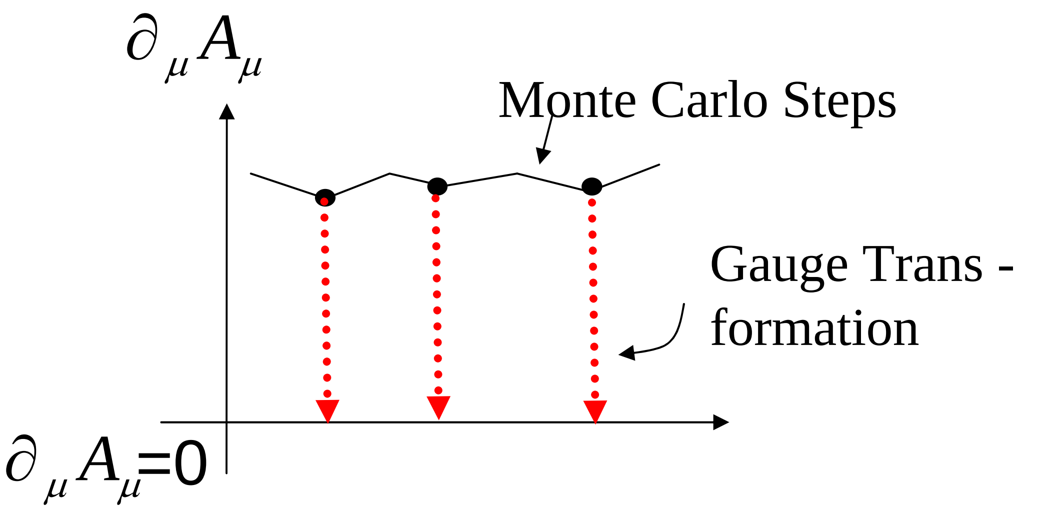


Gauge Fixing on the Lattice

- On the lattice, one needs not fix the gauge, but one can fix the gauge.
  - Wilson, Mandula
- There are Gribov Copies !

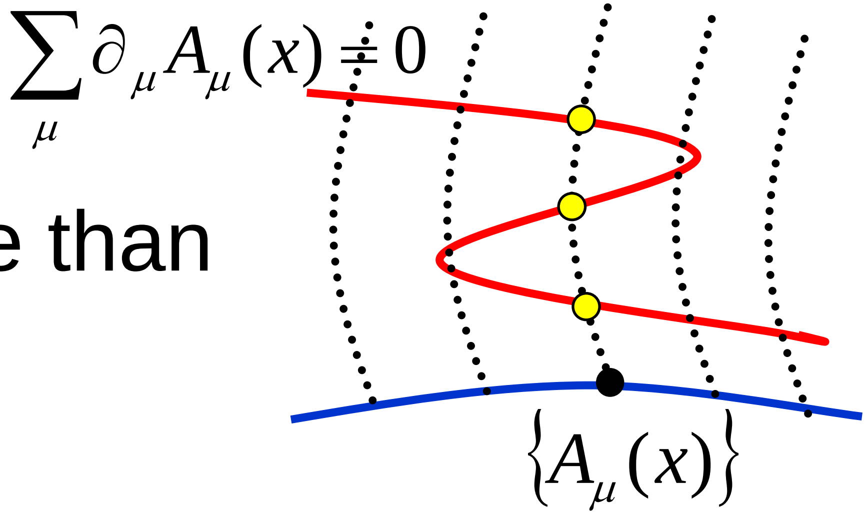
Nakamura and Plewnia, PL B255(1991) 273

A biased review
Nakamura,
Acta Physica Hungaria,
Heavy Ion Physics, (Gribov Memorial Volume) 9 (1999) 121



Wilson-Mandula Method

Gribov Ambiguity



- There can be more than one solution for

$$\sum_\mu \partial_\mu A_\mu(x) = 0$$

⇨ Gribov Copies

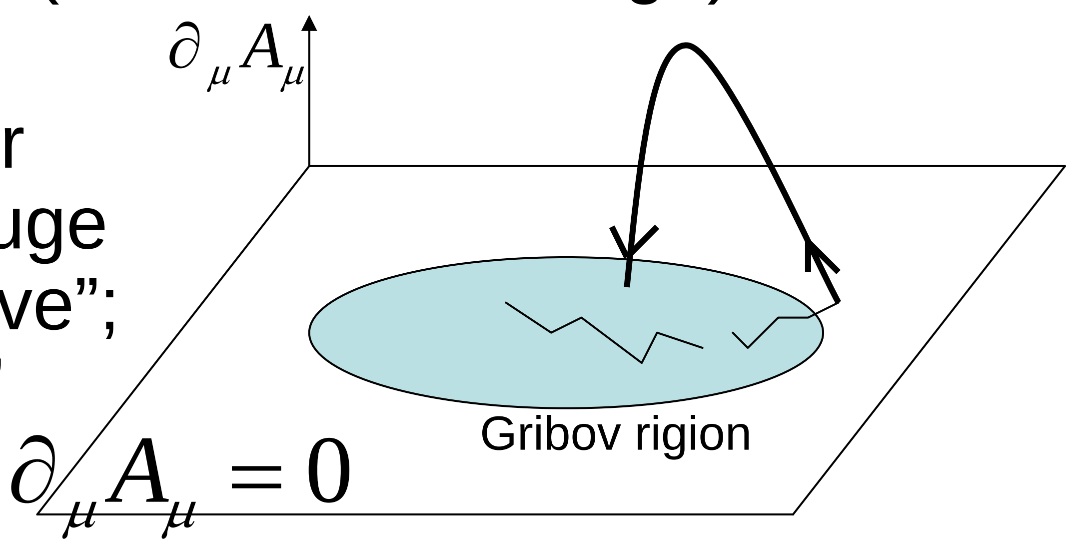
Stochastic Gauge Fixing by Zwanziger

- Langevin update + Gauge Fixing term

$$\frac{dA_\mu^a}{d\tau} = -\frac{\partial S}{\partial A_\mu^a} + \frac{1}{\alpha} D_\mu(A)^{ab} \partial_\nu A_\nu^b + \eta_\mu^a$$

α:gauge parameter (α=0:Landau Gauge)

- Inside the Gribov region (F.P.operator is negative),the Gauge Fixing term “attractive”; Outside: “repulsive”

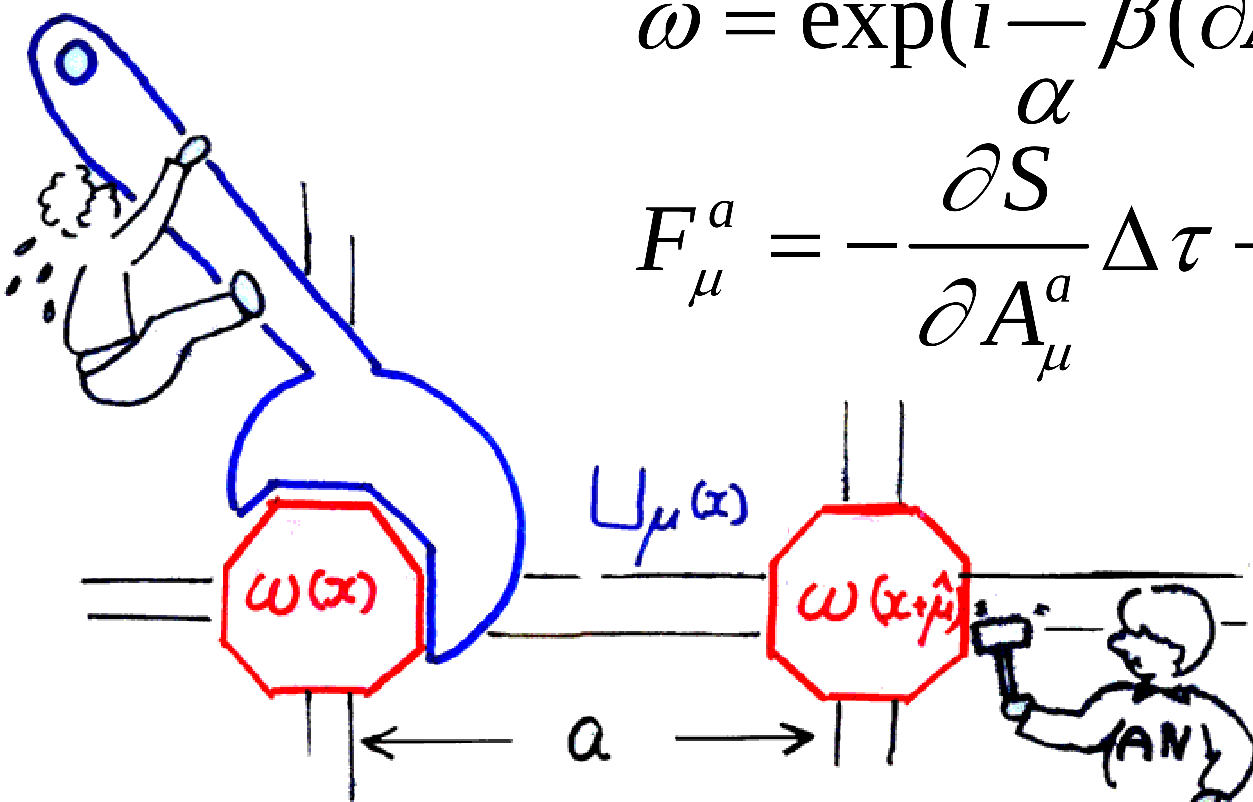


- Lattice version
  - Standard Langevin update step
  - After each step, we gauge-rotate

$$U_\mu(x,\tau+\Delta\tau) = \omega^\dagger(x,\tau) e^{iF_\mu^a t^a} U_\mu(x,\tau)\omega(x+\hat{\mu},\tau)$$

$$\omega = \exp(i\frac{1}{\alpha}\beta(\partial A^a)t^a\Delta\tau)$$

$$F_\mu^a = -\frac{\partial S}{\partial A_\mu^a}\Delta\tau + \eta^a\sqrt{\Delta\tau}$$



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1. Introduction
   – Gauge Fixing
2. **Gluon Propagators**
   – **Zero temperature and Finite temperature**
   – **Landau Gauge and <u>Coulomb gauge</u>**
3. Polyakov Loop Correlations
   – Finite Temperature
   – Zero Temperature (Preliminary)
      • Finite Length Polyakov Loops

Preliminary

Gluon Propagators

$$G_{\mu\nu}(z) = < TrA_\mu(z)A_\nu(0) >$$

Perturbative form

$$G_{\mu\nu}(p) = \frac{Z}{p^2}\left(\delta_{\mu\nu} - \frac{(1-\alpha)\,p_\mu p_\nu}{p^2}\right)$$

Electric

$$G_e(p_x, p_y, z, p_t) \sim G_{tt}(p_x = \frac{2\pi}{N_x}, p_y = 0, p_t = 0, z)$$

Magnetic

$$G_m(p_x, p_y, z, p_t) \sim G_{yy}(p_x = \frac{2\pi}{N_x}, p_y = 0, p_t = 0, z)$$

Gribov Copies

- Singer
  - One cannot define a Gauge Fixing for

$$SU(N) \text{ on } S^d \ \ (d \geq 3)$$

$$(S^d \ : \text{d-Sphere})$$

- Killingback
  - One cannot define a Gauge Fixing for

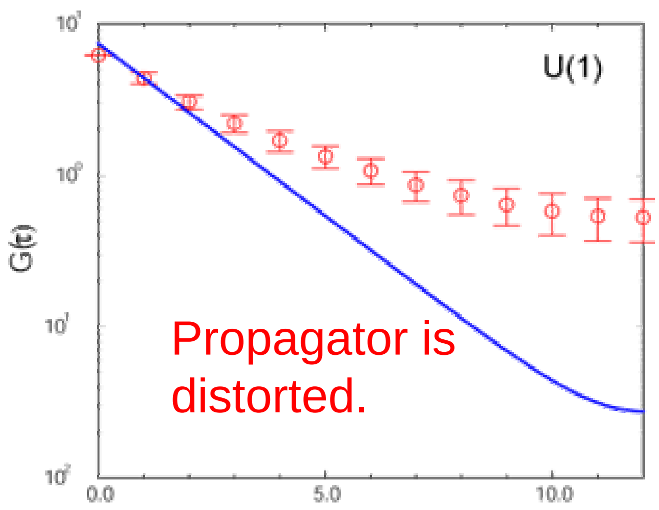
$$SU(N) \text{ on } T^d \ \ (d \geq 3)$$

$$U(1) \text{ on } T^d \ \ (d \geq 2)$$

$$(T^d \ : \text{d-Torus})$$

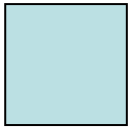
U(1) with Periodic

Boundary Condition ! (Nakamura-Plewnia)



Double Dirac Sheets

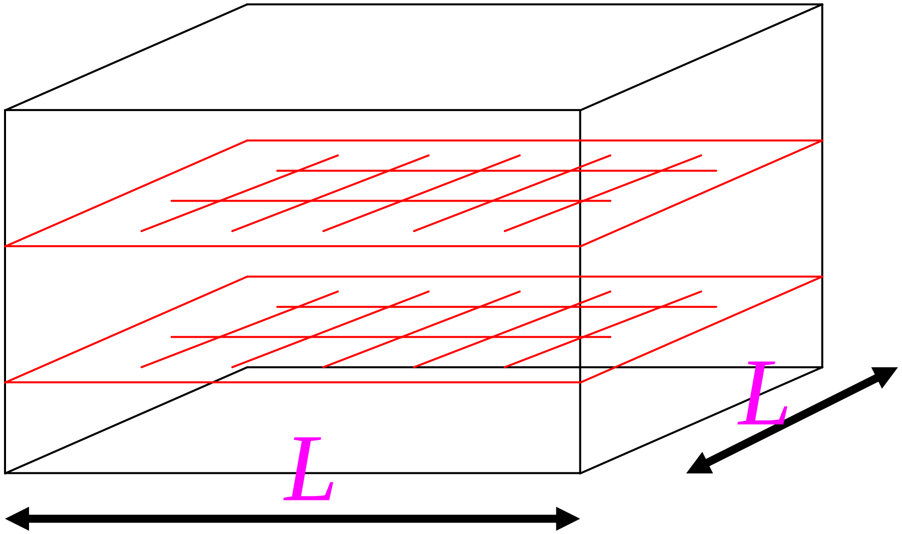
Bornyakov, Mitrjushkin, Muller-Preussker and Pahl, Phys.Lett. B317 (1993) 596

