Neutrino Mass Spectrum, Majorana CP Violation, $(\beta\beta)_{0\nu}$-Decay and Beyond

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Compelling Evidences for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of** $\mu-$**like events**

Dominant $\nu_\mu \longrightarrow \nu_\tau$ K2K, MINOS; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \longrightarrow \nu_{\mu,\tau}$ **BOREXINO; KamLAND..., LowNu**

$-$ LSND

Dominant $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$; **MiniBOONE 11/04/07: negative result**

$$\nu_{l\text{L}} = \sum_{j=1} U_{lj}\, \nu_{j\text{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

The $\nu-$Oscillation Data: 3-ν mixing

$$\nu_{l\mathsf{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathsf{L}} \qquad l = e, \mu, \tau.$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980;
J. Schechter, J.W.F. Valle,1980;
M. Doi, T. Kotani, E. Takasugi,1981

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \rightarrow e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\mathrm{el}} = 0,\ L_l = 0,\ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)-$Dirac, $\chi(x)-$Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0 \ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0 \ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y) \ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta} \ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi \ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP} \ \chi(x) \ U^{-1}_{CP} = \eta_{CP} \ \gamma_0 \ \chi(x'), \quad \eta_{CP} = \pm i \ .$$

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.

S.M. Bilenky, J. Hosek, S.T.P.,1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.6 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.26$ (3σ),
- $|\Delta m^2_{\rm atm}| \equiv |\Delta m^2_{31}| \cong 2.4\ (2.5) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.040\ (0.056)$ 2σ (3σ).

A.Bandyopadhyay *et al.*, arXiv:0804.4857;

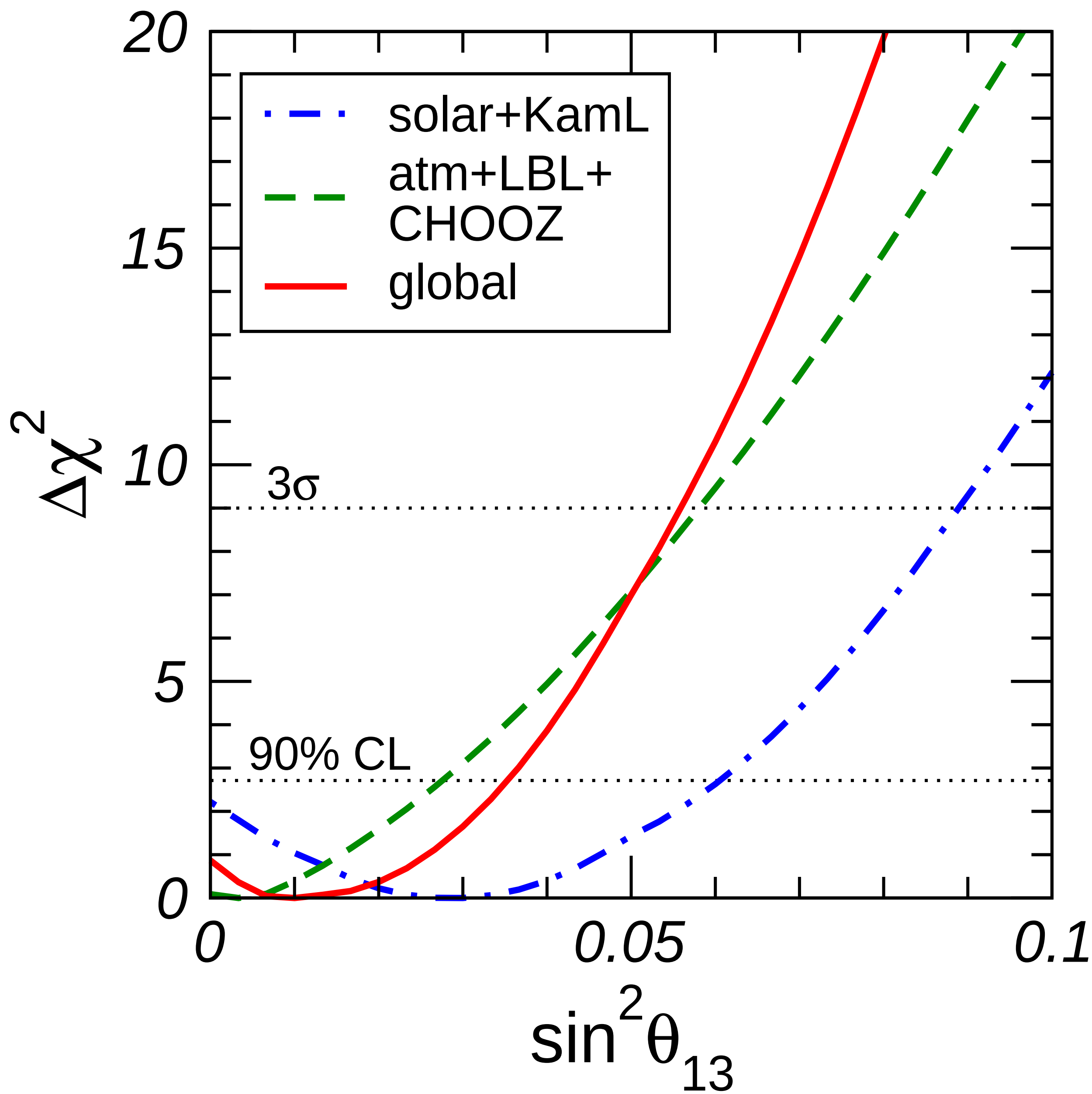
T. Schwetz *et al.*, arXiv:0808.2016

$\sin^2\theta_{13} = 0.016 \pm 0.010$, $\sin\theta_{13} = (0.077 - 0.161)\,,\quad 1\sigma$

E. Lisi *et al.*, arXiv:0806.2649

Atmospheric ν data: $\cos\delta = -1$ favored over $\cos\delta = +1$

J. Escamilla *et al.*, arXiv:0805.2924



T. Schwetz *et al.*, arXiv:0808.2016[hep-ph]

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $\quad m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \; m^2_{1,2,3} >> \Delta m^2_{\text{atm}}, \; \text{QD}; \; m_j \gtrsim 0.10 \text{ eV}.$$

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\text{CP}} \propto J_{\text{CP}} \propto \sin\theta_{13} \sin\delta$
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $\left|< m >\right|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn(Δm^2_{atm}), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

  – the origin of the observed patterns of ν-mixing and ν-masses ;

  – the physical origin of CPV phases in $U_{\rm PMNS}$;

  – Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

  – Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

  – Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

  – Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

- Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

  – Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

If ν_j- Majorana particles, U_{PMNS} contains (3-ν mixing)

δ-Dirac, α_{21}, α_{31} - Majorana physical CPV phases

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- are not sensitive to the nature of ν_j,

S.M. Bilenky et al.,1980;
P. Langacker et al., 1987

- provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + (\text{A}, \text{Z}) \rightarrow \mu^+ + (\text{A}, \text{Z} - 2)$$

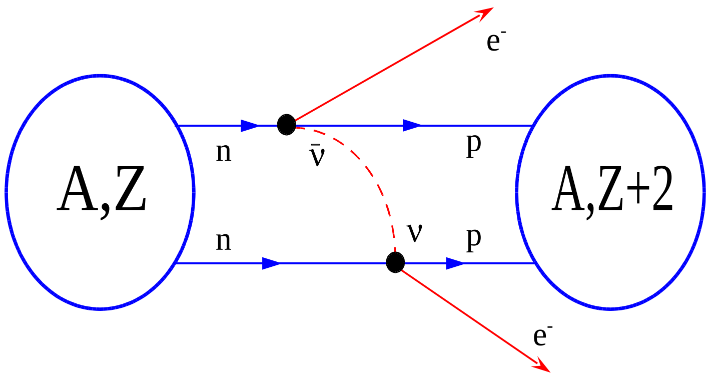
The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$(\text{A}, \text{Z}) \rightarrow (\text{A}, \text{Z} + 2) + e^- + e^-$$

of even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, ^{150}Nd.

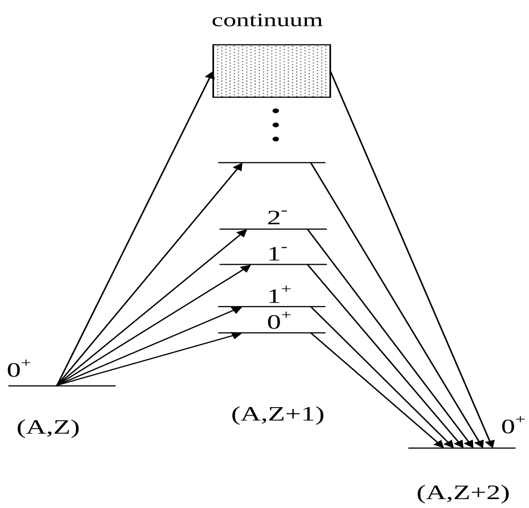
$2n$ from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of (A,Z+2) and two free e^-.

Nuclear $0\nu\beta\beta$-decay



strong in-medium modification of the basic process

$dd \rightarrow uue^- e^- (\overline{\nu}_e \overline{\nu}_e)$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}-$Decay Experiments:

- Majorana nature of ν_j
- Type of $\nu-$mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay , cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j- Dirac or Majorana particles, **fundamental problem**

ν_j-Dirac: **conserved lepton charge exists,** $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j-Majorana: **no lepton charge is exactly conserved,** $\nu_j \equiv \bar{\nu}_j$

The observed patterns of $\nu-$mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j- Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of $\nu-$ oscillations.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\; e^{i\alpha_{21}} + m_3|U_{e3}|^2\; e^{i\alpha'_{31}}\right|$$

$$= \left|m_1\; c_{12}^2\; c_{13}^2 + m_2\; s_{12}^2\; c_{13}^2\; e^{i\alpha_{21}} + m_3\; s_{13}^2\; e^{i\alpha'_{31}}\right|,\quad \theta_{12} \equiv \theta_{\odot},\; \theta_{13}\text{- CHOOZ}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix; $\alpha'_{31} \equiv \alpha_{31} - 2\delta$

CP-invariance: $\alpha_{21} = 0, \pm\pi,\; \alpha_{31} = 0, \pm\pi;$

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1,\quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$|<m>|$: m_j, $\theta_\odot \equiv \theta_{12}$, θ_{13}, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of min(m_j), $\Delta m^2_{\rm atm}$, $\Delta m^2_\odot$

S.T.P., A.Yu. Smirnov, 1994

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$\Delta m^2_\odot \equiv \Delta m^2_{21}, \quad m_2 = \sqrt{m_1^2 + \Delta m^2_\odot},$$

while either

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{\rm atm}}, \quad \text{normal mass ordering,} \quad \text{or}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad m_1 = \sqrt{m_3^2 + |\Delta m^2_{\rm atm}| - \Delta m^2_\odot}, \quad \text{inverted mass ordering}$$

The neutrino mass spectrum –

Normal hierarchical (NH) if $m_1 << m_2 << m_3$,

Inverted hierarchical (IH) if $m_3 << m_1 \cong m_2$,

Quasi-degenerate (QD) if $m_1 \cong m_2 \cong m_3 = m$, $m_j^2 >> |\Delta m^2_{\rm atm}|$; $m_j \gtrsim 0.1$ eV

Given $|\Delta m^2_{\rm atm}|$, $\Delta m^2_\odot$, $\theta_\odot$, θ_{13},

$$|<m>| = |<m>|\,(\mathrm{m_{min}}, \alpha_{21}, \alpha_{31}; \mathrm{S}), \ \mathrm{S} = \mathrm{NO(NH), IO(IH)}.$$

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left|\sqrt{\Delta m^2_{\odot}}\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13}e^{i\beta}\right|, \quad m_1 \ll m_2 \ll m_3 \text{ (NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

$$|<m>| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

$\theta_{12} \equiv \theta_{\odot}$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta + 2\delta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$$|<m>| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV}, \quad \text{IH};$$

$$m\cos 2\theta_{12} \lesssim |<m>| \lesssim m,\ m \gtrsim 0.10 \text{ eV}, \quad \text{QD}.$$

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|<m>|$ $= (0.1 - 0.9)$ eV (99.73% C.L.).

