

# Neutrino Masses:

What Kind?

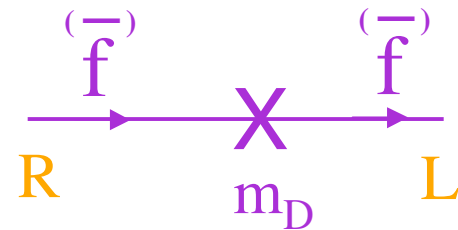
How Big?

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Japan–US Seminar  
September 17, 2005

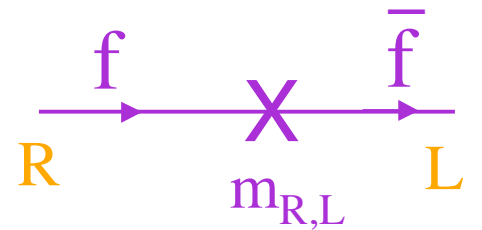
# What Kind of Masses?

There are two kinds of fermion masses:

**Dirac mass:**  $m_D \bar{f}_L f_R$



**Majorana mass:**  $m_R \bar{f}_R^c f_R$  or  $m_L \bar{f}_L^c f_L$



A *quark* or *charged-lepton* **Majorana** mass would not conserve electric charge.

Only a *neutrino* can have a **Majorana** mass.

If neutrinos do have **Majorana** masses, then —

The physics of neutrino mass is **different** from that of the charged lepton, quark, nucleon, human, earth, and galactic masses.

If neutrinos do have **Majorana** masses, then —

Each neutrino mass eigenstate  $\nu_i$  is **identical** to its antiparticle:

$$\bar{\nu}_i(\mathbf{h}) = \nu_i(\mathbf{h})$$

The diagram shows the equation  $\bar{\nu}_i(\mathbf{h}) = \nu_i(\mathbf{h})$  where the  $\mathbf{h}$  in both terms is blue. Below the equation, a horizontal blue line extends from the right towards the center. From the left end of this line, a vertical blue arrow points up to the  $\mathbf{h}$  in  $\bar{\nu}_i(\mathbf{h})$ . From the right end of the line, a vertical blue arrow points up to the  $\mathbf{h}$  in  $\nu_i(\mathbf{h})$ . The word "helicity" is written in blue to the right of the horizontal line.

The other constituents of matter, the quarks and charged leptons, being electrically charged, are *not* identical to their antiparticles.

If neutrinos do have **Majorana** masses, then —

The neutrinos and the physics of their masses are very distinctive.

If neutrinos do have **Majorana** masses, then —

The **Lepton Number L** defined by —

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$$

is not conserved.

- The presence of Majorana masses
- $\bar{\nu}_i = \nu_i$  (Majorana neutrinos)
- L not conserved

— are all equivalent

**Any one implies the other two.**

# Why Do Many Theorists Expect Majorana Masses?

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably Electroweak Isospin Invariance), and its renormalizability.

Anything allowed by the symmetries occurs in nature.

The SM contains no  $\nu_R$  field, only  $\nu_L$ , and no  $\nu$  mass.

This SM conserves the lepton number  $L$ .

We now know that the neutrino does have mass.

If we try to preserve conservation of L, we accommodate this mass by adding to the SM a Dirac, L - conserving, mass term:  $m_D \bar{\nu}_L \nu_R$ .

To add the Dirac mass term, we had to add  $\nu_R$  to the SM.

Unlike  $\nu_L$ ,  $\nu_R$  carries no Electroweak Isospin.

Thus, no SM symmetry prevents the occurrence of the Majorana mass term  $m_R \overline{\nu_R^c} \nu_R$ .

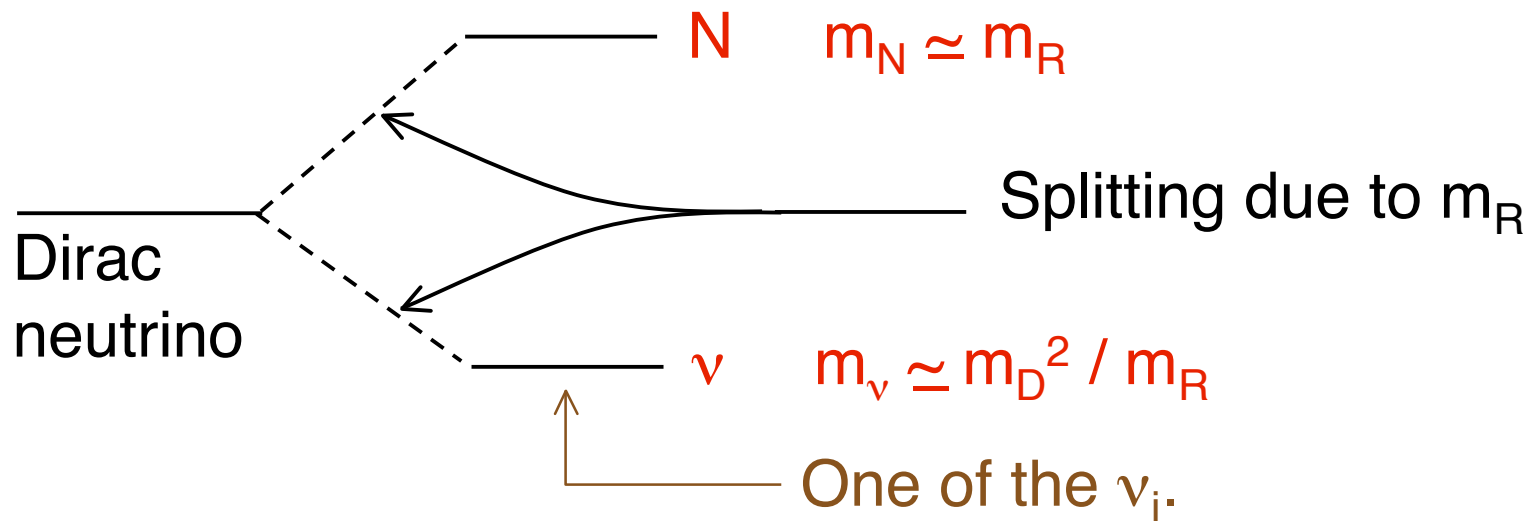
If, in the neutrino-mass sector as elsewhere, nature contains everything allowed by the SM principles, then she contains Majorana neutrino masses.