 $= e^{i\theta}$ $\qquad \theta = \phi + 2\pi n$

$$-\pi < \phi \leq +\pi, n = 0, \pm 1, \pm 2, \cdots$$

$n \neq 0$, Dirac Plaquette

No. of Dirac Plaq. $\geq 2L^2$



Distribution of Double Dirac Plaquettes

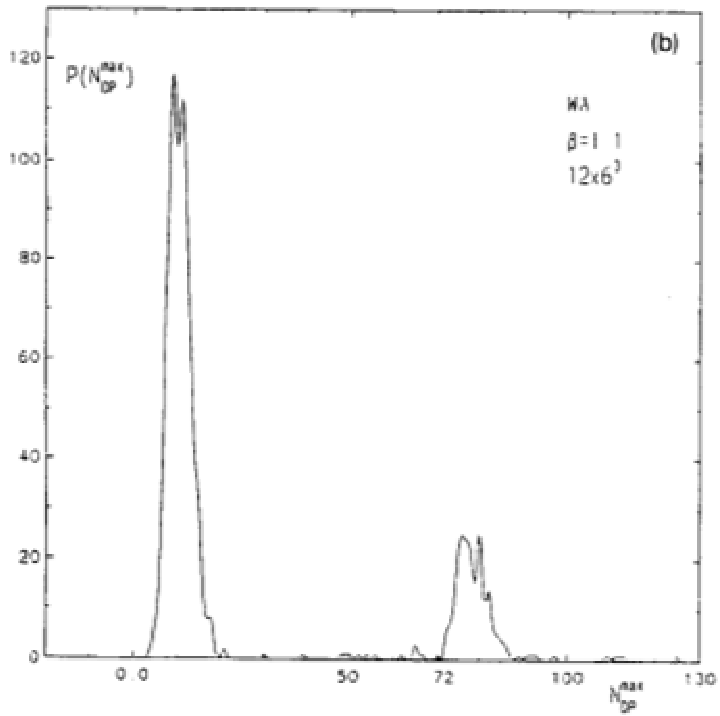
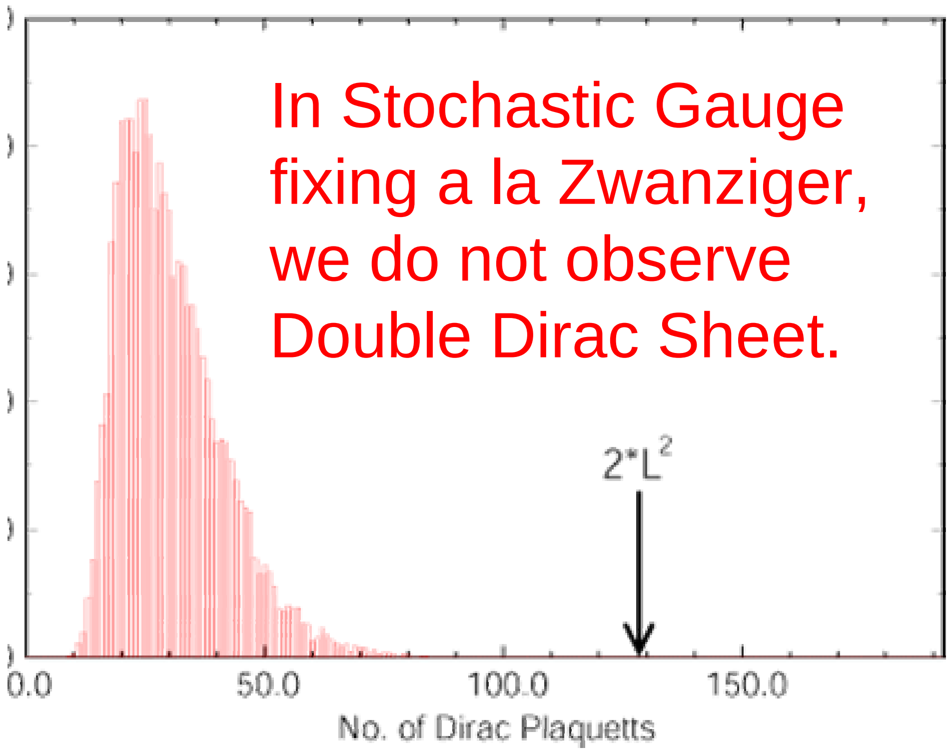
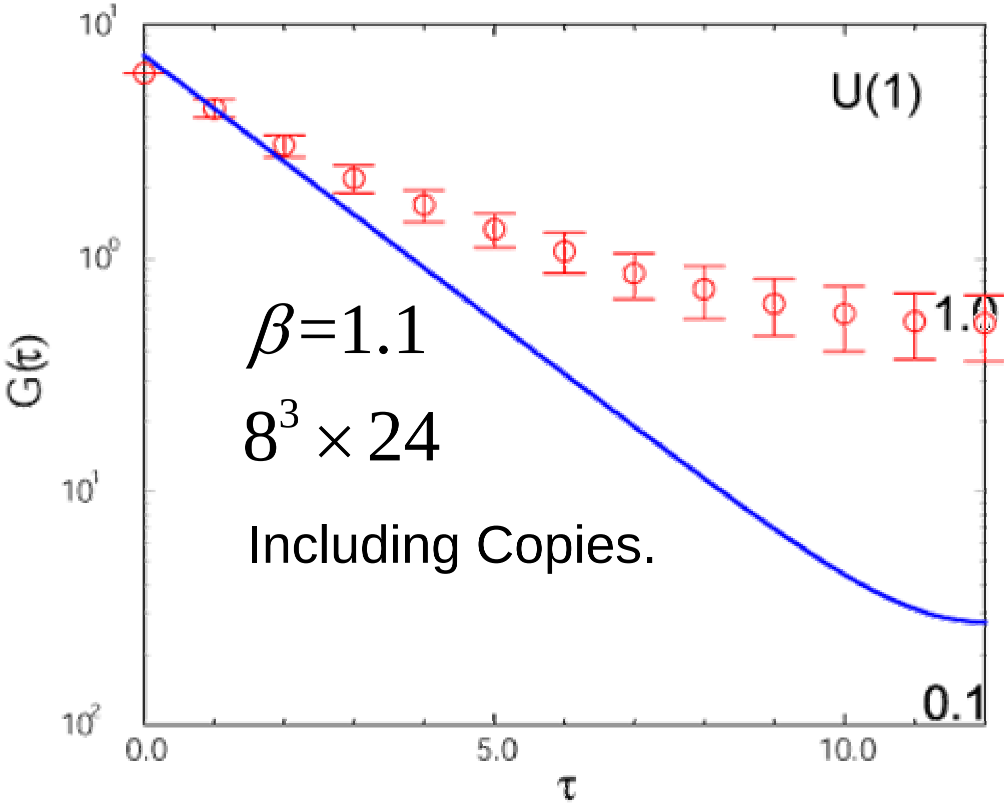
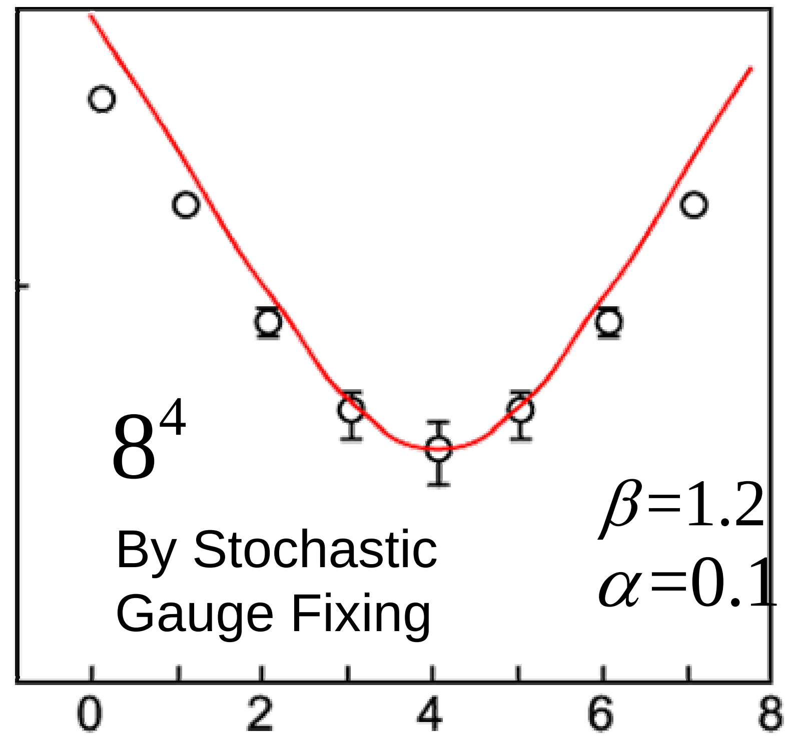


Fig. 1. Distributions of the number of Dirac plaquettes $P(N_{DP})$ on a $12 \cdot 6^3$ lattice at $\beta = 1.1$ for modified action (MA) (a) and for Wilson action (WA) (b). In the latter case, only those plane orientations are selected which contain the maximal number of Dirac plaquettes N_{DP}^{max}.



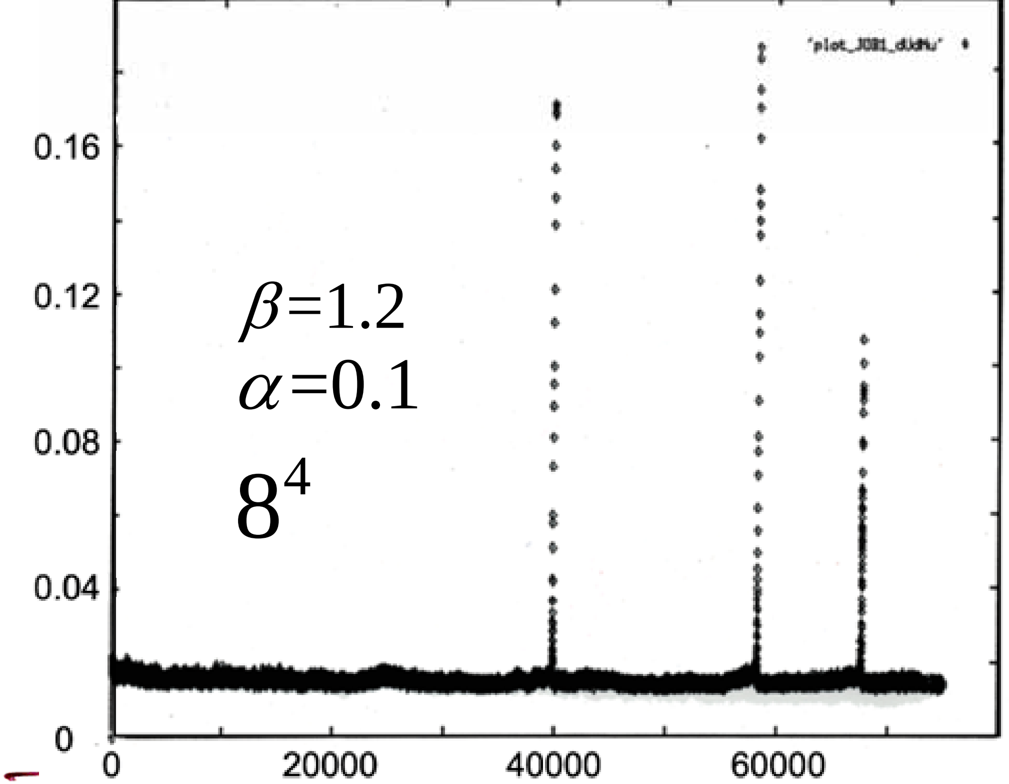
U(1) Propagator

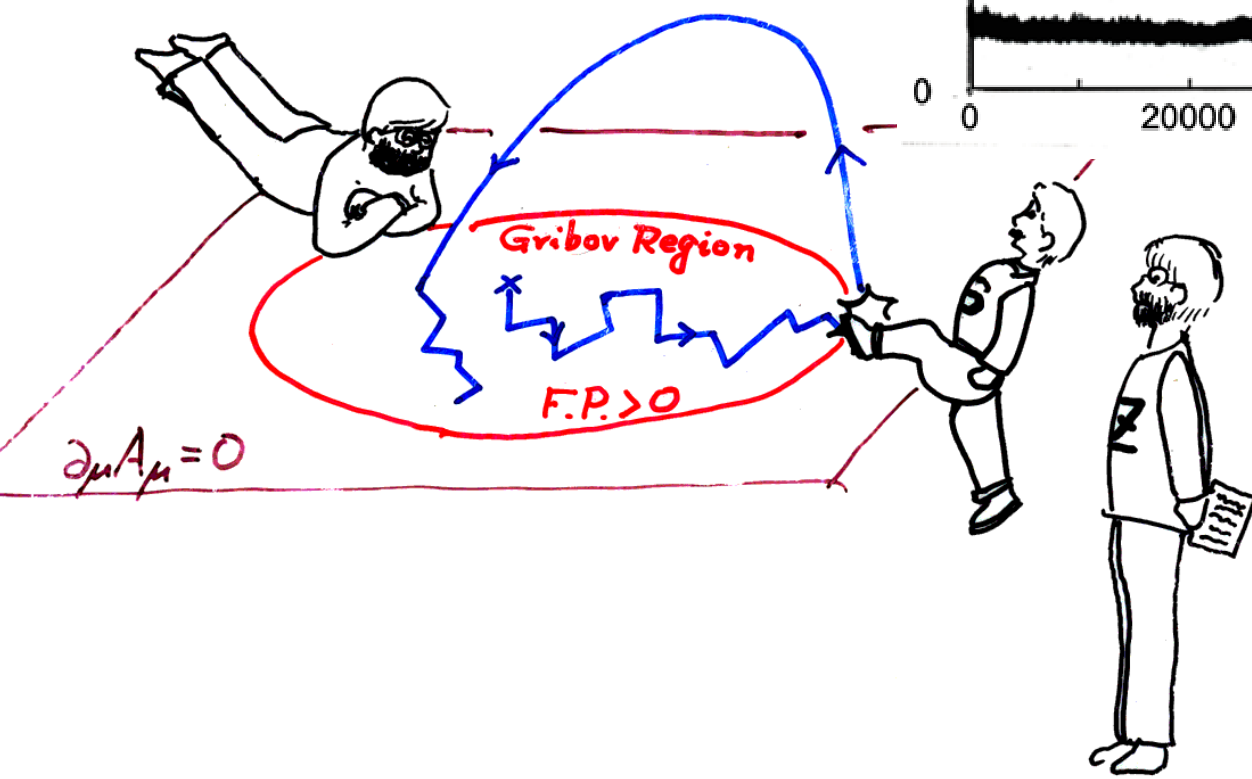




(In SU(3), we do not see this distortion so far.)

