Lepton Number Violating Processes and Neutrino Mass Matrix

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Contents

1. **How to explore neutrino mass matrix?**
   masses, mixing angles, Dirac CP phase, Majorana CP phase
   Lepton number violating processes are important
2. **How to explore the origin of neutrino mass matrix?**
   see-saw mechanism: too many parameters in the model
   Combined analysis of low energy and high energy phenomena is important

Assume
Minimum SUSY with neutrino masses through see-saw mechanism

1.How to explore neutrino mass matrix

$$m_\nu = U D_\nu U^T$$

$$U \sim \begin{pmatrix} c_\odot & -s_\odot & \epsilon e^{-i\delta} \\ \frac{s_\odot}{\sqrt{2}} & \frac{c_\odot}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s_\odot}{\sqrt{2}} & \frac{c_\odot}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \qquad D_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

mixing matrix **Majorana phase matrix**

Bilenky, Hosek. Petcov(80)
Doi, Kotani, Nishiura, Okuda, Takasugi(81)
Schechter, Valle(81)

The Majorana nature of neutrino is the most important for the deep understanding of the origin of neutrinos

Neutrino oscillation

gives information about mixing angles, Dirac phase, and mass squared difference

$\tan^2\theta_\odot \simeq 0.4$ $\quad\quad$ $\Delta m^2_\odot = m^2_2 - m^2_1 \simeq 7 \times 10^{-5}\mathrm{eV}^2$

$\tan^2\theta_{\mathrm{atm}} \simeq 1$ $\quad\quad$ $\Delta m^2_{\mathrm{atm}} \simeq 2.5 \times 10^{-3}\mathrm{eV}^2$

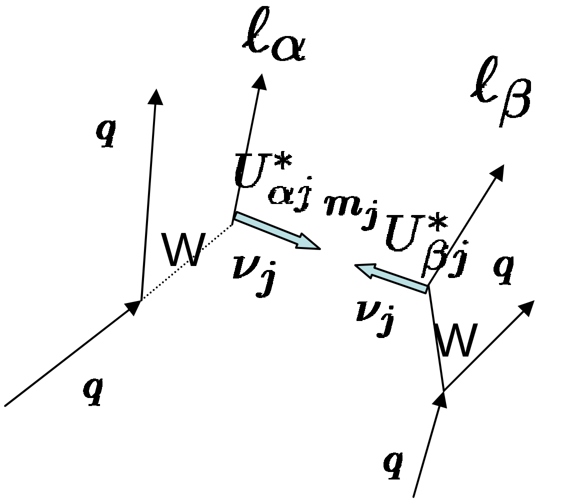
$|U_{13}| = \epsilon << 1$

δ (In the future experimets)

No information on
absolute value of neutrino mass 1
Majorana phases 2

How to explore absolute mass and Majorana phases?

Neutrinoless double beta decay Doi, Kotani, Nishiura, Okuda, Takasugi(81)



Majorana phases are physical and can explored by the $\Delta L = 2$ processes

$$< m_\nu >= |\sum U_{ej}^2 m_j| = |(m_\nu)_{ee}|$$

μ capture

$$G^2(\sum_j U_{\alpha j} U_{\beta j} m_j)\bar{\ell}_\alpha \bar{\ell}_\beta \bar{q} q \bar{q} q$$

$$= G^2(m_\nu)_{\alpha\beta} \bar{\ell}_\alpha \bar{\ell}_\beta \bar{q} q \bar{q} q$$

$\mu^- \to e^+$ $|(m_\nu)_{\mu e}|$

$\mu^- \to \mu^+$ $|(m_\nu)_{\mu\mu}|$

neutrino mass pattern

	hierarchy	**inverse hierarchy**	**democratic**
$\sqrt{\Delta m^2_{\text{atm}}} \sim 0.05\text{eV}$	m_3	m_2, m_1	
$\sqrt{\Delta m^2_{\odot}} \sim 0.008\text{eV}$	m_2		
	m_1 ?	m_3 ?	
			$m_1, m_2, m_3 >\sim 0.05\text{eV}$
		$m_2 - m_1 \sim 0.08\text{eV}$	$m_2 - m_1 \sim 0.08\text{eV}$