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In the See-Saw Mechanism,

$$\mathcal{L}_{\text{mass}} \sim \begin{bmatrix} \bar{\nu}_L, \bar{\nu}_R^c \end{bmatrix} \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L^c \\ \nu_R \end{bmatrix}$$

with  $m_R \gg m_D \sim m_{q \text{ or } l}$ .





# Predictions

- Each  $\bar{\nu}_i = \nu_i$  (Majorana neutrinos)
- The light neutrinos have heavy partners N

How heavy??

$$m_N \sim \frac{m_{\text{top}}^2}{m_\nu} \sim \frac{m_{\text{top}}^2}{0.05 \text{ eV}} \sim 10^{15} \text{ GeV}$$

Near the GUT scale.

# How Can We Demonstrate That $\bar{\nu}_i = \nu_i$ ?

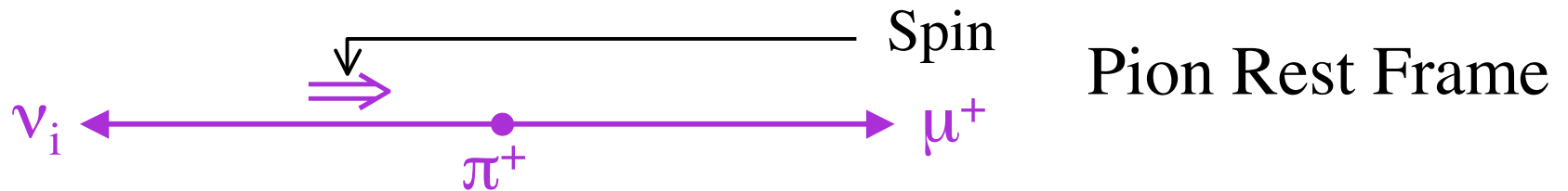
We assume neutrino **interactions** are correctly described by the SM. Then the **interactions** conserve L ( $\nu \rightarrow l^-$ ;  $\bar{\nu} \rightarrow l^+$ ).

## An Idea that Does Not Work

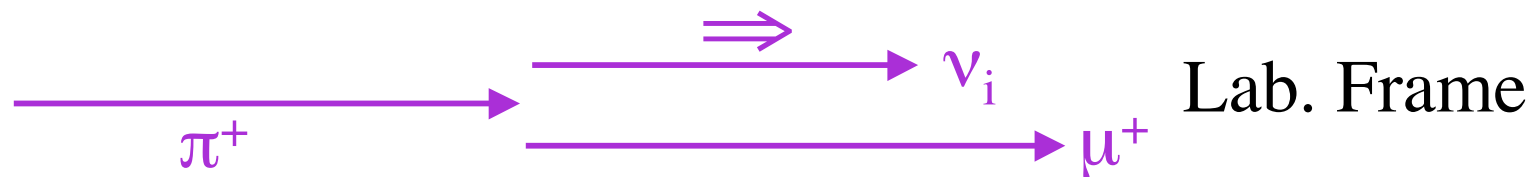
[and illustrates why most ideas do not work]

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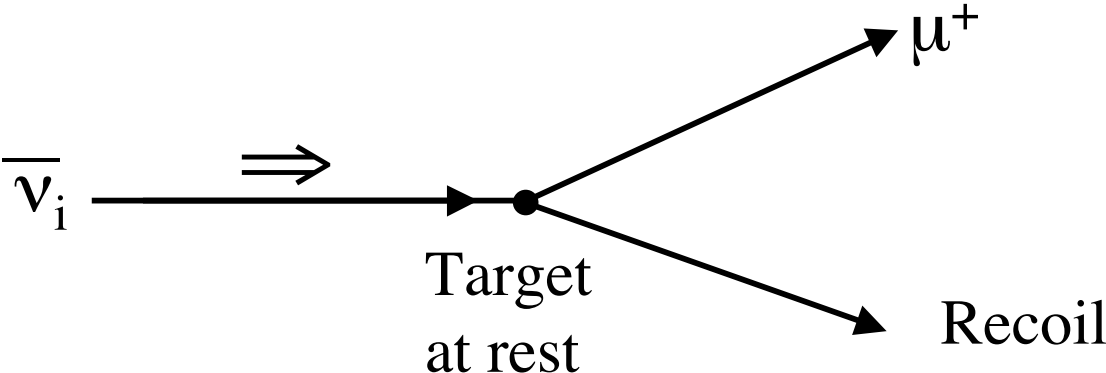
Produce a  $\nu_i$  via—



Give the neutrino a Boost:  
 $\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$



The SM weak interaction causes —



$v_i = \bar{v}_i$  means that  $v_i(h) = \bar{v}_i(h)$ .