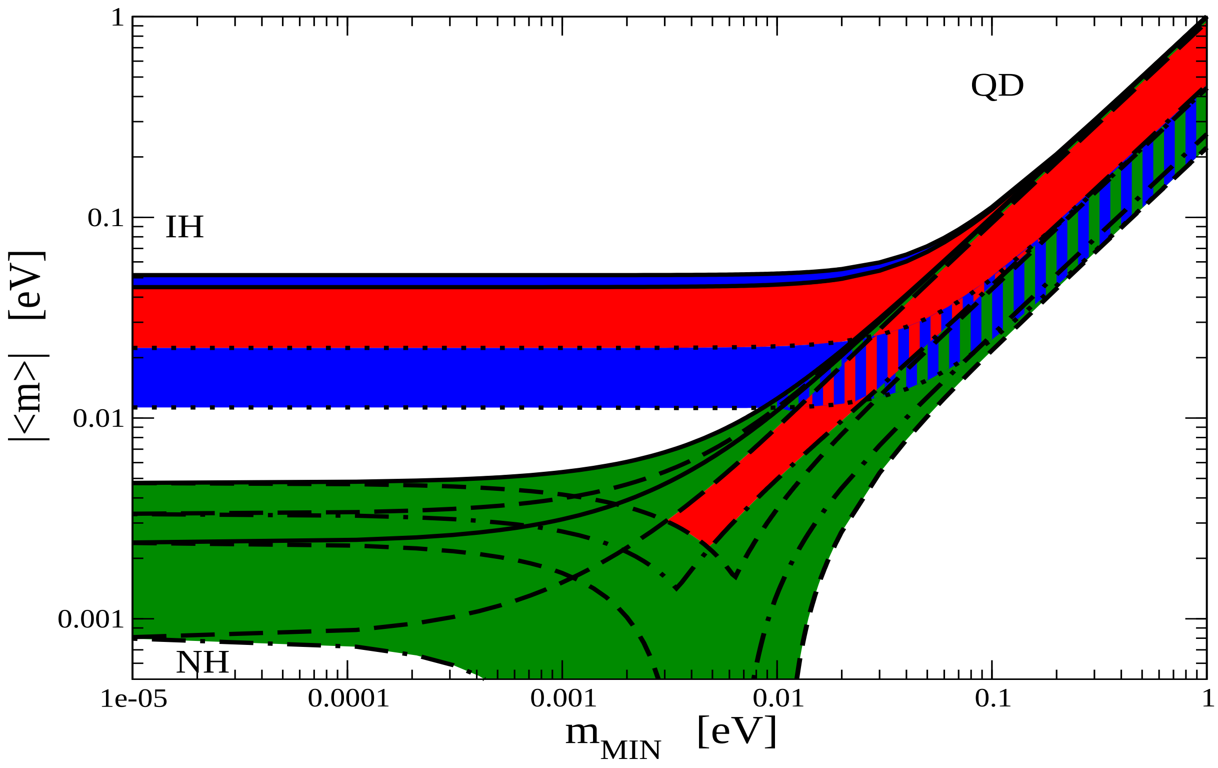
IGEX ^{76}Ge: $|<m>|$ $< (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|<m>|$ <(0.7–1.2) eV, $|<m>|$ <(0.18–0.90) eV (90% C.L.).

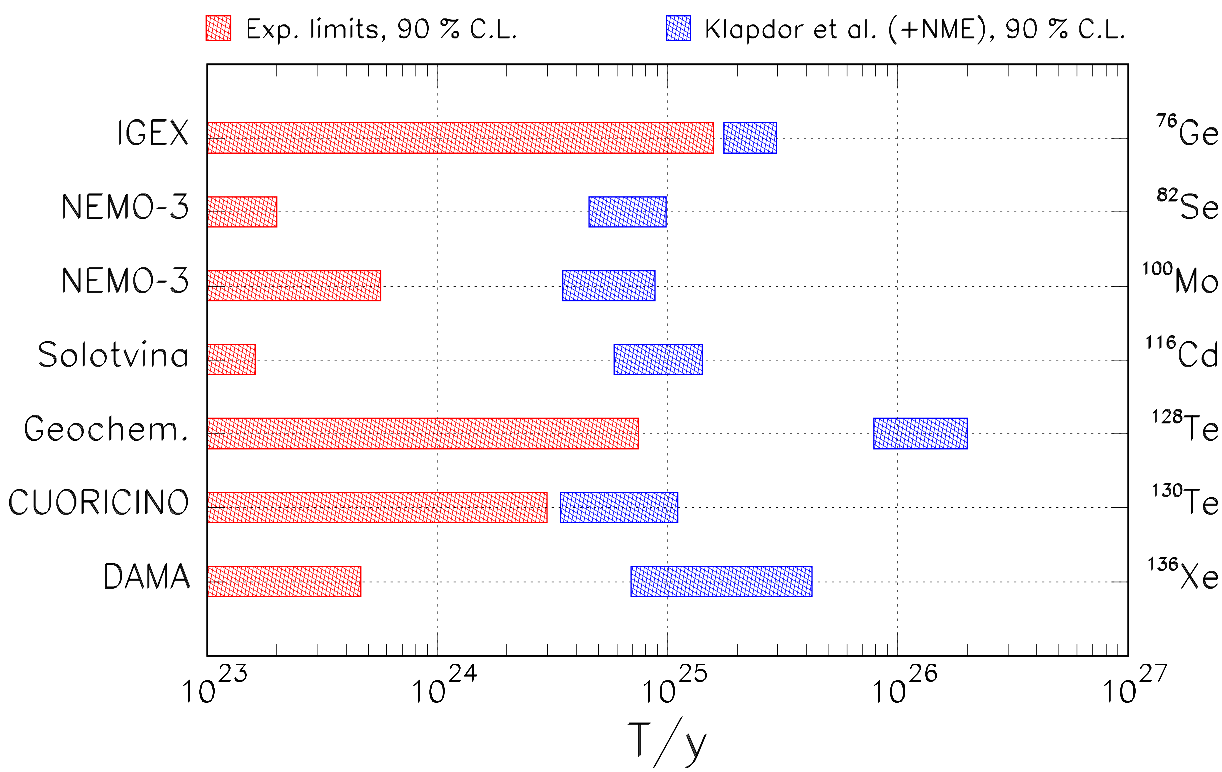
Large number of projects: $|<m>|$ $\sim (0.01 - 0.05)$ eV

CUORE - ^{130}Te;
GERDA - ^{76}Ge;
SuperNEMO - ^{82}Se,...;
COBRA - ^{116}Cd;
EXO - ^{136}Xe;
MAJORANA - ^{76}Ge;
MOON - ^{100}Mo;
CANDLES - ^{48}Ca;
XMASS - ^{136}Xe;
SNO+ - ^{150}Nd; KamLAND+ - ^{136}Xe



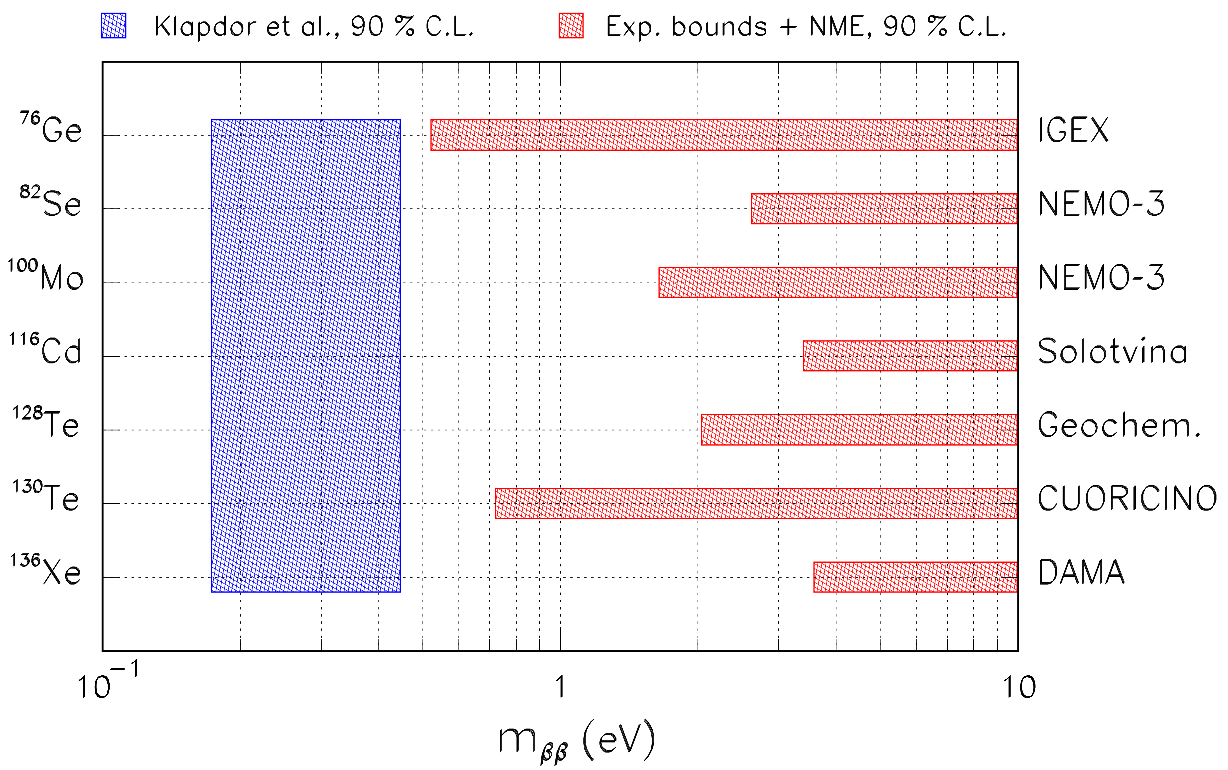
S. Pascoli, S.T.P., 2007

The current 2σ ranges of values of the parameters used.



E. Lisi *et al.*., 2008

The NME of F. Simkovic *et al.*, arXiv:0710.2055, used.