If $m_2 \sim m_1$,

$m_2 > 0.008$

Information obtained from these

In the approximation of $U_{13} = 0$

	H	IH	QD
1. $\rightarrow e^- e^-$ $\quad e^- \rightarrow e^+$ $\lvert (m_\nu)_{ee} \rvert$ $\sim \lvert c_\odot^2 m_1 + s_\odot^2 e^{2i\alpha} m_2 \rvert$	> 0.004	$0.05(1 - 4 s_\odot^2 c_\odot^2 \sin^2\alpha)$ $> 0.05 \lvert \cos 2\theta_\odot \rvert$ ~ 0.03	$> 0.05(1 - 4 s_\odot^2 c_\odot^2 \sin^2\alpha)$ > 0.03
2. $\mu^- \rightarrow e^+$ $\quad e^- \rightarrow \mu^+$ $\quad \rightarrow e^- \mu^-$ $\lvert (m_\nu)_{e\mu} \rvert$ $\sim \frac{1}{\sqrt{2}} \lvert s_\odot c_\odot (m_1 - e^{2i\alpha} m_2) \rvert$	0.003	$0.03 \lvert \sin\alpha \rvert$	$> 0.03 \lvert \sin\alpha \rvert$
3. $\mu^- \rightarrow \mu^+$ $\quad \rightarrow \mu^- \mu^-$ $\lvert (m_\mu)_{\mu\mu} \rvert$ $\sim \frac{1}{2} \lvert s_\odot^2 m_1 + c_\odot^2 e^{2i\alpha} m_2 + e^{2i\beta} m_3 \rvert$	0.03	$0.03(1 - 4 s_\odot^2 c_\odot^2 \sin^2\alpha)$	$> 0.05 \sqrt{1 - \frac{\sin^2\alpha}{4} - \frac{\sin^2\beta}{2} - \frac{\sin^2(\beta-\alpha)}{2}}$

$$\sqrt{\Delta m^2_{\text{atm}}} \sim 0.05\text{eV}$$

Candidates and theoretical estimate

$\rightarrow e^- e^-, \quad e^- \rightarrow e^+$

Double beta decay

hierarchy	>0.004eV
Inverse hierarchy	> 0.03eV
quasi degenerate	>0.03eV

$\mu^- \rightarrow e^+$

$\mu^- + (A,Z) \rightarrow e^+ + (A, Z-2)$ too slow $\frac{\Gamma(\mu^- \to e^+)}{\Gamma(\mu^- \to \nu_\mu)} \sim 1 \times 10^{-26} \left(\frac{|(m_{\mu e}|}{0.1\mathrm{eV}}\right)^2$ $Z \sim 50$

Doi, Kotani, Takasugi(85)

$\mu^- \rightarrow \mu^+$

$\mu^- + (A,Z) \rightarrow \mu^+ + (A, Z-2)$ too slow $\frac{\Gamma(\mu^- {}^{44}\mathrm{Ti} \to \mu^+ {}^{44}\mathrm{Ca})}{\Gamma(\mu^- \to \nu_\mu)} \sim 5 \times 10^{-35} \left(\frac{|(m_{\mu\mu}|}{0.1\mathrm{eV}}\right)^2 \left(\frac{|M|}{1.0}\right)^2$

Missimer, Mohapatra, Mukhopadhyay(94)
Takasugi(03)