Seiler, Stamatescu
and Zwanziger

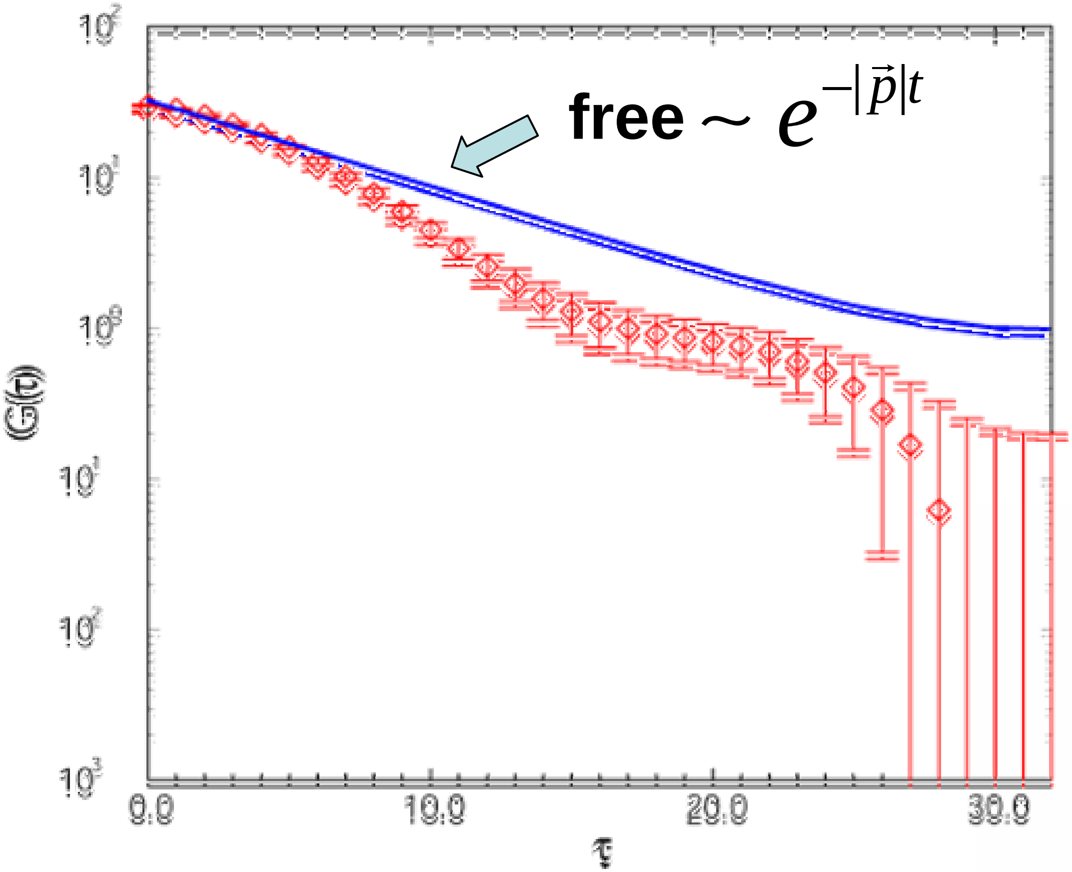




Gluon/Photon Propagators on Lattice (Incomplete) List of Players

- Mandula and M.Ogilvie, Phys.Lett.B185 (1987) 127.
- Nakamura and M.Plewnia, Phys.Letters B255 (1991) 274 (U1)
- Bornyakov, Mitrjushkin, Müller-Preussker and Pahl, Phys.Lett. B317 (1993) 596 (U1)
- Bernardm, Parrinello and Soni, Phys.Rev.D49(1994) 1585.
- Nakamura et al., Nucl. Phys. Proc. Suppl. 42 (1995) 899.
- P.Marenzoni, G. Martinelli and N. Stella, Nucl.Phys. B455 (1995) 339
- D.B.Leinweber, J.I.Skullerud, A.G.Williams and C.Parrinello, Phys.Rev. D60 (1999) 094507
- U.M. Heller, F. Karsch and J. Rank, Phys. Lett. B355 (1995) 511.
- Cucchieri, Phys.Lett. B422 (1998) 233
- H.Nakajima and S.Furui, Nucl.Phys.Proc.Suppl. 73 (1999) 635

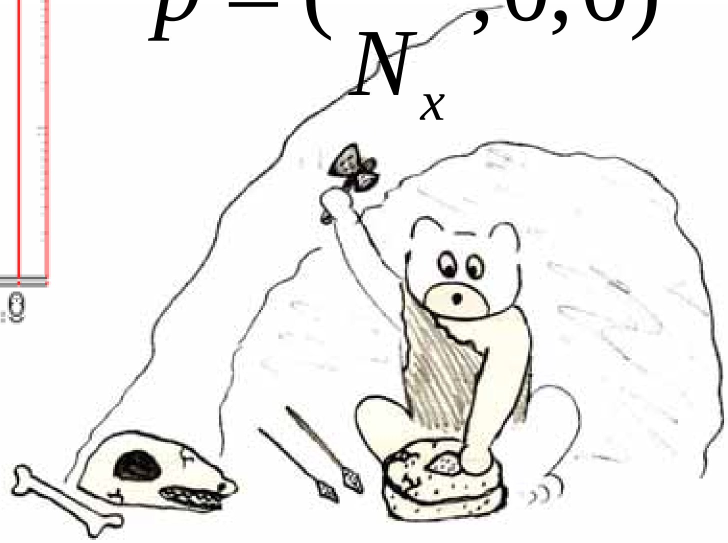
Gluon Propagator in the confinement (Quench, SU(3), Old Days Calculation)



$$48^3 \times 64$$

$$\beta = 6.8$$

$$\vec{p} = (\frac{2\pi}{N_x}, 0, 0)$$



Landau Gauge

Nakamura, 1995

Gribov Conjecture

$$G \sim \frac{1}{p^2 + \dfrac{b^4}{p^2}}$$

$$G \sim \frac{1}{p^2} \text{ for } p^2 \to \infty$$

$$G \sim p^2 \text{ for } p^2 \to 0$$

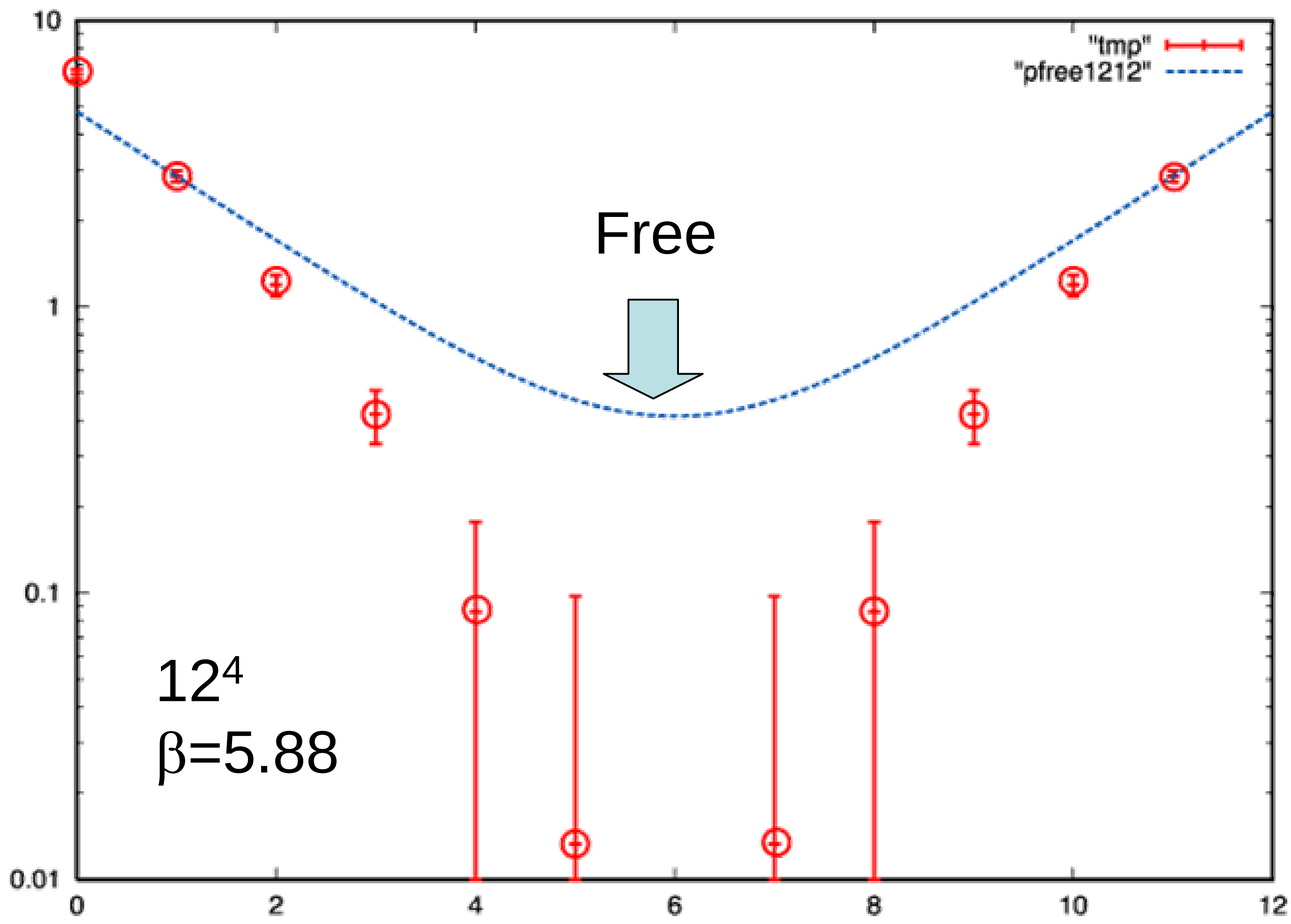
G has no physical pole.