↑      ↑      helicity

If  $v_i \Rightarrow = \bar{v}_i \Rightarrow$  ,

our  $v_i \Rightarrow$  will make  $\mu^+$  too.

# Minor Technical Difficulties

$$\begin{aligned}\beta_{\pi}(\text{Lab}) &> \beta_{\nu}(\pi \text{ Rest Frame}) \\ \Rightarrow \frac{E_{\pi}(\text{Lab})}{m_{\pi}} &> \frac{E_{\nu}(\pi \text{ Rest Frame})}{m_{\nu_i}} \\ \Rightarrow E_{\pi}(\text{Lab}) &\gtrsim 10^5 \text{ TeV if } m_{\nu_i} \sim 0.05 \text{ eV}\end{aligned}$$

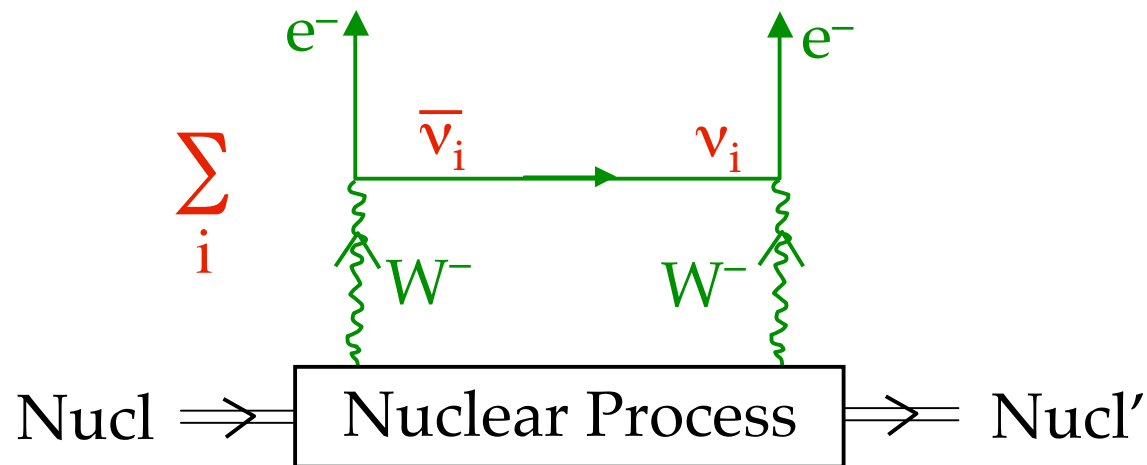
Fraction of all  $\pi$  – decay  $\nu_i$  that get helicity flipped

$$\approx \left( \frac{m_{\nu_i}}{E_{\nu}(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Since L-violation comes only from Majorana neutrino masses, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & Stodolsky)

# The Idea That **Can** Work — Neutrinoless Double Beta Decay [ $0\nu\beta\beta$ ]

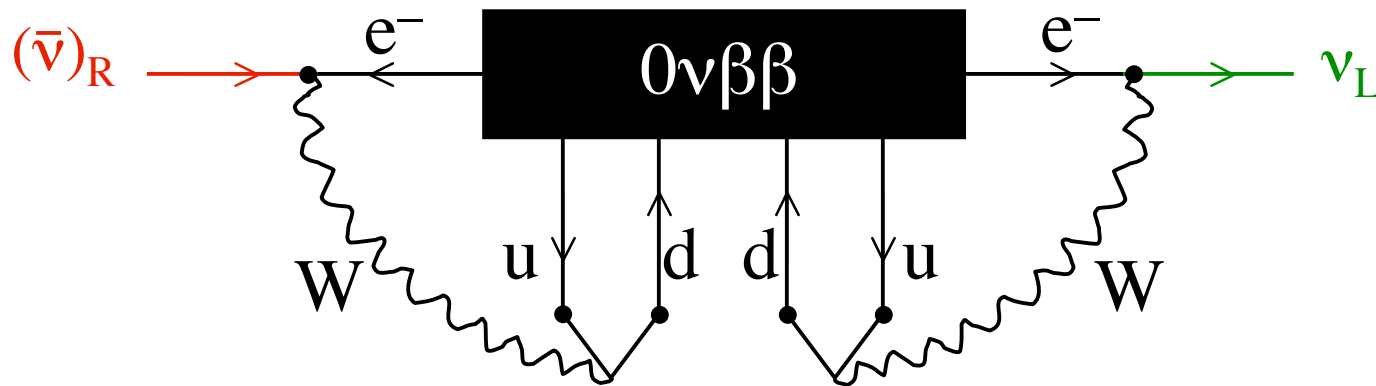


By avoiding competition, this process can cope with the small neutrino masses.