The μ captures rates are the bottom line. If they were found, there must exist new physics other than neutrino mass, such as SUSY (R parity violation term etc)

some others

$$e^- + (A,Z) \to \mu^+ + (A, Z-2)$$

$$\sigma \sim 5 \times 10^{-71} \left(\frac{|(m_\nu)_{e\mu}|}{0.1\mathrm{eV}}\right) \mathrm{cm}^2$$

$$e^- + e^-(atomic) \to \pi^- + \pi^-$$

and others

Lim, Takasugi, Yoshimura(06)

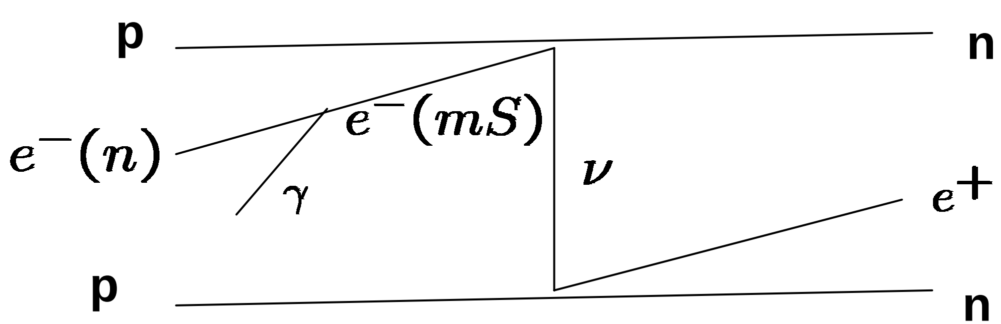
In these reactions, only the double beta decay can reach the sensitivity to explore the neutrino mass matrix

New method is needed

Possible new method

Enhance Lepton number conversion (M. Yoshimura et al)

$$\gamma e^- \to e^+$$



Fine tuned photon energy by intense laser beam
enhanced by resonance effect and large occupancy of photons

can explore   O(1meV)

$$\Gamma_{0\nu}^{(mS)} \frac{|<mS|H_\gamma|n,\gamma>|^2}{(E_\gamma - \Delta\epsilon_{nm})^2 + \Gamma_n^2/4}$$

same Laser medium and target ^{78}Kr

2. Explore the origin of neutrino mass

see-saw mechanism

$$m_\nu(M_X) = m_D^T D_R^{-1} m_D$$

$$m_D = V_R D_D V_L^\dagger$$

3 masses (→ D_R^{-1})

3 masses (→ D_D)

-1 combined

3 angles + 1 Dirac phase
2Majorana phases (→ $V_L^\dagger$)

3 angles + 1 Dirac phase
2Majorana phases (→ V_R)

-2

3+3-1+(3+1)+(3+1)+2+2-2=15

too many parameters

Other experiments
aside from neutrino oscillation, lepton number non conserving processes are needed

Since neutrino mass matrix will arise at the higher energy scale,
some unified story to connect various phenomena is needed

Assume Minimal SUSY with neutrino mass through see-saw mechanism

universal soft breaking terms (no lepton flavor violation aside from neutrino-Yukawa couplings)

Low energy phenomena and high energy phenomena can be connected by the renormalization group

Need to assume some reasonable model and examine whether the model explain all available experimental data

What we can learn to assume the model

hierarchical neutrino masses?

⇒ **Renormalization group gives essentially no effect to the neutrino oscillation parameters,**

⇒ **the model should predict the exact values obtained in the low energy experiments**

should answer: Why solar mixing angle Is not either maximal or small. Why the atmospheric mixing ismaximal.

Inverse hierarchy or quasi-degenerate?

⇒ **mixing angles at GUT can be some extreme values maximal or small (may be realized)**

The renormalization group effect could rotate the solar angle to the experimental value

How to connect various data?