Fourier Transform

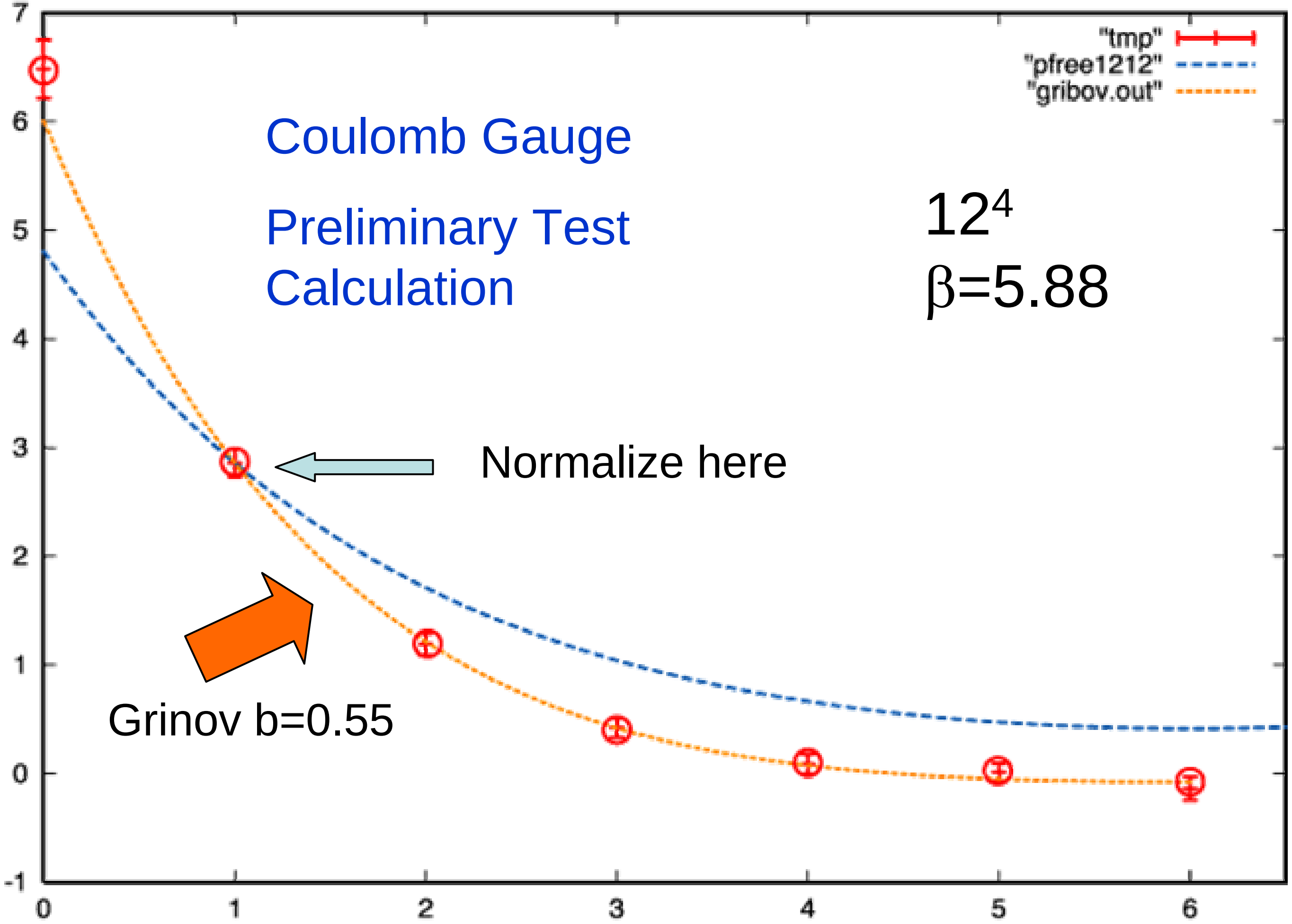
$$G(\tau) \sim \frac{\pi}{r} e^{-r\tau\cos\phi} \cos(r\sin\phi\tau + \phi)$$

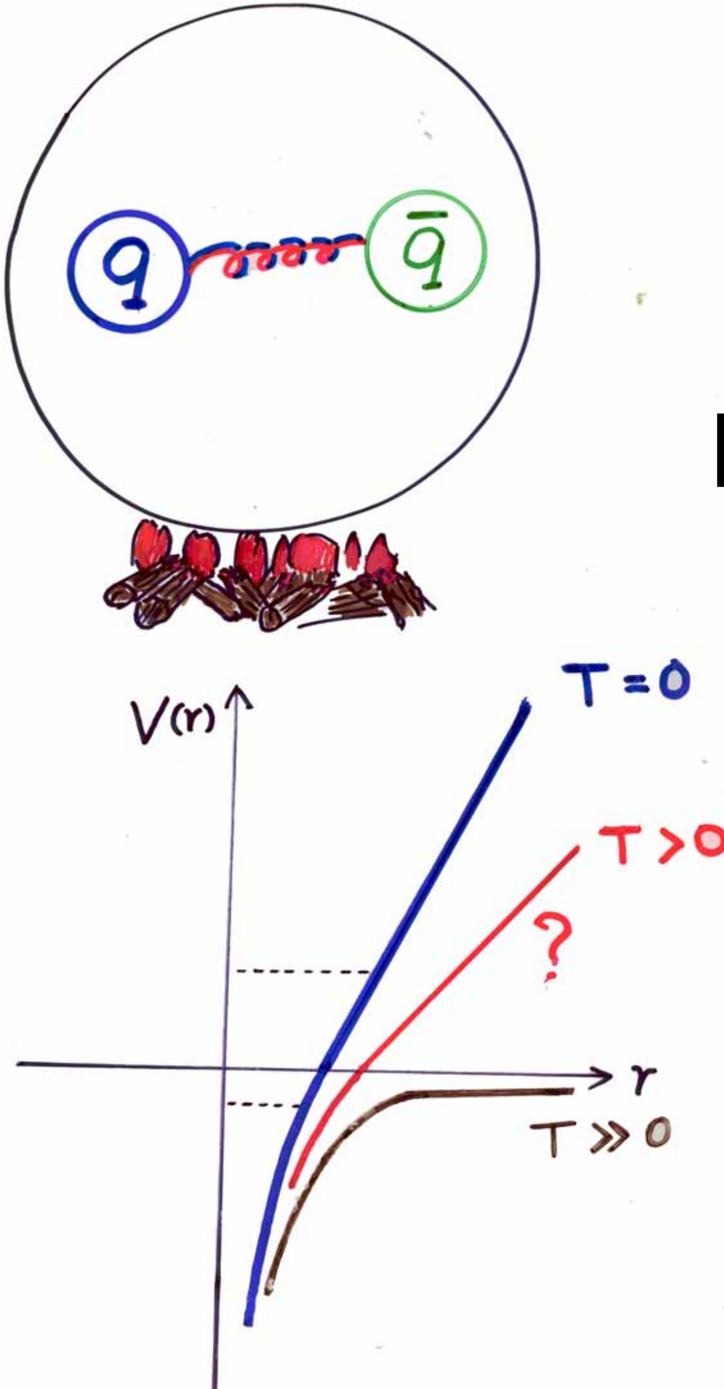
$$r \equiv (\vec{p}^4 + b^4)^{1/4} \quad \phi \equiv \frac{1}{2}\tan^{-1}\frac{b^2}{\vec{p}^2}$$

Gluon Propagator with Coulomb Gauge
(Preliminary and Exploratory)



Comparison with Gribov Conjecture





Glue Dynamics at Finite Temperature

Interesting Regions are

$$T = T_c \sim 5T_c$$

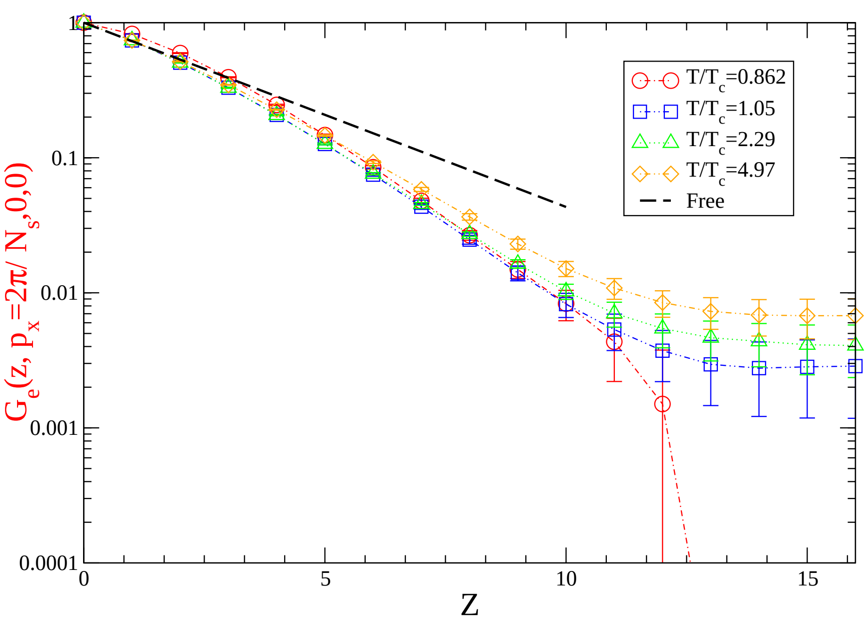
RHIC (and LHC) Region

Gluon Propagators at Finite Temperature

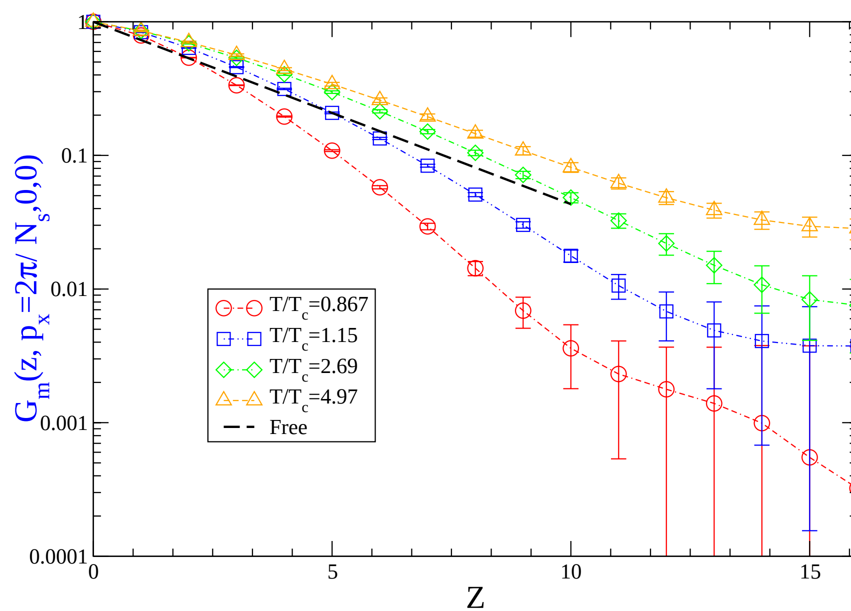
- Heller, Karsch and Rank, PL B355 (1995) 511, PRD57 (1998) 1428
- Cuccieri, Karsch, NP (PS)83 (2000) 357
- Cucchieri, Karsch and Petreczky, PL B497 (2001) 80, PR D64 (2001) 0306001
- A. Nakamura, I. Pushkina, T. Saito, S. Sakai, Phys.Lett. B549 (2002) 133-138.
- A. Nakamura, T. Saito, S. Sakai, PRD69 (2004) 014506

Gluon Propagators at T>0 (Landau Gauge)

Electric propagator



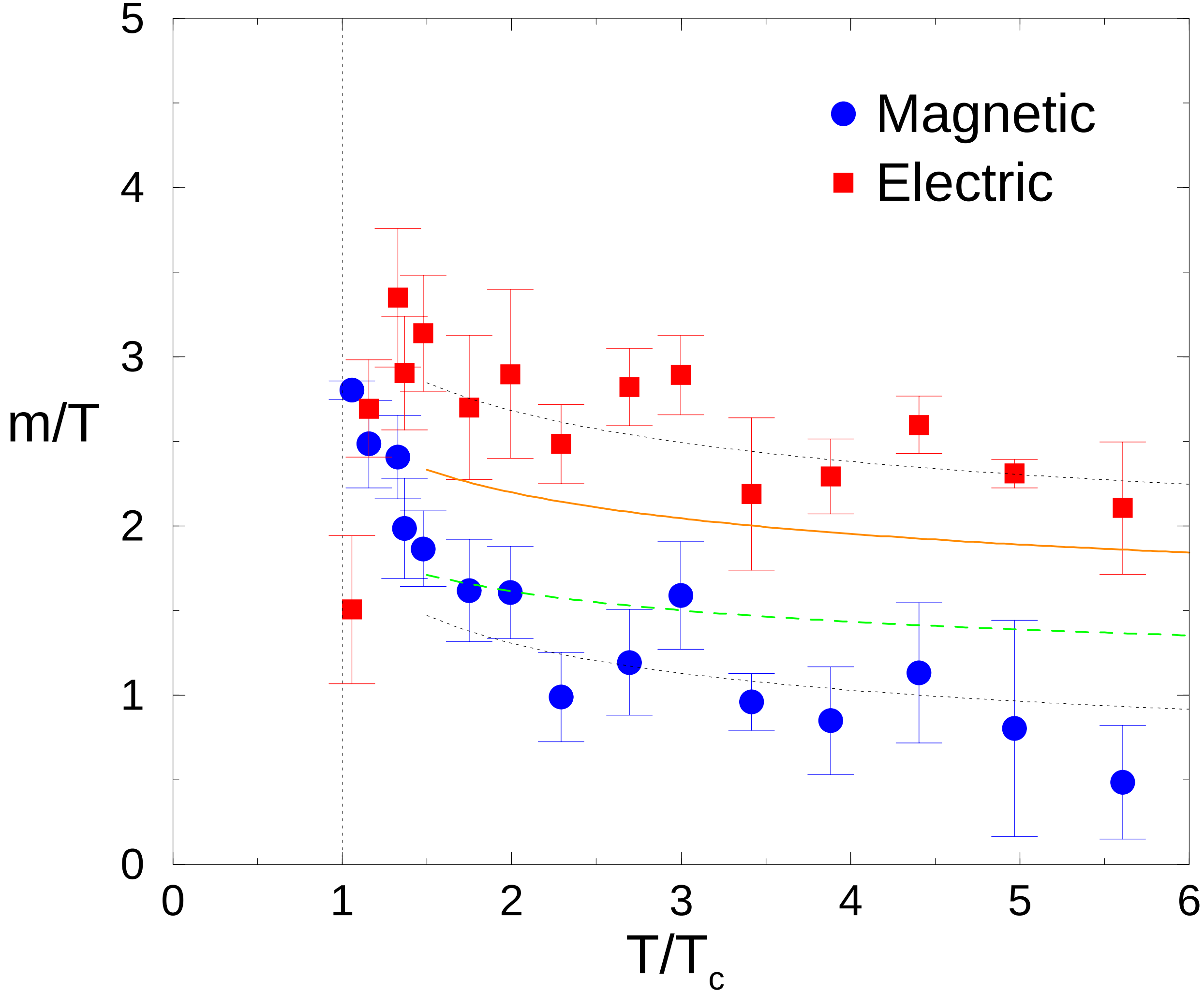
Magnetic propagator



$$20^2 \times 32 \times 6$$

Fitting to extrapolate mass

$$G(z) = C \cdot \cosh(m(z - N_z/2)) \quad \text{at } z > 1/T$$



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   – **Finite Temperature**
   – **Zero Temperature (Preliminary)**
      • **Finite Length Polyakov Loops**