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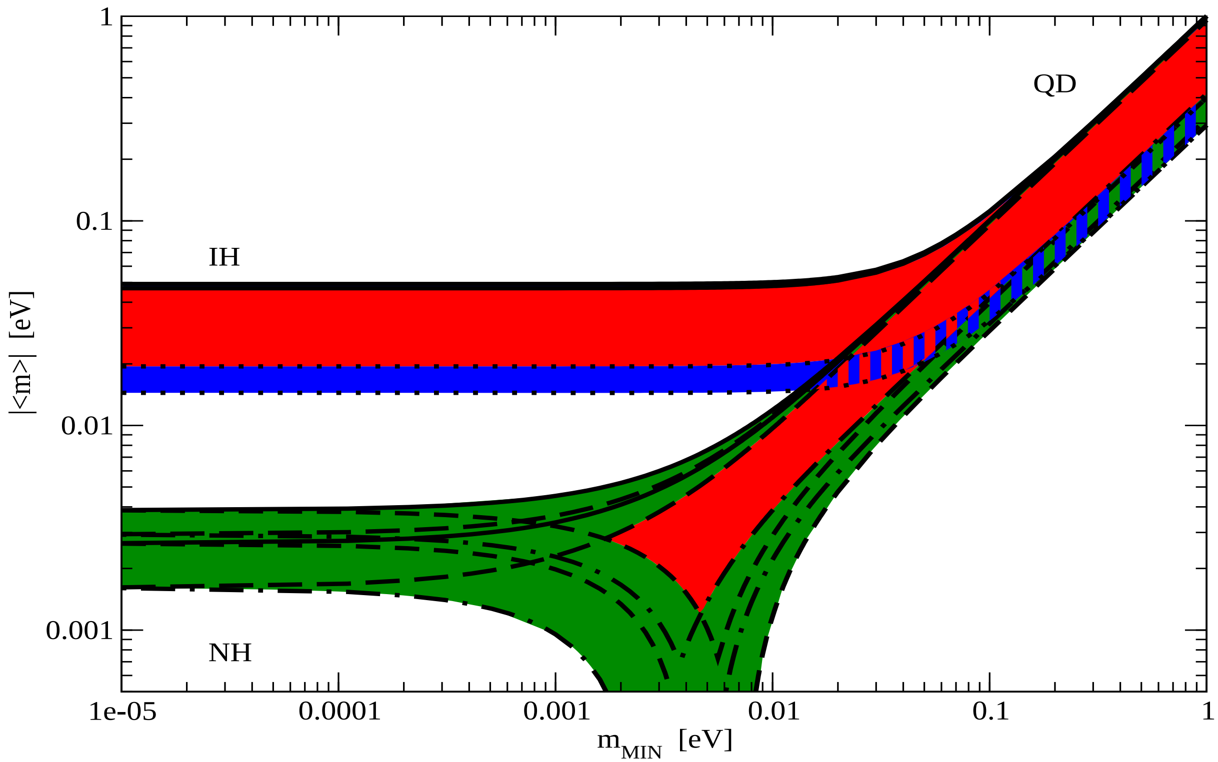


$\sin^2\theta_{13} = 0.010 \pm 0.006;\ 1\sigma(\Delta m^2_{\odot}) = 2\%,\ 1\sigma(\sin^2\theta_{\odot}) = 4\%,\ 1\sigma(|\Delta m^2_{\rm atm}|) = 2\%;$

$2\sigma(|\!<m>\!|\ )$ used.

Majorana CPV Phases and $|{<}\,m\,{>}|$

CPV can be established provided

– $|{<}\,m\,{>}|$ measured with $\Delta \lesssim 15\%$;

– Δm^2_{atm} (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;

– $\xi \lesssim 1.5$;

– α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;

– $\tan^2\theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No "No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"

V. Barger *et al.*, 2002

On the NME Uncertainties

The $(\beta\beta)_{0\nu}$-decay half-life

$$(T^{0\,\nu}_{1/2}(A,Z))^{-1} = |<m>|\,^2\,|M^{0\,\nu}(A,Z)|^2\,G^{0\,\nu}(E_0,Z),$$

$G^{0\nu}(E_0,Z)$, E_0 - known phase-space factor and energy release.

If we use a model M of the calculation of NME,

$$|<m>|\,^2_M(A,Z) = \frac{1}{T^{0\,\nu}_{1/2}(A,Z)\,|M^{0\,\nu}_M(A,Z)|^2\,G^{0\,\nu}(E_0,Z)}.$$

Suppose $(\beta\beta)_{0\nu}-$decay of several nuclei is observed.

$|<m>|$ cannot depend on parent nucleus (A_j,Z_j).

If the light Majorana ν-exchange - dominant mechanism of $(\beta\beta)_{0\nu}$-decay, **model M for NME can be correct only if**

$$|<m>|\,^2_M(A_1,Z_1) \simeq |<m>|\,^2_M(A_2,Z_2) = ...$$

For different models and the same nucleus (A,Z),

$$|<m>|\,^2_{M_1}(A,Z)\ |M^{0\nu}_{M_1}(A,Z)|^2 = |<m>|\,^2_{M_2}(A,Z)\ |M^{0\nu}_{M_2}(A,Z)|^2 = ...,$$

$$|<m>|\,^2_{M_2}(A,Z) = \eta^{M_2;M_1}(A,Z)\,|<m>|\,^2_{M_1}(A,Z)\ ,$$

$$\eta^{M_2;M_1}(A,Z) = \frac{|M^{0\,\nu}_{M_1}(A,Z)|^2}{|M^{0\,\nu}_{M_2}(A,Z)|^2}\ .$$

Nucleus	$\eta^{M_2;M_1}$	$\eta^{M_3;M_1}$	$\eta^{M_2;M_3}$
^{76}Ge	0.37	0.19	1.93
^{82}Se	—	0.38	—
^{100}Mo	—	—	6.56
^{130}Te	0.74	0.10	7.32
^{136}Xe	0.53	0.02	22.42

M_1 (SM): E. Caurier et al., 1999; M_2 (QRPA): V. Rodin et al., 2003;
M_3 (QRPA): O. Civatarese and J. Suhonen, 2003.

The observation of $(\beta\beta)_{0\nu}$-decay of at least 3 nuclei would be important for the solution of the problem of NME.

Table 2 suggests: ^{76}Ge, ^{130}Te, ^{136}Xe .

If for some model M

$$|<m>|\ ^2_M(A_1, Z_1) \simeq |<m>|\ ^2_M(A_2, Z_2) = ... \equiv |<m>|\ ^2_0\ ,$$

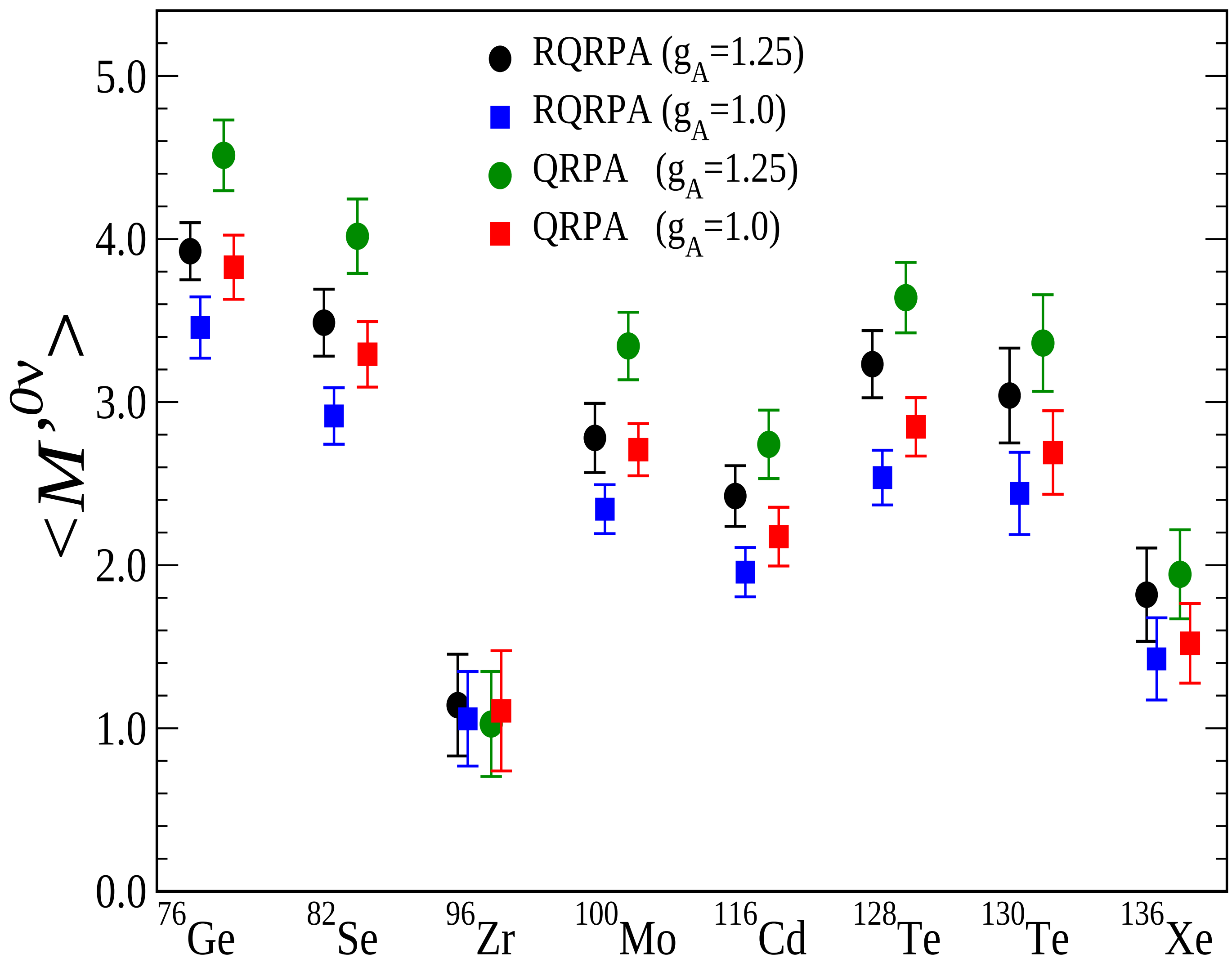
$|<m>|\ _0$ - the true value (most likely).
Strong dependence of NME on (A, Z) - crucial for the test.

L. Wolfenstein, S. Pascoli, S.T.P., 2001;

S. M. Bilenky, S.T.P., 2004

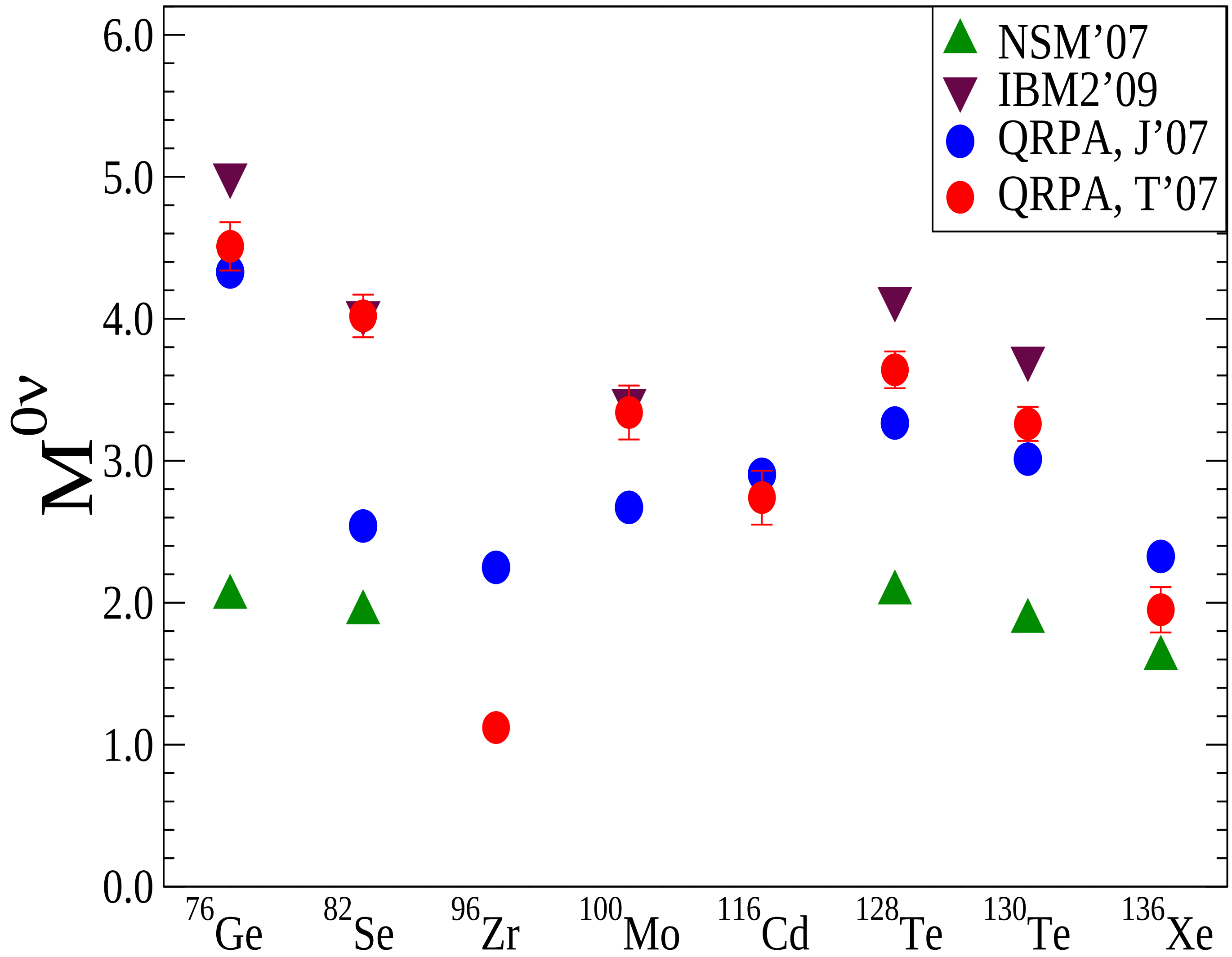
Encouraging results on the problem of calculating the NME ($\xi \lesssim 1.5$) have been obtained recently in

V. A. Rodin, A. Faessler, F. Simkovic, P. Vogel, nucl-th/0503063



V. A. Rodin *et al.*, nucl-th/0503063

The errors have no statistical origin, just illustrate the degree of the variation of the results by changing the basis size. The "systematic error" of the QRPA (due to neglecting many-particle configurations): $(3 \div 5) \times 10\%$, can vary from one nucleus to another.