Observation would imply  $\cancel{X}$  and  $\bar{\nu}_i = \nu_i$ .

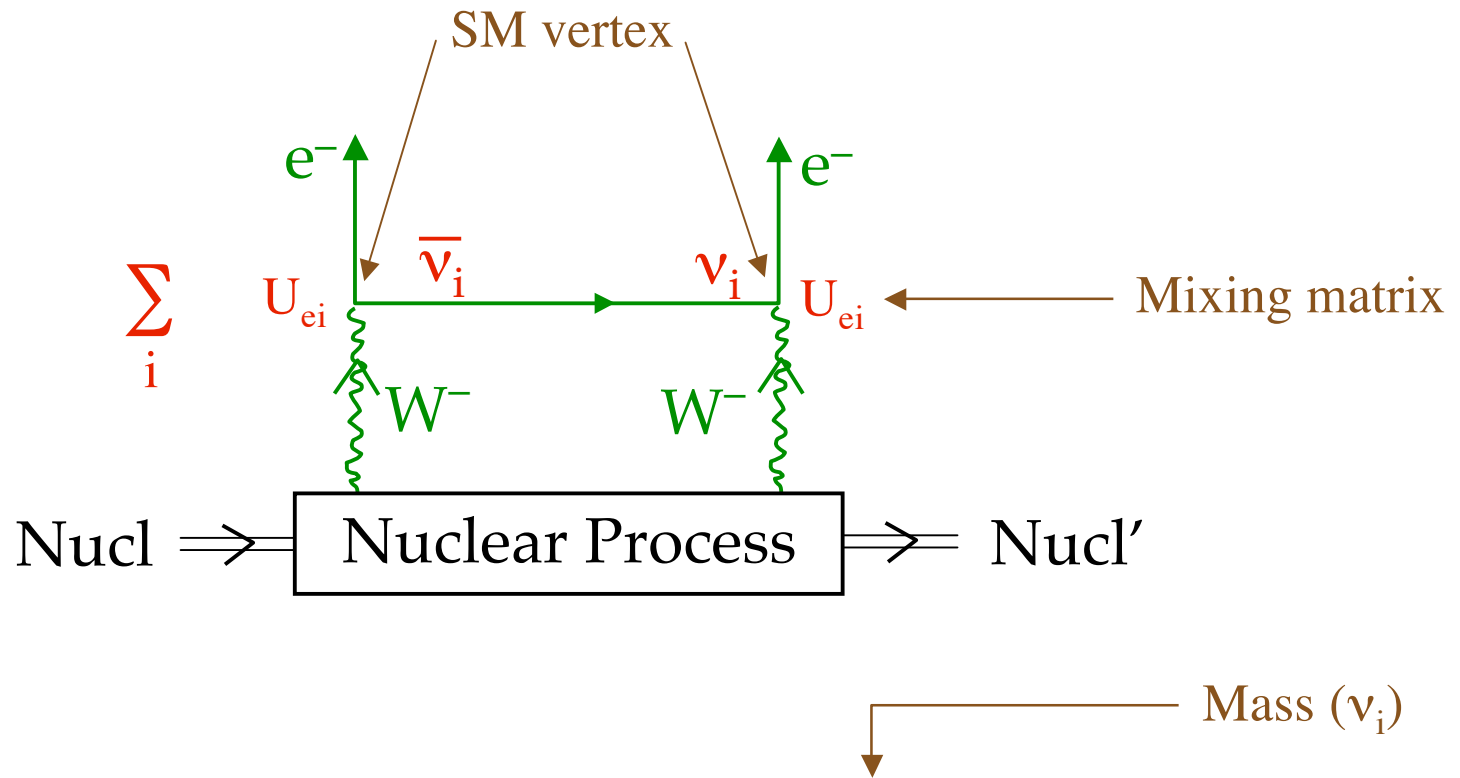
Whatever diagrams cause  $0\nu\beta\beta$ , its observation would imply the existence of a Majorana mass term:

Schechter and Valle



$(\bar{\nu})_R \rightarrow \nu_L$  : A Majorana mass term

In —



the  $\bar{\nu}_i$  is emitted [RH + O{ $m_i/E$ }LH].

Thus, Amp [ $\nu_i$  contribution]  $\propto m_i$

$$\text{Amp}[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^2 \right| \equiv m_{\beta\beta}$$