Polyakov Loop Correlations

- McLerran and Svetisky, Phys.Rev.D24(1981)450
- “Static” quark

$$\left(\frac{1}{i}\frac{\partial}{\partial t} - t^a A_0^a(\vec{x},t)\right)\psi(\vec{x},t) = 0$$

$$\Rightarrow \psi(\vec{x},t) = T\exp\left(i\int_0^t dt' t^a A_0^a(\vec{x},t')\right)\psi(\vec{x},0)$$

$$\sim L(\vec{x})\psi(\vec{x},0)$$

$$L(\vec{x}) \equiv U_t(\vec{x},N_t)U_t(\vec{x},N_t-1)\ldots U_t(\vec{x},1)$$

$TrL(\vec{x})$ **:Polyakov Line**

$\bar{q}q$ state

$$e^{-\beta F_{q\bar{q}}} \sim \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

$$| \phi > = \psi^a(\vec{x},0)^\dagger (\psi^c)^b(\vec{x},0)^\dagger | Gluons >$$

a,b: Color indices ψ^c : anti-quark

$$e^{-\beta F_{q\bar{q}}} \sim \sum_{a,b,gluons} < Gluons | \psi^a(\vec{x}_1,0)(\psi^c)^b(\vec{x}_2,0)$$

$$\times e^{-\beta H} \psi^a(\vec{x}_1,0)^\dagger (\psi^c)^b(\vec{x}_2,0)^\dagger | Gluons >$$

$$= \sum_{a,b,gluons} < Gluons | e^{-\beta H} \psi^a(\vec{x}_1,\beta)\psi^a(\vec{x}_1,0)^\dagger$$

$$\times (\psi^c)^b(\vec{x}_2,\beta)(\psi^c)^b(\vec{x}_2,0)^\dagger | Gluons >$$

$$= \sum_{a,b,gluons} < Gluons | e^{-\beta H} L(\vec{x}_1)^{aa'} \psi^{a'}(\vec{x}_1, 0)$$

$$\times \psi^{a}(\vec{x}_1, 0)^{\dagger} L(\vec{x}_2)^{\dagger bb'} (\psi^{c})^{b'}(\vec{x}_2, 0)(\psi^{c})^{b}(\vec{x}_2, 0)^{\dagger} | Gluons >$$

$$= \sum_{gluons} < Gluons | e^{-\beta H} Tr L(\vec{x}_1) Tr L(\vec{x}_2) | Gluons >$$

$$\sim < Tr L(\vec{x}_1) Tr L^{\dagger}(\vec{x}_2) >$$ Color averaged

Here we used $[\psi^{a}(\vec{x}, 0), \psi^{b}(\vec{x}', 0)^{\dagger}] = \delta_{a,b} \delta_{\vec{x},\vec{x}'}$

and similar relation for anti-quark fields.

Color singlet $\bar{q}q$

- $3 \times 3^* = 1+8$

$$e^{-\beta F_1} \sim \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

$$| \phi > = \sum_{a} \psi^a (\vec{x}_1, 0)^\dagger (\psi^c)^a (\vec{x}_2, 0)^\dagger | Gluons >$$

$$\Rightarrow \quad e^{-\beta F_1} \sim < Tr L(\vec{x}_1) L^\dagger(\vec{x}_2) >$$

Color-dependent Potentials

- McLerran and Svetisky, PRD24(1981)450
- Nadkarni, PRD34 (1986) 3904
- Attig,Karsch,Petersson, Satz and Wolff, PLB209(1988)65
- Gao, PRD41 (1990) 626
- Irbaeck, Lacock, Miller, Petersson and Reisz, NPB363 (1991) 34
- Kaczmarek, Karsch, Laermann and Luetgemeier, PRD62 (2000)034021
- Digal, Petreczky and Satz, PRD64 (2001) 094015
- Philipsen, PLB535(2002)138.
- Muroya, Nakamura and Nonaka, NPB(PS) (2003))119.(hep-lat/0208006)
- Kaczmarek, Ejiri, Karsch, Laermann and Zantow, Prog.Theor.Phys.Suppl. 153 (2004) 287 hep-lat/0312015
- A. Nakamura and T. Saito
Prog. Theor. Phys. Vol. 111, (2004),
Prog. Theor. Phys. 112 (2004) 183
- Jahn and Philipsen, hep-lat/0407042

Color-dependent "potentials"

$$\exp(-\frac{V_1(R)}{T}) = 3\frac{< TrL(R)L^{\dagger}(0) >}{< TrL(0) >^2}$$

(Singlet)

$$\exp(-\frac{V_8(R)}{T}) = \frac{9}{8}\frac{< TrL(R)TrL^{\dagger}(0) >}{< TrL(0) >^2} - \frac{3}{8}\frac{< TrL(R)L^{\dagger}(0) >}{< TrL(0) >^2}$$

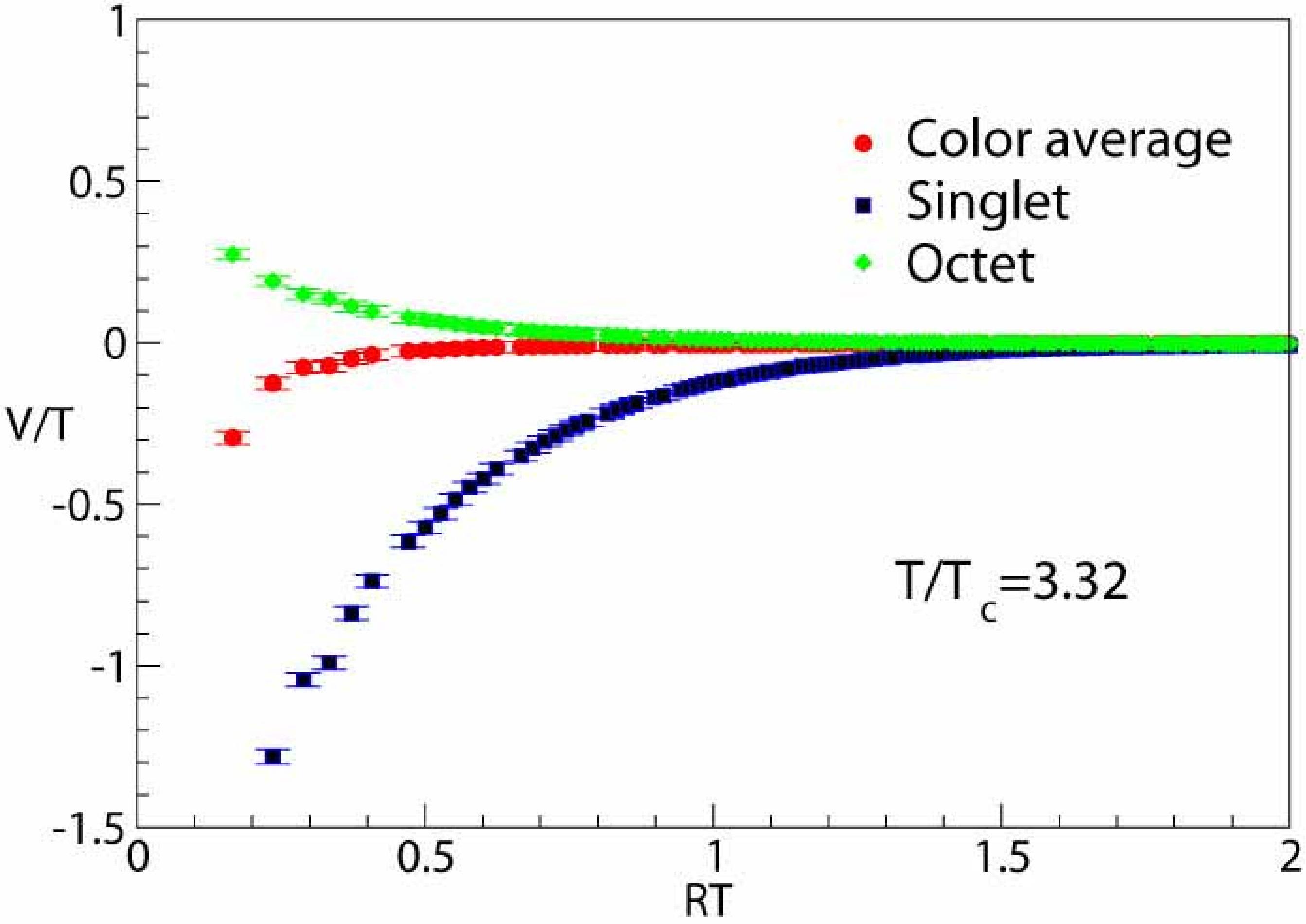
(Octet)

$$\exp(-\frac{V_c(R)}{T}) = \frac{< TrL(R)TrL^{\dagger}(0) >}{< TrL(0) >^2}$$

$$= \frac{1}{9}\left(\exp(-\frac{V_1(R)}{T}) + 8\exp(-\frac{V_8(R)}{T}) \right)$$

(Color-averaged)

Color-dependent Potentials (Landau Gauge)

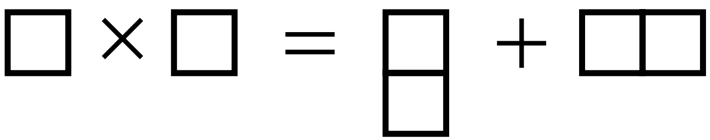


$24^3 \times 6$

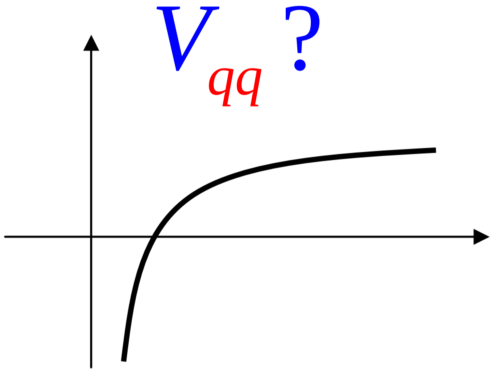
ıench

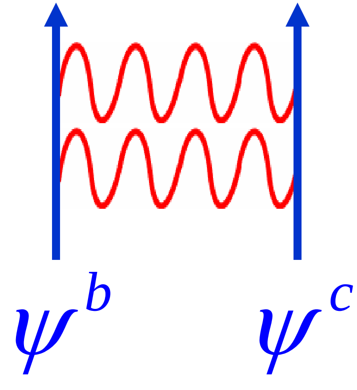
One more thing: Di-quark potential

- qq-Potential is interesting for
  - Color-Super-Conducting
  - Di-quark model
    - e.g., Alford-Jaffe,hep-lat/0306037
- 3✕3=3*+6



$$\varepsilon_{abc}\psi^b\psi^c$$





qq Potentials

Nadkarni 86, Muroya-Nakamura-Nonaka 03

- Same argument for the color-dependent q̄q-potentials

$$e^{-\beta F_{qq}} \sim \sum_{\phi} < \phi | e^{-\beta H} | \phi >$$

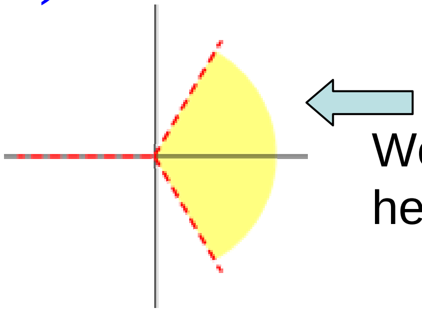
$$| \phi > = \varepsilon_{abc} \psi^{b}(\vec{x}_1, 0)^{\dagger} \psi^{c}(\vec{x}_2, 0)^{\dagger} | Gluons >$$

Using $\varepsilon_{abc}\varepsilon_{ab'c'} = \delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{cb'}$

$$\Rightarrow \quad \exp\left(-\frac{V_{ant.sym.}}{T}\right) = \frac{3}{2} < TrL(R)TrL(0) > - \frac{1}{2} < TrL(R)L(0) >$$