NSM'07
IBM2'09
QRPA, J'07
QRPA, T'07
$M^{0\nu}$
6.0
5.0
4.0
3.0
2.0
1.0
0.0
^{76}Ge
^{82}Se
^{96}Zr
^{100}Mo
^{116}Cd
^{128}Te
^{130}Te
^{136}Xe

Alternative Mechanisms of $(\beta\beta)_{0\nu}$-Decay

- Light neutrino exchange

- R-parity violating SUSY

- Heavy neutrino exchange

- Right-handed weak currents

The alternative mechanisms can also be tested using data on the $(\beta\beta)_{0\nu}$-decay of several nuclei.

H. Paes and F. Deppisch, hep-ph/0612165;
E. Lisi *et al.*, arXiv:0905.1832

SuperNEMO

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.7) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

• Explains the smallness of $\nu-$masses.

• Through **leptogenesis theory** links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.

S. Fukugita, T. Yanagida, 1986.

• In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays

$$\mu \to e + \gamma, \quad \tau \to \mu + \gamma, \quad \tau \to e + \gamma \ , \ \text{etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

• The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\text{lep}}(x) = \mathcal{L}_{\text{CC}}(x) + \mathcal{L}_{\text{Y}}(x) + \mathcal{L}_{\text{M}}^{\text{N}}(x)\,,$$

$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,$$

$$\mathcal{L}_{\text{Y}}(x) = \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \text{h.c.}\,,$$

$$\mathcal{L}_{\text{M}}^{\text{N}}(x) = -\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.$$

ψ_{lL} - LH doublet, $\psi_{lL}^{\text{T}} = (\nu_{lL}\;\; l_L)$, l_R - RH singlet, H - Higgs doublet.
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \text{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$.
m_D generated by the Yukawa interaction:

$$-\mathcal{L}_{\text{Y}}^{\nu} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\;\; v = 174 \text{ GeV},\;\; v\,\lambda = m_D - \text{complex}$$

For M_R - sufficiently large,

$$m_\nu \simeq v^2\;\lambda^T\,M_R^{-1}\,\lambda = U^*_{\text{PMNS}}\;m_\nu^{\text{diag}}\;U^\dagger_{\text{PMNS}}\;.$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\;R\;\sqrt{D_\nu}\;(U_{\text{PMNS}})^\dagger/v_u$, all at M_R ; R-complex, $R^T R = \mathbf{1}$.

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ GeV;

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ GeV, $m_D \sim 1$ GeV.

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j$, $C(\overline{N}_k)^T = N_k$, $j,k = 1,2,3.$

The CP-symmetry transformation:

$$\begin{aligned} U_{\rm CP}\, N_j(x)\, U_{\rm CP}^{\dagger} &= \eta_j^{NCP}\, \gamma_0\, N_j(x')\,, \quad \eta_j^{NCP} = i\rho_j^N = \pm i\,, \\ U_{\rm CP}\, \nu_k(x)\, U_{\rm CP}^{\dagger} &= \eta_k^{\nu CP}\, \gamma_0\, \nu_k(x')\,, \quad \eta_k^{\nu CP} = i\rho_k^{\nu} = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda_{jl}^{*} = \lambda_{jl}\, (\eta_j^{NCP})^{*}\, \eta^{l}\, \eta^{H*}\,, \quad j = 1,2,3,\ l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i$, $\eta^H = 1$ $(\eta^W = 1)$:

$$\begin{aligned} \lambda_{jl}^{*} &= \lambda_{jl}\, \rho_j^N\,, \ \rho_j^N = \pm 1\,, \\ U_{lj}^{*} &= U_{lj}\, \rho_j^{\nu}\,, \ \rho_j^{\nu} = \pm 1\,, \\ R_{jk}^{*} &= R_{jk}\, \rho_j^N\, \rho_k^{\nu}, \quad j,k = 1,2,3,\ l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - either real or purely imaginary.

Relevant quantity:

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U_{lk}^{*}\, U_{lm}\,, \ k \neq m\,,$$

$$CP: \quad P_{jkml}^{*} = P_{jkml}\, (\rho_j^N)^2\, (\rho_k^{\nu})^2\, (\rho_m^{\nu})^2 = P_{jkml}\,, \quad {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\,R_{jm}\,U^{*}_{lk}\,U_{lm}\,,\;\; k \neq m\,,$$

$$CP: \quad P^{*}_{jkml} = P_{jkml}\,(\rho^{N}_{j})^{2}\,(\rho^{\nu}_{k})^{2}\,(\rho^{\nu}_{m})^{2} = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\,R_{13}\,U^{*}_{\tau 2}\,U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^{*}_{\tau 2}\,U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\,R_{13}$ corresponds to CP-violation at "high" E.

Leptogenesis

$$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11} \qquad (n_\gamma:\ \sim 6.3 \times 10^{-10})$$

$$Y_B \cong -10^{-2}\ \ \varepsilon\, \kappa$$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}\,, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\, v^2}{M_1} = \left|\sum_k R_{1k}\, m_k^{1/2}\, U_{lk}^*\right|^2, \quad l = e, \mu, \tau\,.$$

The "one-flavor" approximation - $\mathbf{Y}_{\mathbf{e},\mu,\tau}$ - "small":

Boltzmann eqn. for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$.

$Y_l\ H^c(x)\overline{l_R}(x)\psi_{lL}$- out of equilibrium at $T \sim M_1$.

One-flavor approximation: $M_1 \sim T > 10^{12}$ GeV

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \left|R_{1k}\right|^2}\,,$$

$$\widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k \left|R_{1k}\right|^2\,.$$

Two-Flavour Regime

At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not;

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \rightarrow (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \rightarrow N_1$;

$\tau_L^- + \Phi^0 \rightarrow \tau_R^-$, $\tau_L^- + \tau_L^+ \rightarrow N_1 + \nu_L$, etc.

$\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently.

Three-Flavour Regime

At $M_1 \sim T \sim 10^9$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not.

$\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently.

Thus, at $M_1 \sim 10^9 - 10^{12}$ GeV: L_τ, ΔL_τ - distinguishable;

L_e, L_μ, ΔL_e, ΔL_μ - individually not distinguishable;

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 \ (\sim T) \lesssim 10^{12}$ GeV,

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 \, v^2}{M_1} = \left| \sum_k R_{1k} m_k^{1/2} U_{lk}^* \right|^2 , \quad l = e, \mu, \tau \, .$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*} \left(\epsilon_2 \, \eta \left(\frac{417}{589} \widetilde{m_2} \right) + \epsilon_\tau \, \eta \left(\frac{390}{589} \widetilde{m_\tau} \right) \right) ,$$

$$\eta \left(\widetilde{m_l} \right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25 \times 10^{-3} \, \text{eV}} \right)^{-1} + \left(\frac{0.2 \times 10^{-3} \, \text{eV}}{\widetilde{m_l}} \right)^{-1.16} \right)^{-1} .$$

$Y_{\mathcal{B}} = -(12/37) \, (Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \text{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, \; R_{1j}R_{1k} = \pm\left|R_{1j}R_{1k}\right|, \\
&= \mp \frac{3M_1}{16\pi v^2} \frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j)\, \left|R_{1j} R_{1k}\right| \text{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}, \; R_{1j}R_{1k} = \pm i\left|R_{1j}R_{1k}\right|
\end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: Im $\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$, Re $\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37} \frac{\varepsilon_{1\tau}}{g_*} \left(\eta\left(\frac{390}{589} \widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right)\right)$$

$m_1 \ll m_2 \ll m_3,\ M_1 \ll M_{2,3};\quad R_{12}R_{13}$ – real; $\quad m_1 \cong 0,\ R_{11} \cong 0$ (N_3 decoupling)

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{31}}\right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \times \left(1-\frac{\sqrt{\Delta m^2_{\odot}}}{\sqrt{\Delta m^2_{31}}}\right) \mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3})$$

$$\mathrm{Im}\,(U^*_{\tau 2}U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta-\frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi,\ \delta = 0$: $\quad \mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) = 0,\quad$ CPV due to R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2.8 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.2}\right) \left(\frac{M_1}{10^9\ \text{GeV}}\right) .$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \gtrsim 0.11\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \qquad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ (NH)

Majorana CP-violation

$\delta = 0$, real R_{12}, R_{13} ($\beta_{23} = \pi$ (0));

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}}\right) \left(\frac{M_1}{10^9 \text{ GeV}}\right).$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, or $|\sin \alpha_{32}/2| \gtrsim 0.15$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

$m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;

$|Y_B|$ suppressed by the additional factor $\Delta m^2_\odot / |\Delta m^2_{\rm A}| \cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa |R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - maximise $|\epsilon_\tau|$ and $|Y_B|$:

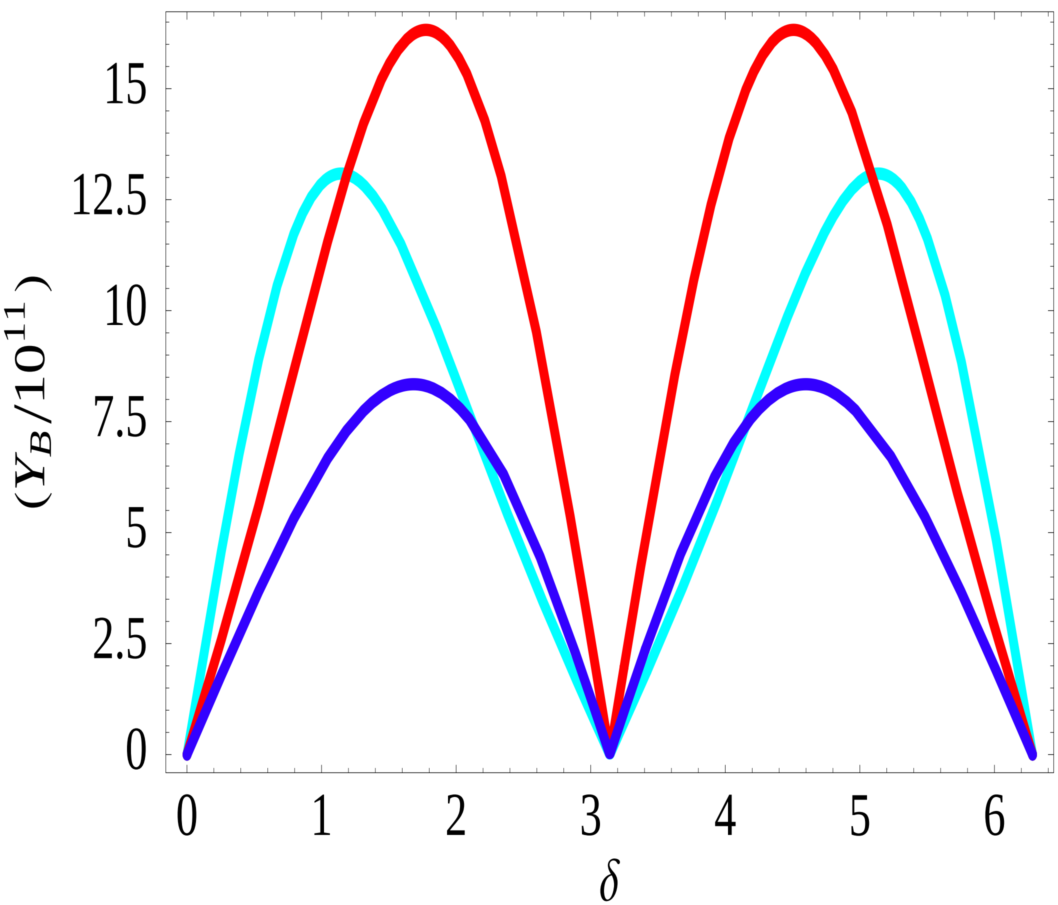
$$|Y_B| \;\cong 8.1 \times 10^{-12}\, |s_{13} \sin\delta| \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,.$$

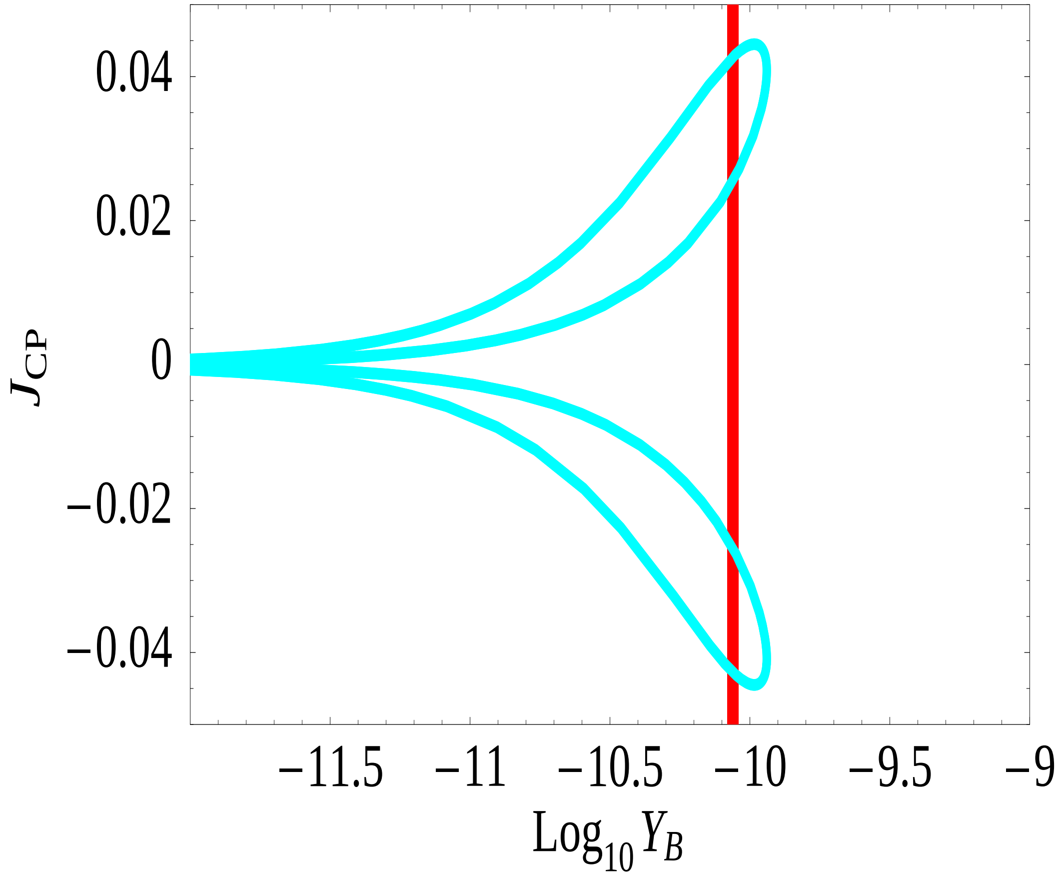
The lower limit corresponds to

$$|J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$$



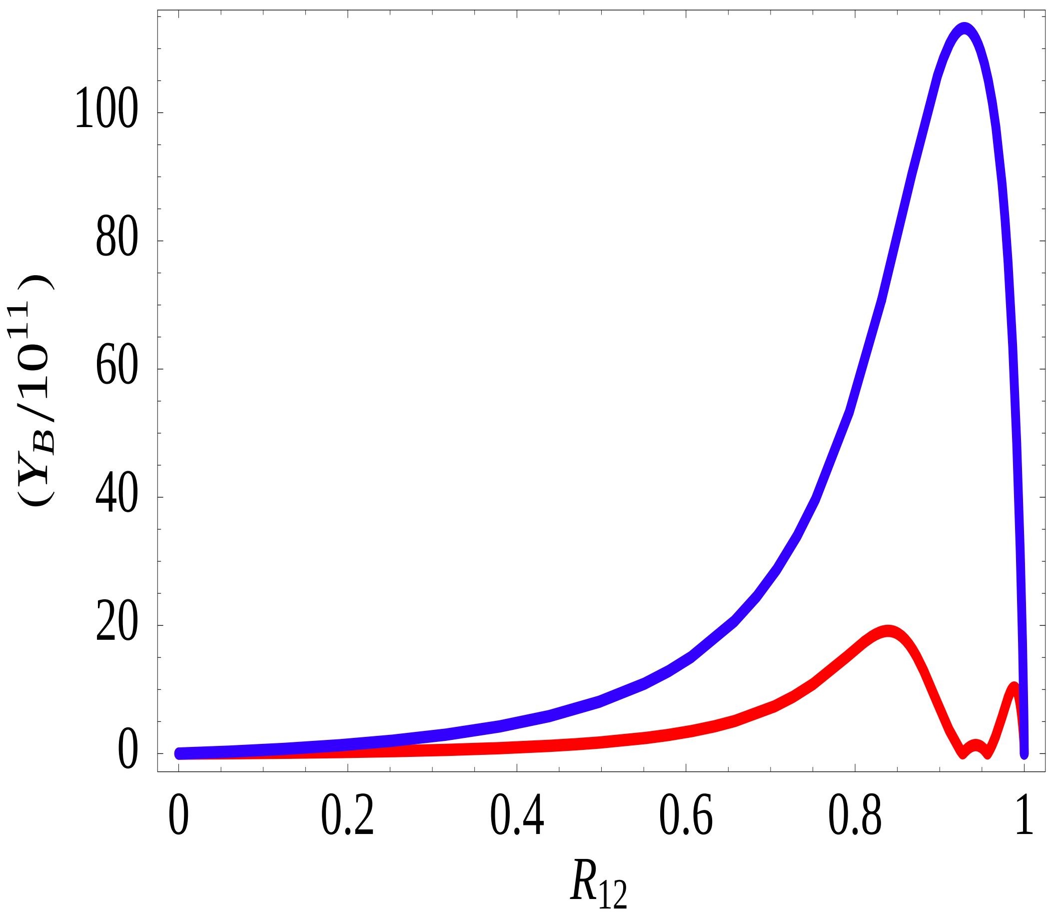
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign$\,(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line);
ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line);
$M_1 = 5 \times 10^{11}$ GeV.

S. Pascoli, S.T.P., A. Riotto, 2006.

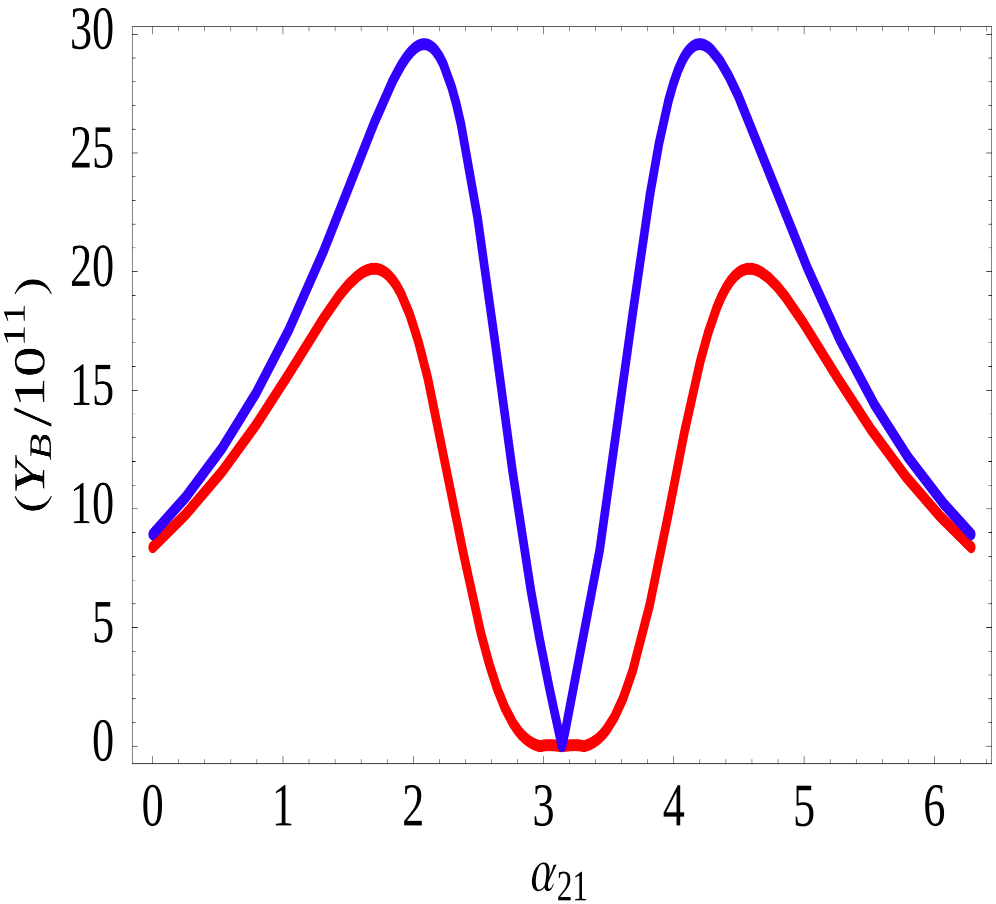


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
Dirac CP-violation, $\alpha_{32} = 0\ (2\pi)$;
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1\ (-1)$ ($\beta_{23} = 0\ (\pi)$, $\kappa' = +1$);
The red region denotes the 2σ allowed range of Y_B.