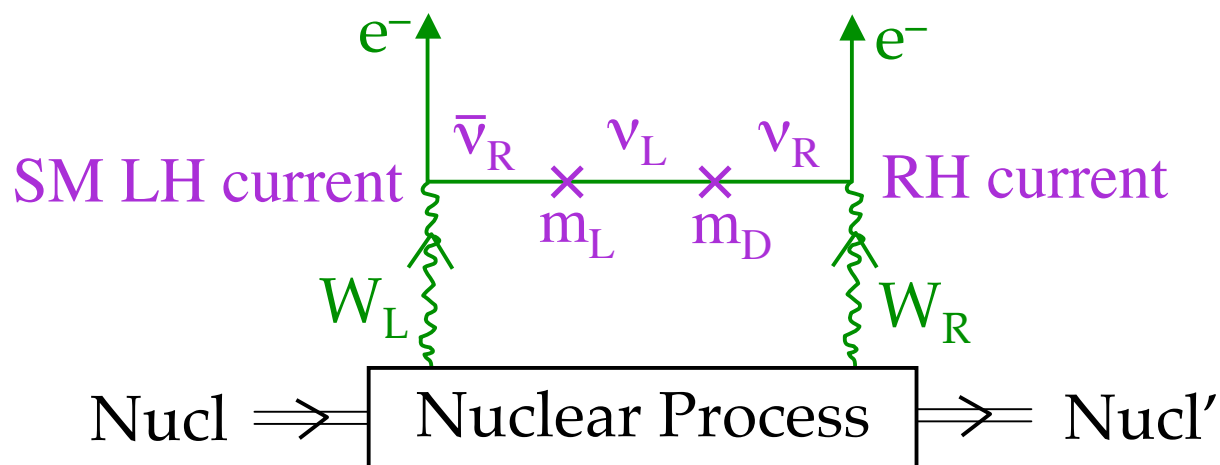
The proportionality of  $0\nu\beta\beta$  to mass is no surprise.

$0\nu\beta\beta$  violates L. But the SM interactions conserve L.

The L – violation in  $0\nu\beta\beta$  comes from underlying  
**Majorana** mass terms.



Wouldn't the dependence on neutrino mass be eliminated by a Right-Handed Current?



The SM LH current does not violate L.

An identical current, but of opposite handedness, wouldn't violate L either.

We still need the L-violating **Majorana neutrino mass** to make this process occur.

With a RH current at one vertex,

$$\text{Amp}[0\nu\beta\beta] \propto (\nu \text{ mass})^2 .$$

Contributions with a RH current at one vertex  
are not likely to be significant.

{ BK, Petcov, Rosen  
Enqvist, Maalampi, Mursula }

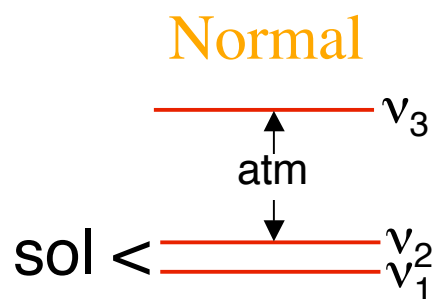
# How Large is $m_{\beta\beta}$ ?

(Minakata)

How sensitive need an experiment be?

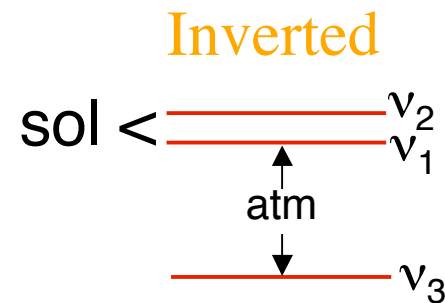
Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —



$$m_{\beta\beta} \geq 0$$

or



$$m_{\beta\beta} \gtrsim 10 \text{ meV} \quad \text{A Goal!}$$

# The Mass Spectrum: $\underline{\underline{=}}$ or $\underline{=}$ ?

Generically, grand unified models (GUTS) favor —

$\underline{=}$

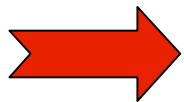
GUTS relate the **Leptons** to the **Quarks**.

$\underline{\underline{=}}$  is un-quark-like, and would probably involve a lepton symmetry with no quark analogue.

# The Four Major Goals of Future Accelerator and Reactor Neutrino Experiments

How big is  $\theta_{13}$ , the small mixing angle?

How big is  $\theta_{23}$ , the very large atmospheric mixing angle? Is it maximal?



$\equiv$  or  $\equiv$  ?

Does neutrino oscillation violate CP?

If we learn the spectrum is inverted, and a secure upper limit well below 10 meV is placed on  $m_{\beta\beta}$ , then neutrinos are Dirac particles ( $\bar{\nu} \neq \nu$ ).

Evidence for  $0\nu\beta\beta$  with  $m_{\beta\beta} = (0.05 - 0.84) \text{ eV}$ ?