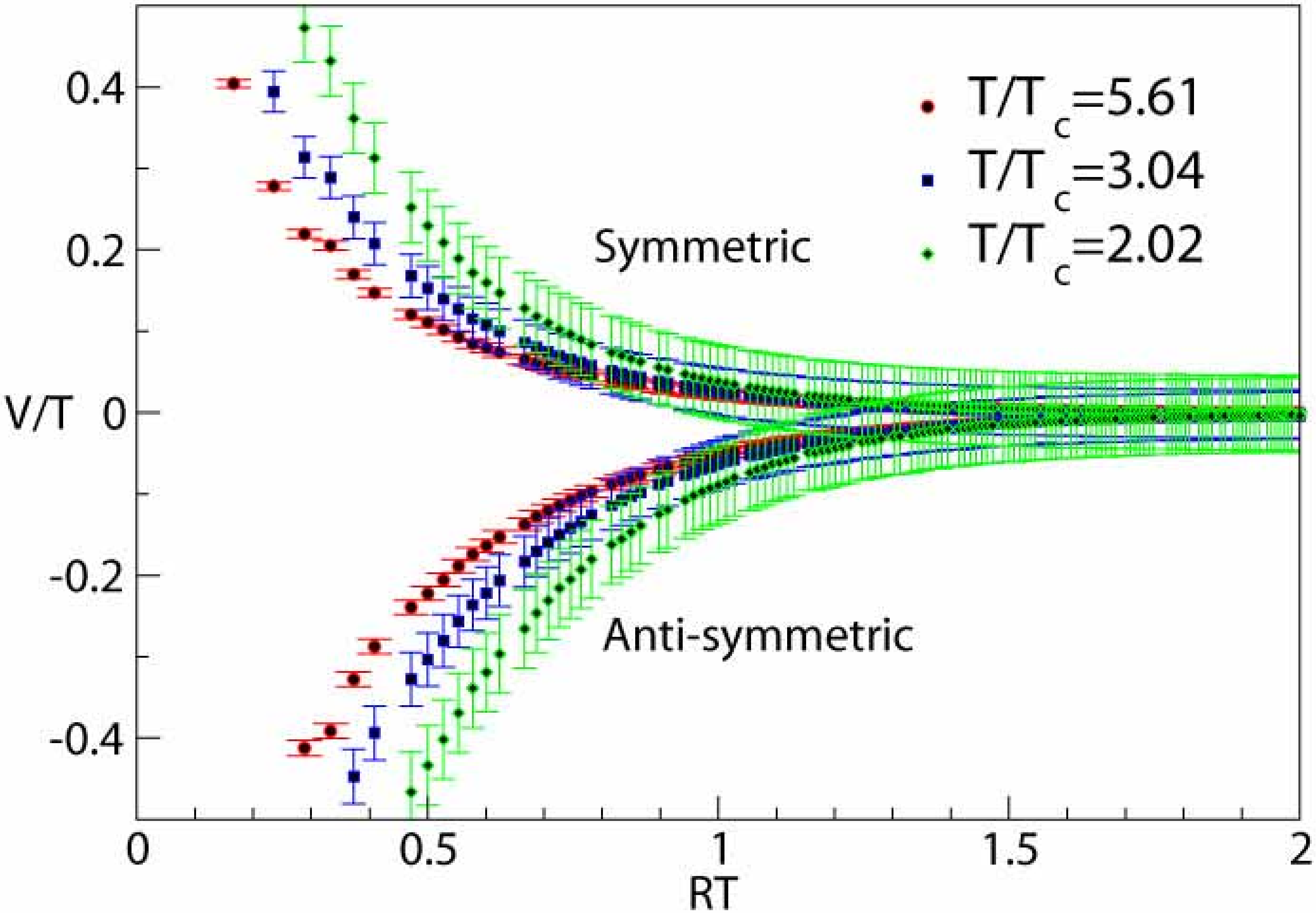
- Problem: $< Tr(L)Tr(L) >, \ < Tr(LL) >$ vanish due to Z3 symmetry in Quench QCD.

$$< Tr(LL) > + < Tr(zLzL) > + < Trz^2Lz^2L > = (1 + z^2 + z) < TrLL > = 0$$



We stay here.

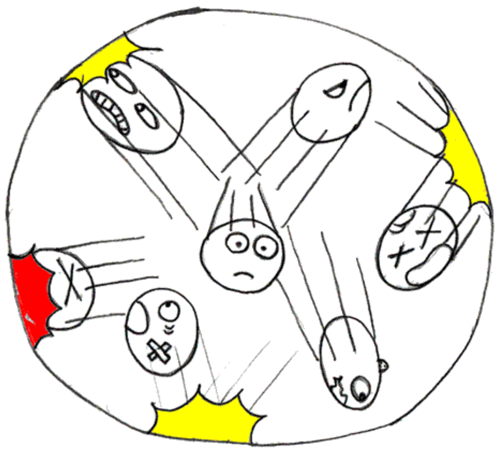
qq Potentials (Landau Gauge)



$24^3 \times 6$

Quench

In the confinement phase, can we study the Potentials by Polykov Line Correlations ?



Almost impossible, because the Polyakov Lines are very small.

Finite Length Polyakov Lines !

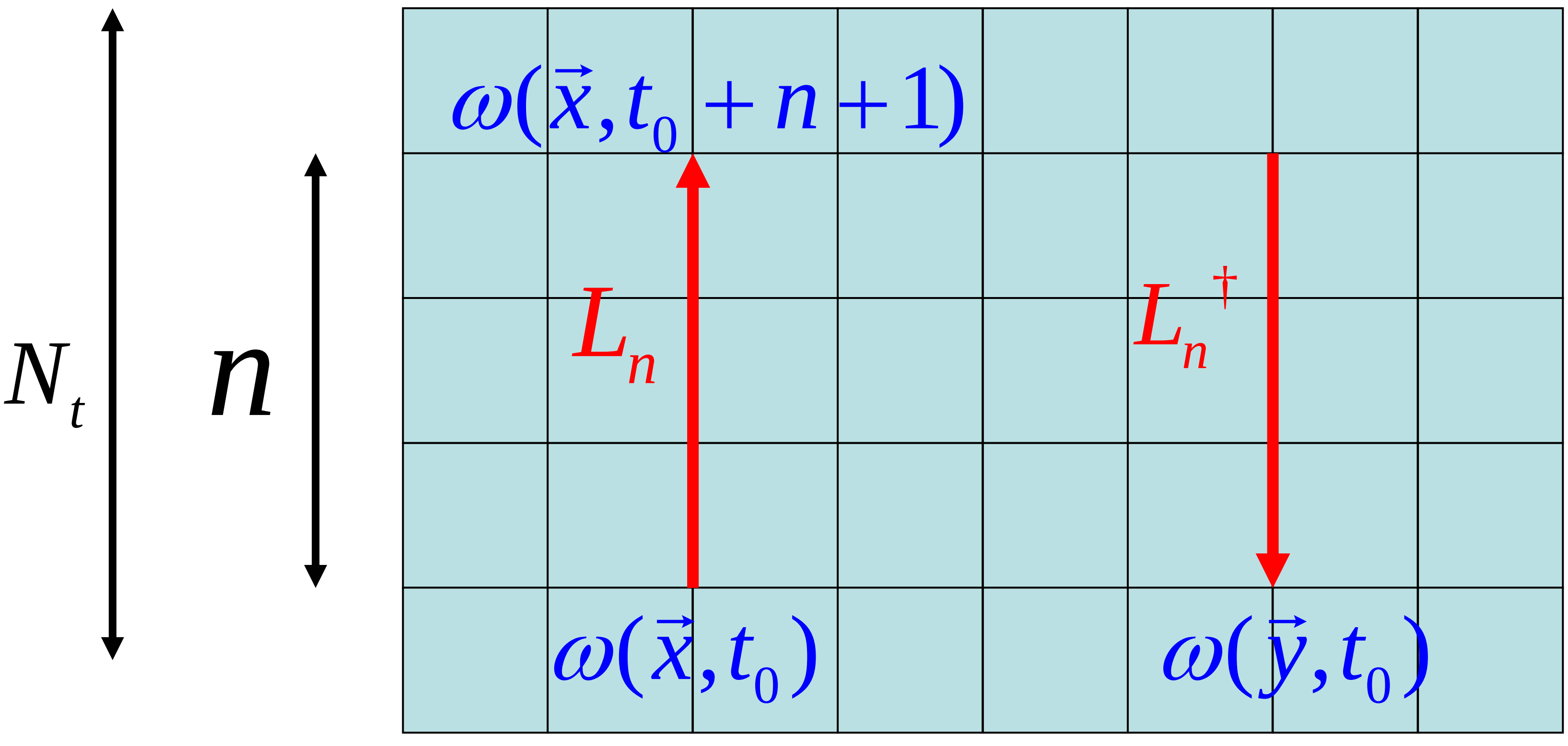
$$L_n(\vec{x},t_0) = U_4(\vec{x},t_0)U_4(\vec{x},t_0+1)\cdots U_4(\vec{x},t_0+n)$$

Marinari, Paciello, Parisi and Taglienti, Phys. Lett. B298 (1993) 400

Greensite, Glejnik and Zwanziger, Phys. Rev. D hep/lat-0401003.

$$L_n(\vec{x},t_0) \to \omega(\vec{x},t_0)^\dagger L_n(\vec{x},t_0)\omega(\vec{x},t_0+n+1)$$

$$\mathrm{Tr}\left(L_n\right), \mathrm{Tr}\left(L_n(\vec{x},t_0)L_n{}^\dagger(\vec{y},t_0)\right)$$ ⇦ Gauge non-invariant



We need Gauge Fixing

$$\mathrm{Tr}\left(L_n(\vec{x},t_0)L_n{}^{\dagger}(\vec{y},t_0)\right)$$ ⇐ Coulomb Gauge

After Coulomb Gauge Fixing, there is still Gauge freedom, i.e., "Global" Gauge Transformation on each Time-Slice

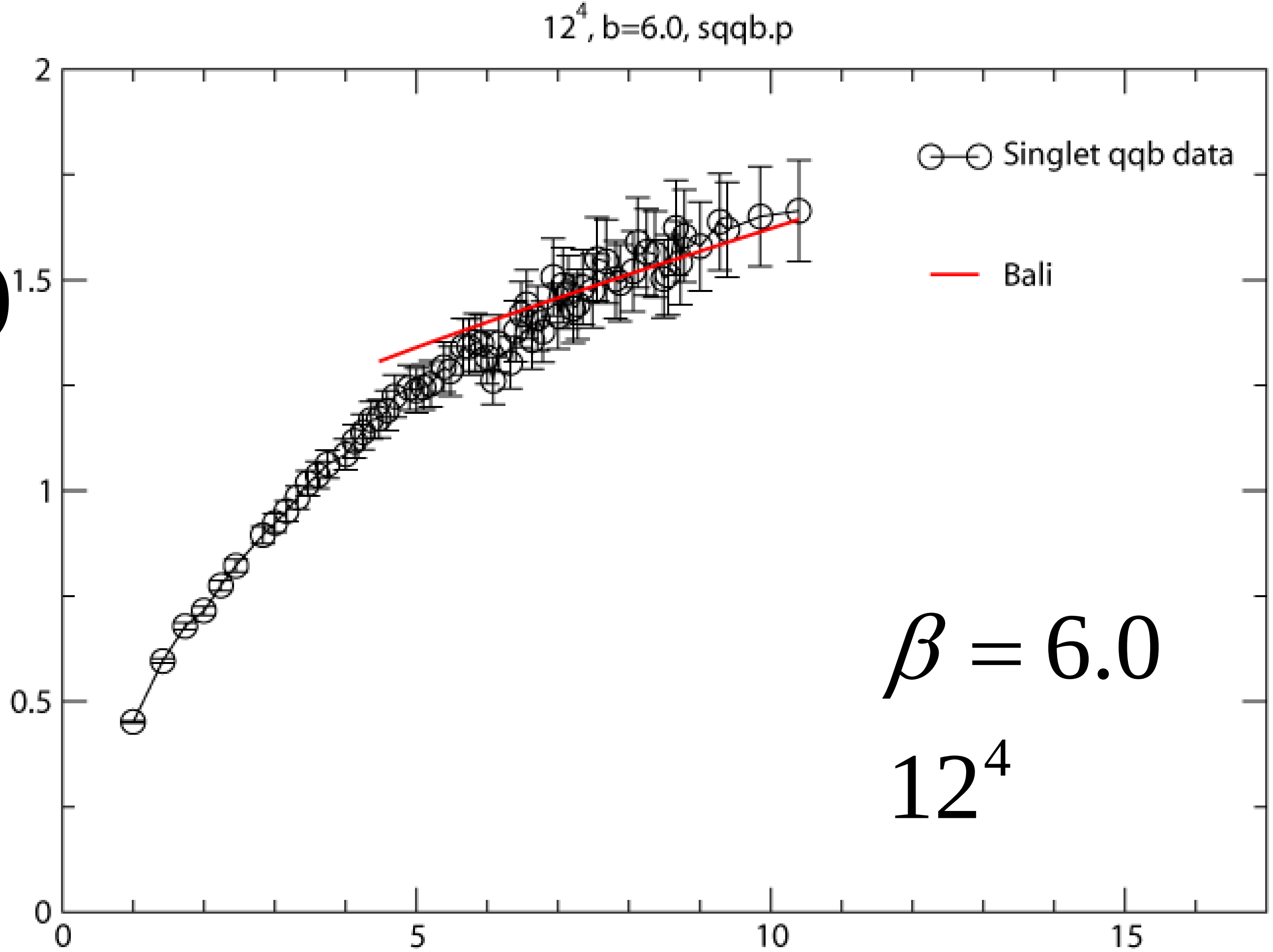
$$\omega(\cancel{\vec{x}},t)$$

Remnant Symmetry

If we fix this remnant symmetry (Temporal Gauge Fixing in Coulomb Gauge), we can calculate also

$$\mathrm{Tr}\left(L_n(\vec{x},t_0)L_n(\vec{y},t_0)\right)$$ Quark-Quark Potential

$\bar{q}$-q Potential (Color Singlet)