S. Pascoli, S.T.P., A. Riotto, 2006.

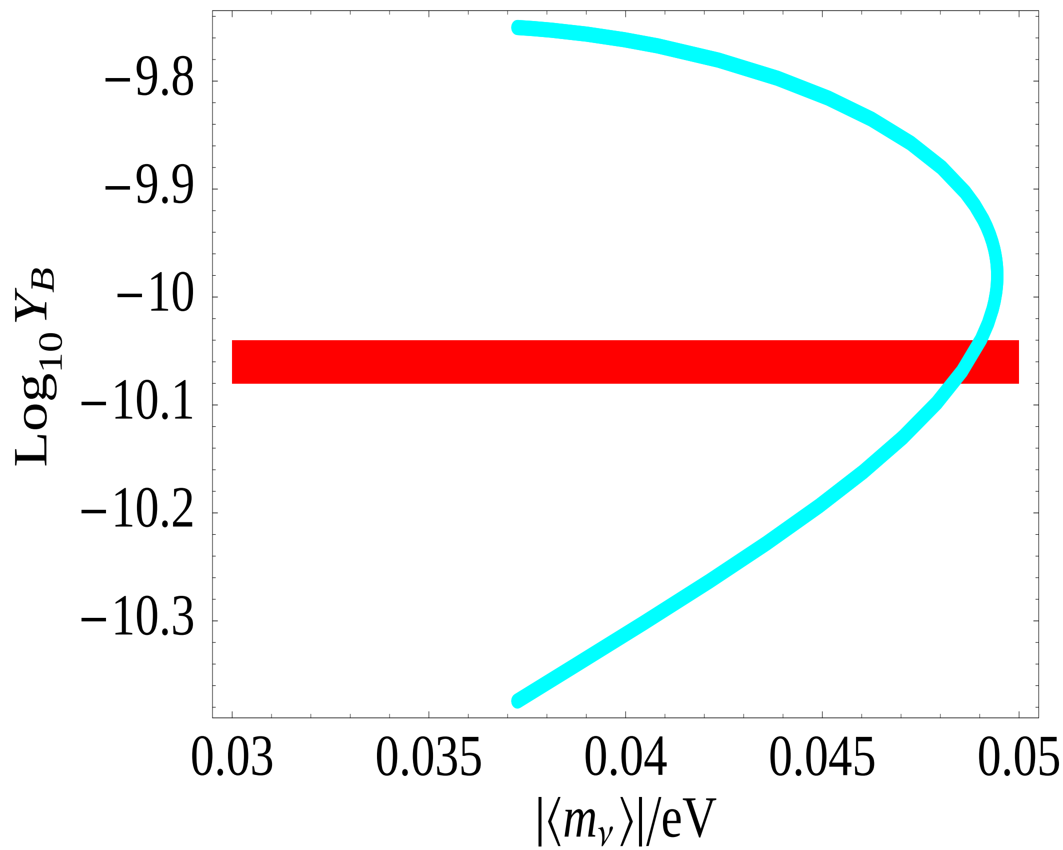


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ GeV;
real R_{12}, R_{13}, sign$\,(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ ($\kappa = +1$);
b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ ($\kappa' = +1$);
$\Delta m^2_{\odot}$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ - fixed at their best fit values.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ (blue line) and 0.2 (red line).

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ GeV;
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ (IH)

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: possible to reproduce Y_B^{obs} for real $R_{12(11)}R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ eV; non-trivial dependence of $|Y_B|$ on m_3

Majorana CPV, $\delta = 0$ (π): requires $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$: typically requires $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $\quad M_1 \lesssim 5 \times 10^{11}$ GeV imply

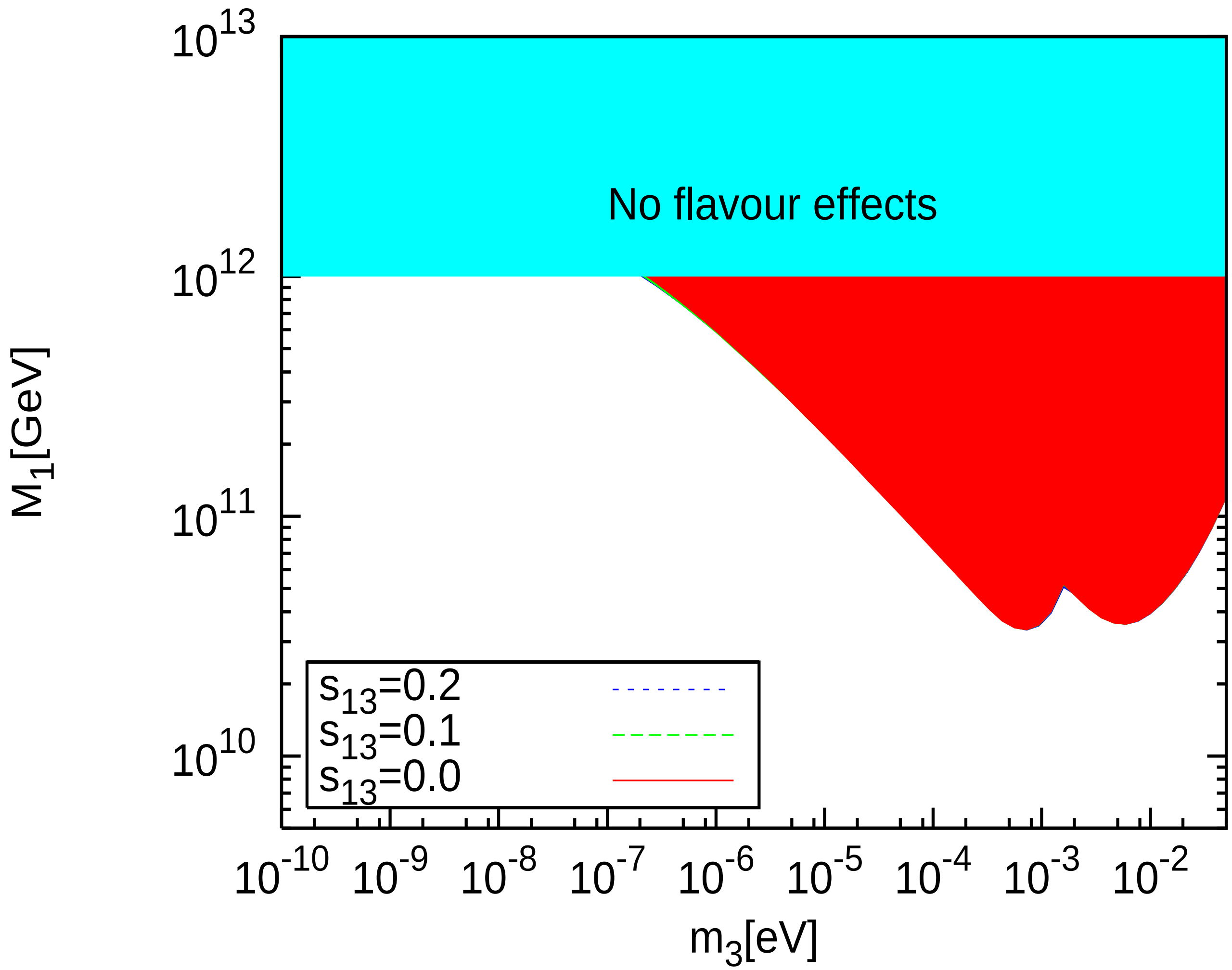
$$|\sin\theta_{13} \sin\delta|, \;\; \sin\theta_{13} \gtrsim (0.04 - 0.09)\,.$$

The lower limit corresponds to

$$|J_{\mathrm{CP}}| \gtrsim (0.009 - 0.02)$$

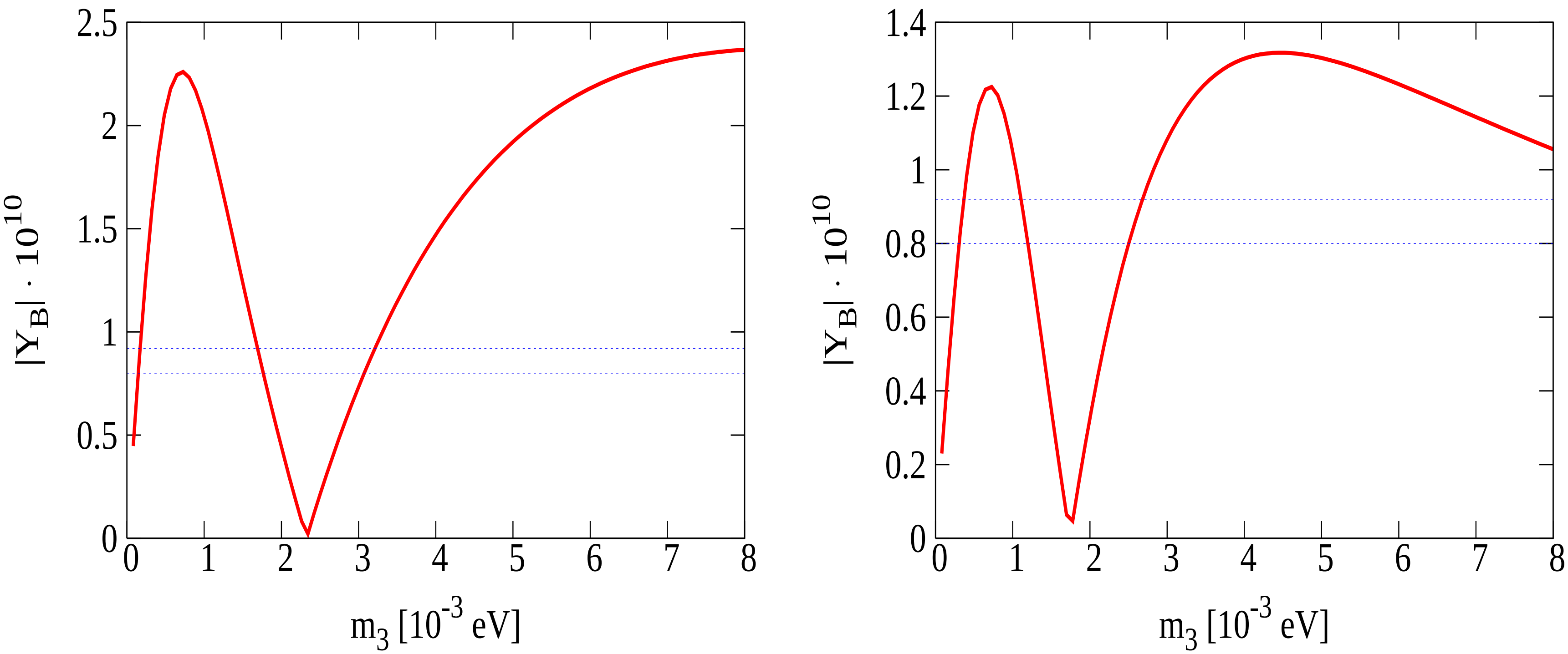
NO (NH) spectrum, $m_1 < \;(\ll)\; m_2 < m_3$: similar dependence of $|Y_B|$ on m_1 if $R_{12} = 0$, $R_{11}R_{13} \neq 0$; non-trivial effects for $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ eV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