Consider one interesting example

suppose that
BiMaximal mixing is realized at GUT in the MSSM
Neutrino mass matrix Is given by the see-saw mechanism

$$m_\nu(M_X) = O_B D_\nu O_B^T \qquad D_\nu = \text{diag}(m_1, m_2, m_3)$$
$$= \text{diag}(|m_1|, |m_2|e^{i\alpha_0}, |m_3|e^{i\beta_0})$$

$$m_\nu(M_X) = m_D^T D_R^{-1} m_D$$

Since $U_{13} = 0$, no Dirac phase
Only 3 unknown, absolute mass and Majonara phases

IF the bi-maximal mixing is realized at GUT

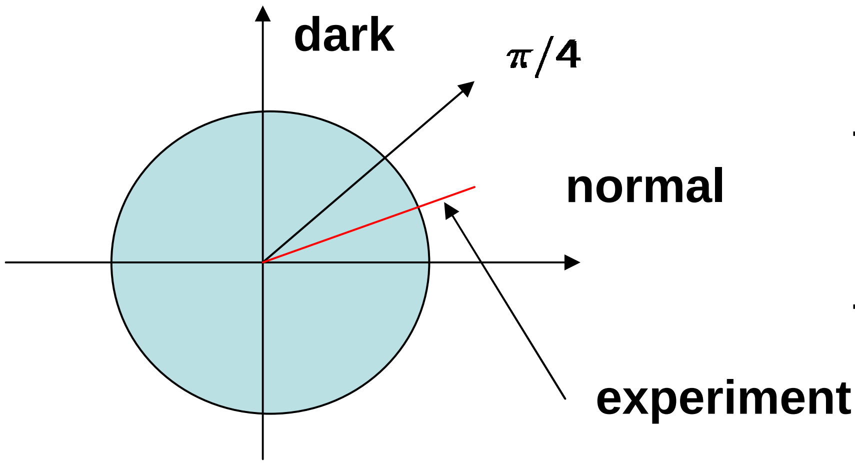
	Renormalization effect	Result
Hierarchical	no effect	no acceptable
Inverse Hierarchical	effect	acceptable*
Quasi degenerate	effect	acceptable*

Possible* if some element of Neutrino-Yukawa couplings is large

$$(Y_\nu^\dagger Y_\nu) \quad \sim m_\tau^2/v^2$$

rotate towards the normal side

rotate towards the dark side



to explain the data, neutrino-Ykawa must be larger than tau-Yukawa

Neutrino-Yukawa couplings should be large

Kanemura, Matsuda, Sato, Takasugi, Tsumura(04)

Large neutrino-Yukawa

$(Y_\nu^\dagger Y_\nu)_{ij}$ leads to the LFV processes $\ell_i \to \ell_j + \gamma$

thorough s-lepton mixing

so that the off diagonal term are small

assujme $(Y_\nu^\dagger Y_\nu)$ is diagonal

assume

no LFV at GUT

⇨ $m_D = V_R D_D$ $m_\nu(M_X) = m_D^T D_R^{-1} m_D$

From the equation

$$m_\nu(M_X) = O_B D_\nu^{-1} O_B^T \quad = m_D^T D_R^{-1} m_D$$

$$= D_D (V_R^* D_R^{-1} V_R^\dagger) D_R$$

$$M_R^{-1} \equiv V_R^* D_R^{-1} V_R^\dagger = D_D^{-1} O_B D_\nu O_B^T D_D^{-1}$$

we obtain V_R and M_i

From D_ν , D_D

5 3 : 8 parameters

(2 Majorana phases)

$$m_D^\dagger m_D \sim \frac{v_d^2}{2}\begin{pmatrix} y_1^2 & 0 & 0 \\ 0 & y_2^2 & 0 \\ 0 & 0 & y_3^2 \end{pmatrix}$$

$$\tan^2\theta_\odot(M_X) = 1$$

solve the renormalization group equation in a good approximation

$$\Rightarrow \quad \tan^2\theta_\odot(m_Z) = \frac{1-(2\epsilon_e-\epsilon_\tau)\cos^2(\alpha_0/2)m_1^2/\Delta m_\odot^2}{1+(2\epsilon_e-\epsilon_\tau)\cos^2(\alpha_0/2)m_1^2/\Delta m_\odot^2}$$

$$\epsilon_e = (y_1^2 - y_2^2)\ln(M_x/M_R)/8\pi^2$$

BiMaximal case
Other case can be given similarly

$$\epsilon_\tau = (y_3^2 - y_2^2)\ln(M_X/M_R)/8\pi^2 + y_\tau^2\ln(M_X/m_Z)/8\pi^2$$