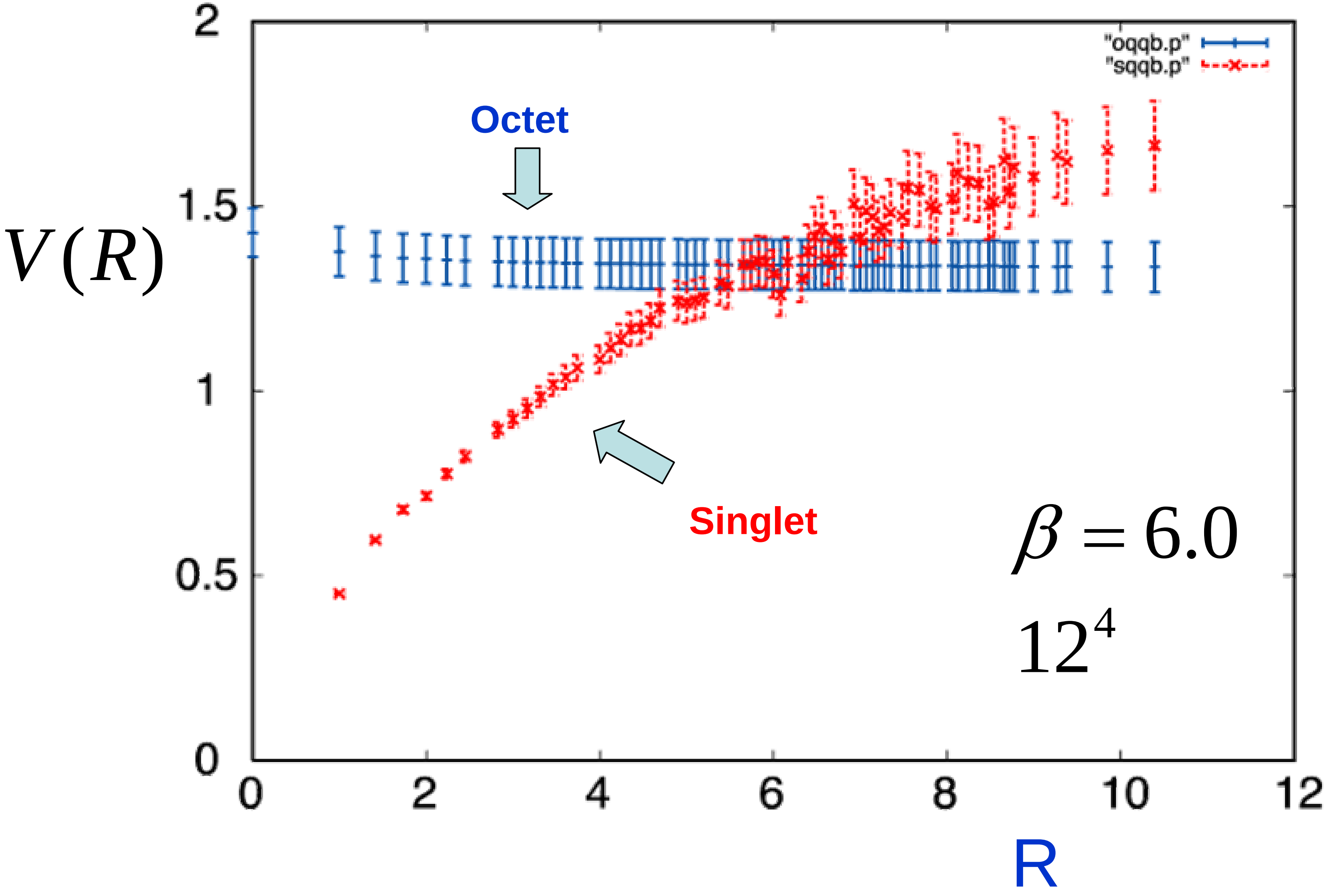
$V(R)$

$\beta = 6.0$

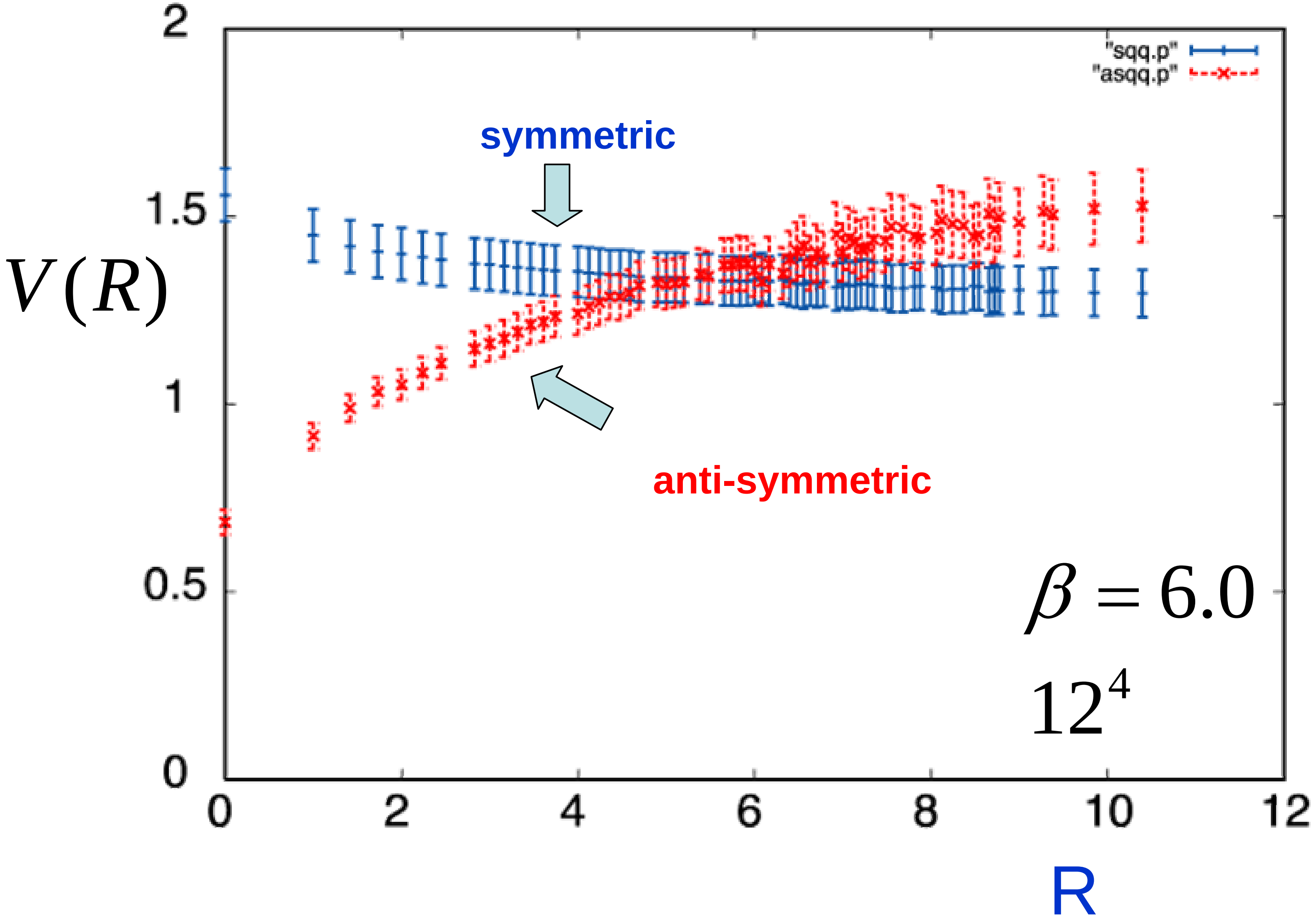
12^4

R

$\bar{q}q$-Potential

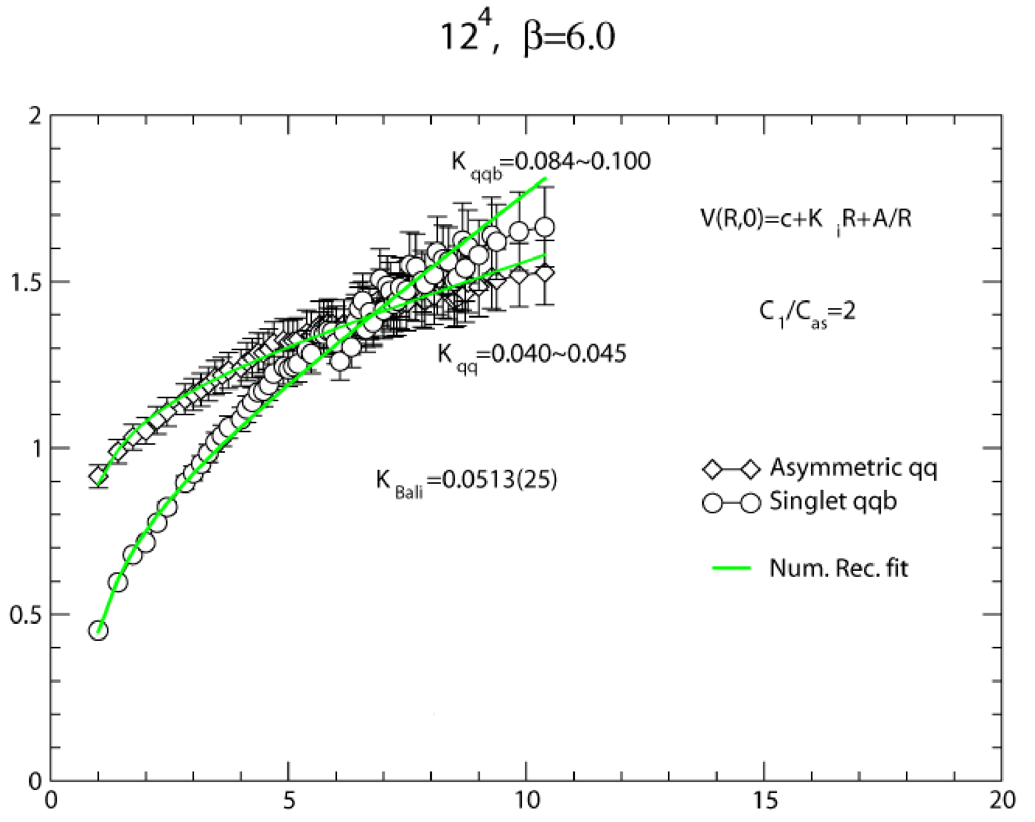


qq-Potential



$\bar{q}$-q and q-q Potentials
Which is stronger ?
Both confinement ?

$V(R)$

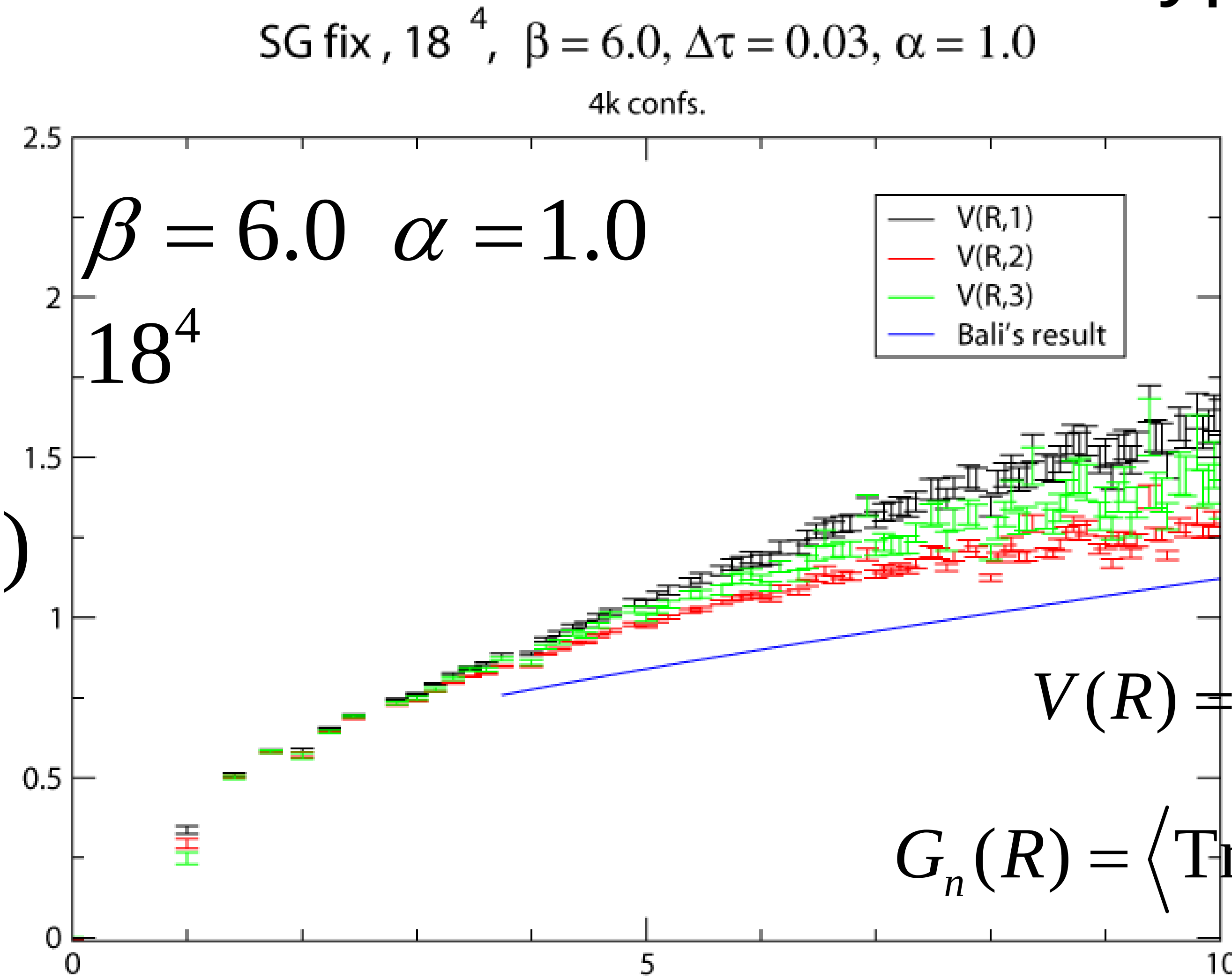


R

$\beta = 6.0$

12^4

We can calculate also “Creutz-Type Ratio”

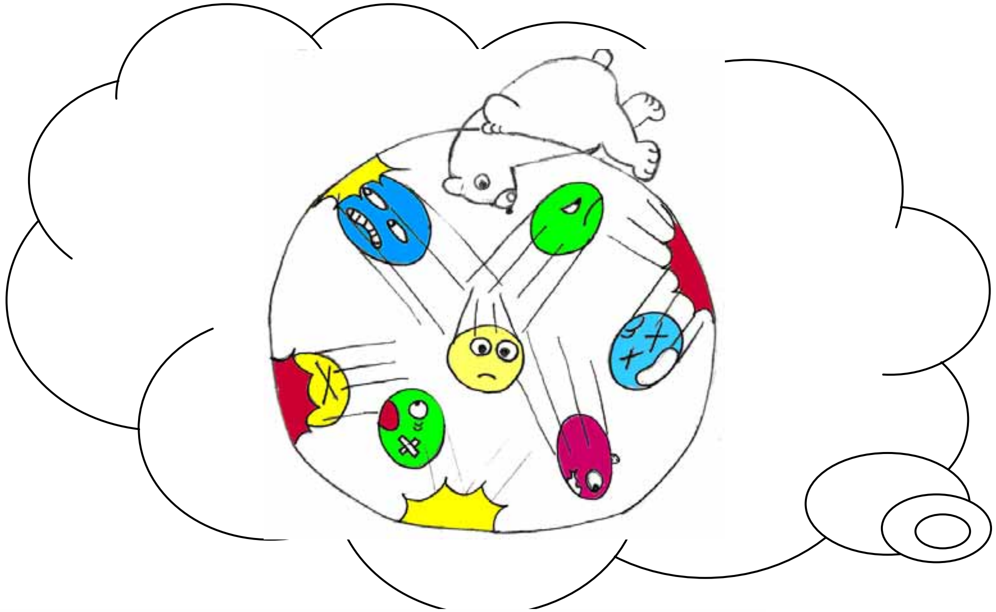


$V(R)$

R

$$V(R) = \log \frac{G_n(R)}{G_{n+1}(R)}$$

$$G_n(R) = \left\langle \mathrm{Tr} L_n(0) L_n(R)^{\dagger} \right\rangle$$



What we saw is
Real or Dream ?