Klapdor-Kleingrothaus

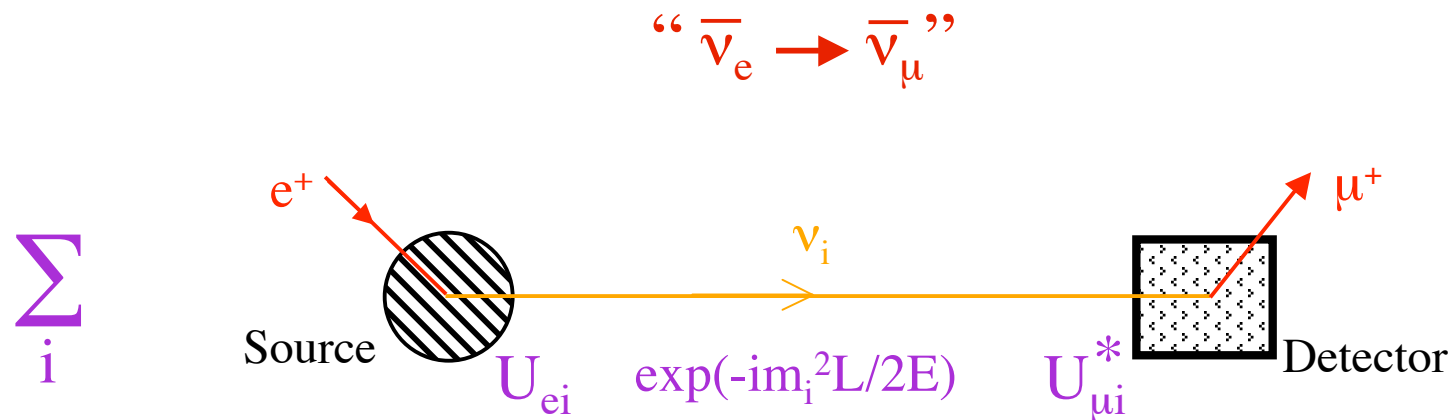
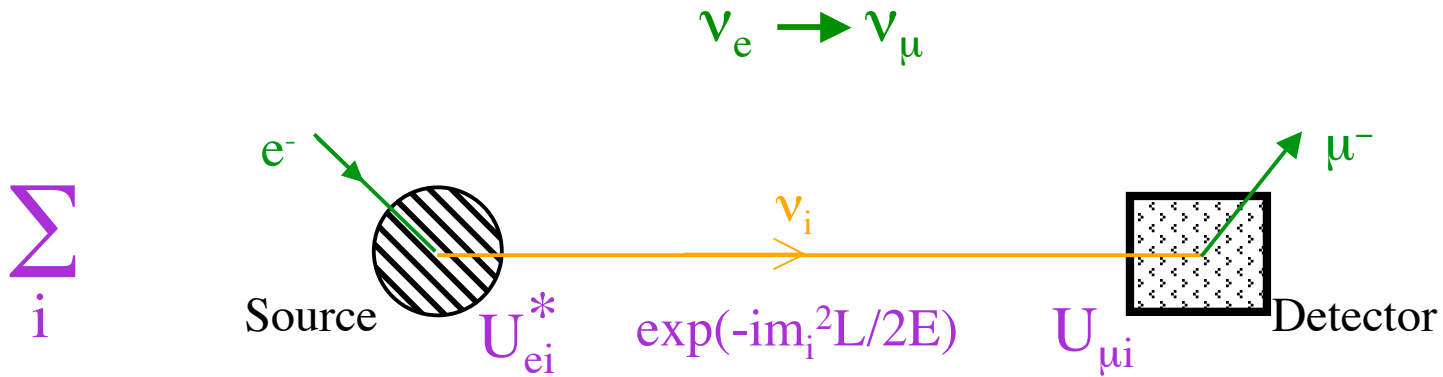
This evidence will be confirmed or refuted  
experimentally.

# If $\bar{\nu} = \nu$ , How Is Neutrino ~~CP~~ Affected?

~~CP~~ in neutrino oscillation is not affected at all.

We can still have  $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$ ,  
even if  $\bar{\nu}_i = \nu_i$ .

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The probabilities can be different!



# Majorana CP-Violating Phases

The 3x3 **quark** mixing matrix: 1 ~~CP~~ phase

When  $\bar{\nu}_i = \nu_i$  —

The 3x3 **lepton** mixing matrix: 3 ~~CP~~ phases

The 2 extra phases,  $\alpha_1$  and  $\alpha_2$ , are called **Majorana phases**.

Each Majorana phase is associated with a particular  $\nu$  mass eigenstate  $\nu_i$ :

$$U_{\rho i} = U_{\rho i}^0 e^{i \frac{\alpha_i}{2}} ; \text{ all } \rho . \quad U = \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \\ \left[ \begin{array}{ccc} U_{e1}^0 e^{i \frac{\alpha_1}{2}} & U_{e2}^0 e^{i \frac{\alpha_2}{2}} & U_{e3}^0 \\ U_{\mu 1}^0 e^{i \frac{\alpha_1}{2}} & U_{\mu 2}^0 e^{i \frac{\alpha_2}{2}} & U_{\mu 3}^0 \\ U_{\tau 1}^0 e^{i \frac{\alpha_1}{2}} & U_{\tau 2}^0 e^{i \frac{\alpha_2}{2}} & U_{\tau 3}^0 \end{array} \right] & \begin{array}{l} e \\ \mu \\ \tau \end{array} \end{array}$$

Bilenky, Hosek, and Petcov; Schechter and Valle, Doi et al. 24

An **L-conserving** process:

$$\begin{aligned}
 & \text{Amp}[e^-W^+ \rightarrow \nu \rightarrow \mu^-W^+] \\
 & \sim \sum_i \underbrace{\langle \mu^-W^+ | H | \nu_i \rangle}_{U_{\mu i}} \text{Propagator}(\nu_i) \underbrace{\langle \nu_i | H | e^-W^+ \rangle}_{U_{ei}^*} \\
 & \sim \sum_i U_{\mu i} \text{Propagator}(\nu_i) U_{ei}^*
 \end{aligned}$$

An **L-nonconserving** process:

$$\begin{aligned}
 & \text{Amp}[e^+W^- \rightarrow \nu \rightarrow \mu^-W^+] \\
 & \sim \sum_i \langle \mu^-W^+ | H | \nu_i \rangle \text{Propagator}(\nu_i) \langle \nu_i | H | e^+W^- \rangle \\
 & \text{CTP: } \langle \nu_i | H | e^+W^- \rangle = \langle \nu_i | H | e^-W^+ \rangle^* = U_{ei} \\
 & \text{So Amp}[ \cancel{L} ] \sim \sum_i U_{\mu i} \text{Propagator}(\nu_i) U_{ei}
 \end{aligned}$$

This is sensitive to Majorana phases.