We can get the normal side solar angle when

$$2\epsilon_e > \epsilon_\tau \quad \Rightarrow \quad y_1^2 >> y_2^2, y_3^2$$

The size of M_3

The condition to reproduce the solar mixing

$$\cos 2\theta_\odot = (2\epsilon_e - \epsilon_\tau)\cos^2(\alpha_0/2)(m_1^2/\Delta m_\odot^2)$$

We substitute ϵ_τ and

$$\epsilon_e = \frac{m_{D3}^2}{4\pi^2(v\sin\beta)^2}\ln\frac{M_X}{M_3}$$

We find M_3 With

$$\begin{aligned} m_{D3}^2 &= 2|m_1 m_2|M_3/|m_1+m_2| \\ &\simeq \frac{|m_1|M_3}{|\cos\alpha_0/2|} \end{aligned}$$

LFV and Leptogenesis

Low energy **M_R** **GUT**

Neutrino oscillation
LFV $\mu \to e + \gamma$
Double beta decay
etc

$\tan^2 \theta_{\odot} \simeq 0.4$

$\sin^2 2\theta_{@} \simeq 1$

leptogenesis

$Y_\nu Y_\nu^\dagger$

BiMaximal
MSSM
see-saw

$\tan^2 \theta_{\odot} = 1$

$\sin^2 2\theta_{@} = 1$

Renormalization equation

off diagonal $(Y_\nu^\dagger Y_\nu)$ no LFV

Neutrino-Yukawa

$m_D = Y_\nu v_u / \sqrt{2}$

$(Y_e^\dagger Y_e)$

Tau-Yukawa

Induce LFV interaction

Special feature: BiMaximal at GUT

1.Inverse hierarchy or quasi-degenerate

2. The effective mass of neutrinoless double beta decay is ~0.03eV

3. Neutrino oscillation data are realized

U_{13} is zero at GUT, but it is induced by the renormalization group.

The value is predicted, but it is small..