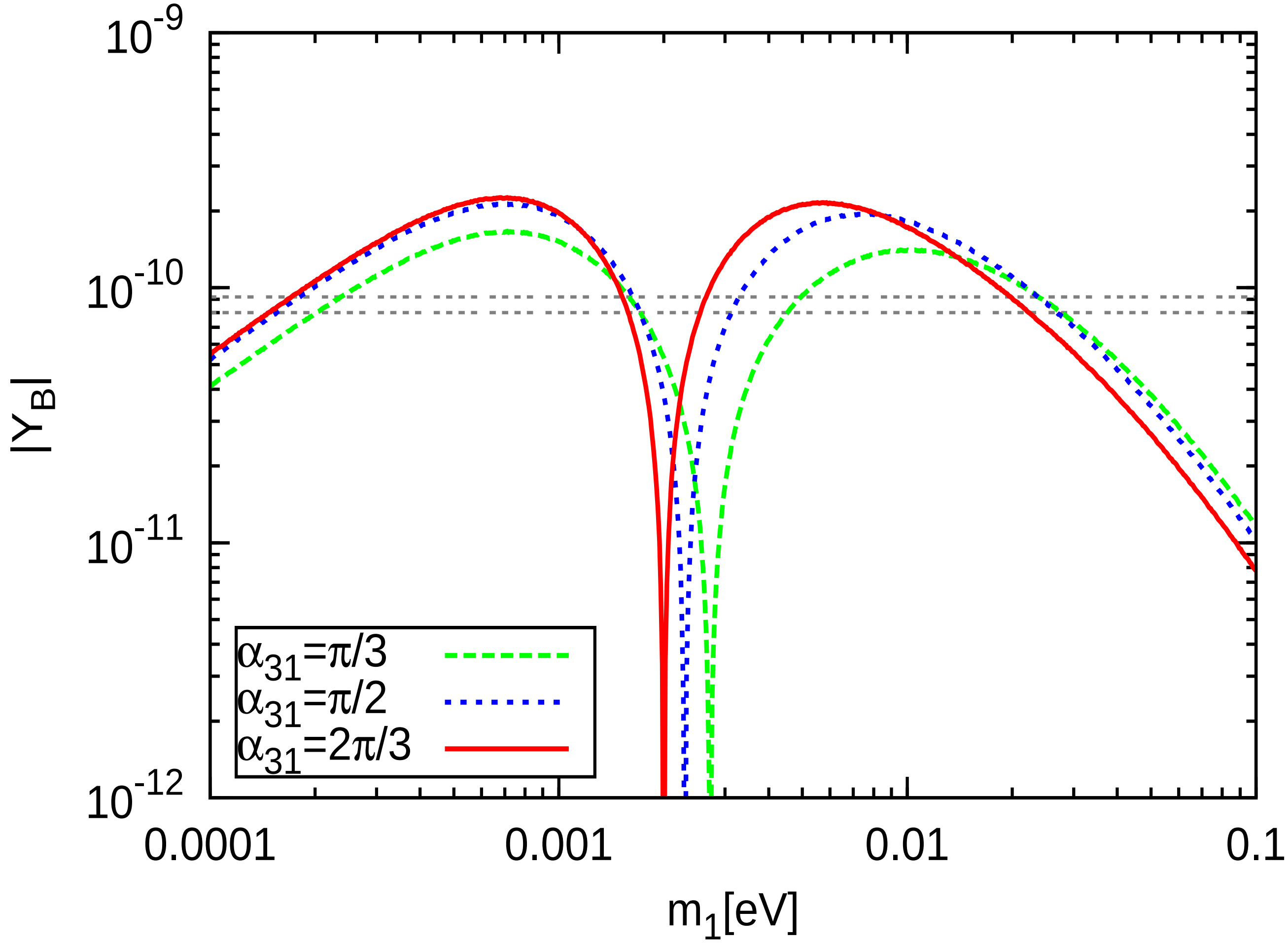
$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, real R_{1j}; $M_1 = (10^9 - 10^{12})$ GeV, $s_{13} = 0.2; 0.1; 0$;
R_{1j} varied within $|R_{13}|^2 + |R_{12}|^2 + |R_{13}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ varied in $[0, 2\pi]$;
min(M_1) for given m_3: $|Y_B| = 8.6 \times 10^{-11}$; absolute minima of M_1:
$m_3 \cong 5.5 \times 10^{-4}$; 5.9×10^{-3} eV, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4\ (3.5) \times 10^{10}$ GeV.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_3 \ll m_1 \ll m_2$ (IH), $R_{11} = 0$, real $R_{12}R_{13}$, Majorana CPV;
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ GeV; $R^2_{12}/R^2_{13} = m_3/m_2$: maximises $|\epsilon_\tau|$;
i) sgn$(R_{12}R_{13}) = +1$; ii) sgn$(R_{12}R_{13}) = -1$.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_1 < m_2 < m_3$ (NO(NH)), $R_{12} = 0$, real $R_{11}R_{13}$, Majorana CPV, $s_{13} = 0$;
sgn($R_{11}R_{13}$) $= -1$, $\sin^2\theta_{23} = 0.50$, $M_1 = 1.5 \times \times 10^{11}$ GeV;
$\alpha_{32} = 2\pi/3; \pi/2; \pi/3$ (red, blue, green lines).

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007

Complex R: $\varepsilon_{1l} \neq 0$, CPV from U and R

$m_1 \ll m_2 < m_3$ (NH), $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ (N_3 decoupling)

$$R_{12}^2 + R_{13}^2 = |R_{12}|^2\, e^{i2\varphi_{12}} + |R_{13}|^2\, e^{i2\varphi_{13}} = 1,$$

$$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0: \ \text{sgn}(\sin 2\varphi_{12}) = -\text{sgn}(\sin 2\varphi_{13})\,.$$

$$\cos 2\varphi_{12} = \tfrac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}\,, \quad \sin 2\varphi_{12} = \pm\sqrt{1-\cos^2 2\varphi_{12}}\,,$$

$$\cos 2\varphi_{13} = \tfrac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}\,, \quad \sin 2\varphi_{13} = \mp\sqrt{1-\cos^2 2\varphi_{13}}\,.$$

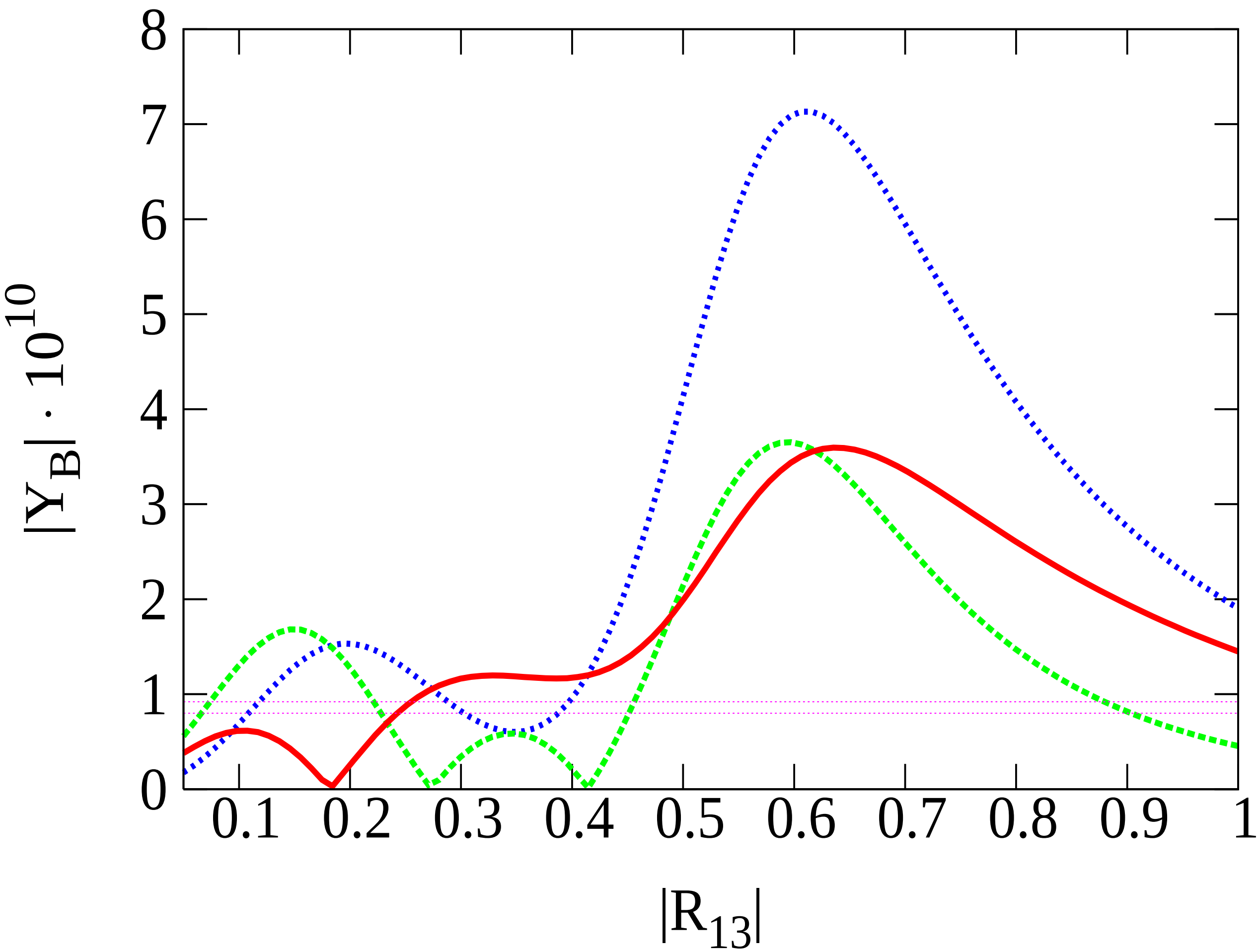
$m_3 \ll m_1 < m_2$ (IH), $M_1 \ll M_{2,3}$; $m_3 \cong 0$, $R_{13} \cong 0$ (N_3 decoupling)

$$R_{11}^2 + R_{12}^2 = |R_{11}|^2\, e^{i2\varphi_{11}} + R_{12}^2\, e^{i2\varphi_{12}} = 1,$$

$$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0\,.$$

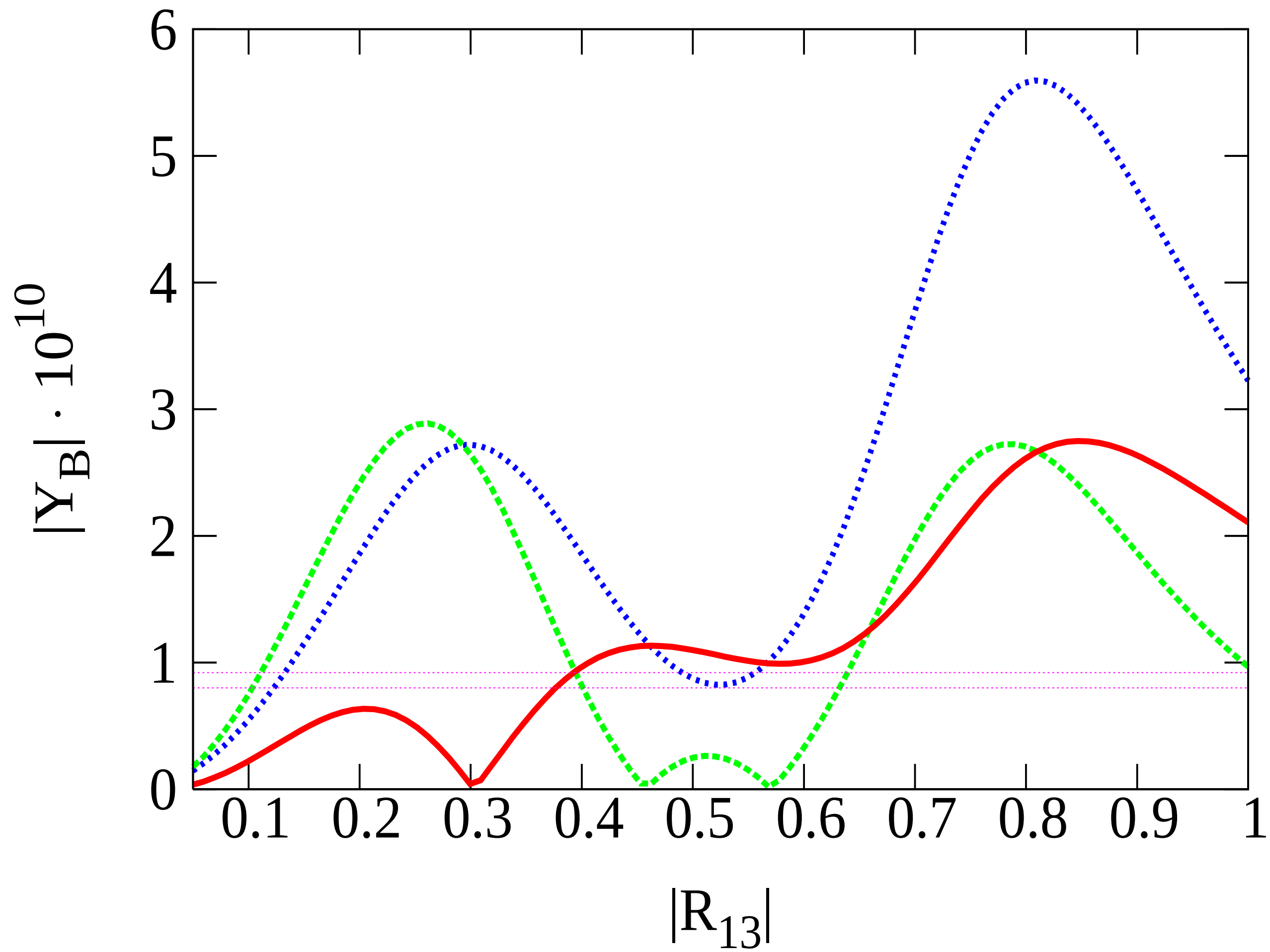
$|Y_B^0 A_{\text{HE}}| \propto |R_{11}|^2 \sin(2\varphi_{11})\,(|U_{\tau 1}|^2 - |U_{\tau 2}|^2)$ - can be suppressed:

$$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2)s_{23}^2 - 4s_{12}c_{12}s_{23}c_{23}s_{13}\cos\delta \cong -0.20 - 0.92\, s_{13}\,\cos\delta\,.$$