Color-dependent Objects are not physical, but help us to understand the QCD mechanism !

Back-up Slides

Screening masses as function of T

$$\frac{m_e}{T} = C_e g(T)$$ electric

$$\frac{m_m}{T} = C_m g^2(T)$$ magnetic

$$g^2(\mu) = \frac{1}{2b_0 \log(\mu/\Lambda)} \left(1 - \frac{b_1}{2b_0} \frac{\log(2\log(\mu/\Lambda))}{\log(\mu/\Lambda)}\right)$$

$$\mu = 2\pi \mathrm{T},\ \Lambda \simeq T_c$$

$$C_e = 1.63(3), \chi^2 = 0.715$$

$$C_m = 0.482(31), \chi^2 = 0.979$$

Observations

- Magnetic mass does not vanish in these regions.
- Electric mass and Magnetic mass behave as

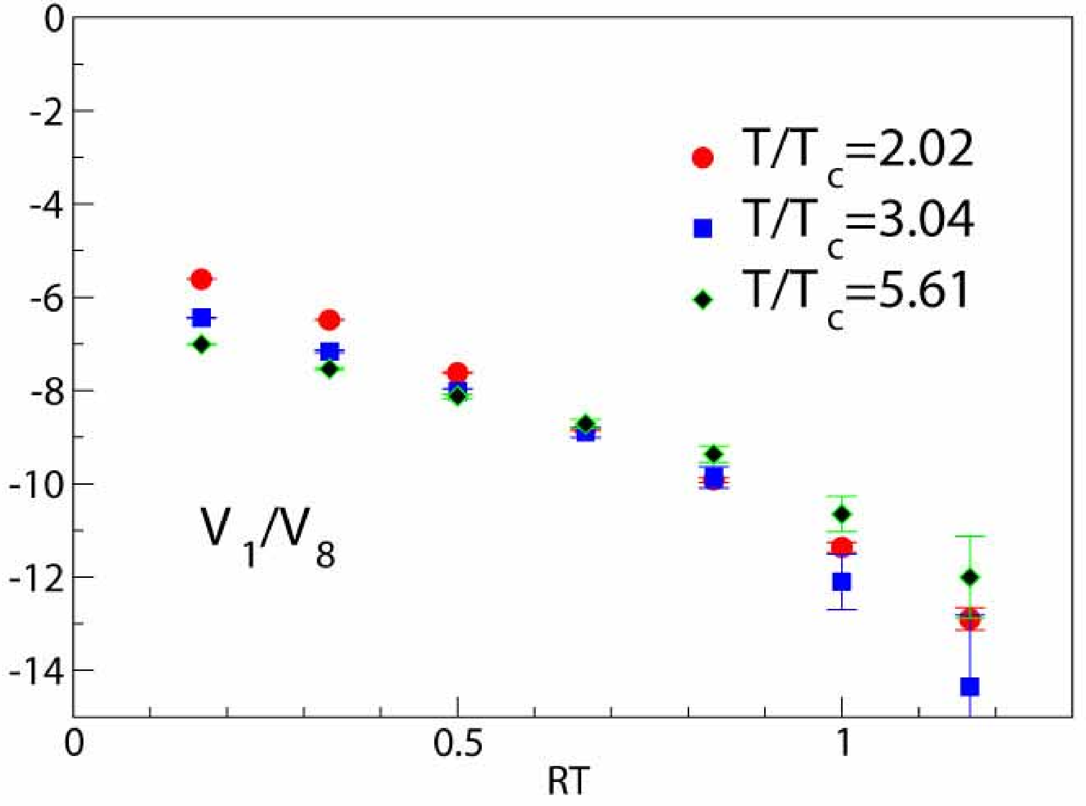
$$\frac{m_e}{T} \sim C_e g \qquad \frac{m_m}{T} \sim C_m g^2$$

- HTL gives better fit than the leading-order perturbation.
- We do not see the gauge parameter α dependence

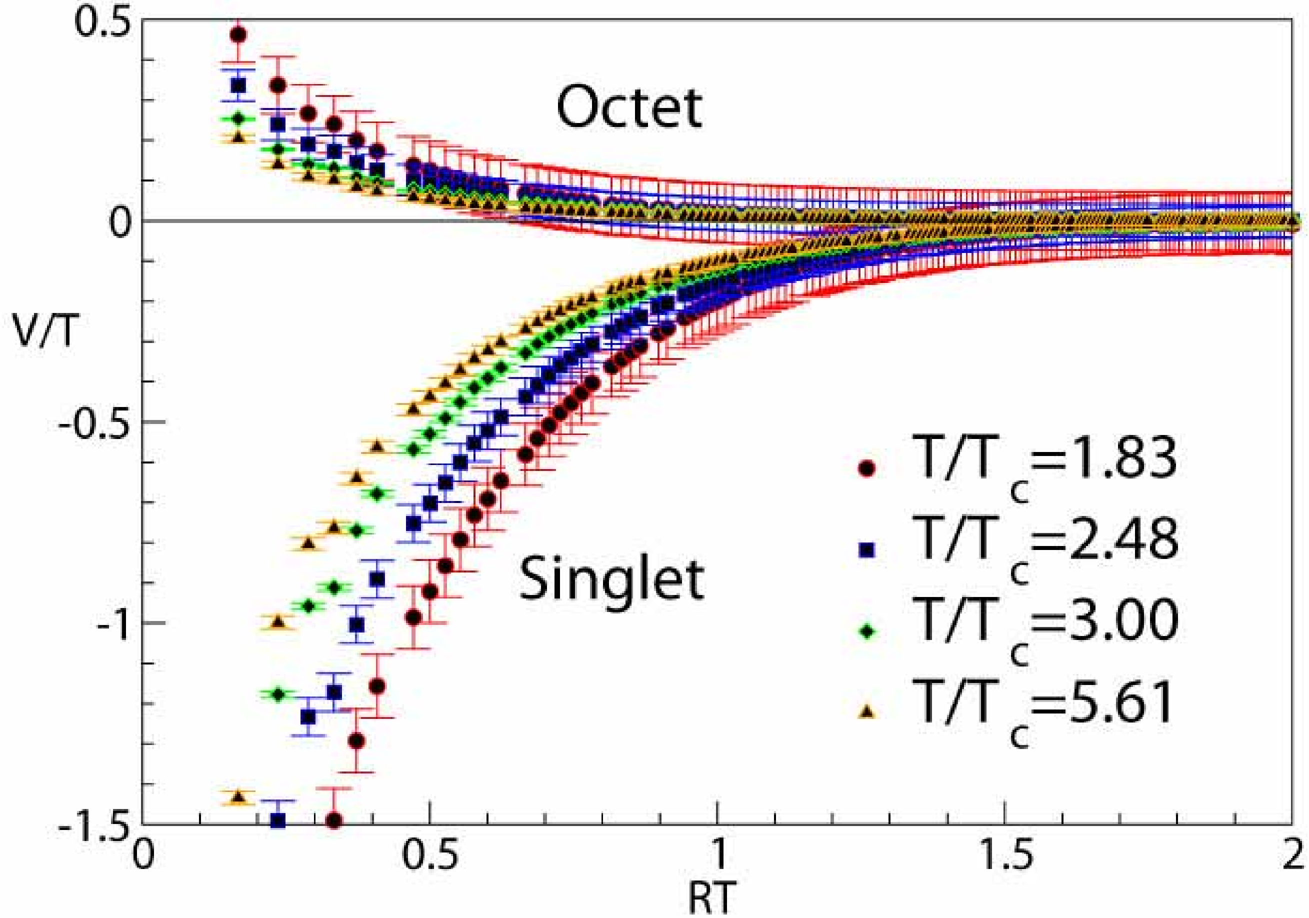
Ratios of V1 and V8

In the continuum perturbation,

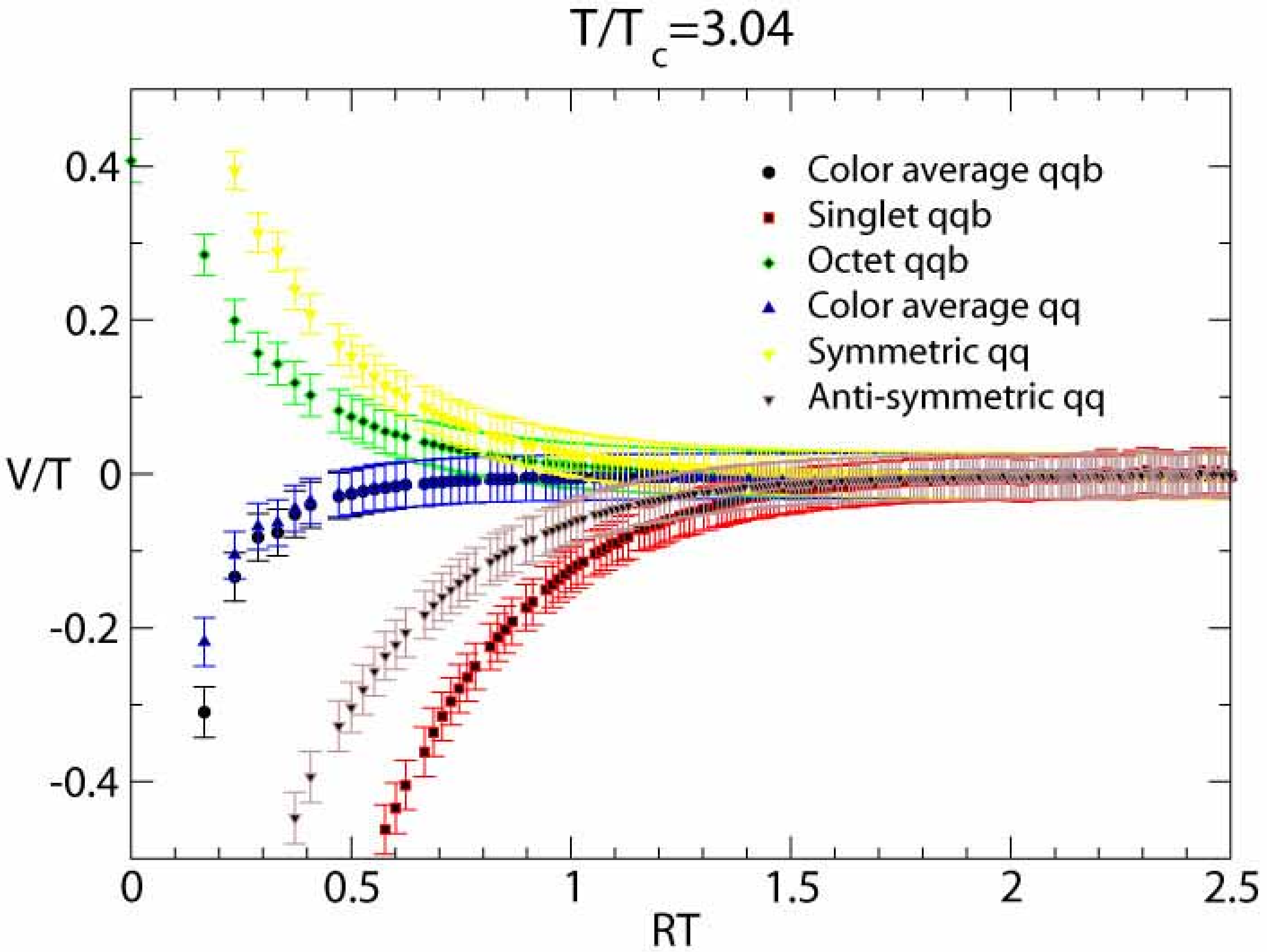
$$\frac{V_1}{V_8} = \frac{C[1]}{C[8]} = -8$$



Temperature dependence



$\bar{q}q$ and qq Potentials



$24^3 \times 6$

Quench