4. LFV $(Y_\nu^\dagger Y_\nu) \to (Y_\nu^\dagger L Y_\nu)$

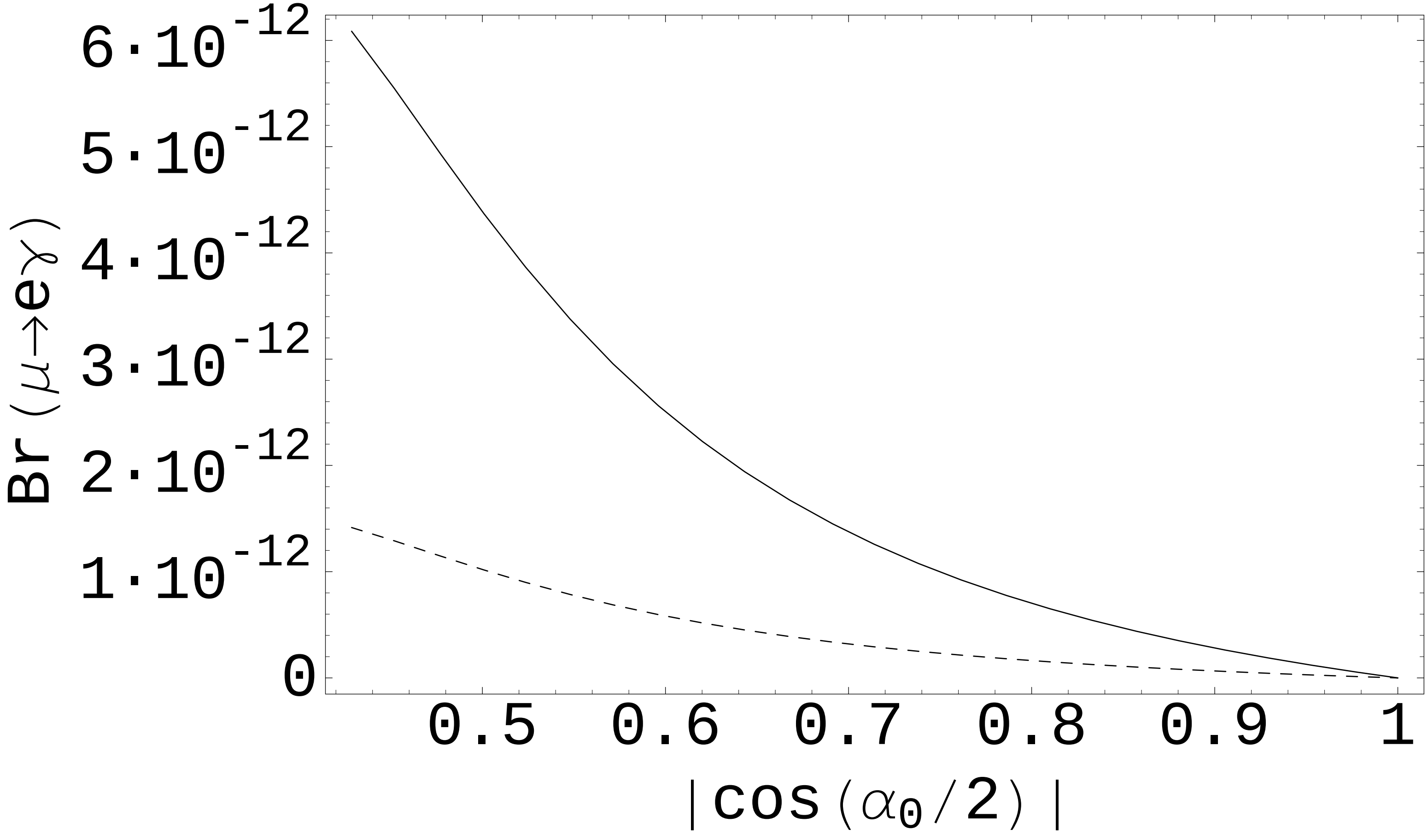
$$L = \mathrm{diag}(\ln(M_X//M_1), \ln(M_X/M_2), \ln(M_X/M_3))$$