Majorana phases have physical consequences,  
but only in physical processes that involve  
violation of L.

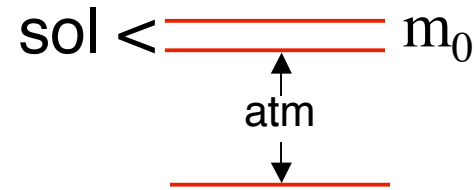
They do not affect  $\nu$  flavor oscillation, but they  
do affect  $0\nu\beta\beta$ :

$$m_{\beta\beta} = \left| \sum_i m_i U_{ei}^2 \right|$$

clearly depends on the relative phase  
of  $U_{e1}^2$  and  $U_{e2}^2$ .

# Can $\Gamma[0\nu\beta\beta]$ Reveal Majorana Phases?

If the spectrum looks like —



then—

$$m_{\beta\beta} \cong m_0 \left[ 1 - \sin^2 2\theta_{\odot} \sin^2 \left( \frac{\alpha_2 - \alpha_1}{2} \right) \right]^{1/2} .$$

With  $\alpha_2 - \alpha_1 \equiv \Delta\alpha$ ,

$$\sin^2 \left( \frac{\Delta\alpha}{2} \right) = \frac{1}{\sin^2 2\theta_{\odot}} \left[ 1 - \left( \frac{m_{\beta\beta}}{m_0} \right)^2 \right]$$

$$\cancel{\mathcal{CP}}: \Delta\alpha \neq 0, \pi. \quad \sin^2(\Delta\alpha/2) \neq 0, 1.$$

Experimentally,  $1/\sin^2 2\theta_{\odot} \cong 1.2$  .

Thus,

$$\sin^2\left(\frac{\Delta\alpha}{2}\right) \cong 1.2 \left[ 1 - \left(\frac{m_{\beta\beta}}{m_0}\right)^2 \right] .$$

Establishing that  $\sin^2(\Delta\alpha/2) \neq 0, 1$  requires —

- ◆ A knowledge of  $m_0$  [Tritium?]
- ◆ Shrinking the present (factor of three)<sup>2</sup> theoretical uncertainty in  $\Gamma[0\nu\beta\beta] / m_{\beta\beta}^2$

### Studies of Observability of $\Delta\alpha \neq 0, \pi$

Barger, Glashow, Langacker, Marfatia;  
Pascoli, Petcov, Rodejohann; Pascoli, Petcov

# How Big Are the Neutrino Masses?

The exploration of this question is **experimentally** driven.

## **What have we learned so far?**

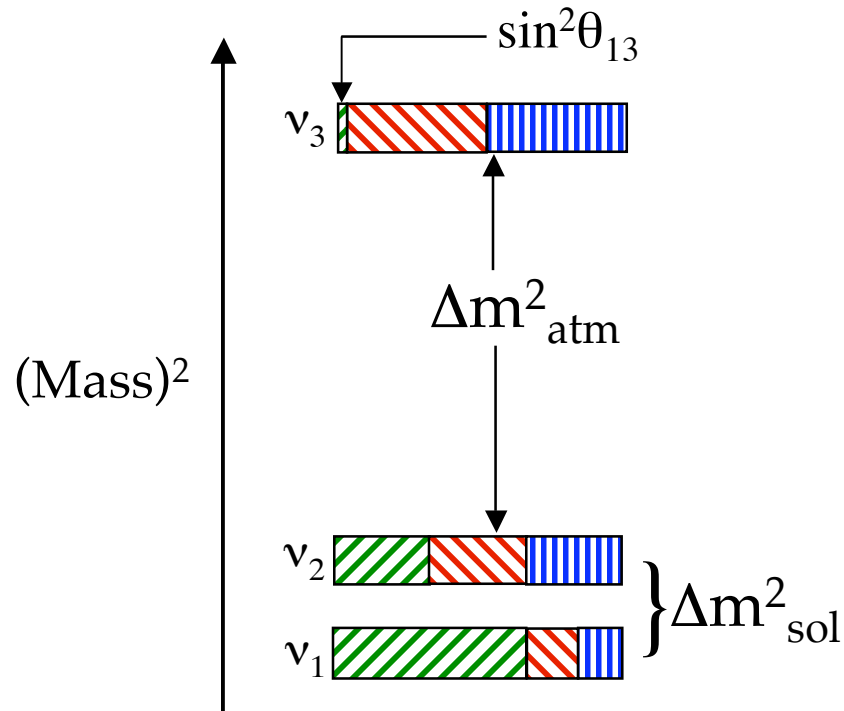
There are *at least 3* neutrino mass eigenstates.

Are there *more* than 3, as LSND suggests?