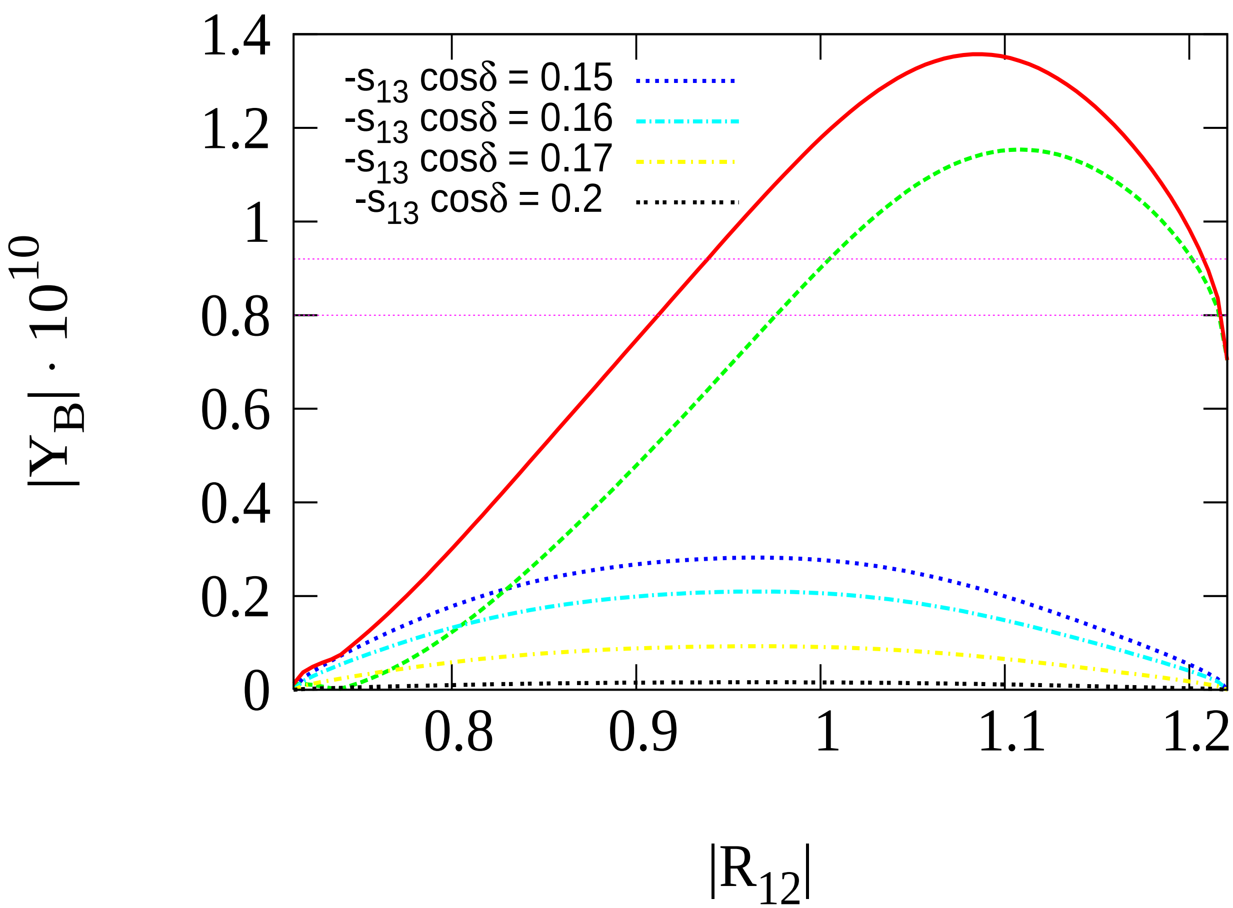
$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U,
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2\theta_{23} = 0.50$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\text{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\text{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line)

E. Molinaro, S.T.P., 2008



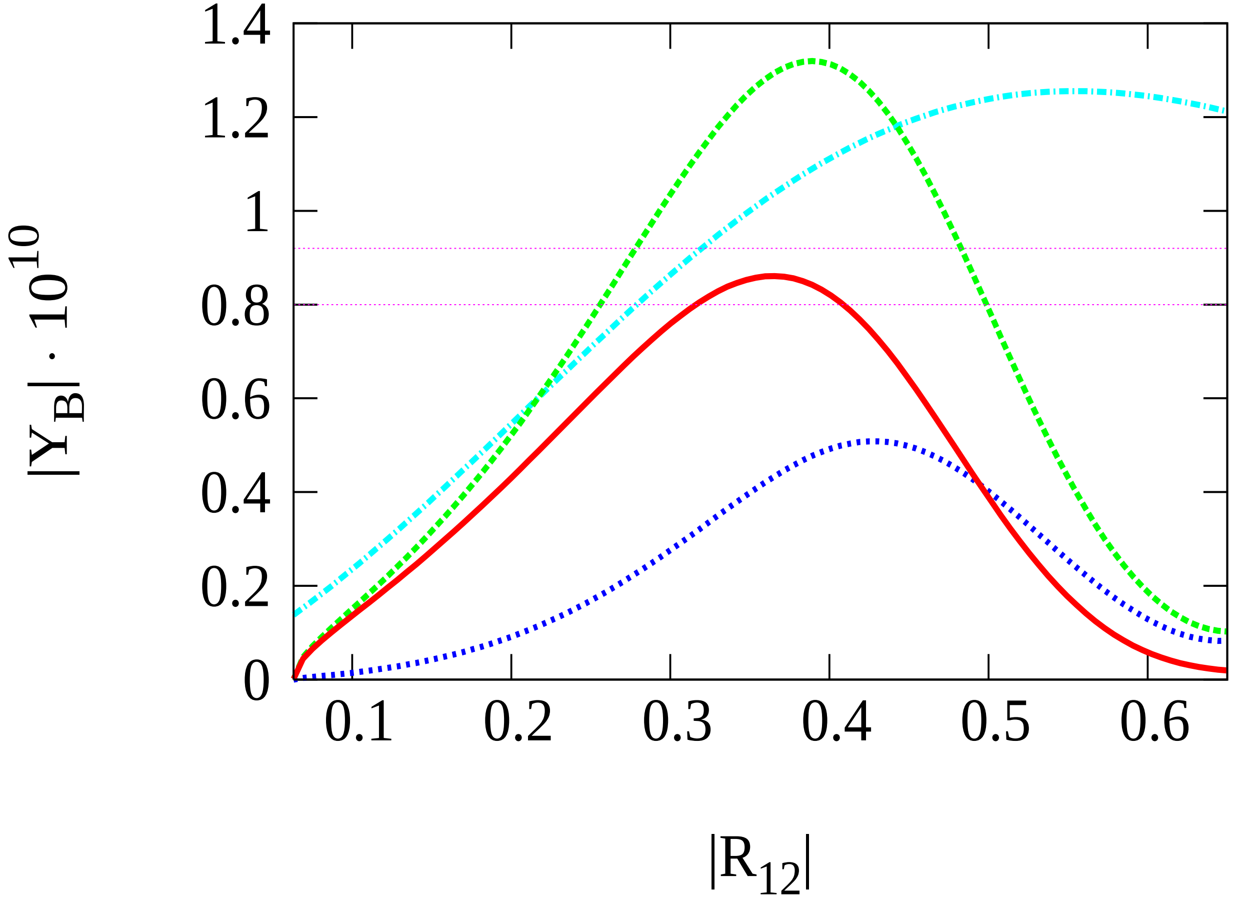
$m_1 < m_2 < m_3$ (NO(NH)), $R_{11} = 0$, CPV due to R and U,
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2\theta_{23} = 0.64$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\rm HE}|$ (R CPV, blue), $|Y_B^0 A_{\rm MIX}|$ (U CPV, green), total $|Y_B|$ (red line)

E. Molinaro, S.T.P., 2008



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV ,
$\alpha_{21} = \pi/2$, $(-s_{13} \cos\delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\text{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\text{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .

E. Molinaro, S.T.P., 2008



$m_3 \ll m_1 < m_2$ (IH)), $R_{13} = 0$, Majorana and R-matrix CPV,
$\alpha_{21} = \pi/2$, $s_{13} = 0$, $|R_{11}| \cong 1$, $M_1 = 10^{11}$ GeV;
$|Y_B^0 A_{\mathrm{HE}}|$ (R CPV, blue), $|Y_B^0 A_{\mathrm{MIX}}|$ (U CPV, green), total $|Y_B|$ (red line) .
Light-blue line: CP-conserving R, $R_{11}R_{12} \equiv ik|R_{11}R_{12}|$, $k = -1$ $|R_{11}|^2 - |R_{12}|^2 = 1$.

E. Molinaro, S.T.P., 2008

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j - heavy RH ν's;
N_j, ν_k - Majorana particles

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ in $U_{\rm PMNS}$, no other sources of CPV (Majorana phases in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 10^{11}$ GeV.

$m_1 \ll m_2 \ll m_3$ (NH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\rm CP}| \gtrsim 2.0\times 10^{-2}$$

$m_3 \ll m_1 < m_2$ (IH):

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \quad |J_{\rm CP}| \gtrsim 4.6\times 10^{-3}$$

B. CP-violation due to the Majorana phases in $U_{\rm PMNS}$, no other sources of CPV (Dirac phase in $U_{\rm PMNS}$ equal to 0, etc.); requires $M_1 \gtrsim 3.5\times 10^{10}$ GeV.

C. CP-violation due to both Dirac and Majorana phases in $U_{\rm PMNS}$.

D. Y_B can depend non-trivially on min$(m_j) \sim (10^{-5} - 10^{-2})$ eV.

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Conclusions

The $(\beta\beta)_{0\nu}$-decay experiments:

– **Can establish the Majorana nature of ν_j**

– **Can provide unique information on the ν mass spectrum**

– **Can provide unique information on the absolute scale of ν masses**

– **Can provide information on the Majorana CPV phases**

The knowledge of the values of the relevant $(\beta\beta)_{0\nu}$−decay NME with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of $\Gamma(\beta\beta)_{0\nu}$.

The precision in the measurement of $\Gamma(\beta\beta)_{0\nu}$ will also be very important for the quantitative interpretation of the data.

Conclusions (contd.)

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The see-saw mechanism provides a link between ν-mass generation and BAU.
Majorana CPV phases in U_{PMNS}: $(\beta\beta)_{0\nu}$-decay, Y_{B}.
Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.