5. Leptogenesis $Y_\nu Y_\nu^\dagger \sim V_R D_D^2 V_R^\dagger$

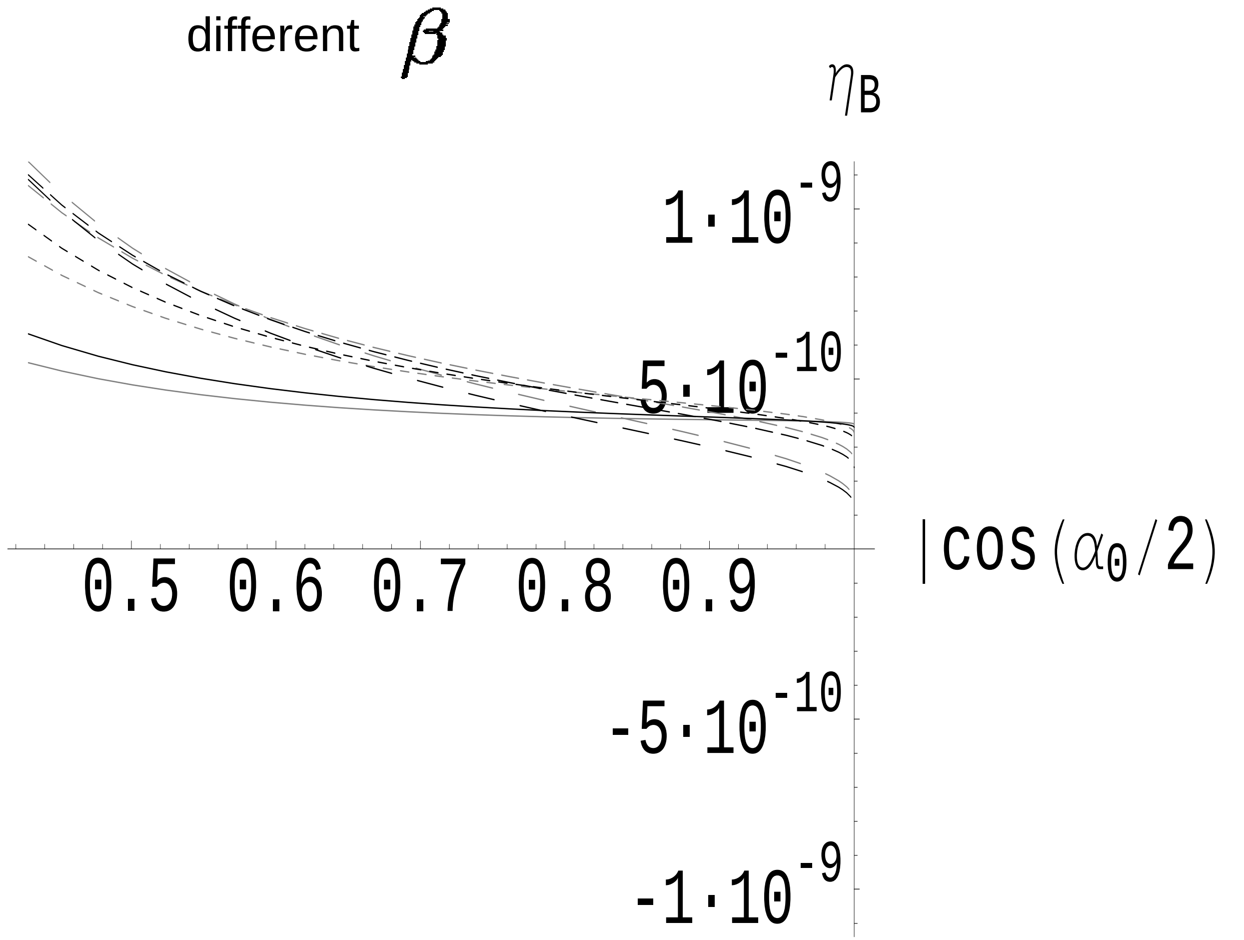
need to assume the mass spectrum of m_{Di} and

Lepton flavor violatgion

α_0 is the Majjorana phase α



Fovorit value is $\alpha_0 \sim 0.5$ $Br(\mu \to e + \gamma) >> Br(\tau \to \mu + \gamma)$


different β
η_B
$1\cdot10^{-9}$
$5\cdot10^{-10}$
$-5\cdot10^{-10}$
$-1\cdot10^{-9}$
0.5
0.6
0.7
0.8
0.9
$|\cos(\alpha_0/2)|$

Concluding remarks

1. Double beta decay is promising for getting information about $|(M_\nu)_{ee}|$,absolute mass scale and Majorana phases

2. It seems quite difficult to obtain the information on their elements $|(M_\nu)_{e\mu}|$ $|(M_\nu)_{\mu\mu}|$

3. New method Is welcomed. Laser assisted lepton flavor violation processes will be effective

4. In order to examine the origin of neutrino mass matrix, that is, the see-saw mechanism, the combined analysis is needed. Since the parameter involved in it is too many to be fixed, some reasonable mode analysis is needed.

5. I showed one example which gives various predictions.