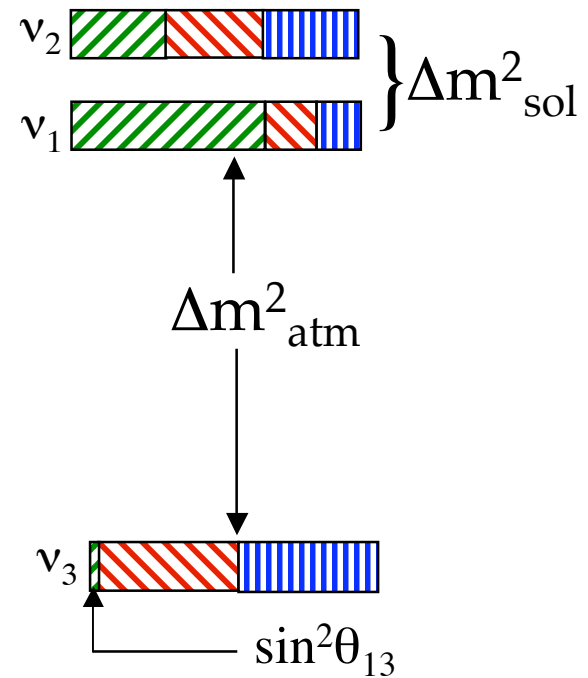
The three-neutrino spectrum is —

Normal



Inverted



or



$$\Delta m^2_{\text{sol}} \cong 8 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \cong 2.5 \times 10^{-3} \text{ eV}^2$$

(Coupling)<sup>2</sup>:     To e                       To  $\mu$                        To  $\tau$

# How Far Above Zero Is The Entire Spectrum?

Oscillation Data  $\Rightarrow \sqrt{\Delta m^2_{\text{atm}}} < \text{Mass}[\text{Heaviest } \nu_i]$

Cosmological Data + **Cosmological Assumptions**  $\Rightarrow$

$$\Sigma m_i < (0.4 - 1.0) \text{ eV} .$$

Mass( $\nu_i$ ) 

(Pastor)

If there are only **3** neutrinos,

$$0.04 \text{ eV} \lesssim \text{Mass}[\text{Heaviest } \nu_i] < (0.2 - 0.4) \text{ eV}$$

  $\sqrt{\Delta m^2_{\text{atm}}}$

Cosmology 



# What Size Neutrino Masses Does Theory Predict?

**There is no firm theoretical guidance  
on the absolute scale of neutrino mass.**

**There are only hints.**

## The See-Saw Hint

Assuming the physics of neutrino mass resides at the Grand Unification (GUT) scale,  $m_{\text{GUT}}$ ,

$$\text{Mass}[\text{Heaviest } \nu_i] \sim \frac{m_{\text{top}}^2}{m_{\text{GUT}}} \sim \frac{(173 \text{ GeV})^2}{10^{16} \text{ GeV}} \sim 0.003 \text{ eV}.$$

# The Extra Dimension Hint

In some models with extra spatial dimensions, only particles with **no non-zero SM quantum numbers** can travel in the extra dimensions.

These special travelers are the **graviton** and the **right-handed, weak isosinglet, neutrinos,  $\nu_R$** .

The mass of a Dirac neutrino,  $\bar{\nu}_L \nu_R$ , is then suppressed by the fact that  $\nu_L$  is confined to 3 dimensions, while  $\nu_R$  is spread out over the extra dimensions.

*Perhaps* the natural scale of neutrino mass in a world with an extra dimension of size  $R$  is  $1/R$ .

From short-distance probes of the law of gravity,  
the present bound is —

$$R \lesssim 0.1\text{mm}.$$

Then —

$$m_\nu \gtrsim 1/(0.1\text{mm}) = 0.002 \text{ eV}.$$

# The Leptogenesis Hint

The hypothesis that the matter-antimatter asymmetry of the universe is due to **leptogenesis** suggests that —

$$\text{Mass[Each } \nu_i] < 0.13 \text{ eV.}$$

(Buchmüller, Di Bari, Plümacher)

## Assumes:

- See-Saw relation between the heavy neutrinos  $N_i$  involved in leptogenesis and the light neutrinos  $\nu_i$
- Hierarchical (non-degenerate)  $N_i$

**Implications for direct neutrino mass searches**

# Can **Cosmology** Determine the Absolute Scale of Neutrino Mass?

Is determination via a **laboratory** experiment unnecessary?

Cosmological determination of neutrino mass is model-dependent.

**Beacom, Bell, and Dodelson:**

Suppose neutrinos couple to an **extra scalar particle  $\varphi$** .

$\nu\bar{\nu} \rightarrow \varphi\varphi$  can eliminate the Big Bang relic neutrinos from the universe before large-scale structure formation.

Then the determination of neutrino mass from large-scale structure is invalid.

**Cosmology** is wonderful, but a **Laboratory** determination of the absolute scale of neutrino mass would be very important.

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# Conclusion

The search for  $0\nu\beta\beta$  is very strongly motivated theoretically.

The observation of  $0\nu\beta\beta$  would establish that neutrinos are *very* distinctive fermions.

There is no firm theoretical guidance on the absolute scale of neutrino mass.

A laboratory determination of this scale would provide very important input to our search for the physics behind neutrino mass.